

Shortcuts to Spherically Symmetric Solutions: A Cautionary Note

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Abstract

Spherically symmetric solutions of generic gravitational models are optimally, and legitimately, obtained by expressing the action in terms of the surviving metric components. This shortcut is not to be overdone, however: a one-function ansatz invalidates it, as illustrated by the incorrect solutions of [1].

It is well-understood since the original work of Palais [2] that the field equations describing highly symmetric geometries can be obtained legitimately by using only candidate metrics endowed with those symmetries. This simplifying procedure does require some care, however, as illustrated in [3]. In particular, one must retain enough dependent variables in order to probe the required range of variation of the action. Specifically, spherically symmetric metrics have two independent components (g_{rr} , g_{00}) in the natural Schwarzschild gauge. While the field equations of, say Einstein gravity imply $g_{00}g_{rr} = -1$, assuming this a priori in the Lagrangian density turns it into a total divergence, with similar inconsistencies in other actions. This

is just a special case of the generic ¹ invalidity of “going on half-shell” in obtaining Euler-Lagrange equations. Actually, there is a deep problem [4] associated with even the Weyl 2-function ansatz – its inability to reproduce Birkhoff’s theorem; given the more immediate difficulties addressed below, we drop it here.

An example of this error is a recent work [1] in this Journal claiming 1-function spherical solutions to Born-Infeld gravity (BIG) models [5]. It is not even necessary to delve into its numerical calculations; the assertion that its single field equation is solved exactly by any monomials r^n , $n = 0, 1, 2, 3$, and that these solutions also solve the full field equations is already false: as is trivially checked, r^3 does *not* solve the original field equations. Neither r^0 nor r^1 are solutions to even the single field equation (14) because an arbitrarily discarded denominator vanishes there. Only r^2 , which represents flat space and does solve BIG, is a solution to both the 1-function and 2-function field equations (but only once the incorrect action (11) is properly rewritten ²).

We need only conclude with the following immediate disproof: BIG contains General Relativity as the first term in its expansion [5]

$$I = \int d^4x \left(\sqrt{-|g_{\mu\nu} + \lambda R_{\mu\nu}|} - \sqrt{-g} \right) \rightarrow \frac{\lambda}{2} \left(\int d^4x \sqrt{-g} R \right) + O(\lambda^2). \quad (1)$$

But we have just seen that the ansatz of [1] misses this term altogether, since it becomes a total divergence (as demonstrated in [3]).

Acknowledgements

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References

- [1] M. N. R. Wohlfarth. Gravity à la Born-Infeld. *Class. Quant. Grav.*, 21:1927, 2004, hep-th/0310067.

¹The few cases that permit this process, such as removal of a constraint variable F , show how restrictive it is: F (but not ϕ !) may be eliminated from $L = F^2 - 2FX(\phi)$, where X is a function of ϕ and its derivatives, by completion of squares.

²The form of (11) does not even correspond to its antecedent (8) because $\sqrt{-g}$ was improperly distributed. This leaves the equation (14) for $D(r)$ unchanged, but alters the ones for the original two components.

- [2] R. S. Palais. The principle of symmetric criticality. *Comm. Math. Physics*, 69:19, 1979.
- [3] S. Deser and B. Tekin. Shortcuts to high symmetry solutions in gravitational theories. *Class. Quant. Grav.*, 20:4877–4884, 2003, gr-qc/0306114.
- [4] S. Deser and J. Franklin. Schwarzschild and Birkhoff a la Weyl. 2004, gr-qc/0408067.
- [5] S. Deser and G. W. Gibbons. Born-Infeld-Einstein actions? *Class. Quant. Grav.*, 15:L35–L39, 1998, hep-th/9803049.