# Fairness in Wireless Networks - Issues, Measures and Challenges

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***Abstract*—Fairness is an important and interdisciplinary topic employed in many fields. This article discusses fairness issues in wireless networks. First, we address various issues in the study on fairness. In general, we formulate the issues in fairness research through the three core questions. Finding answers them leads us to various nuances of fairness studies. Existing fairness models are summarized and compared. We also look into the major fairness research domains in wireless networks. Relationship between fairness, utility and resource allocation are also discussed. At the end of this article, we provide properties of fairness and an example fairness management process. We also state some challenges that point to further work on fairness in wireless networks. Indeed research on fairness is entangled with various other aspects such as performance, utility, optimization and throughput at the network as well as individual (or node) level. While consolidating various contributions in the literature, this article tries to explain the nuances of all these aspects clearly in the domain of wireless networking.**

***Index Terms*—Fairness, Utility, Resource allocation, Wireless networks, Jain's index, Entropy, Max-min, Min-max, Proportional fairness**

## I. INTRODUCTION

The field of wireless networking is experiencing a tremendous growth. Many techniques, protocols, applications and devices have been continuously addressed by industry as well as academics. There are some crucial issues in wireless networking technologies that are brought to fore by researchers and one of them is *fairness*. Fairness is an interdisciplinary research topic which is usually related to resource allocation. For example, in economics, revenues which are to be divided amongst shareholders, economic assistance to the persons in need or resource sharing in a society are all subjected to questions regarding their fairness. We are exposed to concerns regarding fairness in every aspect of our lives, which is also true in technologies. In computer architecture, different computing resources are supposed to be shared fairly amongst all processes and their threads. In computer networks, all nodes expect to gain the bandwidth fairly and also the quality of service (QoS).

To explain the significance of fairness in wireless networks, we start with a simple scenario of an ad hoc network as shown in Fig. 1. Nodes A, B, C, D and E are devices which communicate with each other via wireless links $L_1$ to $L_5$. Node C acts as a gateway while other nodes request the Internet services. Many fairness issues can be explored in this simple scenario. For instance, nodes should get *fair* chance to access the Internet, bandwidth should be *fairly* shared, QoS requirements of the nodes should be *fairly* satisfied, Energy consumption should be the same when the load is similar. Further, the cost, link quality, throughput and other performance aspects should be reasonable and *fair* . These issues show the significance and diversity of fairness issues in wireless networks, and there is no single method or solution taking the lead here. Though this example is a simple we could already see multifariousness of interpretations of the notion of fairness. In this paper we try to capture this aspect of the notion of fairness which is applicable to wireless networking in general.

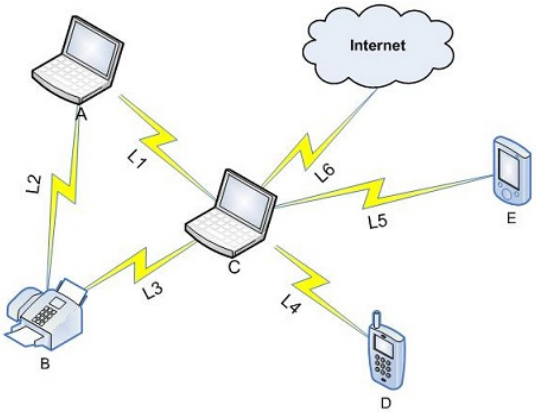

Fig. 1. A simple illustration of a wireless network consisting of five wireless nodes and six wireless links where the objective of the nodes is to access the Internet services over Node C which acts as the gateway.

Next question that arises naturally is, what to do when *unfairness* happens. This is not thoroughly studied. Mostly, researchers focused on measuring the fairness in resource allocations. Devising strategies to mitigate unfairness is also an important aspect of fairness. There are two kinds of strategies when unfairness happens:

- Compensating individuals treated unfairly in the previous resource allocation round and somehow their losses are addressed in the current round of allocations.
- Adjusting the allocation to reach fairness again, without any compensation for the individuals treated unfairly.

To illustrate the above aspects, let us take two strategies. In the first strategy, in Fig. 1, if Node A gets 5% of the network capacity and B gets 35% and others get 20% each in the first hour, it is termed unfair. Upon recognizing that the network capacity allocation is unfair, Node A will get 35% and B will get 5% for the next hour to compensate the unfairness in the first round of assignment. However, in the second strategy, the system provides A with 20% and B with 20% to achieve fairness without requiring any compensation.

There is no single way to rate one strategy over the other. Now, based on the simple strategies mentioned above, we try to address the following three core questions in this paper:

- **Q1** *What is fairness?*
- **Q2** *How do we measure whether a system is fair to all of its individuals?*
- **Q3** *How to make a system fair?*

**Q1** is about the definition of fairness. It is rather difficult to arrive at a consensus to provide a universally accepted definition of *fairness*. **Q2** is about how to measure the fairness in a system. **Q3** implies what to do when *unfairness* happens. We discuss these questions based on the current literature within the scope of wireless networking.

The rest of this paper is organized as follows. We illustrate fairness issues in general in Section II. The definitions and classifications are presented. Section III lists the most used quantitative and qualitative fairness measures and two general fairness models. They are also compared and analyzed. In Section IV, the research trends and major fairness issues are presented. A detailed discussion on the relation between fairness, utility and resource allocation can be seen in Section V. Furthermore, the properties of fairness models, an example fairness management process and the challenges of fairness issues in wireless networks are proposed in Section VI. We conclude in Section VII.

## II. WHAT IS FAIRNESS

In this section, the definition and classification of fairness issues are discussed.

### A. Definition

Definitions of fairness aim at answering **Q1**. In Oxford English Dictionary, the definition of fairness is "..., equitableness, fair dealing, honesty, impartiality, uprightness,..." [1]. There are some other fairness definitions, e.g., "an allocation where no person in the economy prefers anyone else's consumption bundle over his own" [2] and "A fair allocation is free of envy " [3]. Sawyer et al., define fairness as "Equal treatment to equal individuals and reserving preferred treatment for those individuals who are in some sense more deserving [4]". However, these definitions draw our attention to the ambiguity in identifying equal individuals in the first place. Furthermore, equal treatment is another ambiguous term. While all these definitions are fuzzy, they indicate, in general, the essence of impartiality, justice, and satisfaction of individuals in general. Now the task at hand is to translate these values into the domain of wireless networking.

In Wireless networking domain, generally, fairness is attributed to resource sharing or allocation. The consequence of an unfair resource allocation among different individuals may lead to resource starvation, resource wastage or redundant allocation. Fairness has been mostly studied in resource allocation based on *impartial* and *justified* strategies. Fairness strategies allocate system resources *reasonably* to individuals of the system in a distributed or centralized fashion. In this paper, we use the term *Individual* (or *Node*)[1] to refer to the autonomous constituent of a system. A *System* represents the conglomeration of individuals. For instance, in a wireless ad hoc network, *nodes* are the individuals and *ad hoc network* itself is the system.

It is rather difficult to agree on the definition of fairness since it is subjective. When we consider rational individuals, each individual evaluates the share of resources they receive compared to others in the system from their own point of view. Consequently, the definition of fairness or any effort to define fairness is influenced by the *value* ascribed to the resources by the designer of the system or by the individuals of the system. Thus, most of the fairness research is around ascribing a value to the shared resource. However, this is not an easy task since the requirements by individuals are different and the prices paid by those individuals also play a role. Further, the system level resource usage should also be considered. Thus we only track the efforts, hitherto, in defining fairness from the literature. On the whole, finding a universally accepted definition is not an objective of this article.

### B. Classification

Equal opportunity provided to the individuals in resource sharing may not mean equal allocation of resources. On the other hand, a fair allocation may be an outcome of a process where individuals do not have equal opportunity. Therefore, targeted and resultant fairness may not be the same. Furthermore, there may be temporal changes in the allocation in a dynamic system. This suggests that fairness may also have a temporal dimension. Fairness can also be considered from the point of view of both, system and individuals. Moreover, individuals of a system may have to carry out various tasks in which case fairness can be defined per task. Thus in this section, we dwell on these concerns and classify the fairness definitions by providing simple but illustrative examples based on the scenario shown in Fig. 1.

*1) Targeted and Resultant Fairness:* From the point of view of resource allocation and utilization, fairness can be divided into two types: targeted fairness and resultant fairness[2]. Targeted fairness try to achieve fair sharing of resources but resultant fairness aims at fair utilization.

Taking Fig. 1 as an example, all nodes are of the same priority. When each node is assigned the same bandwidth (we also refer it as the percentage of the total network capacity/bandwidth), targeted fairness is achieved. However, if the quality of Link $L_1$ is worse than that of $L_3$, it takes longer time for A than B to access the Internet, which means that A and B do not gain the fair access in the view of resultant fairness. On the other hand, if we only consider the resultant fairness, some individuals may starve to death. In the former example with a contrived assumption that if A's error rate is

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[1] We use the terms *node* and *individual* interchangeably, not to affect the natural flow while discussing the concepts.

[2] In [5], these two terms are mentioned as effort and outcome fairness. We use different terms that could be easily comprehended but carry the same meaning.

extremely large, it will keep requesting for more and more capacity, which will make others to gain lesser and lesser capacity. In an adequate fairness model, both targeted fairness and resultant fairness should be considered, and there should also be a proper balance between them.

*2) Short-term and Long-term Fairness:* Considering the time period, fairness can be categorized into short-term and long-term [5], [6]. Short-term fairness focuses on resource allocation in a very short time period, or in other words, measurement of the fairness is done at certain selected moments. In contrast, long-term fairness measures the resource allocation over a longer time period or at the end of the life cycle. Short-term fairness has a significant impact on QoS, especially in real-time applications because of the focus on the current QoS measurements. Long-term fairness is more important when the resources are scarce. Since short-term fairness is very difficult to be guaranteed with scarce resources when many individuals are vying for it long term measurements are used to fall back on. For example in Fig. 1 all nodes are assumed to have the same priority. Let us say that at 18:00 each node gets 20% of the network capacity, then they reach the short-term fairness at this moment. However, during the previous hour, average capacity of allocation were 10%, 20%, 30%, 5% and 35%, implying that long-term fairness has not been achieved in this period. We shall now take system and individual level fairness.

*3) System and Individual Fairness:* Fairness can be considered both on the system and individual level. The system fairness addresses the overall fairness amongst all individuals in the system, and individual fairness indicates whether a certain individual is treated fairly by the system. For example in Fig. 1, the system fairness of network capacity can be defined as equal allocation (every node gets 20%). However, Node A can be considered to achieve individual fairness when it gets 20% of the network capacity without the concerns of other nodes. Therefore in general, we say if a system reaches system fairness, then all nodes should achieve individual fairness, and if all nodes achieve individual fairness, then the system is also fair. However, if one or more individuals are treated unfairly, then the system is not considered as fair. The above classification is considered to be pragmatic rather than absolute. After this classification, we emphasize that fairness can be very subjective, and researchers have different opinions about resource allocation for a particular scenario.

## III. HOW TO MEASURE FAIRNESS

We came up with three core questions on fairness issues in Section I. In applications, the last two questions (**Q2** and **Q3**) are always termed as fairness measures and fairness optimization models. Fairness measures are tools to measure fairness level, and fairness optimization models distribute resources in a fair way. Based on their quantitative ability, the measures can be classified as quantitative or qualitative. First, in this section, we present the most used quantitative (Jain's index and entropy measure) and qualitative fairness measures (max-min and proportional fairness). Then a more general quantitative fairness measure based on "Tian Lan's" model is introduced. In the sequel, an example of fairness optimization model, envy-based, is discussed. At the end of this section, all measures are compared with each other. We assume that there is one type of resource whose total amount is $x$ and there are $n$ individuals sharing this resource. $\mathbf{X} = (x_1, x_2, ..., x_n)$ implies the allocated resources, where $x_i$ is the amount of resource allocated to individuals $i = 1, 2, \ldots, n$. Sum of individually allocated resources must be less than or equal to the total amount, which can be written as $\sum_{i=1}^{n} x_i \leqslant x$, where $x$ is the total amount of the resource.

### A. Quantitative Fairness Measures

Quantitative fairness measures are usually real valued. We define $f(\mathbf{X}) : \mathbb{R}_n^+ \to \mathbb{R}^+$ as the fairness measure based on resource allocation $\mathbf{X}$, where $n$ is the number of individuals. The basic requirements that a quantitative fairness measure must satisfy are:

R1: $f(\mathbf{X})$ should be continuous on $\mathbf{X} \in \mathbb{R}_n^+$.
R2: $f(\mathbf{X})$ should be independent of $n$.
R3: The range of $f(\mathbf{X})$ should be easily mapped on to $[0, 1]$.
R4: Function $f(\mathbf{X})$ should be easily extendable to multi-resources case.
R5: $f(\mathbf{X})$ should be easy to implement.
R6: $f(\mathbf{X})$ should be sensitive enough to the variation of $\mathbf{X}$.

The requirements R1 and R2 imply the generality of fairness function $f(\mathbf{X})$ with different resource allocations and various number of individuals. R3 shows the scalability of $f(\mathbf{X})$, and it gives intuitive and direct impression on the fairness. Requirements R4 and R5 make $f(\mathbf{X})$ realistic and implementable. In the sequel, we review several frequently used quantitative measures and identify the set of requirements they must satisfy.

*1) Jain's Index:* Jain's index (or simply fairness index) was first proposed in [7] by Rajendra K. Jain. Four desired properties of the fairness index were proposed in [7]:

- Independent of population size: The index should be scalable with number of users (*R2*).
- Independent of scale and metric: The index should not change with measures or metrics used. This property implies that variance can also be a fairness index (*R5*).
- Boundedness: The index should be finite and it can be a ratio between 0 and 1 (*R3*).
- Continuity: The index function should be continuous on allocations (*R1*), and it should have the ability to measure different allocations.

In coherence with the model properties given above, Jain's index is defined in [7] as,

$$f(\mathbf{X}) = \frac{\left[\sum_{i=1}^{n} x_i\right]^2}{n \sum_{i=1}^{n} x_i^2}, \tag{1}$$

where $0 \leqslant f(\mathbf{X}) \leqslant 1$. Jain's index is one of the earliest proposed and widely studied fairness measures. It can be used generally and gives guidelines for fairness study in various domains. Fairness in an allocation can be represented by the index value. A large value of $f(\mathbf{X})$ represents fairer resource allocation from the system perspective. As an example, Table I shows different Jain's index values with different network capacity allocations for the scenario given in Fig. 1. In this table, we can see that the allocation tends to be fairer when Jain's index is closer to 1.

TABLE I
EXAMPLES OF JAIN'S INDEX. (SEE FIG. 1)

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $x_A$ | 0% | 5% | 10% | 20% |
| $x_B$ | 5% | 40% | 30% | 20% |
| $x_C$ | 30% | 50% | 30% | 20% |
| $x_D$ | 0% | 5% | 10% | 20% |
| $x_E$ | 65% | 0% | 20% | 20% |
| $f(\mathbf{X})$ | 0.3883 | 0.4819 | 0.8333 | 1 |

Even though Jain's index provides insight into the overall system fairness, it does not help in identifying the unfairly treated individuals in case of single resource and similarly weighted individuals. It needs complete information of the allocation to compute the fairness index. However, it can be modified and used in multiple resource allocation. In this case $\mathbf{X}$ can be assigned as a combination of various resources. For example in [8], Jain's index is adopted and $\mathbf{X}$ represents throughput, input load, and normalized throughput all together in broad-band fixed wireless access (FWA) systems.

*2) Entropy:* Entropy was introduced by Shannon [9]. Since it also reflects fairness aspects, some researchers, for example in [10]–[12] and [13], employed it as a measure of fairness . It assumes that the proportions of resource are allocated to $n$ individuals $\mathbf{P} = (p_1, p_2, ..., p_n)$, and

$$p_i = \frac{x_i}{\sum_{i=1}^{n} x_i},$$

where $0 \leqslant p_i \leqslant 1$ $(i = 1, 2, ..., n)$ and $\sum_{i=1}^{n} p_i = 1$ [9]. The uncertainty of the distribution $\mathbf{P}$, is called the entropy of the distribution $\mathbf{P}$ and is usually measured by $H(\mathbf{P}) = H(p_1, p_2, ..., p_n)$ as given below [9].

$$H(\mathbf{P}) = \sum_{i=1}^{n} \left(p_i \log_2 {p_i}^{-1}\right). \tag{2}$$

The Shannon's entropy has the following basic characteristics [12]:

- $H(\mathbf{P})$ is a symmetric function of its variables when $n \geqslant 2$.
- $H((p, 1-p))$ is a continuous function of $p$ for $0 \leqslant p \leqslant 1$.
- $H((0.5, 0.5)) = 1$.
- $H((tp_1, (1-t)p_1, p_2, ..., p_n)) = H((p_1, p_2, ..., p_n)) + p_1 H((t, 1-t))$.

When $H(\mathbf{P})$ is used as a fairness measure, it is similar to $f(\mathbf{X})$. Only the absolute resource values of $\mathbf{X}$ are replaced by resource proportions $\mathbf{P}$ in (2).

Four cases based on the scenario in Fig. 1 are given in Table II, and we can find that entropy is larger when the allocations are fairer.

TABLE II
EXAMPLES OF ENTROPY.

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $x_A$ | 1% | 5% | 10% | 20% |
| $x_B$ | 4% | 40% | 30% | 20% |
| $x_C$ | 30% | 50% | 30% | 20% |
| $x_D$ | 1% | 4% | 10% | 20% |
| $x_E$ | 64% | 1% | 20% | 20% |
| $H(\mathbf{P})$ | 1.2518 | 1.4971 | 2.1710 | 2.3219 |

Even though $H(\mathbf{P})$ may be employed as a fairness measure, the quality of measuring the fairness is not clear yet. For example, how sensitive they are to the allocation changes and whether they can locate the unfairness. Similar to Jain's index, complete information of nodes and resource is required in the metric.

*3) Other Measures:* There are some other measures of variability adopted by researchers to measure fairness. For the sake completeness we have listed them here.

- Difference or ratio between the highest and lowest values of particular performance parameter could also as a fairness measure [14], [15].
- Unfairness as a measure [16] can also be used to measure fairness.
- Measures of variability such as Lorenz curve and Gini coefficient can be used since they have the ability to reflect distribution gap [17], [18]. However, they are not as widely employed as other measures in fairness studies.

### *B. Qualitative Fairness Measures*

Qualitative fairness measures are not able to provide a measurement of fairness with a real number representation however, they can judge whether the allocations achieve fairness. Two most representative measures are max-min and proportional fairness. We explicate them in the sequel.

*1) Max-min:* A feasible allocation[3] of resource $x$ to $n$ users, is max-min fair if for each user $i$, $x_i$ cannot be increased (while maintaining the feasibility) without decreasing $x_j$, where $x_j \leqslant x_i$, $(i \neq j)$ [19]. In other words, a system reaches max-min fairness, if it cannot increase any individual's resource without decreasing another individual's resource allocation which is already less than the previous ones. Max-min fairness (or bottleneck optimality) has been studied widely and implemented in many applications, such as flow control, bandwidth sharing, radio channel accessing, etc [19]–[22].

Similar to max-min fairness, we can define min-max fairness which is a feasible allocation of resource $x$ to $n$ users, is min-max fair if for each user $i$, $x_i$ cannot be decreased (while maintaining the feasibility) without increasing $x_j$, where $x_i \leqslant x_j$, $(j \neq i)$. Min-max fairness is contrary to max-min fairness, which is discussed in [19].

In a system with perfect max-min fairness, based on the above definition, every individual gets exactly the same amount of resource. For example in Fig. 1, if nodes do not get the equal network capacity such as 10%, 20%, 30%, 30%

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[3] A *feasible* allocation indicates that in any allocation round, the resource can be re-allocated more than once in order to reach fairness amongst individuals.

and 10% for A, B, C, D and E, respectively, it is not max-min fair, because we can increase A's capacity (10%) by decreasing B's (20%) which is not less than A's. The same results can be seen in min-max fairness. On the other hand, if all nodes obtain equal amount of network capacity (20%), then none of the nodes can increase its capacity without decreasing the capacity of other nodes which is no more than its own capacity.

"weighted max-min fairness" was proposed in [21] by involving various individual weights in allocation. With a weight set $\mathbf{W} = \{w_i | w_i \in \mathbb{R}^+\}$, $\mathbf{X}$ is weighted max-min fair if $x_i$ cannot be increased (while maintaining the feasibility) without decreasing $x_j$, where $x_j/w_j \leqslant x_i/w_i$, $(i \neq j)$. A more general definition was given in [22], where the weight set $\mathbf{W}$ was replaced by a weight function $u(\mathbf{X})$. These definitions provide a flexibility in achieving fairness, but poses other issues such as the assignment of the weight set $\mathbf{W}$ and framing the function $u(\mathbf{X})$.

Max-min and min-max fairness are well-studied topics in wireless networks. Max-min fairness has many modified versions [23]–[34]. Many allocation methods based on max-min fairness has been proposed in the literature. A max-min fairness framework for flow control in ad hoc networks on MAC layer was proposed in [35]. Additionally, several max-min algorithms were provided, which were based on a flow contention graph. Max-min rate allocation always results in strictly equal rates in lifetime-limited ad hoc networks [36], with any of the other constraints such as channel, topology, choice of routes, and power. Trying to achieve equal rates may result in severe inefficiency in wireless networks. However, as shown in [36], in several cases max-min fairness does not lead to equality in wired networks, clustered networks and in long-term average power constraint of wireless networks. It is shown in [19] that max-min fairness does not exist in some cases based on theoretically feasible rate sets. Several max-min and min-max algorithms are proposed both with and without weight. A redefinition of max-min fairness in flow control and rate allocation were proposed in [24] by treating the occupying time of flows as resources. There are still many other formulations by defining different optimization problems based on max-min or min-max fairness. Some examples are multiple resource max-min fairness and distributed max-min fairness [19], [24], [33], [37]–[39]. There are some confusions regarding max-min fairness and maximizing minimal-value problems. Max-min fairness is a measure to judge whether allocations are fair or not. However, the maximizing minimal-value problems are optimization problems trying to equalize the allocations. Here is an example,

$$\max_{\mathbf{X}\in\chi}(\min_{x_i\in\mathbf{X}}(x_i)), \tag{3}$$

where $\chi$ is the set of all possible resource allocation among individuals.

Another variation of maximizing minimal-value problems is the minimizing maximal-value problems, and an example is given in Eq. (4).

$$\min_{\mathbf{X}\in\chi}(\max_{x_i,x_j\in\mathbf{X}}|x_i - x_j|). \tag{4}$$

The purpose of maximizing minimal-value optimizations is to try to improve the resources of the individuals who get the least amount of resource. However, minimizing maximal-value optimizations attempt to decrease the largest gap between individuals in a resource allocation. If $x_i$ is simply the amount of resource, then the optimistic allocation in both these cases will be the equal distribution amongst individuals. Even though max-min fairness is different from the maximizing minimal-value optimizations, the former may be achieved by latter ones in resource allocation.

*2) Proportional fairness:* Proportional fairness was first proposed by Frank Kelly in [40] based on changing rate control for elastic traffic in computer network services. Proportional fairness is proposed in multi-resource allocation. Since proportional fairness was first presented for routing allocations in [40], we rewrite it as a common fairness measure by updating our basic model proposed in the beginning of this section.

Consider a system with a set of resources $\Lambda = \{\Psi_j | j = 1, 2, \cdots, m\}$, and let $C_j$ be the finite capacity of resource $\Psi_j$, and $X_j = (x_{j1}, x_{j2}, \cdots, x_{jn})$ is the allocation of resource $\Psi_j$ among $n$ individuals, and $x_{jk}$ is the amount of resource $\Psi_j$ allocated to the individual $k$. An individual $i$'s allocation can be written as, $x_i = (x_{1i}, x_{2i}, \cdots, x_{mi})$, where $i \in 1, 2, \cdots, n$. Allocation for user $i$ is proportional fair if it satisfies the following three conditions [40].

- $x_{ji} \geqslant 0$.
- $\sum_{i=1}^{n} x_{ji} \leqslant C_j$.
- For any other allocation $x_i^*$, the sum of differences between $x_i^*$ and $x_i$ is zero or negative. That is,

$$\sum_{j=1}^{m} \frac{x_{ji}^* - x_{ji}}{x_{ji}} \leqslant 0. \tag{5}$$

Proportional fairness measures multi-resource allocation and it is based on the view of single individual instead of one kind of resource. Fairness for each individual on the different kinds and amounts of resources allocated to it is measured. Proportional fairness may become max-min fairness under certain conditions, and further details can be found in [40].

A compact comprehensive convex utility set is necessary to achieve the trade-off between fairness and utility [40]. An example of individual utility function for $x_{ij}$ is also given in [40], which is $u(x_{ij}) = log(x_{ij})$. The system utility can be written as $\sum_i \sum_j log(x_{ij})$. There are some other utility functions in the literature along with the proportional fairness [41], [42]. For instance, the utility functions in [40] are generalized to a certain strictly log-convex and non-compact sets with an example of SINR-based utility functions in wireless networks. It is proved in [36] that proportional fairness is robust and can achieve better trade-off between utilities than max-min fairness when particular utility-metrics are involved during rate allocation in ad hoc networks.

There are some variations of proportional fairness in the literature. A $(p, \alpha) - proportional$ fairness[4] was proposed in [43]. Let $P = (p_1, \cdots, p_m)$ be the weight of an individual $i$

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[4] $(p, \alpha) - proportional$ is also named as weighted $\alpha - proportional$ fairness in [10].

and $\alpha$ be a positive number. $x_i$ is $(p, \alpha) - proportional$ fair if it satisfies the following conditions [43]:

- $x_{ji} \geqslant 0$.
- $\sum_{i=1}^{n} x_{ji} \leqslant C_j$.
- For any other allocation $x_i^*$,

$$\sum_j p_j \frac{x_{ji}^* - x_{ji}}{x_{ji}^{\alpha}} \leqslant 0. \tag{6}$$

In $(p, \alpha) - proportional$ fairness, vector $P$ assigns different weights for different resources in multi-resource allocation, which provides flexibility and controllability in the allocation procedure. When $p_i = p_j (i \neq j)$ and $\alpha = 1$, $(p, \alpha) - proportional$ fairness reduces to proportional fairness, and when $\alpha$ becomes very large, it converges to max-min fairness [43].

An optimization problem is discussed in [43] maximizes utilities (as $\sum_i p_i u(x_{ji})$) and guarantees a $(p, \alpha) - proportional$ fairness simultaneously when utility function is as defined below,

$$u_{\alpha}(x) = \begin{cases} \log x & \text{if } \alpha = 1 \\ (1-\alpha)^{-1} x^{1-\alpha} & \text{if } \alpha \neq 1. \end{cases} \tag{7}$$

Mo J. and Walrand J. have proved in [43] that when $u = u_{\alpha}$, the allocation $\mathbf{X}_i$ reaches maximal utility if and only if $\mathbf{X}_i$ is $(p, \alpha) - proportional$ fairness. Furthermore, it is shown in [10] that $(p, \alpha) - proportional$ fairness can be characterized by using the $\alpha - divergence$ measure. Uchida M. et al., proposed a new utility function in [10] by multiplying $log(x)$ by weights. A similar utility function can be seen in [44].

$(p, \alpha) - proportional$ fairness-utility trade-off mechanisms were proved to have some advantages, it is not sure that these utility functions can imply the real network performance. For example, it is hard to predict the influence of rate allocation on packet delay, because there are other factors which influence the packet delay, such as the link quality and computational capacity of hosts. To build a reasonable uniform utility function is even harder. Additionally, proportional fairness requires the whole allocation information, which makes it difficult to be used in distributed wireless networks. Therefore, most work in the literature uses numerical methods.

### *C. Tian Lan's Model*

Tian Lan's model was first proposed in [11]. As a quantitative fairness measure, it is able to converge to Jain's index, entropy, max-min and proportional fairness with different values of its parameters. It provides a general research platform for fairness measures. We rewrite this model with more commonly used symbols to make it coherent with the rest of this paper. We first present the kernel of Tian Lan's fairness. It satisfies five axioms. We list the five axioms of fairness measure $f(\mathbf{X})$ below [11],

- Axiom of Continuity: $f(\mathbf{X})$ is continuous on $\mathbb{R}_n^+$, $\forall n$ (*R1*).
- Axiom of Homogeneity: $f(\mathbf{X})$ is a homogeneous function of zero degree; $f(\mathbf{X}) = f(t\mathbf{X})$, $\forall t > 0$. Without loss of generality, for a single user, we take $|f(\mathbf{X}_1)|$ for all $\mathbf{X}_1 > 0$, i.e., fairness is a constant for $n = 1$.
- Axiom of Asymptotic Saturation: $f(\mathbf{X})$ eventually becomes independent of the number of users,

$$\lim_{n \to \infty} \frac{f(l_{n+1})}{f(l_n)} = 1, \tag{8}$$

  where $l_n$ is a vector of length $n$.
- Axiom of Irrelevance of Partition: If we partition the elements of $\mathbf{X}$ into two parts $\mathbf{X} = [\mathbf{X}^{(1)}, \mathbf{X}^{(2)}]$, the fairness index $f(\mathbf{X}^{(1)}, \mathbf{X}^{(2)})$ can be computed recursively (with respect to a generating function $g(y)$) and is independent of the partition, i.e.,

$$\begin{aligned} f(\mathbf{X}^{(1)}, \mathbf{X}^{(2)}) = f(\omega(\mathbf{X}^{(1)}), \omega(\mathbf{X}^{(2)})) \\ g^{-1}(\sum_{i=1}^{2} s_i g(f(s^{(i)}))), \end{aligned} \tag{9}$$

  where $\omega(\mathbf{X}^{(1)})$ and $\omega(\mathbf{X}^{(2)})$ denote the sum of resource vectors $\mathbf{X}^{(1)}, \mathbf{X}^{(2)}$, and $g(y)$ is a continuous and strictly monotonic function that can generate the following function $h$,

$$h = g^{-1}\left(\sum_{i=1}^{2} s_i g(f(\mathbf{X}^{(i)}))\right), \tag{10}$$

  with positive weights satisfying $\sum_i s_i = 1$ such that $h$ qualifies as a means of $\{f(\mathbf{X}^{(i)}), \forall i\}$.
- Axiom of Monotonicity: For $n = 2$, fairness measure $f(\theta, 1-\theta)$ is monotonically increasing as an absolute difference between the two elements, i.e. $|1 - 2\theta| \to 0$.

Based on these five axioms, a unified representation of fairness measure is proposed in [11],

$$f_{\beta}(\mathbf{X}) = sign(1-\beta) \left[ \sum_{i=1}^{n} \left( \frac{x_i}{\sum_j x_j} \right)^{1-\beta} \right]^{\frac{1}{\beta}}, \tag{11}$$

where the $sign(\cdot)$ is the sign function and $\beta$ is a constant for an allocation. When $\beta \in (0,1) \cup (1, \infty)$, $f_{\beta}(\mathbf{X})$ is proportional fairness. When $\beta \to 0$ it turns out to be the entropy fairness. When $\beta = -1$, it becomes Jain's index. Further details of its convergence results and proofs can be found in [11].

Since five axioms for fairness measure are more general than others, Tian Lan's model provides guidelines for future design of fairness metrics. However, it still requires the complete information of the system, which limits the applicability of distributed network algorithms and their implementation. Besides, it cannot identify the unfairly treated individuals.

### *D. Fairness Optimization Model*

Fairness optimization models focus on methods and strategies to optimize fairness and utilities. These models have a "decision-phase" during resource allocation, and they affect allocations and utilities simultaneously. Envy is a measure of degree of dissatisfaction of individuals to the resource allocation. It was proposed in [45], wherein the measurement of fairness was based on different types of envy and degree, e.g., pairwise envy, individual envy, group envy, *ex post* unfairness/dispersion of envy, and *ex ante* unfairness. All these are based on the individual utility functions $u_i(x_j)$, which is an evaluation function to measure the influence of the allocation to $i$'s utility, and $0 \leqslant u_i(x_j) \leqslant 1$, where $u_i$ is utility of individual $i$, and $x_j$ is the amount of resource allocated to individual $j$.

Pairwise envy reflects the envy degree between two individuals, and is calculated from the utilities of two individuals. Sum of all pairwise envy levels of a node with respect to all other nodes is the individual envy. Group envy, ex post unfairness and ex ante unfairness is related to the dissatisfaction of a group, the distribution of envy, and the sensitivity of envy, respectively. Resource allocation is decided based on both fairness and system utility. The details of envy functions proposed in [45] are listed as follows.

- **Pairwise envy:** taking $u_i(x_i)$ as the utility function, for a given allocation $\mathbf{X}$, the envy of individual $i$ against individual $j$ is defined as,

  $$e_{ij}(\mathbf{X}) = u_j(x_j) - u_i(x_i). \tag{12}$$

  If $e_{ij}(\mathbf{X}) \geqslant 0$, individual $i$ is envious of $j$'s handle of resources compared to its own.
- **Individual envy:** the envy of individual $i$ is,

  $$e_i(\mathbf{X}) = \left( \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} e_{ij}(\mathbf{X}) \right), \tag{13}$$

  where $n$ is the number of individuals. The individual envy is the average of its pairwise envies.
- **Group envy:** $e_G(\mathbf{X})$, the group envy is defined as,

  $$e_G(\mathbf{X}) = \frac{1}{m} \sum_{i \in \eta} e_i(\mathbf{X}), \tag{14}$$

  where $\eta$ is a set of $m$ individuals. The group envy is the average of individual envies in a group.
- **Ex post unfairness/dispersion of envy:** the ex post unfairness is defined as,

  $$I_P(\mathbf{X}) = \frac{4}{n} \sum_i (e_i(\mathbf{X}) - \overline{e}(\mathbf{X}))^2, \tag{15}$$

  where $\overline{e}(\mathbf{X})$ is the mean value of $e_i(\mathbf{X})$. $I_P(\mathbf{X})$ is the variance of individual envies.
- **Ex ante unfairness:** let $A$ be the set of available actions and $S$ be the set of possible states $s$; $a \in A$ is an allocation and $a(s) = (x_1, x_2, \cdots, x_n)$ represents action $a$ resulting in an allocation $(x_1, x_2, \cdots, x_n)$. $p(s)$ is the probability that a particular state $s$ is the best state that can be chosen for allocation. Then the average expected envy may be written as,

  $$E(\overline{e}(a)) = \sum_{s \in S_a} p(s) \overline{e}(a(s)), \tag{16}$$

  where $E[\overline{e}(a)]$ denotes the expected value of envy . Further, $I_A(a)$, the ex ante unfairness, is defined as follows:

  $$I_A(a) = \frac{4}{n} \sum_i (E[e_i(a)] - E[\overline{e}(a)])^2. \tag{17}$$
- **Fairness:** group envy and ex post unfairness are independent additive fairness if ($e_G(\mathbf{X})$, $I_P(\mathbf{X})$; $e_G(\mathbf{X}')$, $I_P(\mathbf{X}')$) and ($e_G(\mathbf{X})$, $I_P(\mathbf{X}')$; $e_G(\mathbf{X}')$, $I_P(\mathbf{X})$) are judged equally fair, where $\mathbf{X}$and $\mathbf{X}'$ are different allocations. Then these two imply the form $G(\mathbf{X}) = -K_D e_G(\mathbf{X}) - K_P I_P(\mathbf{X})$, where $K_D$ and $K_P$ are non-negative scaling constants. $G(\mathbf{X})$ is an intermediate fairness measure for an allocation $\mathbf{X}$.
  Then, fairness function $F(a)$ is defined as,

  $$F(a) = \left[ \sum_{s \in S_a} p(s) G(a(s)) \right] - K_A I_A(a) + 1, \tag{18}$$

  where $K_D + K_P + K_A = 1$ indicates the relative importance of the three parameters defined earlier – $e_G(x)$, $I_P(x)$ and $I_A(a)$, respectively. The range of $F(a)$ is 0 (least fair) to 1 (most fair).
- **Efficiency:** fairness and efficiency are discussed jointly in the envy-based fairness. $V(a)$, the overall evaluation of an action $a$, is expressed as a function of both fairness ($F(a)$) and as a measure of efficiency. The measure of efficiency should satisfy [45]:
  - Pareto Optimality: if the utilities of all individuals $i$ ($i \neq j$) are the same under actions $a_1$ and $a_2$ and the utility for individual $j$ is greater under action $a_2$ than under $a_1$, then the aggregated utility function should imply that $a_2$ is preferred to $a_1$.
  - Anonymity: if $u(\mathbf{X}) = (u_1(x_1), \cdots, u_n(x_n))$ is a permutation of $u(\mathbf{X}') = (u_1(x_1'), \cdots, u_n(x_n'))$, then $u_G(\mathbf{X}) = u_G(\mathbf{X}')$.

  The efficiency is given by,

  $$u_G(\mathbf{X}) = \frac{1}{n} \sum_i u_i(x_i). \tag{19}$$

  Further, efficiency for an action $a$ is given by,

  $$u_G(a) = \sum_{s \in S_a} p(s) u_G(a(s)). \tag{20}$$

  $V(a)$ is a non-decreasing function of fairness $F(a)$ and efficiency $u_G(a)$. An additive form,

  $$V(a) = K_U u_G(a) + K_F F(a), \tag{21}$$

  is the preference for action $a$, which may be appropriate and justified to the group. Other forms (such as multi-linear) of social preference may also be defined.

The procedure of making allocation decisions is presented in Fig. 2 [45]. With a given resource allocation decision, individual utilities are calculated first. Then, pairwise envy and efficiency is obtained based on individual utilities. Computing base of group envy, ex ante unfairness, ex post unfairness, and individual envy is derived from pairwise envy. Finally fairness is found by considering group envy, ex ante unfairness and ex post unfairness. Both efficiency and fairness are employed in the evaluation of allocation. The envy-based fairness is a

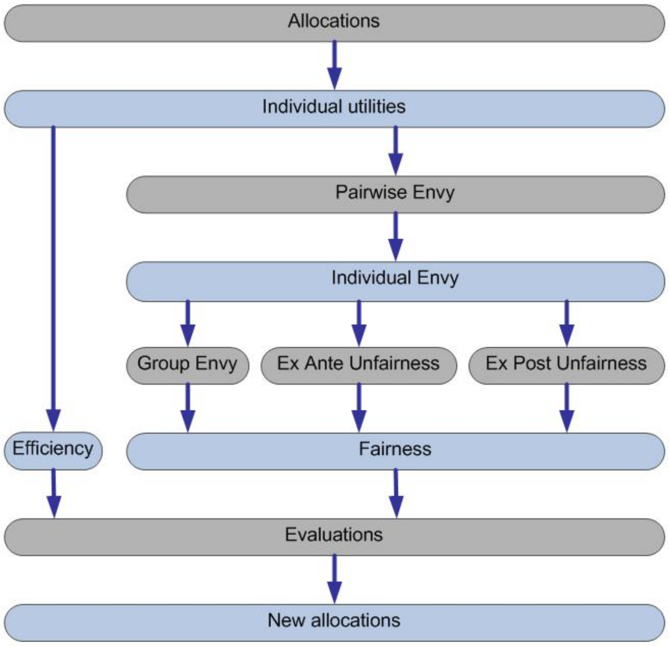

Fig. 2. The working of envy-based fairness (redrawn from [Fig 3 in [45]]).

generic model without specific utility functions and strategies, which give the possibility to be implemented in diverse applications. Trade-off between system utility and fairness is taken into account. Individual weights may be defined by employing different utility functions, and individuals treated unfairly can be identified too. A framework and guidelines for fairness and allocation decisions are demonstrated in this model. However, there are still some open issues, such as how to tune envy-based fairness for specific scenarios, how to find concrete utility functions and select the constants.

### *E. Comparison of Measures*

Both quantitative and qualitative fairness measures have many advantages and shortcomings. For example, the quantitative measures provide a real number revealing the level of allocations. With this it is easy to compare two allocations and choose the fairer one. However, these measures need the complete information, which is not easy in distributed systems. Based on qualitative measures optimization problems, generally, simplify the decision-making procedure for fair allocation. However formulation of utility is still an open issue.

We compare these measures in Table III considering various attributes. The first three attributes, *Definition*, *Measurability* and *Capability* , are based on the core questions **Q1** , **Q2** and **Q3** . *Weight* gives individuals priorities in allocation. *Utility* reflects on whether measures consider trade-off between utility and fairness. *Control* discusses whether the system is distributed or centrally controlled. *Data* indicates whether the measure requires complete information of the system. *Requirements* denotes the requirements (R1 to R6) that are satisfied by measures. These requirements are proposed at the beginning of this section. *Complexity* describes the complexity of the computation of the measures – three levels simple, medium and complex are used here for comparison. Table III shows that none of the existing measures can perform well with respect to all the attributes, thus combination of different measures in resource allocation may be an option.

## IV. FAIRNESS ISSUES IN WIRELESS NETWORKS

Research on fairness in wireless networks has increased rapidly in the last decade. This increase does not only imply the growth of wireless technologies and applications, but also reveals the growth of interest in fairness research. We can find several steps in its evolution.

1) Analysis-phase: Most of fairness discussions in this period treats fairness as equality, and there was no specific measuring method used, for example, in [46]–[48]. Another example is first-in first-out queuing strategy. This is treated as the fairest case in [49] using head-of-line servicing.
2) Notional-phase: Some simple fairness measures for wireless networks were introduced in this period. For instance, in [50] a “uniform fairness”, which is the difference between the maximum and minimum values, was proposed as a constraint in optimal channel-selection scheme. Fairness was defined as the growth of queue in [51] during bandwidth allocation for ATM networks.
3) Development-phase: Many fairness indices and optimal models such as Jain’s index, max-min and proportional fairness are evolved and applied until now. For example, a combination of max-min fairness and spectrum efficiency was adopted in [52]. Proportional fairness was analyzed in multi-channel and multi-rate wireless networks in [44], [53] and [54].

Topics on fairness were studied in both resource allocation and utility optimization. Therefore, we classify these topics from the points of view of resource allocation and utility/performance measure. Some examples are shown in Table. IV. To show explicitly where exactly these fairness issues are used, they are also categorized into different layers: Physical, MAC, LLC, Network, Transport, and Application layers. In Table IV we introduce fairness topics in wireless networks.

Since most of these topics require cross-layer design, we summarize and discuss the research on fairness in wireless networking based on the application domain instead of the layers of OSI model. Fairness needs to be considered in almost all resource allocation related problems in wireless networks. In this paper, fair energy consumption control, power control, topology control, link and flow scheduling, channel assignment, rate allocation, congestion control and routing protocols are discussed. These issues are not isolated from each other, because many of them may co-exist and influence each other. For instance, fair energy consumption may be influenced by all other fairness issues in a multi-channel ad hoc network. However, the ambit of these fairness issues is different. Fair energy consumption is to enhance the network’s life time. Fair power control is to assign powers fairly to wireless nodes. To find the best logical topology in a

TABLE III
COMPARISON OF FAIRNESS MODELS.

| **Models** | **Jain's index** | **Entropy** | **Max-min** | **Proportional** | **Tian Lan's** | **Envy-based** |
|---|---|---|---|---|---|---|
| **Definition** | Yes | No | Yes | Yes | Yes | Yes |
| **Measurability** | Yes | Yes | No | No | Yes | Yes |
| **Locating unfairness** | No | No | No | No | No | Yes |
| **Weight** | No | No | Yes | Yes | No | No |
| **Utility** | No | No | No | Yes | No | Yes |
| **Control** | Centralized | Centralized | Both | Centralized | Centralized | Both |
| **Data** | Full | Full | Both | Full | Full | Both |
| **Requirements** | R1, R2, R3, R5, R6 | R1, R2, R5, R6 | No | No | R1, R2, R3, R6 | R1, R2, R3, R4 |
| **Algorithm** | Simple | Simple | Complex | Complex | Complex | Complex |

TABLE IV
EXAMPLES OF FAIRNESS ISSUES IN WIRELESS NETWORKS.

| Layers | | Fairness issues | |
|---|---|---|---|
| | | Resource allocation | Performance/utility measeurments |
| Application | | Network resources should be shared fairly among different applications such as Internet bandwidth. For example, the on-line video application for two users in a LAN should share the bandwidth fairly. | The applications on one wireless device or different devices should have fair utility in a wireless environment. For instance, two devices running online video application should provide fair continuous realtime playing and quality of videos. |
| Transport | | Flow control at transport layer, multiplexing ports, and data buffering. | Quality of Service (QoS) among different end terminals such as congestion control, reliability, end-to-end communication and data loss rate. |
| Network | | Routing path choices, routing discovery, message forwarding, and flow control on network layer. | Load balancing for wireless routers and security on network layer amongst nodes. |
| Data link | LLC | Flow control on LLC, Error detections, and multiplexing among different upper layer protocols such as IP. | Bit error rates (BER) and packet delay. |
| | MAC | Multiple channel accessing such as collision avoidance using TDMA or CSMA/CA, data packet queuing and scheduling on physical layer, channel sharing among different wireless technologies, such as Bluetooth, WiFi and zigbee, and flow control on MAC layer. | Environment interference among nodes, and transmission power control. |
| Physical | | Transmission power assignment on antennas and battery energy allocation. | Lifetime, energy consumption, and other equipment performance. |

fair way is the main goal of fair topology control. Fair link and flow scheduling focus on fairly allocating links to flows. Fair channel assignment allocates channels fairly to nodes. Fair rate allocation and congestion control balances the rates on links without causing any congestion. Fair routing protocol mainly balances the load amongst nodes acting as routers. Because of different focuses and purposes, these fairness issues are discussed separately to give a clear view of fairness study in wireless networks.

### *A. Fair Energy Consumption Control*

Most of fairness studies are concerned with the energy issues in battery operated networks, in order to achieve increased lifetime, energy balancing and saving strategies are required. For example, to prolong the network lifetime significantly, energy consumption in the backbone nodes is reduced in [55]. Fair energy consumption is mostly studied in energy-efficient routing protocols and combinatorial strategies of sleep scheduling and topology control in wireless networks.

Energy consumption is significantly affected by the routing protocols, especially for routers in wireless networks. Hence, cross-layer design between MAC and network layer is mostly employed. Energy consumption constraints were set for every node in sensor networks [56] in order to balance the energy consumption. Since no flow generation and routing of packets are allowed when the energy consumption exceeds the threshold, nodes save energy and extend the network lifetime. An intra-cluster routing protocol was proposed in [57] for sensor networks. The residual energy in nodes is considered when routing paths are chosen, which brings in fair energy consumption and extends network lifetime. A similar fair energy consumption study can be found in [58], [59]. Similar rate of energy consumption as the fairest case is the goal of these studies.

Sleep scheduling is always considered together with topology control. Four states for nodes are involved: sleep, idle, receive and transmit [60], [61] in most scheduling. In order to guarantee the functionalities of the network while some nodes sleep, clustering strategies are normally adopted. Clustering strategies choose the cluster head periodically and the cluster head changes topologies by scheduling sleep nodes, in order to balance energy consumption. In this case, fairness is treated as one of the performance indices of the network. For example, an energy-efficient MAC protocol could use virtual clusters in an ad hoc sensor network [62]. It has resulted in reduction of per-node fairness and latency, but end-to-end fairness is not affected much. Different cluster heads are chosen over time to balance energy consumption [63]. Combination of cluster head selection and sleeping schedule is discussed in [64]. Fair sleep scheduling, by giving all nodes equal opportunity to be in backbone path is discussed in [65]. Sleeping period of nodes was assigned due to its rate in [66], which partially guarantees the rate-based-weighted fairness. Both rotational sleep scheduling and fair use of residual energy – for energy-efficient operation – is considered in the algorithm proposed in [67] for mobile ad hoc networks. Even though fairness is considered with respect to sleep scheduling by many researchers, it is still treated simply as equal sleep time without involving any advanced models.

In the current literature, notion of fair energy consumption is still being seen as "equal energy consumption". Max-min, proportional fairness, and even Jain's index are seldom used in measuring and reaching energy fairness. We believe that all these fairness measures and the concept of weighted fairness and utilities should be considered in fair energy consumption and control. No significant studies on its impact on the performance of the network, traffic, energy consumption are provided. These aspects should be recognized as general requirements while balancing of energy consumption.

### B. Fair Power Control

Power control assigns transmission power levels to nodes. Network topologies can be changed by power assignment, which influence channel assignment, link scheduling, routing and rate allocation significantly. Therefore, power control may extend lifetime and increase the channel efficiency [68] too. Two types of fair power control studies are seen in the literature most of the time. First type treats fairness as a separate utility. The second one studies fairness properties via fairness of throughput and delay.

In the first type of fair power control, fairness is assigned with a separate utility function. For example, in [69] the utility function for proportional fairness as defined in [43] is adopted. Proportional and max-min fairness are formulated together in [30] with power control and rate allocation by rewriting the utility function in [43]. Some trade-off can also be found between fairness and other utilities, when it is formulated individually. The study in [70] shows that power control may cause unfairness in TDMA ad hoc networks if only the slot utilization is considered. A trade-off between fairness and delay can be seen when power control strategies are adopted [69].

An optimization problem on maximizing network throughput via power control was proposed in [71], and individual throughput fairness was adopted as a constraint. A fairness coefficient was proposed in [71] which is similar to Jain's index, and only measures the overall system fairness. A fairness threshold was set during power control, which guaranteed the system fairness at a certain level, but unfairness and starving of individuals may still happen. Short distance packets are transmitted much more than long distance packets in IEEE 802.11 networks and a Power Controlled Multiple Access Protocol (PCMA) is proposed in [68]. PCMA is based on variable power bounds which can achieve fairness amongst nodes in accessing the channel. Lujun [72] extended PCMA by a novel power function with distances, which provided better results in throughput, delay and fairness at the same time. Fairness was treated as an input of the power control strategies in the form of average link quality constraints in [73]. Zhu et al., believed that the maintenance of equal average link quality ensured fairness between links.

The main challenge in fair power control is that it involves multiple layers in wireless networks, such as network layer dealing with fair routing strategy [25], MAC layer for fair spectrum access [28], [29], [68], [72], and physical layer for transmission power management [26], [27], [38], [74]. Thus, cross-layer design for power control makes it even more difficult and complex for achieving fairness. Another problem is that, even though fairness notions are referred in power control tremendously, only few articles have mentioned advanced fairness measures, such as max-min and proportional fairness. We believe that the involvement of new fairness measures may change power control strategies in the future.

### C. Fair Topology Control

Topology control in wireless networks is done by controlling transmission power and thus managing the range of each node. Both topology and power control have some common objectives: minimizing interference, reducing MAC collisions and energy conservation [75]. Power control focuses on MAC and physical layer, such as power assignment to antennas. While topology control normally works mostly at the network layer. Graph theoretic approach, game theory and other analytical tools are always adopted by topology control mechanism to find the best logical topology for a particular application.

Fair topology control adds fairness as another objective to the general topology control. Unfairness is more widely discovered in small topologies than in large topologies when CSMA/CA based medium access protocols are applied [76]. The unfairness is mainly caused by unfairly treating the border nodes, which indicates the importance of fair topology control. Similar to fair power control, two types of fair topology controls are defined. In the first type fairness is distinguished from other utilities and a separate fairness function is used to measure fairness. The second one measures the fairness of utilities, such as interference fairness and fairness in resource access probability. For example, a separate fairness function was adopted in [28], [43], [77]. A distributed method was proposed in [77] for aloha networks, which only required two-hop information to achieve proportional fairness. Proportional fairness was considered during network topology establishment in [28]. Fairness is treated as a congestion measurement tool in [78] for logical network topology, and max-min fairness is reached by maximizing the minimal free bandwidth on logical links. Jain's index is also adopted in this work as a quantitative fairness measure for the flows, and the fairness function in [73] is also used. A novel method of combining routing protocols and auxiliary graphs based on virtual network topologies are proposed in [79]. Load fairness, as one main purpose of this method, is measured by Jain's index under various offered traffic.

The main challenge of fair topology control is to find the influence of fairness models on wireless topologies, and how to dynamically measure and adjust fairness in topology control. More advanced and particular fairness functions based on the targeted application should also be developed. Cross-layer and joint design with fair power control, link and channel scheduling, routing protocols and congestion control are the other prominent challenges in this regard.

### *D. Fair Link and Flow Scheduling*

Link and flow scheduling mechanisms build interference maps between links and then allocate the collision-free links to flows dynamically. Interference maps are collision maps for links which cannot transmit or receive packets at the same time because of interference. Link allocation tries to allocate as many possible non-interfering links under current data flow requests attempting to reach high throughput and low packet delay. In fair link and flow scheduling, the flows are the individuals that are assigned with links as resource. Hence, the problem is to find whether flows are treated fairly. Two measures of fairness in the literature: fair transmission opportunity and fair bandwidth allocation.

Fair transmission opportunity among flows is known as TXOP in the study of IEEE 802.11 MAC protocols. A rate adaptive mechanism was proposed in the study of MAC protocol in [80]. Packet drop rates in different flows are presented and compared to show the fairness property of the proposed MAC protocol. However, no quantitative measurements were provided and strict equality were believed as fairness in [80]. A similar fairness notion can be found in another rate adaptive TXOP mechanism for IEEE 802.11e in [81], which ensures long-term fairness. However, most of studies of TXOP fairness in the literature simply consider equality problems and hardly discuss the priorities of flows.

Bandwidth allocation is the other aspect of link scheduling, which treats link bandwidth as resources and allocates them amongst flows. For fair bandwidth allocation, flows are trying to get fair link capacity. Strict bandwidth fairness among flows can improve the isolation between nodes, predict network performance more easily and discover bottlenecks as shown in [82]. A relative fairness bound was adopted as fairness measure, which is the maximal bandwidth difference between any two flows. It is a simple bandwidth fairness measurement which tries to achieve equality in allocated bandwidth amongst flows. The same relative fairness bound was employed in [83], and bandwidth fairness were jointly considered in multiple resource allocation. A similar joint bandwidth and resource allocation can be found in [84] however max-min fairness was used. Trade-off between fairness and throughput was discussed during bandwidth allocation in [85]. A demand satisfied factor (DSF) was employed in their algorithm, and fairness is achieved by maximizing the minimal DSF of flows. An algorithm was proposed in [86] to guarantee fairness by allocating the capacity of links amongst flows by setting capacity thresholds for flows and adopting a strategy of slapping penalties. A “rate anomaly” problem in IEEE 802.11 Wireless LAN employing Distributed Coordination Function (DCF) option can be found in [44]. The essence of this problem is the unfair bandwidth usage among wireless devices in the same LAN. Proportional fairness and weights based utility function was proposed, and it ensures fair bandwidth sharing.

Fair link and flow scheduling influence node and network performance significantly and in many aspects, for example network capacity, packet delay, and energy consumption. Therefore, the main challenge in fair link and flow scheduling is the trade-off between fairness and utilities. We also suggest that weighted flows should be considered in fair link and flow scheduling since not each flow is equally important.

### *E. Fair Channel Assignments*

Channel assignment is mostly considered in multi-channel networks, such as cognitive radio networks (CRNs), heterogeneous wireless networks and wireless mesh networks (WMNs). The major function of channel assignment is to allocate radio and channel resources to flows or links and achieving seamless transmission. Fair channel assignment adds the notion of fairness into radio allocation, which guarantees the fair sharing of channels amongst flows and links.

Heterogeneous wireless networks integrate different types of wireless technologies and try to achieve successful migration. Hence, fair channel sharing is required amongst different flows, nodes and networks. Throughput, capacity, delay and other performance should be considered at the same time during channel allocation. For example three channel assignment schemes (least channel, minimizing interference and maximizing capacity) were proposed in [87] considering both internal and external interference, and Jain’s index was adopted to measure the fairness amongst flows. Capacity is regarded as the main goal of the channel assignment scheme in [88], and weighted fairness is adopted as one of the constraints. Channel hopping brings easy migrations among networks, but it may increase the energy consumption, delay and packet drop rate. Therefore trade-off between hopping overhead and flexibility is considered in [89]. Fairness of successfully transmitted flows was simply compared with different throughput in [89], but further study on how to guarantee fairness was not presented. Fair channel assignment is always studied in a cross-layer fashion with routing protocols, bandwidth allocation, rate allocation, etc. For instance in [90], routing protocols were combined with fair channel assignment. A simple fairness constraint for routing was set to prevent starvation. Logical topology control, interference control, and routing protocols are jointly considered in [78]. Cross-layer design of rate allocation, flow scheduling and channel assignment can be seen in [91], which also adopt Jain’s index as the fairness measure.

In cognitive radio networks the radio channels are reused when they are not occupied by primary users. Spectrum management in CRNs allocates white space to secondary users. The sharing and assignment of spectrum becomes a crucial issue in cognitive radio networks [30], [92] since it influences the network performance directly. Two interference models for CRN channel assignment can be found in the literature based on: (a) signal to interference noise power ratio (SINR) and (b) graph theory based.

SINR based channel assignment builds the interference map between channels by fading models. Hence, SINR based channel assignment is always studied together with power control. For example in [93] both SINR and fairness constraints are considered during channel assignment in CRNs and Jain's index measures the fairness of SINR of secondary users. Both power and rate control were jointly studied with channel assignment based on SINR. Optimization problems were formulated based on max-min and proportional fairness were in [30]. A trade-off between fairness and network utilities in SINR based channel assignment was addressed in [94], in which proportional fairness was employed.

Graph theory based channel assignment addresses collisions between channels using graph theoretic approaches, in which traditional graph theoretic algorithms are applied. Fairness in this type of CRN channel assignment studies are mostly treated as maximizing one of the network utilities. For instance, in [95], a maximum matching and multiple coloring algorithm based channel assignment were proposed. It is proved that network adopting max-min fairness could achieve good throughput under congestive traffic. Similarly, application of graph coloring problems can be found in [96] and the access fairness among secondary users are also described. Several graph coloring algorithms are proposed in [97] and the notion of fairness here is to provide equal rates to each user. Similar problem formulations can also be seen in [98]–[101].

There are several major challenges in fair channel assignment. One of them is precise collision modelling (interference mapping), which may combine SINR, graph theoretic approach and dynamic sensing techniques. Furthermore, joint and cross-layer design is required in channel assignment, which has to manage many resources as well as the system utility. It is difficult to address the relationship between fairness, resources allocation and performance due to the unpredictability of wireless networks. Advanced fairness models and measures should also be introduced in fair channel assignment.

### *F. Fair Rate Allocation and Congestion Control*

Rate allocation apportions different flows on links simultaneously without causing congestion and performance degradation in multi-rate wireless networks (or multiple-input-multiple-output, MIMO wireless networks). Link scheduling is different from rate allocation. In link scheduling, links are treated as resources and allocated to flows and the focus is on choosing links. Hence, it is always studied jointly with power control and topology control strategies. While rate allocation treats packets and flows as resources and allocates them simultaneously to already existing links. Hence, they are always studied with congestion control and energy efficient strategies. Fair rate allocation is mostly applied at different levels. Fairness of nodes can be used as a feedback during the congestion control [113]. Packet level fairness was studied via proportional fairness in MIMO wireless networks in [102], and fairness in transmission delay, transmission time, and services were also examined. Fair rate control is applied to links [42], and a modified proportional fairness was proposed to measure the rate of achieving fairness. Channel occupation period of nodes can be treated as a method to achieve fair rate allocation too [106], and two types of unfairness are found in IEEE 802.11 distributed coordination function (DCF): (a) performance anomaly and (b) up/downlink unevenness. Some remedies for unfairness are adjustment of occupation period of nodes at the MAC layer and maintenance of fair transmission rate at the Transport layer.

Some trade-off between fairness and performance can be seen, for example, trade-off between fairness and efficiency in ad hoc networks [36], trade-off between fairness and throughput [103], and trade-off between throughput fairness and total network throughput [105]. Max-min and proportional fairness was studied in ad hoc networks. Proportional fairness achieves better trade-off between fairness and throughput than max-min fairness [103]. Unfairness in rate allocation based on IEEE 802.11 networks was studied and trade-off between short-term and long-term fairness in rate allocation can be seen [104]. An important aspect of fair rate allocation is congestion control. Fairness is always treated as an extra performance metric in congestion control strategies, because unfair rete allocation may cause starvation amongst flows citeSathiaseelan2007Multimedia. During congestion, fairness amongst flows can be guaranteed by dynamic updating of weight, when weighted Jain's index is used [107]. Similar congestion control can be achieved in upstream [108]. Individual unfairness[5] on both TCP and MAC can be caused by TCP congestion control in IEEE 802.11 networks [110], since the cumulative acknowledgment mechanism of TCP tends to assign wider window for sending nodes than receiving nodes. A cross-layer scheme was proposed in [110] to eliminate this unfairness, which took feedback from MAC to adjust the TCP congestion window.

Some studies of fair rate allocation can be found in wireless video services because of their high traffic. Equal average video quality is considered as the main goal in it [111], [111]. Max-min fairness is employed in [112] to ensure the equal video distortions amongst users. Amongst all the fairness studies in rate allocation, simple equality, max-min and proportional fairness measures are the most famous. We do not think simple equality addresses fairness in all the scenarios. The weighted measures might help some times. Even though some advanced max-min fairness models are available in the literature – for instance the price based max-min fairness in [114] – most of the studies show that proportional fairness is much better than max-min fairness during rate allocation. For example, max-min fairness always leads to equality with few exceptions, while proportional fairness can reach better trade-off between fairness and efficiency in energy constrained ad hoc networks [36], [103]. Therefore, in rate allocation studies proportional fairness may be a better choice.

### *G. Fair Routing Protocols*

Routing protocols address path discovery and path selection in wireless networks. Path discovery mainly focuses on building network topology and maintaining the connectivity, while path selection chooses the best paths for flows. It is shown in [115] that routing protocols based on only hop count are not

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[5] Node unfairness was called per-station unfairness in [110]. To keep a coherent view in this paper, we use the term "individual unfairness".

suitable and efficient in wireless networks, since link quality and congestion are not considered. A well-organized summary of routing in wireless networks is available in [75]. We study fairness topic in routing protocols concentrating more on its influence on resource allocation and network performance.

The essential fairness issue in routing protocols is in path selection strategies, which fairly distribute flows and packets to paths. Because path selection in higher layer influences the load on the nodes and links, different load lead to different performance. Furthermore, since energy consumption and QoS is influenced by flows, a joint study of fair routing with other fairness issues is necessary. For example power control may influence the network topology and impact routing eventually. Link scheduling and topology control may completely change routing tables. Fair rate allocation reduces congestion in networks and provides more abate routing environment. A joint study of routing with power control and link scheduling is presented in [116]. It tries to maximize fairness and throughput together by adopting all constraints from link layer and constraints imposed due to power control and routing protocols. A routing protocol in sensor networks with the purpose of prolonging network lifetime is proposed in [117], which also achieves fairness amongst flows. A cross-layer study in [103] combined rate control, routing and scheduling in multi-channel wireless mesh networks altogether and proposed algorithms to maintain fairness.

There are still many open issues in fair routing. An important one is the trade-off between fairness and network throughput. Negotiation and compromise amongst nodes and, between nodes and networks make the situation more complicated. Multi-radio and multi-channel wireless networks require cross-layer design of their routing protocols to make full use of the channels and offer fairness at the same time [118]. Fair routing protocol in distributed wireless networks is more complex because of the negotiation strategies among neighbours and the lack of information about the networks [119]. The effect of routing protocols on fairness makes it difficult to develop a good enough algorithm to achieve fairness with better performance at the same time. Thus, a more advanced fairness model is needed to cope with the cross-layer design issues.

## V. FAIRNESS, RESOURCE ALLOCATION AND UTILITY

In the literature, most of the works do not distinguish fairness, utility and resource allocation. Note that the notion of *utility* may mean performance or efficiency in most of the cases and scenarios. For instance, it is often simply defined as throughput, delay, or other performance metrics [4], [28], [120]. The term “utility” in this paper indicates the metric which can indicate whether the resources allocation satisfies an individual. It is usually converted from performance indices except the fairness.

These three notions (fairness, resource allocation and utility) are different:

- *Fairness* aims at the quality of equal treatment to equal individuals of a system.
- *Resource allocation* tries to distribute resources amongst individuals in a system.
- *Utility* of a resource or multiple resources is mainly related to satisfaction or perceived value of the resource to an individual or the whole system. Rational individuals aim at maximizing the satisfaction of the resources allocated to them which in turn impacts the performance.

Fairness can be measured both in allocation and utility as “allocation fairness” and “utility fairness”. Meanwhile, allocation fairness can be treated as a special case of system utilities. Since resource allocation influences utilities directly, achieving acceptable utility is a fundamental principle in resource allocation. Another essential principle for allocation is to guarantee reasonable fairness. Hence, resource allocation is the action while fairness and utility are two rules that constrain allocation. In this section, we explore the properties for each of them and also the interdependence in detail.

### A. *Resource Allocation*

Resource allocation involves the complete procedure of resource distribution in a system. However, in this paper we mainly focus on the allocation strategies which is tightly bound to fairness. We classify resource allocation as the following types.

- **Possession:** The possession reflects the owners of resource. We adopt two possession types: global and individual resources. Global resources belong to systems and allocated at the system level. By contrast, individual resources are held by individuals and allocated at the individual level. For example, channels in wireless networks are global resources but battery-energy is an individual resource.
- **Consumability:** It indicates whether the resource is consumable. For instance, energy in batteries is consumable whereas channels are non-consumable.
- **Allocability:** Allocability considers whether resources are able to be re-allocated. In wireless networks, bandwidth can be re-allocated to different nodes at the same time. However, some resources are individual in nature such the battery as a resource for a node is only used by that node.
- **Quantity:** Single-resource or multi-resource allocations describe the quantity of resource allocation. When only one type of resource is processed in an allocation, it is a single-resource allocation. In contrast, when several types of resources are considered at the same time in an allocation, it is a multi-resource allocation. For example, channel assignment in wireless networks is a single-resource allocation. However, when it is considered jointly with bandwidth, the allocation becomes multi-resource allocation. Multi-resource allocation usually considers correlated resources at the same time, such as rate, radio and power together, or channels and battery energy together. The complex relation between these resources and their influence on the network performance makes multi-resource allocation more difficult than single-resource allocation.
- **Management:** Allocation can either be controlled centrally or distributedly. In centralized management, a con-

trol unit makes allocation decisions normally based on complete information of the system. Whereas in distributed management, the nodes make allocation decisions themselves, mostly based on partial information of the system. Mostly distributed management is not as fair as centralized control because of the lack of information. However, less management communication overhead, less computation, shorter information collection time can be seen in distributed allocation. Besides, sometimes it is impossible to get complete information. Considering a mobile ad hoc network (MANET) with thousands of nodes, where nodes are mobile, it is not realistic to gather complete details of the whole network.

- **Scope:** Based on the scope of allocation it is classified as global or local. Global allocation allocates resources at the overall system level however the local allocation considers the allocation locally. Global allocations may contain multiple local allocations. For example, channel allocation in a large-area wireless network includes multiple local allocations, because channels can be allocated in different local areas if no interference is caused.
- **Static and Dynamic Individual Set:** Resource allocation can also be classified as static individual and dynamic individual allocations by knowing whether the individual set is static. For instance, in MANETs, wireless nodes join and leave the networks dynamically, which requires the resource allocation to consider topology changes. While in WLAN, the nodes are relatively static.

### B. *Utility*

In wireless networks, utility is often always simply treated as single or multiple performance aspects of network or nodes. However, originally in economics, utility is considered as a measure of satisfaction [121]. We adopt this concept of utility in this article and describe it in the domain of wireless networks.

The notion of utility in this paper is defined as the measurement of satisfaction. Satisfaction in wireless networks is indicated via one or more performance metrics. Node performance and network performance exist in wireless networks. For example, in Fig. 1, the throughput of Node A is the node performance. However, the overall network capacity represents the performance of the whole network. With both of these perspectives, a network can be evaluated. Therefore, utility can be divided into individual and system utility as shown in Fig. 3. Performance metrics used can be merged into single satisfaction term which is individual utility. Similarly, the system utility can be obtained by system performance. Utility can also be classified by the functions used. For example, the functions can either provide an enumeration measurement from performances to utilities, or the order of preference, both of which can describe the satisfaction feature of utilities.

### C. *Fairness, Resource Allocation and Utilities*

The relationship between fairness, resource allocation and utility is shown in Fig. 3. Fairness can be measured both in resource allocation and utility. Fairness can be either targeted or resultant. The targeted fairness measures allocation, while the resultant fairness measures the utilities. Therefore, fairness in resource allocation and utility should be distinguished and measured separately. Furthermore, fairness can also be treated as a type of utility. During the evaluation of utility from performance metrics, fairness can be considered as one of the elements deriving to utilities. On the other hand, in feedback mechanisms, the historical utility information of the system may provide feedback to the fairness mechanisms and influences it. One of the goals of resource allocation is the fair distribution. Especially in wireless networks, starvation due to lack of resources may lead to severe drawback in utility. Another goal of resource allocation is to maximize the system and individual utilities. Resource allocation can influence utilities directly, because different amount of resources allocated may lead to large variation in performance. However, the utility may provide feedback to the resource allocation algorithm in order to achieve higher utility.

Sometimes, it may not be possible to guarantee the individual fairness and the system utility at the same time. For example in Fig. 1, let us take network throughput as the system utility, and network throughput here can be defined as the data transfer per second through Link $L_6$. During Internet access, we assume that Node B does not offer routing service for A and C. We also assume that the quality of Link $L_1$ is worse than $L_3$, e.g., capacity of $L_1$, $L_2$ and $L_6$ are 1 MBps, 2MBps and 8 MBps and the others are 1.5 MBps. The sharing of $L_6$ to access the Internet in three cases is shown in Table V. In Case 1, all nodes get 20% of the 8 MBps capacity of Link $L_6$ to access the Internet. In this case, the network throughput of Node A is $min\{8 \times 20\%(\text{MBps}), 1(\text{MBps})\}$, which is between the amount of $L_6$'s capacity allocated to Node A ($8 \times 20\%(\text{MBps})$) and the capacity of Link $L_1$ ($1(\text{MBps})$). Similarly, the throughput of other nodes can be obtained. Therefore, the network throughput in Case 1 can be calculated as $min\{8 \times 20\%, 1\} + min\{8 \times 20\%, 2\} + 3 \times min\{8 \times 20\%, 1.5\} = 7.1(\text{MBps})$.

TABLE V
EXAMPLES OF UTILITY IN FAIRNESS BASED ON FIG. 1.

| | **Case 1** | **Case 2** | **Case 3** |
|---|---|---|---|
| **Node A (via $L_1$)** | 20% | 5% | 35% |
| **Node B (via $L_3$)** | 20% | 35% | 5% |
| **Node C** | 20% | 20% | 20% |
| **Node D** | 20% | 20% | 20% |
| **Node E** | 20% | 20% | 20% |
| **Throughput** | 7.1 (MBps) | 7.3 (MBps) | 6.3 (MBps) |

In Case 2 and 3, A and B get different shares, and C, D and E get the average share of the link capacity, and the throughput is also shown in Table V. Case 1 is fairer than Case 2 and 3 in a general sense, because all nodes get equal capacity. However, network throughput in Case 1 is larger than 3 but less than Case 2. As a decision maker, Case 3 will be abandoned because of less gain in both fairness and throughput. The question is which one should be picked from the Cases 1 and 2, Case 1 is fairer but throughput is less than that of Case 2. This problem suggests that there is a trade-off between fairness and utility. Another example is the trade-off

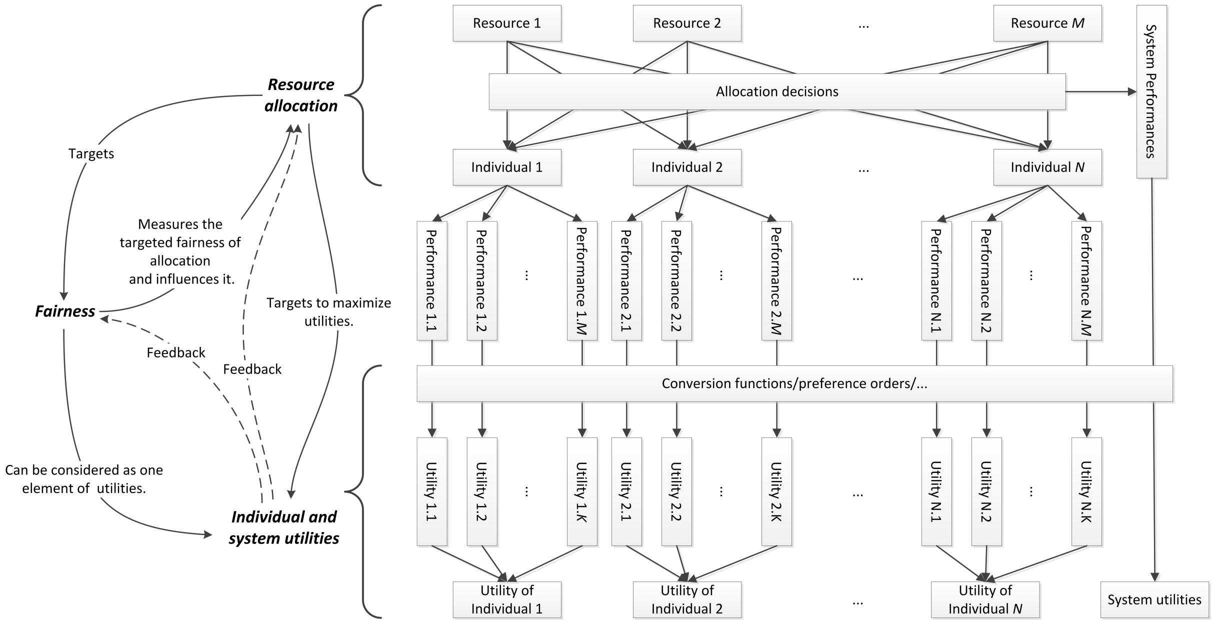


Fig. 3. Fairness, resource allocation and utilities.

between the utilization of time-slots and fairness in ad hoc networks [70].

## VI. DISCUSSIONS

We introduce some basic properties for fairness management, then an example fairness management process is presented.

### A. Properties of Fairness management Process

To allocate the wireless network resources in a fair way, the fairness models in these networks should have the following properties.

*Measurement of Fairness:* A fairness model should provide a real number to imply the fairness experienced by the individuals and the system, which is the basic requirement of a fairness model.

*Identification of Unfairly Treated Individuals:* Unfairness and improper allocations can be identified by a fairness model, then resource allocation can be adjusted.

*Optimization Mechanisms:* Mechanisms should be used to optimize the fairness.

*Assignment of Weights:* Different individuals may have different priorities. Requirements, contributions, historical resource and other elements determine the weights of individuals. Therefore, a fairness model should incorporate a mechanism to assign weights to individuals.

*Distributed Management:* Distributed algorithms should be considered in a fairness model, especially when there is no centralized management mechanism is available.

*Group or Sub-system Fairness Mechanisms:* In some wireless network scenarios groups/clusters and sub-systems/networks act as single individuals during network resource allocation. A fairness model should also consider fairness amongst different clusters and sub-networks.

*Generality:* A generic fairness model in wireless networks can be used in various kinds of resource allocation instead of a tight correspondence between particular model and a particular scenario.

*Adaptability:* Adaptability should be considered with respect to three dynamic aspects – individuals, weights and resources. In some wireless networks, nodes may join or leave the network with time, especially in mobile networks. Their weights should be updated due to requirements of individuals, information history, reputation and other factors dynamically before every channel allocation. Resource availability in wireless networks may change with time too. A fair model should be able to adapt due to the changing of environment quickly.

### B. An Example Fairness Management Process

The process of fairness management fits in the "Plan-Do-Check-Act" model (PDCA or Deming's cycle) very well [122]. PDCA is a four step business model to iteratively control and improve a certain process or a product [122]. Hereby, we adapt the fairness management process into PDCA and illustrate the main functionalities of each stage as shown in Fig. 4. Note that all "individual(s)" can be replaced by group(s)/user cluster(s) or sub-system(s)/network(s) in these modules.

The Plan, Check and Act stages answer the questions **Q1** , **Q2** and **Q3** , respectively. The *Plan* stage mainly collects resource information, allocates weights to individuals, formulates the allocation problem and finds the optimal/accepted solution. Then the allocation solution is handed to the *Do* stage, in which resources are allocated and results are collected. The *Check* stage measures both fairness and utilities based on the actual results. The *Act* stage compares the actual results to the expected results in the *Plan* stage, then it provides suggestions for adjustments. For example reallocating weights, compensating unfairly treated individuals and improving utilities. These suggestions are adopted in next

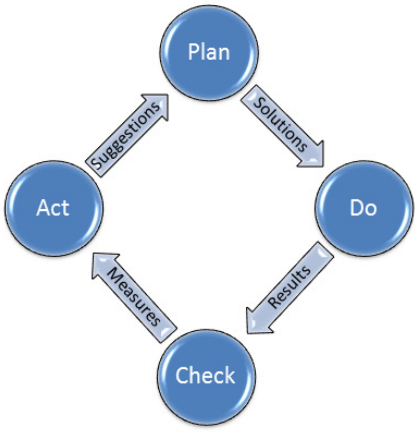


Fig. 4. Fairness management process in PDCA.

Plan stage to obtain better performance iteratively. The detailed functionalities, inputs and outputs of PDCA for fairness management are shown in Table VI.

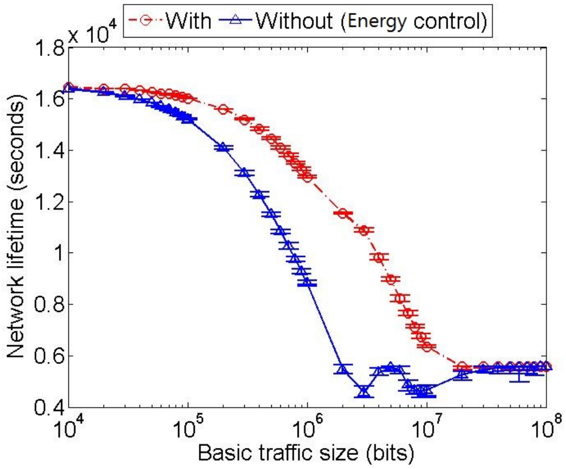
A simple energy control mechanism is simulated based on the process shown in Fig. 4. In this process, energy available at each node is the resource and we "allocate" it by setting thresholds for the energy consumption of nodes in the next minute. When the energy consumption of a node exceeds its threshold, the node turns to "sleep" mode and will wake up at the beginning of the next minute. Fig. 5 shows the network lifetime and network throughput for both with and without energy control mechanisms. When the traffic is low, energy is mainly consumed for maintenance rather than for communication for both cases – with and without energy control. When traffic increases energy is consumed at highest rate for both with and without energy control. Therefore, we see in Fig. 5(a) that the network lifetime for the above two cases join together at the beginning and at the end. Between these two extremes, we can see that network lifetime is much longer when there is energy control, because the fairness mechanism keeps balancing energy consumption between nodes, which extends the network lifetime significantly. In Fig. 5(b), network throughput without energy control is larger than the case with energy control, because the energy consumption threshold limits throughput. However, if traffic reaches the processing limits of nodes or capacity of links, then network throughput achieves the maximal capacity of the network either with or without power control.

The results in Fig. 5 show that the fairness mechanism based on the process (in Fig. 4) maintaining fairness very well with respect to energy consumption.

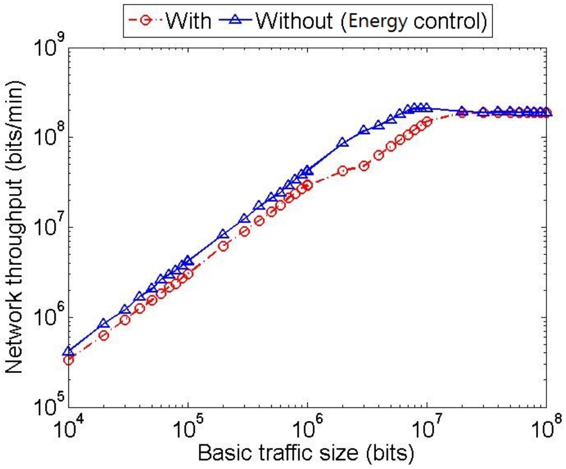
### C. Challenges

Though many studies on Fairness have been reported already, there are still some open questions and challenges. We report significant challenges that could be of interest to the research community. We list them below.

- Multiple dynamic resources and mobile nodes must be considered, instead of single resource and only a certain number of nodes.



(a) Network lifetime, till the first node dies.



(b) Network throughput, sum of all node throughput in one minute.

Fig. 5. Simulation results for the scenario in Fig. 1 with and without energy control mechanism(as in Fig. 4).

- How to identify unfairly treated individuals is not considered by mostly of existing fairness measures, therefore, further study on this issue is needed.
- Corrective strategies when unfairness happens – either re-allocating resources or adjusting current allocations – need to be studied in more depth.
- Weights are the priorities of nodes getting resource in resource allocations. Two more questions come to the fore here: (a) "How to assign weights to individuals, or what factors need to be taken into account while assigning weights to individuals?" and (b) "How to allocate resources according to individual weights?" The former implies the strategies to distribute the weights, while the latter focuses on strategies for resource allocation based on individual weights. Many investigations have been done on the latter one. However, the weight assignment strategies should also be given due attention.
- As far as wireless networks are concerned, distributed scenarios with partial information of the system must be

TABLE VI

INPUT, OUTPUT AND FUNCTIONALITIES OF FAIRNESS MANAGEMENT IN PDCA.

| Stage | Input | Output | Functionalities |
|---|---|---|---|
| **Plan** | Improving suggestions | Resource allocation solutions | 1) Resource management: collecting the information about resources, individuals and requests<br>2) Weight allocation: allocating weights to individuals based on the collected information<br>3) Problem formulation: setting fairness and utility targets, and formulating allocation constrains<br>4) Optimization: finding the best allocation solution |
| **Do** | Resource allocation solutions | Actual results | 1) Resource allocation<br>2) Collection of actual results |
| **Check** | Actual results | Fairness and utility measures | 1) Fairness measurement: measuring the system and individual fairness, and identifying unfairness<br>2) Utility measurement: measuring the system and individual utilities based on the results |
| **Act** | Fairness and utility measures | Improving suggestions | 1) Result comparison: comparing the expected and actual results, then analysing the reasons of the difference<br>2) Suggestions: re-allocating weights, compensating unfairness, and adjusting the accuracy of the optimizing algorithms for the Plan stage |

considered while designing fairness strategies or algorithms.

- The computational complexity of fairness algorithms should be acceptable, since nodes in wireless networks in view of real-time applications always have energy and computational limitations.
- The interference/relationship between fairness and utility is another crucial issue to be looked into in more depth.
- System utility is mostly considered in earlier studies, however influence of system utility on the individual utility is not considered yet. Both system and individual utility should be taken into account during allocation especially in networks which consists of autonomous nodes.

## VII. CONCLUSIONS

In this paper we described, in depth, the fairness issues and in particular its influence on the wireless networking research. We raised three core questions, to begin with, to explore the essence of investigations hitherto with respect to fairness in wireless networks. Based on these questions, we summarized general analytical models of fairness, and compared them. Then, fairness issues in wireless networks were classified and analyzed. We also studied the relationship between fairness, utility and resource allocation. The properties for a fairness model and an example fairness management process were provided to explain various aspects of research on fairness. At the end we also listed challenges in this particular research domain. Many open issues still exist in the study of fairness notion applied to wireless networks. Further studies are needed to address these challenges.

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