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BORN-INFELD EXTENSION OF NEW MASSIVE GRAVITY

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We present a three-dimensional gravitational Born-Infeld theory which reduces to the recently found new massive gravity (NMG) at the quadratic level in the small curvature expansion and at the cubic order reproduces the deformation of NMG obtained from AdS/CFT. Our action provides a remarkable extension of NMG to all orders in the curvature and might define a consistent quantum gravity.

Mass of the graviton is a subtle issue: no satisfactory Higgs-type mechanism seems to exist for spin-2 particles. Therefore, one usually supplies a Pauli-Fierz type hard mass to the graviton, which is quadratic in the fluctuations of the metric around a fixed (maximally symmetric) background with the unique ghost-free combination $m^2\left(h_{\mu\nu}^2 - h^2\right)$. Pauli-Fierz mass comes with a price: general covariance is lost, ghosts appear beyond the tree level and in the interacting theory [1], and $m^2 = 0$ theory is discretely disconnected from $m^2 \to 0$ theory. To solve these problems, one must search for other non-Pauli-Fierz type masses. At least in three dimensions, the theory (NMG) recently introduced by Bergshoeff *et al* [2] provides a nonlinear, generally covariant extension of the Pauli-Fierz massive gravity (for the mostly plus signature):

$$I_{\text{NMG}} = \frac{1}{\kappa^2}\int d^3x\,\sqrt{-\det g}\left[-R + \frac{1}{m^2}\left(R_{\mu\nu}^2 - \frac{3}{8}R^2\right)\right]. \tag{1}$$

This action (or its cosmological extension) defines a massive spin-2 particle around Minkowski (or (anti)-de-Sitter) background. Tree-level ghost freedom, Newtonian limits, classical solutions, supergravity extensions, etc have been worked out in [2–12]. NMG, being the only ghost-free super-renormalizable theory, since its four-dimensional cousin is renormalizable [13], may prove to be a useful laboratory for quantum gravity. Therefore, its possible extensions are invaluable. Here, we present extensions of NMG in terms of determinantal Born-Infeld actions which have appeared in various contexts in the past as means of generalizing Einstein's gravity theory, Maxwell's theory, Yang-Mills theory, and found their natural place in string theory as D-brane actions. [For details see [14].] In the gravitational context, Deser and Gibbons [15] studied conditions on viable Einstein Born-Infeld actions in four dimensions. One such condition is ghost freedom which necessarily removes all the quadratic terms after small curvature expansion. On the other hand, in three dimensions, as we have seen in the NMG case, since a proper combination of quadratic terms yields ghost-free action, one can define a remarkably simple gravitational Born-Infeld action that extends NMG. Below, we present the bare essentials of the theory leaving the details for another work. Let us start with the simplest case where there is no cosmological constant. The action

$$I_{\text{BI}} = -\frac{4m^2}{\kappa^2}\int d^3x\,\left[\sqrt{-\det\left(-\frac{1}{m^2}\mathcal{G}\right)} - \sqrt{-\det g}\right], \tag{2}$$

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with $\mathcal{G}_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - m^2 g_{\mu\nu}$ reduces to (1) upon use of the small curvature expansion

$$[\det(1+A)]^{1/2} = 1 + \frac{1}{2}\mathrm{Tr}A + \frac{1}{8}(\mathrm{Tr}A)^2 - \frac{1}{4}\mathrm{Tr}\left(A^2\right) + \frac{1}{6}\mathrm{Tr}\left(A^3\right) - \frac{1}{8}\mathrm{Tr}\left(A^2\right)\mathrm{Tr}A + \frac{1}{48}(\mathrm{Tr}A)^3 + O\left(A^4\right), \tag{3}$$

up to order $O\left(A^3\right)$ [20]. [We kept $O\left(A^3\right)$ terms in the above expansion for later use below.] Note that to reproduce the Einstein-Hilbert action at the first order, one necessarily uses the cosmological Einstein tensor $\mathcal{G}_{\mu\nu}$. In fact, to be able to make a small curvature expansion, a $g_{\mu\nu}$ is needed in the first determinant. In (2), $\sqrt{-\det g}$ removes a *fixed* cosmological constant coming from the leading-order expansion. To accommodate a general cosmological constant and reproduce the cosmological NMG (CNMG) theory [2]

$$I_{\mathrm{CNMG}} = \frac{1}{\kappa^2}\int d^3x\,\sqrt{-\det g}\left[-(R-2\Lambda) + \frac{1}{m^2}\left(R_{\mu\nu}^2 - \frac{3}{8}R^2\right)\right],$$

one can simply modify (2) to be

$$I_{\mathrm{CBI}} = -\frac{4m^2}{\kappa^2}\int d^3x\left[\sqrt{-\det\left(-\frac{1}{m^2}\mathcal{G}\right)} - \left(\frac{\Lambda}{2m^2}+1\right)\sqrt{-\det g}\right], \tag{4}$$

where $\mathcal{G}$ is given exactly as above. Note that to conform with the second reference in [2], one could set $\Lambda = m^2\lambda$ where λ is a dimensionless number. In summary, (4) defines our minimally Born-Infeld-extended NMG for any (including zero) cosmological constant. What is remarkable about this expression is that only the cosmological Einstein tensor appears in the determinant. It reproduces the only unitary theory NMG at the quadratic level, and as we shall see below it also reproduces at the cubic level the deformation of NMG obtained by the requirement that a holographic c-theorem exists [16]. [More recently, $O\left(R^4\right)$ matching was shown in [17].]

At $O\left(R^3\right)$, let us compute what possible terms are generated by the Born-Infeld action (4). At this order using (3) and defining $\mathcal{G}_{\mu\nu} \equiv G_{\mu\nu} - m^2 g_{\mu\nu}$, after a straightforward computation, one obtains

$$\begin{aligned} O\left(R^3\right): \quad & -\frac{1}{m^6}\left[\frac{1}{6}\mathrm{Tr}\left(g^{-1}Gg^{-1}Gg^{-1}G\right) - \frac{1}{8}\mathrm{Tr}\left(g^{-1}Gg^{-1}G\right)\mathrm{Tr}\left(g^{-1}G\right) + \frac{1}{48}\left[\mathrm{Tr}\left(g^{-1}G\right)\right]^3\right] \\ & = -\frac{1}{6m^6}\left[G^{\mu\nu}G_{\nu\alpha}G^{\alpha}{}_{\mu} - \frac{3}{4}G_{\mu\nu}^2 G^{\alpha}{}_{\alpha} + \frac{1}{8}\left(G^{\alpha}{}_{\alpha}\right)^3\right], \end{aligned}$$

where $G^{\alpha}{}_{\alpha} = -\frac{R}{2}$, $G_{\mu\nu}^2 = R_{\mu\nu}^2 - \frac{1}{4}R^2$ and $G^{\mu\nu}G_{\nu\alpha}G^{\alpha}{}_{\mu} = R^{\mu\nu}R_{\nu\alpha}R^{\alpha}{}_{\mu} - \frac{3}{2}RR_{\mu\nu}^2 + \frac{3}{8}R^3$. Collecting all the terms together, we have

$$I_{\text{NMG-ext}} = \frac{1}{\kappa^2}\int d^3x\,\sqrt{-\det g}\left[-(R-2\Lambda) + \frac{1}{m^2}\left(R_{\mu\nu}^2 - \frac{3}{8}R^2\right) + \frac{2}{3m^4}\left(R^{\mu\nu}R_{\nu}{}^{\alpha}R_{\alpha\mu} - \frac{9}{8}RR_{\mu\nu}^2 + \frac{17}{64}R^3\right) + O\left(R^4\right)\right], \tag{5}$$

which matches the result of Sinha [16] obtained with the help of AdS/CFT and the existence of a c-theorem in $(1+1)$-dimensional CFT. This matching is highly non-trivial and lends support that (4) might define a consistent quantum gravity.

Now, let us discuss the non-minimal extensions. Using just the curvature, and not its derivatives, and staying at the quadratic level one can define the most general non-minimal Born-Infeld extension of the CNMG in the following way:

$$I_{\text{BI-non-minimal}} = -\frac{1}{b}\int d^3x\,\sqrt{-\det(g+X)},$$

where $X_{\mu\nu} \equiv a\left(R_{\mu\nu} + cg_{\mu\nu}R\right) + d\left(R_{\mu\alpha}R^{\alpha}{}_{\nu} + eg_{\mu\nu}R^2_{\alpha\beta} + fR_{\mu\nu}R + lg_{\mu\nu}R^2\right)$. Observe that we have dropped the $\sqrt{-\det g}$ term. [Adding some derivative terms here gives a quite interesting result [18]]. Working out the expansions at the quadratic level, one has various choices in trying to match the dimensional constants the κ^2, m^2, Λ and the dimensionless ratio $-\frac{3}{8}$ of CNMG with the dimensional constants a, b, d and the dimensionless constants; c, e, f, and l of the non-minimal Born-Infeld theory. Here, for the sake of simplicity, we make our choice to be

$$I_{\text{BI-ext}} = -\frac{1}{b}\int d^3x\,\sqrt{-\det\left[g_{\mu\nu} + a\left(R_{\mu\nu} + cg_{\mu\nu}R\right) + dR_{\mu\alpha}R^{\alpha}{}_{\nu}\right]}, \tag{6}$$

where a, b, c and d are

$$a^2 = -\frac{1}{2\Lambda}\left(\frac{9}{2m^2} + \frac{1}{\Lambda}\right), \qquad b = -\frac{\kappa^2}{2\Lambda}, \qquad c = -\frac{1}{3}\left(1 + \frac{1}{a\Lambda}\right), \qquad d = \frac{a^2}{2} + \frac{1}{\Lambda m^2}.$$

So, (6) is also an extension of CNMG, perhaps not as elegant as (4) and with the constraint $\Lambda < -\frac{2m^2}{9}$. Therefore, with such an extension de-Sitter background is ruled out. Of course, one may remove this constraint by considering the other quadratic terms in the $X_{\mu\nu}$ tensor.

In conclusion, we have found the minimal (4) and a non-minimal (6) Born-Infeld extensions of the (cosmological) new massive gravity in three dimensions. Having more powers of curvature, these theories are much better behaved in the ultraviolet. In fact, at $O\left(R^3\right)$ our theory coincides with the one obtained quite non-trivially from the AdS/CFT correspondence with the requirement that a c-theorem exists [16]. Further work with our action in the context of AdS/CFT is needed. It would also be interesting to see if these theories and their supersymmetric generalizations can be derived from string theory D-brane actions. In this communication, we have presented the basics of the model. Elsewhere, we will return to discussions of classical solutions such as BIonic solitons without singularities [see [19] for a related discussion on curing singularities with the BI-type actions], linearization of the theory around (anti)-de-Sitter spacetime and the effects of cubic terms (5) on the mass of the graviton and more importantly on the unitarity of the theory. Around the flat space, there is no problem about the unitarity at the tree level. One can also couple matter fields to the metric: for example, spin-1 Abelian gauge fields can simply be introduced with the substitution $\det\left(g - \frac{1}{m^2}G\right) \to \det\left(g - \frac{1}{m^2}G + \alpha' F\right)$.

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$$I=-\frac{4m^2}{\kappa^2}\int d^3x\left\{\sqrt{-\det\left[g_{\mu\nu}+\frac{1}{m^2}\left(R_{\mu\nu}-\frac{1}{6}g_{\mu\nu}R\right)\right]}-\sqrt{-\det\left(g_{\mu\nu}\right)}\right\}.$$