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Measuring $\left|\frac{V_{td}}{V_{ub}}\right|$ through $B \to M\nu\bar{\nu}$ $(M = \pi, K, \rho, K^*)$ decays

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Abstract

We propose a new method for precise determination of $\left|\frac{V_{td}}{V_{ub}}\right|$ from the ratios of branching ratios $\frac{\mathcal{B}(B\to\rho\nu\bar{\nu})}{\mathcal{B}(B\to\rho l\nu)}$ and $\frac{\mathcal{B}(B\to\pi\nu\bar{\nu})}{\mathcal{B}(B\to\pi l\nu)}$. These ratios depend only on the ratio of the Cabibbo-Kobayashi-Maskawa (CKM) elements $\left|\frac{V_{td}}{V_{ub}}\right|$ with little theoretical uncertainty, when very small isospin breaking effects are neglected. As is well known, $\left|\frac{V_{td}}{V_{ub}}\right|$ equals to $\left(\frac{\sin\gamma}{\sin\beta}\right)$ for the CKM version of CP-violation within the Standard Model. We also give in detail analytical and numerical results on the differential decay width $\frac{d\Gamma(B\to K^*\nu\bar{\nu})}{dq^2}$ and the ratio of the differential rates $\frac{d\mathcal{B}(B\to\rho\nu\bar{\nu})/dq^2}{d\mathcal{B}(B\to K^*\nu\bar{\nu})/dq^2}$ as well as $\frac{\mathcal{B}(B\to\rho\nu\bar{\nu})}{\mathcal{B}(B\to K^*\nu\bar{\nu})}$ and $\frac{\mathcal{B}(B\to\pi\nu\bar{\nu})}{\mathcal{B}(B\to K\nu\bar{\nu})}$.

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1 Introduction

The determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is one of the most important issues of quark flavor physics. The precise determination of V_{td} and V_{ub} elements has principal meaning, since they are solely responsible for the origin of CP violation in the CKM version of CP-violation within the Standard Model (SM). Furthermore, the accurate knowledge of these matrix elements can be useful in relating them to the fermion masses and also in searches for hints of new physics beyond the SM. Therefore, strategies for the accurate determination of V_{td} and V_{ub} are urgently required. In the existing literature, we can find proposals of different methods for precise determination of V_{ub} and V_{td} from inclusive and exclusive, semileptonic and non leptonic decays of B meson (see [1] for a recent review).

The quantity $|V_{ub}/V_{cb}|$ has been historically measured by looking at the endpoint of the inclusive lepton spectrum in semileptonic B decays, or from the exclusive semileptonic decays $B \to \rho l \nu$. It has been suggested that the measurements of hadronic invariant mass spectrum [2, 3] as well as hadronic energy spectrum [4] in the inclusive $B \to X_{c(u)} l \nu$ decays can be useful in extracting $|V_{ub}|$ with better theoretical understanding. The measurement of the ratio $|V_{ub}/V_{ts}|$ from the differential decay widths of the processes $B \to \rho l \nu$ and $B \to K^* l \bar{l}$ by using $SU(3)$-flavor symmetry and heavy quark symmetry has also been proposed [5]. There has also been recent theoretical progress on the exclusive $b \to u$ semileptonic decay form factors using HQET-based scaling laws to extrapolate the form factors from semileptonic D meson decays [6]. The element V_{td} can be extracted indirectly from $B_d - \overline{B_d}$ mixing. However, in $B_d - \overline{B_d}$ mixing the large uncertainty of hadronic matrix elements prevents one from extracting V_{td} with good accuracy. A better extraction of $|V_{td}/V_{ts}|$ can be made if $B_s - \overline{B_s}$ mixing is measured as well, since the ratio $(f_{B_d}^2 B_{B_d})/(f_{B_s}^2 B_{B_s})$ can be determined much better. Another method to determine $|V_{td}/V_{ts}|$ comes from the analysis of the invariant dilepton mass distributions of $B \to X_{d,s} l^+ l^-$ decays [7]. An interesting strategy for measuring $|V_{td}/V_{us}|$ was proposed in [8], which uses isospin symmetry to relate the decay $K^+ \to \pi^+ \nu \bar{\nu}$ to the well measured decay $K^+ \to \pi^0 l \nu$.

In this work we propose a new method to determine the ratio $|V_{td}/V_{ub}|$ from an analysis of exclusive $B \to M \nu \overline{\nu}$ decays, where M means pseudoscalar π, K and vector ρ, K^* mesons. The inclusive $B \to X_q \nu \overline{\nu}$ decay is theoretically very clean because of the absence of any long distance effects and very small QCD corrections ($\sim 3\%$) [1, 9], and is therefore practically free from the scale (μ) dependence. However, in spite of such theoretical advantages, it would be very difficult to detect this inclusive decay in experiments because the final state contains two missing neutrinos and (many) hadrons.

This paper is organized as follows. In Section 2 we give the necessary theoretical framework to describe $B \to M\nu\bar{\nu}$ decays. In Section 3 we study the ratios of branching fractions

$$\mathcal{B}(B \to \rho\nu\bar{\nu})/\mathcal{B}(B \to \rho l\nu) \quad \text{and} \quad \mathcal{B}(B \to \pi\nu\bar{\nu})/\mathcal{B}(B \to \pi l\nu).$$

We also study the q^2 dependence of the differential decay rate of $B \to K^*\nu\bar{\nu}$, and the ratio of the differential decay rates

$$\frac{d\Gamma(B \to \rho\nu\bar{\nu})}{dq^2} \Big/ \frac{d\Gamma(B \to K^*\nu\bar{\nu})}{dq^2},$$

as well as

$$\mathcal{B}(B \to \rho\nu\bar{\nu})/\mathcal{B}(B \to K^*\nu\bar{\nu}) \quad \text{and} \quad \mathcal{B}(B \to \pi\nu\bar{\nu})/\mathcal{B}(B \to K\nu\bar{\nu}).$$

Section 4 is devoted to a discussion of our results and conclusion.

2 Theory of $B \to M\nu\bar{\nu}$ $(M = \pi, K, \rho, K^*)$ decays

In the Standard Model (SM), the process $B \to M\nu\bar{\nu}$ is described at quark level by the $b \to q\nu\bar{\nu}$ transition, and receives contributions from Z-penguin and box diagrams, where dominant contributions come from intermediate top quarks. The effective Hamiltonian responsible for $b \to q\nu\bar{\nu}$ decays is described by only one Wilson coefficient, namely C_{10}^{ν}, and its explicit form is

$$H_{eff} = \frac{G_F \alpha}{2\pi\sqrt{2}}\, C_{10}^{\nu}\, (V_{tb}V_{tq}^*)\, \overline{q}\gamma^{\mu}(1-\gamma_5)b\, \overline{\nu}\gamma_{\mu}(1-\gamma_5)\nu, \tag{1}$$

where G_F is the Fermi constant, α is the fine structure constant (at the Z mass scale), and V_{ij} are elements of the CKM matrix. In Eq. (1), the Wilson coefficient C_{10}^{ν} has the following form, including $\mathcal{O}(\alpha_s)$ corrections:

$$C_{10}^{\nu} = \frac{X(x_t)}{\sin^2\theta_w}, \tag{2}$$

where

$$X(x_t) = X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t). \tag{3}$$

In Eq. (3),

$$X_0(x_t) = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\ln(x_t)\right]$$

is the Inami-Lim function [10], and

$$X_1(x_t) = \frac{4x_t^3 - 5x_t^2 - 23x_t}{3(x_t-1)^2} - \frac{x_t^4 + x_t^3 - 11x_t^2 + x_t}{(x_t-1)^3}\ln(x_t) + \frac{x_t^4 - x_t^3 - 4x_t^2 - 8x_t}{2(x_t-1)^3}\ln^2(x_t) + \frac{x_t^3 - 4x_t}{(x_t-1)^2}Li_2(1-x_t) + 8x_t\frac{\partial X_0(x_t)}{\partial x_t}\ln(x_\mu),$$

where

$$Li_2(1-x_t) = \int_1^{x_t} dt \frac{\ln(t)}{1-t},$$

is the Spence function, and

$$x_t = \frac{{m_t}^2}{m_W^2}, \quad \text{and} \quad x_\mu = \frac{\mu^2}{m_W^2}.$$

Here μ describes the scale dependence when leading QCD corrections are taken into account. The term $X_1(x_t)$ is calculated in Ref. [9]. The presence of only one operator in the effective Hamiltonian makes the process $b \to q\nu\bar{\nu}$ very attractive, because the estimated theoretical uncertainty is related only to the value of the Wilson coefficient C_{10}^ν (*i.e.* the uncertainty due to the top quark mass), contrary to the $b \to ql^+l^-$ decay, where the uncertainties are described by three independent Wilson coefficients, C_7, C_9 and C_{10}. Another favorable property of this decay is the absence of any long distance effects, which make the $b \to ql^+l^-$ process considerably more complicated. In spite of such theoretical advantages, in practice the inclusive channel $B \to X_q\nu\bar{\nu}$ would be very difficult to detect in experiments. Only exclusive channels, namely $B \to M\nu\bar{\nu}$, may be studied experimentally.

At this point we consider the problem of computing the matrix elements of the effective Hamiltonian (1) between B and M states. This problem is related to the non-perturbative sector of QCD, and it can be solved only by using non-perturbative methods. The matrix element $< M|H_{eff}|B >$ has been investigated through different approaches, such as chiral perturbation theory [11], three point QCD sum rules [12], relativistic quark model by the light front formalism [13], effective heavy quark theory [14], light-cone QCD sum rules [15]-[17], *etc.*

The hadronic matrix elements for $B \to P\nu\bar{\nu}$ (P is a pseudoscalar meson, π or K) decays can be parametrized in terms of the form-factors $f_+^P(q^2)$ and $f_-^P(q^2)$ in the following way;

$$< P(p_2)|\overline{q}\gamma_\mu(1-\gamma_5)b|B(p_1) >= p_\mu f_+^P(q^2) + q_\mu f_-^P(q^2), \tag{4}$$

where $p = p_1 + p_2$ and $q = p_1 - p_2$. For $B \to V\nu\overline{\nu}$ (V is the vector ρ or K^* meson) decays, the hadronic matrix element can be written in terms of five form-factors:

$$< V(p_2,\varepsilon)|\overline{q}\gamma_\mu(1 - \gamma_5)b|B(p_1) >= -\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_2^\alpha q^\beta\frac{2V(q^2)}{m_B + m_V} - i[\varepsilon_\mu^*(m_B + m_V)A_1(q^2)$$

$$- \quad (\varepsilon^* q)(p_1+p_2)_\mu \frac{A_2(q^2)}{m_{_B}+m_{_V}} - q_\mu(\varepsilon^* q)\frac{2m_{_V}}{q^2}(A_3(q^2)-A_0(q^2))] \tag{5}$$

with condition

$$A_3(q^2=0)=A_0(q^2=0). \tag{6}$$

Note that after using the equations of motion the form-factor $A_3(q^2)$ can be written as a linear combination of the form-factors A_1 and A_2 (for more details see the first reference in [12]):

$$A_3(q^2)=\frac{1}{2m_{_V}}[(m_{_B}+m_{_V})A_1(q^2)-(m_{_B}-m_{_V})A_2(q^2)]. \tag{7}$$

In Eq. (5), ε_μ, p_2 and $m_{_V}$ are the polarization 4–vector, 4–momentum and mass of the vector particle, respectively. Using Eqs. (1), (4) and (5), and after performing summation over vector meson polarization and taking into account the number of light neutrinos $N_\nu=3$, we have:

$$\frac{d\Gamma}{dq^2}(B^\pm \to P^\pm \nu\bar\nu)=\frac{G_{_F}^2\alpha^2}{2^8\pi^5}|V_{tq}V_{tb}^*|^2\lambda^{3/2}(1,r_{_P},s)m_{_B}^3|C_{10}^\nu|^2|f_p^+(q^2)|^2 \tag{8}$$

and

$$\begin{aligned}\frac{d\Gamma}{dq^2}(B^\pm \to V^\pm \nu\bar\nu) &= \frac{G_{_F}^2\alpha^2}{2^{10}\pi^5}|V_{tq}V_{tb}^*|^2\lambda^{1/2}(1,r_{_V},s)m_{_B}^3|C_{10}^\nu|^2 \\ &\times \left(8\lambda s\frac{V^2}{(1+\sqrt{r_{_V}})^2}+\frac{1}{r_{_V}}\left[\lambda^2\frac{A_2^2}{(1+\sqrt{r_{_V}})^2}\right.\right. \\ &+ \left.\left.(1+\sqrt{r_{_V}})^2(\lambda+12r_{_V}s)A_1^2-2\lambda(1-r_{_V}-s)Re(A_1A_2)\right]\right).. \end{aligned} \tag{9}$$

In Eqs. (8) and (9), $\lambda(1,r_{_M},s)$ is the usual triangle function

$$\lambda(1,r_{_M},s)=1+r_{_M}^2+s^2-2r_{_M}-2s-2r_{_M}s \qquad \text{with } r_{_M}=\frac{m_{_M}^2}{m_{_B}^2}, \quad s=\frac{q^2}{m_{_B}^2}.$$

Similarly, calculations for the $B^\pm \to M^0 e^\pm \nu$ decay lead to the following results:

$$\frac{d\Gamma}{dq^2}(B^\pm \to P^0 e^\pm\nu)=\frac{G_{_F}^2}{192\,\pi^3}|V_{qb}|^2\lambda^{3/2}(1,r_{_P},s)m_{_B}^3|f_p^+(q^2)|^2, \tag{10}$$

and

$$\begin{aligned}\frac{d\Gamma}{dq^2}(B^\pm \to V^0 e^\pm\nu) &= \frac{G_{_F}^2|V_{qb}|^2\lambda^{1/2}m_{_B}^3}{768\pi^3}\left(8\lambda s\frac{V^2}{(1+\sqrt{r_{_V}})^2}+\frac{1}{r_{_V}}\left[\lambda^2\frac{A_2^2}{(1+\sqrt{r_{_V}})^2}\right.\right. \\ &+ \left.\left.(1+\sqrt{r_{_V}})^2(\lambda+12r_{_V}s)A_1^2-2\lambda(1-r_{_V}-s)Re(A_1A_2)\right]\right). \end{aligned} \tag{11}$$

3 Numerical analysis

In deriving Eqs.(8)–(11), we set the masses of M^+ and M^0 equal and the electron mass is neglected. Using isospin symmetry the branching ratio for $B^\pm \to \rho^\pm \overline{\nu}\nu$ can be related to that for $B^\pm \to \rho^0 \overline{e}\nu$. It is clear that their ratio is independent of form-factors, *i.e.* free of hadronic long–distance uncertainties in the limit $m_{\rho^\pm} = m_{\rho^0}$. Corrections to the strict isospin symmetry, which come from phase space factors due to the difference of masses of $\rho^\pm$ and ρ^0, isospin violation in the $B \to \rho$ form-factors and electromagnetic radiative corrections to the $b \to q e\nu$ transition, are all small. In the following discussions we shall neglect these small isospin violation effects. Also note that these corrections for $K \to \pi$ transition have been calculated in [18] and found to be small, $\sim 5\%$.

Now we relate the branching ratio $\mathcal{B}(B^\pm \to \rho^\pm \overline{\nu}\nu)$ with $\mathcal{B}(B^\pm \to \rho^0 e^\pm \nu)$. From Eqs. (9) and (11), we have

$$\frac{\mathcal{B}(B^\pm \to \rho^\pm \overline{\nu}\nu)}{\mathcal{B}(B^\pm \to \rho^0 e^\pm \nu)} = 6\frac{\alpha^2}{4\pi^2}|C_{10}^\nu|^2 \left|\frac{V_{td}}{V_{ub}}\right|^2 . \tag{12}$$

Here the numerical factor 6 comes from the number of light neutrinos, and isospin symmetry relation between the form-factors of $B^\pm \to \rho^\pm$ and $B^\pm \to \rho^0$. In Eq. (12), we also put $|V_{tb}| = 1$. From Eq. (12), we get

$$\left|\frac{V_{td}}{V_{ub}}\right|^2 = \frac{1}{6C}\frac{\mathcal{B}_{exp}(B^\pm \to \rho^\pm \nu\bar{\nu})}{\mathcal{B}_{exp}(B^\pm \to \rho^0 e^\pm \nu)} = \left(\frac{\sin\gamma}{\sin\beta}\right)^2 , \tag{13}$$

where

$$C = \frac{\alpha^2}{4\pi^2}\,|C_{10}^\nu|^2 .$$

The second relation in (13) holds only for the CKM version of CP-violation within the SM.

From Eq. (13), we can see that measurements of the ratio of the branching fractions allow to determine the ratio of $\sin\gamma$ and $\sin\beta$. Up to now[3], various methods for measuring each angle separately have been proposed, *e.g.*., the angle β will be measured from $B \to J/\psi K_s$ decay with high accuracy, and angle γ is from the charged B decay $B^\pm \to DK^\pm$ with larger uncertainty. As follows from Eq. (13), one can measure the angle γ with small theoretical uncertainty, if $\sin\beta$ is measured independently with high accuracy. The following relations will also be useful for extracting the phase angle γ precisely:

$$\frac{\mathcal{B}(B^0 \to \rho^0 \nu\bar{\nu})}{\mathcal{B}(B^0 \to \rho^\pm e^\mp \nu)} = \frac{3}{2}\left(\frac{\sin\gamma}{\sin\beta}\right)^2 C, \tag{14}$$

[3] See also the recent work [19] on the simultaneous determination of $\sin\alpha$ and $\sin\gamma$ from $B^0_{d,s} \to K, \pi$ decays.

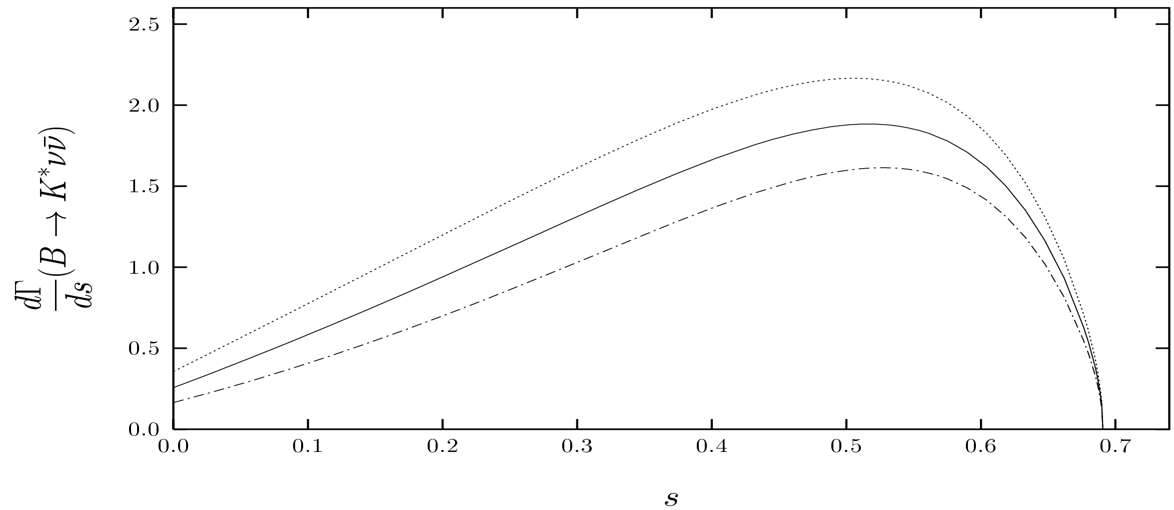

Fig.1

Figure 1: *Differential decay width* $\frac{d\Gamma}{ds}(B \to K^* \nu\bar{\nu})$ *as a function of the normalized momentum transfer square,* $s \equiv q^2/m_B^2$*, in units of* $\left[8.84 \times 10^{-18} \left(\frac{|V_{tb}V_{ts}^*|}{0.045}\right)^2\right]$ GeV*. Dotted and dash-dotted curves correspond to the cases when the uncertainty is added and subtracted from the central values of all form-factors, respectively.*

$$\frac{\mathcal{B}(B^\pm \to \pi^\pm \nu\bar{\nu})}{\mathcal{B}(B^\pm \to \pi^0 e^\pm \bar{\nu})} = 6\left(\frac{\sin\gamma}{\sin\beta}\right)^2 C, \tag{15}$$

$$\frac{\mathcal{B}(B^0 \to \pi^0 \nu\bar{\nu})}{\mathcal{B}(B^0 \to \pi^\pm e^\mp \nu)} = \frac{3}{2}\left(\frac{\sin\gamma}{\sin\beta}\right)^2 C, \tag{16}$$

and

$$\frac{\mathcal{B}(B^\pm \to K^{*\pm} \nu\bar{\nu})}{\mathcal{B}(B^\pm \to \rho^0 e^\pm \nu)} \approx 6\left|\frac{V_{ts}}{V_{ub}}\right|^2 C. \tag{17}$$

In derivation (14)-(17), we assumed that the mass of charged and neutral final states mesons are equal.

Now we consider the differential decay widths, $\frac{d\Gamma}{dq^2}(B \to \rho, K^* + \nu + \bar{\nu})$. For the hadronic form-factors we have used the results of the works [15]-[17], *i.e.* the monopole type form-factors based on light cone QCD sum rules. The values of the form-factors at $q^2 = 0$ are (see also Ref. [20]):

$$\begin{aligned} A_1^{B\to K^*}(0) &= 0.36 \pm 0.05, \\ A_2^{B\to K^*}(0) &= 0.40 \pm 0.05, \\ V^{B\to K^*}(0) &= 0.55 \pm 0.08, \\ A_1^{B\to\rho}(0) &= 0.30 \pm 0.05, \end{aligned}$$

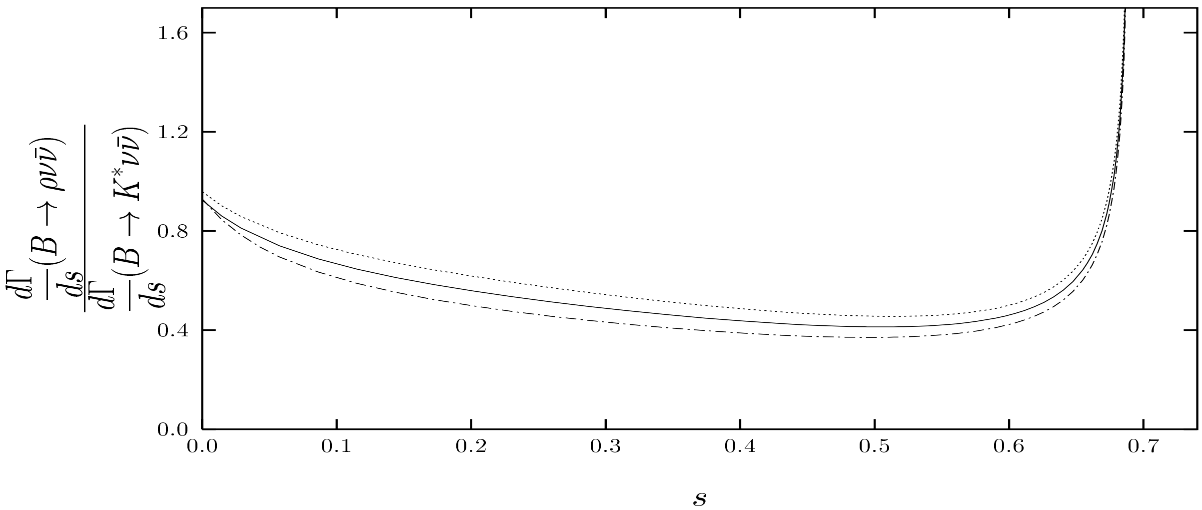

Fig.2

Figure 2: *Ratio of the differential decay rates $B \to \rho\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$, in units of $\left|\frac{V_{td}}{V_{ts}}\right|^2$, as a function of the normalized momentum transfer square, $s \equiv q^2/m_B^2$.*

$$\begin{aligned} A_2^{B\to\rho}(0) &= 0.325 \pm 0.05, \\ V^{B\to\rho}(0) &= 0.37 \pm 0.07, \\ f_+^{B\to K}(0) &= 0.29 \pm 0.05, \\ \text{and} \quad f_+^{B\to\pi}(0) &= 0.32 \pm 0.05. \end{aligned} \tag{18}$$

Note that all errors, which come from the uncertainties of the b quark mass, the Borel parameter variation, wave functions, non-inclusion of higher twists and radiative corrections, are added in quadrature.

In Fig. 1, we present the differential decay width $d\Gamma/ds(B \to K^*\nu\bar{\nu})$ as a function of the normalized momentum transfer square, $s \equiv q^2/m_B^2$. In Fig. 2, we show the q^2 dependence of the ratio of the differential decay rates $B \to \rho\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$, normalized to $\left|\frac{V_{td}}{V_{ts}}\right|^2$. In these figures, dotted and dash-dotted curves correspond to the cases when the uncertainty is added and subtracted from the central values of all form-factors, respectively. For the central solid curve we use the central values of form-factors. We note that the errors in the differential decay width of Fig. 1 due to the form-factors uncertainties are about $\sim \pm 20\%$. However, the errors in the ratio of Fig. 2 are reduced to about $\sim \pm 10\%$. We conclude that even though the errors from uncertainties of the form-factors for each channel are substantial, those in the corresponding ratio are comparatively small, and that for precise determination of the elements of the CKM matrix the investigation of the corresponding ratio is very suitable. We also note that the uncertainties for our main results, Eqs. (12)–(16), where we only assume flavor $SU(2)$ (isospin), should be even much smaller than that shown in Fig. 2, since there we had to assume flavor $SU(3)$ symmetry.

For completeness we present the integrated value for the branching fractions of $B \to K^*\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ as well as the value of the ratio $\mathcal{B}(B \to \rho\nu\bar{\nu})/\mathcal{B}(B \to K^*\nu\bar{\nu})$ and $\mathcal{B}(B \to \pi\nu\bar{\nu})/\mathcal{B}(B \to K\nu\bar{\nu})$:

$$\begin{aligned} \mathcal{B}(B \to K^*\nu\bar{\nu}) &= 1.7 \times (1 \pm 0.16) \cdot 10^{-5} \left|\frac{V_{ts}V_{tb}}{0.045}\right|^2, \\ \frac{\mathcal{B}(B \to \rho\nu\bar{\nu})}{\mathcal{B}(B \to K^*\nu\bar{\nu})} &= 0.52 \times (1 \pm 0.1) \left|\frac{V_{td}}{V_{ts}}\right|^2, \\ \mathcal{B}(B \to K\nu\bar{\nu}) &= 7.8 \times (1 \pm 0.25) \cdot 10^{-6} \left|\frac{V_{ts}V_{tb}}{0.045}\right|^2, \\ \text{and} \quad \frac{\mathcal{B}(B \to \pi\nu\bar{\nu})}{\mathcal{B}(B \to K\nu\bar{\nu})} &= 1.29 \times (1 \pm 0.2) \left|\frac{V_{td}}{V_{ts}}\right|^2. \end{aligned} \tag{19}$$

The values of the main input parameters, which appear in the expressions for the decay widths are

$$m_b = (4.8 \pm 0.1)\ \text{GeV}, \quad m_\rho \approx 0.77\ \text{GeV}, \quad m_{K^*} = 0.892\ \text{GeV}.$$

For the B meson life time, we take $\tau(B_d) = 1.56 \cdot 10^{-12}\ sec$ [21].

4 Discussions and conclusions

We proposed a new method for the precise determination of $\left|\frac{V_{td}}{V_{ub}}\right|$ from the ratios of the branching fractions

$$\mathcal{R}_\rho = \frac{\mathcal{B}(B \to \rho\nu\bar{\nu})}{\mathcal{B}(B \to \rho\nu e)} \quad \text{and} \quad \mathcal{R}_\pi = \frac{\mathcal{B}(B \to \pi\nu\bar{\nu})}{\mathcal{B}(B \to \pi e\nu)}.$$

As is well known, each partial decay width depends very strongly on hadronic form-factors. However, as also shown in Eqs. (9)–(13), these ratios, $\mathcal{R}_\rho, \mathcal{R}_\pi$, are free of any hadronic uncertainties, if small isospin breaking effects are neglected. Measurements of $\mathcal{R}_{\rho,\pi}$ allow to determine $\left|\frac{V_{td}}{V_{ub}}\right|$ with little theoretical error, which equals $(\frac{\sin\gamma}{\sin\beta})$ for the CKM version of CP-violation within the Standard Model. Therefore, $\mathcal{R}_{\rho,\pi}$ measures a relation between two different phases angles, which can be measured separately by experiments. We also found that each exclusive channel $B \to (K, K^*, \rho, \pi)\nu\bar{\nu}$ has rather large theoretical uncertainties due to the unknown hadronic form-factors, as shown in Fig. 1. In order to reduce these uncertainties we have considered the ratio of the corresponding exclusive channels, *e.g.* $(B \to \rho\nu\bar{\nu})/(B \to K^*\nu\bar{\nu})$, as shown in Fig. 2.

A few words about experimental statistics for detecting the $B \to \rho\nu\bar{\nu}$ decay follow: Future symmetric and asymmetric B factories should produce much more than $\sim 10^9$ $B-\overline{B}$ mesons by the year 2010. With 10^9 B mesons effectively reconstructed, the number of expected events for $B^\pm \to \rho^\pm\nu\bar{\nu}$ channel is

$$N \equiv \mathcal{B}(B \to \rho\nu\bar{\nu}) \times 10^9 \sim 100 \quad (\text{and} \;\; N(B \to K^*\nu\bar{\nu}) \sim 2 \times 10^4).$$

And the statistically estimated error for $B \to \rho\nu\bar{\nu}$ decay is approximately

$$\frac{1}{\sqrt{N}} \approx \frac{1}{10} = 10\%.$$

We argue that within the next decade the decay channel $B^\pm \to \rho^\pm\nu\bar{\nu}$ has a good chance for being detected in future B factories.

Note that the inclusive channels $B \to X_{d,s}\nu\bar{\nu}$ are also free of any theoretical uncertainties. However, measuring inclusive channels in experiments would be very difficult because of the two missing neutrinos and (many) hadrons. For completeness, we give here the summarized results for the inclusive decays in the lowest order:

$$\frac{\mathcal{B}(B \to X\nu\bar{\nu})}{\mathcal{B}(B \to Xe^-\nu)} \approx \frac{\mathcal{B}(B \to X_s\nu\bar{\nu})}{\mathcal{B}(B \to X_c e^-\nu)} = 3\left|\frac{V_{ts}}{V_{cb}}\right|^2 C, \tag{20}$$

$$\frac{\mathcal{B}(B \to X_d\nu\bar{\nu})}{\mathcal{B}(B \to X_u e^-\nu)} = 3\left(\frac{\sin\gamma}{\sin\beta}\right)^2 C,$$

and

$$\begin{aligned} \mathcal{B}(B \to X_s\nu\bar{\nu}) &\sim 3 \times 10^{-5}, \\ \mathcal{B}(B \to X_d\nu\bar{\nu}) &\sim 5 \times 10^{-7}. \end{aligned}$$

In derivation of Eq. (20) we have neglected the charm quark mass.

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