# The flavor changing $t \rightarrow c l_{1}^{-} l_{2}^{+}$decay in the general two Higgs doublet model. 

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#### Abstract

We study the flavor changing $t \rightarrow c l_{1}^{-} l_{2}^{+}$decay in the framework of the general two Higgs doublet model, so called model III. We predict the branching ratio for $l_{1}=\tau, l_{2}=\mu$ at the order of the magnitude of $B R \sim 10^{-8}$.


[^0]
## 1 Introduction

The top quark has a large mass and therefore it breaks the $S U(2) \times U(1)$ symmetry maximally. Richness of the decay products stimulates one to study its decays to test the standard model (SM) and to get some clues about the new physics, beyond. The rare decays of the top quark have been studied in the literature, in the framework of the SM and beyond [1]- [10]; the oneloop flavor changing transitions $t \rightarrow c g(\gamma, Z)$ in [4, 7], $t \rightarrow c V(V V)$ in [6] and $t \rightarrow c H^{0}$ in [2, 7, 8, 9, 10].

These decays are strongly suppressed in the SM and the predicted values of the branching ratio $(B R)$ of the process $t \rightarrow c g(\gamma, Z)$ is $4 \times 10^{-11}\left(5 \times 10^{-13}, 1.3 \times 10^{-13}\right)$ [2], the $B R$ for $t \rightarrow c H^{0}$ is at the order of the magnitude of $10^{-14}-10^{-13}$, in the SM [8]. These prediction are so small that it is not possible to measure them even at the highest luminosity accelerators. This forces one to go beyond the SM and study these rare decays in the framework of new physics. $t \rightarrow c H^{0}$ decay has been studied in the general two Higgs doublet model (model III) [10] and it has been found that the $B R$ of this process could reach to the values of order $10^{-6}$, playing with the free parameters of the model III, respecting the existing experimental restrictions. This is a strong enhancement, almost seven orders larger compared to the one in the SM.

The present work is devoted to the analysis of the flavor changing (FC) $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$ decay in the framework of the general two Higgs doublet model (model III). This decay occurs in the tree level since the FC transitions in the quark and leptonic sector are permitted in the model III. Here, the Yukawa couplings for $t-c$ and $l_{1}^{-}-l_{2}^{+}$transitions play the main role and they exist with the help of the internal neutral Higgs bosons, $h^{0}$ and $A^{0}$. In the process, it is possible to get $h^{0}$ and $A^{0}$ resonances since the kinematical region is large enough and this difficulty can be solved by choosing the appropriate propagator for $h^{0}$ and $A^{0}$ (see section 2). In the tree level, the $B R$ of the $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$decay for $l_{1}=\tau$ and $l_{2}=\mu$ is predicted as $10^{-8}-10^{-7}$. We also calculate the one loop effects related with the interactions due to the internal mediating charged Higgs boson (see Fig. 目 b,c,d) and observe that their contribution to the $B R$ is negligible, namely $10^{-11}-10^{-10}$.

The paper is organized as follows: In Section 2, we present the $B R$ of the decay $t \rightarrow$ $c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$in the framework of model III. Section 3 is devoted to discussion and our conclusions.

## 2 The flavor changing $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$decay in the framework of the general two Higgs Doublet model

The flavor changing transition $t \rightarrow c l_{1}^{-} l_{2}^{+}$is forbidden in the SM. Such transitions would be possible in the case that the Higgs sector is extended and the flavor changing neutral currents (FCNC) in the tree level are permitted. The simplest model which obeys these features is the model III version of the two Higgs doublet model (2HDM). This section is devoted to the calculation of the $B R$, in the model III. In this model, there are various new parameters, such as complex Yukawa couplings, masses of new Higgs bosons., etc... and they should be restricted by using the present experimental results.

The $t \rightarrow c l_{1}^{-} l_{2}^{+}$process is controlled by the Yukawa interaction and, in the model III, it reads

$$
\begin{align*}
\mathcal{L}_{Y} & =\eta_{i j}^{U} \bar{Q}_{i L} \tilde{\phi}_{1} U_{j R}+\eta_{i j}^{D} \bar{Q}_{i L} \phi_{1} D_{j R}+\xi_{i j}^{U \dagger} \bar{Q}_{i L} \tilde{\phi}_{2} U_{j R}+\xi_{i j}^{D} \bar{Q}_{i L} \phi_{2} D_{j R} \\
& +\eta_{i j}^{E} \bar{l}_{i L} \phi_{1} E_{j R}+\xi_{i j}^{E} \overline{\bar{l}}_{i L} \phi_{2} E_{j R}+h . c . \tag{1}
\end{align*}
$$

where $L$ and $R$ denote chiral projections $L(R)=1 / 2\left(1 \mp \gamma_{5}\right), \phi_{i}$ for $i=1,2$, are the two scalar doublets, $\bar{Q}_{i L}$ are left handed quark doublets, $U_{j R}\left(D_{j R}\right)$ are right handed up (down) quark singlets, $l_{i L}\left(E_{j R}\right)$ are lepton doublets (singlets), with family indices $i, j$. The Yukawa matrices $\xi_{i j}^{U, D}$ and $\xi_{i j}^{E}$ have in general complex entries. It is possible to collect SM particles in the first doublet and new particles in the second one by choosing the parametrization for $\phi_{1}$ and $\phi_{2}$ as

$$
\begin{equation*}
\phi_{1}=\frac{1}{\sqrt{2}}\left[\binom{0}{v+H^{0}}+\binom{\sqrt{2} \chi^{+}}{i \chi^{0}}\right] ; \phi_{2}=\frac{1}{\sqrt{2}}\binom{\sqrt{2} H^{+}}{H_{1}+i H_{2}} . \tag{2}
\end{equation*}
$$

with the vacuum expectation values,

$$
\begin{equation*}
<\phi_{1}>=\frac{1}{\sqrt{2}}\binom{0}{v} ;<\phi_{2}>=0 \tag{3}
\end{equation*}
$$

and considering the gauge and $C P$ invariant Higgs potential which spontaneously breaks $S U(2) \times U(1)$ down to $U(1)$ as:

$$
\begin{align*}
V\left(\phi_{1}, \phi_{2}, \phi_{3}\right) & =c_{1}\left(\phi_{1}^{+} \phi_{1}-v^{2} / 2\right)^{2}+c_{2}\left(\phi_{2}^{+} \phi_{2}\right)^{2} \\
& ++c_{3}\left[\left(\phi_{1}^{+} \phi_{1}-v^{2} / 2\right)+\phi_{2}^{+} \phi_{2}\right]^{2}+c_{4}\left[\left(\phi_{1}^{+} \phi_{1}\right)\left(\phi_{2}^{+} \phi_{2}\right)-\left(\phi_{1}^{+} \phi_{2}\right)\left(\phi_{2}^{+} \phi_{1}\right)\right] \\
& +c_{5}\left[\operatorname{Re}\left(\phi_{1}^{+} \phi_{2}\right)\right]^{2}+c_{6}\left[\operatorname{Im}\left(\phi_{1}^{+} \phi_{2}\right)\right]^{2}+c_{7}, \tag{4}
\end{align*}
$$

with constants $c_{i}, i=1, \ldots, 7$. Here, $H_{1}$ and $H_{2}$ are the mass eigenstates $h^{0}$ and $A^{0}$ respectively, since no mixing occurs between two CP-even neutral bosons $H^{0}$ and $h^{0}$ in the tree level, for our choice.

The Flavor Changing (FC) interaction can be obtained as

$$
\begin{equation*}
\mathcal{L}_{Y, F C}=\xi_{i j}^{U \dagger} \bar{Q}_{i L} \tilde{\phi}_{2} U_{j R}+\xi_{i j}^{D} \bar{Q}_{i L} \phi_{2} D_{j R}+\xi_{i j}^{E} \bar{l}_{i L} \phi_{2} E_{j R}+h . c . \tag{5}
\end{equation*}
$$

where the couplings $\xi^{U, D}$ for the FC charged interactions are

$$
\begin{align*}
\xi_{c h}^{U} & =\xi_{N}^{U} V_{C K M}, \\
\xi_{c h}^{D} & =V_{C K M} \xi_{N}^{D}, \tag{6}
\end{align*}
$$

and $\xi_{N}^{U, D}$ is defined by the expression

$$
\begin{equation*}
\xi_{N}^{U(D)}=\left(V_{R(L)}^{U(D)}\right)^{-1} \xi^{U,(D)} V_{L(R)}^{U(D)} . \tag{7}
\end{equation*}
$$

Here the index " N " in $\xi_{N}^{U, D}$ denotes the word "neutral". Notice that, in the following, we replace $\xi^{U, D, E}$ with $\xi_{N}^{U, D, E}$ where "N" denotes the word "neutral" and define $\bar{\xi}_{N}^{U, D, E}$ which satisfies the equation $\xi_{N}^{U, D, E}=\sqrt{\frac{4 G_{F}}{\sqrt{2}}} \bar{\xi}_{N}^{U, D, E}$.

In the model III, the $t \rightarrow c l_{1}^{-} l_{2}^{+}$decay exists in the tree level, by taking non-zero $t-c\left(l_{1}^{-}-l_{2}^{+}\right)$ transition with the help of the neutral bosons $h^{0}$ and $A^{0}$. For completeness, we also take the one loop contributions into account (see Fig. []) and, we use the onshell renormalization scheme to get rid of the existing divergences. The method is to obtain the renormalized $t \rightarrow c h^{0 *}\left(A^{0 *}\right)$ transition vertex function

$$
\begin{align*}
& \Gamma_{R E N}^{h^{0 *}}=\Gamma_{0}^{h^{0 *}}+\Gamma_{C}^{h^{0}} \\
& \Gamma_{R E N}^{A^{0 *}}=\Gamma_{0}^{A^{0 *}}+\Gamma_{C}^{A^{0}} \tag{8}
\end{align*}
$$

by using

$$
\begin{align*}
\left.\Gamma_{R E N}^{h^{0}}\right|_{\text {onshell }} & =\frac{i}{2 \sqrt{2}}\left(\left(\xi_{N, t c}^{U}+\xi_{N, c t}^{U *}\right)+\left(\xi_{N, t c}^{U}-\xi_{N, c t}^{U *}\right) \gamma_{5}\right) \\
\left.\Gamma_{R E N}^{A^{0}}\right|_{\text {onshell }} & =-\frac{1}{2 \sqrt{2}}\left(\left(\xi_{N, t c}^{U}-\xi_{N, c t}^{U *}\right)+\left(\xi_{N, t c}^{U}+\xi_{N, c t}^{U *}\right) \gamma_{5}\right) \tag{9}
\end{align*}
$$

and the counter term

$$
\begin{align*}
\Gamma_{C}^{h^{0}} & =\left.\Gamma_{R E N}^{h^{0}}\right|_{\text {onshell }}-\left.\Gamma_{0}^{h^{0}}\right|_{\text {onshell }} \\
\Gamma_{C}^{A^{0}} & =\left.\Gamma_{R E N}^{A^{0}}\right|_{\text {onshell }}-\left.\Gamma_{0}^{A^{0}}\right|_{\text {onshell }} . \tag{10}
\end{align*}
$$

where $\Gamma_{0}^{h^{0}}$ is the bare vertex function. Here, we take the loop diagrams (see Fig. \#) including $H^{ \pm}$intermediate boson for FC interaction (Fig. 1: b, c, d) in the quark sector, since $\xi_{N, b b}^{D}$ and $\xi_{N, t t}^{U}$ are dominant couplings in the loop effects. Therefore, we neglect all the Yukawa couplings
except $\xi_{N, b b}^{D}$ and $\xi_{N, t t}^{U}$ in the loop contributions. Notice that the self energy diagrams do not give any contribution in the onshell renormalization scheme.

The renormalized vertex function is connected to the $l_{1}^{-} l_{2}^{+}$out going leptons by intermediate $h^{0}$ and $A^{0}$ bosons as shown in the Fig. 1 and for the matrix element square of the process $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$we get

$$
\begin{align*}
|M|^{2} & =8 m_{t}^{2}(1-s) \sum_{S=h^{0}, A^{0}}\left|p_{S}\right|^{2}\left(\left|a_{S}^{(q)}\right|^{2}+\left|a_{S}^{\prime(q)}\right|^{2}\right)\left(\left(s m_{t}^{2}-\left(m_{l_{1}^{-}}-m_{l_{2}^{+}}\right)^{2}\right)\left|a_{S}^{(l)}\right|^{2}\right. \\
& \left.+\left(s m_{t}^{2}-\left(m_{l_{1}^{-}}+m_{l_{2}^{+}}\right)^{2}\right)\left|a_{S}^{\prime(l)}\right|^{2}\right) \\
& +16 m_{t}^{2}(1-s)\left(\left(s m_{t}^{2}-\left(m_{l_{1}^{-}}-m_{l_{2}^{+}}\right)^{2}\right) \operatorname{Re}\left[p_{h^{0}} p_{A^{0}}^{*} a_{h^{0}}^{(l)} a_{A^{0}}^{*(l)}\left(a_{h^{0}}^{(q)} a_{A^{0}}^{*(q)}+a_{h^{0}}^{\prime(q)} a_{A^{0}}^{\prime *(q)}\right)\right]\right. \\
& \left.+\left(s m_{t}^{2}-\left(m_{l_{1}^{-}}+m_{l_{2}^{+}}\right)^{2}\right) \operatorname{Re}\left[p_{h^{0}} p_{A^{0}}^{*} a_{h^{0}}^{\prime(l)} a_{A^{0}}^{\prime *(l)}\left(a_{h^{0}}^{(q)} a_{A^{0}}^{*(q)}+a_{h^{0}}^{\prime(q)} a_{A^{0}}^{\prime *(q)}\right)\right]\right) \tag{11}
\end{align*}
$$

where

$$
\begin{equation*}
p_{S}=\frac{i}{s m_{t}^{2}-m_{S}^{2}+i m_{S} \Gamma_{t o t}^{S}} \tag{12}
\end{equation*}
$$

$\Gamma_{\text {tot }}^{S}$ is the total decay width of $S$ boson, for $S=h^{0} A^{0}$. Here, the parameter $s$ is $s=\frac{q^{2}}{m_{t}^{2}}$, and $q^{2}$ is the intermediate $S$ boson momentum square. In eq. (11) the functions $a_{h^{0}, A^{0}}^{(l)}, a_{h^{0}, A^{0}}^{\prime(l)}$ have tree level contributions and $a_{h^{0}, A^{0}}^{(q)}, a_{h^{0}, A^{0}}^{\prime(q)}$ are the combinations of tree level and one-loop level contributions,

$$
\begin{align*}
a_{h^{0}, A^{0}}^{(l)} & =a_{h^{0}, A^{0}}^{\text {Tree }(l)}, \\
a_{h^{0}, A^{0}}^{(q)} & =a_{h^{0}, A^{0}(q)}^{\text {Tree }}+a_{h^{0}, A^{0}}^{\text {Loop }(q)}, \\
a_{h^{0}, A^{0}}^{\prime(l)} & =a_{h^{0}, A^{0}}^{\prime \text { Tree }(l)}, \\
a_{h^{0}, A^{0}}^{\prime(q)} & =a_{h^{0}, A^{0}}^{\prime \text { Tree }(q)}+a_{h^{0}, A^{0}}^{\text {Loop }}(q) \tag{13}
\end{align*}
$$

and they read

$$
\begin{aligned}
a_{h^{0}}^{\text {Tree }(l)} & =-\frac{i}{2 \sqrt{2}}\left(\xi_{N, l_{1} l_{2}}^{E}+\xi_{N, l_{2} l_{1}}^{* E}\right), \\
a_{A^{0}}^{T r e e}(l) & =\frac{1}{2 \sqrt{2}}\left(\xi_{N, l_{1} l_{2}}^{E}-\xi_{N, l_{2} l_{1}}^{* E}\right), \\
a_{h^{0}}^{\prime \text { Tree }(l)} & =-\frac{i}{2 \sqrt{2}}\left(\xi_{N, l_{1} l_{2}}^{E}-\xi_{N, l_{2} l_{1}}^{* E}\right), \\
a_{A^{0}}^{\prime \text { Tree }(l)} & =\frac{1}{2 \sqrt{2}}\left(\xi_{N, l_{1} l_{2}}^{E}+\xi_{N, l_{2} l_{1}}^{* E}\right), \\
a_{h^{0}}^{\text {Tree }(q)} & =\frac{i}{2 \sqrt{2}}\left(\xi_{N, t c}^{U}+\xi_{N, c t}^{* U}\right),
\end{aligned}
$$

$$
\begin{align*}
& a_{A^{0}}^{\text {Tree }(q)}=-\frac{1}{2 \sqrt{2}}\left(\xi_{N, t c}^{U}-\xi_{N, c t}^{* U}\right), \\
& a_{h^{0}}^{\prime \text { Tree }(q)}=\frac{i}{2 \sqrt{2}}\left(\xi_{N, t c}^{U}-\xi_{N, c t}^{* U}\right), \\
& a_{A^{0}}^{\prime \text { Tree }(q)}=-\frac{1}{2 \sqrt{2}}\left(\xi_{N, t c}^{U}+\xi_{N, c t}^{* U}\right), \\
& a_{h^{0}}^{\text {Loop }(q)}=-\frac{i}{32 \sqrt{2} \pi^{2}} V_{c b} V_{t b}^{*} \xi_{N, b b}^{D}\left(m_{b}^{2} \xi_{N, b b}^{D} \xi_{N, t t}^{U *} \int_{0}^{1} d x \int_{0}^{1-x} d y f_{1}^{h^{0}}(x, y)\right. \\
&+m_{b} m_{t}\left(\xi_{N, b b}^{D *}\right)^{2} \int_{0}^{1} d x \int_{0}^{1-x} d y\left((1-x-y) f_{1}^{h^{0}}(x, y)\right) \\
&-m_{b} m_{t}\left|\xi_{N, b b}^{D}\right|^{2} \int_{0}^{1} d x \int_{0}^{1-x} d y\left((x+y) f_{1}^{h^{0}}(x, y)\right) \\
&\left.-\xi_{N, b b}^{D *} \xi_{N, t t}^{U *} \int_{0}^{1} d x \int_{0}^{1-x} d y f_{2}^{h^{0}}(x, y)\right), \\
&=\frac{1}{32 \sqrt{2} \pi^{2}} V_{c b} V_{t b}^{*} \xi_{N, b b}^{D}\left(m_{b}^{2} \xi_{N, b b}^{D} \xi_{N, t t}^{U *} \int_{0}^{1} d x \int_{0}^{1-x} d y f_{1}^{A^{0}}(x, y)\right. \\
& m_{b} m_{t}\left(\xi_{N, b b}^{D *}\right)^{2} \int_{0}^{1} d x \int_{0}^{1-x} d y\left((1-x-y) f_{1}^{A^{0}}(x, y)\right) \\
& a_{A^{0}}^{\text {Loop }(q)} \\
&-m_{b} m_{t}\left|\xi_{N, b b}^{D}\right|^{2} \int_{0}^{1} d x \int_{0}^{1-x} d y\left((x+y) f_{1}^{A^{0}}(x, y)\right) \\
&\left.+\xi_{N, b b}^{D *} \xi_{N, t t}^{U *} \int_{0}^{1} d x \int_{0}^{1-x} d y f_{2}^{A^{0}}(x, y)\right), \\
& a_{h^{0}}^{\text {Loop }(q)},  \tag{14}\\
& a_{h^{0}}^{\prime \operatorname{Loop}(q)} \\
& a_{A^{0}}^{\prime \text { Loop }(q)}=a_{A^{0}}^{L o o p(q)},
\end{align*}
$$

where

$$
\begin{align*}
f_{1}^{S} & =\frac{1}{L^{S}\left(m_{S}\right)}-\frac{1}{L^{S}(s)} \\
f_{2}^{S} & =(1-x-y)\left(\frac{\left.m_{t}^{2} x+m_{S}^{2} y\right)}{L^{S}\left(m_{S}\right)}-\frac{m_{t}^{2}(x+s y)}{L^{S}(s)}\right)+2 \ln \frac{L^{S}(s)}{L^{S}\left(m_{S}\right)} \tag{15}
\end{align*}
$$

with

$$
\begin{align*}
L^{S}(s) & =m_{b}^{2}(x-1)+m_{H^{ \pm}}^{2} x+m_{t}^{2}(-1+x+y)(x+s y) \\
L^{S}\left(m_{S}\right) & =m_{b}^{2}(x-1)+m_{H^{ \pm}}^{2} x+(-1+x+y)\left(m_{t}^{2} x+m_{S}^{2} y\right) \tag{16}
\end{align*}
$$

Finally, the differential decay width (dDW) $\frac{d \Gamma}{d s}\left(t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)\right)$is obtained by using the expression

$$
\begin{equation*}
\frac{d \Gamma}{d s}=\frac{1}{256 N_{c} \pi^{3}} \lambda|M|^{2}, \tag{17}
\end{equation*}
$$

where $\lambda$ is:
$\lambda=\frac{\sqrt{\left(m_{t}^{2}(s-1)^{2}-4 m_{c}^{2}\right)\left(m_{c}^{4}+m_{l_{1}}^{4}+\left(m_{l_{2}}^{2}-m_{t}^{2} s\right)^{2}-2 m_{c}^{2}\left(m_{l_{1}}^{2}+m_{l_{2}}^{2}-m_{t}^{2} s\right)-2 m_{l_{1}}^{2}\left(m_{l_{2}}^{2}+m_{t}^{2} s\right)\right)}}{2 m_{t}^{2} s}$. Here the parameter
$s$ is restricted into the region $\frac{\left(m_{l_{1}}+m_{l_{2}}\right)^{2}}{m_{t}^{2}} \leq s \leq \frac{\left(m_{t}-m_{c}\right)^{2}}{m_{t}^{2}}$. Notice that we use the parametrization $\xi_{N, l_{1} l_{2}}^{E}=\left|\xi_{N, l_{1} l_{2}}^{E}\right| e^{i \theta_{l_{1} l_{2}}}$ for the leptonic part, in the numerical calculations.

## 3 Discussion

This section is devoted to the analyses of the differential $B R(d B R)$ and the $B R$ of the process $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$in the tree level and also in the one loop level, in the model III. The Yukawa couplings $\xi_{N, t c}^{U}$ and $\xi_{N, l_{1} l_{2}}^{E}$ play the main role in the tree level and new couplings, especially $\xi_{N, b b}^{D}, \xi_{N, t t}^{U}$, enter into calculations if one goes to the loop level. Since these couplings are free parameters of the model used, it is necessary to restrict them, using appropriate experimental results. We use the constraint region by restricting the Wilson coefficient $C_{7}^{e f f}$, which is the effective coefficient of the operator $O_{7}=\frac{e}{16 \pi^{2}} \bar{s}_{\alpha} \sigma_{\mu \nu}\left(m_{b} R+m_{s} L\right) b_{\alpha} \mathcal{F}^{\mu \nu}$ (see 11 and references therein), in the region $0.257 \leq\left|C_{7}^{e f f}\right| \leq 0.439$. Here upper and lower limits were calculated using the CLEO measurement 12

$$
\begin{equation*}
B R\left(B \rightarrow X_{s} \gamma\right)=(3.15 \pm 0.35 \pm 0.32) 10^{-4} \tag{18}
\end{equation*}
$$

and all possible uncertainities in the calculation of $C_{7}^{e f f}$ [1]. The above restriction ensures to get upper and lower limits for $\xi_{N, b b}^{D}, \xi_{N, t t}^{U}$ and also for $\xi_{N, t c}^{U}$ (see (11 for details). In our numerical calculations we choose the upper limit for $C_{7}^{e f f}>0$, fix $\xi_{N, b b}^{D}=30 m_{b}$ and take $\xi_{N, t c}^{U} \sim 0.01 \xi_{N, t t}^{U} \sim 0.0025$, respecting the constraints mentioned. Furthermore, the couplings $\xi_{N, l_{1} l_{2}}^{E}$ in the leptonic part are restricted by using the experimental results, such as, anomalous magnetic moment of muon, dipole moments of leptons, rare leptonic decays. For $l_{1}=\tau$ and $l_{2}=\mu$, we take the upper limit obtained by using experimental result of anomalous magnetic moment of muon [13]. For $l_{1}=\tau$ and $l_{2}=e$, we use the numerical result obtained for the couplings $\xi_{N, \tau e}^{E}$ in [14], based on the experimental measurement of the leptonic process $\mu \rightarrow e \gamma$ [15]. The total decay widths of $h^{0}$ and $A^{0}$ are unknown parameters and we expect that they are at the same order of magnitude of $\Gamma_{\text {tot }}^{H^{0}} \sim(0.1-1.0) G e V$, where $H^{0}$ is the SM Higgs boson. Notice that, we take the value of the total decay width $\Gamma_{T} \sim \Gamma(t \rightarrow b W)$ as $\Gamma_{T}=1.55 \mathrm{GeV}$ and choose the numerical values $m_{h^{0}}=80 \mathrm{GeV}$ and $m_{A^{0}}=90 \mathrm{GeV}$, for the calculation of the $B R$.

In Fig. 日, we plot the dBR for the $t \rightarrow c\left(\tau^{-} \mu^{+}+\tau^{+} \mu^{-}\right)$decay with respect to $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$ for $\sin \theta_{\tau \mu}=0.5$, different $s$ values, $s=\left(\frac{10}{175}\right)^{2},\left(\frac{50}{175}\right)^{2}, s=\left(\frac{150}{175}\right)^{2}$. Here, we choose $\bar{\xi}_{N, t c}^{U}$ real and $\Gamma_{\text {tot }}^{h^{0}}=\Gamma_{\text {tot }}^{A^{0}}=0.1 \mathrm{GeV}$. The solid (dashed, small dashed) line represents the case for $s=\left(\frac{10}{175}\right)^{2}\left(\left(\frac{50}{175}\right)^{2},\left(\frac{150}{175}\right)^{2}\right)$. From the figure, it is seen that the dBR is at the order of the
magnitude of $10^{-8}$ for $s=\left(\frac{50}{175}\right)^{2}$ and $\left|\bar{\xi}_{N, \tau \mu}^{E}\right| \sim 5 \mathrm{GeV}$. dBR is less than $10^{-8}$ for $s=\left(\frac{10}{175}\right)^{2}$ and $s=\left(\frac{150}{175}\right)^{2}$ and it reaches extremely small values for $\left|\bar{\xi}_{N, \tau \mu}^{E}\right| \leq 1 \mathrm{GeV}$. Increasing $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$ causes to enhance the dBR, as expected. Fig. 园 is devoted to the same dependence for $s=\left(\frac{80}{175}\right)^{2}$ (solid line), $\left(\frac{90}{175}\right)^{2}$ (dashed line), where the values of $s$ are taken at the $h^{0}$ and $A^{0}$ resonances. The dBR is at the order of the magnitude of $10^{-6}$ for the small values of the coupling $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$ and increases extremely with the increasing values of this coupling.

In Fig. 4, we plot the dBR with respect to $s$, for $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|=10 \mathrm{GeV}$, $\sin \theta_{\tau \mu}=0.5$ and $\Gamma_{\text {tot }}^{h^{0}}=\Gamma_{\text {tot }}^{A^{0}}=0.1 \mathrm{GeV}$. It is observed that dBR has a strong $s$ dependence.

Finally, in Fig. 5 we present the $B R$ for the process $t \rightarrow c\left(\tau^{-} \mu^{+}+\tau^{+} \mu^{-}\right)$with respect to $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$ for $\sin \theta_{\tau \mu}=0.5$ and $\Gamma_{\text {tot }}^{h^{0}}=\Gamma_{\text {tot }}^{A^{0}}=0.1 \mathrm{GeV}$. The $B R$ is at the order of the magnitude of $10^{-8}$ for $\left|\bar{\xi}_{N, \tau \mu}^{E}\right| \sim 2(G e V)$ and increases to the values $10^{-7}$ with increasing $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$. Notice that the one loop effects are at the order of the magnitude of $0.1 \%$ of the tree level result and therefore their contribution is negligible.

In the case of outgoing $\tau$ and $e$ leptons, the $B R$ is predicted at the order of the magnitude of $10^{-14}-10^{-15}$, respecting the numerical values of the coupling $\left|\bar{\xi}_{N, \tau e}^{E}\right|=\left(10^{-4}-10^{-3}\right) \mathrm{GeV}$, obtained in [14], based on the experimental measurement of the leptonic process $\mu \rightarrow e \gamma$. For the outgoing $\mu$ and $e$ leptons, we believe that the $B R$ is extremely small, too difficult to be measured.

At this stage we would like to summarize our results:

- The $B R$ of the flavor changing process $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$is forbidden in the SM and the extended Higgs sector can bring considerable contribution to the $B R$ in the tree level, at the order of the magnitude of $10^{-8}-10^{-7}$, for $l_{1}=\tau$ and $l_{1}=\mu$. A measurement of such a $B R$ will be highly non-trivial due to efficiency problems in measuring the $\tau$-lepton and in identifying a c-quark jet. Moreover, one will have to overcome the problem of isolating the signal from possibly large reducible background by applying clever kinematical cuts which will further degrade the signal. However, the possible enhancement of the $B R$ of the given process in the model III forces one to search new models to get a measurable $B R$ theoretically. The $B R$ is sensitive to Yukawa coupling $\xi_{N, l_{1} l_{2}}^{E}$ and, respecting the experimental limits on the relevant couplings, this results in extremely smaller $B R$ 's of $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+l_{1}^{+} l_{2}^{-}\right)$, for $l_{1}=\tau, l_{2}=e$ and $l_{1}=\mu, l_{2}=e$, compared to the one for $l_{1}=\tau, l_{2}=\mu$. Notice that the loop effects are negligibly small.

Therefore, the future theoretical and experimental investigations of the process $t \rightarrow c\left(l_{1}^{-} l_{2}^{+}+\right.$ $l_{1}^{+} l_{2}^{-}$), especially for $l_{1}=\tau, l_{2}=\mu$, would play an important role in the determination the
physics beyond the SM.

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Figure 1: Tree level and one loop level diagrams contribute to the decay $t \rightarrow c l_{1}^{-} l_{2}^{+}$. Dashed lines represent the $h^{0}, A^{0}, \phi^{ \pm}, W^{ \pm}, H^{ \pm}$fields.


Figure 2: $\mathrm{dBR}\left(t \rightarrow c\left(\tau^{-} \mu^{+}+\tau^{+} \mu^{-}\right)\right)$as a function of $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$ for $m_{h^{0}}=80 \mathrm{GeV}, m_{A^{0}}=90 \mathrm{GeV}$, $\sin \theta_{\tau \mu}=0.5$, real $\bar{\xi}_{N, t c}^{U}$ and $\Gamma_{\text {tot }}^{h^{0}}=\Gamma_{t o t}^{A^{0}}=0.1 \mathrm{GeV}$. The solid (dashed, dash-dotted) line represents the case for $s=\left(\frac{10}{175}\right)^{2}\left(\left(\frac{50}{175}\right)^{2},\left(\frac{150}{175}\right)^{2}\right)$.


Figure 3: The same as Fig. 2 but for $s=\left(\frac{80}{175}\right)^{2}$ and $\left(\left(\frac{90}{175}\right)^{2}\right.$. The solid (dashed) line represents the case for $s=\left(\frac{80}{175}\right)^{2}\left(\left(\frac{90}{175}\right)^{2}\right)$.


Figure 4: $\mathrm{dBR}\left(t \rightarrow c\left(\tau^{-} \mu^{+}+\tau^{+} \mu^{-}\right)\right)$as a function of $s$ for $m_{h^{0}}=80 \mathrm{GeV}, m_{A^{0}}=90 \mathrm{GeV}$, $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|=10 \mathrm{GeV}, \sin \theta_{\tau \mu}=0.5$, real $\bar{\xi}_{N, t c}^{U}$ and $\Gamma_{\text {tot }}^{h^{0}}=\Gamma_{\text {tot }}^{A^{0}}=0.1 \mathrm{GeV}$.


Figure 5: $B R\left(t \rightarrow c\left(\tau^{-} \mu^{+}+\tau^{+} \mu^{-}\right)\right)$as a function of $\left|\bar{\xi}_{N, \tau \mu}^{E}\right|$ for $m_{h^{0}}=80 \mathrm{GeV}, m_{A^{0}}=90 \mathrm{GeV}$, $\sin \theta_{\tau \mu}=0.5$, real $\bar{\xi}_{N, t c}^{U}$ and $\Gamma_{t o t}^{h^{0}}=\Gamma_{\text {tot }}^{A^{0}}=0.1 \mathrm{GeV}$.


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