Magnetic moments of $J^P = \frac{3}{2}^-$ baryons in QCD

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Abstract

The magnetic moments of the low lying, negative parity, spin-3/2 baryons are calculated within the light cone QCD sum rules method. The contributions coming from the positive parity, spin-3/2 baryons, as well as from the positive and negative parity spin-1/2 baryons are eliminated by constructing combinations of various invariant amplitudes corresponding to the coefficients of the different Lorentz structures.

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1 Introduction

The study of the structure and properties of hadrons represents one of the main directions in particle and nuclear physics. There are different approaches in investigation of the structure of hadrons, especially, the most promising one in this direction is the study of the radiative decays and electromagnetic properties of the hadrons and their excitations (for a review, see for example [1]). Negative parity partners of the octet baryons arise from increasing of orbital angular momentum by one unit.

The investigation of the magnetic moment of the negative parity baryons can provide us useful information about their internal structure. Note that the study of the magnetic moment of the N^* baryon has already been planned at Mainz Microtron (MAMI) facility [2–4] and Jefferson Laboratory (JLAB) [5]. The main difficulty in measurement of the magnetic moments of the excited baryons is can be attributed to the considerably large width they posses. Due to this fact the magnetic moments can be measured from the polarization observables of the decay products of the excited resonances. The very first effort in measuring the magnetic moment of the $\Delta^+(1232)$ in the reaction $\gamma p \to p\pi^0\gamma$ has been realized at MAMI, and future program to get information about the magnetic moment of the $N^*(1535)$ baryon in the reaction $\gamma p \to p\eta\gamma$ has already been planned. The magnetic moments of the spin-1/2 negative parity baryons have been calculated within the nonrelativistic quark model [6], simple quark model [7], lattice QCD [8], chiral perturbation theory [9], light cone QCD sum rules (LCSR) [10], and effective Hamiltonian model [11].

In the present work we calculate the magnetic moments of the negative parity, spin-3/2 partners of the octet baryons in framework of the LCSR method (more about LCSR formalism can be found in [12]). It should be reminded here that, theoretically there exists only one work where magnetic moment of the negative parity, spin-3/2 Λ-baryon is calculated in framework of the chiral quark model [13].

The paper is organized as follows. In section 2 we introduce the interpolating currents for the spin-3/2 partners of the octet baryons and derive sum rules for the magnetic moments of negative parity baryons. In the same section, we also calculate the two-point correlators in order to obtain the sum rules for mass and residues of the negative parity, spin-3/2 Section 3 is devoted to numerical analysis of the obtained sum rules for the magnetic moments of these baryons, and discussion and conclusion.

2 Calculation of the magnetic moments of negative parity, spin-3/2 baryons from LCSR

The starting point in calculation of the magnetic moment of the negative parity, spin-3/2 baryons from the QCD side is the consideration of the following correlation function:

$$\Pi_{\mu\nu} = i \int d^4x e^{ipx} \left\langle 0 \left| \mathrm{T}\left\{ \eta_\mu^B(x) \bar{\eta}_\nu^B(0) \right\} \right| 0 \right\rangle_F \ , \tag{1}$$

where η_μ^B is the interpolating current for the corresponding baryon, F refers to the external electromagnetic field, and T is the time ordering operator.

In order to obtain the sum rules for the magnetic moments, the correlation function is calculated in terms of hadrons and the quark-gluon degrees of freedom. Using then the quark-hadron duality ansatz these two forms of the correlation function are related by using the analytical continuation.

The phenomenological part of the correlation function can be obtained by saturating it with the complete states of single particle hadronic states carrying the same quantum numbers as the interpolating currents. We get from Eq.(1):

$$\begin{aligned}
\Pi_{\mu\nu} &= \frac{\langle 0|\eta_\mu|\frac{3}{2}^+(p_2)\rangle\langle\frac{3}{2}^+(p_2)\gamma(q)|\frac{3}{2}^+(p_1)\rangle\langle\frac{3}{2}^+(p_1)|\bar{\eta}_\nu|0\rangle}{(m^2_{\frac{3}{2}^+}-p_1^2)(m^2_{\frac{3}{2}^+}-p_2^2)} \\
&+ \frac{\langle 0|\eta_\mu|\frac{3}{2}^-(p_2)\rangle\langle\frac{3}{2}^-(p_2)\gamma(q)|\frac{3}{2}^+(p_1)\rangle\langle\frac{3}{2}^+(p_1)|\bar{\eta}_\nu|0\rangle}{(m^2_{\frac{3}{2}^+}-p_1^2)(m^2_{\frac{3}{2}^-}-p_2^2)} \\
&+ \frac{\langle 0|\eta_\mu|\frac{3}{2}^+(p_2)\rangle\langle\frac{3}{2}^+(p_2)\gamma(q)|\frac{3}{2}^-(p_1)\rangle\langle\frac{3}{2}^-(p_1)|\bar{\eta}_\nu|0\rangle}{(m^2_{\frac{3}{2}^-}-p_1^2)(m^2_{\frac{3}{2}^+}-p_2^2)} \\
&+ \frac{\langle 0|\eta_\mu|\frac{3}{2}^-(p_2)\rangle\langle\frac{3}{2}^-(p_2)\gamma(q)|\frac{3}{2}^-(p_1)\rangle\langle\frac{3}{2}^-(p_1)|\bar{\eta}_\nu|0\rangle}{(m^2_{\frac{3}{2}^-}-p_1^2)(m^2_{\frac{3}{2}^-}-p_2^2)}\ . \qquad (2)
\end{aligned}$$

In general the current η_μ interacts not only with spin-3/2 states, but also with spin-1/2 states. Since our aim is to calculate the magnetic moments of the negative parity, spin-3/2 baryons, the contributions coming spin-1/2 baryons must be eliminated. The prescription to eliminate these unwanted contributions can be summarized as follows.

The general form of the matrix element of η_μ between spin-1/2 and vacuum states can be written as,

$$\langle 0\,|\eta_\mu|\,\tfrac{1}{2}(p)\rangle = (Ap_\mu + B\gamma_\mu)u(p)\ . \qquad (3)$$

Multiplying both sides with γ^μ, and using condition $\eta_\mu\gamma^\mu = 0$ we get,

$$\langle 0\,|\eta_\mu|\,\tfrac{1}{2}^+(p)\rangle = B\bigg(-\frac{4}{m_{\frac{1}{2}^+}}p_\mu + \gamma_\mu\bigg)u(p)\ , \qquad (4)$$

and similarly,

$$\langle 0\,|\eta_\mu|\,\tfrac{1}{2}^-(p)\rangle = B\gamma_5\bigg(-\frac{4}{m_{\frac{1}{2}^-}}p_\mu + \gamma_\mu\bigg)u(p)\ . \qquad (5)$$

We see from Eqs. (4) and (5) that the unwanted spin-1/2 state contributions are proportional to either p_μ or γ_μ, and they must be eliminated. For this goal an ordering procedure of the Dirac matrices is needed which is necessary in obtaining the independent structures. In the present study the ordering of the Dirac matrices chosen is $\gamma_\mu \not\varepsilon \not q \not p \gamma_\nu$. In the light of these remarks we can now proceed to calculate $\Pi_{\mu\nu}$ given in Eq. (2) in terms of hadrons, for which the following matrix elements are needed,

$$\begin{aligned}
\langle 0\,|\eta_\mu|\,\tfrac{3}{2}^+(p)\rangle &= \lambda_{\frac{3}{2}^+}u_\mu(p)\ , \\
\langle 0\,|\eta_\mu|\,\tfrac{3}{2}^-(p)\rangle &= \lambda_{\frac{3}{2}^-}\gamma_5 u_\mu(p)\ , \\
\langle \tfrac{3}{2}^+(p)\,|\bar{\eta}_\nu|\,0\rangle &= \lambda_{\frac{3}{2}^+}\bar{u}_\nu(p)\ , \\
\langle \tfrac{3}{2}^-(p)\,|\bar{\eta}_\nu|\,0\rangle &= -\lambda_{\frac{3}{2}^-}\bar{u}_\nu(p)\gamma_5\ , \qquad (6)
\end{aligned}$$

where $u_\mu(p)$ is the Rarita-Schwinger spinor for the spin-3/2 particles. The matrix element $\langle B_{\frac{3}{2}^+}(p_2)|B_{\frac{3}{2}^+}(p_1)\rangle_\gamma$ is determined in the following way:

$$\begin{aligned}\langle B_{\frac{3}{2}^+}(p_2)|B_{\frac{3}{2}^+}(p_1)\rangle_\gamma &= \varepsilon_\rho \bar{u}_\alpha(p_2)\mathcal{O}^{\alpha\rho\beta}u_\beta(p_1) \equiv \\ & \varepsilon_\rho \bar{u}_\alpha(p_2)\Bigg\{ - g^{\alpha\beta}\left[\gamma^\rho(f_1^+ + f_2^+) + \frac{(p_1+p_2)^\rho}{2m_{\frac{3}{2}^+}} f_2^+ + q^\rho f_3^+\right] \\ & - \frac{q^\alpha q^\beta}{2m^2_{\frac{3}{2}^+}}\left[\gamma^\rho(g_1^+ + g_2^+) + \frac{(p_1+p_2)^\rho}{2m_{\frac{3}{2}^+}} g_2^+ + q^\rho g_3^+\right]\Bigg\} u_\beta(p_1)~, \end{aligned} \qquad (7)$$

where f_i and g_i are the form factors whose values are needed only at the point $q^2 = 0$. The matrix elements $\langle B_{\frac{3}{2}^-}(p_2)|B_{\frac{3}{2}^-}(p_1)\rangle_\gamma$ and $\langle B_{\frac{3}{2}^+}(p_2)|B_{\frac{3}{2}^-}(p_1)\rangle_\gamma$ can be obtained from Eq. (7) by making the replacements of the form factors as,

$$f_i \to f_i^*, \quad g_i \to g_i^*, \text{ and } f_i \to f_i^{tr}, \quad g_i \to g_i^{tr},$$

respectively. For the $3^+/2 \to 3^-/2$ transition the matrix element such as given in Eq. (7) should contain γ_5 matrix before the Rarita-Schwinger spinor $u_\beta(p)$, which follows from the parity consideration. Note that for the real photons the terms multiplying f_3 and g_3 can be neglected since $\varepsilon_\rho q^\rho = 0$. Summation over spin-3/2 states is performed according to the following formula:

$$\sum_s u_\mu(p,s)\bar{u}_\alpha(p,s) = -(\not{p} + m)\left[g_{\mu\alpha} - \frac{1}{3}\gamma_\mu\gamma_\alpha - \frac{2p_\mu p_\alpha}{3m^2} + \frac{p_\mu\gamma_\alpha - p_\alpha\gamma_\mu}{3m}\right]~. \qquad (8)$$

It should be remembered again that the terms proportional to γ_μ on the left and those to γ_α on the right; as well as to $p_{1\alpha}$ and $p_{2\mu}$ contain contributions from spin-1/2 states (see Eqs. (4) and (5)), and therefore can be neglected. After elimination of the spin-1/2 states, the only structure that contains contributions from spin-3/2 states is $g_{\mu\alpha}$. As a result, the hadronic part of the correlation function containing only spin-3/2 contributions can be written as,

$$\begin{aligned}\Pi_{\mu\nu} =& \frac{\lambda^2_{\frac{3}{2}^+}}{(m^2_{\frac{3}{2}^+} - p_1^2)(m^2_{\frac{3}{2}^+} - p_2^2)}(\not{p}_2 + m_{\frac{3}{2}^+})g_{\mu\alpha}\varepsilon_\rho\Bigg\{ - g^{\alpha\beta}\left[\gamma^\rho(f_1^+ + f_2^+) + \frac{(p_1+p_2)^\rho}{2m_{\frac{3}{2}^+}} f_2^+\right] \\ & - \frac{q^\alpha q^\beta}{2m^2_{\frac{3}{2}^+}}\left[\gamma^\rho(g_1^+ + g_2^+) + \frac{(p_1+p_2)^\rho}{2m_{\frac{3}{2}^+}} g_2^+\right]\Bigg\}(\not{p}_1 + m_{\frac{3}{2}^+})g_{\nu\beta} \\ & - \frac{\lambda_{\frac{3}{2}^+}\lambda_{\frac{3}{2}^-}}{(m^2_{\frac{3}{2}^-} - p_1^2)(m^2_{\frac{3}{2}^+} - p_2^2)}(\not{p}_2 + m_{\frac{3}{2}^+})g_{\mu\alpha}\varepsilon_\rho\Bigg\{ - g^{\alpha\beta}\left[\gamma^\rho(f_1^{tr} + f_2^{tr}) + \frac{(p_1+p_2)^\rho}{m_{\frac{3}{2}^+} + m_{\frac{3}{2}^-}} f_2^{tr}\right] \\ & - \frac{q^\alpha q^\beta}{2m^2_{\frac{3}{2}^+}}\left[\gamma^\rho(g_1^{tr} + g_2^{tr}) + \frac{(p_1+p_2)^\rho}{m_{\frac{3}{2}^+} + m_{\frac{3}{2}^-}} g_2^{tr}\right]\Bigg\}\gamma_5(\not{p}_1 + m_{\frac{3}{2}^-})\gamma_5 g_{\nu\beta} \\ & + \frac{\lambda_{\frac{3}{2}^-}\lambda_{\frac{3}{2}^+}}{(m^2_{\frac{3}{2}^+} - p_1^2)(m^2_{\frac{3}{2}^-} - p_2^2)}\gamma_5(\not{p}_2 + m_{\frac{3}{2}^-})g_{\mu\alpha}\varepsilon_\rho\Bigg\{ - g^{\alpha\beta}\left[\gamma^\rho(f_1^{tr^*} + f_2^{tr^*}) + \frac{(p_1+p_2)^\rho}{m_{\frac{3}{2}^+} + m_{\frac{3}{2}^-}} f_2^{tr^*}\right] \\ & - \frac{q^\alpha q^\beta}{m^2_{\frac{3}{2}^+}}\left[\gamma^\rho(g_1^{tr^*} + g_2^{tr^*}) + \frac{(p_1+p_2)^\rho}{m_{\frac{3}{2}^+} + m_{\frac{3}{2}^-}} g_2^{tr^*}\right]\Bigg\}\gamma_5(\not{p}_1 + m_{\frac{3}{2}^+})g_{\nu\beta}\end{aligned}$$

$$
- \frac{\lambda^2_{\frac{3}{2}^-}}{(m^2_{\frac{3}{2}^-} - p_1^2)(m^2_{\frac{3}{2}^-} - p_2^2)} \gamma_5 (\not{p}_2 + m_{\frac{3}{2}^-}) g_{\mu\alpha} \varepsilon_\rho \Bigg\{ - g^{\alpha\beta} \left[\gamma^\rho (f_1^- + f_2^-) + \frac{(p_1+p_2)^\rho}{2m_{\frac{3}{2}^-}} f_2^- \right]
- \frac{q^\alpha q^\beta}{2m^2_{\frac{3}{2}^-}} \left[\gamma^\rho (g_1^- + g_2^-) + \frac{(p_1+p_2)^\rho}{2m_{\frac{3}{2}^-}} g_2^+ \right] \Bigg\} (\not{p}_1 + m_{\frac{3}{2}^-}) \gamma_5 g_{\nu\beta} \ . \tag{9}
$$

It is shown in [14] that, in the nonrelativistic limit, in the presence of external uniform magnetic field the maximum energy of the baryon is equal to $3(f_1 + f_2)B$, where B is the magnitude of the field. In other words, $3(f_1 + f_2)$ is equal to the magnetic moment at $q^2 = 0$. Therefore, among many structures we chose the one which multiply the coefficient $(f_1 + f_2)$. Using this fact, the hadronic part of the correlation function given in Eq.(9) takes the following form:

$$
\begin{aligned}
\Pi_{\mu\nu} &= \frac{\lambda^2_{\frac{3}{2}^+}}{(m^2_{\frac{3}{2}^+} - p_1^2)(m^2_{\frac{3}{2}^+} - p_2^2)} (\not{p}_2 + m_{\frac{3}{2}^+}) \Big[- g_{\mu\nu} \not{\varepsilon} (f_1^+ + f_2^+) \Big] (\not{p}_1 + m_{\frac{3}{2}^+}) \\
&+ \frac{\lambda_{\frac{3}{2}^+} \lambda_{\frac{3}{2}^-}}{(m^2_{\frac{3}{2}^-} - p_1^2)(m^2_{\frac{3}{2}^+} - p_2^2)} (\not{p}_2 + m_{\frac{3}{2}^+}) \Big[- g_{\mu\nu} \not{\varepsilon} (f_1^{tr} + f_2^{tr}) \Big] (\not{p}_1 - m_{\frac{3}{2}^-}) \\
&+ \frac{\lambda_{\frac{3}{2}^-} \lambda_{\frac{3}{2}^+}}{(m^2_{\frac{3}{2}^+} - p_1^2)(m^2_{\frac{3}{2}^-} - p_2^2)} (\not{p}_2 - m_{\frac{3}{2}^-}) \Big[- g_{\mu\nu} \not{\varepsilon} (f_1^{tr^*} + f_2^{tr^*}) \Big] (\not{p}_1 + m_{\frac{3}{2}^+}) \\
&+ \frac{\lambda^2_{\frac{3}{2}^-}}{(m^2_{\frac{3}{2}^-} - p_1^2)(m^2_{\frac{3}{2}^-} - p_2^2)} (\not{p}_2 - m_{\frac{3}{2}^-}) \Big[- g_{\mu\nu} \not{\varepsilon} (f_1^- + f_2^-) \Big] (\not{p}_1 - m_{\frac{3}{2}^-}) \ .
\end{aligned} \tag{10}
$$

It follows from Eq.(10) that only the last term describes the magnetic moment of the negative parity baryons. Denoting by,

$$
\begin{aligned}
A &= \frac{\lambda^2_{\frac{3}{2}^+}(f_1^+ + f_2^+)}{(m^2_{\frac{3}{2}^+} - p_1^2)(m^2_{\frac{3}{2}^+} - p_2^2)} \\
B &= \frac{\lambda_{\frac{3}{2}^+}\lambda_{\frac{3}{2}^-}(f_1^{tr} + f_2^{tr})}{(m^2_{\frac{3}{2}^-} - p_1^2)(m^2_{\frac{3}{2}^+} - p_2^2)} \\
C &= \frac{\lambda_{\frac{3}{2}^-}\lambda_{\frac{3}{2}^+}(f_1^{tr^*} + f_2^{tr^*})}{(m^2_{\frac{3}{2}^+} - p_1^2)(m^2_{\frac{3}{2}^-} - p_2^2)} \\
D &= \frac{\lambda^2_{\frac{3}{2}^-}(f_1^- + f_2^-)}{(m^2_{\frac{3}{2}^-} - p_1^2)(m^2_{\frac{3}{2}^-} - p_2^2)} \ .
\end{aligned} \tag{11}
$$

On the hadronic side, for the invariant function with the structure $g_{\mu\nu}$, we have

$$
\Big[A(\not{p}_2 + m_{\frac{3}{2}^+})(-\not{\varepsilon})(\not{p}_1 + m_{\frac{3}{2}^+}) + B(\not{p}_2 + m_{\frac{3}{2}^+})(\not{\varepsilon})(\not{p}_1 - m_{\frac{3}{2}^-})
+ C(\not{p}_2 - m_{\frac{3}{2}^-})(\not{\varepsilon})(\not{p}_1 + m_{\frac{3}{2}^+}) + D(\not{p}_2 - m_{\frac{3}{2}^-})(\not{\varepsilon})(\not{p}_1 - m_{\frac{3}{2}^-}) \Big] \ , \tag{12}
$$

where $p_2 = p$ and $p_1 = p + q$. Note that in determination of the magnetic moments we need the values of the form factors only at $q^2 = 0$.

Now let us turn our attention to the calculation of the correlator function from the QCD side. For this goal, as has already been noted, we need the form of the interpolating currents of the excited state baryons. The interpolating currents for the spin-3/2, positive parity baryons are given as [15],

$$
\begin{aligned}
\eta_\mu^{p*} &= \varepsilon^{abc}\Big[\Big(u^{aT}C\sigma_{\rho\lambda}d^b\Big)\sigma^{\rho\lambda}\gamma_\mu u^c - \Big(u^{aT}C\sigma_{\rho\lambda}u^b\Big)\sigma^{\rho\lambda}\gamma_\mu d^c\Big] ~, \\
\eta_\mu^{n*} &= \eta_\mu^{p*}(u \leftrightarrow d) ~, \\
\eta_\mu^{\Sigma^{*+}} &= \varepsilon^{abc}\Big[\Big(u^{aT}C\sigma_{\rho\lambda}s^b\Big)\sigma^{\rho\lambda}\gamma_\mu u^c - \Big(u^{aT}C\sigma_{\rho\lambda}u^b\Big)\sigma^{\rho\lambda}\gamma_\mu s^c\Big] ~, \\
\eta_\mu^{\Sigma^{*-}} &= \Sigma^{*+}(u \leftrightarrow d) ~, \\
\eta_\mu^{\Sigma^{*0}} &= \frac{\varepsilon^{abc}}{\sqrt{2}}\Big[\Big(u^{aT}C\sigma_{\rho\lambda}s^b\Big)\sigma^{\rho\lambda}\gamma_\mu d^c - \Big(u^{aT}C\sigma_{\rho\lambda}d^b\Big)\sigma^{\rho\lambda}\gamma_\mu s^c \\
&+ \Big(d^{aT}C\sigma_{\rho\lambda}s^b\Big)\sigma^{\rho\lambda}\gamma_\mu u^c - \Big(d^{aT}C\sigma_{\rho\lambda}u^b\Big)\sigma^{\rho\lambda}\gamma_\mu s^c\Big] \\
\eta_\mu^{\Xi^{*0}} &= \varepsilon^{abc}\Big[\Big(s^{aT}C\sigma_{\rho\lambda}u^b\Big)\sigma^{\rho\lambda}\gamma_\mu s^c - \Big(s^{aT}C\sigma_{\rho\lambda}s^b\Big)\sigma^{\rho\lambda}\gamma_\mu u^c\Big] ~, \\
\eta_\mu^{\Xi^{*-}} &= \eta_\mu^{\Xi^{*0}}(u \leftrightarrow d) ~. \qquad (13)
\end{aligned}
$$

As an example we present the result for the correlation function for the Σ^{*+} baryon from the QCD side, which can be written as:

$$
\Pi_{\mu\nu}^{\Sigma^{*+}} = \int d^4x e^{ipx} \left\langle \gamma(q) \left| \eta_\mu^{\Sigma^{*+}}(x) \bar{\eta}_\nu^{\Sigma^{*+}}(0) \right| 0 \right\rangle ~, \qquad (14)
$$

where a, b, c, a', b', c' are the color indices, S_q is the light quark propagator. In further numerical calculations involving excited baryons containing strange quark, we take into account only linear terms in strange quark mass m_s. The results for the Σ^{*-}, Σ^{*0}, Ξ^{*0} and Ξ^{*-} excited baryons can be obtained from Eq. (14) with the help of the following replacements,

$$
\begin{aligned}
\Pi_{\mu\nu}^{\Sigma^{*-}} &= \Pi_{\mu\nu}^{\Sigma^{*+}}(u \leftrightarrow d) ~, \\
\Pi_{\mu\nu}^{\Sigma^{*0}} &= \frac{1}{2}\Big(\Pi_{\mu\nu}^{\Sigma^{*+}} + \Pi_{\mu\nu}^{\Sigma^{*-}}\Big) ~, \\
\Pi_{\mu\nu}^{\Xi^{*0}} &= \Pi_{\mu\nu}^{\Sigma^{*+}}(u \leftrightarrow s) ~, \\
\Pi_{\mu\nu}^{\Xi^{*-}} &= \Pi_{\mu\nu}^{\Xi^{*0}}(u \leftrightarrow d) ~. \qquad (15)
\end{aligned}
$$

In order to calculate the correlation function from the QCD side, we need to know explicit expression of the light quark propagators, which has the following form:

$$
\begin{aligned}
S_q(x) &= \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12}\Big(1 - \frac{i m_q}{4}\not{x}\Big) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \Big(1 - \frac{i m_q}{6}\not{x}\Big) \\
&- i g_s \int_0^1 dv \Big[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(vx)\sigma^{\mu\nu} - v x^\mu G_{\mu\nu}(vx)\gamma^\nu \frac{i}{4\pi^2 x^2} \\
&- \frac{i m_q}{32\pi^2} G_{\mu\nu}(vx)\sigma^{\mu\nu}\Big(\ln\frac{-x^2\Lambda^2}{4} + 2\gamma_E\Big)\Big] ~, \qquad (16)
\end{aligned}
$$

where Λ is the energy scale in order to separate the perturbative and nonperturbative sectors. We should note that in numerical calculations two-gluon and four-quark operators are neglected due to their small contributions.

In order to obtain perturbative contributions it is enough to replace one of the free quark operators (the first two terms in Eq. (16)) by,

$$S = -\frac{1}{2}\int dy y_\nu \mathcal{F}^{\mu\nu} S^{free}(x-y)\gamma_\mu S^{free}(y) , \tag{17}$$

where $\mathcal{A}_\mu = \frac{1}{2}\mathcal{F}^{\mu\nu} y_\nu$ satisfying $x_\mu \mathcal{A}^\mu = 0$ in the Schwinger gauge, and $\mathcal{F}^{\mu\nu}$ is the electromagnetic field strength tensor, S^{free} is the free quark propagator. The remaining two propagators are taken as are given in Eq. (16). The nonperturbative contribution describing the case when a photon interacts with a quark field nonperturbatively can again be obtained by replacing one of the propagators by,

$$S^{ab}_{\rho\sigma} = -\frac{1}{4}\Big[\bar{q}^a \Gamma_j q^b \Gamma_j\Big]_{\rho\sigma} ,$$

where Γ_j are the full set of Dirac matrices, and the remaining two propagators are taken from Eq. (16).

These contributions, obviously, are described by the matrix elements of the nonlocal operators $\bar{q}\Gamma_j q$ and $\bar{q}G_{\mu\nu}\Gamma_j q$ between the vacuum and photon states. These matrix elements are described in terms of the photon distribution amplitudes (DAs), whose explicit expressions are given in [16].

Using Eq. (14) and the explicit expression for the light quark operator given in Eq. (16), and the definitions of the nonlocal quark operators between the photon and vacuum states in terms of the photon DAs, we can perform the calculation for the theoretical part of the correlation function. It follows from Eq. (12) that, in order to determine the coefficient D, which contain the magnetic moment of the negative parity, spin-3/2 baryons, we need four equations. For this aim we choose the structures $g_{\mu\nu}(\varepsilon p)$, $(\varepsilon p)\not{p} g_{\mu\nu}$, $\not{\varepsilon}\not{p} g_{\mu\nu}$ and $\not{\varepsilon} g_{\mu\nu}$. Denoting the invariant functions of these structures by Π_1, Π_2, Π_3 and Π_4, respectively, we get the following four equations,

$$\begin{aligned}
-2m_+(A+B) + 2m_-(C+D) &= \Pi_1 \\
-2(A+B+C+D) &= \Pi_2 \\
(B-C)(m_+ + m_-) &= \Pi_3 \\
(Cm_- + Bm_+)(m_+ + m_-) &= \Pi_4 \ .
\end{aligned} \tag{18}$$

Solving these equations for D, we get

$$D = \frac{1}{2(m_+ + m_-)^2}\Big[(m_+ + m_-)\Pi_1 - m_+\Pi_2 + 2m_+\Pi_3 - 2\Pi_4\Big] . \tag{19}$$

The final step for obtaining the sum rules for the magnetic moments is performing double Borel transformation with respect to the variables $-(p+q)^2$ and $-p^2$, and subtracting higher states and continuum contributions. Performing all these operations in the appropriate order, for the magnetic moment of negative parity baryons we get,

$$\frac{\mu}{3} = \frac{e^{m_-^2/M^2}}{\lambda_-^2}\frac{1}{2(m_+ + m_-)^2}\Big[(m_+ + m_-)\Pi_1^B - m_+\Pi_2^B + 2m_+\Pi_3^B - 2\Pi_4^B\Big] , \tag{20}$$

where we take $M_1^2 = 2M^2$, $M_2^2 = 2M^2$, with M^2 is being the Borel mass parameter; and Π_i^B are the expressions of the corresponding invariant functions after performing the Borel transformation. The expressions of Π_i^B quite lengthy, and for this reason we do not present their explicit expressions.

Having obtained the final result for determination of the magnetic moment of the negative parity, spin-3/2 baryons, which is given in Eq. (19), we now need to calculate their overlap amplitudes (residues). These residues are determined from from the two-point correlation function given as,

$$\Pi_{\mu\nu}(p^2) = i \int d^4x e^{ipx} \left\langle 0 \left| \mathrm{T}\left\{ \eta_\mu^B(x) \bar{\eta}_\nu^B(0) \right\} \right| 0 \right\rangle_. \tag{21}$$

Saturating this correlation function with positive and negative baryons, and choosing the structures $g_{\mu\nu}\not{p}$ and $g_{\mu\nu}$ that contain only spin-3/2 baryons, we get,

$$\begin{aligned} \frac{\lambda_+^2}{m_+^2 - p^2} + \frac{\lambda_-^2}{m_-^2 - p^2} &= T_1 \\ \frac{\lambda_+^2 m_+}{m_+^2 - p^2} + \frac{\lambda_-^2 m_-}{m_-^2 - p^2} &= T_2 \ , \end{aligned} \tag{22}$$

where T_1 and T_2 are the invariant functions on the theoretical side, multiplying the structures $g_{\mu\nu}\not{p}$ and $g_{\mu\nu}$, respectively. Performing Borel transformation over the variable $-p^2$, and continuum subtraction procedure, for the mass and residues of the negative parity baryons we get,

$$\begin{aligned} m_-^2 &= \frac{\dfrac{d}{d(1/M^2)} \left[m_+ T_1^B - T_2^B \right]}{m_+ T_1^B - T_2^B} \ , \\ \lambda_-^2 &= \frac{e^{m_-^2/M^2}}{m_+ + m_-} \left[m_+ T_1^B - T_2^B \right] \ . \end{aligned}$$

The explicit expressions of T_1 and T_2 for the excited spin-3/2 baryons are calculated in [15].

3 Numerical calculations

This section is devoted to the numerical analysis of the sum rules derived for the negative parity, spin-3/2 baryons. The values of the input parameters which enter to the sum rules for the magnetic moments are: $\langle \bar{u}u \rangle(\mu = 1~GeV) = \langle \bar{d}d \rangle(\mu = 1~GeV) = -(0.243)^3~GeV^3$, $\langle \bar{s}s \rangle(\mu = 1~GeV) = 0.8 \langle \bar{u}u \rangle(\mu = 1~GeV)$, $f_{3\gamma} = -0.039$ [16], $\Lambda = (0.5 \pm 1.0)~GeV$ [17], $m_s(\mu = 2~GeV) = 111 \pm 6~MeV$ [18], and the magnetic susceptibility $\chi(\mu = 1~GeV) = -2.85 \pm 0.5~GeV^{-2}$ [19].

The main ingredient of the light cone sum rules are the DAs. In our problem we need the photon DAs, whose analytic expressions are presented in [16]. In addition to the above-presented input parameters and photon DAs, sum rules involve the Borel mass parameter M^2 and continuum threshold s_0 as well. Since these two parameters are the auxiliary ones, the magnetic moments should be independent of them. The continuum threshold is not totally arbitrary, and it is correlated to the energy of the first excited states. It is usually chosen in the region $(m_- + 0.4)^2 \leq s_0 \leq (m_- + 0.5)^2~GeV^2$. The working region of the Borel mass parameter M^2 is determined in the following way. In order to obtain the upper bound of M^2 we require that the continuum and higher states contributions are less than, say, 30% of the perturbative contributions. The lower bound of M^2 is determined from the requirement that the contribution of the highest power of $1/M^2$ terms should be less than 25% of the highest M^2 contributions. These two requirements lead the following results for the working regions of M^2:

$$
\begin{aligned}
&1 \leq M^2 \leq 3~GeV^2 \text{ for } p^*,\ n^* \text{ and } \Sigma^*, \\
&1.5 \leq M^2 \leq 3.5~GeV^2 \text{ , for } \Xi^*.
\end{aligned} \tag{23}
$$

In these regions of the Borel mass parameter M^2, the results for the magnetic moments of the negative parity, spin-3/2 baryons are very weakly dependent on M^2 and s_0. As an example, we present in Fig. (1) the dependence of the magnetic moment for the $p^*(3/2)$ state on M^2, at three fixed values of the continuum threshold s_0. We deduce from this figure that $\mu_{p^*} = (1.2 \pm 0.2)\mu_N$.

The results for the other excited states of negative parity, spin-3/2 baryons are presented in Table 1.

μ_{p^*}	$(1.2 \pm 0.2)\mu_N$
μ_{n^*}	$(0.9 \pm 0.1)\mu_N$
$\mu_{\Sigma^{+*}}$	$(1.2 \pm 0.2)\mu_N$
$\mu_{\Sigma^{-*}}$	$(-1.5 \pm 0.1)\mu_N$
$\mu_{\Sigma^{0*}}$	$(-0.22 \pm 0.02)\mu_N$
$\mu_{\Xi^{0*}}$	$(0.36 \pm 0.06)\mu_N$
$\mu_{\Xi^{-*}}$	$(-1.8 \pm 0.3)\mu_N$

Table 1:

We see from this table that, the value of the magnetic moments of Σ^{+*} and p^*, as well as, Ξ^{-*} and Σ^{-*} are very close to each other, which follows from the small difference in magnetic moments due to the SU(3) symmetry breaking effects. Moreover, in exact SU(3) symmetry, the values of the magnetic moments of Σ^{0*} and Ξ^{0*} should be equal to zero. Indeed, our results show that they have quite small values, and these small nonzero values of the magnetic moments of the relevant baryons can be attributed to the SU(3) symmetry breaking. It should finally be mentioned here that, in exact SU(3) symmetry limit, the relation

$$
\mu_{\Sigma^{0*}} = \frac{1}{2}\Big(\mu_{\Sigma^{+*}} + \mu_{\Sigma^{-*}}\Big)
$$

should be satisfied, which is also confirmed by our results. The apparent small deviation can again be attributed to the SU(3) symmetry breaking effects.

In conclusion, the magnetic moments of the negative parity, spin-3/2 baryons are estimated within the LCSR. Checking our predictions in future experiments could be very useful for understanding the dynamics of the negative parity baryons.

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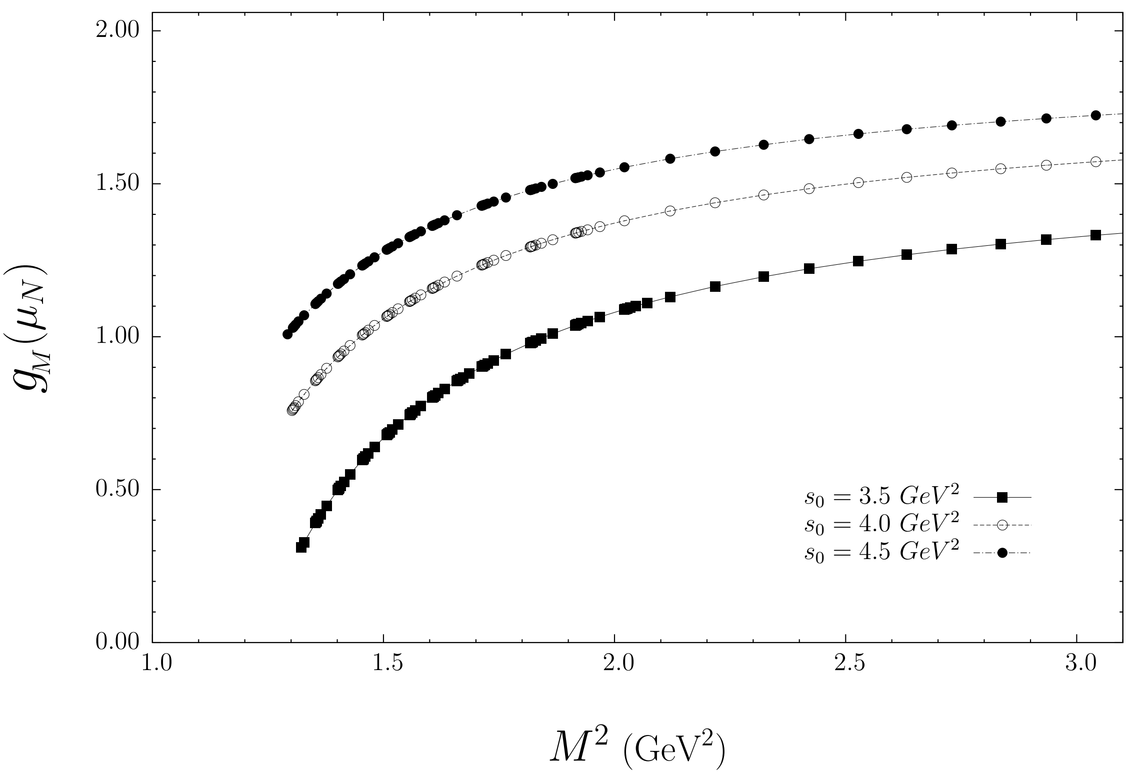
Figure captions

Fig. (1) The dependence of the magnetic moment for the $p^*(3/2)$ state on M^2, at three fixed values of the continuum threshold s_0.



2.00
1.50
1.00
0.50
0.00
1.0
1.5
2.0
2.5
3.0
$g_M(\mu_N)$
M^2 (GeV^2)
$s_0 = 3.5\ GeV^2$
$s_0 = 4.0\ GeV^2$
$s_0 = 4.5\ GeV^2$

Figure 1: