



Electric quadrupole and magnetic octupole moments of the light decuplet baryons within light cone QCD sum rules

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ABSTRACT

The electric quadrupole and magnetic octupole moments of the light decuplet baryons are calculated in the framework of the light cone QCD sum rules. The obtained non-vanishing values for the electric quadrupole and magnetic octupole moments of these baryons show nonspherical charge distribution. The sign of electric quadrupole moment is positive for Ω^- , Ξ^{*-} , Σ^{*-} and negative for Σ^{*+} , which correspond to the prolate and oblate charge distributions, respectively. A comparison of the obtained results with the predictions of non-covariant quark model which shows a good consistency between two approaches is also presented. Comparison of the obtained results on the multipole moments of the decuplet baryons containing strange quark with those of Δ baryons shows a large SU(3) flavor symmetry breaking.

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1. Introduction

Detailed study of the electromagnetic properties of baryons, such as electromagnetic multipole moments and electromagnetic form factors, can give essential information about the nonperturbative structure of QCD. These multipole moments are related to the spatial charge and current distributions in baryons. Therefore, calculating these parameters could provide valuable insight on the internal structure as well as the geometric shape of baryons. The dominant elastic form factors of decuplet baryons are the charge G_{E_0} and magnetic dipole G_{M_1} . The subdominant form factors are the electric quadrupole G_{E_2} and magnetic octupole G_{M_3} (all these form factors are defined below). Note that, at $q^2 = 0$, the form factors G_{M_1} , G_{E_2} and G_{M_3} give the magnetic dipole μ_B , electric quadrupole Q_B , and the magnetic octupole moments O_B , respectively [1]. The size of the higher multipole moments Q_B and O_B provide information about the deformation of the baryon and its direction.

Few words about the experimental prospects for measurement of the multipole moments are in order. There are two types of transitions for studying the multipole moments of the ground state decuplet baryons: diagonal transitions between them and off diagonal transitions between the decuplet and octet baryons, i.e., $\Delta \rightarrow N$, $\Sigma^* \rightarrow \Sigma$, $\Sigma^* \rightarrow \Lambda$ and $\Xi^* \rightarrow \Xi$. The couplings of diagonal decuplet–decuplet–photon transitions, obviously, can be measured only by virtual photon exchange. The magnetic moment of Δ^+ has been measured via $\gamma p \rightarrow \pi^0 \gamma' p$ reaction [2]. However, measurement of the electric quadrupole by studying the diagonal transition is practically hopeless. This is due to the fact that the electric quadrupole operator is T-odd quantity and matrix element of this operator between the same initial and final states is equal to zero. Therefore, for the experimental study of the electric quadrupole moment, the suitable place is off-diagonal transitions. For example, the E_2 transition can be measured in reaction *octet baryons* + $X \rightarrow$ *decuplet baryons* + X [3], where X is heavy nucleons and also kaon photoproduction experiments $\gamma p \rightarrow K + \text{decuplet} \rightarrow K + \text{octet} + \gamma$ [4]. Analysis of the electron–proton and photon–proton scattering experiments leads to a nonzero quadrupole moment of $p \rightarrow \Delta^+$ transition [5].

There are large number of works in literature which are devoted to the investigation of the magnetic moment of hadrons, but unfortunately relatively little is known about the other multipole moments. Therefore, further detailed analysis is needed in studying higher multipole moments of the hadrons. Since obtaining direct experimental information about the electromagnetic multipole moments of these baryons is very limited, the theoretical studies play important role in this respect. The electric quadrupole and magnetic octupole

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moments of the Δ baryons have been calculated within the frame work of the light cone QCD sum rules (LCSR) in [6]. It is obtained that both quadrupole and magnetic octupole moments have nonzero values and negative sign, for example, for Δ^+ , implying that the quadrupole and octupole moment distributions of Δ^+ are oblate and have the same geometric shape as the charge distribution. The same result has also been obtained by analyzing the quadrupole and octupole moments of Δ baryons in spectator quark model [7]. These multipole moments for Δ baryons have also been discussed in constituent quark model with configuration mixing but no exchange currents (impulse approximation), and constituent quark model with exchange currents but no configuration mixing [8].

Present work is devoted to the calculation of the electric quadrupole and magnetic octupole moments of the decuplet baryons in the framework of the light cone QCD sum rules. As has already been noted, these multipole moments of the Δ baryons have been calculated in [6] in the same framework. Here, we extend the calculation of the multipole moments to the other members of the decuplet spin 3/2 baryons, i.e., $\Sigma^{*+,0,-}$, $\Xi^{*0,-}$ and Ω^{*-} . Note that, the magnetic octupole moments of these baryons have been calculated in non-covariant quark model (NCQM) [9]. Recently, the electromagnetic form factors of decuplet baryons have been calculated in lattice QCD in [10]. Here, also we stress that the magnetic moments of the decuplet baryons have been studied in [11] within light cone QCD sum rules. The main difference between the present study and [11] is that, in the present work we calculate additional form factors corresponding to different kinematical structures which are related to the higher multipole moments such as quadrupole and octupole. The outline of the Letter is as follows: in Section 2, the light cone QCD sum rules for the electromagnetic form factors are obtained in LCSR. Section 3 encompasses the numerical analysis of the form factors, a comparison of the results with the predictions of the other approaches and discussion.

2. Light cone QCD sum rules for electric quadrupole and magnetic octupole moments of the decuplet baryons

For study the properties of hadrons in the sum rule formalism, the main working tool is the correlation function. To calculate the multipole form factors of the decuplet baryons, we consider the following correlation function:

$$T_{\mu\nu} = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_\mu(x) \bar{\eta}_\nu(0) \} | 0 \rangle_\gamma, \quad (1)$$

where η_μ is the interpolating current for the decuplet baryons, and γ denotes the electromagnetic field. In the sum rule method, the above-mentioned correlation function is calculated in two different ways: on the phenomenological or physical side, it is saturated by a tower of baryons with the same quantum numbers as their interpolating current. On the QCD or theoretical side, it is calculated using the operator product expansion (OPE), where the short- and long-distance quark–gluon interactions are separated. The former is calculated using QCD perturbation theory, whereas the latter are parameterized in terms of the light-cone distribution amplitudes of the photon in light cone version of QCD sum rules. The electromagnetic form factors are determined by matching these two representations of the correlation function.

First, let us calculate the physical part of the correlation function. By isolating the contributions of the ground state baryons from Eq. (1), we obtain

$$T_{\mu\nu} = \frac{\langle 0 | \eta_\mu | B(p_2) \rangle \langle B(p_2) | B(p_1) \rangle_\gamma \langle B(p_1) | \bar{\eta}_\nu | 0 \rangle}{p_2^2 - m_B^2} + \dots, \quad (2)$$

where $p_1 = p + q$, $p_2 = p$ and q is the momentum of a photon. The dots mean contributions of the higher states and continuum.

It follows from Eq. (2) that, for calculation of the physical part, we need to know the matrix element of the interpolating current between the vacuum and the decuplet baryon state as well as transition matrix element, $\langle B(p_2) | B(p_1) \rangle_\gamma$. The $\langle 0 | \eta_\mu(0) | B(p, s) \rangle$ is defined in terms of the residue of the corresponding decuplet baryons, λ_B as:

$$\langle 0 | \eta_\mu(0) | B(p, s) \rangle = \lambda_B u_\mu(p, s), \quad (3)$$

where $u_\mu(p, s)$ is the Rarita–Schwinger spinor. The transition matrix element $\langle B(p_2) | B(p_1) \rangle_\gamma$ can be parameterized in terms of four form factors as [7,12,13]:

$$\langle B(p_2) | B(p_1) \rangle_\gamma = -\varepsilon \bar{u}_\mu(p_2) \left\{ F_1 g^{\mu\nu} \not{\varepsilon} - \frac{1}{2m_B} \left[F_2 g^{\mu\nu} + F_4 \frac{q^\mu q^\nu}{(2m_B)^2} \right] \not{\varepsilon} \not{q} + F_3 \frac{1}{(2m_B)^2} q^\mu q^\nu \not{\varepsilon} \right\} u_\nu(p_1), \quad (4)$$

where ε is the polarization vector of the photon and F_i are form factors as functions of transfer momentum square $q^2 = (p_1 - p_2)^2$. For obtaining the expression for the correlation function from physical side, summation over spins of the spin 3/2 particles is performed using

$$\sum_s u_\mu(p, s) \bar{u}_\nu(p, s) = (\not{p} + m_B) \left\{ -g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2p_\mu p_\nu}{3m_B^2} - \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3m_B} \right\}. \quad (5)$$

In principle, using the above equations, we can obtain the final expression of the physical side of the correlation function, but we come across with two difficulties. (a) Not only spin 3/2, but spin 1/2 particles contribute to the correlation function, i.e., the matrix element of the current η_μ of the spin 3/2 particles between vacuum and spin 1/2 states is nonzero. This matrix element in general form can be written as

$$\langle 0 | \eta_\mu(0) | B(p, s = 1/2) \rangle = (A p_\mu + B \gamma_\mu) u(p, s = 1/2). \quad (6)$$

Using the condition $\gamma_\mu \eta^\mu = 0$, one can immediately obtain that $B = -\frac{A}{4} m$. (b) All Lorentz structures are not independent (for more details, see [14]).

In order to eliminate the unwanted spin 1/2 contributions and obtain only independent structures, the ordering procedure of Dirac matrices are applied and in the present work, we choose it as $\gamma_\mu \not{p} \not{q} \gamma_\nu$. After this ordering procedure, we obtain the final expression of the physical side of the correlation function as follows:

$$\begin{aligned}
T_{\mu\nu} = & \lambda_B^2 \frac{1}{(p_1^2 - m_B^2)(p_2^2 - m_B^2)} \left[2(\varepsilon \cdot p) g_{\mu\nu} \not{p} F_1 + \frac{1}{m_B} (\varepsilon \cdot p) g_{\mu\nu} \not{p} \not{q} F_2 \right. \\
& + \frac{1}{2m_B^2} (\varepsilon \cdot p) q_\mu q_\nu \not{p} F_3 + \frac{1}{4m_B^2} (\varepsilon \cdot p) q_\mu q_\nu \not{q} F_4 + \text{other independent structures} \\
& \left. + \text{structures with } \gamma_\mu \text{ at the beginning and } \gamma_\nu \text{ at the end or which are proportional to } p_{2\mu} \text{ or } p_{1\nu} \right]. \quad (7)
\end{aligned}$$

For calculation of the four form factors, we need four structures. We will choose the structures $(\varepsilon \cdot p) g_{\mu\nu} \not{p}$, $(\varepsilon \cdot p) g_{\mu\nu} \not{p} \not{q}$, $(\varepsilon \cdot p) q_\mu q_\nu \not{p}$ and $(\varepsilon \cdot p) q_\mu q_\nu \not{q}$ for determination of the form factors F_1 , F_2 , F_3 and F_4 , respectively. In the experiments the multipole form factors, G_{E_0} (charge), G_{M_1} (magnetic dipole), G_{E_2} (electric quadrupole) and G_{M_3} (magnetic octupole) are usually measured. Therefore, we need relations between two sets of form factors. The multipole form factors are defined in terms of the form factors $F_i(q^2)$ as [7,12,13,15]:

$$\begin{aligned}
G_{E_0}(q^2) &= [F_1(q^2) - xF_2(q^2)] \left(1 + \frac{2}{3}x\right) - [F_3(q^2) - xF_4(q^2)] \frac{x}{3}(1+x), \\
G_{M_1}(q^2) &= [F_1(q^2) + F_2(q^2)] \left(1 + \frac{4}{5}x\right) - \frac{2}{5}[F_3(q^2) + F_4(q^2)]x(1+x), \\
G_{E_2}(q^2) &= [F_1(q^2) - xF_2(q^2)] - \frac{1}{2}[F_3(q^2) - xF_4(q^2)](1+x), \\
G_{M_3}(q^2) &= [F_1(q^2) + F_2(q^2)] - \frac{1}{2}[F_3(q^2) + F_4(q^2)](1+x), \quad (8)
\end{aligned}$$

where $x = -q^2/4m_B^2$. At $q^2 = 0$, we obtain

$$G_{M_1}(0) = F_1(0) + F_2(0), \quad G_{E_2}(0) = F_1(0) - \frac{1}{2}F_3(0), \quad G_{M_3}(0) = F_1(0) + F_2(0) - \frac{1}{2}[F_3(0) + F_4(0)]. \quad (9)$$

The magnetic dipole μ_B , the electric quadrupole \mathcal{Q}_B , and the magnetic octupole \mathcal{O}_B moments are defined in terms of these form factors at $q^2 = 0$ in the following way:

$$\mu_B = \frac{e}{2m_B} G_{M_1}(0), \quad \mathcal{Q}_B = \frac{e}{m_B^2} G_{E_2}(0), \quad \mathcal{O}_B = \frac{e}{2m_B^3} G_{M_3}(0). \quad (10)$$

The QCD side of the correlation function, on the other hand, can be calculated by the help of the OPE in deep Euclidean region where $p^2 \ll 0$ and $(p+q)^2 \ll 0$. For this aim we need to know the explicit expressions of the interpolating currents of the corresponding baryons. The interpolating currents for decuplet baryons are [16]

$$\begin{aligned}
\eta_\mu^{\Sigma^{*0}} &= \sqrt{\frac{2}{3}} \epsilon^{abc} [(u^{aT} C \gamma_\mu d^b) s^c + (d^{aT} C \gamma_\mu s^b) u^c + (s^{aT} C \gamma_\mu u^b) d^c], \quad \eta_\mu^{\Sigma^{*+}} = \frac{1}{\sqrt{2}} \eta_\mu^{\Sigma^{*0}}(d \rightarrow u), \\
\eta_\mu^{\Sigma^{*-}} &= \frac{1}{\sqrt{2}} \eta_\mu^{\Sigma^{*0}}(u \rightarrow d), \quad \eta_\mu^{\Xi^{*0}} = \eta_\mu^{\Sigma^{*-}}(s \rightarrow u)(d \rightarrow s), \quad \eta_\mu^{\Xi^{*-}} = \eta_\mu^{\Xi^{*0}}(u \rightarrow d), \quad \eta_\mu^{\Omega^{*-}} = \frac{1}{\sqrt{3}} \eta_\mu^{\Sigma^{*+}}(u \rightarrow s), \quad (11)
\end{aligned}$$

where a , b and c are color indices and C is the charge conjugation operator. After contracting out all quark pairs in Eq. (1) using the Wick's theorem, we obtain the following expression for the correlation function of the $\Sigma^{*0} \rightarrow \Sigma^{*0} \gamma$ transition in terms of the quark propagators:

$$\begin{aligned}
\Pi_{\mu\nu}^{\Sigma^{*0} \rightarrow \Sigma^{*0} \gamma} &= -\frac{2i}{3} \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{ipx} \langle \gamma(q) | \{ S_d^{ca'} \gamma_\nu S_u^{bb'} \gamma_\mu S_d^{ac'} + S_d^{cb'} \gamma_\nu S_s^{aa'} \gamma_\mu S_u^{bc'} \\
&+ S_s^{ca'} \gamma_\nu S_d^{bb'} \gamma_\mu S_u^{ac'} + S_s^{cb'} \gamma_\nu S_u^{aa'} \gamma_\mu S_d^{bc'} + S_u^{cb'} \gamma_\nu S_d^{aa'} \gamma_\mu S_s^{bc'} + S_u^{ca'} \gamma_\nu S_s^{bb'} \gamma_\mu S_d^{ac'} \\
&+ \text{Tr}(\gamma_\mu S_s^{ab'} \gamma_\nu S_u^{ba'}) S_d^{cc'} + \text{Tr}(\gamma_\mu S_u^{ab'} \gamma_\nu S_d^{ba'}) S_s^{cc'} + \text{Tr}(\gamma_\mu S_d^{ab'} \gamma_\nu S_s^{ba'}) S_u^{cc'} \} | 0 \rangle, \quad (12)
\end{aligned}$$

where $S' = CS^T C$ and $S_{u,d,s}$ are the light quark propagators. The correlation functions for other transitions can be obtained by the replacements mentioned in Eq. (11). The expression of the light quark propagator in the external field is calculated in [17,18]:

$$\begin{aligned}
S_q(x) &= S^{\text{free}}(x) - \frac{\langle qq \rangle}{12} \left(1 - i \frac{m_q}{4} \not{x}\right) - \frac{x^2}{192} m_0^2 \langle qq \rangle \left(1 - i \frac{m_q}{6} \not{x}\right) \\
&- ig_s \int_0^1 du \left\{ \frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma^{\mu\nu} - ux^\mu G_{\mu\nu}(ux) \gamma^\nu \frac{i}{4\pi^2 x^2} - i \frac{m_q}{32\pi^2} G_{\mu\nu}(ux) \sigma^{\mu\nu} \left[\ln \left(-\frac{x^2 \Lambda^2}{4} + 2\gamma_E \right) \right] \right\}, \quad (13)
\end{aligned}$$

where Λ is the scale parameter, and following [19], we choose it at the factorization scale $\Lambda = 0.5\text{--}1.0$ GeV. The correlation function contain three pieces: (a) Short distance contributions, (b) "Mixed" contributions, (c) Large distance contributions when a photon is radiated at long distance.

Different terms in Eq. (13) give contributions to the different pieces of the correlation function. The short distance contributions can easily be obtained from Eq. (12) by replacing one of the propagators by

Table 1Results of the electric quadrupole moment (in units of fm^2) and magnetic octupole moments (in units of fm^3) of the decuplet baryons.

	Quadrupole \mathcal{Q} (fm^2)	Octupole \mathcal{O} (fm^3)	
	Present work	Present work	NCQM [9]
Ω^-	0.12 ± 0.04	0.016 ± 0.004	$0.003-0.012$
Σ^{*-}	0.03 ± 0.01	0.013 ± 0.004	$0.008-0.012$
Σ^{*0}	0.0012 ± 0.0004	-0.001 ± 0.0003	$0.000-0.002$
Σ^{*+}	-0.028 ± 0.009	-0.015 ± 0.005	$-0.004-(-0.012)$
Ξ^{*-}	0.045 ± 0.015	0.020 ± 0.006	$0.005-0.012$
Ξ^{*0}	0.0025 ± 0.0008	-0.0014 ± 0.0005	$0.000-0.002$

$$S_{\alpha\beta}^{ab} = \left\{ \int d^4y S^{\text{free}}(x-y) A S^{\text{free}}(y) \right\}_{\alpha\beta}^{ab}, \quad (14)$$

where S^{free} is the light quark propagator given as

$$S^{\text{free}}(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} \quad (15)$$

and two other quark propagators are replaced by the free quark propagator.

In the “mixed” contributions case, a photon interacts with quark fields perturbatively. Therefore, one of the quark propagators is replaced by Eq. (14), two other propagators either both are replaced by

$$S_{\alpha\beta}^{ab} = -\frac{1}{4} \bar{q}^a \Gamma_j q^b (\Gamma_j)_{\alpha\beta}, \quad (16)$$

which both can form quark condensates, or one of them is replaced by Eq. (16) and second one by the free quark propagator. In Eq. (16), Γ_j is the full set of Dirac matrices.

The large distance contributions can be obtained from Eq. (12) by following replacements: One of the quark propagators is replaced by Eq. (16) and a photon interacts with the quark fields at large distance, i.e., the matrix elements of the nonlocal operators $\bar{q}(x_1) \Gamma q'(x_2)$ and $\bar{q}(x_1) G_{\mu\nu} \Gamma q'(x_2)$ appear between the vacuum and the vector meson states, which is parameterized in terms of photon distribution amplitudes (DAs). Two other propagators are both replaced by free quark propagator, or one of them is replaced by free quark propagator and second one is replaced by Eq. (16) and then it interact with QCD vacuum, i.e., it forms a quark condensate, or both of propagators are replaced by Eq. (16) and then they form quark condensates.

Using the expressions of the light propagators and the photon DAs and separating the coefficient of the structures mentioned before and applying double Borel transformation with respect to the variables $p_2^2 = p^2$ and $p_1^2 = (p+q)^2$ to suppress the contributions of the higher states and continuum, sum rules for the form factors F_1 , F_2 , F_3 and F_4 are obtained. The explicit expressions of the sum rules for these form factors are given in Appendix A. From the expressions of the form factors it is clear that, to obtain form factors, we need to know the explicit expressions of residues of the corresponding baryons. The explicit expressions for these residues are given in [20,23].

3. Numerical analysis

Present section is devoted to the numerical analysis for the, electric quadrupole and magnetic octupole moments of the light spin 3/2 baryons. The values for input parameters used in the analysis of the sum rules for the F_1 , F_2 , F_3 and F_4 are: $\langle \bar{u}u(1 \text{ GeV}) \rangle = \langle \bar{d}d(1 \text{ GeV}) \rangle = -(1.65 \pm 0.15) \times 10^{-2} \text{ GeV}^3$ [21], $\langle \bar{s}s(1 \text{ GeV}) \rangle = 0.8 \langle \bar{u}u(1 \text{ GeV}) \rangle$, $m_s(2 \text{ GeV}) = (111 \pm 6) \text{ MeV}$ at $\Lambda_{\text{QCD}} = 330 \text{ MeV}$ [22], $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [23] and $f_{3\gamma} = -0.0039 \text{ GeV}^2$ [24]. The value of the magnetic susceptibility is taken to be $\chi(1 \text{ GeV}) = -3.15 \pm 0.3 \text{ GeV}^{-2}$ [24]. As has already be noted, the main input parameters in light cone sum rules are the DAs. The explicit expression of the photon DAs are given in [24].

The sum rules for the electromagnetic form factors also contain two auxiliary parameters: Borel mass parameter M^2 and continuum threshold s_0 . The physical quantities should be independent of these parameters. Therefore, we look for a region for these parameters such that the electromagnetic form factors are independent of them. The working region for M^2 are found requiring that not only the contributions of the higher states and continuum should be less than the ground state contribution, but the highest power of $1/M^2$ be less than say 30% of the highest power of M^2 . These conditions are satisfied in the regions $1.1 \text{ GeV}^2 \leq M^2 \leq 1.6 \text{ GeV}^2$, $1.2 \text{ GeV}^2 \leq M^2 \leq 1.7 \text{ GeV}^2$ and $1.4 \text{ GeV}^2 \leq M^2 \leq 2.4 \text{ GeV}^2$ for Σ^* , Ξ^* and Ω^* baryons, respectively. In the numerical analysis, $s_0 = (m_B + 0.5)^2 \text{ GeV}^2$ has been used for value of the continuum threshold.

Our final results on the electric quadrupole Q_B and magnetic octupole \mathcal{O}_B moments of decuplet baryons are presented in Table 1. The quoted errors in Table 1 can be attributed to the uncertainties in the variation of the Borel parameter M^2 , the continuum threshold s_0 , as well as the uncertainties in the determination of the other input parameters entering the sum rules. A comparison of our predictions on magnetic octupole moment with the results obtained in NCQM is also presented in Table 1. The results for magnetic octupole moments show a good consistency between our predictions and those of the NCQM [9]. As has already been noted, the electromagnetic form factors of the decuplet baryons have been calculated at $q^2 \neq 0$ in [10], so we cannot compare our results with theirs. However, the order of magnitude of our results are in good agreement with their predictions at low q^2 . Comparison between our results on electric quadrupole and magnetic octupole moments of Σ^{*+-} , Ξ^{*-} , Ω^- and the predictions of [6] for Δ baryons, shows a large SU(3) flavor symmetry breaking. This violation is larger for Ξ^{*-} and Ω^- baryons which contain two and three strange quarks, respectively. In the case of the strange baryons, the results are very sensitive to the strange quark mass. This sensitivity together with the different working regions of Borel mass parameter, M^2 , and continuum threshold, s_0 lead to the different values of multipole moments for Σ^* , Ξ^* , Ω baryons.

In conclusion, the electric quadrupole and magnetic octupole moments of decuplet baryons were calculated in the framework of the light cone QCD sum rules. We obtained non-vanishing values for the electric quadrupole and magnetic octupole moments of these baryons which mean nonspherical charge distribution. The sign of electric quadrupole moment is positive for Ω^- , Ξ^{*-} , Σ^{*-} and negative for Σ^{*+} , which correspond to the prolate and oblate charge distributions, respectively. The obtained results are in good consistency with the predictions of the non-covariant quark model. Comparison of the obtained results on the multipole moments of the decuplet baryons containing strange quark with those of Δ baryons presents a large SU(3) flavor symmetry breaking.

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Appendix A

In this appendix, we present the sum rules for the form factors, $F_1(0)$, $F_2(0)$, $F_3(0)$ and $F_4(0)$.

$$\begin{aligned}
 F_1(q^2 = 0) = & \frac{1}{2\lambda_{\Sigma^{*0}}^2} e^{m_{\Sigma^{*0}}^2/M^2} \left\{ \frac{1}{40\pi^4} (e_u + e_d + e_s) M^6 - \frac{1}{6\pi^2} M^2 m_s [2(e_d + e_s) \langle \bar{u}u \rangle + 2(e_u + e_s) \langle \bar{d}d \rangle - (e_u + e_d) \langle \bar{s}s \rangle] \right. \\
 & - \frac{1}{36\pi^2 M^2} m_s \langle g_s^2 G^2 \rangle (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \left(\gamma_E + \ln \frac{\Lambda^2}{M^2} \right) + \frac{1}{54\pi^2 M^2} m_s \langle g_s^2 G^2 \rangle (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
 & - \frac{4}{9M^2} m_0^2 (e_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle + e_d \langle \bar{u}u \rangle \langle \bar{s}s \rangle + e_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle) - \frac{1}{144\pi^2 M^4} m_0^2 m_s \langle g_s^2 G^2 \rangle (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
 & + \frac{1}{54\pi^2} m_0^2 m_s [(9e_d + 10e_s) \langle \bar{u}u \rangle + (9e_u + 10e_s) \langle \bar{d}d \rangle - 4(e_u + e_d) \langle \bar{s}s \rangle] \\
 & \left. + \frac{8}{9} (e_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle + e_d \langle \bar{u}u \rangle \langle \bar{s}s \rangle + e_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle) \right\}, \tag{A.1}
 \end{aligned}$$

$$\begin{aligned}
 F_2(q^2 = 0) = & \frac{m_{\Sigma^{*0}}}{\lambda_{\Sigma^{*0}}^2} e^{m_{\Sigma^{*0}}^2/M^2} \left\{ -\frac{1}{1152\pi^4 M^2} m_s \left(\gamma_E + \ln \frac{\Lambda^2}{M^2} \right) \{ [3M^2 + 2\pi^2 f_{3\gamma} \psi_a(u_0)] \langle g_s^2 G^2 \rangle (e_u + e_d) + 24e_s M^6 \} \right. \\
 & - \frac{1}{288\pi^4} M^4 [m_s (3e_u + 3e_d + 11e_s) + 8\pi^2 \chi (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle + e_s \langle \bar{s}s \rangle) \phi_\gamma(u_0)] \\
 & + \frac{1}{144\pi^2} M^2 \{ 6[(e_u + e_d) \langle \bar{s}s \rangle + (e_u + e_s) \langle \bar{d}d \rangle + (e_d + e_s) \langle \bar{u}u \rangle] \\
 & + (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle + e_s \langle \bar{s}s \rangle) (3\mathbb{A}(u_0) - 4[i_2(\mathcal{S}, 1) + i_2(\tilde{\mathcal{S}}, 3 - 4\nu) \\
 & + i_2(\mathcal{T}_2, 1 - 2\nu) + 2i_2(\mathcal{T}_3, 1 - 2\nu) - i_2(\mathcal{T}_4, 1 - 2\nu) + 8\tilde{i}_3(h_\gamma)]) - 3m_s f_{3\gamma} (e_u + e_d) \psi_a(u_0) \} \\
 & - \frac{1}{54M^2} m_s \langle \bar{s}s \rangle (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) [i_2(\mathcal{S}, 1) + i_2(\tilde{\mathcal{S}}, 3 - 4\nu) + i_2(\mathcal{T}_2, 1 - 2\nu) \\
 & + 2i_2(\mathcal{T}_3, 1 - 2\nu) - i_2(\mathcal{T}_4, 1 - 2\nu)] + \frac{1}{36M^2} m_s [2(e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \langle \bar{s}s \rangle \\
 & + (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle \mathbb{A}(u_0) + 16(e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \tilde{i}_3(h_\gamma)] \\
 & + \frac{1}{864\pi^2 M^2} m_s f_{3\gamma} \langle g_s^2 G^2 \rangle (e_u + e_d) \psi_a(u_0) + \frac{1}{648M^2} m_0^2 \{ 24m_s \langle \bar{s}s \rangle \chi (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) \phi_\gamma(u_0) \\
 & - 11f_{3\gamma} [(e_d + e_s) \langle \bar{u}u \rangle + (e_u + e_s) \langle \bar{d}d \rangle + (e_u + e_d) \langle \bar{s}s \rangle] \psi_a(u_0) \} \\
 & + \frac{2}{81M^4} m_0^2 m_s [(e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle - 2(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle] \tilde{i}_3(h_\gamma) \\
 & + \frac{1}{54M^6} m_s \langle g_s^2 G^2 \rangle (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \tilde{i}_3(h_\gamma) + \frac{1}{108M^8} m_s m_0^2 \langle g_s^2 G^2 \rangle (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \tilde{i}_3(h_\gamma) \\
 & - \frac{1}{1152\pi^4} m_s \langle g_s^2 G^2 \rangle (e_u + e_d) - \frac{5}{432\pi^2} m_0^2 [(e_d + e_s) \langle \bar{u}u \rangle + (e_u + e_s) \langle \bar{d}d \rangle + (e_u + e_d) \langle \bar{s}s \rangle] \\
 & + \frac{1}{18} \{ -2m_s \chi (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle \phi_\gamma(u_0) + f_{3\gamma} [(e_d + e_s) \langle \bar{u}u \rangle + (e_u + e_s) \langle \bar{d}d \rangle \\
 & \left. + (e_u + e_d) \langle \bar{s}s \rangle] \psi_a(u_0) \}, \tag{A.2}
 \end{aligned}$$

$$\begin{aligned}
 F_3(q^2 = 0) = & \frac{2m_{\Sigma^{*0}}^2}{\lambda_{\Sigma^{*0}}^2} e^{m_{\Sigma^{*0}}^2/M^2} \left\{ -\frac{7}{960\pi^4} M^4 (e_u + e_d + e_s) \right. \\
 & \left. - \frac{1}{36\pi^2} M^2 f_{3\gamma} (e_u + e_d + e_s) [(2i_2(\mathcal{A}, 5 - 4\nu) + 4i_2(\mathcal{V}, 1 - 2\nu) - \psi_a(u_0))] \right\}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{36\pi^2 M^4} m_s (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) (4M^4 [5i_1(\mathcal{T}_1 + \mathcal{T}_2, 1) - 3i_1(\mathcal{T}_3 + \mathcal{T}_4, 1)] - \langle g_s^2 G^2 \rangle \tilde{i}_3(h_\gamma)) \left(\gamma_E + \ln \frac{\Lambda^2}{M^2} \right) \\
& + \frac{1}{54M^2 \pi^2} m_0^2 m_s \langle \bar{s}s \rangle (e_u + e_d) + \frac{4}{27M^2} [(e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle + (e_u + e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle \\
& + (e_d + e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle] [i_1(3\mathcal{T}_1 + 4\mathcal{T}_2 - \mathcal{T}_4, 1) + 6\tilde{i}_3(h_\gamma)] \\
& - \frac{1}{27M^2} m_s f_{3\gamma} \langle \bar{s}s \rangle (e_u + e_d) [i_2(\mathcal{A}, 5 - 4\nu) + 2i_2(\mathcal{V}, 1 - 2\nu) - 3\psi_a(u_0)] \\
& + \frac{1}{216M^4 \pi^2} m_s \langle g_s^2 G^2 \rangle (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) [i_1(3\mathcal{T}_1 + 4\mathcal{T}_2 - \mathcal{T}_4, 1) + 4\tilde{i}_3(h_\gamma)] \\
& - \frac{2}{81M^4} m_0^2 (10[(e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle + (e_u + e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle + (e_d + e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle] \tilde{i}_3(h_\gamma) + m_s f_{3\gamma} (e_u + e_d) \langle \bar{s}s \rangle \psi_a(u_0)) \\
& - \frac{1}{72\pi^2} m_s (3(e_u + e_d) \langle \bar{s}s \rangle - 8(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle)) [i_1(2\mathcal{T}_1 + \mathcal{T}_2 - 3\mathcal{T}_3 - 2\mathcal{T}_4, 1) - 3\tilde{i}_3(h_\gamma)] \Big\}, \tag{A.3}
\end{aligned}$$

$$\begin{aligned}
F_4(q^2 = 0) & = \frac{4m_{\Sigma^*0}^2}{\lambda_{\Sigma^*0}^2} e^{m_{\Sigma^*0}^2/M^2} \left\{ -\frac{1}{160\pi^4} M^4 (e_u + e_d + e_s) \right. \\
& - \frac{1}{144\pi^2} M^2 f_{3\gamma} (e_u + e_d + e_s) (4[4i_2(\mathcal{A}, 1 + \nu) + 4i_2(\mathcal{V}, 1 - \nu) + \tilde{i}_3(\psi_\nu)] - 3\psi_a(u_0)) \\
& + \frac{1}{72M^4 \pi^2} m_s (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) [16M^4 i_1(\mathcal{T}_1 + \mathcal{T}_2 - \mathcal{T}_3 - \mathcal{T}_4, 1) \\
& - \langle g_s^2 G^2 \rangle \tilde{i}_3(h_\gamma)] \left(\gamma_E + \ln \frac{\Lambda^2}{M^2} \right) + \frac{1}{108M^2 \pi^2} m_0^2 m_s \langle \bar{s}s \rangle (e_u + e_d) \\
& + \frac{4}{27M^2} [(e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle + (e_u + e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle + (e_d + e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle] [4i_1(\mathcal{T}_2 - \mathcal{T}_4, \nu) + 3\tilde{i}_3(h_\gamma)] \\
& - \frac{1}{108M^2} m_s f_{3\gamma} \langle \bar{s}s \rangle (e_u + e_d) [8i_2(\mathcal{A}, 1 + \nu) + 8i_2(\mathcal{V}, 1 - \nu) + 12\tilde{i}_3(\psi_\nu) - 9\psi_a(u_0)] \\
& + \frac{1}{108M^4 \pi^2} m_s \langle g_s^2 G^2 \rangle (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) [2i_1(\mathcal{T}_2 - \mathcal{T}_4, \nu) + \tilde{i}_3(h_\gamma)] \\
& - \frac{1}{81M^4} m_0^2 (10[(e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle + (e_u + e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle + (e_d + e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle] \tilde{i}_3(h_\gamma) + m_s f_{3\gamma} (e_u + e_d) \langle \bar{s}s \rangle \psi_a(u_0)) \\
& \left. - \frac{1}{72\pi^2} m_s (3(e_u + e_d) \langle \bar{s}s \rangle - 4(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle)) [4i_1(\mathcal{T}_1 - \mathcal{T}_3, 1) + 4i_1(\mathcal{T}_2 - \mathcal{T}_4, 1 - 2\nu) - 3\tilde{i}_3(h_\gamma)] \right\} \tag{A.4}
\end{aligned}$$

where the Borel parameter M^2 is defined as $M^2 = M_1^2 M_2^2 / M_1^2 + M_2^2$ and $u_0 = M_1^2 / (M_1^2 + M_2^2)$. Since the masses of the initial and final baryons are the same, we have set $M_1^2 = M_2^2$ and $u_0 = 1/2$. The continuum subtractions have been made via $M^{2n} \rightarrow M^{2n} E_n(x)$, where $E_n(x) = 1 - e^{-x} \sum_{i=0}^{n-1} \frac{x^i}{i!}$ with $x = s_0/M^2$.

The functions i_n , \tilde{i}_3 and \tilde{i}_3 are also defined as

$$\begin{aligned}
i_1(\phi, f(\nu)) & = \int \mathcal{D}\alpha_i \int_0^1 d\nu \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(\nu) \theta(k - u_0), & i_2(\phi, f(\nu)) & = \int \mathcal{D}\alpha_i \int_0^1 d\nu \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(\nu) \delta(k - u_0), \\
\tilde{i}_3(f(u)) & = \int_{u_0}^1 du f(u), & \tilde{i}_3(f(u)) & = \int_{u_0}^1 du (u - u_0) f(u), \tag{A.5}
\end{aligned}$$

where $k = \alpha_q + \alpha_g \bar{\nu}$.

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