Semileptonic transition of Σ_b to Σ in Light Cone QCD Sum Rules

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Abstract

We use distribution amplitudes of the light Σ baryon and the most general form of the interpolating current for heavy Σ_b baryon to investigate the semileptonic $\Sigma_b \to \Sigma l^+ l^-$ transition in light cone QCD sum rules. We calculate all twelve form factors responsible for this transition and use them to evaluate the branching ratio of the considered channel. The order of branching fraction shows that this channel can be detected at LHC.

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1 Introduction

The systems involving heavy quarks decays are important frameworks to restrict the standard model (SM) parameters as well as search for new physics effects. Especially, the flavor changing neutral current (FCNC) transition of $b \to s\bar{\ell}\ell$, which is underlying transition of $\Sigma_b \to \Sigma l^+ l^-$ decay at the quark level, is known to be sensitive to new physics effects. This process can also be used in exact determination of the V_{tb} and V_{ts} as elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and answering some fundamental questions such as CP violation.

In the last decade, important experimental progress has been made in identification and spectroscopy of the heavy baryons with single heavy quark [1–8]. It is expected that the LHC will open new horizons not only in the identification and spectroscopy of these baryons, but also it will provide possibility to study the weak, strong and electromagnetic decays of heavy baryons.

In accordance with this experimental progress, there is an increasing interest on calculation of parameters of the heavy baryons and investigation of their decay modes theoretically. The masses of these baryons have been calculated using various methods such as quark models [9–17], heavy quark effective theory [18–24] and QCD sum rules [25–33]. Besides the mass spectrum, their weak, strong and electromagnetic decays have also received special attention, recently (for instance see [34–43] and references therein).

In the present work, we analyze the semileptonic $\Sigma_b \to \Sigma l^+ l^-$ transition in the framework of the light cone QCD sum rules. The main ingredients in analysis of this channel are form factors entering the transition matrix elements. Using the most general form of the interpolating field for the Σ_b heavy baryon as well as the distribution amplitudes (DA's) of the light Σ baryon, we first calculate all twelve form factors in full theory. Then, we use these form factors to calculate the total decay rate as well as the branching ratio of the considered decay channel.

The paper is organized in three sections. In the next section, we obtain QCD sum rules for the form factors. In section 3, we numerically analyze the form factors and use them to calculate the related decay rate and branching fraction.

2 light cone QCD sum rules for form factors

In this section, we focus on the calculation of the form factors corresponding to $\Sigma_b \to \Sigma l^+ l^-$ semileptonic decay which proceeds via $b \to s$ transition at quark level. The effective Hamiltonian describing this transition is written as:

$$\mathcal{H}_{eff} = \frac{G_F\ \alpha_{em} V_{tb}\ V_{ts}^*}{2\sqrt{2}\ \pi}\Big\{ C_9^{eff}\ \bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu l + C_{10}\ \bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu\gamma_5 l - 2m_b\ C_7\frac{1}{q^2}\ \bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\bar{l}\gamma^\mu l\Big\}\ . \quad (1)$$

The amplitude of the transition can be obtained by sandwiching the effective Hamiltonian between the initial and final states,

$$\mathcal{M} = \frac{G_F\ \alpha_{em} V_{tb}\ V_{ts}^*}{2\sqrt{2}\ \pi}\Bigg\{ C_9^{eff}\ \langle \Sigma|\bar{s}\gamma_\mu(1-\gamma_5)b|\Sigma_b\rangle \bar{l}\gamma^\mu l + C_{10}\ \langle \Sigma|\bar{s}\gamma_\mu(1-\gamma_5)b|\Sigma_b\rangle \bar{l}\gamma^\mu\gamma_5 l$$
$$-\ 2m_b\ C_7 \frac{1}{q^2}\ \langle \Sigma|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\Sigma_b\rangle \bar{l}\gamma^\mu l \Bigg\}\ . \tag{2}$$

From this equation, it is obvious that the transition matrix elements $\langle \Sigma(p)|\bar{s}\gamma_\mu(1-\gamma_5)b|\Sigma_b(p+q)\rangle$ and $\langle \Sigma(p)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\Sigma_b(p+q)\rangle$ are required. These matrix elements are expressed in terms of twelve form factors f_i, g_i, f_i^T and g_i^T (i running from 1 to 3) as follows:

$$\langle \Sigma(p)\ |\ \bar{s}\gamma_\mu(1-\gamma_5)b\ |\ \Sigma_b(p+q)\rangle = \bar{u}_\Sigma(p)\Big[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}q^\nu f_2(q^2) + q^\mu f_3(q^2)$$
$$-\ \gamma_\mu\gamma_5 g_1(q^2) - i\sigma_{\mu\nu}\gamma_5 q^\nu g_2(q^2) - q^\mu\gamma_5 g_3(q^2)\Big] u_{\Sigma_b}(p+q)\ , \tag{3}$$

and

$$\langle \Sigma(p)\ |\ \bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\ |\ \Sigma_b(p+q)\rangle = \bar{u}_\Sigma(p)\Big[\gamma_\mu f_1^T(q^2) + i\sigma_{\mu\nu}q^\nu f_2^T(q^2) + q^\mu f_3^T(q^2)$$
$$+\ \gamma_\mu\gamma_5 g_1^T(q^2) + i\sigma_{\mu\nu}\gamma_5 q^\nu g_2^T(q^2) + q^\mu\gamma_5 g_3^T(q^2)\Big] u_{\Sigma_b}(p+q)\ , \tag{4}$$

where u_{Σ_b} and u_Σ are the spinors of Σ_b and Σ baryons, respectively, and q denotes transferred momentum.

Our main task is to calculate the aforesaid transition form factors. In accordance with the philosophy of QCD sum rules, we start considering the following correlation functions:

$$\Pi_\mu^I(p,q) = i\int d^4x e^{-iqx}\langle 0\ |\ T\{J^{\Sigma_b}(0), \bar{b}(x)\gamma_\mu(1-\gamma_5)s(x))\}\ |\ \Sigma(p)\rangle\ ,$$
$$\Pi_\mu^{II}(p,q) = i\int d^4x e^{-iqx}\langle 0\ |\ T\{J^{\Sigma_b}(0), \bar{b}(x)i\sigma_{\mu\nu}q^\nu(1+\gamma_5)s(x)\}\ |\ \Sigma(p)\rangle\ , \tag{5}$$

where J^{Σ_b} stands for interpolating current of Σ_b. The interpolating current should be chosen as a composite operator that has the same quantum numbers as the baryon under study. For Σ_b baryon, there are two possible choices for such a current that does not contain any derivatives or auxiliary four vectors. The most general form for the interpolating current is a superposition of these two choices. Hence, for the interpolating current of the Σ_b baryon, the operator

$$J^{\Sigma_b}(x) = \frac{-1}{\sqrt{2}}\epsilon_{abc}\Big\{[u_1^{aT}(x)Cb^b(x)]\gamma_5 d^c(x) + \beta[u_1^{aT}(x)C\gamma_5 b^b(x)]d^c(x)$$
$$-\ [b^{aT}(x)Cd^b(x)]\gamma_5 u^c(x) - \beta[b^{aT}(x)C\gamma_5 d^b(x)]u^c(x)\Big\}\ , \tag{6}$$

is chosen. Here C is the charge conjugation operator, β is an arbitrary parameter, and a, b, and c, are the color indices. Taking $\beta = -1$ corresponds to the Ioffe current.

The correlation function can be calculated both in terms of the hadronic parameters, such as the form factors, and also in terms of the QCD parameters. The expression in terms of the QCD parameters is evaluated by expanding the time ordered product of the currents in terms of the Σ distribution amplitudes via operator product expansion (OPE) in deep Euclidean region. On the other hand, the physical counterpart is calculated by inserting a complete set of intermediate states. The two expression are then matched using dispersion relations.

To begin with, let us evaluate the correlation function in terms of hadronic parameters. After inserting the complete set of intermediate states into the correlation functions and isolating the ground state contribution we obtain

$$\Pi_\mu^I(p,q) = \sum_s \frac{\langle 0 \mid J^{\Sigma_b}(0) \mid \Sigma_b(p+q,s)\rangle\langle\Sigma_b(p+q,s) \mid \bar{b}\gamma_\mu(1-\gamma_5)s \mid \Sigma(p)\rangle}{m_{\Sigma_b}^2-(p+q)^2} + \cdots ~, \tag{7}$$

$$\Pi_\mu^{II}(p,q) = \sum_s \frac{\langle 0 \mid J^{\Sigma_b}(0) \mid \Sigma_b(p+q,s)\rangle\langle\Sigma_b(p+q,s) \mid \bar{b}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)s \mid \Sigma(p)\rangle}{m_{\Sigma_b}^2-(p+q)^2} + \cdots ~, \tag{8}$$

where the $\cdots$ stands for contributions of the higher states and continuum, and the sum is over the polarizations of the Σ_b baryon. The matrix element of the interpolating current between the vacuum and the Σ_b baryon appearing in Eqs. (7) and (8), $\langle 0 \mid J^{\Sigma_b}(0) \mid \Sigma_b(p+q,s)\rangle$, can be expressed in terms of the residue of the Σ_b baryon defined as:

$$\langle 0 \mid J^{\Sigma_b}(0) \mid \Sigma_b(p+q,s)\rangle = \lambda_{\Sigma_b} u_{\Sigma_b}(p+q,s) ~. \tag{9}$$

The other matrix elements in Eqs. (7) and (8) are defined in terms of the form factors as previously shown. Combining Eqs. (3), (4), and (7)–(9) and summing over the polarization of the Σ_b baryon using the expression

$$\sum_s u_{\Sigma_b}(p+q,s)\overline{u}_{\Sigma_b}(p+q,s) = \not\!p + \not\!q + m_{\Sigma_b} ~, \tag{10}$$

the correlation functions can be expressed as:

$$\begin{aligned}\Pi_\mu^I(p,q) &= \lambda_{\Sigma_b}\frac{\not\!p+\not\!q+m_{\Sigma_b}}{m_{\Sigma_b}^2-(p+q)^2}\Big\{\gamma_\mu f_1 - i\sigma_{\mu\nu}q^\nu f_2 + q_\mu f_3 \\ &\quad -\gamma_\mu\gamma_5 g_1 - i\sigma_{\mu\nu}q^\nu\gamma_5 g_2 + q_\mu\gamma_5 g_3\Big\}u_\Sigma(p) ~,\end{aligned} \tag{11}$$

$$\begin{aligned}\Pi_\mu^{II}(p,q) &= \lambda_{\Sigma_b}\frac{\not\!p+\not\!q+m_{\Sigma_b}}{m_{\Sigma_b}^2-(p+q)^2}\Big\{\gamma_\mu f_1^T - i\sigma_{\mu\nu}q^\nu f_2^T + q_\mu f_3^T \\ &\quad + \gamma_\mu\gamma_5 g_1^T + i\sigma_{\mu\nu}\gamma_5 q^\nu g_2^T - q_\mu\gamma_5 g_3^T\Big\}u_\Sigma(p) ~.\end{aligned} \tag{12}$$

Commuting $\not\!p$ all the way to the right and using the equation of motion to write $\not\!p u_\Sigma(p) = m_\Sigma u_\Sigma(p)$, Eqs. (11) and (12) lead to the final expressions for the phenomenological side:

$$\Pi_\mu^I(p,q) = \frac{\lambda_{\Sigma_b}}{m_{\Sigma_b}^2-(p+q)^2}\Big\{2f_1(q^2)p_\mu + 2f_2(q^2)p_\mu \not\!q + \Big[f_2(q^2)+f_3(q^2)\Big]q_\mu \not\!q$$

$$- 2g_1(q^2)p_\mu\gamma_5 + 2g_2(q^2)p_\mu \not{q}\gamma_5 + \Big[g_2(q^2) + g_3(q^2)\Big]q_\mu \not{q}\gamma_5$$
$$+ \text{other structures}\Big\}u_\Sigma(p) \ , \quad (13)$$

$$\Pi^{II}_\mu(p,q) = \frac{\lambda_{\Sigma_b}}{m^2_{\Sigma_b} - (p+q)^2}\Big\{2f^T_1(q^2)p_\mu + 2f^T_2(q^2)p_\mu \not{q} + \Big[f^T_2(q^2) + f^T_3(q^2)\Big]q_\mu \not{q}$$
$$+ 2g^T_1(q^2)p_\mu\gamma_5 - 2g^T_2(q^2)p_\mu \not{q}\gamma_5 - \Big[g^T_2(q^2) + g^T_3(q^2)\Big]q_\mu \not{q}\gamma_5$$
$$+ \text{other structures}\Big\}u_\Sigma(p) \ . \quad (14)$$

In these two expressions only the independent structures, p_μ, $p_\mu\not{q}$, $q_\mu\not{q}$, $p_\mu\gamma_5$, $p_\mu\not{q}\gamma_5$, and $q_\mu\not{q}\gamma_5$, are presented explicitly, owing to their sufficiency to determine the aimed form factors, $f_1(f^T_1)$, $f_2(f^T_2)$, $f_2 + f_3(f^T_2 + f^T_3)$, $g_1(g^T_1)$, $g_2(g^T_2)$ and $g_2 + g_3(g^T_2 + g^T_3)$.

After completing the evaluation of the correlation function in terms of the hadronic parameters, now let us focus our attention on evaluating the correlation function in terms of the QCD parameters and the DA's of the Σ baryon . After placing the explicit expression of interpolating current given in Eq. (6) into Eq. (5) and contracting out the heavy quark operators, we attain the following representation of the correlators in QCD side:

$$\Pi^I_\mu = \frac{-i}{\sqrt{2}}\epsilon^{abc}\int d^4x e^{-iqx}\Bigg\{\Bigg(\Big[(C)_{\eta\beta}(\gamma_5)_{\rho\phi} - (C)_{\beta\phi}(\gamma_5)_{\rho\eta}\Big] + \beta\Big[(C\gamma_5)_{\eta\beta}(I)_{\rho\phi}$$
$$- (C\gamma_5)_{\beta\phi}(I)_{\rho\eta}\Big]\Bigg)\Big[\gamma_\mu(1-\gamma_5)\Big]_{\sigma\theta}\Bigg\}S_Q(-x)_{\beta\sigma}\langle 0|u^a_\eta(0)s^b_\theta(x)d^c_\phi(0)|\Sigma(p)\rangle \ , \quad (15)$$

$$\Pi^{II}_\mu = \frac{-i}{\sqrt{2}}\epsilon^{abc}\int d^4x e^{-iqx}\Bigg\{\Bigg(\Big[(C)_{\eta\beta}(\gamma_5)_{\rho\phi} - (C)_{\beta\phi}(\gamma_5)_{\rho\eta}\Big] + \beta\Big[(C\gamma_5)_{\eta\beta}(I)_{\rho\phi}$$
$$- (C\gamma_5)_{\beta\phi}(I)_{\rho\eta}\Big]\Bigg)\Big[i\sigma_{\mu\nu}q^\nu(1+\gamma_5)\Big]_{\sigma\theta}\Bigg\}S_Q(-x)_{\beta\sigma}\langle 0|u^a_\eta(0)s^b_\theta(x)d^c_\phi(0)|\Sigma(p)\rangle \ , \quad (16)$$

The heavy quark propagator, $S_Q(x)$ is calculated in [44]:

$$S_Q(x) = S^{free}_Q(x) - ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv\Bigg[\frac{\not{k} + m_Q}{(m^2_Q - k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu}$$
$$+ \frac{1}{m^2_Q - k^2}vx_\mu G^{\mu\nu}\gamma_\nu\Bigg] \ . \quad (17)$$

where,

$$S^{free}_Q = \frac{m^2_Q}{4\pi^2}\frac{K_1(m_b\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m^2_Q \not{x}}{4\pi^2x^2}K_2(m_b\sqrt{-x^2}) \ , \quad (18)$$

and K_i are the Bessel functions. Note that S_Q^{free} represents the free propagation of the heavy quark, and the remaining terms represent the interaction of the heavy quark with the external gluon field. The calculation of the contributions of the latter effects require the four- and five-particle baryons DA's which are currently unknown. But since they are higher order contributions, they are expected to be small [45–47] and we ignore them in the present work. In [48], it is also found that the form factors entering the semileptonic decays of the heavy Λ baryons turn out to receive only a very small contribution from the gluon condensate.

The matrix element $\epsilon^{abc}\langle 0|u_\eta^a(0)s_\theta^b(x)d_\phi^c(0)|\Sigma(p)\rangle$ can be expressed in terms of Σ baryon's wave functions and are given in [49], and for completeness explicit form of them are presented in the Appendix. After evaluating the Fourier transform, the correlation function is expressed in terms of the QCD parameters and the DA's of the Σ.

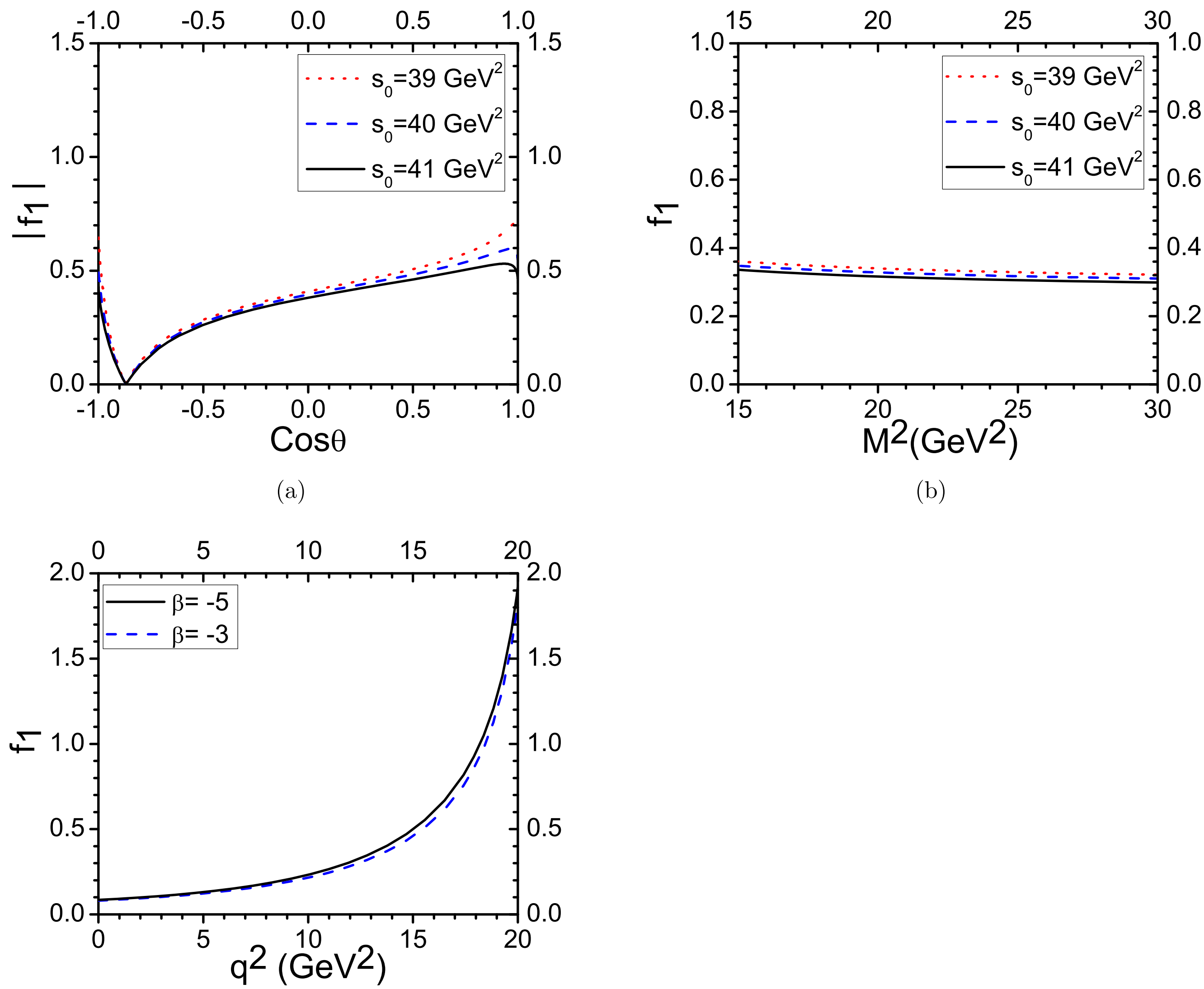
The sum rules are obtained by first Borel transforming both expression of the correlation functions and then equating the coefficient of various structures. Finally, the contributions of the higher states and the continuum are subtracted using quark hadron duality. To extract the numerical value of the form factors, value of the residue is also required. The residue of the Σ_b baryon is calculated in [50].

3 Numerical Analysis

In this section, we perform numerical analysis of the form factors and use them to predict the decay rate and the branching ratio. The masses of the Σ_b, Σ baryons and the b quark are taken as $m_{\Sigma_b} = (5807.8 \pm 2.7)\ MeV$ [52], $m_\Sigma = (1192.642 \pm 0.024)\ MeV$, and $m_b = (4.7 \pm 0.1)\ GeV$, respectively. For CKM matrix element entering into the transition amplitude, $|V_{tb}V_{ts}^*| = 0.041$ is used. The main input parameters of QCD sum rules for the form factors are DA's of the Σ baryon, whose explicit expression are presented in the Appendix. Here, we should make the following remark. In the matrix element, $\epsilon^{abc}\langle 0|u_\eta^a(0)s_\theta^b(x)d_\phi^c(0)|\Sigma(p)\rangle$, besides the functions presented in the Appendix, there appear also the functions $\mathcal{A}_1^M$, $\mathcal{V}_1^M$ and $\mathcal{T}_1^M$ whose explicit forms are unknown for the Σ baryon. Considering the $SU(3)_f$ symmetry, we get them from the nucleon DA's. Our calculations show that their contribution constitutes only few percent of the final results, so we neglect their contribution in the present work.

Besides these input parameters, there appear also three auxiliary parameters in the sum rules, i.e. Borel mass parameter M^2, continuum threshold s_0, and the general parameter β arising in the interpolating current of the Σ_b baryon. These parameters should not effect the values of the form factors, so one should obtain working regions of them for which the form factors show weak dependence on these parameters.

Lowering the value of the Borel mass increases the contribution of the higher twist DA's, hence requiring that the twist expansion converges leads to a lower limit on the Borel mass; on the other hand, increasing the value of the Borel mass increases the contribution of the higher states and the continuum. Hence requiring that the contribution of the higher states and continuum to the correlation function is less than half the total contribution yields an upper bound on the Borel mass. Both of these conditions are met if the Borel mass is chosen in the interval $15\ GeV^2 \leq M_B^2 \leq 30\ GeV^2$. The continuum threshold s_0 is not totally arbitrary but it is correlated to the energy of the first excited state. Our analysis shows that in the region $(m_{\Sigma_b} + 0.3\ GeV)^2 \leq s_0 \leq (m_{\Sigma_b} + 0.7\ GeV)^2$, the dependences of the form factors on this parameter are weak. Finally, to find the working region of β, the dependence of the form factors on $\cos\theta$ in the interval $-1 \leq \cos\theta \leq 1$, where $\tan\theta = \beta$ is considered. Our numerical calculations lead to the working region, $-0.5 \leq \cos\theta \leq 0.7$.



(c)

Figure 1: Analysis of the sum rules for the form factor $f_1(q^2)$

As an example, in Fig. 1, we depict the dependences of the form factor $f_1(q^2)$ on auxiliary parameters as well as q^2. Fig. (1a) shows the dependence of this form factor on $\cos\theta$ at the fixed values $q^2 = 13\ GeV^2$, $M^2 = 20\ GeV^2$ and $s_0 = (40 \pm 1)\ GeV^2$. As it is seen, there is a stable region in the interval $-0.5 \leq \cos\theta \leq 0.7$. In Fig (1b), the Borel mass dependence of the same form factor is depicted at $q^2 = 13\ GeV^2$ and the same value of the continuum threshold. In the chosen working region of the Borel mass, our predictions change by approximately 5%. From these two figures, it is also seen that our predictions are almost independent of the continuum threshold. Finally, in Fig (1c), we show the dependence of the form factor f_1 on q^2 at two fixed values of β, and $M^2 = 20\ GeV^2$ and $s_0 = 40\ GeV^2$.

The sum rules predictions are only reliable for the region $q^2 \ll m_b^2$, where in the decay of Σ_b, the allowed range of q^2 extends until $(m_{\Sigma_b} - m_\Sigma)^2$. To extend the sum rules predictions to the whole physical region, the sum rules predictions are fitted to the following function:

$$f_i(q^2)[g_i(q^2)] = \frac{a}{(1 - \frac{q^2}{m_{fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit}^2})^2}. \tag{19}$$

The central values of the fit parameters a, b, and m_{fit} are presented in Table 1. This table also exhibits values of the form factors at $q^2 = 0$. These errors presented in this table are due to the variation of the auxiliary parameters, M^2, s_0, and β, as well as the errors in the input parameters. Note that for all form factors, the fit mass is always in the range $m_{fit} = (5.1 - 5.4)\ GeV$. In a vector dominance model, although the double pole structure would not be expected, these form factors would have poles at the masses of the (axial)vector meson that couples to the transition currents. The observed (axial)vector B_s mesons have masses in the range $(5.4-5.8)\ GeV$. Although the fit mass tends to be slightly smaller than the observed masses, considering the uncertainties inherent in the sum rules calculations, the results are reasonable. To improve the results one should consider the α_s corrections to the distributions amplitudes and more accurately determine the DA's of Σ baryon.

Finally we calculate the differential and total decay rate of the $\Sigma_b \to \Sigma \ell^+ \ell^-$ transition. The general form of the differential rate for the rare baryonic weak decay is given by [53] :

$$\frac{d\Gamma}{ds} = \frac{G^2 \alpha_{em}^2 m_{\Lambda_b}}{8192\pi^5} |V_{tb} V_{ts}^*|^2 v \sqrt{\lambda} \left[\Theta(s) + \frac{1}{3} \Delta(s) \right], \tag{20}$$

where $s = q^2/m_{\Sigma_b}^2$, $G = 1.17 \times 10^{-5}\ \text{GeV}^{-2}$ is the Fermi coupling constant and $\lambda = \lambda(1, r, s)$ with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the usual triangle function. Here, $v = \sqrt{1 - \frac{4m_\ell^2}{q^2}}$ is the lepton velocity. The functions $\Theta(s)$ and $\Delta(s)$ are given as:

$$\begin{aligned}
\Theta(s) &= 32 m_\ell^2 m_{\Sigma_b}^4 s(1 + r - s) \left(|D_3|^2 + |E_3|^2\right) \\
&+ 64 m_\ell^2 m_{\Sigma_b}^3 (1 - r - s) \,\mathrm{Re}[D_1^* E_3 + D_3 E_1^*] \\
&+ 64 m_{\Sigma_b}^2 \sqrt{r} (6m_\ell^2 - m_{\Sigma_b}^2 s) \mathrm{Re}[D_1^* E_1] \\
&+ 64 m_\ell^2 m_{\Sigma_b}^3 \sqrt{r} \Big(2 m_{\Sigma_b} s \mathrm{Re}[D_3^* E_3] + (1 - r + s) \mathrm{Re}[D_1^* D_3 + E_1^* E_3] \Big) \\
&+ 32 m_{\Sigma_b}^2 (2m_\ell^2 + m_{\Sigma_b}^2 s) \Big\{ (1 - r + s) m_{\Sigma_b} \sqrt{r} \,\mathrm{Re}[A_1^* A_2 + B_1^* B_2] \\
&- m_{\Sigma_b} (1 - r - s) \,\mathrm{Re}[A_1^* B_2 + A_2^* B_1] - 2\sqrt{r} \Big(\mathrm{Re}[A_1^* B_1] + m_{\Sigma_b}^2 s \,\mathrm{Re}[A_2^* B_2] \Big) \Big\} \\
&+ 8 m_{\Sigma_b}^2 \Big\{ 4 m_\ell^2 (1 + r - s) + m_{\Sigma_b}^2 \Big[(1 - r)^2 - s^2 \Big] \Big\} \left(|A_1|^2 + |B_1|^2\right)
\end{aligned}$$

$$
\begin{aligned}
&+ 8m_{\Sigma_b}^4\Big\{4m_\ell^2\Big[\lambda+(1+r-s)s\Big]+m_{\Sigma_b}^2 s\Big[(1-r)^2-s^2\Big]\Big\}\left(|A_2|^2+|B_2|^2\right)\\
&- 8m_{\Sigma_b}^2\Big\{4m_\ell^2(1+r-s)-m_{\Sigma_b}^2\Big[(1-r)^2-s^2\Big]\Big\}\left(|D_1|^2+|E_1|^2\right)\\
&+ 8m_{\Sigma_b}^5 sv^2\Big\{-8m_{\Sigma_b}s\sqrt{r}\,\mathrm{Re}[D_2^* E_2]+4(1-r+s)\sqrt{r}\,\mathrm{Re}[D_1^* D_2+E_1^* E_2]\\
&- 4(1-r-s)\,\mathrm{Re}[D_1^* E_2+D_2^* E_1]+m_{\Sigma_b}\Big[(1-r)^2-s^2\Big]\left(|D_2|^2+|E_2|^2\right)\Big\}\ , \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
\Delta(s) &= -8m_{\Sigma_b}^4 v^2\lambda\left(|A_1|^2+|B_1|^2+|D_1|^2+|E_1|^2\right)\\
&+ 8m_{\Sigma_b}^6 sv^2\lambda\Big(|A_2|^2+|B_2|^2+|D_2|^2+|E_2|^2\Big)\ , \qquad (22)
\end{aligned}
$$

where $r=m_\Sigma^2/m_{\Sigma_b}^2$ and

$$
\begin{aligned}
A_1 &= \frac{1}{q^2}\left(f_1^T+g_1^T\right)\left(-2m_bC_7\right)+\left(f_1-g_1\right)C_9^{eff}\\
A_2 &= A_1\left(1\rightarrow 2\right)\ ,\\
A_3 &= A_1\left(1\rightarrow 3\right)\ ,\\
B_1 &= A_1\left(g_1\rightarrow -g_1;\ g_1^T\rightarrow -g_1^T\right)\ ,\\
B_2 &= B_1\left(1\rightarrow 2\right)\ ,\\
B_3 &= B_1\left(1\rightarrow 3\right)\ ,\\
D_1 &= \left(f_1-g_1\right)C_{10}\ ,\\
D_2 &= D_1\left(1\rightarrow 2\right)\ , \qquad (23)\\
D_3 &= D_1\left(1\rightarrow 3\right)\ ,\\
E_1 &= D_1\left(g_1\rightarrow -g_1\right)\ ,\\
E_2 &= E_1\left(1\rightarrow 2\right)\ ,\\
E_3 &= E_1\left(1\rightarrow 3\right)\ . \qquad (24)
\end{aligned}
$$

Integrating the differential decay rate over s in the interval, $4m_\ell^2/m_{\Sigma_b}^2 \le s \le (1-\sqrt{r})^2$, we get the total decay rates presented in the Table 2. Finally, to obtain the branching ratios, one needs the lifetime of the Σ_b baryon. Although there is no exact information about the lifetime of this baryon, it may be informative to take this lifetime approximately at the same order of the b-baryon admixture, $(\Lambda_b, \Xi_b, \Sigma_b, \Omega_b)$ which is $\tau = 1.391^{+0.039}_{-0.038}\times 10^{-12}$ s [52]. The results of the branching ratios for different leptons are also presented in Table 2. It is seen that the branching ratio for decays into the electrons or muons are more or less the same, while the branching ratio for decay into final states containing τ lepton is reduced by approximately a factor of four. The order of branching fractions show that these channels can be detected at LHC. Comparing the presented results in this work with the results of any measurements, one can obtain useful information about the nature of Σ_b baryon as well as new physics effects beyond the SM.

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	a	b	m_{fit}	$q^2 = 0$
f_1	-0.035	0.13	5.1	0.095 ± 0.017
f_2	0.026	-0.081	5.2	-0.055 ± 0.012
f_3	0.013	-0.065	5.3	-0.052 ± 0.016
g_1	-0.031	0.15	5.3	0.12 ± 0.03
g_2	0.015	-0.040	5.3	-0.025 ± 0.008
g_3	0.012	-0.047	5.4	-0.035 ± 0.009
f_1^T	1.0	-1.0	5.4	0.0 ± 0.0
f_2^T	-0.29	0.42	5.4	0.13 ± 0.04
f_3^T	-0.24	0.41	5.4	0.17 ± 0.05
g_1^T	0.45	-0.46	5.4	-0.010 ± 0.003
g_2^T	0.031	0.055	5.4	0.086 ± 0.024
g_3^T	-0.011	-0.18	5.4	-0.19 ± 0.06

Table 1: Parameters appearing in the fit function of the form factors, f_1, f_2, f_3, g_1, g_2, g_3, f_1^T, f_2^T, f_3^T, g_1^T, g_2^T and g_3^T in full theory for $\Sigma_b \to \Sigma \ell^+ \ell^-$ and the values of the form factors at $q^2 = 0$. In this Table only central values of the parameters are presented.

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	$\Gamma(GeV)$	BR
$\Sigma_b \to \Sigma e^+e^-$	$(4.30 \pm 0.82) \times 10^{-18}$	$(9.09 \pm 1.73) \times 10^{-6}$
$\Sigma_b \to \Sigma \mu^+\mu^-$	$(4.29 \pm 0.82) \times 10^{-18}$	$(9.06 \pm 1.72) \times 10^{-6}$
$\Sigma_b \to \Sigma \tau^+\tau^-$	$(1.30 \pm 0.42) \times 10^{-18}$	$(2.75 \pm 0.88) \times 10^{-6}$

Table 2: The values of the decay rate and branching ratios for $\Sigma_b \to \Sigma \ell^+\ell^-$ for different leptons. In the case of decays into and electron positron pair, a lower cut-off of $q^2 \geq 0.04\ GeV^2$ is imposed to avoid the resonance due to a real photon creating the electron-positron pair.

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Appendix A

In this Appendix, the general decomposition of the matrix element, $\epsilon^{abc}\langle 0|u^a_\eta(0)d^b_\theta(x)s^c_\phi(0)|\Sigma(p)\rangle$ as well as the DA's of Σ [49] are presented. Considering Lorentz and parity invariances, the matrix element can be decomposed into various Lorentz structures as:

$$
\begin{aligned}
& 4\langle 0|\epsilon^{abc}u^a_\alpha(a_1x)s^b_\beta(a_2x)d^c_\gamma(a_3x)|\Sigma(p)\rangle \\
= & \ \mathcal{S}_1 m_\Sigma C_{\alpha\beta}(\gamma_5\Sigma)_\gamma + \mathcal{S}_2 m_\Sigma^2 C_{\alpha\beta}(\not{x}\gamma_5\Sigma)_\gamma \\
+ & \ \mathcal{P}_1 m_\Sigma(\gamma_5 C)_{\alpha\beta}\Sigma_\gamma + \mathcal{P}_2 m_\Sigma^2(\gamma_5 C)_{\alpha\beta}(\not{x}\Sigma)_\gamma + (\mathcal{V}_1 + \frac{x^2 m_\Sigma^2}{4}\mathcal{V}_1^M)(\not{p}C)_{\alpha\beta}(\gamma_5\Sigma)_\gamma \\
+ & \ \mathcal{V}_2 m_\Sigma(\not{p}C)_{\alpha\beta}(\not{x}\gamma_5\Sigma)_\gamma + \mathcal{V}_3 m_\Sigma(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu\gamma_5\Sigma)_\gamma + \mathcal{V}_4 m_\Sigma^2(\not{x}C)_{\alpha\beta}(\gamma_5\Sigma)_\gamma \\
+ & \ \mathcal{V}_5 m_\Sigma^2(\gamma_\mu C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu\gamma_5\Sigma)_\gamma + \mathcal{V}_6 m_\Sigma^3(\not{x}C)_{\alpha\beta}(\not{x}\gamma_5\Sigma)_\gamma + (\mathcal{A}_1 \\
+ & \ \frac{x^2 m_\Sigma^2}{4}\mathcal{A}_1^M)(\not{p}\gamma_5 C)_{\alpha\beta}\Sigma_\gamma + \mathcal{A}_2 m_\Sigma(\not{p}\gamma_5 C)_{\alpha\beta}(\not{x}\Sigma)_\gamma + \mathcal{A}_3 m_\Sigma(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma^\mu\Sigma)_\gamma \\
+ & \ \mathcal{A}_4 m_\Sigma^2(\not{x}\gamma_5 C)_{\alpha\beta}\Sigma_\gamma + \mathcal{A}_5 m_\Sigma^2(\gamma_\mu\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu\Sigma)_\gamma + \mathcal{A}_6 m_\Sigma^3(\not{x}\gamma_5 C)_{\alpha\beta}(\not{x}\Sigma)_\gamma \\
+ & \ (\mathcal{T}_1 + \frac{x^2 m_\Sigma^2}{4}\mathcal{T}_1^M)(p^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5\Sigma)_\gamma + \mathcal{T}_2 m_\Sigma(x^\mu p^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma_5\Sigma)_\gamma \\
+ & \ \mathcal{T}_3 m_\Sigma(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\gamma_5\Sigma)_\gamma + \mathcal{T}_4 m_\Sigma(p^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho\gamma_5\Sigma)_\gamma \\
+ & \ \mathcal{T}_5 m_\Sigma^2(x^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5\Sigma)_\gamma + \mathcal{T}_6 m_\Sigma^2(x^\mu p^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\not{x}\gamma_5\Sigma)_\gamma \\
+ & \ \mathcal{T}_7 m_\Sigma^2(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\not{x}\gamma_5\Sigma)_\gamma + \mathcal{T}_8 m_\Sigma^3(x^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho\gamma_5\Sigma)_\gamma \ . \qquad (A.1)
\end{aligned}
$$

The calligraphic functions in the above expression do not have definite twists but they can be written in terms of the Σ distribution amplitudes (DA's) with definite and increasing twists via the scalar product px and the parameters a_i, $i = 1, 2, 3$. The relationship between the calligraphic functions appearing in the above equation and scalar, pseudo-scalar, vector, axial vector and tensor DA's for Σ are given in Tables 3, 4, 5, 6 and 7, respectively.

$\mathcal{S}_1 = S_1$
$2px\mathcal{S}_2 = S_1 - S_2$

Table 3: Relations between the calligraphic functions and Σ scalar DA's.

$\mathcal{P}_1 = P_1$
$2px\mathcal{P}_2 = P_1 - P_2$

Table 4: Relations between the calligraphic functions and Σ pseudo-scalar DA's.

Every distribution amplitude $F(a_ipx)$= S_i, P_i, V_i, A_i, T_i can be represented as:

$$F(a_ipx) = \int dx_1dx_2dx_3\delta(x_1 + x_2 + x_3 - 1)e^{-ipx\sum_i x_ia_i}F(x_i) \ . \qquad (A.2)$$

where, x_i with $i = 1,\ 2$ and 3 are longitudinal momentum fractions carried by the participating quarks.

$\mathcal{V}_1 = V_1$
$2px\mathcal{V}_2 = V_1 - V_2 - V_3$
$2\mathcal{V}_3 = V_3$
$4px\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5$
$4px\mathcal{V}_5 = V_4 - V_3$
$4(px)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6$

Table 5: Relations between the calligraphic functions and Σ vector DA's.

$\mathcal{A}_1 = A_1$
$2px\mathcal{A}_2 = -A_1 + A_2 - A_3$
$2\mathcal{A}_3 = A_3$
$4px\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5$
$4px\mathcal{A}_5 = A_3 - A_4$
$4(px)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6$

Table 6: Relations between the calligraphic functions and Σ axial vector DA's.

The explicit expressions for the Σ DA's up to twists 6 are given as follows [49]:
Twist-3 distribution amplitudes:

$$
\begin{aligned}
V_1(x_i) &= 120x_1x_2x_3\phi_3^0\,, & A_1(x_i) = 0\,,\\
T_1(x_i) &= 120x_1x_2x_3\phi_3^{'0}\,. & \qquad (25)
\end{aligned}
$$

Twist-4 distribution amplitudes:

$$
\begin{aligned}
S_1(x_i) &= 6(x_2-x_1)x_3(\xi_4^0+\xi_4^{'0})\,, & P_1(x_i) = 6(x_2-x_1)x_3(\xi_4^0-\xi_4^{'0})\,,\\
V_2(x_i) &= 24x_1x_2\phi_4^0\,, & A_2(x_i) = 0\,,\\
V_3(x_i) &= 12x_3(1-x_3)\psi_4^0\,, & A_3(x_i) = -12x_3(x_1-x_2)\psi_4^0\,,\\
T_2(x_i) &= 24x_1x_2\phi_4^{'0}\,, & T_3(x_i) = 6x_3(1-x_3)(\xi_4^0+\xi_4^{'0})\,,\\
T_7(x_i) &= 6x_3(1-x_3)(\xi_4^{'0}-\xi_4^0)\,. & \qquad (26)
\end{aligned}
$$

Twist-5 distribution amplitudes:

$$
\begin{aligned}
S_2(x_i) &= \frac{3}{2}(x_1-x_2)(\xi_5^0+\xi_5^{'0})\,, & P_2(x_i) = \frac{3}{2}(x_1-x_2)(\xi_5^0-\xi_5^{'0})\,,\\
V_4(x_i) &= 3(1-x_3)\psi_5^0\,, & A_4(x_i) = 3(x_1-x_2)\psi_5^0\,,\\
V_5(x_i) &= 6x_3\phi_5^0\,, & A_5(x_i) = 0\,,\\
T_4(x_i) &= -\frac{3}{2}(x_1+x_2)(\xi_5^{'0}+\xi_5^0)\,, & T_5(x_i) = 6x_3\phi_5^{'0}\,,\\
T_8(x_i) &= \frac{3}{2}(x_1+x_2)(\xi_5^{'0}-\xi_5^0)\,. & \qquad (27)
\end{aligned}
$$

Finally, twist-6 distribution amplitudes:

$$
\begin{aligned}
V_6(x_i) &= 2\phi_6^0\,, & A_6(x_i) = 0\,,\\
T_6(x_i) &= 2\phi_6^{'0}\,. & \qquad (28)
\end{aligned}
$$

$\mathcal{T}_1 = T_1$
$2px\mathcal{T}_2 = T_1 + T_2 - 2T_3$
$2\mathcal{T}_3 = T_7$
$2px\mathcal{T}_4 = T_1 - T_2 - 2T_7$
$2px\mathcal{T}_5 = -T_1 + T_5 + 2T_8$
$4(px)^2\mathcal{T}_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8$
$4px\mathcal{T}_7 = T_7 - T_8$
$4(px)^2\mathcal{T}_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8$

Table 7: Relations between the calligraphic functions and Σ tensor DA's.

$$\begin{aligned}
\phi_3^0 &= \phi_6^0 = f_{\Sigma^+}, & \psi_4^0 = \psi_5^0 = \frac{1}{2}(f_{\Sigma^+} - \lambda_1)\,, \\
\phi_4^0 &= \phi_5^0 = \frac{1}{2}(f_{\Sigma^+} + \lambda_1), & \phi_3'^0 = \phi_6'^0 = -\xi_5^0 = \frac{1}{6}(4\lambda_3 - \lambda_2)\,, \\
\phi_4'^0 &= \xi_4^0 = \frac{1}{6}(8\lambda_3 - 3\lambda_2), & \phi_5'^0 = -\xi_5'^0 = \frac{1}{6}\lambda_2\,, \\
\xi_4'^0 &= \frac{1}{6}(12\lambda_3 - 5\lambda_2)\,, \qquad (29)
\end{aligned}$$

where,

$$\begin{aligned}
f_\Sigma &= (9.4 \pm 0.4) \times 10^{-3}\ \mathrm{GeV}^2, & \lambda_1 = -(2.5 \pm 0.1) \times 10^{-2}\ \mathrm{GeV}^2, \\
\lambda_2 &= (4.4 \pm 0.1) \times 10^{-2}\ \mathrm{GeV}^2, & \lambda_3 = (2.0 \pm 0.1) \times 10^{-2}\ \mathrm{GeV}^2. \qquad (30)
\end{aligned}$$