RARE $B \to K^* l^+ l^-$ DECAY IN LIGHT CONE QCD

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Abstract

We investigate the transition formfactors for the $B \to K^* l^+ l^- (l = \mu,\ \tau)$ decay in the light cone QCD. It is found that the light cone and 3-point QCD sum rules analyses for some of the formfactors for the decay $B \to K^* l^+ l^-$ lead to absolutely different q^2 dependence. The invariant dilepton mass distributions for the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays and final lepton longitudinal polarization asymmetry, which includes both short and long-distance contributions, are also calculated.

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1 Introduction

Experimental observation [1] of the inclusive and exclusive radiative decays $B \to X_s\gamma$ and $B \to K^*\gamma$ stimulated the study of rare B decays on a new footing. These flavor changing neutral current (FCNC) $b \to s$ transitions in the SM do not occur at tree level and appear only at loop level. Therefore the study of these rare B-meson decays can provide a means of testing the detailed structure of the SM at the loop level. These decays are also very useful for extracting the values of the Cabibbo-Kobayashi-Maskava (CKM) matrix elements [2], as well as for establishing new physics beyond the Standard Model [3].

Currently, the main interest on rare B-meson decays is focused on decays for which the SM predicts large branching ratios and can be potentially measurable in the near future. The rare $B \to Kl^+l^-$ and $B \to K^*l^+l^-$ decays are such decays. The experimental situation for these decays is very promising [4], with e^+e^- and hadron colliders focusing only on the observation of exclusive modes with $l = e,\ \mu$ and τ final states, respectively. At quark level the process $b \to sl^+l^-$ takes place via electromagnetic and Z penguins and W box diagrams and are described by three independent Wilson coefficients $C_7,\ C_9$ and C_{10}. Investigations allow us to study different structures, described by the above mentioned Wilson coefficients. In the SM, the measurement of the forward-backward asymmetry and invariant dilepton mass distribution in $b \to ql^+l^-$ $(q = s,\ d)$ provide information on the short distance contributions dominated by the top quark loops and are essential in separating the short distance FCNC process from the contributing long distance effects [5] and also are very sensitive to the contributions from new physics [6]. Recently it has been emphasized by Hewett [7] that the longitudinal lepton polarization, which is another parity violating observable, is also an important asymmetry and that the lepton polarization in $b \to sl^+l^-$ will be measurable with the high statistics available at the B-factories currently under construction. However, in calculating the Branching ratios and other observables in hadron level, i.e. for $B \to K^*l^+l^-$ decay, we have the problem of computing the matrix element of the effective Hamiltonian, $\mathcal{H}_{eff}$, between the states B and K^*. But this problem is related to the non-perturbative sector of QCD.

These matrix elements, in the framework of different approaches such as chiral theory [8], three point QCD sum rules method [9], relativistic quark model by the light-front formalism [10, 11], have been investigated. The aim of this work is the calculation of these matrix elements in light cone QCD sum rules method and to study the lepton polarization asymmetry for the exclusive $B \to K^*l^+l^-$ decays.

The effective Hamiltonian for the $b \to sl^+l^-$ decay, including QCD corrections [12-14] can be written as

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \tag{1}$$

which is evolved from the electroweak scale down to $\mu \sim m_b$ by the renormalization group equations. Here V_{ij} represent the relevant CKM matrix elements, and O_i are a complete set of renormalized dimension 5 and 6 operators involving light fields which govern the $b \to s$ transitions and $C_i(\mu)$ are the Wilson coefficients for the corresponding operators.

The explicit forms of $C_i(\mu)$ and $O_i(\mu)$ can be found in [12-14]. For $b \to s l^+ l^-$ decay, this effective Hamiltonian leads to the matrix element

$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[C_9^{eff} \bar{s}_L \gamma_\mu b_L \; \bar{l} \gamma^\mu l + C_{10} \bar{s}_L \gamma_\mu b_L \; \bar{l} \gamma^\mu \gamma_5 l - 2 \frac{C_7}{q^2} \bar{s} i \sigma_{\mu\nu} q^\nu (m_b R + m_s L) b \; \bar{l} \gamma^\mu l \right] \quad (2)$$

where q^2 is the invariant dilepton mass, and $L(R) = [1 - (+)\gamma_5]/2$ are the projection operators. The coefficient $C_9^{eff}(\mu,\ q^2) \equiv C_9(\mu) + Y(\mu,\ q^2)$, where the function Y contains the contributions from the one loop matrix element of the four-quark operators and can be found in [12-14]. Note that the function $Y(\mu,\ q^2)$ contains both real and imaginary parts (the imaginary part arises when the c-quark in the loop is on the mass shell).

The $B \to K^* l^+ l^-$ decay also receives large long distance contributions from the cascade process $B \to K^* \psi^{(\prime)} \to K^* l^+ l^-$. These contributions are taken into account by introducing a Breit-Wigner form of the resonance propogator and this procedure leads to an additional contribution to C_9^{eff} of the form [15]

$$-\frac{2\pi}{\alpha^2} \sum_{V=\psi,\ \psi'} \frac{m_V \Gamma(V \to l^+ l^-)}{(q^2 - m_V^2) - i m_V \Gamma_V}$$

As we noted earlier, for the calculation of the branching ratios for the exclusive $B \to K^* l^+ l^-$ decays, first of all, we must calculate the matrix elements $\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) q | B \rangle$ and $\langle K^* | \bar{s} i \sigma_{\mu\nu} p^\nu (1 + \gamma_5) q | B \rangle$. These matrix elements can be parametrized in terms of the formfactors as follows (see also [9]):

$$\begin{aligned}
\langle K^*(p,\epsilon) \left| \bar{s} \gamma_\mu (1 - \gamma_5) q \right| B(p+q) \rangle \;=\; & -\epsilon_{\mu\nu\rho\sigma}\, \epsilon^{*\nu} p^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} - \\
& - i\, \epsilon_\mu^* \, (m_B + m_{K^*})\, A_1(q^2) + \\
& + i\, (\epsilon^* q) P_\mu \frac{A_2(q^2)}{m_B + m_{K^*}} + \\
& + i\ (\epsilon^* q)\, \frac{2 m_{K^*}}{q^2} \left[A_3(q^2) - A_0(q^2) \right] q_\mu\ , \qquad (3)
\end{aligned}$$

$$\begin{aligned}
\langle K^*(p,\epsilon) \left| \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) q \right| B(p+q) \rangle \;=\; & 4\, \epsilon_{\mu\nu\rho\sigma}\, \epsilon^{*\nu} p^\rho q^\sigma \, T_1(q^2) + \\
& + 2\, i \left[\epsilon_\mu^* (Pq) - (\epsilon^* q) P_\mu \right] T_2(q^2) + \\
& + 2\, i\, (\epsilon^* q) \left[q_\mu - \frac{q^2}{Pq} P_\mu \right] T_3(q^2)\ ,
\end{aligned} \qquad (4)$$

where ϵ_μ^* is the polarization vector of K^*, $p + q$ and p are the momentum of B and K^* and $P_\mu = (2p+q)_\mu$. The formfactor $A_3(q^2)$ can be written as a linear combination of the formfactors A_1 and A_2 (see [9]):

$$A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2) \ , \tag{5}$$

with the condition $A_3(0) = A_0(0)$. In calculating these formfactors we employ the light cone QCD sum rules.

2 QCD Sum Rules for Formfactors

According to the QCD sum rules ideology, in order to calculate the formfactors we start by considering the representation of a suitable correlator function in terms of hadron language and quark-gluon language. Equating these representations we get the sum rules. For this purpose we choose the following correlators.

$$\Pi^{(1)}_\mu(p,q) = i \int d^4x e^{iqx} \langle K^*(p)|\bar{s}(x)\gamma_\mu(1-\gamma_5)b(x)\bar{b}(0)i\gamma_5 q(0)|0\rangle \ , \tag{6}$$

$$\Pi^{(2)}_\mu(p,q) = i \int d^4x e^{iqx} \langle K^*(p)|\bar{s}(x)i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b(x)\bar{b}(0)i\gamma_5 q(0)|0\rangle \ . \tag{7}$$

Here the first correlator is relevant for the calculation of the formfactors $V(q^2),\ A_1(q^2),\ A_2(q^2)$ and $A_0(q^2)$ and the second one for $T_1,\ T_2$ and T_3.

The main task in QCD is the calculation of the correlation functions (6) and (7). This problem can be solved in the deep Euclidean region, where both virtualities q^2 and $(p+q)^2$ are large and negative. The virtuality of the heavy quark in the correlators (6) and (7) is large, of order $m_b^2-(p+q)^2$, and one can use the perturbative expansion of its propagator in the external field of slowly varying fluctuations inside the vector meson. Then, the leading contribution is

$$\Pi^{(1)}_\mu(p,q) = i \int \frac{d^4x\, d^4k}{(2\pi)^4} \frac{e^{i(q-k)x}}{(m_b^2-k^2)} \langle K^*|\bar{s}(x)\gamma_\mu(1-\gamma_5)(\not{k}+m_b)\gamma_5 q(0)|0\rangle \ , \tag{8}$$

$$\Pi^{(2)}_\mu(p,q) = -\int \frac{d^4x\, d^4k}{(2\pi)^4} \frac{e^{i(q-k)x}}{(m_b^2-k^2)} q^\nu \langle K^*|\bar{s}(x)\sigma_{\mu\nu}(1+\gamma_5)(\not{k}+m_b)\gamma_5 q(0)|0\rangle \ . \tag{9}$$

It is obvious from the above expressions that the problem is reduced to the calculation of the matrix elements of the gauge-invariant non-local operators, sandwiched in between the vacuum and the meson states. These matrix elements define the vector meson light cone wave functions. Following [16, 17] we define the meson wave functions as:

$$\langle 0|\bar{q}(0)\sigma_{\mu\nu}q(x)|K^*(p,\epsilon)\rangle = i\ (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)\, f^\perp_{K^*} \int_0^1 du\, e^{-iupx}\phi_\perp(u,\mu^2) \ , \tag{10}$$

$$\begin{aligned}\langle 0|\bar{q}(0)\gamma_\mu q(x)|K^*(p,\epsilon)\rangle &= p_\mu \frac{\epsilon x}{px} f_{K^*} m_{K^*} \int_0^1 du\, e^{-iupx}\phi_\parallel(u,\mu^2) + \\ &+ \left(\epsilon_\mu - p_\mu \frac{\epsilon x}{px}\right) f_{K^*} m_{K^*} \int_0^1 du\, e^{-iupx} g^{(v)}_\perp(u,\mu^2) \ ,\end{aligned} \tag{11}$$

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 q(x)|K^*(p,\epsilon)\rangle = -\frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu p^\rho x^\sigma f_{K^*} m_{K^*} \int_0^1 du\, e^{-iupx} g^{(a)}_\perp(u,\mu^2) \ . \tag{12}$$

The functions $\phi_\perp(u,\ \mu^2)$ and $\phi_\parallel(u,\mu^2)$ give the leading twist distributions in the fraction of total momentum carried by the quark in the transversaly and longitudinally polarized meson, respectively. In [17] it was shown that

$$g_\perp^v(u) = \frac{3}{4}\left[1+(2u-1)^2\right] \ , \tag{13}$$
$$g_\perp^a(u) = 6u(1-u) \ , \tag{14}$$

which we use in the numerical analysis. For the explicit form of $\phi_\perp(u,\ \mu^2)$ we shall use the results of [17]:

$$\begin{aligned}\phi_\perp(u,\ \mu^2) &= 6u(1-u)\Big\{1+a_1(\mu)(2u-1)+a_2(\mu)\left[(2u-1)^2-\frac{1}{5}\right]+ \\ &+ a_3(\mu)\left[\frac{7}{3}(2u-1)^3-(2u-1)\right]+...\Big\} \ ,\end{aligned} \tag{15}$$

$$a_n(\mu) = a_n(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_n}{b}} \ . \tag{16}$$

Here $b=\frac{11}{3}N_C-\frac{2}{3}n_f$, and

$$\gamma_n = C_F\left(1+4\sum_{j=2}^{n+1}\frac{1}{j}\right) \ , \tag{17}$$

where $C_F=\frac{N_C^2-1}{2N_C}$.
As in [17], we will use the following values for the parameters appearing in eqs.(10)-(12) and eq.(15) :

$$\begin{aligned}&f_{K^*}^\perp = 210\ MeV, \quad a_1^{K^*}(\mu=m_b)=0.57, \quad a_2^{K^*}(\mu=m_b)=-1.35 \\ &\text{and} \ \ a_3^{K^*}(\mu=m_b)=0.46 \ , \\ &\phi_\parallel(u,\ \mu^2)=6u(1-u).\end{aligned} \tag{18}$$

Using eqs.(10-12), we get the following results from eq.(8) and eq.(9) for the theoretical part of the sum rules:

$$\begin{aligned}\Pi_\mu^{(1)} &= -\,im_bf_{K^*}m_{K^*}\int_0^1\frac{du}{\Delta}\left[\epsilon_\mu^* g_\perp^{(v)}+2(q\epsilon^*)p_\mu\frac{1}{\Delta}(\Phi_\parallel-G_\perp^{(v)})\right]- \\ &-\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho q^\sigma\left[\frac{m_b}{2}f_{K^*}m_{K^*}\int_0^1\frac{du}{\Delta^2}g_\perp^{(a)}+f_{K^*}^\perp\int_0^1 du\frac{\phi_\perp}{\Delta}\right]- \\ &-if_{K^*}^\perp\int_0^1 du\frac{\phi_\perp}{\Delta}\left[\epsilon_\mu^*(pq+p^2u)-p_\mu(q\epsilon^*)\right]\end{aligned} \tag{19}$$

$$
\begin{aligned}
\Pi_\mu^{(2)} &= \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho q^\sigma \left\{ m_b f_{K^*}^\perp \int_0^1 \frac{du}{\Delta}\phi_\perp - m_{K^*} f_{K^*} \left[\int_0^1 \frac{du}{\Delta}\left(\Phi_\parallel - G_\perp^{(v)}\right) - \right.\right. \\
&\left.\left. - \int_0^1 \frac{du}{\Delta} u g_\perp^{(v)} - \int_0^1 \frac{du}{2\Delta^2} g_\perp^{(a)} \left(\Delta + q^2 + 2pqu\right)\right]\right\} + \\
&+ i\left[\epsilon_\mu^*(pq) - (q\epsilon^*)p_\mu\right] \left\{ m_b f_{K^*}^\perp \int_0^1 \frac{du}{\Delta}\phi_\perp + \right. \\
&\left. + m_{K^*} f_{K^*} \int_0^1 \frac{du}{\Delta}\left[-\left(\Phi_\parallel - G_\perp^{(v)}\right) + u g_\perp^{(v)} + \frac{g_\perp^{(a)}}{2} + \frac{g_\perp^{(a)} u(qp)}{2\Delta}\right]\right\} + \\
&+ i\, m_{K^*} f_{K^*} \left[\epsilon_\mu^*(q^2) - (q\epsilon^*)q_\mu\right] \int_0^1 \frac{du}{\Delta}\left[g_\perp^{(v)} - \frac{p^2 u}{2\Delta} g_\perp^{(a)}\right] + \\
&+ 2i m_{K^*} f_{K^*} (q\epsilon^*) \left[p_\mu(q^2) - (pq)q_\mu\right] \int_0^1 \frac{du}{\Delta^2}\left(\Phi_\parallel - G_\perp^{(v)}\right) \ , \qquad (20)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_\parallel(u) &= - \int_0^u \phi_\parallel(v) dv \ , \\
G_\perp^{(v)}(u) &= - \int_0^u g_\perp^{(v)}(v) dv \ , \qquad (21)
\end{aligned}
$$

and

$$
\Delta = m_b^2 - (q + pu)^2 \ .
$$

Let us turn our attention to the physical part of the correlator functions (6) and (7). Writing a dispersion relation in the variable $(p+q)^2$, one can separate the B meson ground state contribution to the correlator functions $\Pi_\mu^{(1)}$ and $\Pi_\mu^{(2)}$, by inserting a complete set of states between the currents in (6) and (7) focusing on the term $|B\rangle\,\langle B|$:

$$
\Pi_\mu^{(1)} = \frac{f_B m_B^2}{m_b\left[m_B^2 - (q+p)^2\right]} \langle K^*(p)|\bar{s}\gamma_\mu(1-\gamma_5)q|B(p+q)\rangle \qquad (22)
$$

$$
\Pi_\mu^{(2)} = \frac{f_B m_B^2}{m_b\left[m_B^2 - (q+p)^2\right]} \langle K^*(p)|\bar{s}i\sigma_{\mu\alpha}q^\alpha(1+\gamma_5)q|B(p+q)\rangle \qquad (23)
$$

Using the definitions of the formfactors (see eqs.(3) and (4)) in (22) and (23) and equating these expressions to eqs.(19) and (20), we get the sum rules for the formfactors. The remaining part of the calculation follows from the QCD sum rules procedure: perform the Borel transformation on the variable $(p+q)^2$ and subtract the continuum and higher states contributions invoking quark-hadron duality. (Details of these procedures can be found in [17-19]). After this procedure we obtain the following sum rules for the formfactors:

$$
\begin{aligned}
V(q^2) &= \frac{m_B+m_{K^*}}{2}\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 du\,exp\left(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\right)\times \\
&\times\left\{m_b f_{K^*} m_{K^*}\frac{g_\perp^{(a)}}{2u^2M^2}+\frac{f_{K^*}^\perp\phi_\perp}{u}\right\}\ , \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
A_1(q^2) &= \frac{1}{m_B+m_{K^*}}\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 du\,exp\left(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\right)\times \\
&\times\left\{m_b f_{K^*} m_{K^*}\frac{g_\perp^{(v)}}{u}+\frac{f_{K^*}^\perp\phi_\perp(m_b^2-q^2+p^2u^2)}{2u^2}\right\}\ , \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
A_2(q^2) &= -\,(m_B+m_{K^*})\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 du\,exp\left(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\right)\times \\
&\times\left\{\frac{m_b f_{K^*} m_{K^*}}{u^2M^2}\left(\Phi_\parallel-G_\perp^{(v)}\right)-\frac{1}{2}\,f_{K^*}^\perp\frac{\phi_\perp}{u}\right\}\ , \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
A_3(q^2)-A_0(q^2) &= \frac{q^2}{2m_{K^*}}\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 du\,exp\left(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\right)\times \\
&\times\left\{\frac{m_b f_{K^*} m_{K^*}}{u^2M^2}\left(\Phi_\parallel-G_\perp^{(v)}\right)-\frac{1}{2}\,f_{K^*}^\perp\frac{\phi_\perp}{u}\right\}\ . \qquad (27)
\end{aligned}
$$

From eq.(26) and eq.(27) we get a new relation between formfactors $A_3,\ A_0$ and A_2:

$$
A_3(q^2)-A_0(q^2)=-\frac{A_2(q^2)q^2}{2m_{K^*}(m_B+m_{K^*})}\ . \qquad (28)
$$

For the formfactors $T_1,\ T_2$, and T_3, we get the following sum rules:

$$
\begin{aligned}
T_1(q^2) &= \frac{1}{4}\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 \frac{du}{u}\,exp\left(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\right)\left\{m_b f_{K^*}^\perp\phi_\perp-\right. \\
&\left.-\,f_{K^*}m_{K^*}\left[\Phi_\parallel-G_\perp^{(v)}-ug_\perp^{(v)}-\frac{g_\perp^{(a)}}{4}-\frac{g_\perp^{a}(m_b^2+q^2-p^2u^2)}{4uM^2}\right]\right\}\ , \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
T_2(q^2) &= \frac{1}{2(m_{B^*}^2-m_{K^*}^2)}\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 \frac{du}{u}\,exp(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2})\times \\
&\times\Bigg\{f_{K^*}m_{K^*}\left[g_\perp^{(v)}-\frac{p^2}{2M^2}g_\perp^{(a)}\right]q^2+\frac{m_b f_{K^*}^\perp\phi_\perp}{2u}(m_b^2-q^2-p^2u^2)\;+ \\
&+f_{K^*}m_{K^*}\Bigg[\frac{(m_b^2-q^2-p^2u^2)}{2u}\times \\
&\times\Bigg(-\left[\Phi_\parallel-G_\perp^{(v)}\right]+ug_\perp^{(v)}+\frac{(m_b^2-q^2-p^2u^2)g_\perp^{(a)}}{4uM^2}\Bigg)\Bigg]\Bigg\}\;, \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
T_3(q^2) &= \frac{1}{4}\,\frac{m_b}{f_B m_B^2}\,e^{\frac{m_B^2}{M^2}}\int_\delta^1 \frac{du}{u}\,exp\left(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\right)\times \\
&\times\Bigg\{m_{K^*}f_{K^*}\left[\frac{g_\perp^{(a)}}{4}+\frac{(m_b^2-q^2-p^2u^2)}{4uM^2}g_\perp^{(a)}\right]- \\
&-2m_{K^*}f_{K*}\left[\frac{g_\perp^{(v)}}{2}(2-u)-\frac{p^2g_\perp^{(a)}}{2M^2}\right]- \\
&-2m_{K^*}f_{K*}\Bigg[\frac{\Phi_\parallel-G_\perp^{(v)}}{uM^2}\Bigg(\frac{m_b^2-q^2-p^2u^2}{u}\;+ \\
&+q^2-M^2+\frac{uM^2}{2}\Bigg)\Bigg]+m_b f_{K^*}^\perp\phi_\perp\Bigg\}\;. \qquad (31)
\end{aligned}
$$

Using the equation of motion we can relate T_3 and $A_3 - A_0$ by:

$$
T_3(q^2) = m_{K^*}(m_b-m_s)\frac{A_3(q^2)-A_0(q^2)}{q^2}\;. \qquad (32)
$$

Here M is the Borel mass parameter. The lower integration limit $\delta = \frac{m_b^2-p^2}{s_0-p^2}$ depends on the effective threshold s_0 above which the contributions from higher states to the dispersion relation (22) and (23) are cancelled against the corresponding piece in the QCD representation (19) and (20) Note that the sum rules for $V(q^2)$ and $A_1(q^2)$ and $T_1(q^2)$ in the light cone QCD are derived in [17]. Our results agree with that of [17]. The region of applicability of these sum rules is restricted by the requirement that the value of $q^2 - m_b^2$ be sufficiently less than zero. In order not to introduce an additional scale, we require that $q^2-m_b^2 \le (p+q)^2-m_b^2$ which translates to the condition that $m_b^2-q^2$ is of the order of the typical Borel parameter $M^2 \sim 5 \div 8\ GeV^2$. From this condition we obtain that the region of applicability of the sum rules is $q^2 < 15 \div 17\ GeV^2$, which is few GeV^2 below the zero recoil point.

Finally we calculate the differential decay rate with longitudinal polarization of the final leptons. The differential decay rate is given by:

$$
\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{G^2\alpha^2}{2^{12}\pi^5}\frac{|V_{tb}V_{ts}^*|^2\sqrt{\lambda}v}{3m_B}\Bigg\{(2m_l^2+m_B^2 s)\Bigg[16\left(|A|^2+|C|^2\right)m_B^4\lambda + \\
&+ 2\left(|B_1|^2+|D_1|^2\right)\frac{\lambda+12rs}{rs} + 2\left(|B_2|^2+|D_2|^2\right)\frac{m_B^4\lambda^2}{rs} - \\
&- 4\left[Re\left(B_1B_2^*\right)+Re\left(D_1D_2^*\right)\right]\frac{m_B^2\lambda}{rs}(1-r-s)\Bigg] + \\
&+ 6m_l^2\Bigg[-\ 16|C|^2m_B^4\lambda + 4Re\left(D_1D_3^*\right)\frac{m_B^2\lambda}{r} - \\
&- 4Re\left(D_2D_3^*\right)\frac{m_B^4(1-r)\lambda}{r} + 2|D_3|^2\frac{m_B^4 s\lambda}{r} - 4Re\left(D_1D_2^*\right)\frac{m_B^2\lambda}{r} - \\
&- 24|D_1|^2 + 2|D_2|^2\frac{m_B^4\lambda}{r}(2+2r-s)\Bigg] - \\
&- 4v\xi\Bigg[8Re\left(AC^*\right)\lambda m_B^6 s - \left[Re\left(B_1^*D_2\right)+Re\left(B_2^*D_1\right)\right]\frac{m_B^4\lambda}{r}(1-r-s) + \\
&+ Re\left(B_2^*D_2\right)\frac{m_B^6\lambda^2}{r} +\ Re\left(B_1^*D_1\right)m_B^2\frac{\lambda+12rs}{r}\Bigg]\Bigg\}\ ,
\end{aligned}
\tag{33}
$$

where $\lambda = 1+r^2+s^2-2r-2s-2rs$, $r=\frac{m_{K^*}^2}{m_B^2}$, $s=\frac{q^2}{m_B^2}$, ξ is the longitudinal polarization of the final lepton, m_l and $v=\sqrt{1-\frac{4m_l^2}{q^2}}$ are its mass and velocity, respectively. In eq.(33) $A,\ B_1,\ B_2,\ B_3,\ C,\ D_1,\ D_2$, and D_3 are defined as follows:

$$
\begin{aligned}
A &= C_9^{eff}\frac{V}{m_B+m_{K^*}} + 4C_7\frac{m_b}{q^2}T_1\ , \\
B_1 &= C_9^{eff}(m_B+m_{K^*})A_1 + 4C_7\frac{m_b}{q^2}(m_B^2-m_{K^*}^2)\ , \\
B_2 &= C_9^{eff}\frac{A_2}{m_B+m_{K^*}} + 4C_7\frac{m_b}{q^2}\left(T_2+\frac{q^2}{m_B^2-m_{K^*}^2}T_3\right)\ , \\
B_3 &= -C_9^{eff}\frac{2m_{K^*}}{q^2}(A_3-A_0) + 4C_7\frac{m_b}{q^2}T_3\ , \\
C &= C_{10}\frac{V}{m_B+m_{K^*}}\ , \\
D_1 &= C_{10}(m_B+m_{K^*})A_1\ , \\
D_2 &= C_{10}\frac{A_2}{m_B+m_{K^*}}\ ,
\end{aligned}
$$

$$D_3 = -C_{10}\frac{2m_{K^*}}{q^2}(A_3 - A_0) \ .$$

For the dileptonic decays of the B mesons, the longitudinal polarization asymmetry, P_L, of the final lepton, l, is defined as

$$P_L(q^2) = \frac{\frac{d\Gamma}{dq^2}(\xi=-1) - \frac{d\Gamma}{dq^2}(\xi=1)}{\frac{d\Gamma}{dq^2}(\xi=-1) + \frac{d\Gamma}{dq^2}(\xi=1)} \ , \qquad (34)$$

where $\xi = -1(+1)$ corresponds to the left (right) handed lepton in the final state. In the Standard Model, this polarization asymmetry comes from the interference of the vector or magnetic moment and axial vector operators. If in eq.(33) the lepton mass is equated to zero, our results coincide with the results in [20] and if $m_l \neq 0$ they coincide with the results in [11].

3 Numerical Analysis

For the input parameters which enter the sum rules for the formfactors and the expressions of the decay width we have used the following values :

$$m_b = 4.8\ GeV,\ \ m_c = 1.35\ GeV,\ m_\mu = 0.106\ GeV,\ m_\tau = 1.78\ GeV,$$

$$\Lambda_{QCD} = 225\ MeV,\ \ m_B = 5.28\ GeV,\ \ m_{K^*} = 0.892\ GeV,\ \ s_0 = 36\ GeV^2,\ \ M^2 = 8\ GeV^2$$

In Fig.1 we present the q^2 dependence of the formfactors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$ and $A_0(q^2)$ (the formfactor A_3 can be easily obtained from eq.(28)). All these formfactors increase with q^2. From these figures we see that $A_2(q^2)$ increases with q^2 strongly, but $A_1(q^2)$ and $A_0(q^2)$ do so smoothly. At this point let us compare our results on these formfactors with the results which are obtained from 3-point QCD sum rules analysis in [9]. In our case $A_1(q^2)$ increases with q^2, but in [9] it decreases with q^2. The behaviour of the other formfactors are similar.

In Fig.2 we depict the dependence of the formfactors T_1, T_2, and T_3 on q^2. In this case also all formfactors increase with q^2. For formfactors T_2 and T_3, our predictions on their q^2 dependence also differ from the predictions of [9]. In [9], T_2 is positive and smoothly decreases, the value of T_3 is negative for all q^2. Note that our predictions on the q^2 dependence of all formfactors coincide with relativistic quark model predictions [11]. The source of discrepeancy of our results with the predictions of [9] on A_1, T_2, and T_3 should be carefully analysed. This lies out of the scope of this paper. We are planning to come back to the analysis of these points in our forthcoming works.

In Fig.3(4) we present the q^2 dependence of the branching ratios for $B \to K^*\mu^+\mu^-$ ($B \to K^*\tau^+\tau^-$) decay, with and without the long distance effects, respectively.

In Fig.5 we plot the longitudinal polarization asymmetry P_L as a function of q^2 for $B \to K^*\mu^+\mu^-$ and $B \to K^*\tau^+\tau^-$, with $m_t = 176\ GeV$, with and without the long distance effects. From this figure we see that P_L vanishes at the threshold due to the kinematical factor v and that the value of P_L for $B \to K^* \mu^+ \mu^-$ decay varies in the region $(-0.5 \div +0.5)$, when the resonance ψ, ψ' mass region is excluded. In the $B \to K^* \tau^+ \tau^-$ decay case, without long distance effects P_L is negative for all values of q^2, and only in the resonance ψ' mass region P_L become positive. Therefore the study of the longitudinal polarization P_L can be very useful for understanding the relative roles of the long and short distance contributions in the $B \to K^* l^+ l^-$ decay.

At the end of this section let us compare our results on the Branching ratio of the $B \to K^* l^+ l^-$ decay with those in [9, 11]. The value of the branching ratio is close to the results of [9], but about 30 times smaller than that of [11]. In our opinion this is due to the over estimation of the formfactors in [11].

4 Conclusions

We calculate the transition formfactors for the exclusive $B \to K^* l^+ l^- (l = \mu,\ \tau)$ decay in the framework of the lightcone QCD sum rules, and investigate the longitudinal polarization asymmetries of the muon and tau in this decay. It is shown that some of the formfactors in light cone and 3-point QCD sum rules have absolutely different q^2 dependence. It is found that the value of the longitudinal polarization $P_L(\mu)$ in the region $(-0.5; +0.5)$ and $P_L(\tau)$ in $(0; -0.6)$. We also calculate the integral branching ratios and find that they are $\mathrm{Br}(B \to K^* \mu^+ \mu^-) = 12.06$ and $\mathrm{Br}(B \to K^* \tau^+ \tau^-) = 0.217$.

Few words about the possibility of the experimental observation of this decay are in order. Experimentally, to observe an asymmetry P_L of a decay with the branching ratio Br at the $n\sigma$ level, the required number of events is $N = \frac{n^2}{Br\, P_L^2}$ (see [11]). For example, to observe the τ lepton polarization at the exclusive channel $B \to K^* \tau^+ \tau^-$ at the 3σ level, one needs at least $N = 1.66 \times 10^9$ $B\bar{B}$ decays. Since in the future B-factories, it is expected that $\sim$ 10^9 B-mesons would be created per year, it is possible to measure the longitudinal polarization asymmetry of the τ lepton.

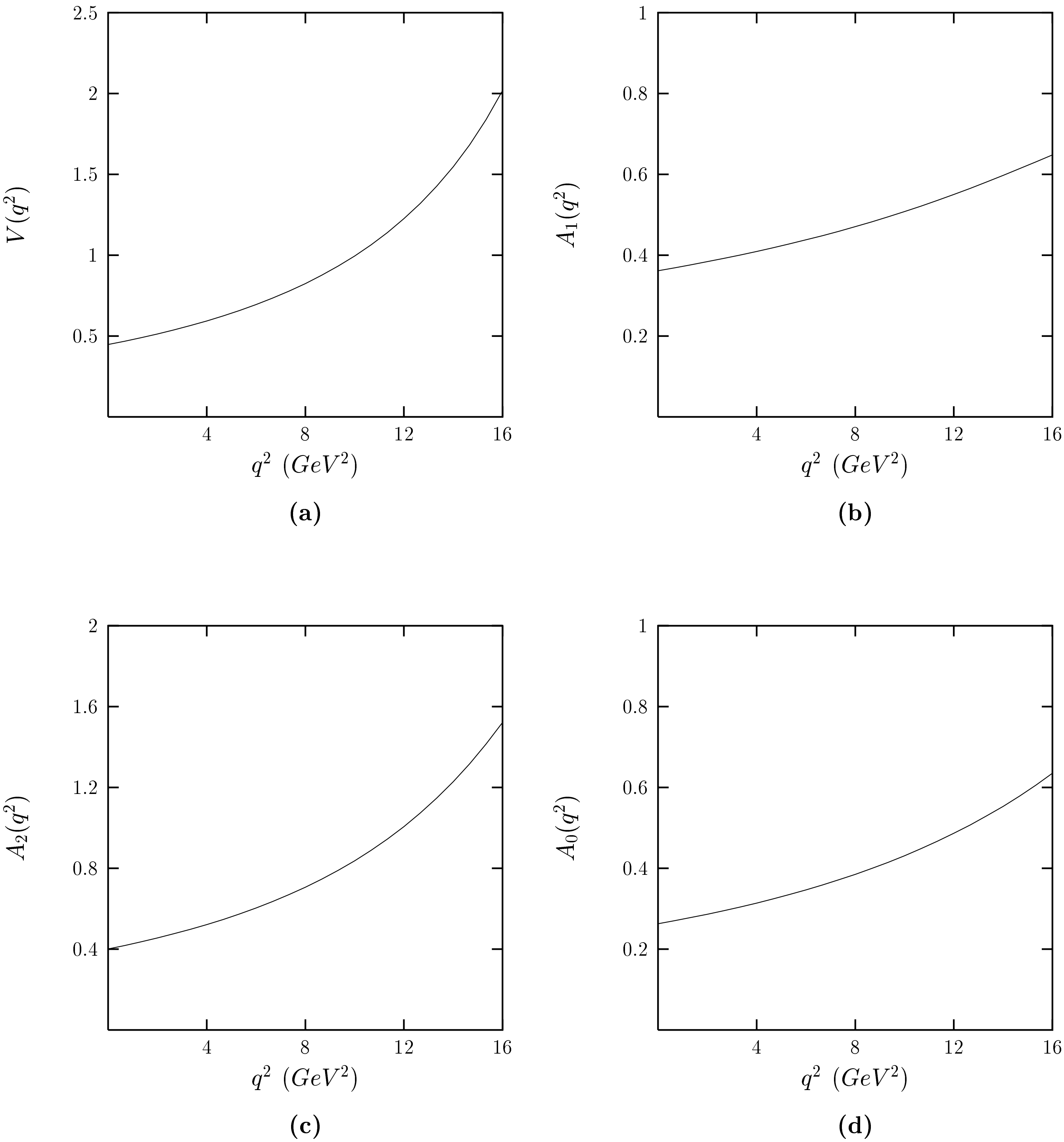
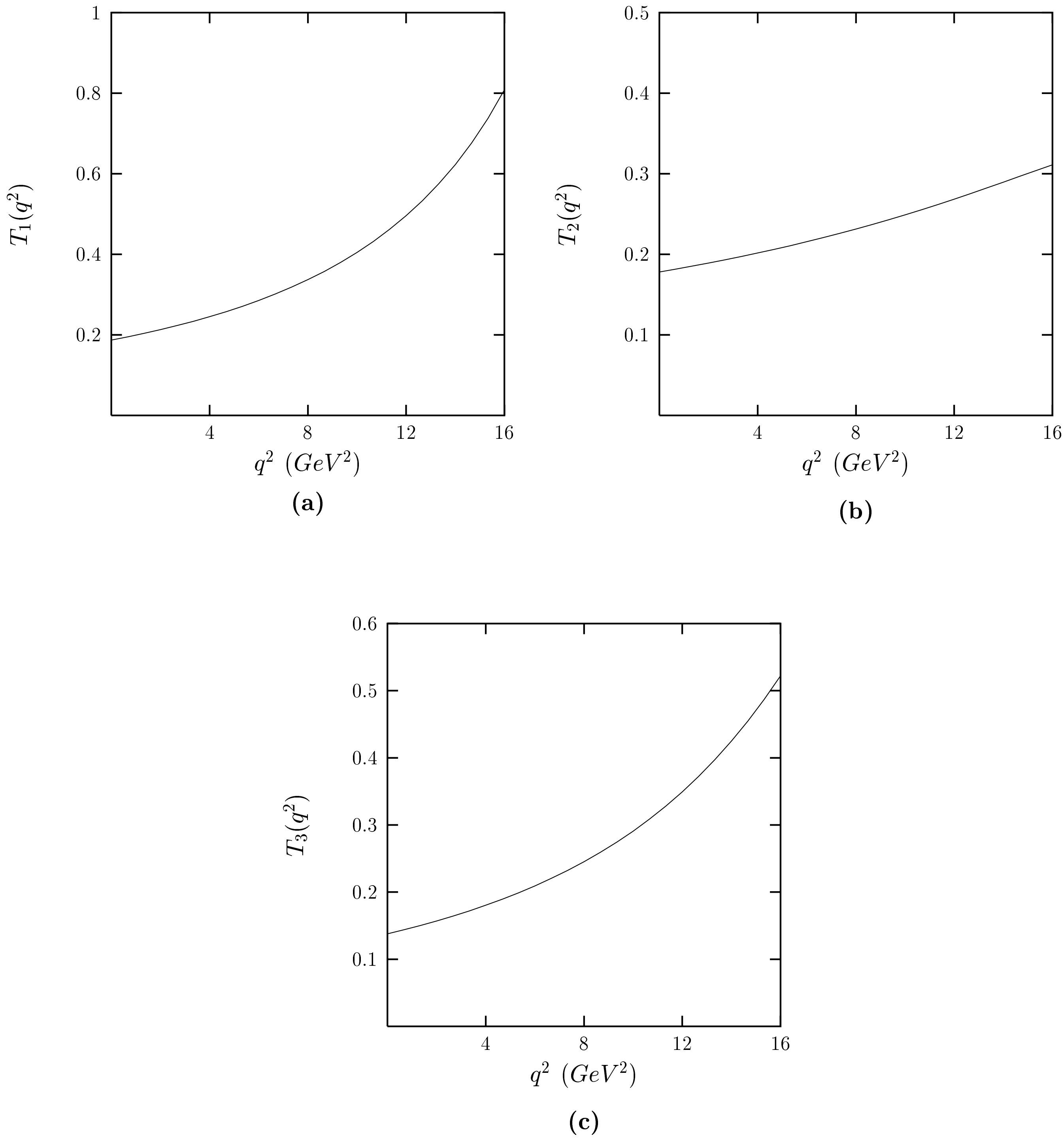
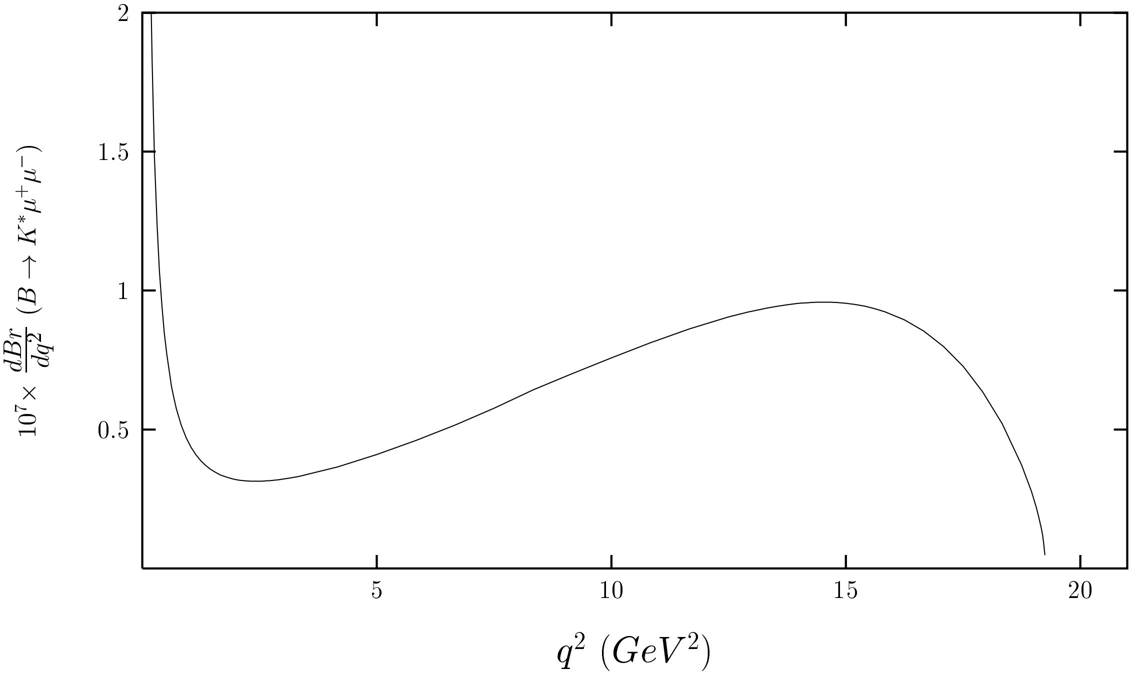

2.5
2
1.5
1
0.5
$V(q^2)$
4
8
12
16
$q^2\ (GeV^2)$
(a)
1
0.8
0.6
0.4
0.2
$A_1(q^2)$
4
8
12
16
$q^2\ (GeV^2)$
(b)
2
1.6
1.2
0.8
0.4
$A_2(q^2)$
4
8
12
16
$q^2\ (GeV^2)$
(c)
1
0.8
0.6
0.4
0.2
$A_0(q^2)$
4
8
12
16
$q^2\ (GeV^2)$
(d)

Figure 1



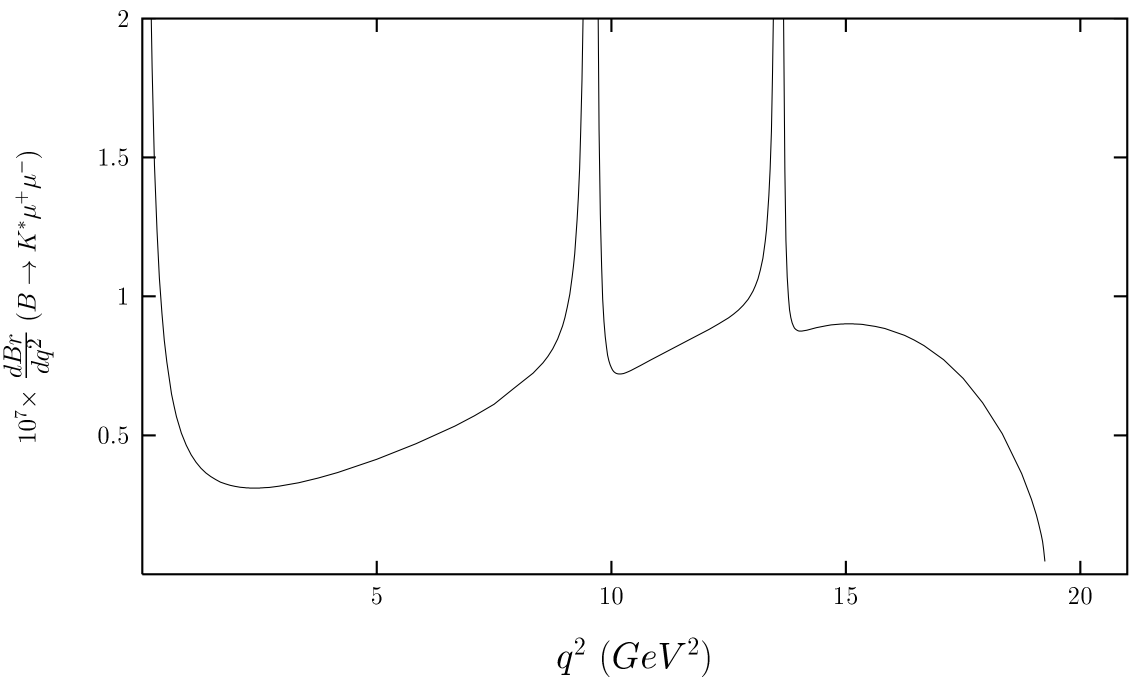
1
0.8
0.6
0.4
0.2
$T_1(q^2)$
4
8
12
16
q^2 (GeV^2)
(a)
0.5
0.4
0.3
0.2
0.1
$T_2(q^2)$
4
8
12
16
q^2 (GeV^2)
(b)
0.6
0.5
0.4
0.3
0.2
0.1
$T_3(q^2)$
4
8
12
16
q^2 (GeV^2)
(c)

Figure 2



$10^7 \times \frac{dBr}{dq^2}\ (B \to K^* \mu^+ \mu^-)$
2
1.5
1
0.5
5
10
15
20
$q^2\ (GeV^2)$

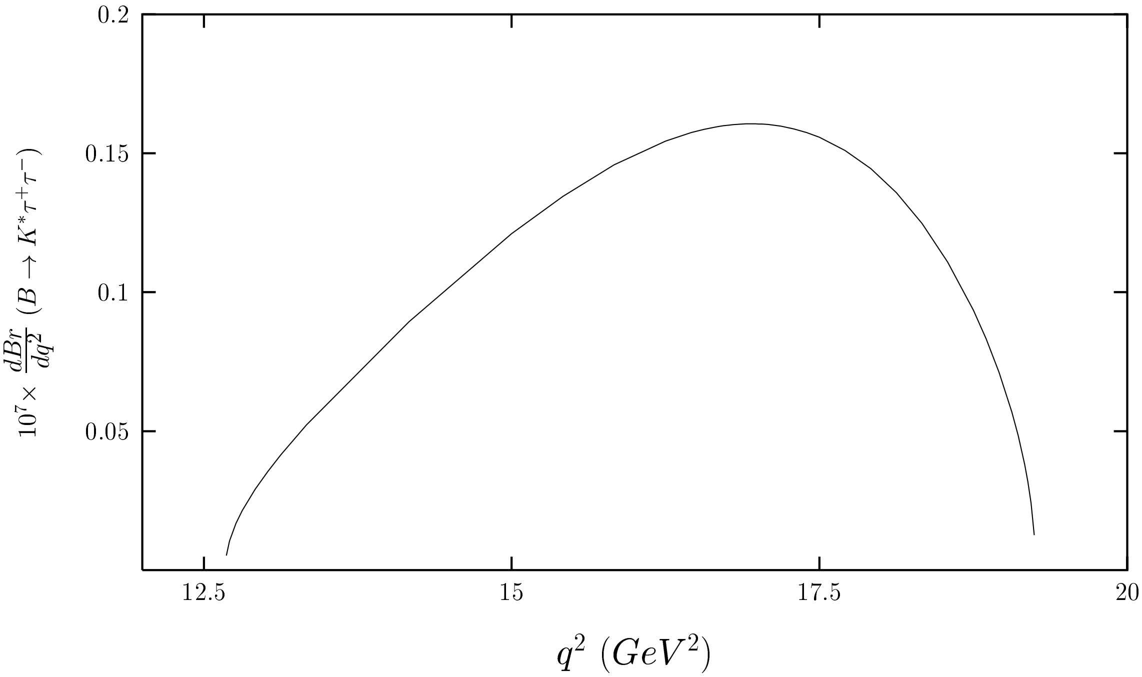
(a)



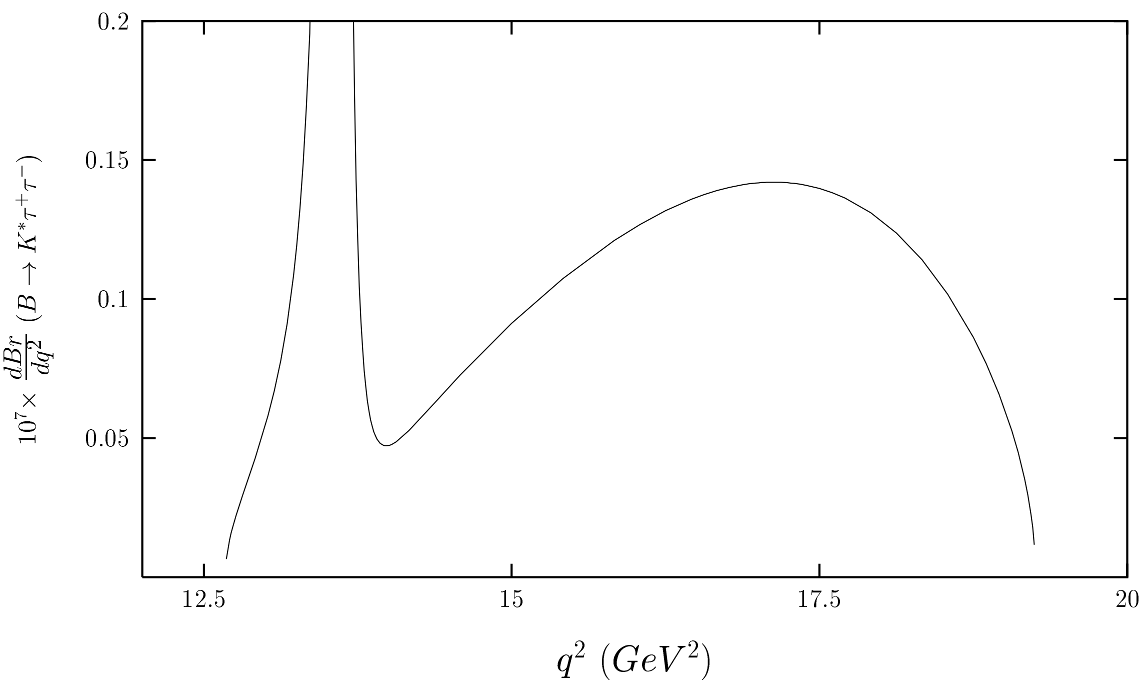
$10^7 \times \frac{dBr}{dq^2}\ (B \to K^* \mu^+ \mu^-)$
2
1.5
1
0.5
5
10
15
20
$q^2\ (GeV^2)$

(b)

Figure 3

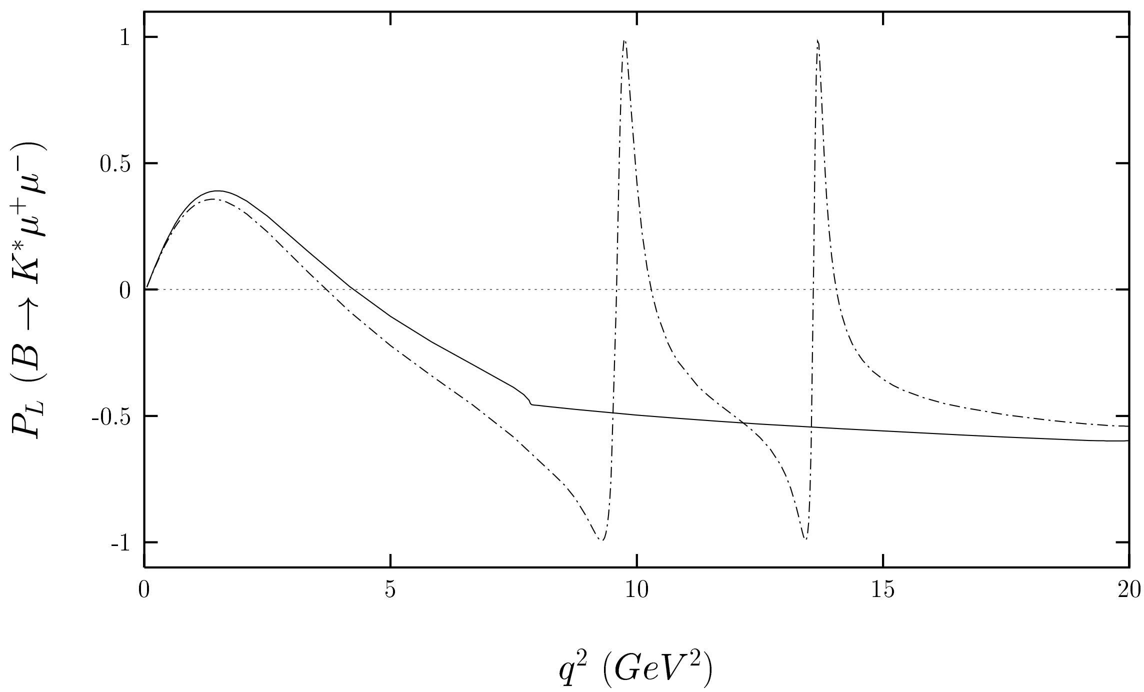

0.2
0.15
0.1
0.05
$10^7 \times \frac{dBr}{dq^2}$ $(B \to K^* \tau^+ \tau^-)$
12.5
15
17.5
20
q^2 (GeV^2)

(a)


0.2
0.15
0.1
0.05
$10^7 \times \frac{dBr}{dq^2}$ $(B \to K^* \tau^+ \tau^-)$
12.5
15
17.5
20
q^2 (GeV^2)

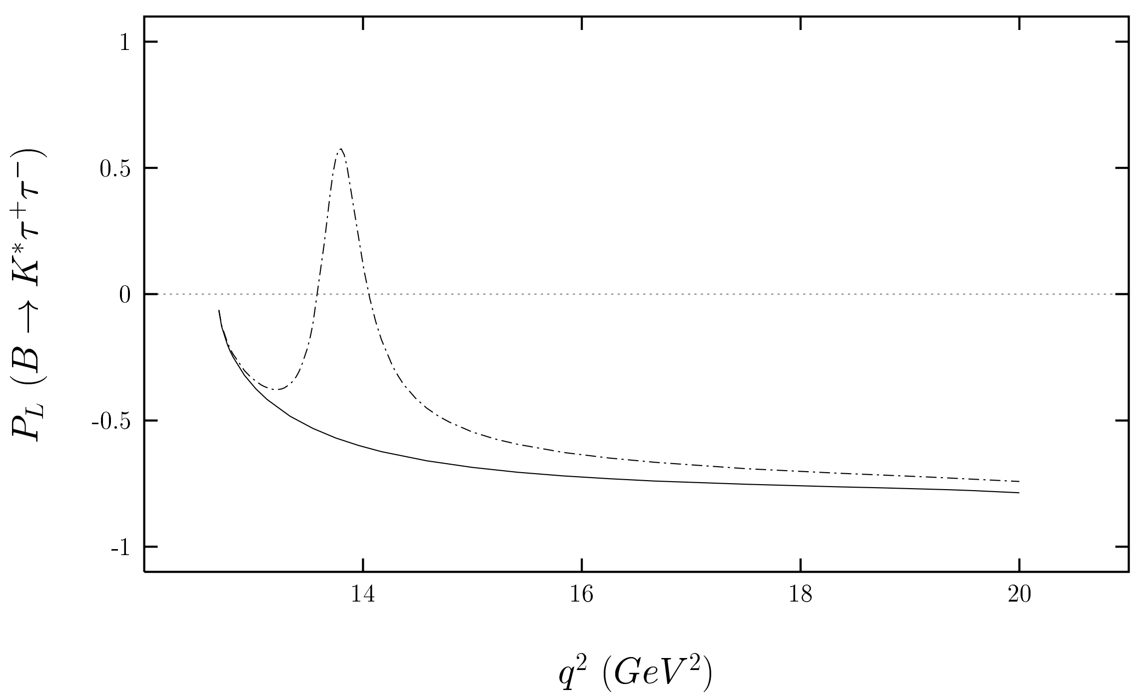
(b)

Figure 4



1
0.5
0
-0.5
-1
0
5
10
15
20
$P_L\ (B \to K^* \mu^+ \mu^-)$
$q^2\ (GeV^2)$

(a)



1
0.5
0
-0.5
-1
14
16
18
20
$P_L\ (B \to K^* \tau^+ \tau^-)$
$q^2\ (GeV^2)$

(b)

Figure 5

Figure Captions

1. The q^2 dependence of the formfactors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$ and $A_0(q^2)$.

2. The q^2 dependence of the formfactors $T_1(q^2)$, $T_2(q^2)$ and $T_3(q^2)$.

3. a) Invariant mass squared distribution of the lepton pair for the decay $B \to K^* \mu^+ \mu^-$ which includes only the short distance contributions.
b) The same as in a) but including long distance effects, too.

4. The same as in Fig.3, but for $B \to K^* \tau^+ \tau^-$ decay.

5. a) The longitudinal polarization asymmetry P_L for the $B \to K^* \mu^+ \mu^-$ decay.
b) The same as in a), but for $B \to K^* \tau^+ \tau^-$ decay.
In these figures, solid line corresponds to the short distance contributions only and dashed to the sum of both short and long distance contributions.

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