Forward–backward asymmetries in $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay beyond the standard model

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Abstract

We study the doubly–polarized lepton pair forward–backward asymmetries in $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay using a general, model independent form of the effective Hamiltonian. We present the general expression for nine double–polarization forward–backward asymmetries. It is observed that, the study of the forward–backward asymmetries of the doubly–polarized lepton pair is a very useful tool for establishing new physics beyond the standard model. Moreover, the correlation between forward–backward asymmetry and branching ratio is investigated. It is shown that there exist certain certain regions of the new Wilson coefficients where new physics can be established by measuring the polarized forward–backward asymmetry only.

PACS numbers: 12.60.–i, 13.30.–a. 13.88.+e

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1 Introduction

With the B–factories at work, search of rare decays induced by the flavor–changing neutral current (FCNC) $b \to s(d)\ell^+\ell^-$ is now entering a new interesting era. These transitions provide an important consistency check of the standard model (SM) at loop level, since FCNC transitions are forbidden in the SM at tree level. These decays induced by the FCNC are very sensitive to the new physics beyond the SM. New physics appear in rare decays through the Wilson coefficients which can take values different from their SM counterpart or through the new operator structures in an effective Hamiltonian.

First measurements of the $B \to X_s\gamma$ decay were reported by CLEO Collaboration [1] and more precise measurements are currently being carried out in the experiments at B factories (see for example [2]). Exclusive decay involving the $b \to s\gamma$ transition has been measured in [3, 4]. After these measurements of the radiative decay induced by the $b \to s\gamma$ transition, main interest has been focused on the rare decays induced by the $b \to s\ell^+\ell^-$ transitions, which have relatively large branching ratio in the SM. These decays have been extensively studied in the SM and its various extensions [5]–[13].

The exclusive $B \to K^*(K)\ell^+\ell^-$ decays, which are described by $b \to s\ell^+\ell^-$ transition at quark level, have been studied comprehensively in literature (see [11]–[17] and references therein). Recently, BELLE and BaBar Collaborations announced the following results for the branching ratios of both decays:

$$\mathcal{B}(B \to K^*\ell^+\ell^-) = \begin{cases} \Big(11.5^{+2.6}_{-2.4} \pm 0.8 \pm 0.2\Big) \times 10^{-7} & [18]\ , \\[2ex] \Big(0.88^{+0.33}_{-0.29}\Big) \times 10^{-6} & [19]\ , \end{cases}$$

$$\mathcal{B}(B \to K\ell^+\ell^-) = \begin{cases} \Big(4.8^{+1.0}_{-0.9} \pm 0.3 \pm 0.1\Big) \times 10^{-7} & [18]\ , \\[2ex] \Big(0.65^{+0.14}_{-0.13} \pm 0.04\Big) \times 10^{-6} & [19]\ . \end{cases}$$

Another exclusive decay which is described at inclusive level by the $b \to s\ell^+\ell^-$ transition is the baryonic $\Lambda_b \to \Lambda\ell^+\ell^-$ decay. Interest to the baryonic decays can be attributed to the fact that, unlike mesonic decays, they could maintain the helicity structure of the effective Hamiltonian for the $b \to s$ transition. Note that, new physics effects in the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay are studied in [20].

Experimentally measurable quantities such as branching ratio [21], Λ polarization and single lepton polarization have already been studied in [22] and [23], respectively. Study of such quantities can give useful information for more precise determination of the SM parameters and looking for new physics beyond the SM. It has been pointed out in [24] that the study of the polarizations of both leptons provides measurement of many more observables which would be useful in further improvement of the parameters of the SM and probing new physics beyond the SM.

In the present work we analyze the possibility of searching for new physics in the baryonic $\Lambda_b \to \Lambda\ell^+\ell^-$ decay by studying the polarized forward–backward asymmetry of various double–lepton polarizations, using a general form of the effective Hamiltonian, including all possible forms of interactions. It should be mentioned here that the sensitivity of polarized forward–backward asymmetry to the new Wilson coefficients for the meson→meson transition has been investigated in [25]. Naturally, one is forced to ask what happens to this sensitivity in the case of baryon→baryon transition, i.e., which polarized asymmetry is sensitive to which new Wilson coefficients for the baryon→baryon transition. A detailed investigation of this problem is the main goal of the present work.

The paper is organized as follows. In section 2, using the general, model independent form of the effective Hamiltonian, the matrix element for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ is obtained. In section 3 the analytic expressions for the polarized forward–backward asymmetry are derived. Section 4 is devoted to the numerical analysis, discussions and conclusions.

2 Matrix element for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay

In this section we derive the matrix element for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay using the general, model independent form of the effective Hamiltonian. At quark level, the matrix element of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is described by the $b \to s \ell^+ \ell^-$ transition. The effective Hamiltonian for the $b \to s \ell^+ \ell^-$ transition can be written in terms of twelve model independent four–Fermi interactions as [15, 26]:

$$
\begin{aligned}
\mathcal{M} = \frac{G\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Bigg\{ & C_{SL} \bar{s}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_L \bar{\ell} \gamma^\mu \ell + C_{BR} \bar{s}_L i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \bar{\ell} \gamma^\mu \ell + C_{LL}^{tot} \bar{s}_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \ell_L \\
&+ C_{LR}^{tot} \bar{s}_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_R + C_{RL} \bar{s}_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \ell_L + C_{RR} \bar{s}_R \gamma_\mu b_R \bar{\ell}_R \gamma^\mu \ell_R \\
&+ C_{LRLR} \bar{s}_L b_R \bar{\ell}_L \ell_R + C_{RLLR} \bar{s}_R b_L \bar{\ell}_L \ell_R + C_{LRRL} \bar{s}_L b_R \bar{\ell}_R \ell_L + C_{RLRL} \bar{s}_R b_L \bar{\ell}_R \ell_L \\
&+ C_T \bar{s} \sigma_{\mu\nu} b \bar{\ell} \sigma^{\mu\nu} \ell + i C_{TE} \epsilon_{\mu\nu\alpha\beta} \bar{s} \sigma^{\mu\nu} b \bar{\ell} \sigma^{\alpha\beta} \ell \Bigg\} ~, \qquad (1)
\end{aligned}
$$

where L and R are the chiral operators defined as $L = (1-\gamma_5)/2$ and $R = (1+\gamma_5)/2$. The coefficients of the first two terms, C_{SL} and C_{BR} describe the penguin contributions, which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The next four terms in Eq. (1) with coefficients C_{LL}^{tot}, C_{LR}^{tot}, C_{RL} and C_{RR} describe vector type interactions. Two of these coefficients C_{LL}^{tot} and C_{LR}^{tot} contain SM results in the form $C_9^{eff} - C_{10}$ and $C_9^{eff} - C_{10}$, respectively. Therefore, C_{LL}^{tot} and C_{LR}^{tot} can be written in the following form:

$$
\begin{aligned}
C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} ~, \\
C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} ~, \qquad (2)
\end{aligned}
$$

where C_{LL} and C_{LR} describe the contributions of new physics. The following four terms in Eq. (1) with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} represent the scalar type interactions. The remaining last two terms led by the coefficients C_T and C_{TE} are the tensor type interactions.

The amplitude of the exclusive $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is obtained by calculating the matrix element of $\mathcal{H}_{eff}$ for the $b \to s \ell^+ \ell^-$ transition between initial and final baryon states $\langle \Lambda \left| \mathcal{H}_{eff} \right| \Lambda_b \rangle$. We see from Eq. (1) that for calculating the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay amplitude, the following matrix elements are needed:

$$\langle \Lambda \,|\bar{s}\gamma_\mu(1 \mp \gamma_5)b|\, \Lambda_b\rangle \ ,$$
$$\langle \Lambda \,|\bar{s}\sigma_{\mu\nu}(1 \mp \gamma_5)b|\, \Lambda_b\rangle \ ,$$
$$\langle \Lambda \,|\bar{s}(1 \mp \gamma_5)b|\, \Lambda_b\rangle \ .$$

The relevant matrix elements parametrized in terms of the form factors are as follows (see [22, 27])

$$\langle \Lambda \,|\bar{s}\gamma_\mu b|\, \Lambda_b\rangle = \bar{u}_\Lambda\Big[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu\Big]u_{\Lambda_b} \ , \tag{3}$$
$$\langle \Lambda \,|\bar{s}\gamma_\mu\gamma_5 b|\, \Lambda_b\rangle = \bar{u}_\Lambda\Big[g_1\gamma_\mu\gamma_5 + ig_2\sigma_{\mu\nu}\gamma_5 q^\nu + g_3 q_\mu\gamma_5\Big]u_{\Lambda_b} \ , \tag{4}$$
$$\langle \Lambda \,|\bar{s}\sigma_{\mu\nu} b|\, \Lambda_b\rangle = \bar{u}_\Lambda\Big[f_T\sigma_{\mu\nu} - if_T^V\left(\gamma_\mu q^\nu - \gamma_\nu q^\mu\right) - if_T^S\left(P_\mu q^\nu - P_\nu q^\mu\right)\Big]u_{\Lambda_b} \ , \tag{5}$$
$$\langle \Lambda \,|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\, \Lambda_b\rangle = \bar{u}_\Lambda\Big[g_T\sigma_{\mu\nu} - ig_T^V\left(\gamma_\mu q^\nu - \gamma_\nu q^\mu\right) - ig_T^S\left(P_\mu q^\nu - P_\nu q^\mu\right)\Big]\gamma_5 u_{\Lambda_b} \ , \tag{6}$$

where $P = p_{\Lambda_b} + p_\Lambda$ and $q = p_{\Lambda_b} - p_\Lambda$.

The form factors of the magnetic dipole operators are defined as

$$\begin{aligned}
\langle \Lambda \,|\bar{s}i\sigma_{\mu\nu}q^\nu b|\, \Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1^T\gamma_\mu + if_2^T\sigma_{\mu\nu}q^\nu + f_3^T q_\mu\Big]u_{\Lambda_b} \ , \\
\langle \Lambda \,|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\, \Lambda_b\rangle &= \bar{u}_\Lambda\Big[g_1^T\gamma_\mu\gamma_5 + ig_2^T\sigma_{\mu\nu}\gamma_5 q^\nu + g_3^T q_\mu\gamma_5\Big]u_{\Lambda_b} \ .
\end{aligned} \tag{7}$$

Using the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta} \ ,$$

and Eq. (5), the last expression in Eq. (7) can be written as

$$\langle \Lambda \,|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\, \Lambda_b\rangle = \bar{u}_\Lambda\Big[f_T i\sigma_{\mu\nu}\gamma_5 q^\nu\Big]u_{\Lambda_b} \ .$$

Multiplying (5) and (6) by iq^ν and comparing with (7), one can easily obtain the following relations between the form factors

$$\begin{aligned}
f_2^T &= f_T + f_T^S q^2 \ , \\
f_1^T &= \Big[f_T^V + f_T^S\left(m_{\Lambda_b} + m_\Lambda\right)\Big]q^2 \ = -\frac{q^2}{m_{\Lambda_b} - m_\Lambda}f_3^T \ , \\
g_2^T &= g_T + g_T^S q^2 \ , \\
g_1^T &= \Big[g_T^V - g_T^S\left(m_{\Lambda_b} - m_\Lambda\right)\Big]q^2 = \frac{q^2}{m_{\Lambda_b} + m_\Lambda}g_3^T \ .
\end{aligned} \tag{8}$$

The matrix element of the scalar $\bar{s}b$ and pseudoscalar $\bar{s}\gamma_5 b$ operators can be obtained from (3) and (4) by multiplying both sides to q^μ and using equation of motion. Neglecting the mass of the strange quark, we get

$$\langle \Lambda \,|\bar{s}b|\, \Lambda_b\rangle = \frac{1}{m_b}\bar{u}_\Lambda\Big[f_1\left(m_{\Lambda_b} - m_\Lambda\right) + f_3 q^2\Big]u_{\Lambda_b} \ , \tag{9}$$
$$\langle \Lambda \,|\bar{s}\gamma_5 b|\, \Lambda_b\rangle = \frac{1}{m_b}\bar{u}_\Lambda\Big[g_1\left(m_{\Lambda_b} + m_\Lambda\right)\gamma_5 - g_3 q^2\gamma_5\Big]u_{\Lambda_b} \ . \tag{10}$$

Using these definitions of the form factors, for the matrix element of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ we get [22]

$$
\begin{aligned}
\mathcal{M} = & \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \Bigg\{ \bar{\ell}\gamma^\mu \ell \, \bar{u}_\Lambda \Big[A_1 \gamma_\mu (1+\gamma_5) + B_1 \gamma_\mu (1-\gamma_5) \\
& + i\sigma_{\mu\nu} q^\nu [A_2(1+\gamma_5) + B_2(1-\gamma_5)] + q_\mu [A_3(1+\gamma_5) + B_3(1-\gamma_5)] \Big] u_{\Lambda_b} \\
& + \bar{\ell}\gamma^\mu \gamma_5 \ell \, \bar{u}_\Lambda \Big[D_1 \gamma_\mu (1+\gamma_5) + E_1 \gamma_\mu (1-\gamma_5) + i\sigma_{\mu\nu} q^\nu [D_2(1+\gamma_5) + E_2(1-\gamma_5)] \\
& + q_\mu [D_3(1+\gamma_5) + E_3(1-\gamma_5)] \Big] u_{\Lambda_b} + \bar{\ell}\ell \, \bar{u}_\Lambda (N_1 + H_1 \gamma_5) u_{\Lambda_b} + \bar{\ell}\gamma_5 \ell \, \bar{u}_\Lambda (N_2 + H_2 \gamma_5) u_{\Lambda_b} \\
& + 4 C_T \bar{\ell} \sigma^{\mu\nu} \ell \, \bar{u}_\Lambda \Big[f_T \sigma_{\mu\nu} - i f_T^V (q_\nu \gamma_\mu - q_\mu \gamma_\nu) - i f_T^S (P_\mu q_\nu - P_\nu q_\mu) \Big] u_{\Lambda_b} \\
& + 4 C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{\ell} \sigma_{\alpha\beta} \ell \, i \bar{u}_\Lambda \Big[f_T \sigma_{\mu\nu} - i f_T^V (q_\nu \gamma_\mu - q_\mu \gamma_\nu) - i f_T^S (P_\mu q_\nu - P_\nu q_\mu) \Big] u_{\Lambda_b} \Bigg\} ,
\end{aligned}
\tag{11}
$$

where the explicit forms of the functions $A_i,\ B_i,\ D_i,\ E_i,\ H_j$ and N_j ($i = 1,2,3$ and $j = 1,2$) can be written as [22]

$$
\begin{aligned}
A_1 &= \frac{1}{q^2}\left(f_1^T - g_1^T\right) C_{SL} + \frac{1}{q^2}\left(f_1^T + g_1^T\right) C_{BR} + \frac{1}{2}\left(f_1 - g_1\right)\left(C_{LL}^{tot} + C_{LR}^{tot}\right) \\
&\quad + \frac{1}{2}\left(f_1 + g_1\right)\left(C_{RL} + C_{RR}\right) , \\
A_2 &= A_1 \left(1 \to 2\right) , \\
A_3 &= A_1 \left(1 \to 3\right) , \\
B_1 &= A_1 \left(g_1 \to -g_1;\ g_1^T \to -g_1^T\right) , \\
B_2 &= B_1 \left(1 \to 2\right) , \\
B_3 &= B_1 \left(1 \to 3\right) , \\
D_1 &= \frac{1}{2}\left(C_{RR} - C_{RL}\right)\left(f_1 + g_1\right) + \frac{1}{2}\left(C_{LR}^{tot} - C_{LL}^{tot}\right)\left(f_1 - g_1\right) , \\
D_2 &= D_1 \left(1 \to 2\right) , \\
D_3 &= D_1 \left(1 \to 3\right) , \\
E_1 &= D_1 \left(g_1 \to -g_1\right) , \\
E_2 &= E_1 \left(1 \to 2\right) , \\
E_3 &= E_1 \left(1 \to 3\right) , \\
N_1 &= \frac{1}{m_b}\Big(f_1\left(m_{\Lambda_b} - m_\Lambda\right) + f_3 q^2\Big)\Big(C_{LRLR} + C_{RLLR} + C_{LRRL} + C_{RLRL}\Big) , \\
N_2 &= N_1 \left(C_{LRRL} \to -C_{LRRL};\ C_{RLRL} \to -C_{RLRL}\right) , \\
H_1 &= \frac{1}{m_b}\Big(g_1\left(m_{\Lambda_b} + m_\Lambda\right) - g_3 q^2\Big)\Big(C_{LRLR} - C_{RLLR} + C_{LRRL} - C_{RLRL}\Big) , \\
H_2 &= H_1 \left(C_{LRRL} \to -C_{LRRL};\ C_{RLRL} \to -C_{RLRL}\right) .
\end{aligned}
\tag{12}
$$

From these expressions it follows that $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is described in terms of many form factors. It is shown in [28] that when HQET is applied the number of independent form factors reduces to two (F_1 and F_2) irrelevant of the Dirac structure of the corresponding operators, i.e.,

$$\left\langle \Lambda(p_\Lambda) \left| \bar{s}\Gamma b \right| \Lambda(p_{\Lambda_b}) \right\rangle = \bar{u}_\Lambda \Big[F_1(q^2) + \not{v} F_2(q^2) \Big] \Gamma u_{\Lambda_b} ~, \tag{13}$$

where Γ is an arbitrary Dirac structure and $v^\mu = p^\mu_{\Lambda_b}/m_{\Lambda_b}$ is the four–velocity of Λ_b. Comparing the general form of the form factors given in Eqs. (3)–(10) with (13), one can easily obtain the following relations among them (see also [27])

$$\begin{aligned}
g_1 &= f_1 = f_2^T = g_2^T = F_1 + \sqrt{\hat{r}_\Lambda} F_2 ~, \\
g_2 &= f_2 = g_3 = f_3 = g_T^V = f_T^V = \frac{F_2}{m_{\Lambda_b}} ~, \\
g_T^S &= f_T^S = 0 ~, \\
g_1^T &= f_1^T = \frac{F_2}{m_{\Lambda_b}} q^2 ~, \\
g_3^T &= \frac{F_2}{m_{\Lambda_b}} \left(m_{\Lambda_b} + m_\Lambda \right) ~, \\
f_3^T &= - \frac{F_2}{m_{\Lambda_b}} \left(m_{\Lambda_b} - m_\Lambda \right) ~,
\end{aligned} \tag{14}$$

where $\hat{r}_\Lambda = m^2_\Lambda / m^2_{\Lambda_b}$.

From Eq. (11), we get for the unpolarized decay width

$$\left(\frac{d\Gamma}{d\hat{s}} \right)_0 = \frac{G^2 \alpha^2}{8192 \pi^5} \left| V_{tb} V_{ts}^* \right|^2 \lambda^{1/2}(1, \hat{r}_\Lambda, \hat{s}) v \left[\mathcal{T}_0(\hat{s}) + \frac{1}{3} \mathcal{T}_2(\hat{s}) \right] ~, \tag{15}$$

where $\lambda(1, \hat{r}_\Lambda, \hat{s}) = 1 + \hat{r}_\Lambda^2 + \hat{s}^2 - 2\hat{r}_\Lambda - 2\hat{s} - 2\hat{r}_\Lambda \hat{s}$ is the triangle function, $\hat{s} = q^2/m^2_{\Lambda_b}$ and $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the lepton velocity, with $\hat{m}_\ell = m_\ell/m_{\Lambda_b}$. The explicit expressions for $\mathcal{T}_0$ and $\mathcal{T}_2$ can be found in [22]. The expressions for the doubly–polarized lepton pair forward–backward asymmetry will be presented in the next section.

3 Polarized forward–backward asymmetries of leptons in the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay

In this section we calculate the polarized forward–backward asymmetries, and for this aim we define the following orthogonal unit vectors $s_i^{\pm\mu}$ in the rest frame of $\ell^\pm$ ($i = L, T$ or N, stand for longitudinal, transversal or normal polarizations, respectively. See also [15], [29]–[31])

$$\begin{aligned}
s_L^{-\mu} &= \left(0, \vec{e}_L^{\,-} \right) = \left(0, \frac{\vec{p}_-}{\left| \vec{p}_- \right|} \right) ~, \\
s_N^{-\mu} &= \left(0, \vec{e}_N^{\,-} \right) = \left(0, \frac{\vec{p}_\Lambda \times \vec{p}_-}{\left| \vec{p}_\Lambda \times \vec{p}_- \right|} \right) ~,
\end{aligned}$$

$$
\begin{aligned}
s_T^{-\mu} &= \left(0, \vec{e}_T^{\,-}\right) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right) ~, \\
s_L^{+\mu} &= \left(0, \vec{e}_L^{\,+}\right) = \left(0, \frac{\vec{p}_+}{\left|\vec{p}_+\right|}\right) ~, \\
s_N^{+\mu} &= \left(0, \vec{e}_N^{\,+}\right) = \left(0, \frac{\vec{p}_\Lambda \times \vec{p}_+}{\left|\vec{p}_\Lambda \times \vec{p}_+\right|}\right) ~, \\
s_T^{+\mu} &= \left(0, \vec{e}_T^{\,+}\right) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right) ~,
\end{aligned}
\tag{16}
$$

where $\vec{p}_\mp$ and $\vec{p}_\Lambda$ are the three–momenta of the leptons $\ell^\mp$ and Λ baryon in the center of mass frame (CM) of $\ell^- \ell^+$ system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be accomplished by the Lorentz boost. Boosting of the longitudinal unit vectors $s_L^{\pm\mu}$ yields

$$
\left(s_L^{\mp\mu}\right)_{CM} = \left(\frac{\left|\vec{p}_\mp\right|}{m_\ell}, \frac{E_\ell \vec{p}_\mp}{m_\ell \left|\vec{p}_\mp\right|}\right) ~, \tag{17}
$$

where $\vec{p}_+ = -\vec{p}_-$, E_ℓ and m_ℓ are the energy and mass of leptons in the CM frame, respectively. The remaining two unit vectors $s_N^{\pm\mu}$, $s_T^{\pm\mu}$ are unchanged under Lorentz boost.

The definition of the normalized, unpolarized differential forward–backward asymmetry is

$$
\mathcal{A}_{FB} = \frac{\displaystyle\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz}}{\displaystyle\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} + \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz}} ~, \tag{18}
$$

where $z = \cos\theta$ is the angle between Λ_b meson and ℓ^- in the center mass frame of leptons. When the spins of both leptons are taken into account, the $\mathcal{A}_{FB}$ will be a function of the spins of the final leptons and it is defined as

$$
\begin{aligned}
\mathcal{A}_{FB}^{ij}(\hat{s}) &= \left(\frac{d\Gamma(\hat{s})}{d\hat{s}}\right)^{-1} \left\{\int_0^1 dz - \int_{-1}^0 dz\right\} \left\{\left[\frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = -\vec{j})}{d\hat{s}dz}\right]\right. \\
&- \left.\left[\frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = -\vec{j})}{d\hat{s}dz}\right]\right\} ~, \\
&= \mathcal{A}_{FB}(\vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = \vec{j}) - \mathcal{A}_{FB}(\vec{s}^{\,-} = \vec{i}, \vec{s}^{\,+} = -\vec{j}) - \mathcal{A}_{FB}(\vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = \vec{j}) \\
&+ \mathcal{A}_{FB}(\vec{s}^{\,-} = -\vec{i}, \vec{s}^{\,+} = -\vec{j}) ~.
\end{aligned}
\tag{19}
$$

Using these definitions for the double polarized FB asymmetries, we get the following results:

$$
\begin{aligned}
\mathcal{A}_{FB}^{LL} &= \frac{16}{\Delta} m_{\Lambda_b}^4 \sqrt{\lambda} v \mathrm{Re}\Big[- \hat{m}_\ell \Big(1 - \sqrt{\hat{r}_\Lambda}\Big)(A_1 - B_1) H_1^* \\
&+ \hat{m}_\ell \Big(1 + \sqrt{\hat{r}_\Lambda}\Big)(A_1 + B_1) F_1^* \\
&+ 8\hat{m}_\ell \Big\{ 2\Big(1 - \sqrt{\hat{r}_\Lambda}\Big)(D_1 + E_1) f_T^* C_{TE}^* + \Big(1 + \sqrt{\hat{r}_\Lambda}\Big)(D_1 - E_1) f_T^* C_T^* \Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+\ 16m_{\Lambda_b}\hat{m}_\ell(1-\hat{r}_\Lambda)\Big\{(D_2-E_2)f_T^*C_T^*-2(D_2+E_2)f_T^*C_{TE}^*\Big\}\\
&+\ 16m_{\Lambda_b}^2\hat{m}_\ell\Big(1-\sqrt{\hat{r}_\Lambda}\Big)\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)(D_1+E_1)f_T^{S*}C_{TE}^*\\
&+\ 2\hat{s}\Big\{A_1D_1^*-B_1E_1^*-4(F_2+H_1)f_T^*C_{TE}^*+2(F_1+H_2)f_T^*C_T^*\Big\}\\
&-\ m_{\Lambda_b}\hat{s}\Big\{2(A_1E_2^*-A_2E_1^*-B_1D_2^*+B_2D_1^*)+\hat{m}_\ell(A_2+B_2)F_1^*\Big\}\\
&+\ 8m_{\Lambda_b}\hat{m}_\ell\hat{s}\Big\{(D_3-E_3)f_T^*C_T^*-2(D_3+E_3)f_T^*C_{TE}^*\Big\}\\
&-\ m_{\Lambda_b}\hat{m}_\ell\hat{s}(A_2-B_2)H_1^*\\
&-\ 16m_{\Lambda_b}^2\hat{m}_\ell\hat{s}\Big(1-\sqrt{\hat{r}_\Lambda}\Big)(D_2-E_2)f_T^{V*}C_T^*\\
&-\ 4m_{\Lambda_b}\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\Big(F_1f_T^{V*}C_T^*-2F_2f_T^{V*}C_{TE}^*\Big)\\
&+\ 16m_{\Lambda_b}^2\hat{m}_\ell\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)(D_3+E_3)f_T^{V*}C_{TE}^*\\
&-\ 2m_{\Lambda_b}\hat{s}\Big\{m_{\Lambda_b}(1-\hat{r}_\Lambda)(A_2D_2^*-B_2E_2^*)+\sqrt{\hat{r}_\Lambda}(A_1D_2^*+A_2D_1^*)\Big\}\\
&+\ 2m_{\Lambda_b}\hat{s}\sqrt{\hat{r}_\Lambda}(B_1E_2^*+B_2E_1^*)\\
&+\ 16m_{\Lambda_b}^3\hat{m}_\ell\hat{s}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)(D_3+E_3)f_T^{S*}C_{TE}^*\\
&-\ 4m_{\Lambda_b}^2\hat{s}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\Big(F_1f_T^{S*}C_T^*-2F_2f_T^{S*}C_{TE}^*\Big)\\
&+\ 16m_{\Lambda_b}\hat{m}_\ell\Big\{(1-\hat{r}_\Lambda)(D_1+E_1)f_T^{V*}C_{TE}^*-\hat{s}(D_1-E_1)f_T^{V*}C_T^*\Big\}\Big]\ , \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{LT} &= \frac{64}{3\sqrt{\hat{s}}\Delta}m_{\Lambda_b}^4\lambda\mathrm{Re}\Big[-\hat{m}_\ell\Big\{\,|A_1|^2+|B_1|^2-32\Big(4\,|C_{TE}|^2+|C_T|^2\Big)\,|f_T|^2\,\Big\}\\
&+\ m_{\Lambda_b}^2\hat{m}_\ell\hat{s}\Big\{\,|A_2|^2+|B_2|^2+32\,|C_T|^2\,\Big(2f_Tf_T^{S*}-\Big|f_T^V\Big|^2\,\Big)\Big\}\\
&-\ 8m_{\Lambda_b}\hat{s}v^2\Big\{(A_2+B_2)f_T^*C_T^*-2(A_2-B_2)f_T^*C_{TE}^*-A_1f_T^{V*}C_T^*+m_{\Lambda_b}^2\hat{s}(A_2+B_2)f_T^{S*}C_T^*\Big\}\\
&+\ 8m_{\Lambda_b}\hat{s}v^2\Big\{B_1f_T^{V*}C_T^*+m_{\Lambda_b}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)A_1f_T^{S*}C_T^*\Big\}\\
&+\ 8m_{\Lambda_b}^2\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\Big(-32m_{\Lambda_b}\hat{m}_\ell\,|C_T|^2\,f_T^Sf_T^{V*}+v^2B_1C_T^*f_T^{S*}\Big)\\
&+\ 8m_{\Lambda_b}\hat{s}v^2\Big\{(D_2-E_2)f_T^*C_T^*-2(D_2+E_2)f_T^*C_{TE}^*+2(D_1+E_1)f_T^{V*}C_{TE}^*\Big\}\\
&+\ 16m_{\Lambda_b}^2\hat{s}v^2\Big(1+\sqrt{\hat{r}_\Lambda}\Big)(D_1+E_1)f_T^{S*}C_{TE}^*\\
&-\ 16m_{\Lambda_b}^3\hat{s}^2v^2(D_2+E_2)f_T^{S*}C_{TE}^*\\
&-\ 128m_{\Lambda_b}^4\hat{m}_\ell\hat{s}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\,|C_T|^2\,\Big|f_T^{S*}\Big|^2\,\Big]\ , \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{TL} &= \frac{64}{3\sqrt{\hat{s}}\Delta}m_{\Lambda_b}^4\lambda\mathrm{Re}\Big[\hat{m}_\ell\Big\{\,|A_1|^2+|B_1|^2-32\Big(4\,|C_{TE}|^2+|C_T|^2\Big)\,|f_T|^2\,\Big\}\\
&-\ m_{\Lambda_b}^2\hat{m}_\ell\hat{s}\Big\{\,|A_2|^2+|B_2|^2+32\,|C_T|^2\,\Big(2f_Tf_T^{S*}-\Big|f_T^V\Big|^2\,\Big)\Big\}\\
&+\ 8m_{\Lambda_b}\hat{s}v^2\Big\{(A_2+B_2)f_T^*C_T^*-2(A_2-B_2)f_T^*C_{TE}^*-A_1f_T^{V*}C_T^*+m_{\Lambda_b}^2\hat{s}(A_2+B_2)f_T^{S*}C_T^*\Big\}\\
&-\ 8m_{\Lambda_b}\hat{s}v^2\Big\{B_1f_T^{V*}C_T^*+m_{\Lambda_b}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)A_1f_T^{S*}C_T^*\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&- 8m_{\Lambda_b}^2\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\Big(-32m_{\Lambda_b}\hat{m}_\ell\left|C_T\right|^2 f_T^S f_T^{V*} + v^2 B_1 C_T^* f_T^{S*}\Big)\\
&+ 8m_{\Lambda_b}\hat{s}v^2\Big\{(D_2-E_2)f_T^* C_T^* - 2(D_2+E_2)f_T^* C_{TE}^* + 2(D_1+E_1)f_T^{V*}C_{TE}^*\Big\}\\
&+ 16m_{\Lambda_b}^2\hat{s}v^2\Big(1+\sqrt{\hat{r}_\Lambda}\Big)(D_1+E_1)f_T^{S*}C_{TE}^*\\
&- 16m_{\Lambda_b}^3\hat{s}^2v^2(D_2+E_2)f_T^{S*}C_{TE}^*\\
&+ 128m_{\Lambda_b}^4\hat{m}_\ell\hat{s}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\left|C_T\right|^2\left|f_T^{S*}\right|^2\Big]\ , \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{LN} = &\ \frac{64}{3\sqrt{\hat{s}}\Delta}m_{\Lambda_b}^4\lambda v\,\mathrm{Im}\Big[-\hat{m}_\ell(A_1D_1^*+B_1E_1^*)\\
&- 2m_{\Lambda_b}\hat{s}\Big\{(A_2-D_2)f_T^*(C_T^*-2C_{TE}^*)-(B_2+E_2)f_T^*(C_T^*+2C_{TE}^*)\Big\}\\
&- 2m_{\Lambda_b}\hat{s}\Big\{2(A_1+B_1)f_T^{V*}C_{TE}^*+(D_1+E_1)f_T^{V*}C_T^*\Big\}\\
&+ m_{\Lambda_b}^2\hat{m}_\ell\hat{s}(A_2D_2^*+B_2E_2^*)\\
&- 64m_{\Lambda_b}^2\hat{m}_\ell\hat{s}\Big(2\mathrm{Re}\Big[f_Tf_T^{S*}\Big]-\left|f_T^V\right|^2\Big)C_TC_{TE}^*\\
&- 2m_{\Lambda_b}^2\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\Big\{2(A_1+B_1)f_T^{S*}C_{TE}^*+(D_1+E_1)f_T^{S*}C_T^*\Big\}\\
&+ 2m_{\Lambda_b}^3\hat{s}^2\Big\{2(A_2+B_2)f_T^{S*}C_{TE}^*+(D_2+E_2)f_T^{S*}C_T^*\Big\}\\
&+ 64m_{\Lambda_b}^3\hat{m}_\ell\hat{s}\Big\{2\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\mathrm{Re}\Big[f_T^Sf_T^{V*}\Big]+m_{\Lambda_b}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\left|f_T^S\right|^2\Big\}C_TC_{TE}^*\Big]\ , \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NL} = &\ \frac{64}{3\sqrt{\hat{s}}\Delta}m_{\Lambda_b}^4\lambda v\,\mathrm{Im}\Big[-\hat{m}_\ell(A_1D_1^*+B_1E_1^*)\\
&+ 2m_{\Lambda_b}\hat{s}\Big\{(A_2+D_2)f_T^*(C_T^*-2C_{TE}^*)-(B_2-E_2)f_T^*(C_T^*+2C_{TE}^*)\Big\}\\
&+ 2m_{\Lambda_b}\hat{s}\Big\{2(A_1+B_1)f_T^{V*}C_{TE}^*-(D_1+E_1)f_T^{V*}C_T^*\Big\}\\
&+ m_{\Lambda_b}^2\hat{m}_\ell\hat{s}(A_2D_2^*+B_2E_2^*)\\
&+ 64m_{\Lambda_b}^2\hat{m}_\ell\hat{s}\Big(2\mathrm{Re}\Big[f_Tf_T^{S*}\Big]-\left|f_T^V\right|^2\Big)C_TC_{TE}^*\\
&+ 2m_{\Lambda_b}^2\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\Big\{2(A_1+B_1)f_T^{S*}C_{TE}^*-(D_1+E_1)f_T^{S*}C_T^*\Big\}\\
&- 2m_{\Lambda_b}^3\hat{s}^2\Big\{2(A_2+B_2)f_T^{S*}C_{TE}^*-(D_2+E_2)f_T^{S*}C_T^*\Big\}\\
&- 64m_{\Lambda_b}^3\hat{m}_\ell\hat{s}\Big\{2\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\mathrm{Re}\Big[f_T^Sf_T^{V*}\Big]+m_{\Lambda_b}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\left|f_T^S\right|^2\Big\}C_TC_{TE}^*\Big]\ , \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NT} &= \mathcal{A}_{FB}^{TN}\\
&= \frac{16}{\Delta}m_{\Lambda_b}^4\sqrt{\lambda}\,\mathrm{Im}\Big[4m_{\Lambda_b}\hat{m}_\ell^2\Big\{A_1E_3^*-A_2E_1^*+B_1D_3^*-B_2D_1^*\Big\}\\
&- \hat{m}_\ell\Big(1-\sqrt{\hat{r}_\Lambda}\Big)(A_1-B_1)H_2^*\\
&+ \hat{m}_\ell\Big(1+\sqrt{\hat{r}_\Lambda}\Big)(A_1+B_1)F_2^*\\
&- 8\hat{m}_\ell\Big\{\Big(1-\sqrt{\hat{r}_\Lambda}\Big)(D_1+E_1)f_T^*C_T^*+2\Big(1+\sqrt{\hat{r}_\Lambda}\Big)(D_1-E_1)f_T^*C_{TE}^*\Big\}\\
&+ 8m_{\Lambda_b}\hat{m}_\ell(1-\hat{r}_\Lambda)(D_1+E_1)f_T^{V^*}C_T^*
\end{aligned}
$$

$$
\begin{aligned}
&+ 4m_{\Lambda_b}\hat{m}_\ell^2\sqrt{\hat{r}_\Lambda}(A_1D_3^\ast + A_2D_1^\ast + B_1E_3^\ast + B_2E_1^\ast)\\
&+ 8m_{\Lambda_b}^2\hat{m}_\ell\Big(1-\sqrt{\hat{r}_\Lambda}\Big)\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)(D_1+E_1)f_T^{S\ast}C_T^\ast\\
&+ \frac{4}{\hat{s}}\hat{m}_\ell^2(1-\hat{r}_\Lambda)(A_1D_1^\ast + B_1E_1^\ast)\\
&- m_{\Lambda_b}\hat{m}_\ell\hat{s}\Big\{(A_2+B_2)F_2^\ast + (A_2-B_2)H_2^\ast\Big\}\\
&+ 4\hat{s}\Big[2\{H_2 + 2m_{\Lambda_b}\hat{m}_\ell(D_3-E_3)\}f_T^\ast C_{TE}^\ast - \{F_2 + 2m_{\Lambda_b}\hat{m}_\ell(D_3+E_3)\}f_T^\ast C_T^\ast\Big]\\
&- 4m_{\Lambda_b}^2\hat{m}_\ell^2\hat{s}(A_2D_3^\ast + B_2E_3^\ast)\\
&+ 4m_{\Lambda_b}\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\{F_2 + 2m_{\Lambda_b}\hat{m}_\ell(D_3+E_3)\}f_T^{V\ast}C_T^\ast\\
&+ 4m_{\Lambda_b}^2\hat{s}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\{F_2 + 2m_{\Lambda_b}\hat{m}_\ell(D_3+E_3)\}f_T^{S\ast}C_T^\ast\\
&- 4\hat{s}v^2\Big(2F_1f_T^\ast C_{TE}^\ast - H_1f_T^\ast C_T^\ast\Big)\\
&+ 8m_{\Lambda_b}\hat{s}v^2\Big\{\Big(1+\sqrt{\hat{r}_\Lambda}\Big)F_1f_T^{V\ast}C_{TE}^\ast + m_{\Lambda_b}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)F_1f_T^{S\ast}C_{TE}^\ast\Big\}\Big]~, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NN} &= -\mathcal{A}_{FB}^{TT}\\
&= \frac{16}{\Delta}m_{\Lambda_b}^4\sqrt{\lambda}v\mathrm{Re}\Big[-\hat{m}_\ell\Big\{(A_1+B_1)F_1^\ast - (A_1-B_1)H_1^\ast + \sqrt{\hat{r}_\Lambda}\{(A_1+B_1)F_1^\ast + (A_1-B_1)H_1^\ast\}\Big\}\\
&- 8\hat{m}_\ell\Big\{\Big(1+\sqrt{\hat{r}_\Lambda}\Big)(D_1-E_1)f_T^\ast C_T^\ast + 2\Big(1-\sqrt{\hat{r}_\Lambda}\Big)(D_1+E_1)f_T^\ast C_{TE}^\ast\Big\}\\
&+ 16m_{\Lambda_b}\hat{m}_\ell(1-\hat{r}_\Lambda)(D_1+E_1)f_T^{V\ast}C_{TE}^\ast\\
&+ 16m_{\Lambda_b}^2\hat{m}_\ell\Big(1-\sqrt{\hat{r}_\Lambda}\Big)\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)(D_1+E_1)f_T^{S\ast}C_{TE}^\ast\\
&- 4\hat{s}\Big\{(F_1-H_2)f_T^\ast C_T^\ast + 2(F_2-H_1)f_T^\ast C_{TE}^\ast\Big\}\\
&+ m_{\Lambda_b}\hat{m}_\ell\hat{s}\Big\{(A_2+B_2)F_1^\ast + (A_2-B_2)H_1^\ast\Big\}\\
&+ 8m_{\Lambda_b}\hat{m}_\ell\hat{s}\Big\{(D_3-E_3)f_T^\ast C_T^\ast - 2(D_3+E_3)f_T^\ast C_{TE}^\ast\Big\}\\
&+ 4m_{\Lambda_b}\hat{s}\Big(1+\sqrt{\hat{r}_\Lambda}\Big)\Big(F_1f_T^{V\ast}C_T^\ast + 2\{F_2 + 2m_{\Lambda_b}\hat{m}_\ell(D_3+E_3)\}f_T^{V\ast}C_{TE}^\ast\Big)\\
&+ 4m_{\Lambda_b}^2\hat{s}\Big(1+2\sqrt{\hat{r}_\Lambda}+\hat{r}_\Lambda-\hat{s}\Big)\Big(F_1f_T^{S\ast}C_T^\ast + 2\{F_2 + 2m_{\Lambda_b}\hat{m}_\ell(D_3+E_3)\}f_T^{S\ast}C_{TE}^\ast\Big)\Big]~. \qquad (26)
\end{aligned}
$$

In the expressions for $\mathcal{A}_{FB}^{ij}$, the superscript indices i and j correspond to the lepton and anti–lepton polarizations, respectively.

At the end of this section, we would like to remind the interested reader that, the doubly polarized forward–backward asymmetry is calculated in a model independent way for the $B \to K^\ast \ell^+\ell^-$ decay, in SUSY theories for the $B \to K^\ast \tau^+\tau^-$ decay and $b \to s\tau^+\tau^-$ transition in [25], [32] and [33], respectively.

4 Numerical analysis

In this section we study the influence of the new Wilson coefficients on the polarized forward–backward asymmetry. Before performing numerical calculations, we present the values of the input parameters.$|V_{tb}V_{ts}^\ast| = 0.0385$, $m_\tau = 1.77~GeV$, $m_\mu = 0.106~GeV$.

$m_b = 4.8\ GeV$. For the values of the Wilson coefficients in SM we use $C_7^{SM} = -0.313$, $C_9^{SM} = 4.344$ and $C_{10}^{SM} = -4.669$. The value of C_9^{SM} we use corresponds to the short distance contributions. It is well known that, in addition to the short distance contributions, C_9 receives long distance contributions, coming from the production of $\bar{c}c$ pair at intermediate states. In the present work we neglect such long distance effects. From the expressions of $\mathcal{A}_{FB}^{ij}$, it can easily be seen that, one of the most important input parameters necessary in the numerical calculations are the form factors. So far, the calculations for all of the form factors of the $\Lambda_b \to \Lambda$ transition are absent. Therefore, we will use the results from QCD sum rules in corporation with HQET [28, 34]. As has already been noted, HQET allows us to establish relations among the form factors and reduces the number of independent form factors into two. In [28, 34], the q^2 dependence of these form factors are given as follows

$$F(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2} \ .$$

The values of the parameters $F(0)$, a_F and b_F are given in table 1.

	$F(0)$	a_F	b_F
F_1	0.462	-0.0182	-0.000176
F_2	-0.077	-0.0685	0.00146

Table 1: Form factors for $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in a three parameter fit.

In further numerical analysis, the values of the new Wilson coefficients which describe new physics beyond the SM, are needed. In numerical calculations we will vary all new Wilson coefficients in the range $-\left|C_{10}^{SM}\right| \le C_X \le \left|C_{10}^{SM}\right|$. The experimental results on the branching ratio of the $B \to K^* \ell^+ \ell^-$ decay [18, 19] and the bound on the branching ratio of $B \to \mu^+ \mu^-$ [3, 35] suggest that this is the right order of magnitude for the vector and scalar interaction coefficients. Here, we emphasize that the existing experimental results on the $B \to K^* \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$ decays put stronger restrictions on some of the new Wilson coefficients. For example, $-2 \le C_{LL} \le 0$, $0 \le C_{RL} \le 2.3$, $-1.5 \le C_T \le 1.5$ and $-3.3 \le C_{TE} \le 2.6$, and all of the remaining Wilson coefficients vary in the region $-\left|C_{10}^{SM}\right| \le C_X \le \left|C_{10}^{SM}\right|$.

In Figs. 1(2), the dependence of the $\mathcal{A}_{FB}^{LL}$ on q^2 for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay at five different values of $C_{LL}(C_{LR})$, are presented. We observe from these figures that zero position of $\mathcal{A}_{FB}^{LL}$ is shifted compared to that of the SM result, and this behavior of $\mathcal{A}_{FB}^{LL}$ is very similar for both new Wilson coefficients C_{LL} and C_{LR}. When both these coefficients get positive (negative) values, the zero position of $\mathcal{A}_{FB}^{LL}$ shifts to the left(right) compared to that of the SM case.

It should be noted here that, for the above–mentioned cases $\mathcal{A}_{FB}^{LL}$ passes through zero for $q^2 < 7\ GeV^2$. Therefore, this zero position of $\mathcal{A}_{FB}^{LL}$ is free from long distance J/ψ contributions.

Our detailed numerical analysis shows that, the zero position of $\mathcal{A}_{FB}^{LL}$ for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay is practically independent of the tensor type interactions, and also we observe that the value of the forward–backward asymmetry is quite small for the scalar type interactions. Therefore, we do not present the dependence of $\mathcal{A}_{FB}^{LL}$ on q^2 at fixed values of scalar and tensor interaction coefficients. Moreover, for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay, the dependencies of $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ on q^2 are very similar to that of $\mathcal{A}_{FB}^{LL}$ case, i.e., zero position of $\mathcal{A}_{FB}^{LT}$ is shifted to the right(left) compared to that of the SM result when the new Wilson coefficients C_{LL} and C_{LR} are negative(positive).

The zero position of the forward–backward asymmetry for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay is absent for all Wilson coefficients (except the $q^2_{max} = (m_{\Lambda_b} - m_\Lambda)^2$ point), and hence it is insensitive to the new physics beyond the SM. But, it should be noted here that the values of $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ at all q^2 have opposite sign in the presence of tensor interaction, when compared to that of the SM result. Therefore, if $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ are measured far from zero position in the future–planned experiments, the sign of these asymmetries can give unambiguous information about new physics beyond the SM, more precisely, about the existence of the tensor type interaction (see Figs. (3) and (4)).

We can get additional information by studying the dependence of $\mathcal{A}_{FB}^{NN}$ (and $\mathcal{A}_{FB}^{TT} = -\mathcal{A}_{FB}^{NN}$) on q^2. First of all, this asymmetry gets positive(negative) value when C_T is negative(positive) compared to the same case in the SM. The behavior of $\mathcal{A}_{FB}^{NN}$ is opposite to the above one in the presence of C_{TE}. Remember that $\mathcal{A}_{FB}^{NN} \approx 0$ in the SM (see Fig. (5)). Also, $\mathcal{A}_{FB}^{NN}$ is very sensitive to the presence of the scalar interactions. When Wilson coefficients of scalar interactions, C_{LRRL} and C_{LRLR}, are negative(positive), the value of $\mathcal{A}_{FB}^{NN}$ becomes positive(negative) for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay (see Fig.(6)).

Obviously, it follows from the explicit expressions of $\mathcal{A}_{FB}^{ij}$ that they all depend both on q^2 and the new Wilson coefficients. Therefore there may appear difficulties in simultaneous study of the dependence of the physical observables on both parameters. In order to avoid such difficulties, we must eliminate the dependence of $\mathcal{A}_{FB}^{ij}$ on one of these parameters. In the present work, the q^2 dependence of $\mathcal{A}_{FB}^{ij}$ is eliminated by performing integration over q^2 in the allowed kinematical region, i.e., we average the polarized $\mathcal{A}_{FB}^{ij}$ which is defined as

$$\left\langle \mathcal{A}_{FB}^{ij} \right\rangle = \frac{\displaystyle\int_{4m_\ell^2}^{(m_{\Lambda_b}-m_\Lambda)^2} \mathcal{A}_{FB}^{ij} \frac{d\mathcal{B}}{dq^2} dq^2}{\displaystyle\int_{4m_\ell^2}^{(m_{\Lambda_b}-m_\Lambda)^2} \frac{d\mathcal{B}}{dq^2} dq^2} \,. \qquad (27)$$

In Fig.(7) we depict the dependence of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on C_X for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay. The intersection of all curves corresponds to the SM case. It follows from this figure that, $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ has symmetric behavior in its dependence on the tensor type interactions and except C_{RR} it remains smaller compared to the SM result at negative values of all type of interactions. At positive values of the new Wilson coefficients $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle > \left\langle \mathcal{A}_{FB}^{LL} \right\rangle_{SM}$ for all type of scalar interactions and for the vector interactions C_{LL}, C_{LR} and C_{RR}.

Depicted in Fig. (8) is the dependence of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ on C_X for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay. We observe from this figure that $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ depends more strongly on the tensor interaction coefficients C_T and C_{TE}. When $C_T < 0$, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ is positive and when $-4 < C_T < -2$, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ reaches its maximum value (about $\sim 5\%$). On the other hand when C_T is positive, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ changes sign and gets negative values. The situation is different for the other tensor interaction coefficient, namely, C_{TE}. When $-4 < C_{TE} < -2$, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ is positive, and when $-2 < C_{TE} < 0$, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ is negative. $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ attains at positive values again when $C_{TE} > 0$. Therefore measurement of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ in experiments can give essential information about the existence of tensor interactions.

The dependence of $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$ on the new Wilson coefficients, for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay, is vice versa of the behavior of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ explained above (see Fig. (9)).

In Fig. (10) we present the dependence of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on the new Wilson coefficients for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay. From this figure one can easily conclude that $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ is sensitive to the presence of the new Wilson coefficients, except C_{RR}, C_{LL}, C_{LRLR}, C_{LRRL} and C_{RLLR}. Practically, at all negative(positive) values of the new Wilson coefficients (except the regions $-1 \le C_T \le 0$ and $0 \le C_{TE} \le 0.4$) $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ is smaller(larger) compared to $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle_{SM}$ predicted by the SM. Along the same lines, the dependence of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay is presented in Fig. (11). In this case $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ seems to be strongly dependent on C_T, C_{TE}, C_{LR} and C_{LRRL}. In the case of tensor interaction, at all values of $C_T(C_{TE})$ we observe that $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle > \left\langle \mathcal{A}_{FB}^{LT} \right\rangle_{SM}$, while for the coefficient C_{LR}, the magnitude of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ is smaller(larger) compared to the SM result when C_{LR} gets negative(positive) values. For the scalar type interaction induced by C_{LRRL}, the situation is vice versa when compared to the C_{LR} case.

As far as the dependence of $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$ on C_X is concerned, apart from an overall sign, practically, its behavior is almost the same as that of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay (see Figs. (11) and (12)).

In Fig. (13) we present the dependence of $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle = -\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$ on the Wilson coefficients for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay, and we observe that it is strongly dependent on C_T, C_{TE} and the scalar type interactions with the coefficients C_{LRLR} and C_{LRRL}.

From these results it follows that several of the double–lepton polarization forward–backward asymmetries demonstrate sizable departure from the SM results and they are sensitive to the existence of different types of new interactions. For this reason, study of these observables can be very useful in looking for new physics beyond the SM.

At the end of this section we want to discuss the following problem. Obviously, if new physics beyond the SM exists, it effects not only the polarized $\mathcal{A}_{FB}$, but also the branching ratio. The measurement of the branching ratio in experiments is easier and therefore its investigation is more convenient for establishing new physics. The main question is, whether there could appear situations in which the value of the branching ratio coincides with that of the SM result, while polarized $\mathcal{A}_{FB}$ does not. To find out an answer to this question we study the correlation between the averaged, polarized $\langle \mathcal{A}_{FB} \rangle$ and the branching ratio. In further analysis we vary the branching ratio of the $\Lambda_b \to \Lambda \mu^+ \mu^-$ ($\Lambda \tau^+ \tau^-$) decay between the values $(3-6) \times 10^{-6}$ $[(3-6) \times 10^{-7}]$, which is very close to the SM results.

Our comment on the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay, as far as the above–mentioned correlated relation is concerned, can briefly be summarized as follows (remember that, the intersection of all curves corresponds to the SM value):

- for $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$, such a region exists only for C_{LR}.

The situation is much richer for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay. In Figs. (14)–(17), we depict the dependence of the averaged, forward–backward polarized asymmetries $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$; $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle \approx -\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$; $\left\langle \mathcal{A}_{FB}^{NT} \right\rangle = \left\langle \mathcal{A}_{FB}^{TN} \right\rangle$; $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle = -\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$, on branching ratio. It follows from these figures that, indeed, there exist certain regions of the new Wilson coefficients in where the study of the doubly polarized $\mathcal{A}_{FB}$ can establish new physics beyond the SM.

In conclusion, we present the analysis for the forward–backward asymmetries using a general, model independent form of the effective Hamiltonian. We obtain that the determination of the zero position of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ can serve as a good probe for establishing new physics beyond the SM. Finally we obtain that there exist certain regions of the new Wilson coefficients for which, only study of the polarized forward–backward asymmetry gives invaluable information in search of new physics beyond the SM.

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Figure captions

Fig. (1) The dependence of the double–lepton polarization asymmetry $\mathcal{A}_{FB}^{LL}$ on q^2 at four fixed values of C_{LL}, for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay.

Fig. (2) The same as in Fig. (1), but at four fixed values of C_{LR}.

Fig. (3) The dependence of the double–lepton polarization asymmetry $\mathcal{A}_{FB}^{LT}$ on q^2 at four fixed values of C_T, for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (4) The same as in Fig. (3), but for $\mathcal{A}_{FB}^{TL}$.

Fig. (5) The same as in Fig. (3), but for $\mathcal{A}_{FB}^{NN}$.

Fig. (6) The same as in Fig. (5), but at four fixed values of C_{LRRL}.

Fig. (7) The dependence of the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on the new Wilson coefficients C_X, for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay.

Fig. (8) The same as in Fig. (7), but for $\mathcal{A}_{FB}^{LT}$.

Fig. (9) The same as in Fig. (8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$.

Fig. (10) The same as in Fig. (7), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (11) The same as in Fig. (8), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (12) The same as in Fig. (9), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

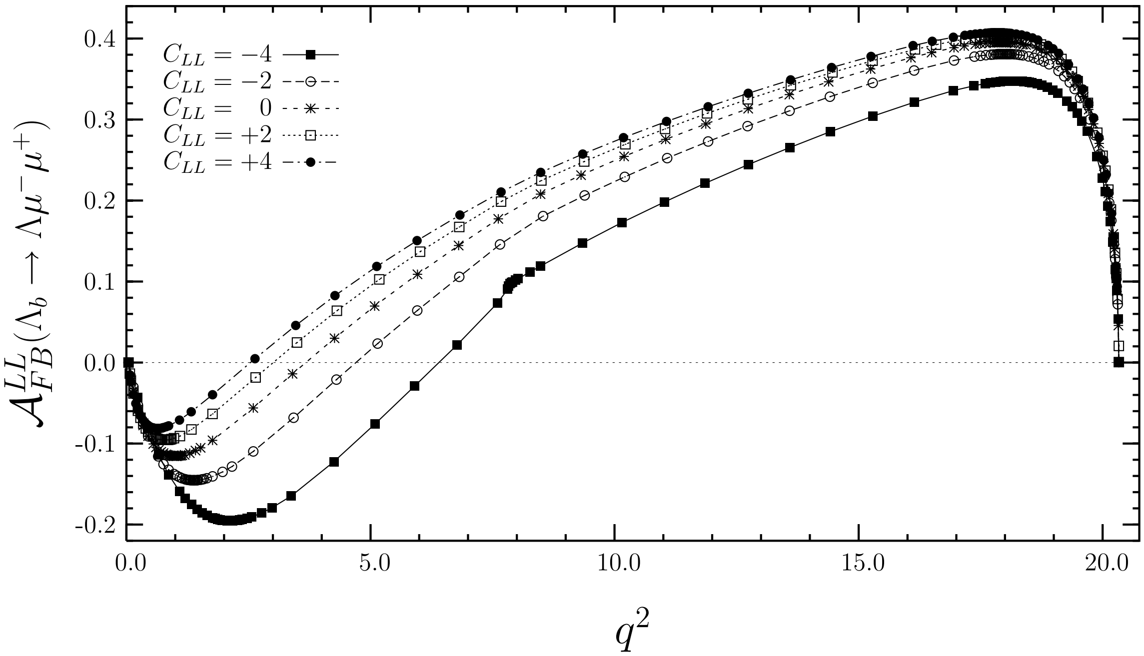
Fig. (13) The same as in Fig. (12), but for the $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle = -\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$.

Fig. (14) Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ and the branching ratio for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (15) The same as in Fig. (14), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and the branching ratio.

Fig. (16) The same as in Fig. (15), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NT} \right\rangle = \left\langle \mathcal{A}_{FB}^{TN} \right\rangle$ and the branching ratio.

Fig. (17) The same as in Fig. (16), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle = -\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$ and the branching ratio.



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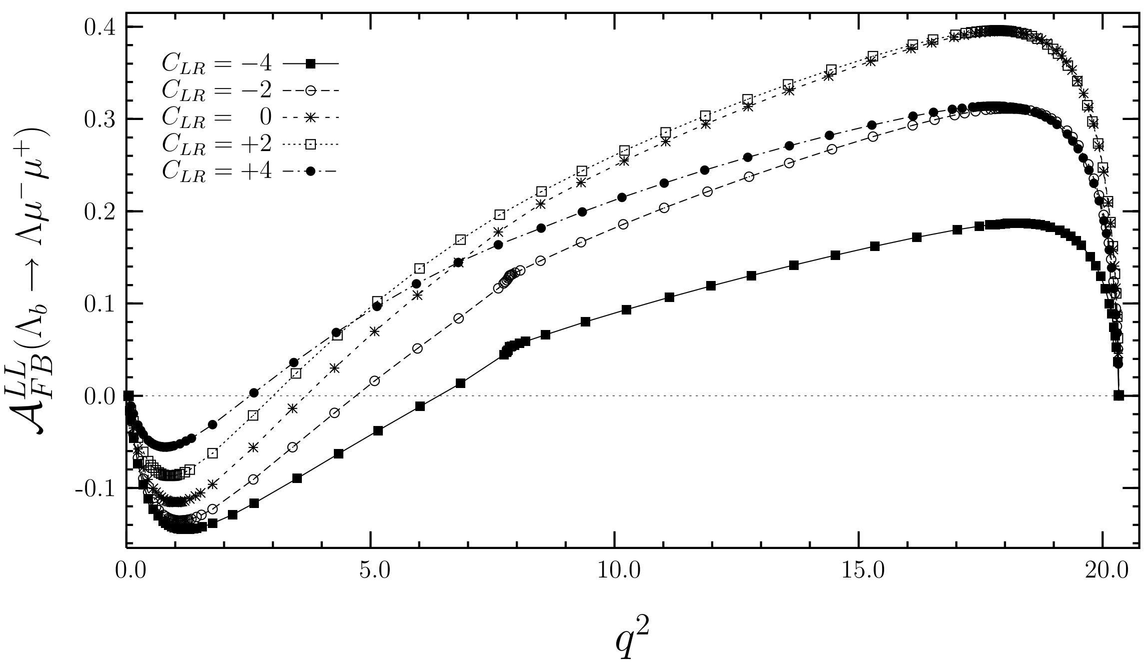

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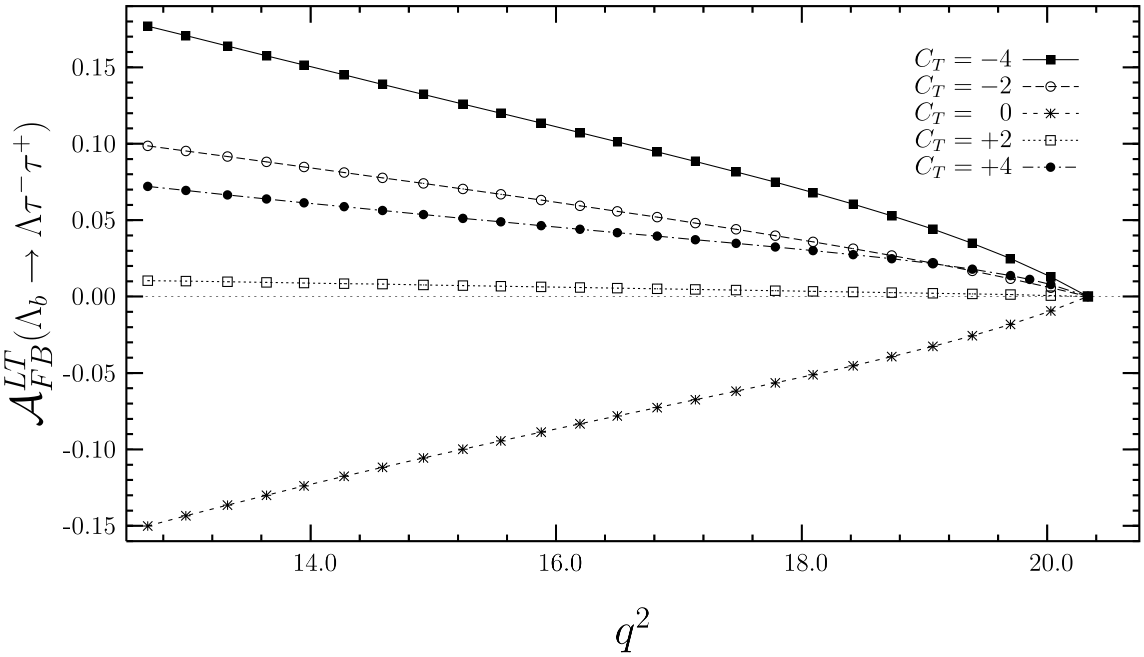

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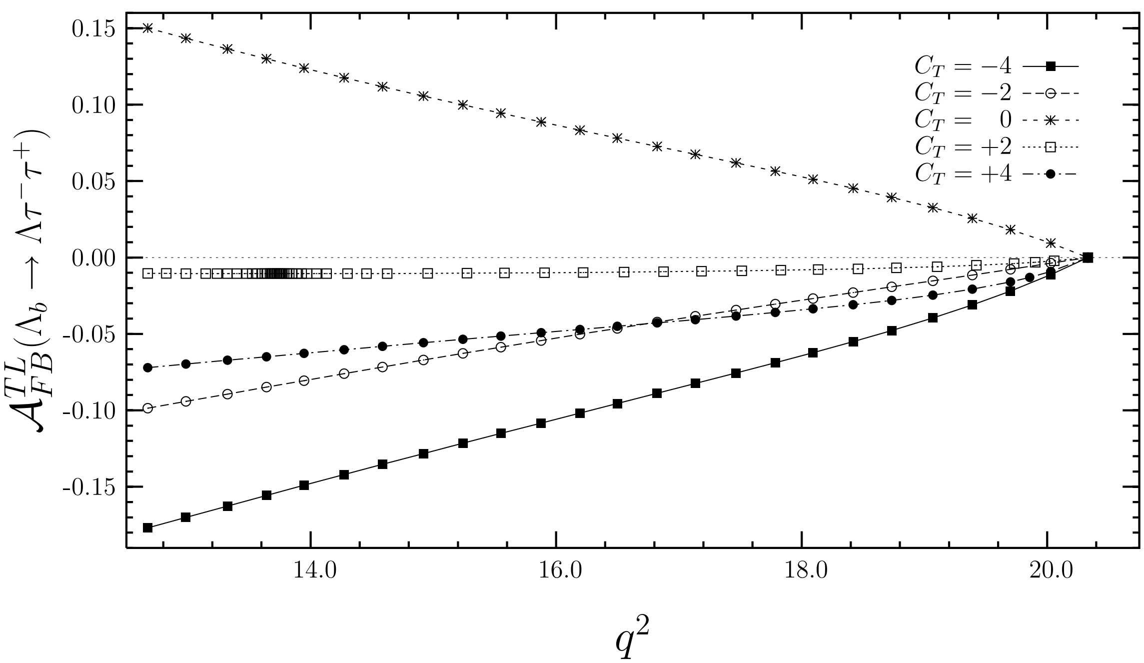

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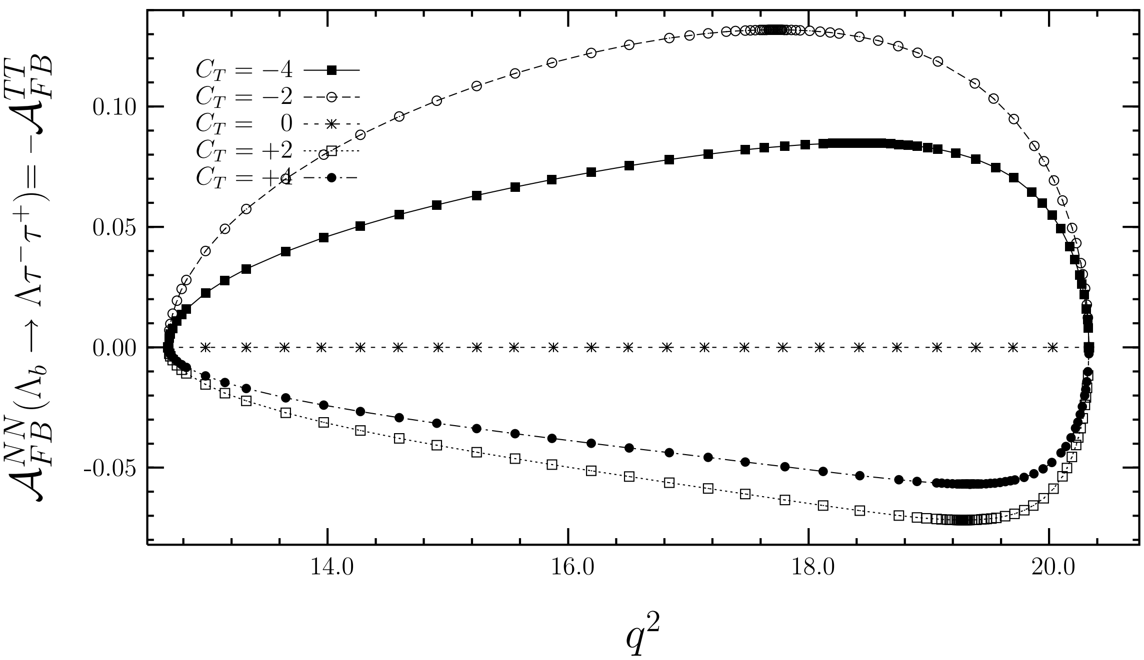

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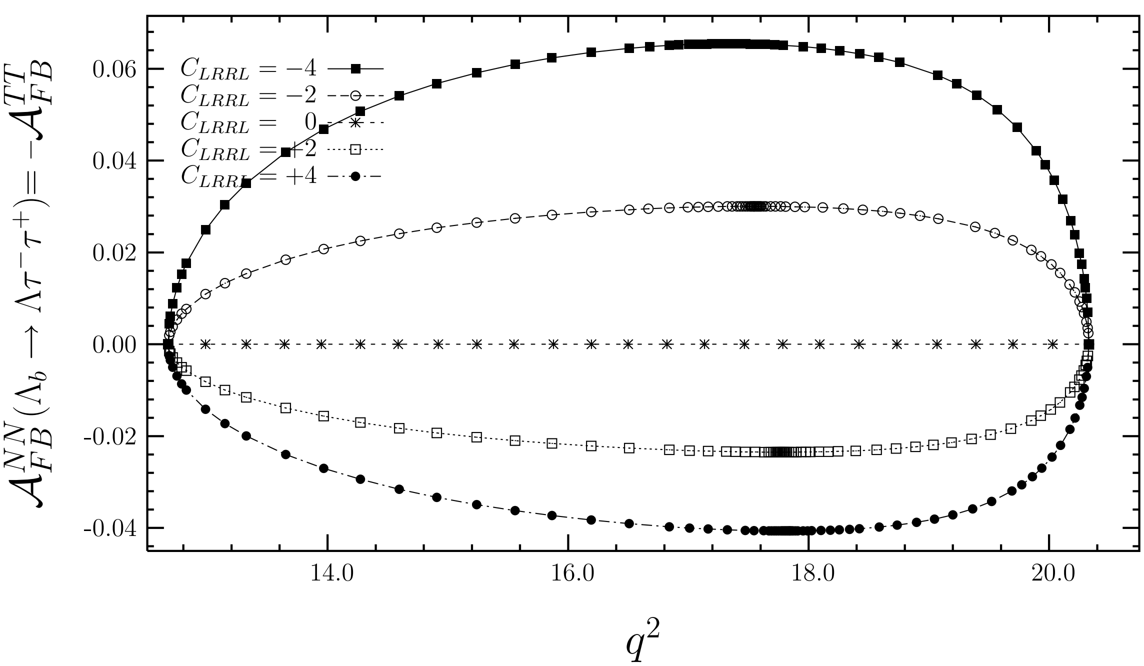

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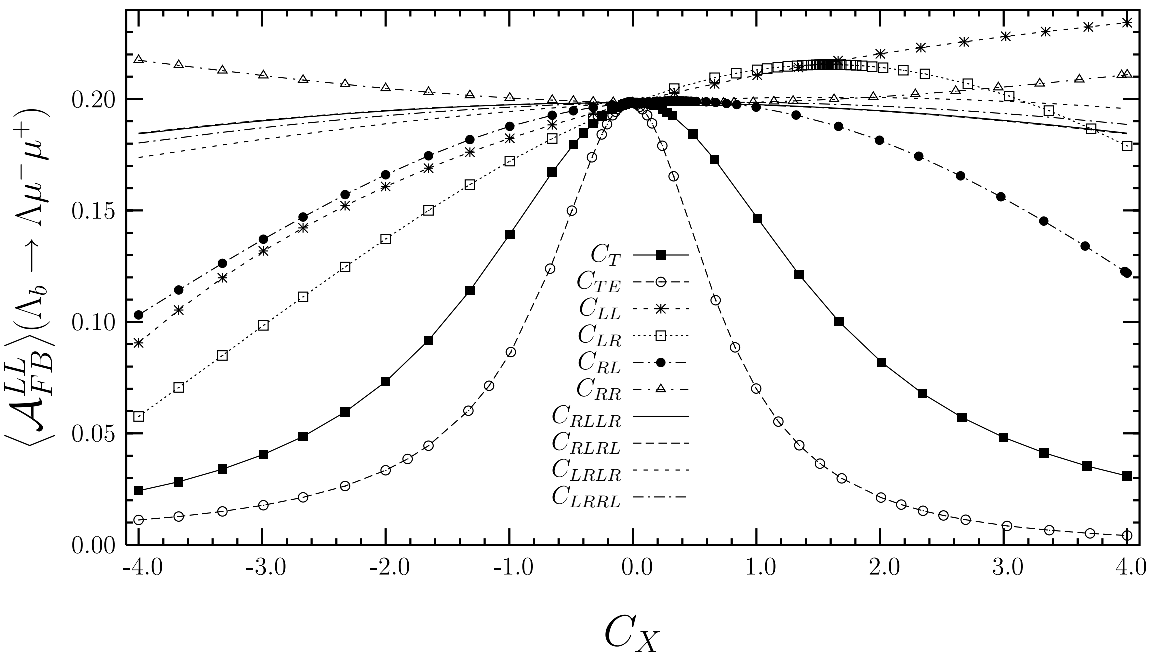

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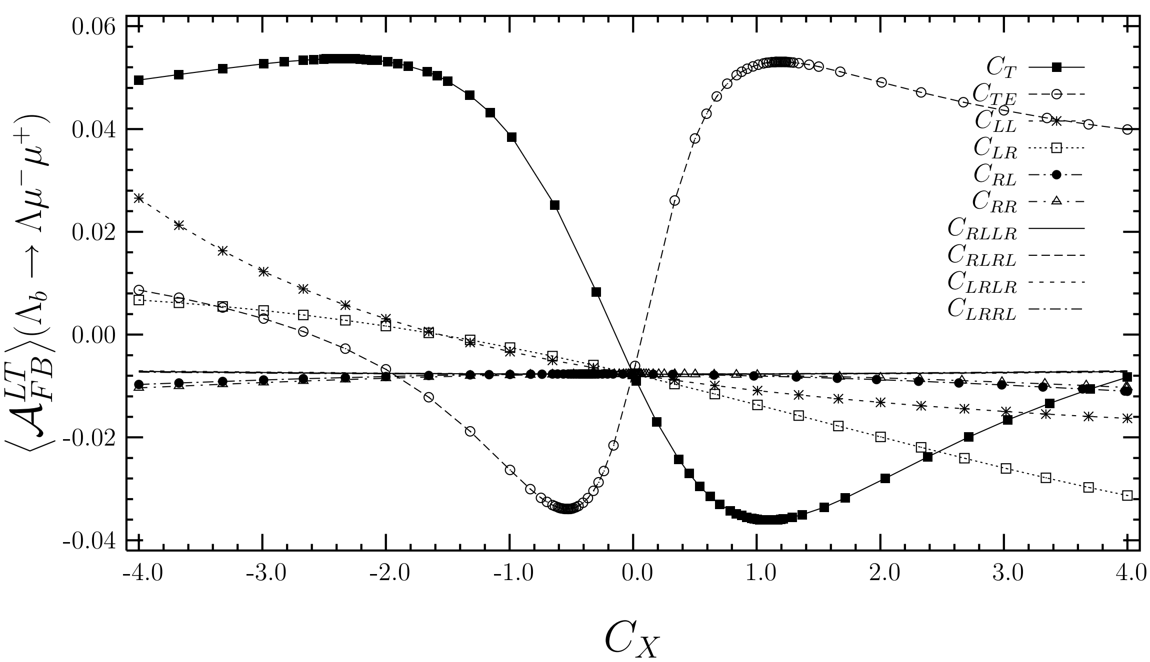

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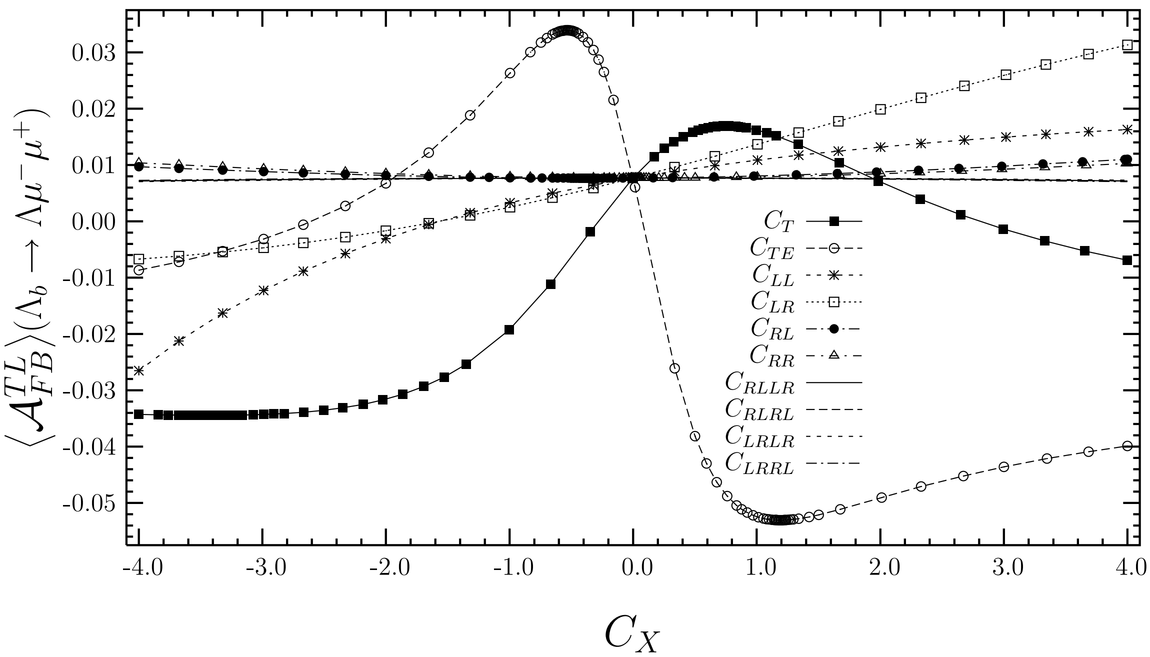

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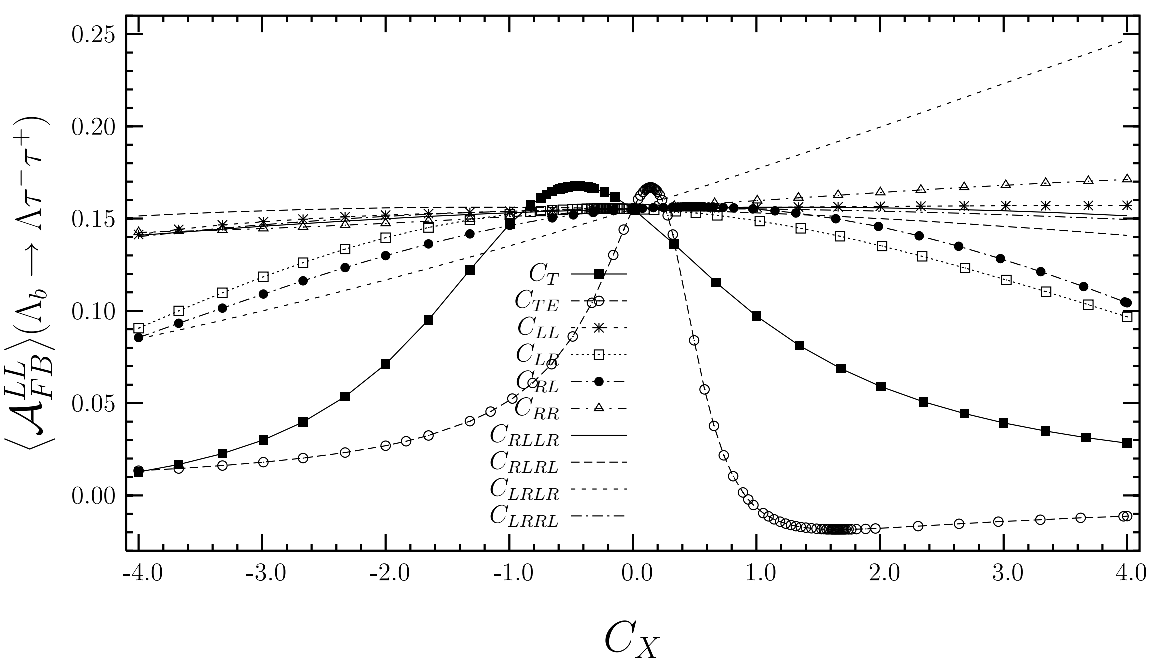

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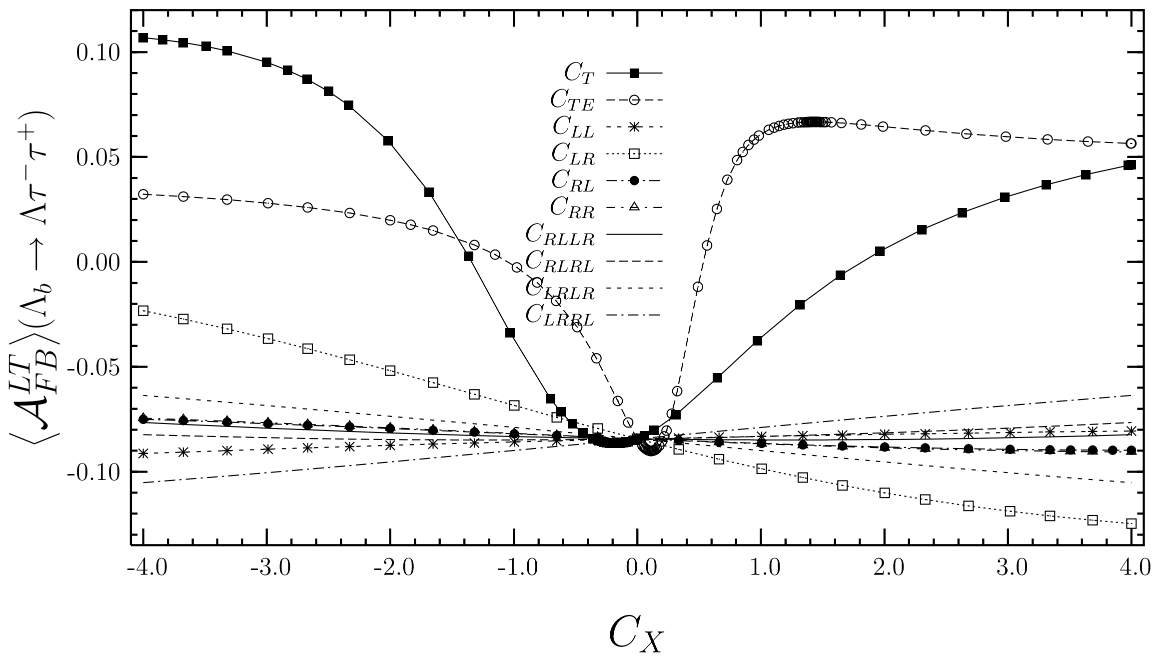

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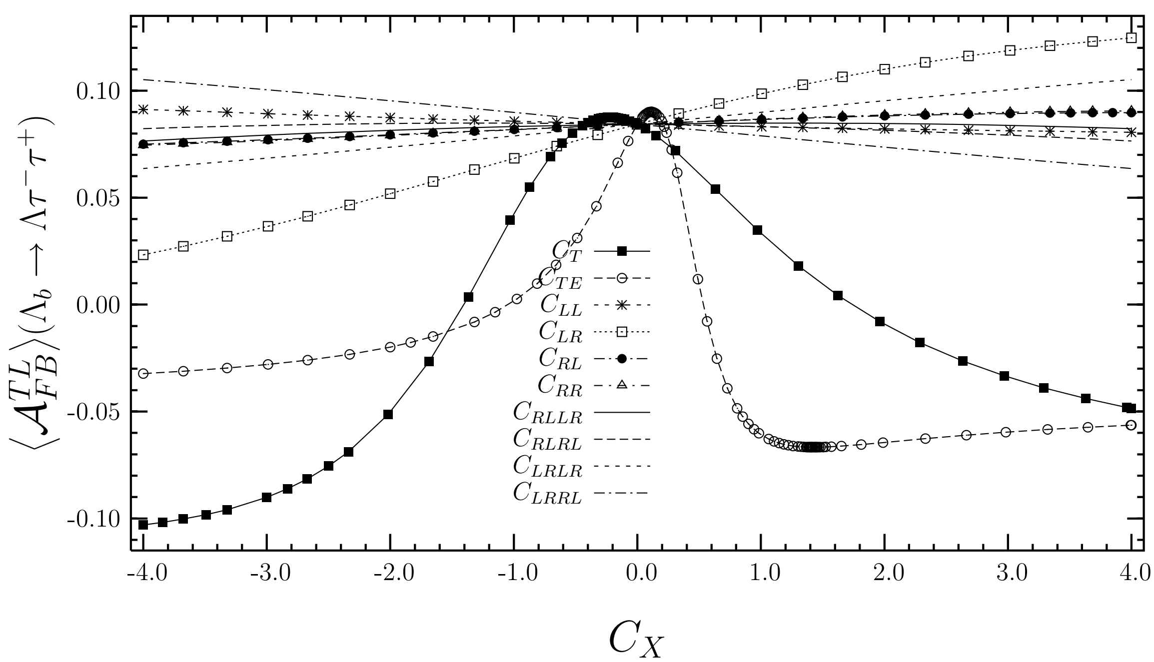

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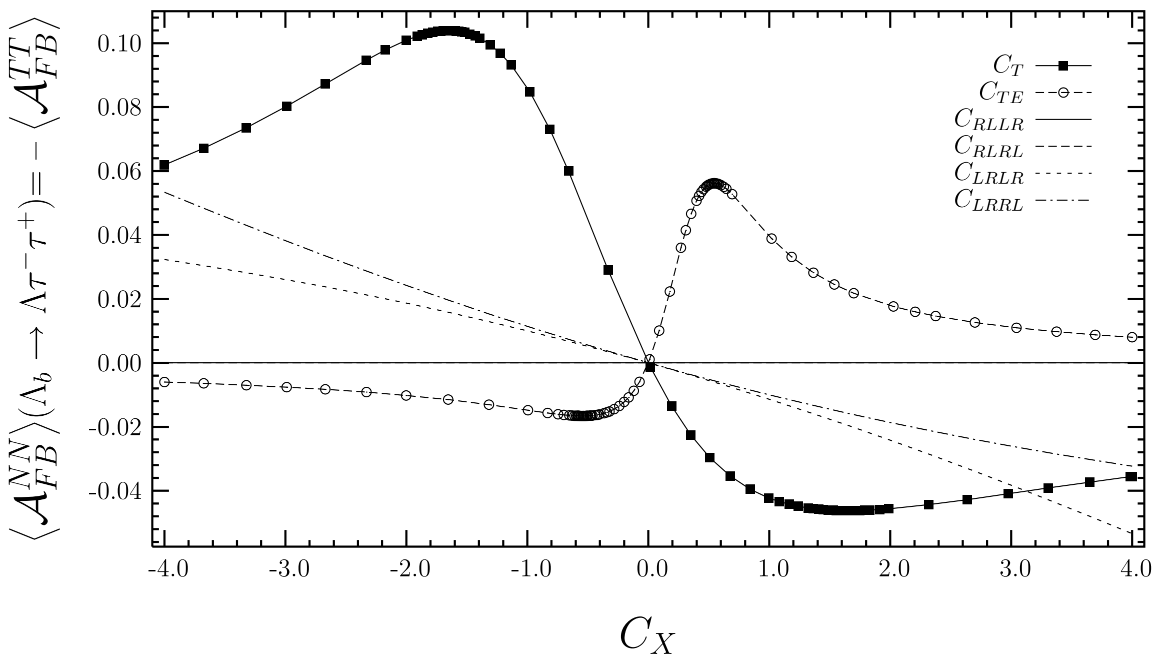
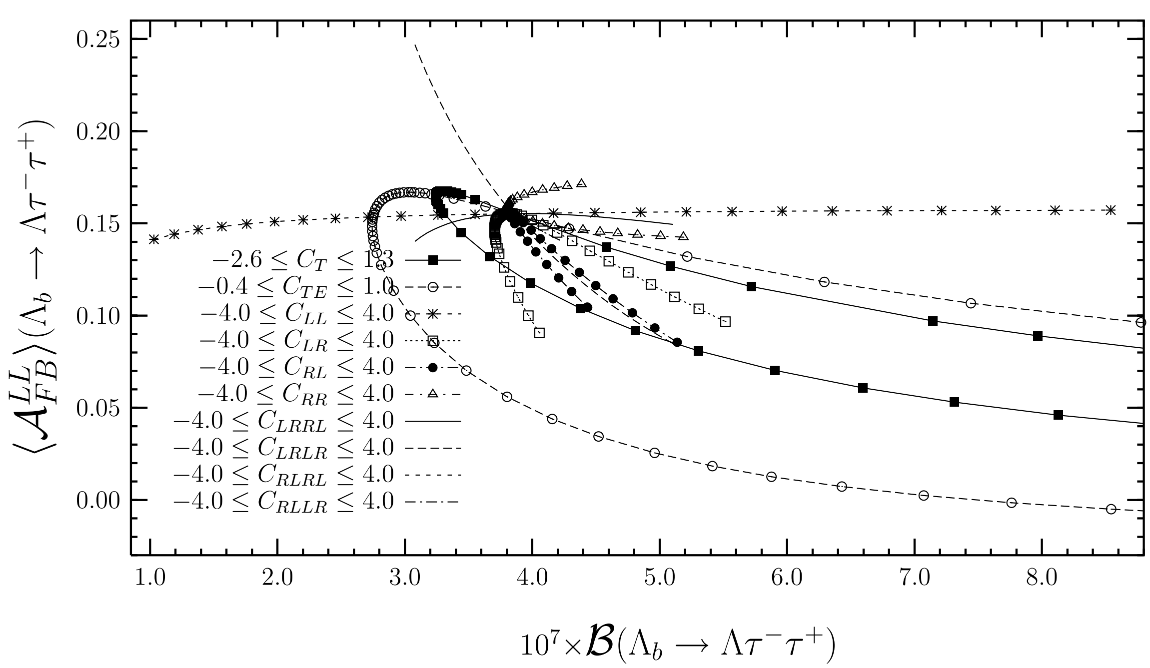


Figure 13:



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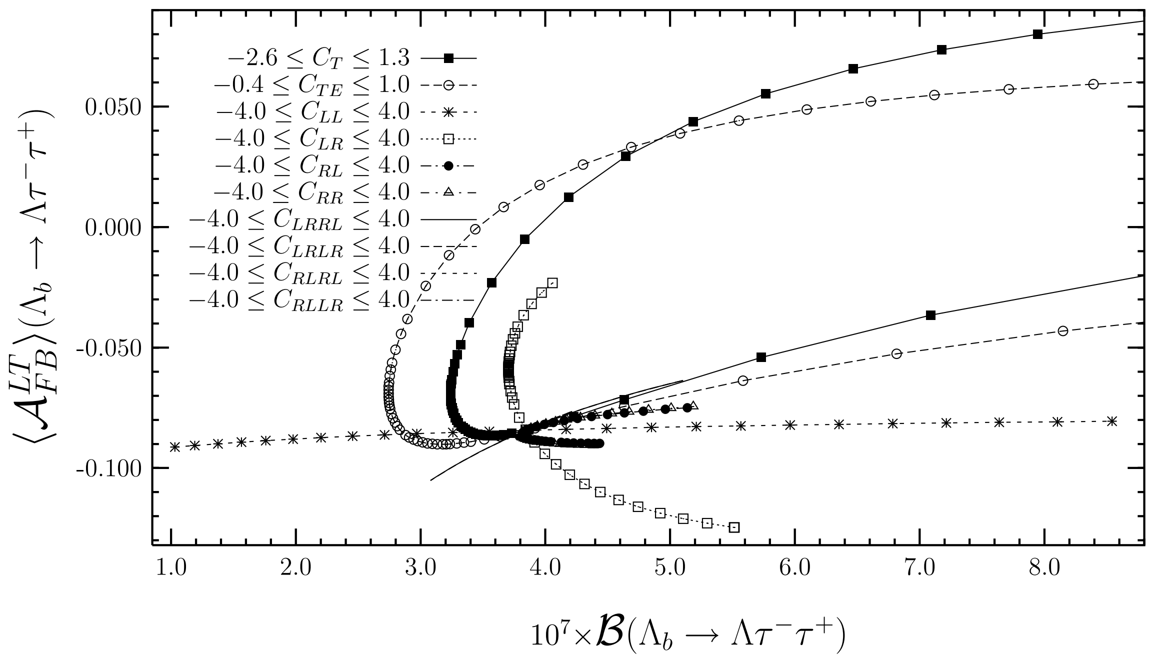

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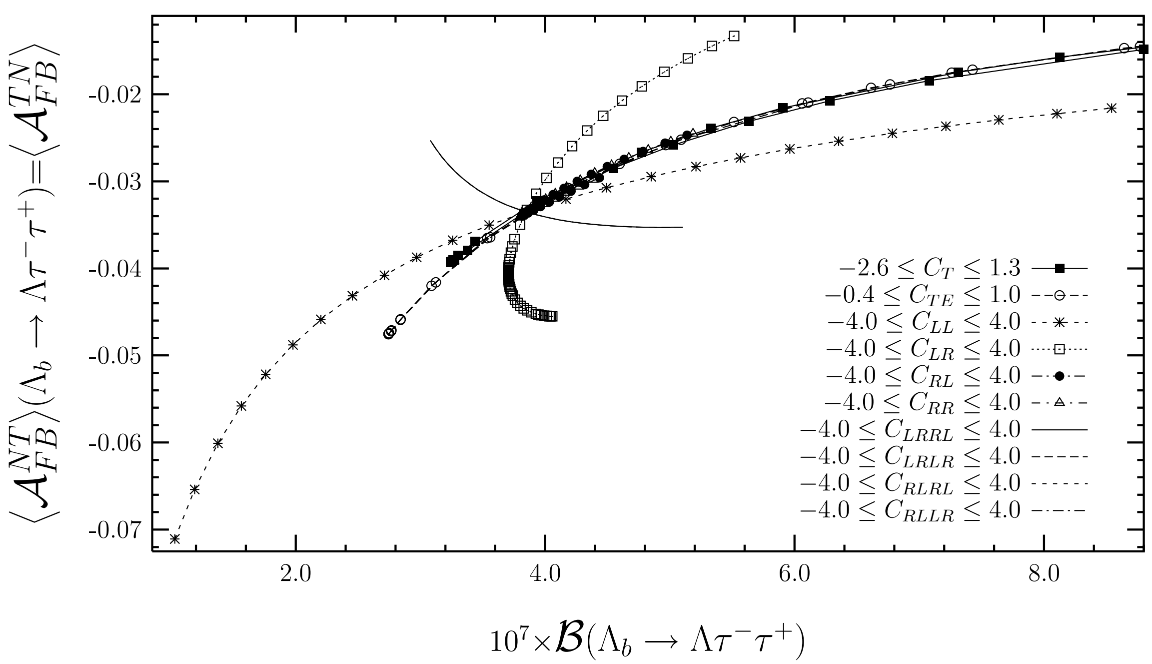

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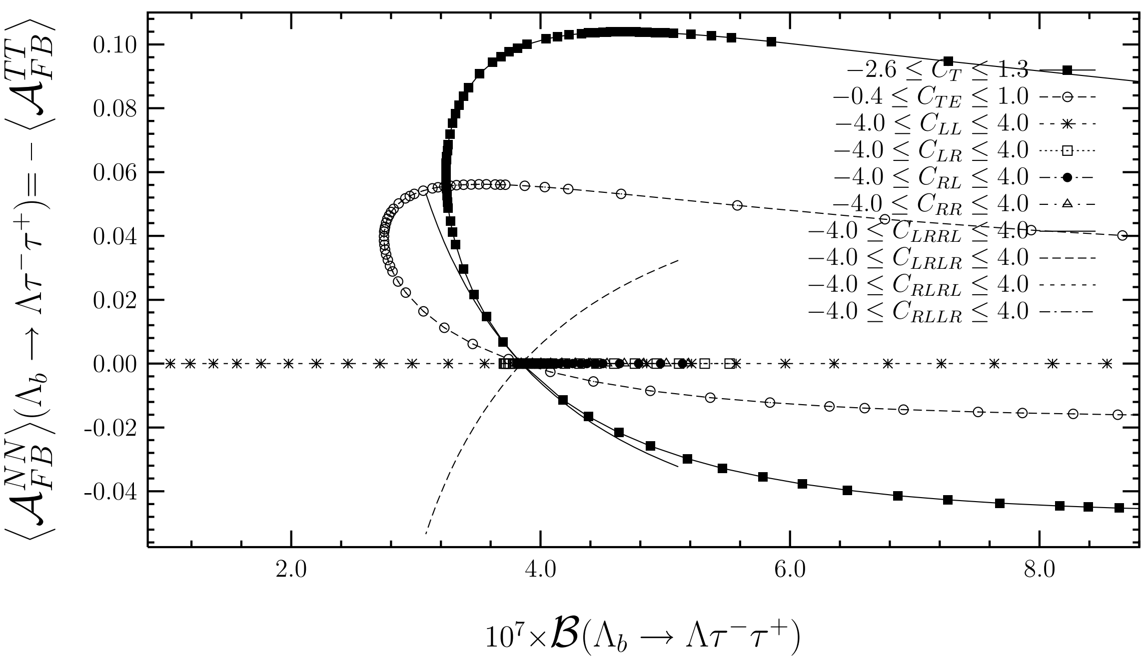
$\langle \mathcal{A}_{FB}^{NN} \rangle (\Lambda_b \to \Lambda \tau^- \tau^+) = -\langle \mathcal{A}_{FB}^{TT} \rangle$
$10^7 \times \mathcal{B}(\Lambda_b \to \Lambda \tau^- \tau^+)$
$-2.6 \le C_T \le 1.3$
$-0.4 \le C_{TE} \le 1.0$
$-4.0 \le C_{LL} \le 4.0$
$-4.0 \le C_{LR} \le 4.0$
$-4.0 \le C_{RL} \le 4.0$
$-4.0 \le C_{RR} \le 4.0$
$-4.0 \le C_{LRRL} \le 4.0$
$-4.0 \le C_{LRLR} \le 4.0$
$-4.0 \le C_{RLRL} \le 4.0$
$-4.0 \le C_{RLLR} \le 4.0$
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
2.0
4.0
6.0
8.0

Figure 17: