# Exclusive $B \to \rho \ell^+ \ell^-$ decay and Polarized lepton pair forward–backward asymmetries

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**Abstract**

The polarized lepton pair forward–backward asymmetries in $B \to \rho \ell^+ \ell^-$ decay using a general, model independent form of the effective Hamiltonian is studied. The general expression for nine double–polarization forward–backward asymmetries are calculated. The study of the forward–backward asymmetries of the doubly–polarized lepton pair proves to be very useful tool in looking for new physics beyond the standard model.

PACS numbers: 13.20.He, 12.60.–i, 13.88.+e

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# 1 Introduction

Rare $B$ meson decays, induced by flavor changing neutral current (FCNC) $b \to s(d)\ell^+\ell^-$ transitions provide a promising ground for testing the gauge structure of the Standard Model (SM). These decays which are forbidden in the SM at tree level, occur at loop level and allow us to check the prediction of the theory at quantum level. Moreover, these decays are also quite sensitive to the existence of new physics beyond the SM, since loops with new particles can give contribution to these decays. The new physics effects in rare decays can appear in two ways; one via modification of the existing Wilson coefficients in the SM, or through the introduction of some new operators with new coefficients. Theoretical study of the $B \to X_s\ell^+\ell^-$ decays are relatively more clean compared to their exclusive counterparts, since they are not spoiled by nonperturbative long distance effects, while the corresponding exclusive channels are easier to measure experimentally. Some of the most important exclusive FCNC decays are $B \to K^*\gamma$ and $B \to (\pi,\rho,K,K^*)\ell^+\ell^-$ decays. The latter provides potentially a very rich set of experimental observables, such as, lepton pair forward–backward (FB) asymmetry, lepton polarizations, etc. Various kinematical distributions of such processes as $B \to K(K^*)\ell^+\ell^-$ [1–3], $B \to \pi(\rho)\ell^+\ell^-$ [4], $B_{s,d} \to \ell^+\ell^-$ [5] and $B_{s,d} \to \gamma\ell^+\ell^-$ [6] have already been studied. Study of the forward–backward asymmetry, single polarization asymmetry, etc., which are experimentally measurable quantities, is an efficient approach that have been already been investigated in detail for the $B \to K(K^*)\ell^+\ell^-$ decay in [1, 7–12] in fitting the parameters of the SM and put constraints on new physics. It has been pointed out in [13] that some of the single lepton polarization asymmetries might be quite small to be observed and might nor provide sufficient number of observables in checking the structure of the effective Hamiltonian. In [10] the maximum number of independent observables are constructed by taking both lepton polarizations into account simultaneously. It is clear that, measurement of many more observables which would be useful in further improvement of the parameters of the SM probing new physics beyond the SM. It should be noted here that both lepton polarizations in the $B \to K^*\tau^+\tau^-$ and $B \to K\ell^+\ell^-$ decays are studied in [14] and [15], respectively. As has already been noted, one efficient way of establishing new physics effects is studying forward–backward asymmetry in semileptonic $B \to K^*\ell^+\ell^-$ decay, since, $\mathcal{A}_{FB}$ vanishes at specific values of the dilepton invariant mass, and more essential than that, this zero position of $\mathcal{A}_{FB}$ is known to be practically free of hadronic uncertainties [12]. The decays of $B$ mesons induced by the $b \to d\ell^+\ell^-$ transition are promising in looking for CP violation since the CKM factors $V_{tb}V_{td}^*$, $V_{ub}V_{ud}^*$ and $V_{cb}V_{cd}^*$ in the SM are all of the same order. For this reason CP violation is much more considerable in the decays induced by $b \to d$ transition. So, study of the exclusive decays $B_d \to (\pi,\rho,\eta)\ell^+\ell^-$ are quite promising for the confirmation of the CP violation and these decays have extensively been investigated in the SM [13] and beyond [4].

The aim of the present work is studying the polarized forward–backward asymmetry in the exclusive $B \to \rho\ell^+\ell^-$ decay using a general form of the effective Hamiltonian, including all possible forms of interactions. Here we would like to remind the reader that the influence of new Wilson coefficients on various kinematical variables, such as branching ratios, lepton pair forward–backward asymmetries and single lepton polarization asymmetries for the inclusive $B \to X_{s(d)}\ell^+\ell^-$ decays (see first references in [11, 14, 17]) and exclusive $B \to K\ell^+\ell^-$, $\rho\ell^+\ell^-$, $\gamma\ell^+\ell^-$, $\pi\ell^+\ell^-$, $\rho\ell^+\ell^-$ [1, 2, 6, 9, 18, 19] and pure leptonic $B \to \ell^+\ell^-$ decays

[5, 20] have been studied comprehensively.

The paper is organized as follows. In section 2, using a general form of the effective Hamiltonian, we obtain the matrix element in terms of the form factors of the $B \to \rho$ transition. In section 3 we derive the analytical results for the polarized forward–backward asymmetry. Last section is devoted to the numerical analysis, discussion and conclusions.

# 2 Calculation of double lepton polarizations in $B \to \rho\ell^+\ell^-$ decay

In this section we calculate the double lepton polarizations using a general form of the effective Hamiltonian. The $B \to \rho\ell^+\ell^-$ process is governed by $b \to d\ell^+\ell^-$ transition at quark level. The matrix element for the $b \to d\ell^+\ell^-$ can be written in terms of the twelve model independent four–Fermi interactions in the following form:

$$\begin{aligned}
\mathcal{H}_{eff} &= \frac{G_F\alpha}{\sqrt{2}\pi} V_{td}V_{tb}^* \Bigg\{ C_{SL}\, \bar{d} i\sigma_{\mu\nu}\frac{q^\nu}{q^2}\, L\, b\, \bar{\ell}\gamma^\mu\ell + C_{BR}\, \bar{d} i\sigma_{\mu\nu}\frac{q^\nu}{q^2}\, R\, b\, \bar{\ell}\gamma^\mu\ell \\
&+ C_{LL}^{tot}\, \bar{d}_L\gamma_\mu b_L\, \bar{\ell}_L\gamma^\mu\ell_L + C_{LR}^{tot}\, \bar{d}_L\gamma_\mu b_L\, \bar{\ell}_R\gamma^\mu\ell_R + C_{RL}\, \bar{d}_R\gamma_\mu b_R\, \bar{\ell}_L\gamma^\mu\ell_L \\
&+ C_{RR}\, \bar{d}_R\gamma_\mu b_R\, \bar{\ell}_R\gamma^\mu\ell_R + C_{LRLR}\, \bar{d}_L b_R\, \bar{\ell}_L\ell_R + C_{RLLR}\, \bar{d}_R b_L\, \bar{\ell}_L\ell_R \\
&+ C_{LRRL}\, \bar{d}_L b_R\, \bar{\ell}_R\ell_L + C_{RLRL}\, \bar{d}_R b_L\, \bar{\ell}_R\ell_L + C_T\, \bar{d}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu}\ell \\
&+ iC_{TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{d}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta}\ell \Bigg\} , \qquad (1)
\end{aligned}$$

where $L$ and $R$ in (1) are defined as

$$L = \frac{1-\gamma_5}{2} , \qquad R = \frac{1+\gamma_5}{2} ,$$

and $C_X$ are the coefficients of the four–Fermi interactions. The first two coefficients in Eq. (1), $C_{SL}$ and $C_{BR}$, are the nonlocal Fermi interactions, which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The following four terms with coefficients $C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$ are the vector type interactions. Two of these vector interactions containing $C_{LL}^{tot}$ and $C_{LR}^{tot}$ do already exist in the SM in the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$. Therefore, $C_{LL}^{tot}$ and $C_{LR}^{tot}$ can be written as

$$\begin{aligned}
C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} , \\
C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} ,
\end{aligned}$$

where $C_{LL}$ and $C_{LR}$ describe the contributions of the new physics. The terms with coefficients $C_{LRLR}$, $C_{RLLR}$, $C_{LRRL}$ and $C_{RLRL}$ describe the scalar type interactions. The remaining last two terms lead by the coefficients $C_T$ and $C_{TE}$, obviously, describe the tensor type interactions.

It should be noted here that, in further analysis we will assume that all new Wilson coefficients are real, as is the case in the SM, while only $C_9^{eff}$ contains imaginary part and it is parametrized in the following form

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2 , \qquad (2)$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} ~,$$

and

$$\begin{aligned}
\xi_1 &= 4.128 + 0.138\omega(\hat{s}) + g(\hat{m}_c,\hat{s})C_0(\hat{m}_b) - \frac{1}{2}g(\hat{m}_d,\hat{s})(C_3+C_4) \\
&- \frac{1}{2}g(\hat{m}_b,\hat{s})(4C_3+4C_4+3C_5+C_6) + \frac{2}{9}(3C_3+C_4+3C_5+C_6) ~, \\
\xi_2 &= [g(\hat{m}_c,\hat{s}) - g(\hat{m}_u,\hat{s})](3C_1+C_2) ~,
\end{aligned} \tag{3}$$

where $\hat{m}_q = m_q/m_b$, $\hat{s} = q^2$, $C_0(\mu) = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$, and

$$\begin{aligned}
\omega(\hat{s}) &= -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2(\hat{s}) - \frac{2}{3}\ln(\hat{s})\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s}) \\
&- \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln(\hat{s}) + \frac{5+9\hat{s}-6\hat{s}^2}{3(1-\hat{s})(1+2\hat{s})} ~,
\end{aligned} \tag{4}$$

represents the $O(\alpha_s)$ correction coming from one gluon exchange in the matrix element of the operator $\mathcal{O}_9$ [21], while the function $g(\hat{m}_q,\hat{s})$ represents one–loop corrections to the four–quark operators $O_1$–$O_6$ [22], whose form is

$$\begin{aligned}
g(\hat{m}_q,\hat{s}) &= -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2+y_q) \\
&- \sqrt{|1-y_q|}\left\{\theta(1-y_q)\left[\ln\left(\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}}\right) - i\pi\right] + \theta(y_q-1)\arctan\left(\frac{1}{\sqrt{y_q-1}}\right)\right\} ~,
\end{aligned} \tag{5}$$

where $y_q = 4\hat{m}_q^2/\hat{s}$.

In addition to the short distance contributions, $B \to X_d \ell^+\ell^-$ decay also receives long distance contributions, which have their origin in the real $\bar{u}u$, $\bar{d}d$ and $\bar{c}c$ intermediate states, i.e., $\rho$, $\omega$ and $J/\psi$ family. There are four different approaches in taking long distance contributions into consideration: a) HQET based approach [23], b) AMM approach [24], c) LSW approach [25], and d) KS approach [26]. In the present work we choose the AMM approach, in which these resonance contributions are parametrized using the Breit–Wigner form for the resonant states. The effective coefficient $C_9^{eff}$ including the $\rho$, $\omega$ and $J/\psi$ resonances are defined as

$$C_9^{eff} = C_9(\mu) + Y_{res}(\hat{s}) ~, \tag{6}$$

where

$$\begin{aligned}
Y_{res} &= -\frac{3\pi}{\alpha^2}\Bigg\{ \left(C^{(0)}(\mu) + \lambda_u\left[3C_1(\mu) + C_2(\mu)\right]\right) \sum_{V_i=\psi} K_i \frac{\Gamma(V_i \to \ell^+\ell^-)M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i}\Gamma_{V_i}} \\
&- \lambda_u g(\hat{m}_u,\hat{s})\left[3C_1(\mu) + C_2(\mu)\right] \sum_{V_i=\rho,\omega} \frac{\Gamma(V_i \to \ell^+\ell^-)M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i}\Gamma_{V_i}} \Bigg\} ~.
\end{aligned} \tag{7}$$

The phenomenological factor $K_i$ has the universal value for the inclusive $B \to X_{s(d)} \ell^+ \ell^-$ decay $K_i \simeq 2.3$ [27], which we use in our calculations.

The exclusive $B \to \rho \ell^+ \ell^-$ decay is obtained from the matrix elements of the quark operators in Eq. (1) over meson states, which can be parametrized in terms of the form factors. Obviously, the following matrix elements

$$
\begin{aligned}
&\left\langle \rho \left| \bar{d} \gamma_\mu (1 \pm \gamma_5) b \right| B \right\rangle ~, \\
&\left\langle \rho \left| \bar{d} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b \right| B \right\rangle ~, \\
&\left\langle \rho \left| \bar{d} (1 \pm \gamma_5) b \right| B \right\rangle ~, \\
&\left\langle \rho \left| \bar{d} \sigma_{\mu\nu} b \right| B \right\rangle ~,
\end{aligned}
$$

are needed in obtaining the decay amplitude of the $B \to \rho \ell^+ \ell^-$ decay. These matrix elements are defined as follows:

$$
\begin{aligned}
&\left\langle \rho(p_\rho, \varepsilon) \left| \bar{d} \gamma_\mu (1 \pm \gamma_5) b \right| B(p_B) \right\rangle = \\
&\quad - \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma \frac{2 V(q^2)}{m_B + m_\rho} \pm i \varepsilon_\mu^* (m_B + m_\rho) A_1(q^2) \\
&\quad \mp i (p_B + p_\rho)_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_\rho} \mp i q_\mu \frac{2 m_\rho}{q^2} (\varepsilon^* q) \left[ A_3(q^2) - A_0(q^2) \right] ~,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&\left\langle \rho(p_\rho, \varepsilon) \left| \bar{d} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b \right| B(p_B) \right\rangle = \\
&\quad 4 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma T_1(q^2) \pm 2i \left[ \varepsilon_\mu^* (m_B^2 - m_\rho^2) - (p_B + p_\rho)_\mu (\varepsilon^* q) \right] T_2(q^2) \\
&\quad \pm 2i (\varepsilon^* q) \left[ q_\mu - (p_B + p_\rho)_\mu \frac{q^2}{m_B^2 - m_\rho^2} \right] T_3(q^2) ~,
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
&\left\langle \rho(p_\rho, \varepsilon) \left| \bar{d} \sigma_{\mu\nu} b \right| B(p_B) \right\rangle = \\
&\quad i \epsilon_{\mu\nu\lambda\sigma} \Bigg\{ - 2 T_1(q^2) \varepsilon^{*\lambda} (p_B + p_\rho)^\sigma + \frac{2}{q^2} (m_B^2 - m_\rho^2) \Big[ T_1(q^2) - T_2(q^2) \Big] \varepsilon^{*\lambda} q^\sigma \\
&\quad - \frac{4}{q^2} \left[ T_1(q^2) - T_2(q^2) - \frac{q^2}{m_B^2 - m_\rho^2} T_3(q^2) \right] (\varepsilon^* q) p_\rho^\lambda q^\sigma \Bigg\} ~.
\end{aligned} \tag{10}
$$

where $q = p_B - p_\rho$ is the momentum transfer and $\varepsilon$ is the polarization vector of $\rho$ meson. In order to ensure finiteness of (8) and (10) at $q^2 = 0$, we assume that $A_3(q^2 = 0) = A_0(q^2 = 0)$ and $T_1(q^2 = 0) = T_2(q^2 = 0)$. The matrix element $\left\langle \rho \left| \bar{d}(1 \pm \gamma_5) b \right| B \right\rangle$ can be calculated by contracting both sides of Eq. (8) with $q^\mu$ and using equation of motion. Neglecting the mass of the $d$ quark we get

$$
\left\langle \rho(p_\rho, \varepsilon) \left| \bar{d} (1 \pm \gamma_5) b \right| B(p_B) \right\rangle = \frac{1}{m_b} \Big[ \mp 2 i m_\rho (\varepsilon^* q) A_0(q^2) \Big] ~. \tag{11}
$$

In deriving Eq. (11) we have used the relationship

$$
2 m_\rho A_3(q^2) = (m_B + m_\rho) A_1(q^2) - (m_B - m_\rho) A_2(q^2) ~,
$$

which follows from the equations of motion.

Using the definition of the form factors, as given above, the amplitude of the $B \to \rho \ell^+ \ell^-$ decay can be written as

$$
\begin{aligned}
\mathcal{M}(B \to \rho \ell^+ \ell^-) &= \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{td}^* \\
&\times \Bigg\{ \bar{\ell}\gamma^\mu(1-\gamma_5)\ell \Big[ -2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma - iB_1 \varepsilon_\mu^* + iB_2 (\varepsilon^* q)(p_B + p_\rho)_\mu + iB_3 (\varepsilon^* q) q_\mu \Big] \\
&+ \bar{\ell}\gamma^\mu(1+\gamma_5)\ell \Big[ -2C_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma - iD_1 \varepsilon_\mu^* + iD_2 (\varepsilon^* q)(p_B + p_\rho)_\mu + iD_3 (\varepsilon^* q) q_\mu \Big] \\
&+ \bar{\ell}(1-\gamma_5)\ell \Big[ iB_4 (\varepsilon^* q) \Big] + \bar{\ell}(1+\gamma_5)\ell \Big[ iB_5 (\varepsilon^* q) \Big] \\
&+ 4\bar{\ell}\sigma^{\mu\nu}\ell \Big( iC_T \epsilon_{\mu\nu\lambda\sigma} \Big) \Big[ -2T_1 \varepsilon^{*\lambda} (p_B + p_\rho)^\sigma + B_6 \varepsilon^{*\lambda} q^\sigma - B_7 (\varepsilon^* q) {p_\rho}^\lambda q^\sigma \Big] \\
&+ 16 C_{TE} \bar{\ell}\sigma_{\mu\nu}\ell \Big[ -2T_1 \varepsilon^{*\mu} (p_B + p_\rho)^\nu + B_6 \varepsilon^{*\mu} q^\nu - B_7 (\varepsilon^* q) {p_\rho}^\mu q^\nu \Bigg\} ,
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
A_1 &= (C_{LL}^{tot} + C_{RL}) \frac{V}{m_B + m_\rho} - 2(C_{BR} + C_{SL}) \frac{T_1}{q^2} , \\
B_1 &= (C_{LL}^{tot} - C_{RL})(m_B + m_\rho) A_1 - 2(C_{BR} - C_{SL})(m_B^2 - m_\rho^2) \frac{T_2}{q^2} , \\
B_2 &= \frac{C_{LL}^{tot} - C_{RL}}{m_B + m_\rho} A_2 - 2(C_{BR} - C_{SL}) \frac{1}{q^2} \left[ T_2 + \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right] , \\
B_3 &= 2(C_{LL}^{tot} - C_{RL}) m_\rho \frac{A_3 - A_0}{q^2} + 2(C_{BR} - C_{SL}) \frac{T_3}{q^2} , \\
C_1 &= A_1 (C_{LL}^{tot} \to C_{LR}^{tot} , \quad C_{RL} \to C_{RR}) , \\
D_1 &= B_1 (C_{LL}^{tot} \to C_{LR}^{tot} , \quad C_{RL} \to C_{RR}) , \\
D_2 &= B_2 (C_{LL}^{tot} \to C_{LR}^{tot} , \quad C_{RL} \to C_{RR}) , \\
D_3 &= B_3 (C_{LL}^{tot} \to C_{LR}^{tot} , \quad C_{RL} \to C_{RR}) , \\
B_4 &= -2(C_{LRRL} - C_{RLRL}) \frac{m_\rho}{m_b} A_0 , \\
B_5 &= -2(C_{LRLR} - C_{RLLR}) \frac{m_\rho}{m_b} A_0 , \\
B_6 &= 2(m_B^2 - m_\rho^2) \frac{T_1 - T_2}{q^2} , \\
B_7 &= \frac{4}{q^2} \left( T_1 - T_2 - \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right) .
\end{aligned}
\tag{13}
$$

From this expression of the decay amplitude, for the differential decay width we get the following result:

$$
\frac{d\Gamma}{d\hat{s}}(B \to \rho \ell^+ \ell^-) = \frac{G^2 \alpha^2 m_B}{2^{14} \pi^5} \left| V_{tb} V_{td}^* \right|^2 \lambda^{1/2}(1, \hat{r}, \hat{s}) v \Delta(\hat{s}) , \tag{14}
$$

with

$$
\begin{aligned}
\Delta \;=\; & \frac{2}{3\hat{r}_\rho \hat{s}} m_B^2 \,\mathrm{Re}\Big[ -6 m_B \hat{m}_\ell \hat{s}\lambda (B_1 - D_1)(B_4^* - B_5^*) \\
& - 12 m_B^2 \hat{m}_\ell^2 \hat{s}\lambda \Big\{ B_4 B_5^* + (B_3 - D_2 - D_3) B_1^* - (B_2 + B_3 - D_3) D_1^* \Big\} \\
& + 6 m_B^3 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho) \lambda (B_2 - D_2)(B_4^* - B_5^*) \\
& + 12 m_B^4 \hat{m}_\ell^2 \hat{s} (1 - \hat{r}_\rho) \lambda (B_2 - D_2)(B_3^* - D_3^*) \\
& + 6 m_B^3 \hat{m}_\ell \lambda \hat{s}^2 (B_4 - B_5)(B_3^* - D_3^*) \\
& + 48 \hat{m}_\ell^2 \hat{r}_\rho \hat{s} \Big\{ 3 B_1 D_1^* + 2 m_B^4 \lambda A_1 C_1^* \Big\} \\
& + 48 m_B^5 \hat{m}_\ell \hat{s} \lambda^2 (B_2 + D_2) B_7^* C_{TE}^* \\
& - 16 m_B^4 \hat{r}_\rho \hat{s} (\hat{m}_\ell^2 - \hat{s}) \lambda \Big\{ \left| A_1 \right|^2 + \left| C_1 \right|^2 \Big\} \\
& - m_B^2 \hat{s} (2\hat{m}_\ell^2 - \hat{s}) \lambda \Big\{ \left| B_4 \right|^2 + \left| B_5 \right|^2 \Big\} \\
& - 48 m_B^3 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho - \hat{s}) \lambda \Big\{ (B_1 + D_1) B_7^* C_{TE}^* + 2 (B_2 + D_2) B_6^* C_{TE}^* \Big\} \\
& - 6 m_B^4 \hat{m}_\ell^2 \hat{s} \lambda \Big\{ 2 (2 + 2\hat{r}_\rho - \hat{s}) B_2 D_2^* - \hat{s} \left| (B_3 - D_3) \right|^2 \Big\} \\
& + 96 m_B \hat{m}_\ell \hat{s} (\lambda + 12 \hat{r}_\rho \hat{s}) (B_1 + D_1) B_6^* C_{TE}^* \\
& + 8 m_B^2 \hat{s}^2 \Big\{ v^2 \left| C_T \right|^2 + 4 (3 - 2v^2) \left| C_{TE} \right|^2 \Big\} \Big\{ 4 (\lambda + 12 \hat{r}_\rho \hat{s}) \left| B_6 \right|^2 \\
& - 4 m_B^2 \lambda (1 - \hat{r}_\rho - \hat{s}) B_6 B_7^* + m_B^4 \lambda^2 \left| B_7 \right|^2 \Big\} \\
& - 4 m_B^2 \lambda \Big\{ \hat{m}_\ell^2 (2 - 2\hat{r}_\rho + \hat{s}) + \hat{s} (1 - \hat{r}_\rho - \hat{s}) \Big\} (B_1 B_2^* + D_1 D_2^*) \\
& + \hat{s} \Big\{ 6 \hat{r}_\rho \hat{s} (3 + v^2) + \lambda (3 - v^2) \Big\} \Big\{ \left| B_1 \right|^2 + \left| D_1 \right|^2 \Big\} \\
& - 2 m_B^4 \lambda \Big\{ \hat{m}_\ell^2 [\lambda - 3 (1 - \hat{r}_\rho)^2] - \lambda \hat{s} \Big\} \Big\{ \left| B_2 \right|^2 + \left| D_2 \right|^2 \Big\} \\
& + 128 m_B^2 \Big\{ 4 \hat{m}_\ell^2 [20 \hat{r}_\rho \lambda - 12 \hat{r}_\rho (1 - \hat{r}_\rho)^2 - \lambda \hat{s}] \\
& + \hat{s} [4 \hat{r}_\rho \lambda + 12 \hat{r}_\rho (1 - \hat{r}_\rho)^2 + \lambda \hat{s}] \Big\} \left| C_T \right|^2 \left| T_1 \right|^2 \\
& + 512 m_B^2 \Big\{ \hat{s} [4 \hat{r}_\rho \lambda + 12 \hat{r}_\rho (1 - \hat{r}_\rho)^2 + \lambda \hat{s}] \\
& + 8 \hat{m}_\ell^2 [12 \hat{r}_\rho (1 - \hat{r}_\rho)^2 + \lambda (\hat{s} - 8 \hat{r}_\rho)] \Big\} \left| C_{TE} \right|^2 \left| T_1 \right|^2 \\
& - 64 m_B^2 \hat{s}^2 \Big\{ v^2 \left| C_T \right|^2 + 4 (3 - 2v^2) \left| C_{TE} \right|^2 \Big\} \Big\{ 2 [\lambda + 12 \hat{r}_\rho (1 - \hat{r}_\rho)] B_6 T_1^* \\
& - m_B^2 \lambda (1 + 3\hat{r}_\rho - \hat{s}) B_7 T_1^* \Big\} \\
& + 768 m_B^3 \hat{m}_\ell \hat{r}_\rho \hat{s} \lambda (A_1 + C_1) C_T^* T_1^* \\
& - 192 m_B \hat{m}_\ell \hat{s} [\lambda + 12 \hat{r}_\rho (1 - \hat{r}_\rho)] (B_1 + D_1) C_{TE}^* T_1^* \\
& + 192 m_B^3 \hat{m}_\ell \hat{s} \lambda (1 + 3\hat{r}_\rho - \hat{s}) \lambda (B_2 + D_2) C_{TE}^* T_1^* \Big] \;, \qquad (15)
\end{aligned}
$$

where $\hat{s} = q^2/m_B^2$, $\hat{r}_\rho = m_\rho^2/m_B^2$ and $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, $\hat{m}_\ell = m_\ell/m_B$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity.

Using the matrix element for the $B \to \rho \ell^+ \ell^-$ decay, our next problem is to calculate the polarized $FB$ asymmetries. For this purpose, we define the following orthogonal unit vectors $s_i^{\pm\mu}$ in the rest frame of $\ell^\pm$, where $i = L, N$ or $T$ correspond to longitudinal, normal, transversal polarization directions, respectively (see also [1, 8, 10, 15]),

$$\begin{aligned}
s_L^{-\mu} &= \left(0, \vec{e}_L^{\,-}\right) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) ~, \\
s_N^{-\mu} &= \left(0, \vec{e}_N^{\,-}\right) = \left(0, \frac{\vec{p}_K \times \vec{p}_-}{|\vec{p}_K \times \vec{p}_-|}\right) ~, \\
s_T^{-\mu} &= \left(0, \vec{e}_T^{\,-}\right) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right) ~, \\
s_L^{+\mu} &= \left(0, \vec{e}_L^{\,+}\right) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) ~, \\
s_N^{+\mu} &= \left(0, \vec{e}_N^{\,+}\right) = \left(0, \frac{\vec{p}_K \times \vec{p}_+}{|\vec{p}_K \times \vec{p}_+|}\right) ~, \\
s_T^{+\mu} &= \left(0, \vec{e}_T^{\,+}\right) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right) ~, \qquad (16)
\end{aligned}$$

where $\vec{p}_\mp$ and $\vec{p}_K$ are the three–momenta of the leptons $\ell^\mp$ and $\rho$ meson in the center of mass frame (CM) of $\ell^- \ell^+$ system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be done by the Lorentz boost. Boosting of the longitudinal unit vectors $s_L^{\pm\mu}$ yields

$$\left(s_L^{\mp\mu}\right)_{CM} = \left(\frac{|\vec{p}_\mp|}{m_\ell} ~, \frac{E_\ell \vec{p}_\mp}{m_\ell \, |\vec{p}_\mp|}\right) ~, \qquad (17)$$

where $\vec{p}_+ = -\vec{p}_-$, $E_\ell$ and $m_\ell$ are the energy and mass of leptons in the CM frame, respectively. The remaining two unit vectors $s_N^{\pm\mu}$, $s_T^{\pm\mu}$ are unchanged under Lorentz boost.

The definition of the unpolarized and normalized differential forward–backward asymmetry is (see for example [28])

$$\mathcal{A}_{FB} = \frac{\displaystyle\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} dz - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz} dz}{\displaystyle\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} dz + \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz} dz} ~, \qquad (18)$$

where $z = \cos\theta$ is the angle between $B$ meson and $\ell^-$ in the center mass frame of leptons. When the spins of both leptons are taken into account, the $\mathcal{A}_{FB}$ will be a function of the spins of the final leptons and it is defined as

$$\begin{aligned}
\mathcal{A}_{FB}^{ij}(\hat{s}) &= \left(\frac{d\Gamma(\hat{s})}{d\hat{s}}\right)^{-1} \left\{ \int_0^1 dz - \int_{-1}^0 dz \right\} \left\{ \left[ \frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = -\vec{j})}{d\hat{s}dz} \right] \right. \\
&- \left. \left[ \frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = -\vec{j})}{d\hat{s}dz} \right] \right\} ~, \\
&= \mathcal{A}_{FB}(\vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = \vec{j}) - \mathcal{A}_{FB}(\vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = -\vec{j}) - \mathcal{A}_{FB}(\vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = \vec{j}) \\
&+ \mathcal{A}_{FB}(\vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = -\vec{j}) ~. \qquad (19)
\end{aligned}$$

Using these definitions for the double polarized $FB$ asymmetries, we get the following results:

$$
\begin{aligned}
\mathcal{A}_{FB}^{LL} &= \frac{2}{\hat{r}_\rho \Delta} m_B^3 \sqrt{\lambda} v \,\mathrm{Re}\Big[ - m_B^3 \hat{m}_\ell \lambda \Big\{ 4(B_1 - D_1) B_7^* C_T^* - (B_4 + B_5)(B_2^* + D_2^*) \Big\} \\
&+ 4 m_B^4 \hat{m}_\ell \lambda \Big\{ (1 - \hat{r}_\rho)(B_2 - D_2) B_7^* C_T^* + \hat{s}(B_3 - D_3) B_7^* C_T^* \Big\} \\
&- \hat{m}_\ell (1 - \hat{r}_\rho - \hat{s}) \Big\{ B_1^* (B_4 + B_5 - 8 B_6 C_T) + D_1^* (B_4 + B_5 + 8 B_6 C_T) \Big\} \\
&+ 8 m_B \hat{r}_\rho \hat{s} (A_1 B_1^* - C_1 D_1^*) + 128 m_B^2 \hat{m}_\ell \hat{r}_\rho \hat{s} (A_1 - C_1) B_6^* C_{TE}^* \\
&+ 2 m_B^3 \hat{s} \lambda \Big\{ (B_4 - B_5) B_7^* C_T^* + 2(B_4 + B_5) B_7^* C_{TE}^* \Big\} \\
&- 8 m_B^2 \hat{m}_\ell (1 - \hat{r}_\rho)(1 - \hat{r}_\rho - \hat{s})(B_2 - D_2) B_6^* C_T^* \\
&- 4 m_B (1 - \hat{r}_\rho - \hat{s}) \hat{s} \Big\{ (B_4 - B_5) B_6^* C_T^* + 2(B_4 + B_5) B_6^* C_{TE}^* \\
&+ 2 m_B \hat{m}_\ell (B_3 - D_3) B_6^* C_T^* \Big\} - 256 m_B^5 \hat{m}_\ell \hat{r}_\rho (1 - \hat{r}_\rho)(A_1 - C_1) T_1^* C_{TE}^* \\
&- 16 \hat{m}_\ell (1 - 5\hat{r}_\rho - \hat{s})(B_1 - D_1) T_1^* C_T^* \\
&+ 16 m_B^2 \hat{m}_\ell (1 - \hat{r}_\rho)(1 + 3\hat{r}_\rho - \hat{s})(B_2 - D_2) T_1^* C_T^* \\
&+ 8 m_B (1 + 3\hat{r}_\rho - \hat{s}) \hat{s} \Big\{ 2(B_4 + B_5) T_1^* C_{TE}^* + (B_4 - B_5) T_1^* C_T^* \\
&+ 2 m_B \hat{m}_\ell (B_3 - D_3) T_1^* C_T^* \Big\} \Big] \;, \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{LN} &= \frac{8}{3 \hat{r}_\rho \hat{s} \Delta} m_B^2 \sqrt{\hat{s}} \lambda v \,\mathrm{Im}\Big[ - \hat{m}_\ell (B_1 D_1^* + m_B^4 \lambda B_2 D_2^*) + 4 m_B^4 \hat{m}_\ell \hat{r}_\rho \sqrt{\hat{s}} A_1 C_1^* \\
&- 2 m_B \hat{s} \Big\{ B_6 (C_T - 2 C_{TE}) B_1^* + B_6 (C_T + 2 C_{TE}) D_1^* \Big\} \\
&- m_B^5 \hat{s} \lambda \Big\{ B_7 (C_T - 2 C_{TE}) B_2^* + B_7 (C_T + 2 C_{TE}) D_2^* \Big\} \\
&- 16 m_B^2 \hat{m}_\ell \hat{s} \Big( 4 \left| B_6 \right|^2 + m_B^4 \lambda \left| B_7 \right|^2 \Big) C_T C_{TE}^* \\
&+ m_B^2 \hat{m}_\ell (1 - \hat{r}_\rho - \hat{s})(B_1 D_2^* + B_2 D_1^*) \\
&+ m_B^3 \hat{s} (1 - \hat{r}_\rho - \hat{s}) \Big\{ (B_1^* B_7 + 2 B_2^* B_6)(C_T - 2 C_{TE}) \\
&+ (D_1^* B_7 + 2 D_2^* B_6)(C_T + 2 C_{TE}) \Big\} \\
&- 64 m_B^2 \hat{m}_\ell \hat{s} \Big\{ - m_B^2 (1 - \hat{r}_\rho - \hat{s}) \mathrm{Re}[B_6 B_7^*] + 4 \left| T_1 \right|^2 - 4 \mathrm{Re}[B_6 T_1^*] \\
&+ 2 m_B^2 (1 + 3\hat{r}_\rho - \hat{s}) \mathrm{Re}[B_7 T_1^*] \Big\} C_T C_{TE}^* \\
&+ 16 m_B^3 \hat{r}_\rho \hat{s} \Big\{ (A_1 - C_1) C_T^* T_1^* - 2(A_1 + C_1) C_{TE}^* T_1^* \Big\} \\
&+ 4 m_B \hat{s} \Big\{ B_1^* (C_T - 2 C_{TE}) T_1 + D_1^* (C_T + 2 C_{TE}) T_1 \Big\} \\
&- 4 m_B^3 \hat{s} (1 + 3\hat{r}_\rho - \hat{s}) \Big\{ B_2^* (C_T - 2 C_{TE}) T_1 + D_2^* (C_T + 2 C_{TE}) T_1 \Big\} \Big] \;, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NL} &= \frac{8}{3 \hat{r}_\rho \hat{s} \Delta} m_B^2 \sqrt{\hat{s}} \lambda v \,\mathrm{Im}\Big[ - \hat{m}_\ell (B_1 D_1^*] + m_B^4 \lambda B_2 D_2^*) + 4 m_B^2 \hat{m}_\ell \hat{r}_\rho \hat{s} A_1 C_1^* \\
&+ 2 m_B \hat{s} \Big\{ B_6 (C_T + 2 C_{TE}) B_1^* + B_6 (C_T - 2 C_{TE}) D_1^* \Big\} \\
&+ m_B^5 \hat{s} \lambda \Big\{ B_7 (C_T + 2 C_{TE}) B_2^* + B_7 (C_T - 2 C_{TE}) D_2^* \Big\} \\
&+ 16 m_B^2 \hat{m}_\ell \hat{s} \Big( 4 \left| B_6 \right|^2 + m_B^4 \lambda \left| B_7 \right|^2 \Big) C_T C_{TE}^*
\end{aligned}
$$

$$
\begin{aligned}
&+\ m_B^2\hat{m}_\ell(1-\hat{r}_\rho-\hat{s})(B_1D_2^*+B_2D_1^*)\\
&-\ m_B^3\hat{s}(1-\hat{r}_\rho-\hat{s})\Big\{(B_1^*B_7+2B_2^*B_6)(C_T+2C_{TE})\\
&+\ (D_1^*B_7+2D_2^*B_6)(C_T-2C_{TE})\Big\}\\
&+\ 64m_B^2\hat{m}_\ell\hat{s}\Big\{-m_B^2(1-\hat{r}_\rho-\hat{s})\mathrm{Re}[B_6B_7^*]+4\left|T_1\right|^2-4\mathrm{Re}[B_6T_1^*]\\
&+\ 2m_B^2(1+3\hat{r}_\rho-\hat{s})\mathrm{Re}[B_7T_1^*]\Big\}C_TC_{TE}^*\\
&+\ 16m_B^3\hat{r}_\rho\hat{s}\Big\{(A_1-C_1)C_T^*T_1^*+2(A_1+C_1)C_{TE}^*T_1^*\Big\}\\
&-\ 4m_B\hat{s}\Big\{B_1^*(C_T+2C_{TE})T_1+D_1^*(C_T-2C_{TE})T_1\Big\}\\
&+\ 4m_B^3\hat{s}(1+3\hat{r}_\rho-\hat{s})\Big\{B_2^*(C_T+2C_{TE})T_1+D_2^*(C_T-2C_{TE})T_1\Big\}\Big]\ ,
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{LT} =&\ \frac{4}{3\hat{r}_\rho\hat{s}\Delta}m_B^2\sqrt{\hat{s}}\lambda\,\mathrm{Re}\Big[-\hat{m}_\ell\Big\{\left|B_1+D_1\right|^2+m_B^4\lambda\left|B_2+D_2\right|^2\Big\}\\
&+\ 4m_B^4\hat{m}_\ell\hat{r}_\rho\hat{s}\Big\{\left|A_1+C_1\right|^2\Big\}\\
&-\ 64m_B^2\hat{m}_\ell\hat{s}\left|C_{TE}\right|^2\Big\{4\left|B_6\right|^2+m_B^4\lambda\left|B_7\right|^2-4m_B^2(1-\hat{r}_\rho-\hat{s})B_6B_7^*\Big\}\\
&+\ 2m_B^2\hat{m}_\ell(1-\hat{r}_\rho-\hat{s})(B_1+D_1)(B_2^*+D_2^*)\\
&+\ 2m_B^3(1-\hat{r}_\rho-\hat{s})\Big\{4\hat{m}_\ell^2(2B_2^*B_6+B_1^*B_7)(C_T+2C_{TE})\\
&-\ \hat{s}(2B_2^*B_6+B_1^*B_7)(C_T-2C_{TE})\Big\}\\
&-\ 4m_B\Big\{4\hat{m}_\ell^2\Big[B_1^*B_6(C_T+2C_{TE})-B_6D_1^*(C_T-2C_{TE})\Big]\\
&-\ \hat{s}\Big[B_1^*B_6(C_T-2C_{TE})-B_6D_1^*(C_T+2C_{TE})\Big]\Big\}\\
&-\ 2m_B^5\lambda\Big\{4\hat{m}_\ell^2\Big[B_2^*B_7(C_T+2C_{TE})-B_7D_2^*(C_T-2C_{TE})\Big]\\
&-\ \hat{s}\Big[B_2^*B_7(C_T-2C_{TE})-B_7D_2^*(C_T+2C_{TE})\Big]\Big\}\\
&-\ 2m_B^3(1-\hat{r}_\rho-\hat{s})\Big\{4\hat{m}_\ell^2(2B_6D_2^*+B_7D_1^*)(C_T-2C_{TE})\\
&-\ \hat{s}(2B_6D_2^*+B_7D_1^*)(C_T+2C_{TE})\Big\}\\
&+\ 256m_B^2\hat{m}_\ell\Big\{2\hat{s}\left|C_{TE}\right|^2\Big[2B_6T_1^*-m_B^2(1+3\hat{r}_\rho-\hat{s})B_7T_1^*\Big]\\
&+\ 4\left|T_1\right|^2\Big[\hat{r}_\rho\left|C_T\right|^2+(4\hat{r}_\rho-\hat{s})\left|C_{TE}\right|^2\Big]\Big\}\\
&+\ 32m_B^3\hat{r}_\rho\Big\{4\hat{m}_\ell^2\Big[(A_1+C_1)C_T^*T_1^*+2(A_1-C_1)C_{TE}^*T_1^*\Big]\\
&+\ \hat{s}\Big[A_1^*(C_T-2C_{TE})T_1+C_1^*(C_T+2C_{TE})T_1\Big]\Big\}\\
&+\ 8m_B\Big\{4\hat{m}_\ell^2(C_T+2C_{TE})-\hat{s}(C_T-2C_{TE})\Big\}\Big\{B_1^*-m_B^2(1+3\hat{r}_\rho-\hat{s})B_2^*\Big\}T_1\\
&-\ 8m_B\Big\{4\hat{m}_\ell^2(C_T-2C_{TE})-\hat{s}(C_T+2C_{TE})\Big\}\Big\{D_1^*-m_B^2(1+3\hat{r}_\rho-\hat{s})D_2^*\Big\}T_1\Big]\ ,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{TL} =&\ \frac{4}{3\hat{r}_\rho\hat{s}\Delta}m_B^2\sqrt{\hat{s}}\lambda\,\mathrm{Re}\Big[\hat{m}_\ell\Big\{\left|B_1+D_1\right|^2+m_B^4\lambda\left|B_2+D_2\right|^2\Big\}\\
&-\ 4m_B^4\hat{m}_\ell\hat{r}_\rho\Big\{\left|A_1+C_1\right|^2\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+\ 64m_B^2\hat{m}_\ell\hat{s}\,|C_{TE}|^2\Big\{4\,|B_6|^2+m_B^4\lambda\,|B_7|^2-4m_B^2(1-\hat{r}_\rho-\hat{s})B_6B_7^*\Big\}\\
&-\ 2m_B^2\hat{m}_\ell(1-\hat{r}_\rho-\hat{s})(B_1+D_1)(B_2^*+D_2^*)\\
&+\ 2m_B^3(1-\hat{r}_\rho-\hat{s})\Big\{4\hat{m}_\ell^2(2B_2^*B_6+B_1^*B_7)(C_T-2C_{TE})\\
&-\ \hat{s}(2B_2^*B_6+B_1^*B_7)(C_T+2C_{TE})\Big\}\\
&-\ 4m_B\Big\{4\hat{m}_\ell^2\Big[B_1^*B_6(C_T-2C_{TE})-B_6D_1^*(C_T+2C_{TE})\Big]\\
&-\ \hat{s}\Big[B_1^*B_6(C_T+2C_{TE})-B_6D_1^*(C_T-2C_{TE})\Big]\Big\}\\
&-\ 2m_B^5\lambda\Big\{4\hat{m}_\ell^2\Big[B_2^*B_7(C_T-2C_{TE})-B_7D_2^*(C_T+2C_{TE})\Big]\\
&-\ \hat{s}\Big[B_2^*B_7(C_T+2C_{TE})-B_7D_2^*(C_T-2C_{TE})\Big]\Big\}\\
&-\ 2m_B^3(1-\hat{r}_\rho-\hat{s})\Big\{4\hat{m}_\ell^2(2B_6D_2^*+B_7D_1^*)(C_T+2C_{TE})\\
&-\ \hat{s}(2B_6D_2^*+B_7D_1^*)(C_T-2C_{TE})\Big\}\\
&-\ 256m_B^2\hat{m}_\ell\Big\{2\hat{s}\,|C_{TE}|^2\Big[2B_6T_1^*-m_B^2(1+3\hat{r}_\rho-\hat{s})B_7T_1^*\Big]\\
&+\ 4\,|T_1|^2\Big[\hat{r}_\rho\,|C_T|^2+(4\hat{r}_\rho-\hat{s})\,|C_{TE}|^2\Big]\Big\}\\
&-\ 32m_B^3\hat{r}_\rho\Big\{4\hat{m}_\ell^2\Big[(A_1+C_1)C_T^*T_1^*-2(A_1-C_1)C_{TE}^*T_1^*\Big]\\
&+\ \hat{s}\Big[A_1^*(C_T+2C_{TE})T_1+C_1^*(C_T-2C_{TE})T_1\Big]\Big\}\\
&-\ 8m_B\Big\{4\hat{m}_\ell^2(C_T+2C_{TE})-\hat{s}(C_T-2C_{TE})\Big\}\Big\{D_1^*-m_B^2(1+3\hat{r}_\rho-\hat{s})D_2^*\Big\}T_1\\
&+\ 8m_B\Big\{4\hat{m}_\ell^2(C_T-2C_{TE})-\hat{s}(C_T+2C_{TE})\Big\}\Big\{B_1^*-m_B^2(1+3\hat{r}_\rho-\hat{s})B_2^*\Big\}\ ,T_1\Big]
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NT} &= \mathcal{A}_{FB}^{TN}\\
&= \frac{2}{\hat{r}_\rho\hat{s}\Delta}m_B^2\sqrt{\lambda}\,\mathrm{Im}\Big[m_B^3\hat{m}_\ell\hat{s}\lambda\Big\{(B_4-B_5)(B_2^*+D_2^*)+8B_7C_{TE}(B_1^*-D_1^*)\\
&+\ 8m_B^2\hat{s}B_7^*C_{TE}^*(B_3-D_3)\Big\}\\
&-\ 2m_B^4\hat{m}_\ell^2\hat{s}\lambda(B_2+D_2)(B_3^*-D_3^*)\\
&+\ 4m_B^4\hat{m}_\ell(1-\hat{r}_\rho)\lambda\Big\{2m_B\hat{s}B_7^*C_{TE}^*(B_2-D_2)+\hat{m}_\ell B_2D_2^*\Big\}\\
&+\ 2m_B^2\hat{m}_\ell^2\hat{s}(1+3\hat{r}_\rho-\hat{s})(B_1B_2^*-D_1D_2^*)\\
&+\ \hat{m}_\ell(1-\hat{r}_\rho-\hat{s})\Big\{m_B\hat{s}\Big[-B_1^*(B_4-B_5+16B_6C_{TE})\\
&-\ D_1^*(B_4-B_5-16B_6C_{TE})+2m_B\hat{m}_\ell(B_1+D_1)(B_3^*-D_3^*)\Big]\\
&+\ 4\Big[\hat{m}_\ell B_1D_1^*+4m_B^3\hat{s}^2B_6C_{TE}(B_3^*-D_3^*)\Big]\Big\}\\
&-\ 16m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_\rho)(1-\hat{r}_\rho-\hat{s})(B_2-D_2)B_6^*C_{TE}^*\\
&+\ 2m_B^2\hat{m}_\ell^2[\lambda+(1-\hat{r}_\rho)(1-\hat{r}_\rho-\hat{s})](B_1^*D_2+B_2^*D_1)\\
&+\ 32m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_\rho)(1+3\hat{r}_\rho-\hat{s})(B_2-D_2)C_{TE}^*T_1^*\\
&-\ 8m_B\hat{s}(1+3\hat{r}_\rho-\hat{s})\Big\{4\hat{m}_\ell(B_1-D_1)C_{TE}^*T_1^*-2m_B\hat{s}(B_4-B_5)C_{TE}^*T_1^*\\
&-\ 4m_B^2\hat{m}_\ell\hat{s}(B_3-D_3)C_{TE}^*T_1^*+m_B\hat{s}v^2(B_4+B_5)C_T^*T_1^*\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&- 4m_B^2\hat{s}^2(1-\hat{r}_\rho-\hat{s})\Big\{2(B_4-B_5)B_6^*C_{TE}^* - v^2(B_4+B_5)B_6^*C_T^*\Big\}\\
&+ 2m_B^4\hat{s}^2\lambda\Big\{2(B_4-B_5)B_7^*C_{TE}^* - v^2(B_4+B_5)B_7^*C_T^*\Big\}\Big]~, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
{\cal A}_{FB}^{NN} &= -{\cal A}_{FB}^{TT}\\
&= \frac{2}{\hat{r}_\rho\Delta} m_B^3 \sqrt{\lambda} v\, \mathrm{Re}\Big[ - m_B^2 \hat{m}_\ell \lambda \Big\{4(B_1-D_1)B_7^*C_T^* + (B_2+D_2)(B_4^*+B_5^*)\Big\}\\
&+ 4m_B^4\hat{m}_\ell\lambda\Big\{(1-\hat{r}_\rho)(B_2-D_2)B_7^*C_T^* + \hat{s}(B_3-D_3)B_7^*C_T^*\Big\}\\
&+ 2m_B^3\hat{s}\lambda\Big\{(B_4-B_5)B_7^*C_T^* - 2(B_4+B_5)B_7^*C_{TE}^*\Big\}\\
&+ \hat{m}_\ell(1-\hat{r}_\rho-\hat{s})\Big\{B_1^*(B_4+B_5+8B_6C_T)\\
&+ D_1^*(B_4+B_5-8B_6C_T)\Big\}\\
&- 8m_B^2\hat{m}_\ell(1-\hat{r}_\rho)(1-\hat{r}_\rho-\hat{s})(B_2-D_2)B_6^*C_T^*\\
&- 4m_B\hat{s}(1-\hat{r}_\rho-\hat{s})\Big\{(B_4-B_5)B_6^*C_T^* - 2(B_4+B_5)B_6^*C_{TE}^*\\
&+ 2m_B\hat{m}_\ell(B_3-D_3)B_6^*C_T^*\Big\}\\
&+ 16m_B^2\hat{m}_\ell(1-\hat{r}_\rho)(1+3\hat{r}_\rho-\hat{s})(B_2-D_2)C_T^*T_1^*\\
&+ 8m_B\hat{s}(1+3\hat{r}_\rho-\hat{s})\Big\{(B_4-B_5)C_T^*T_1^* - 2(B_4+B_5)C_{TE}^*T_1^*\Big\}\\
&- 16\hat{m}_\ell(1+3\hat{r}_\rho-\hat{s})(B_1-D_1)C_T^*T_1^*\\
&+ 16m_B^2\hat{m}_\ell\hat{s}(1+3\hat{r}_\rho-\hat{s})(B_3-D_3)C_T^*T_1^*\Big]~. \qquad (26)
\end{aligned}
$$

In these expressions for ${\cal A}_{FB}^{ij}$, the first index in the superscript describes the polarization of lepton and the second index describes that of anti–lepton.

# 3 Numerical analysis

In this section we analyze the effects of the Wilson coefficients on the polarized $FB$ asymmetry. The input parameters we use in our numerical calculations are: $m_\rho = 0.77~GeV$, $m_\tau = 1.77~GeV$, $m_\mu = 0.106~GeV$, $m_b = 4.8~GeV$, $m_B = 5.26~GeV$ and $\Gamma_B = 4.22 \times 10^{-13}~GeV$. For the values of the Wilson coefficients we use $C_7^{SM} = -0.313$, $C_9^{SM} = 4.344$ and $C_{10}^{SM} = -4.669$. It should be noted that the above–presented value for $C_9^{SM}$ corresponds only to short distance contributions. In addition to the short distance contributions, it receives long distance contributions which result from the conversion of $\bar{u}u$, $\bar{d}d$ and $\bar{c}c$ to the lepton pair. In order to minimize the hadronic uncertainties we will discard the regions around low lying resonances $\rho$, $w$, $J/\psi$, $\psi'$, $\psi''$, by dividing the $q^2$ region to low and high dilepton mass intervals:

$$
\begin{aligned}
&\text{Region I:} && 1~GeV^2 \le q^2 \le 8~GeV^2~,\\
&\text{Region II:} && 14.5~GeV^2 \le q^2 \le (m_B - m_\rho)^2~,
\end{aligned}
$$

where the contributions of the higher $\psi$ resonances do still exist in the second region. For the form factors we have used the light cone QCD sum rules results [24, 29]. As a result of the analysis carried out in this scheme, the $q^2$ dependence of the form factors can be represented in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2} ~,$$

where the values of parameters $F(0)$, $a_F$ and $b_F$ for the $B \to \rho$ decay are listed in Table 1.

| | $F(0)$ | $a_F$ | $b_F$ |
|---|---|---|---|
| $A_0^{B\to\rho}$ | $0.372 \pm 0.04$ | 1.40 | 0.437 |
| $A_1^{B\to\rho}$ | $0.261 \pm 0.04$ | 0.29 | $-0.415$ |
| $A_2^{B\to\rho}$ | $0.223 \pm 0.03$ | 0.93 | $-0.092$ |
| $V^{B\to\rho}$ | $0.338 \pm 0.05$ | 1.37 | 0.315 |
| $T_1^{B\to\rho}$ | $0.285 \pm 0.04$ | 1.41 | 0.361 |
| $T_2^{B\to\rho}$ | $0.285 \pm 0.04$ | 0.28 | $-0.500$ |
| $T_3^{B\to\rho}$ | $0.202 \pm 0.04$ | 1.06 | $-0.076$ |

Table 1: $B$ meson decay form factors in a three-parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account.

In further numerical analysis, the values of the new Wilson coefficients are needed, and in the present analysis we will vary them in the range $-|C_{10}| \le |C_i| \le |C_{10}|$. The experimental value of the branching ratio of the $B \to K^* \ell^+ \ell^-$ decay [30, 31] and the bound on the branching ratio of the $B \to \mu^+ \mu^-$ [32] suggest that this is the right order of magnitude for the vector and scalar interaction coefficients. It should be noted here that the experimental results lead to stronger restrictions on some of the Wilson coefficients, namely $-1.5 \le C_T \le 1.5$, $-3.3 \le C_{TE} \le 2.6$, $-2 \le C_{LL}$, $C_{RL} \le 2.3$, while the remaining coefficients vary in the range $-4 \le C_X \le 4$.

As is obvious from the explicit expressions of the forward–backward asymmetries, they depend both on $q^2$ and the new Wilson coefficients $C_X$. As a result of this, it might be difficult to study the dependence of the polarized forward–backward asymmetries $\mathcal{A}_{FB}^{ij}$ on these parameters simultaneously. Therefore, it is necessary to eliminate the dependence of $\mathcal{A}_{FB}^{ij}$ on one of the parameters. We eliminate the dependence of the polarized $\mathcal{A}_{FB}^{ij}$ on $q^2$ by performing integration over $q^2$ in the kinematically allowed region, so that the polarized forward–backward asymmetry is said to be averaged. The averaged polarized forward–backward asymmetry is defined as

$$\left\langle \mathcal{A}_{FB}^{ij} \right\rangle = \frac{\displaystyle\int_{q^2_{min}}^{q^2_{max}} \mathcal{A}_{FB}^{ij} \frac{d\mathcal{B}}{dq^2} dq^2}{\displaystyle\int_{q^2_{min}}^{q^2_{max}} \frac{d\mathcal{B}}{dq^2} dq^2} ~.$$

In Figs. (1) and (2), we present the dependence of $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$ on $C_X$ for the $B \to \rho\mu^+\mu^-$ decay, in the Regions I and II, respectively. The common intersection point of all curves corresponds to the SM case. We observe from this figure that, $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$ has practically symmetric behavior in regard to its dependence on $C_T$ and $C_{TE}$ with respect to zero position. We see from Fig. (1) that, $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$ for the $B \to \rho\mu^+\mu^-$ decay is very strongly dependent on $C_{LL}$, $C_{LR}$, $C_T$, $C_{TE}$ and on $C_{RL}$ when $C_{RL} > 1$. There are certain regions of $C_X$ where the magnitude of $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$ is, more or less, two times larger than compared to its value in the SM. This fact is a direct indication of the confirmation of new physics beyond the SM which can be attributed to the existence of new vector type interaction. $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$ behaves in the same way in region II as it does in Region I, except for the scalar interactions and vector interaction with coefficient $C_{RR}$ (see Fig. (2)), and being quite sensitive to the rest of the remaining new Wilson coefficients. It is interesting to observe that the sign of $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$ is negative (positive) in Region I (Region II) for all values of $C_X$.

Figs. (3) and (4) depict the dependence of $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ on $C_X$ for the $B \to \rho\mu^+\mu^-$ decay, in Regions I and II, respectively. We observe from Fig. (3) that, except scalar interactions, $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ is quite sensitive to the existence of the remaining ones. More important than that is the presence of regions of the new Wilson coefficients where $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ changes its sign, while in the SM case the sign of the $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ is never switched. So, study of the magnitude and sign of $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ can serve as a good test for looking new physics beyond the SM. In region II, $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ is strongly dependent only on tensor interactions (see Fig. (4)).

The dependence of $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$ on $C_X$ for the $B \to \rho\mu^+\mu^-$ decay, is given in Fig. (5) in Region I, and Fig. (6) in Region II, respectively. We observe from these figures that $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$ exhibits strong dependence only on tensor interactions, and especially there is a region of $C_{TE}$ where $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$ exceeds the SM prediction more than one order of magnitude. Moreover, when $C_T$ and $C_{TE}$ is negative (positive) the sign of $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$ is positive (negative). Hence, determination of its magnitude and sign is an unambiguous confirmation of the existence of tensor interaction. Similarly, Fig. (6) depicts strong dependence of $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$ on tensor interactions. When $C_{TE}$ is negative the sign of $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$ is positive, and when $C_{TE}$ is positive it is negative. Analogous behavior is observed for $C_T$, except for quite a narrow region.

All remaining forward–backward asymmetries for the $B \to \rho\mu^+\mu^-$ decay are numerically very small and for this reason we do not present them.

The study of the dependence of forward–backward asymmetry for the $B \to \rho\tau^+\tau^-$ decay gives richer information. In Figs. (7), (8), (9) and (10) we present the dependence of $\left\langle\mathcal{A}_{FB}^{LL}\right\rangle$, $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$, $\left\langle\mathcal{A}_{FB}^{TL}\right\rangle$, $\left\langle\mathcal{A}_{FB}^{NT}\right\rangle = \left\langle\mathcal{A}_{FB}^{TN}\right\rangle$ and $\left\langle\mathcal{A}_{FB}^{NN}\right\rangle = -\left\langle\mathcal{A}_{FB}^{TT}\right\rangle$ on new Wilson coefficients, respectively. We see from Fig. (7) that when $C_X$ is negative, $\left\langle\mathcal{A}^{LL}\right\rangle > \left\langle\mathcal{A}_{SM}^{LL}\right\rangle$ only for the scalar interaction $C_{RLLR}$. Also, when $C_X$ is positive, $\left\langle\mathcal{A}^{LL}\right\rangle > \left\langle\mathcal{A}_{SM}^{LL}\right\rangle$ for the coefficients $C_{RL}$, $C_{LRLR}$. Fig. (8) depicts that $\left\langle\mathcal{A}_{FB}^{LT}\right\rangle$ is strongly dependent on tensor interactions, as well as on vector interactions with the coefficients $C_{RL}$ and $C_{RR}$ when they get positive values. We observe from Fig. (9) that, $\left\langle\mathcal{A}_{FB}^{NT}\right\rangle = \left\langle\mathcal{A}_{FB}^{TN}\right\rangle$ both exhibit strong dependence on all new Wilson coefficients. As can easily be observed from Fig. (10), $\left\langle\mathcal{A}_{FB}^{NN}\right\rangle = -\left\langle\mathcal{A}_{FB}^{TT}\right\rangle$ both are very sensitive to the presence of tensor and scalar interactions.

It follows from these results that few of the polarized forward–backward asymmetries show considerable departure from the SM predictions and these ones are strongly dependent on different types of interactions. Hence, the study of these quantities can play crucial role in establishing new physics beyond the SM.

At the end of this section, we would like to discuss the following problem. It is clear that the existence of new physics can more easily be checked through branching ratio measurements. In this connection there follows the question: could there be a situation in which the branching ratio coincides with that of the SM result, while polarized forward–backward asymmetry does not? In order to answer this question we study the correlation between the $\left\langle \mathcal{A}_{FB}^{ij} \right\rangle$ and the branching ratio $\mathcal{B}$. We can briefly summarize the results of our numerical analysis as follows: As far as $B \to \rho \mu^+ \mu^-$ decay is concerned, except for a very narrow region of $C_{RR}$, such a region is absent for all new Wilson coefficients for the asymmetries $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$.

The $B \to \rho \tau^+ \tau^-$ decay is more informative for this aim. In Figs. (11) and (12) we present the dependence of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ and $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ on the branching ratio. It follows from these figures that, there indeed exists certain regions of $C_X$ for which the polarized forward–backward asymmetry differs from the SM prediction, while the branching ratio coincides with that of the SM result. We also note that, such a region exists for the asymmetries $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle = -\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$ as well, only for the tensor interaction.

In conclusion, in this work we investigate the forward–backward asymmetries when both leptons are polarized, using a general, model independent form of the effective Hamiltonian. We see that the study of the zero position of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ can give unambiguous conformation of the new physics beyond the SM, since when new physics effects are taken into account, the results are shifted with respect to their zero positions in the SM. We also find out that the polarized $\mathcal{A}_{FB}$ is quite sensitive to the existence of some of the new Wilson coefficients. We see that there exist certain regions of some of the new Wilson coefficients for which, only study of the polarized forward–backward asymmetry gives invaluable information in establishing new physics beyond the SM.

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# Figure captions

**Fig. (1)** The dependence of the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on the new Wilson coefficients $C_X$, for the $B \to \rho \mu^+ \mu^-$ decay, in Region I.

**Fig. (2)** The same as in Fig. (1), but in Region II.

**Fig. (3)** The same as in Fig. (1), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$.

**Fig. (4)** The same as in Fig. (3), but in Region II.

**Fig. (5)** The same as in Fig. (1), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$.

**Fig. (6)** The same as in Fig. (5), but in Region II.

**Fig. (7)** The dependence of the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on the new Wilson coefficients $C_X$, for the $B \to \rho \tau^+ \tau^-$ decay, in Region II.

**Fig. (8)** The same as in Fig. (7), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$.

**Fig. (9)** The same as in Fig. (7), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NT} \right\rangle = \left\langle \mathcal{A}_{FB}^{TN} \right\rangle$.

**Fig. (10)** The same as in Fig. (7), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle = -\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$.

**Fig. (11)** Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ and the branching ratio for the $B \to \rho \tau^+ \tau^-$ decay, in Region II.

**Fig. (12)** Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and the branching ratio for the $B \to \rho \tau^+ \tau^-$ decay, in Region II.

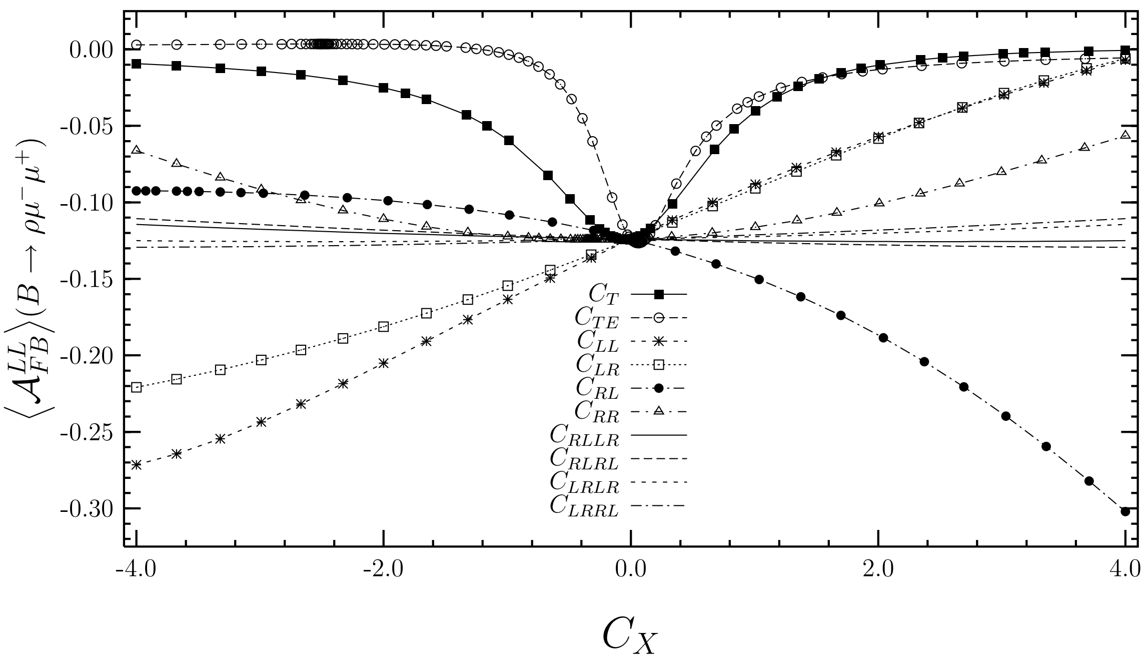
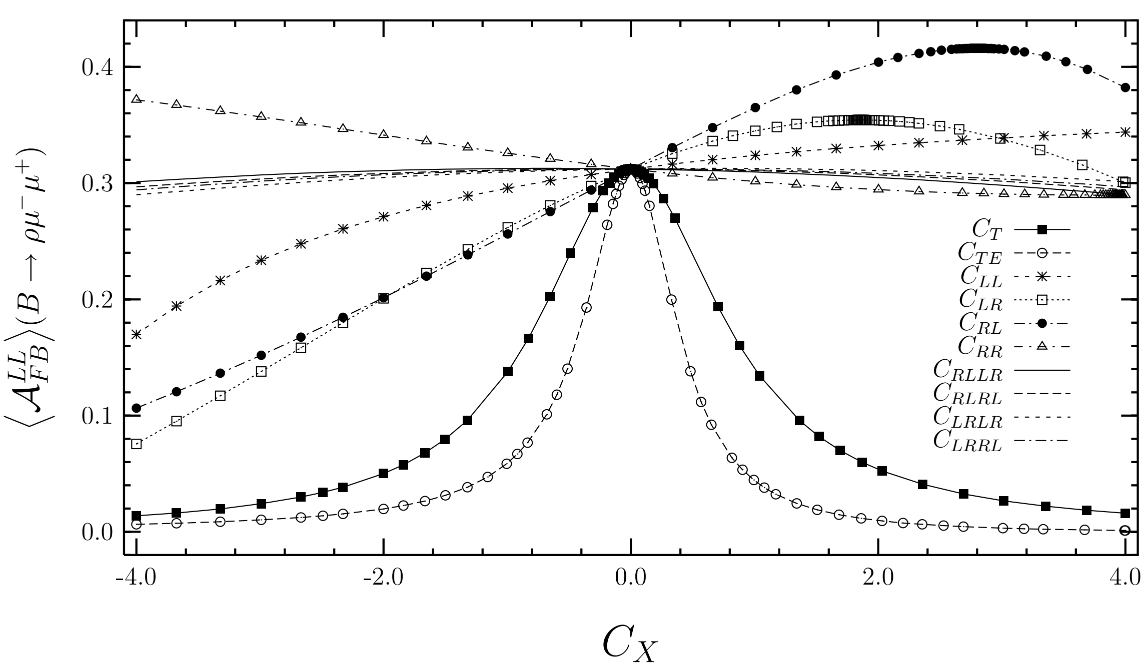



Figure 1:

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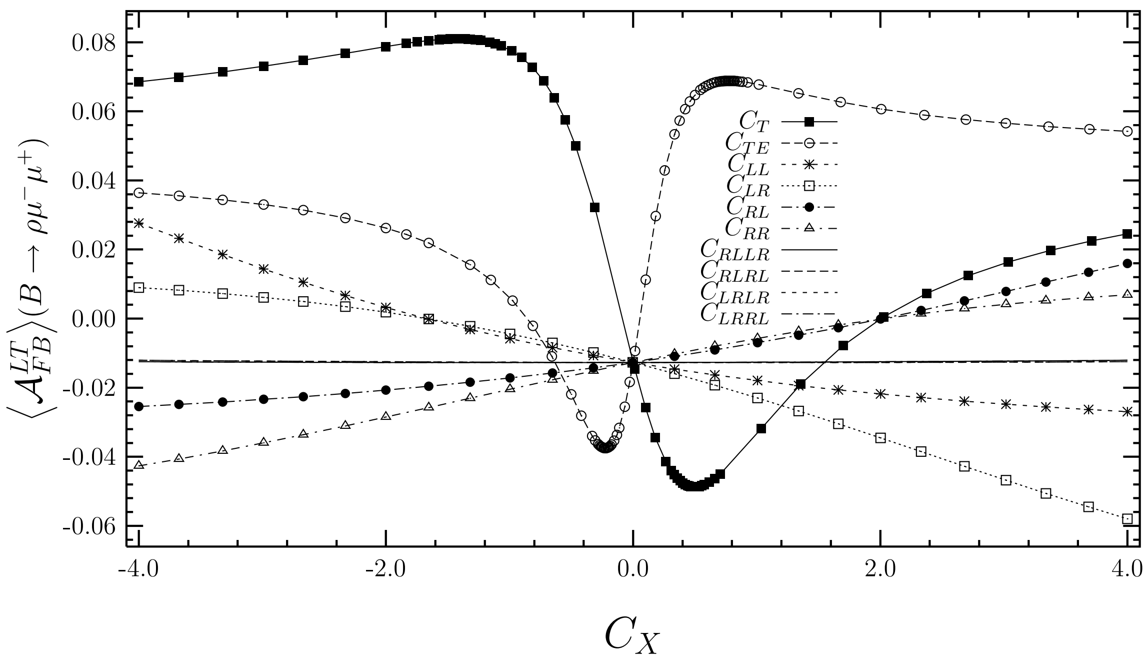


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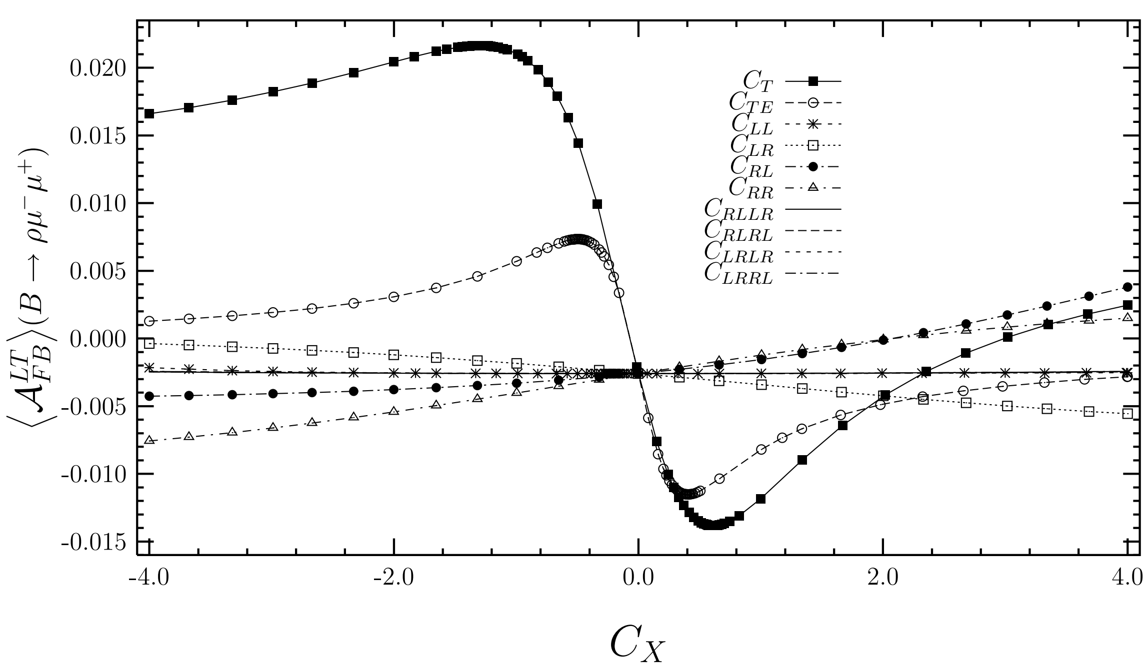


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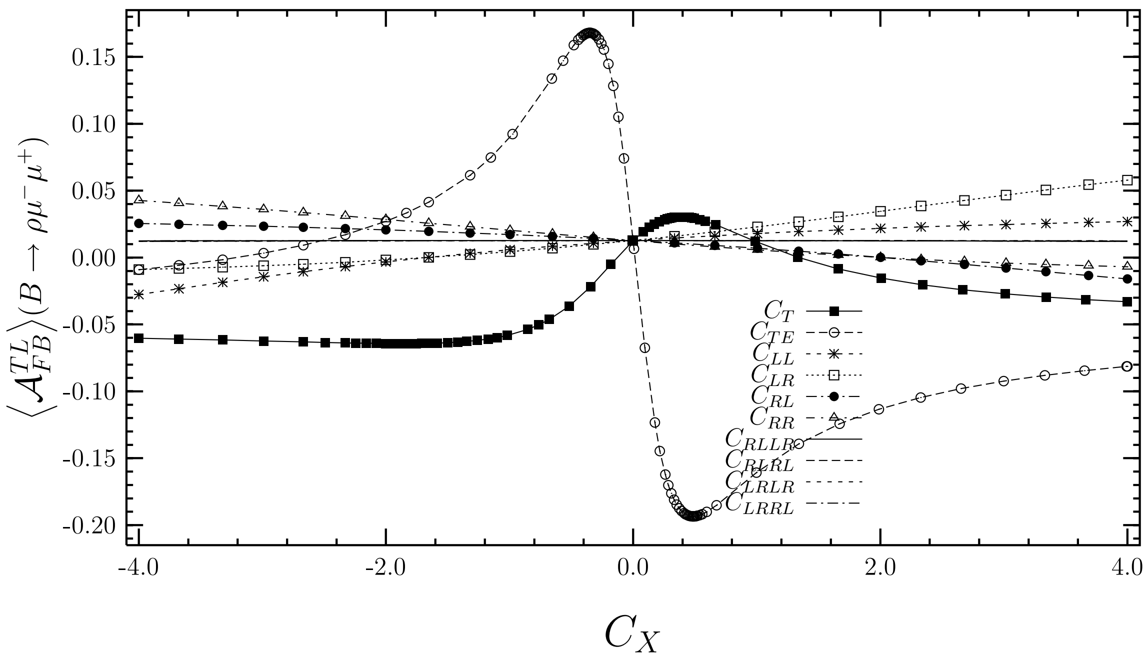


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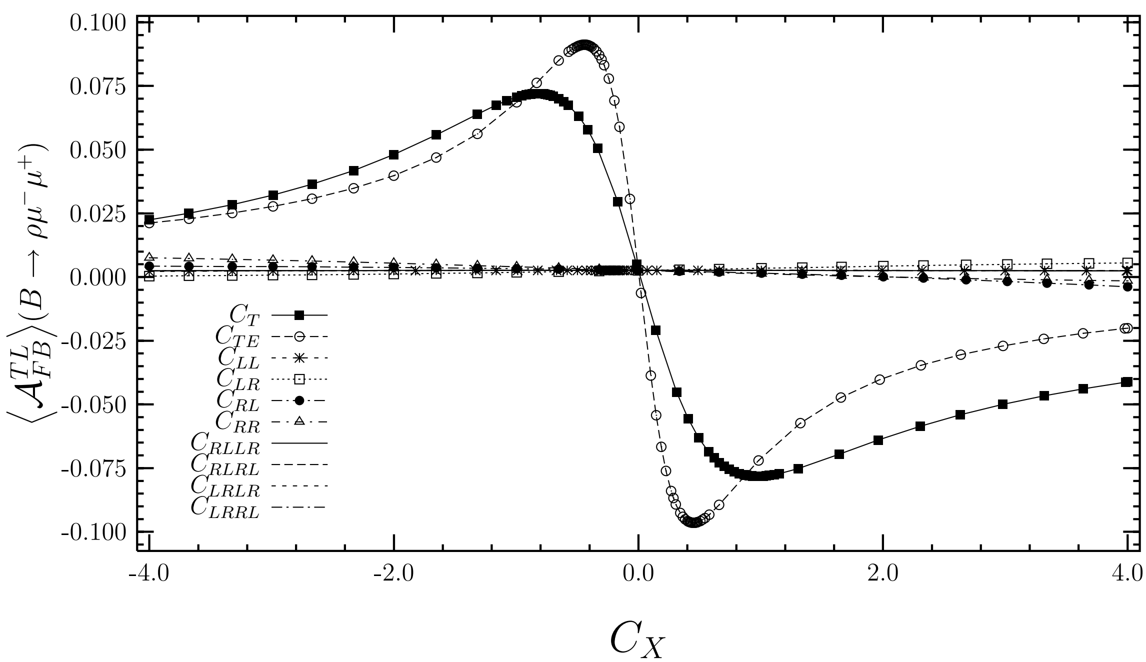


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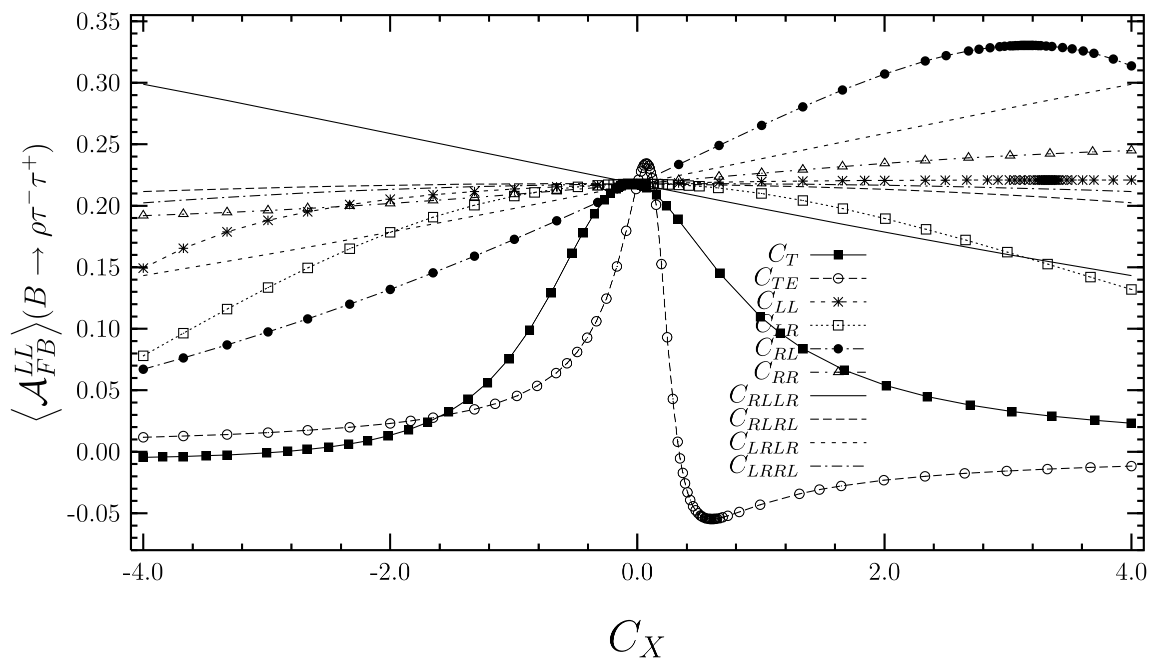


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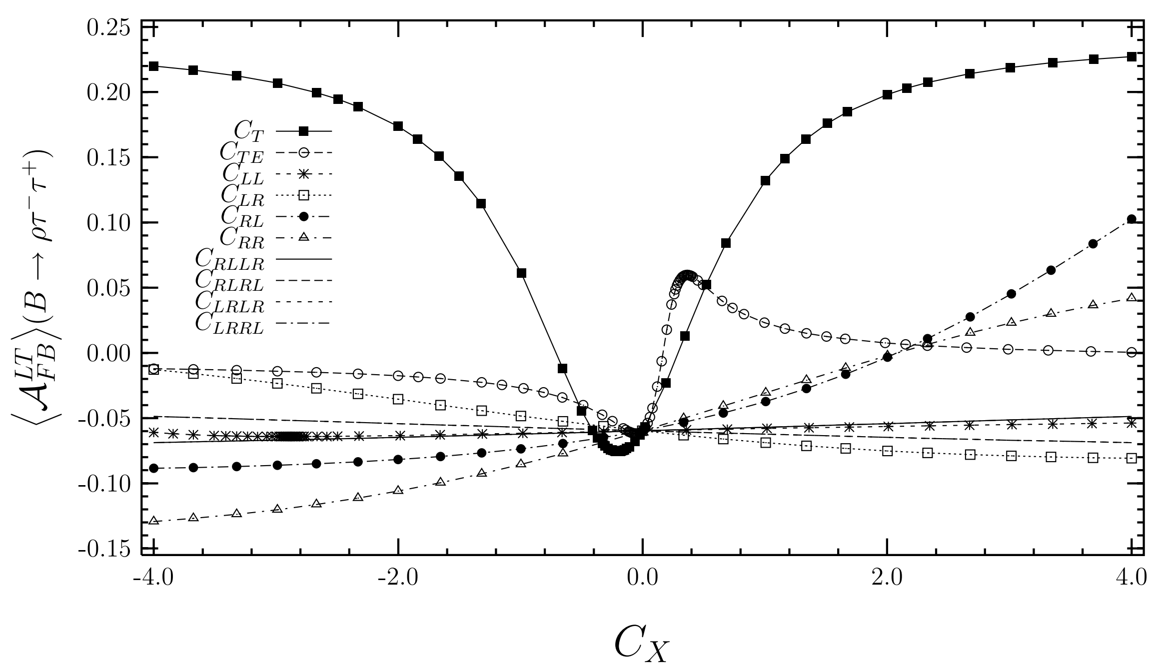


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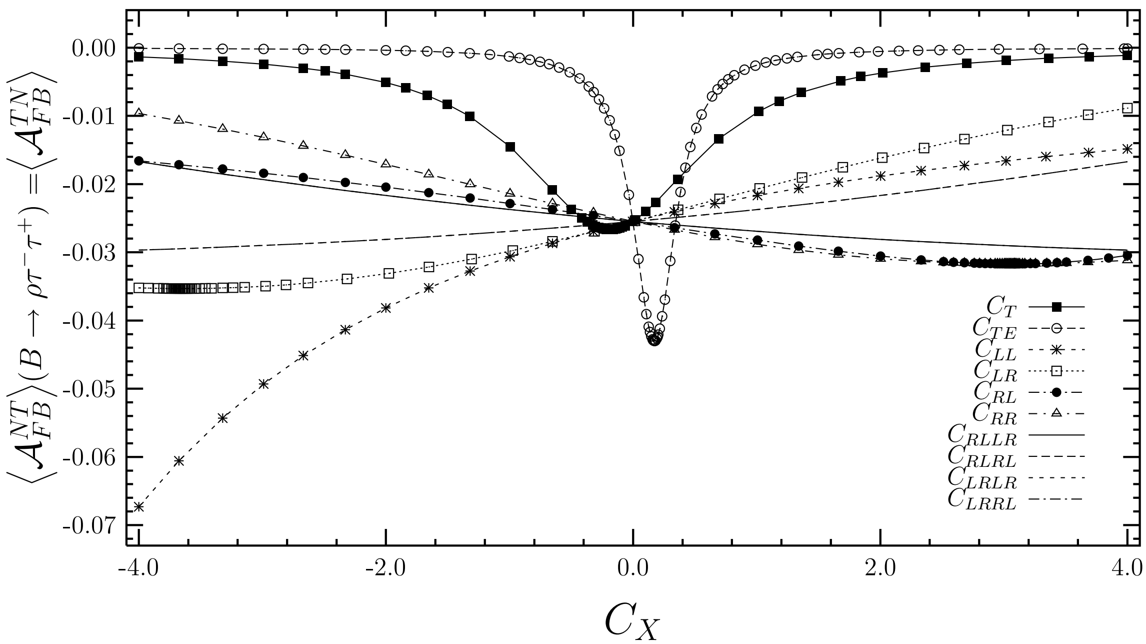


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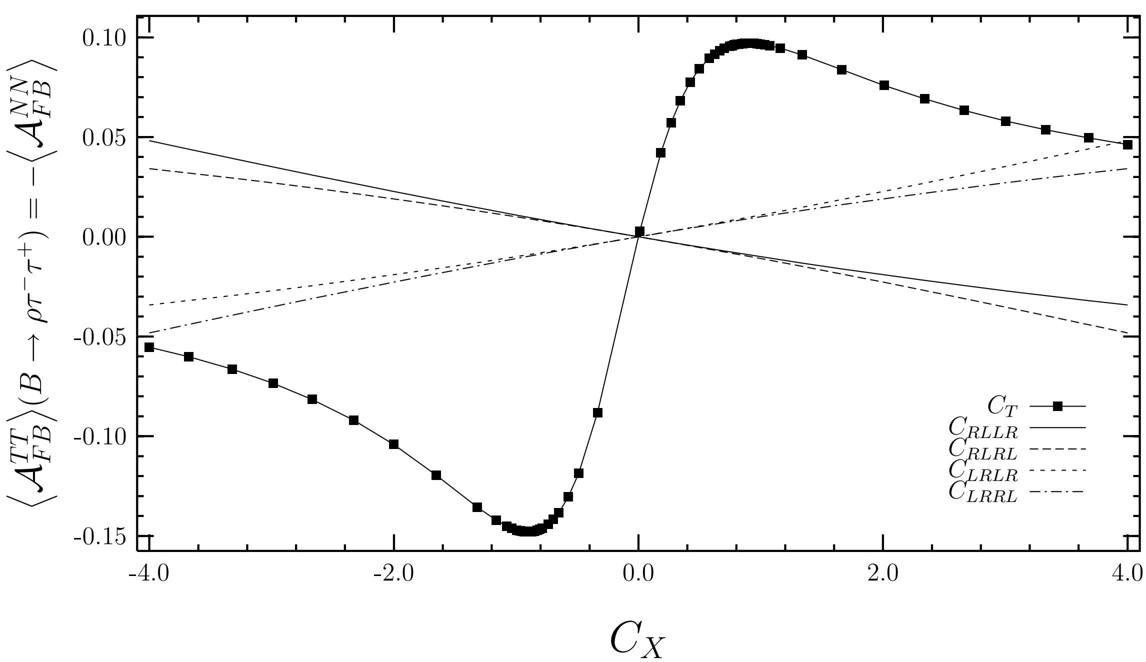


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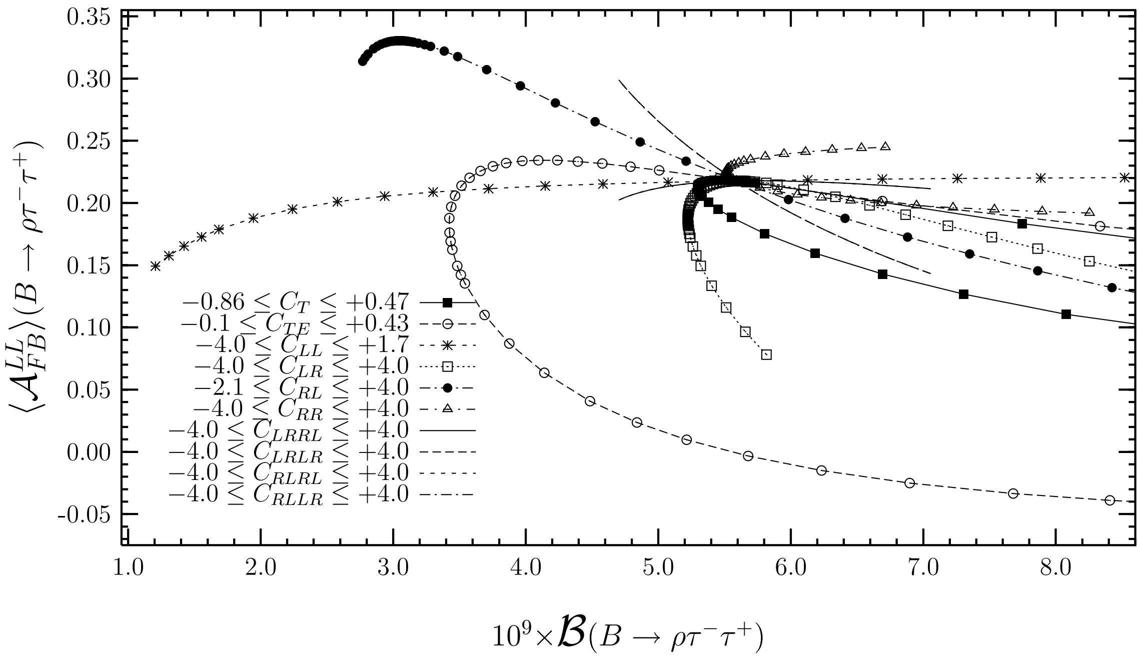


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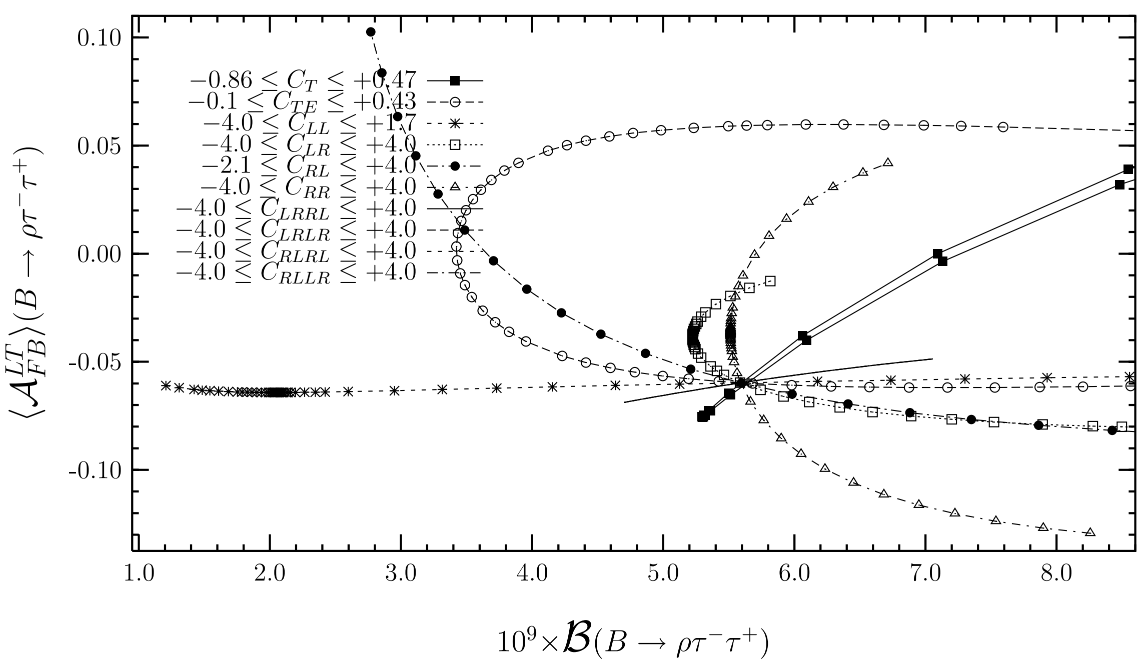


Figure 12: