SERVICIZING AS AN ALTERNATIVE TO SELLING FOR A DURABLE GOODS MANUFACTURER

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ABSTRACT

SERVICIZING AS AN ALTERNATIVE TO SELLING FOR A DURABLE GOODS MANUFACTURER

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Servicizing is the activity of selling the services provided by the product rather than the product itself. It has been considered as an alternative business model that might be environmentally superior to the conventional selling business model. Servicizing promises pooling of consumer use and products with higher durability. However, it can also inflate demand/consumption and result in a bigger environmental impact overall. This thesis compares selling and servicizing business models for a monopolist durable-goods manufacturer. Durability and price decisions of the firm, and usage level decisions of consumers are modeled as endogenous. Under this setting, we compare the profitability and environmental performance of the two business models. We find that servicizing business model leads to more durable product design. We identify the conditions which enable servicizing to be superior regarding profitability and environmental impact.

Keywords: Servicizing, Environmental impact, Profitability, Durability

ÖZ

DAYANIKLI MAL ÜRETİCİSİ İÇİN SATIŞA ALTERNATİF OLARAK HİZMETLEŞTİRME

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Hizmetleştirme, ürünün kendisi yerine hizmetinin satılması faaliyetidir. Bu faaliyet, geleneksel satış iş modeline kıyasla hem çevresel etki yönünden hem de karlılık yönünden daha üstün bir iş modeli olarak görülmektedir. Hizmetleştirme, tüketici kullanımını ortaklama ve daha dayanıklı ürünler vaat eder. Ancak talep ve tüketimi arttırarak toplamda daha büyük bir çevresel etkiye sebep olabilir. Bu çalışma, dayanıklı mal üreticisi bir tekelci firma üzerinden satış ve hizmetleştirme iş modellerini kıyaslamaktadır. Firmanın ürün dayanıklılığı ve fiyat kararı ile tüketicilerin kullanım miktarı kararları endojen olarak modellenmiştir. Bu kurulum ile iki iş modelinin karlılığı ve çevresel etki performansı kıyaslanmaktadır. Hizmetleştirme iş modelinin daha dayanıklı mal üretimine sebep olduğuna ulaşılmış, hizmetleştirmenin karlılık ve çevresel etki yönünden üstün olduğu koşullar belirlenmiştir.

Anahtar Kelimeler: Hizmetleştirme, Çevresel etki, Karlılık, Dayanıklılık

To my family

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CHAPTER 1

INTRODUCTION

In recent years, environmental concerns have been growing across the globe due to drained natural resources, increased carbon emissions and the consequent climate change threat. These concerns channel researchers' interests into discovering business models that have environmentally better outcomes. One fundamental change that is discussed is the transition from a linear economy, where a product is manufactured, sold and consumed, and delivered to landfill at end-of-life, to a circular economy where the product is designed, sold and consumed for reuse, recycle and/or remanufacture. To facilitate this transition to a circular economy, alternative business models are discussed and analyzed (Stahel, W. R. 2016, Agrawal et al. 2018).

Currently, servicizing, as a different form of leasing, has emerged as an alternative business strategy in a number of industries. In a leasing strategy, the firm essentially sells the availability of the product for a certain duration. Customers who have leased the product are charged for the duration, which is relatively long, regardless of their usage amount. Commonly, operating cost is also paid by the customer under a leasing business model. On the other hand, in a servitization or servicizing strategy, the firm charges the customers per use or per duration of use. For example, a servicizing firm prices usage of a car by each mile driven or each hour of usage while covering gas, maintenance and insurance costs. In a servicizing business model, charged duration might be very short (per hour or minute) while this period is generally long in a leasing business model (per day). Under servicizing business model, all costs related with usage, such as maintenance and operating costs, are generally incurred by the firm. Customers only pay a service fee.

Especially in IT businesses, many firms are turning into service providers rather than hardware sellers. This transition can be observed in major firms like IBM, Apple, Asus and several others that increasingly promote cloud services rather than hardware sales. According to Gartner Inc., cloud services market is expected to grow more than 50% by 2022 (Gartner, 2019). In addition, a number of other major industries contain significant applications of servicizing. For example, Bridgestone servicize its tires under a program called "Mileage Sales" where it charges customers by per mile driven (Bridgestone, 2019). Xerox, apart from selling printers, offers a service called "PagePack" in which customers are charged by per page printed (Xerox, 2010). In warewashing industry, Winterhalter provides customers with pay per wash service (Winterhalter, 2019). In transportation industry, ZipCar (ZipCar, 2019) and Car2Go (Car2Go, 2019) enables customers to drive private vehicles without customers' ownership. Compared to conventional leasing firms, they allow customers to pay for very short durations and distances covering all costs which converges their business strategies to servicizing.

Servicizing has been recognized as a potentially greener alternative by important organizations (White et al. (1999), Fischer et al. (2012)). It is argued to have better environmental features compared to traditional selling of a durable good. First, compared to selling, or leasing , servicizing is capable of enabling pooling of consumer demand on few products rather than satisfying each consumer demand with a separate product. It may lower production and natural resource extraction. Second, due to firm's ownership of the product and performance-based profit, it is suggested that firms will design products of higher durability under a servicizing business model. Third, operating efficiency is expected to be higher in a servicizing firm's product since operating cost is born by the firm. Lastly, considering recent developments in data science, IoT and sensor technology; servicizing offers firms an important data analysis opportunity that may lead to significant cost saving advantage.

To investigate the validity of these arguments, we study and compare the servicizing and selling business strategies for a monopolistic manufacturer of a durable product. We evaluate the two business models in terms of profitability, the durability decision, and the eventual profit and environmental performance. In our problem setting, durability of a product represents the maximum usage a product can endure before reaching its end of life. In other words, we define durability as the cumulative usage limit in a product lifetime. For a product of given durability, we study the firm's pricing and the customer's use/purchase decisions under both selling and servicizing models. Customers are heterogeneous and determine their usage based on pricing and/or the durability of the product. We later characterize the durability choice of the manufacturer under each model and assess the periodic environmental impact that follows.

In our results we can analytically characterize the conditions of each strategy's relative profitability, environmental impact, optimal price/service fee and total use amounts for a given durability level. We can also provide with analytical results of optimal durability decision under servicizing. We conduct numerical analysis to find the optimal durability choice of a selling firm. Through numerical comparisons we find that servicizing firm always has greater product durability.

We find that compared to selling, servicizing tends to be more profitable when product related costs are high. However, for such high cost products, servicizing is not a more profitable option when customer valuation differences are also high in the market. We also see that in most cases, servicizing is environmentally a better choice. Selling tends to be a greener business model when product related costs are at an intermediate level and high segment customer proportion is low.

This thesis also shows the conditions when servicizing is more profitable and greener. Our findings suggest that servicizing cannot be both greener and more profitable alternative for low cost products. For high cost products, selling cannot reach the same equilibrium.

In the next chapter, we summarize the literature related to this study. Chapter 3 and 4 gives the details of the modeling for servicizing and selling business models. Then, in Chapter 5, we analytically compare the profitability and environmental impact of the two alternative business models for a given durability level and we numerically check and discuss the results. Next, taking durability as endogenous, we conduct numerical experiments for the comparisons regarding durability, profitability and environmental performance. Finally, in Chapter 7, we summarize main findings of this thesis and specify related insights. All proofs are available in the Appendix of the study.

CHAPTER 2

LITERATURE REVIEW

Servicizing emerged from the idea of "leasing" that was proposed as an alternative to selling in the durable goods market. Initial studies compare leasing with selling in terms of various factors such as "time inconsistency," manufacturer's profitability, and product durability.

Bulow (1982) studies the leasing business model to investigate if the "time inconsistency" problem is eliminated. Time inconsistency, as coined by Coase (1972), is the issue of unstable pricing that arises when durable goods are sold by a monopolist. The problem arises when the monopolist firm, starting from selling the durable goods with the monopoly price, prefers to continue producing and selling more products. As production continues, prices are decreased until the price reaches the competitive market level. Rational customers realizing or anticipating this price decrease wouldn't want to pay more than the competitive price level. It is therefore possible for the firm to lose the monopoly power. Bulow (1982) uses a two-period model where in each period the firm determines production quantity. The products produced in the first period remains in the market in the second period. The market demand is linear and decreasing in price. The study compares three cases: competitive market, selling monopolist and renting monopolist. Bulow (1982) proposes that leasing eliminates the time inconsistency problem and the leasing monopolist earns more profit than the selling monopolist. Bulow (1982) studies leasing, and is focused on the time inconsistency aspect observed in a multi-period setting. Our study, however, focuses on servicizing, and is more concerned with the environmental and economic comparisons of the two business models.

In the search of more profitable business models for durable goods, Desai and Purohit

(1998) and Bashkaran and Gilbert (2005) study selling and leasing business models. Similar to Bulow (1982), to capture the characteristics of a durable good market, both studies use a two-period model at the end of which, product lifetime ends.

In Desai and Purohit (1998), the monopolist firm determines the product quantity and the consequent price for two periods while customers choose their two-period strategies. Desai and Purohit (1998) analyze the effect of different depreciation rates for selling and leasing by exogenously inserting depreciation in the consumer utility model. Considering different depreciation rates also implicitly enables the study to comment on the quality choice of the manufacturer firms. Contrary to common view, the study suggests that the optimal behaviour for the firm is selling/leasing low quality products to consumers with high valuation and high quality products to consumers with low valuation. Authors also suggest that concurrent leasing and selling is optimal for the manufacturer.

Bashkaran and Gilbert (2005) create a setting of two monopolist firms manufacturing a durable good and its complementary product separately. In this setting, besides the production quantity decisions, the durable good manufacturer also decides the fraction of products allocated for leasing. Influenced by the complementarity between products, customers determine purchase quantity of the complementary product and they decide their best strategy for two periods such as lease-lease, lease- do not use, buy - hold and etc. The authors investigate whether pure selling, pure leasing, or a hybrid approach is optimal for the durable good manufacturer when there are complementary products in the market. They find that strong complementarity may change the manufacturer's preference from leasing to selling.

Bashkaran and Gilbert (2009) also do a similar comparison of leasing with selling when there exist intermediaries between manufacturers and consumers. In a market with uniformly distributed customer valuations, the study compares the two forms of setting with intermediaries: in the first one, the manufacturer sells to the dealer which can either sell or lease it; in the second one, while retaining the ownership of the product, the manufacturer brokers the product to the dealer for leasing and charges a margin for each product leased. The manufacturer decides to broker or sell to the dealer in each period, while in return the dealer determines the quantities that are released to the market. Authors also analyze the case where there are multiple competitive dealers, where dealer-specific wholesale prices are applied. The study proposes that when the number of dealers is high, lease brokering would be more attractive to the manufacturer. When there is only one dealer, manufacturer would prefer selling to the dealer which in turn leases the product.

Desai and Purohit (1998) and Bashkaran and Gilbert (2005, 2009) commonly study the profitability of selling and leasing business models. By adopting the two-period model used by Coase (1972) and Bulow (1982), in these studies, durability is mainly considered for the purpose of capturing the time inconsistency, and the product lifetime is fixed. This thesis, however, does not take into account the time inconsistency problem, but compares servicizing with selling by extending it to the durability decision, and the environmental and economic consequences that follow for each model.

In more recent works, besides profitability, selling and servicizing are also compared in terms of environmental performance. Selling is first compared with leasing in Agrawal et al. (2012). Later, Avci et al. (2015), Bellos et al. (2016), Agrawal and Bellos (2016) and Orsdemir et al. (2018) focus on the selling and servicizing comparison.

Agrawal et al. (2012) characterize the conditions which guarantee that pure leasing option is environmentally superior to pure selling. In a setting with a single durable product is sold or leased by a monopolist firm, the authors develop a dynamic sequential game of infinite time horizon. The product lasts for two periods. Products in the first and the second (last) periods of their lifetime are categorized as new and old respectively. In each period, the product can be sold or leased for a period. Independent from the modeled lifetime of the product (i.e., two periods), durability is exogeneous and only defined as a factor of consumer's willingness to pay. Old products are valued less because they have lower durability compared to new products. While new product valuations are the same, old products have different valuations under selling and leasing due to different durability levels. The authors explain this difference by pointing to depreciation differences under each strategy. Each period, after the firm determines the quantities of products to be sold (or leased), consumers decide to purchase (or lease) based on the prices which is the consequence of manufactured quantity. Environmental impact comprises of three phases in a product's lifetime; i.e, production phase, use phase and disposal phase, is inserted into the model. The study concludes that unlike the common belief, leasing may not be environmentally preferable in many cases.

While durability is exogenous in the model, through numerical analysis, authors argue that leasing leads to higher durability levels. Agrawal et al. (2012) models the durability only as a factor of willingness to pay. On the other hand, in this thesis, we define durability as a lifetime usage capacity of a product which enables us to emphasize the relation between usage level and production volume. While the use phase environmental impact is scaled by the sales (or lease) quantity in Agrawal et al. (2012), we scale the use phase impact with the aggregate usage amount.

Agrawal and Bellos (2016) examine pay-per-use pricing systems and analyze how servicizing can be economically and environmentally preferable. A sequential game is designed to compare servicizing and selling. First, the monopolist manufacturing firm determines the operating efficiency (i.e., energy or resource consumption) of the product and which business model(s) to adopt. The firm may choose pure selling, pure servicizing or a hybrid model. While choosing the business model(s), the firm also determines the sales price or servicizing fee depending on the choice. Observing the outcomes of the firm decisions, consumers decide on their options offered by the firm which can be servicizing, purchasing or remaining inactive. Heterogeneous customers also determine their usage level that will maximize their utility. As product efficiency increases marginal production cost increases while marginal operating cost decreases. Regardless of the business model chosen, customers always incur the operating cost. Under servicizing, multiple customers may share the same product to satisfy their needs which is defined as "pooling" in the study. Two phases of the product life cycle is considered in the environmental impact model: production phase and use phase. While the environmental impact due to production is scaled by the production quantity, unlike Agrawal et al. (2012), the environmental impact due to usage is scaled by the usage amount. The study suggests that under strong pooling; a hybrid model, where servicizing and selling are both available options for customers, is environmentally preferable to a pure sales model. Another important finding is that while under strong pooling, servicizing causes more efficient products to be designed; under no pooling, it causes lower efficiency. Authors propose the conditions for servicizing's environmental success over selling.

This paper, as in this thesis, focuses on a manufacturer's design decision, pricing, and consequently customers' usage amount of the product. In this framework, they compare the two business models in terms of environmental performance and profitability as in this thesis. Our study is similar to Agrawal and Bellos (2016) also in the way environmental impact is modeled. The main difference is that the design choice in the paper is the operating efficiency of the product whereas we focus on a product's durability; i.e., its total use capacity. Therefore, unlike our work, Agrawal and Bellos (2016) takes durability as exogenous. Another different aspect is that we only consider pure selling or servicizing models while Agrawal and Bellos (2016) also considers hybrid business model.

Orsdemir et al. (2018) also study environmental and economic aspects of servicizing. Unlike Agrawal and Bellos (2016), under servicizing, the firm offers service contract options rather than using a pay-per-use pricing structure. In this business model, the firm determines the product durability level and offers two types of service contracts that are options of low and high use duration. The firm bears the operating cost under the servicizing business model. Customers then decide on taking one of the service contracts or not receiving any service. Under the selling business model, the firm determines the selling price along with the durability level while customers decide whether to purchase or not. If they purchase, customers determine their usage duration considering operating cost per use duration. Environmental impact is modeled as it involves three phases of the lifetime of a product: use, production and disposal phases. However, as opposed to Agrawal and Bellos (2016), environmental impact of the use phase is defined in terms of the duration that product is used for rather than the usage amount.

The findings suggest that when environmental impact of usage is high for the product, servicizing is both more profitable and more environmentally friendly only if consumer valuation is more homogeneous and the firm's operating cost is higher compared to the customers'. For the low use impact products, this result can be achieved when the firm's operating cost is relatively lower. Orsdemir et al. (2018) also argue that when the same market segments are served under both business models, servicizing leads to higher durability levels. When more consumer segments are served under servicizing, it leads to lower durability.

To the best of our knowledge, Orsdemir et al. (2018) is the only study that endogenizes durability choice in the context of environmental comparison of servicizing and selling business strategies. This thesis, also, studies the durability choice differences under the two business models. However, in terms of durability, there are two key differences between our work and Orsdemir et al. (2018). First, our durability definition is directly linked to the usage amount; i.e., the usage behaviour of consumers determines how long the product endures. Since Orsdemir et al. (2018) does not relate durability with usage, it bases the analysis for only one generation of products released to the market. Our way of modeling durability allows us to consider the link between the firm's durability choice and the periodic manufacturing volume. Second, in our study, besides its operating cost effects, customers take durability into account in their purchasing decision as it represents the product's useful time period. In Orsdemir et al. (2018), durability does not have a direct effect on the purchasing decision. While deciding on purchasing, customers evaluate durability indirectly, through its influence on the maintenance and operating cost.

In the context of transportation, Avci et al. (2015) and Bellos et al. (2017) study servicizing practices in the automobile industry as an alternative to selling. Both studies cover environmental and economic analysis of servicizing.

Avci et al. (2015) investigate the environmental performance of a proposed system for electric vehicle usage in which batteries are servicized by the firm rather than sold as installed in a vehicle. With the consideration of a broad network of battery switching stations in the model, drivers can switch their depleted batteries with the charged ones paying for the miles driven. Ownership of the batteries are retained by the firm. Avci et al. (2015) analyze the effects of battery switching systems on the adoption of electric vehicles, total miles driven and eventually total emissions due to fossil fuel consumption. The study makes comparisons with the current fossil fuel vehicle system as a base case. For the battery switching station case, while customers (vehicle owners) decide their best option to drive and their daily milage, the firm determines the battery stock quantity, fixed fee and a per-mile fee that will be charged to the customers. Authors suggest that per mile driven pricing system (servicizing) with battery-switching stations increases the electric vehicle adoption. However, the study remarks that depending on the electricity grid mix of the country, the battery-switching system can cause increased emissions since it leads to more usage.

Bellos et al. (2017) study an automobile manufacturer company's decision to introduce servicizing business model via a car sharing program. Besides evaluating the effectiveness of an environmental regulation, the study also investigates whether a servicizing model like a car sharing program is more profitable and more environmentally beneficial. Parallel to the related literature, environmental impact is modeled covering the production and use phases. In the model, the manufacturer firm decides either to servicize its cars by a car sharing program along with the selling business or continuing to a pure sales business. In a Stackelberg game, the firm first decides whether to introduce a car sharing program with consequent decisions of fuel efficiency and price (and usage fee if necessary). Customers, who have differing valuations of driving performance, decide their mobility options: ownership, car sharing service or outside option. The study shows that if the firm introduces the car sharing option, low segment consumers always prefer car sharing while the high segment prefers owning a car. The firm then can discriminate the price and fuel efficiency for different consumer segments. Efficient cars are used for servicizing while less efficient ones are purchased by the high segment customers due to their valuation of driving performance. The study shows that due to the pooling effect ,servicizing leads to a relatively low number of efficient cars in the fleet of a firm that manufactures less efficient cars for selling. This can cause worse CAFE (Corporate Average Fuel Economy) levels since fuel efficient cars will be highly outnumbered by the less efficient ones. It is argued that car sharing can be environmentally superior; however, CAFE regulations disincentivize it.

As Orsdemir et al.(2018), Bellos et al. (2017) consider two type of customers with high and low product valuation, this thesis also presents customer heterogeneity in the same way. While Bellos et al. (2017) and Avci et al. (2015) focus on the automobile industry, our model represents a durable good from a wider context. Additionally, we consider the implied durability decisions and differences that may arise between the two business models. These studies do not consider durability differences between the two business models.

CHAPTER 3

SERVICIZING BUSINESS MODEL

In this chapter, we introduce the details of the servicizing business model and present the related analysis.

3.1 Model Details

We consider a monopolist firm producing a single durable product. The firm can either servicize or sell its product to a market populated with M customers. We assume that the customers are heterogeneous in their usage needs which makes their valuations of usage different. As Orsdemir et al. (2018) and Bellos et al. (2016), we model the heterogeneity with two types of customers: customers with high and low usage needs. Proportion of high segment customers in the population is defined as $\beta \in (0,1)$. Therefore, the number of high segment customers is βM, while the number of low segment customers is $(1-\beta)M$. It is assumed that the market size M does not change in time.

For one unit of usage amount (denoted by q), customer type $i = H, L$ gains a gross utility of θ_i where $\theta_H > \theta_L$. Due to different usage needs, two customer types yield different utilities, that we denote by $U_i(q)$, from using the durable product by an amount of q for one period. Similar to Agrawal & Bellos (2016) and Orsdemir et al. (2018), marginal consumer utility is modeled as decreasing in usage amount q. This approach reflects the fact that consumers wouldn't want to use a durable product infinitely many times. For example, considering a fixed period of time, independent of the financial costs, after a certain mileage or duration, traveling or driving will cause disutiliy due to human conditions. Under servicizing, covering all of the costs, the firm charges customers a usage fee f for a unit of usage. Customers using the product q amounts, will be charged qf.

Net customer utility depends on the type of the customer, i.e. usage need θ_i , which could be high or low (θ_H, θ_L), the usage amount of the customer in a period and the usage fee determined by the firm. Customers of type $i = H, L$ decide their per period usage level q_i by maximizing their per period utility as defined below:

$$U_i(q) = \theta_i q - \frac{1}{2}q^2 - qf \quad \text{for } i = H, L \tag{3.1}$$

This formulation reflects diminishing marginal utility since $U' = \frac{dU}{dq}$ is decreasing in q. To analyze usage level in the market, we define aggregate usage Ω as the total usage level of all customers for a period.

As a product gets more durable, it enables more usage until the end of its lifetime. Therefore, we define durability δ as the maximum amount of usage a product can offer before it reaches end of life. In other words, after δ amount of usage, the product can no longer be used. This definition also implies that when perodic usage amount is high, it leads to a shorter product lifetime. There is a cost of operating and maintaining the product which is borne by the firm under the servicizing business model. Considering limited amount of usage a durable product offers, this cost can be defined in terms of per use. While an operating cost is incurred each time the product is used, a maintenance cost is also accumulated to be charged later. Regular maintenance of a car that is repeated after a certain amount of mileage can be regarded as an example for usage-dependent maintenance cost. Accordingly, as a per use cost, we define operating and maintenance cost with a single term $m(\delta)$. We model this cost as a function of durability δ. Considering that maintenance is generally the activity of component replacement, it is reasonable to assume that as products get more durable, having more durable components, they require less maintenance for the same amount of usage. Operating cost, on the other hand, may or may not be influenced by the durability design choice. In the scope of our study, we consider durable products, operating cost of which does not increase with higher durability. Thus, similar to Orsdemir et al. (2018), we define $m(\delta)$ as a decreasing function of δ.

We now formulate the characteristics of the firm profit. In a servicizing business model, per period revenue of the firm depends on per period aggregate usage Ω and the usage fee f.

Taking aggregate usage amount into account, on average, the firm produces Q units for a period each of which can endure δ amount of usage.

During manufacturing, the firm incurs a manufacturing cost $c(\delta)$ for each product. Since it is more costly to produce a more durable product, parallel to the literature, $c(\delta)$ is an increasing function in δ which can simply be defined as $c(\delta) = c\delta^2$. Thus, per period profit of a servicizing firm consists of the elements below.

$$\text{Revenue} = \Omega f$$
$$\text{Manufacturing Cost} = Qc(\delta)$$
$$\text{Maint. and Op. Cost} = \Omega m(\delta)$$

Below table summarizes the notation used throughout the study:

Table 3.1: Notation

Variables	**Description**
f	Fee per usage
δ	Maximum usage amount a product can endure
Q	Number of products produced per period
q	Per period usage amount of a customer
Parameters	**Description**
θ_L, θ_H	Gross marginal utility from usage for a customer of type H, L
M	Number of users
β	Fraction of customers with high usage need
$m(\delta)$	Unit maintenance and operating cost
$c(\delta)$	Unit manufacturing cost

Setting of the model is as follows: Monopolist firm, aiming to maximize its profit, sets the durability level δ and then the usage fee f. Observing firm decisions, customers with different usage needs determine per period usage amount q (possibly zero) to maximize their utility.

3.2 Analysis

By backward induction, we first analyze the customer response to the decisions of the monopolist firm. While deciding the usage level, utility function in 3.1 is maximized by each customer. The next lemma shows the characteristic of the usage behaviour.

Lemma 3.2.1. *For a given durability level δ, and a usage fee f, the optimal usage amount of customer type i under servicizing is as follows:*

$$q_i^*(\delta, f) = (\theta_i - f)^+ \tag{3.2}$$

The customers will prefer to use the product when their optimal usage level is positive. This decision depends on the service fee charged by the firm and the usage need of consumer segments. As in Agrawal & Bellos (2016), Lemma 3.2.1 shows that the optimal usage amount of a customer cannot exceed the usage need θ_i even when the service fee is zero. Knowing individual usage levels, we can also define total usage amount in the market.

Lemma 3.2.2. *The aggregate usage under servicizing business model is as follows:*

$$\Omega = \begin{cases} (\beta\theta_H + (1-\beta)\theta_L - f)M & \text{if } \theta_L \geq f \\ \beta(\theta_H - f)M & \text{if } \theta_H \geq f > \theta_L \\ 0 & \text{if } f > \theta_H \end{cases}$$

The lemma shows how the monopolist firm, by determining its usage fee f, determines whether it will serve to both low and high customer segments or to the high segment only. When usage fee is low enough, both customer segments derive utility from using the product. As fee goes higher, the firm loses potential customers which shows that the market demand curve for usage is decreasing as expected.

The market coverage decision and the consequent aggregate usage amount have an impact on Q, the product quantity needed per period. From the firm's perspective, anticipating the usage need of consumers provides the firm with information about how much product lifetime is going to be implied from a customer usage level. Note that with our definition of durability, product lifetime depends on the durability; i.e., the maximum amount of usage a product can endure, and the usage amount the product is exposed to. Considering these factors, the following lemma defines the formulation of per period product quantity needed Q.

Lemma 3.2.3. *In a servicizing business model, the monopolist firm on average manufactures $\frac{\Omega}{\delta}$ units per period.*

$$Q = \frac{\Omega}{\delta} \tag{3.3}$$

Taking customers' usage level decisions and the consequent aggregate usage into account, the profit function of the firm can be defined as:

$$\Pi_{serv} = \begin{cases} M(\theta_{avg} - f)(f - \delta c - m(\delta)) & \text{if } 0 < f < \theta_L \\ \beta M(\theta_H - f)(f - \delta c - m(\delta)) & \text{if } \theta_L \leq f < \theta_H \\ 0 & \text{if } \theta_H \leq f \end{cases}$$

where $\theta_{avg} = \beta\theta_H + (1 - \beta)\theta_L$.

Throughout the study, we will use θ_{avg} as a short-cut expression to refer to the weighted average $\beta\theta_H + (1 - \beta)\theta_L$; i.e., the average usage need of customers in the market.

Based on the profit function defined above, we can determine the optimal service fee for the firm.

Proposition 3.2.4. *In a servicizing business model, for a given durability level, δ, the firm decides the profit maximizing service fee as follows:*

$$f^* = \begin{cases} \frac{\theta_{avg}+m(\delta)+c\delta}{2} & \textit{if} \quad m(\delta) + c\delta \leq \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \\ \frac{\theta_H+m(\delta)+c\delta}{2} & \textit{if} \quad \theta_L - \sqrt{\beta}(\theta_H - \theta_L) < m(\delta) + c\delta < \theta_H \\ [\theta_H, \infty) & \textit{if} \quad m(\delta) + c\delta \geq \theta_H \end{cases}$$

Corollary 3.2.5. *For a given durability level, the servicizing firm's decisions can be summarized as below:*

Table 3.2: Equilibrium results for a given durability level under servicizing

Equilibrium Results			**Conditions**
The firm serves to	f^*	Ω^*	$m(\delta)+c\delta$
both segments	$\frac{\theta_{avg}+m(\delta)+c\delta}{2}$	$M\frac{\theta_{avg}-m(\delta)-c\delta}{2}$	$\left(0\ ,\ \theta_L-\sqrt{\beta}(\theta_H-\theta_L)\right]$
only high segment	$\frac{\theta_H+m(\delta)+c\delta}{2}$	$M\beta\frac{\theta_H-m(\delta)-c\delta}{2}$	$\left(\theta_L-\sqrt{\beta}(\theta_H-\theta_L)\ ,\ \theta_H\right)$
no segments	$[\theta_H,\infty)$	0	$[\theta_H,\infty)$

The firm's market coverage decisions hinge on three factors; marginal cost of usage $(m(\delta)+c\delta)$, proportion of high segment population β, and the usage need heterogeneity of the customers $(\theta_H-\theta_L)$. When the marginal cost of usage is higher than the usage need of high segment customers θ_H, the firm refuses to serve to the market since it is not profitable. When it is in an intermediate range the firm serves only to the high segment. When this cost is low enough, the firm covers the whole market.

Corollary 3.2.5 also demonstrates results related to the relative customer segment population. As high segment customer proportion β increases, the firm gets more inclined to serve only to the high segment. However, when marginal cost of usage is less than $\theta_L-(\theta_H-\theta_L)$, the firm serves to the whole market regardless of the high segment population. At that cost level, it is not preferable to lose even a single customer due to the high profit margin. We can also observe that when the marginal cost exceeds θ_L; i.e., the usage need of low segment customers, the firm will no longer serve to the low segment customers regardless of the low segment population. In that case, low segment customers cannot derive utility due to the high usage fees caused by high usage related costs. The relative population of customer segments is important to the market coverage decision only when marginal cost of usage is less than θ_L and higher than $\theta_L-(\theta_H-\theta_L)$. When $m(\delta)+c\delta$ is in that interval, the firm will determine its market coverage based on the high segment proportion β.

The effect of consumer heterogeneity $(\theta_H-\theta_L)$ on market coverage decision is exposed within the conditions of Corollary 3.2.5. As consumer heterogeneity increases the firm will be more inclined to serve only to the high segment. It is important to point out that the effect of consumer heterogeneity is scaled by the market's high segment proportion β. When high segment proportion gets close to zero; i.e., the low segment dominance is extreme in the market, consumer heterogeneity has very little effect. Here, both consumer heterogeneity and high segment proportion factors translated as the value for forgoing the low segment customer to charge a higher fee and serve to the high segment only.

Proposition 3.2.6. *For $m(\delta) = \frac{m}{\delta}$, a servicizing firm will choose the optimal durability level as below:*

$$\delta^* = \begin{cases} \sqrt{\frac{m}{c}} & \text{if} \quad 2\sqrt{mc} < \theta_H \\ (0, \infty) & \text{if} \quad 2\sqrt{mc} \geq \theta_H \end{cases}$$

Corollary 3.2.7. *For $m(\delta) = \frac{m}{\delta}$, servicizing firm's market coverage, durability and service fee decisions and its resulting profit are as below:*

Table 3.3: Equilibrium results with optimal durability level under servicizing

Decisions			**Conditions**	**Profit**
The firm serves to	δ^*	$f(\delta^*)^*$	$2\sqrt{mc}$	
both segments	$\sqrt{\frac{m}{c}}$	$\frac{\theta_{avg}}{2} + \sqrt{mc}$	$\left(0 \, , \, \theta_L - \sqrt{\beta}(\theta_H - \theta_L)\right]$	$M\frac{(\theta_{avg}-2\sqrt{mc})^2}{4}$
only high segment		$\frac{\theta_H}{2} + \sqrt{mc}$	$\left(\theta_L - \sqrt{\beta}(\theta_H - \theta_L) \, , \, \theta_H\right)$	$M\beta\frac{(\theta_H-2\sqrt{mc})^2}{4}$
no segments	$(0, \infty)$	$[\theta_H, \infty)$	$[\theta_H, \infty)$	0

It can be noticed that the optimal durability decision of a servicizing firm does not directly depend on the market coverage level. The firm decides on the optimal durability level only considering the product cost parameters under both full and partial market coverage conditions. It might be a result of the profit generating mechanism of servicizing. Under servicizing, durability is not directly a part of product valuation for customers. Customers do not pay higher fees for a more durable product because it allows more usage. Therefore, the firm's focus is only on the product related costs which also determines usage fee. By evaluating the product cost parameters, the firm decides the durability level that will maximize its marginal profit per use. This rule does not change under different market coverage levels.

Also, the formulation of optimal durability level shows that the durability level increases when m increases. Since $m(\delta) = \frac{m}{\delta}$, it can be argued that the firm increases durability to balance the unit maintenance and operating cost. Similarly, since unit manufacturing cost is $c(\delta) = c\delta^2$, an increase in c will lead to decrease in δ in order to reduce the effect of cost inflation.

CHAPTER 4

SELLING BUSINESS MODEL

4.1 Model Details

Unlike under servicizing, in a selling business model the product has a price for ownership. When customers purchase the product, it allows them to use the product, until it expires at total usage of δ, without any extra fee. However, due to the ownership, customers are the ones that incur the maintenance and operating cost $m(\delta)$. Thus, customers' per period utility is defined as below:

$$U_i = \theta_i q_i - \frac{1}{2} q_i^2 - q_i m(\delta) \qquad \text{for } i = H, L \tag{4.1}$$

Customers pay purchasing price p in return of the product ownership. The price paid for the product is not included in the consumer utility because once the product is purchased, p becomes a sunk cost which does not affect the usage decision or derived utility. The price is evaluated by the customers while deciding on purchasing.

We assume that under selling, each product is purchased and used by a single customer only. This assumption emphasizes the distinction of servitization's product sharing feature for the purposes of our analysis.

Revenue of the firm depends on the number of products produced per period Q, and the selling price p. Each product is dedicated to one customer, total amount of products to be produced may be as high as the market size M. Customers do not purchase a new product until the old one's lifetime is completely exhausted. Note that, all customers in the market do not necessarily purchase at the same time. Per period profit function of a firm can be defined as below:

$$\Pi = Q(p - c(\delta)) \tag{4.2}$$

Under a selling business model, the firm, aiming to maximize its profit, first decides the durability level, δ, of its product. Then, it decides how much selling price p to ask for the product.[1] Customers, observing the durability and price levels, decide whether or not to purchase the product. They consider their utility maximizing usage levels, which will be realized each period if the product is bought, to determine if the product is worth purchasing or not. The details of the customer decisions are explained in the next section.

4.2 Analysis

Under this setting, customers determine their optimal usage decisions that will maximize their utility. This decision depends on the usage needs and maintenance & operating cost covered by the customer. Next lemma shows the formulation of corresponding usage behavior.

Lemma 4.2.1. *A customer of type i, who purchases the product with durability δ, determines the optimal usage level as:*

$$q_i^* = (\theta_i - m(\delta))^+ \qquad \textit{for } i = H, L \tag{4.3}$$

In a selling business model, to make a buying decision, customers try to anticipate the utility they will derive throughout the lifetime of the durable product. Customers' expected lifetime period for a durable product depends on their per period usage amount q^* and the product durability δ. Therefore, anticipated lifetime of the durable product is defined as $\alpha(\frac{\delta}{q_i^*})$ for $i = H, L$. Since the anticipation may not be exact, a coefficient of $\alpha \in (0.5, 2)$ is defined.

Anticipating the utility to be derived during the whole lifetime of the product, customers evaluate if the price is worth paying or not. Customers purchase the product if the lifetime utility of a product is greater than the selling price.[2]

$$U_i^*(q_i^*)\frac{\alpha\delta}{q_i^*} \geq p \tag{4.4}$$

Note that the coefficient α reflects the myopic approach of customers regarding the durability period and serves as a discounting factor for future utility as well.

[1] Note that the price is restricted to be nonnegative in the model.

[2] Note that when the periodic usage amount q_i^* equals zero, the customer will automatically reject purchasing.

Lemma 4.2.2. *Customer segment $i = H, L$ purchases the product if*

$$\frac{\alpha\delta(\theta_i - m(\delta))}{2} \geq p \tag{4.5}$$

Based on the purchasing conditions of each customer segment, we can now formulate the total usage amount in the market. Depending on the price offered, aggregate usage amount in a selling business model can be characterized as below:

Lemma 4.2.3. *The aggregate usage under the selling business model is as follows:*

$$\Omega = \begin{cases} (1-\beta)(\theta_L - m(\delta))M + \beta(\theta_H - m(\delta))M & \text{if } \frac{\alpha\delta(\theta_L - m(\delta))}{2} \geq p \\ \beta(\theta_H - m(\delta))M & \text{if } \frac{\alpha\delta(\theta_H - m(\delta))}{2} \geq p > \frac{\alpha\delta(\theta_L - m(\delta))}{2} \\ 0 & \text{if } p > \frac{\alpha\delta(\theta_H - m(\delta))}{2} \end{cases}$$

Lemma 4.2.3 summarizes the firm's market coverage conditions with respect to price. When the price is low enough, the firm will sell both type of customers. When it is in an intermediate range, the firm will sell only to the high segment since price is too high for low segment customers. When the price is above a certain limit, the firm cannot sell to any customer. It can also be observed that under a selling business model aggregate usage levels do not depend on price level since individual usage levels are independent of the purchasing price. Next lemma characterizes the per period production quantity.

Lemma 4.2.4. *In a selling business model, the monopolist firm on average manufactures $\frac{\Omega}{\delta}$ units per period.*

$$Q = \frac{\Omega}{\delta} \tag{4.6}$$

The lemma shows that formulation of per period manufacturing quantity is the same for the two business models (see Lemma 3.2.3). When product utilization is low in each period as in selling strategy, the number of products in circulation is higher; however, these products also last longer. When product utilization is high as in the servicizing model, few products can satisfy the demand but their lifetime will be shorter. In steady state, per period product quantity will be determined by the aggregate use level and product durability. After characterizing the average production (sales) quantity, we can characterize per period profit in a selling business model as below:

$$\Pi_{sell} = \begin{cases} \Pi^1_{sell} = \frac{M(\theta_{avg}-m(\delta))}{\delta}\left(p - c\delta^2\right) & \text{if } \frac{\delta\alpha(\theta_L-m(\delta))}{2} \geq p \\ \Pi^2_{sell} = \frac{M\beta(\theta_H-m(\delta))}{\delta}\left(p - c\delta^2\right) & \text{if } \frac{\delta\alpha(\theta_H-m(\delta))}{2} \geq p > \frac{\delta(\theta_L-m(\delta))}{2} \\ \Pi^3_{sell} = 0 & \text{if } p > \frac{\delta\alpha(\theta_H-m(\delta))}{2} \end{cases}$$

Here, the profit seeking firm would want to increase the product price as long as customers are still willing to purchase. Therefore, a firm selling to both customer segments will set its price to the highest level that low segment customers are willing to purchase. Likewise, a firm selling to the high segment only, will set its price to the maximum amount that the segment is willing to pay.

Proposition 4.2.5. *In a selling business model, for a given durability level, δ, the firm decides the profit maximizing product price as follows:*

Table 4.1: Optimal price decision

Price Decision	**Conditions**	
p^*	β	$m(\delta) + c\alpha\delta$
$\frac{\alpha\delta(\theta_L-m(\delta))}{2}$	$\left(0\ ,\ \frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\alpha\delta)}{((\theta_L-m(\delta))(\theta_L-m(\delta)-2\alpha c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)}\right)$	$\left(0\ ,\ \theta_L - c\alpha\delta\right)$
$\frac{\alpha\delta(\theta_H-m(\delta))}{2}$	$\left[\frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\alpha\delta)}{((\theta_L-m(\delta))(\theta_L-m(\delta)-2\alpha c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)}\ ,\ 1\right)$	$\left(0\ ,\ \theta_L - c\alpha\delta\right)$
		$\left[\theta_L - c\alpha\delta, \theta_H - c\alpha\delta\right)$
$\left[\frac{\alpha\delta(\theta_H-m(\delta))}{2}, \infty\right)$		$\left[\theta_H - c\alpha\delta, \infty\right)$

Corollary 4.2.6. *For a given durability level, the selling firm's decisions can be summarized as below:*

Table 4.2: Equilibrium results for a given durability level under selling

Equilibrium Results			**Conditions**	
The firm sells to	p^*	Ω^*	β	$m(\delta) + c\alpha\delta$
both segments	$\frac{\alpha\delta(\theta_L-m(\delta))}{2}$	$M(\theta_{avg} - m(\delta))$	$(0, \beta_L]$	$\left(0\ ,\ \theta_L - c\alpha\delta\right]$
high segment	$\frac{\alpha\delta(\theta_H-m(\delta))}{2}$	$\beta M(\theta_H - m(\delta))$	$(\beta_L, 1)$	$\left(0\ ,\ \theta_L - c\alpha\delta\right]$
				$\left(\theta_L - c\alpha\delta\ ,\ \theta_H - c\alpha\delta\right)$
no segment	$\left[\frac{\alpha\delta(\theta_H-m(\delta))}{2}, \infty\right)$	0		$\left[\theta_H - c\alpha\delta\ ,\ \infty\right)$

$(\beta_L = \frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\alpha\delta)}{(\theta_L-m(\delta))(\theta_L-m(\delta)-2\alpha c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)})$

When $m(\delta) + c\alpha\delta$ is greater than $\theta_H - c\alpha\delta$, market is not profitable. When it is lower than $\theta_H - c\alpha\delta$ but greater than $\theta_L - c\alpha\delta$, it is not profitable to sell to the low segment customers. When $m(\delta) + c\alpha\delta$ is lower than $\theta_L - c\alpha\delta$, the firm's (implicit) market coverage decision depends on the high segment customer proportion β.

Similar to that in the servicizing business model, when the high segment population increases; i.e., β increases, the firm prefers to sell only to the high segment customers. Evaluating the threshold level β_L enables us to look into what drives the firm to prefer the high segment only. As previously observed, when customer heterogeneity $\theta_H - \theta_L$ increases, the threshold level β_L decreases. Therefore, when customer heterogeneity is high, the firm sells to the high segment even when its population is at moderate levels. When customer usage needs are close to homogeneous, the firm would want to serve the whole market unless the population of high segment customers are extremely high.

4.3 Durability Decision under the Selling Business Model

In this section, we will conduct numerical analysis to portray the selling firm's durability decision. We study the sequential game where the firm determines the durability level δ in the first stage, followed by the price decision p specifically for the selling model in the second stage, and generates sales and profit accordingly.

Note that there is a complex relation between the durability decision and the profit function of the selling firm. Durability is involved in both the function and the piecewise conditions. Also, the profit function cannot be reduced to a version defined by any simple terms of δ. Due to this complexity, optimal durability decision of the selling firm couldn't be analytically found in this thesis.

For numerical analysis, we use the parameter values given in Table 4.3 to represent the base case, and characterize the change in the firm's durability decision with respect to one parameter at a time.

Before analyzing the optimal durability decision, we first provide examples on how the firm's profit changes with respect to durability. As can be observed in Figure 4.1, it is piecewise in nature and has an interior optimal level. This result is also verified with several different parameter sets. It is seen that there are two break points in the function which indicate shifting points between selling to the high segment and both segments.

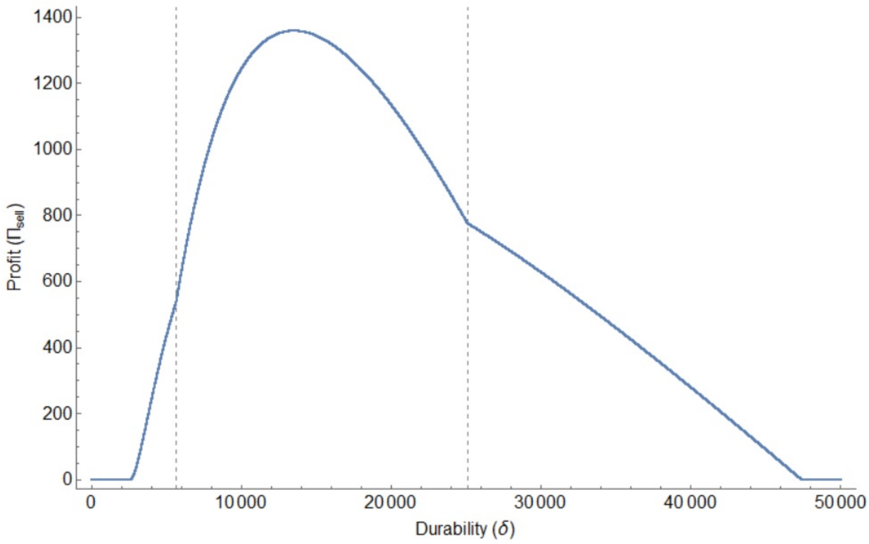


Figure 4.1: Profit vs. durability under selling (M = 100, θ_H = 12, θ_L = 9, m = 30000, c = 0.00012, β = 0.3)

In the second part of the analysis, we will discuss the effect of parameter changes on the durability decision of the firm through numerical examples. We use below values as a base in our analysis.

Table 4.3: Base Parameter Values for Numerical Analysis

M	θ_H	θ_L	m	c	β	p	δ
100	12	9	30000	0.00012	0.3	(0,100000]	(0,50000]

Durability decision of the selling firm depends on several parameters. We investigate the effect of customer heterogeneity, proportion of the high segment population, unit maintenance & operating cost and unit manufacturing cost on the durability decision of the selling firm. Figure 4.2 shows how equilibrium durability level (δ^*) is influenced by the change of $(\theta_H - \theta_L)$, i.e.; customer heterogeneity. Note that in our model, the effect of customer heterogeneity $(\theta_H - \theta_L)$ can be analyzed only when the firm sells to both customer segments. Therefore, the figure below reflects full market coverage cases only.

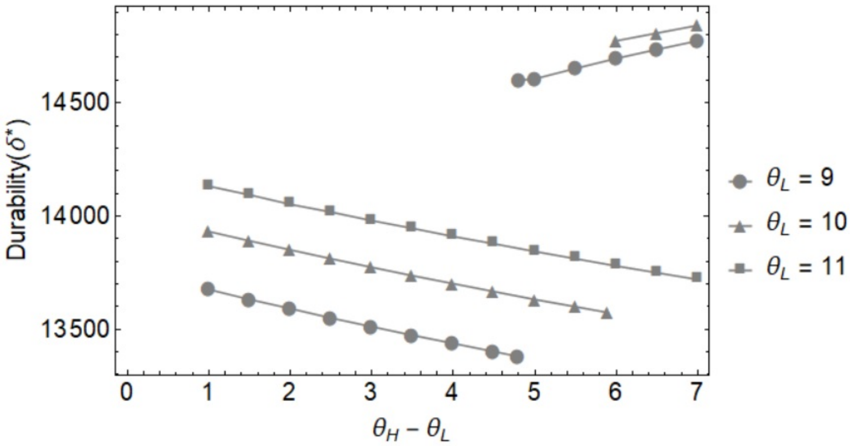


Figure 4.2: Effect of customer heterogeneity on durability under selling (M= 100, m = 30000, c = 0.00012, β = 0.3)

In Figure 4.2, we see that when θ_L is fixed, increasing market heterogeneity; i.e., increasing θ_H, causes a decrease in equilibrium durability. To understand the reason behind this effect, we look for the influence of durability change on the components of profit. Within the scope of our numerical analysis, we see that a lower durability increases the number of products sold. However, it also causes a decrease in the marginal profit of the firm. Therefore, there is a tradeoff between these two factors of profit. When θ_H is higher, the firm knows that in the long term the number of products sold will be higher. At this point, our model shows that slightly decreasing the durability, and by this way increasing the sales quantity, benefits the firm, even though profit margin is decreased. Figure 4.2 also shows the change when the firm serves only to the high segment. When the serves only to the high segment, increase in θ_H leads to higher willingness to pay. Thus, increasing part in the figure shows that the firm chooses higher durability (and price) when customer willingness to pay increases.

In the case where θ_H is fixed, we see the same durability reducing effect of increased customer heterogeneity with a different shape. In such a change, the firm is pushed to decrease durability since low segment customers are willing to pay less when θ_L is lower. To satisfy the demand, the firm reduces the price by allowing a decrease in product durability. Next, we analyze the effect of maintenance and operating cost. In our numerical analysis, we use $m(\delta) = \frac{m}{\delta}$.

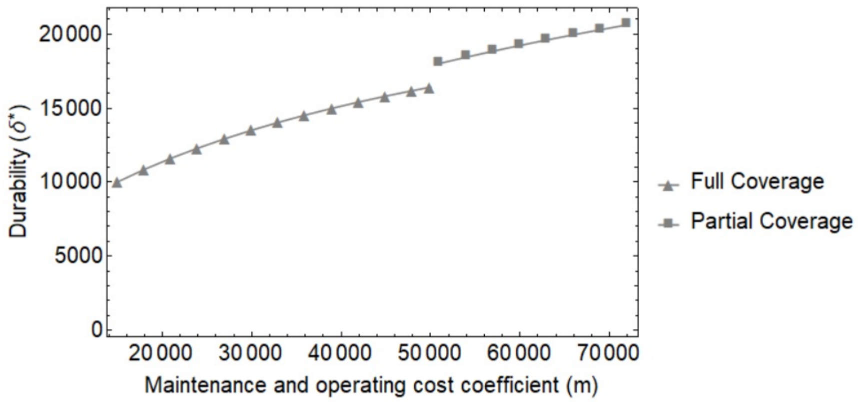


Figure 4.3: Effect of maintenance & operating cost coefficient (m) on durability under selling (M= 100, θ_H = 12, θ_L = 9, c = 0.00012, β = 0.3)

Figure 4.3 shows that when maintenance and operating cost coefficient (m) is higher for the product, equilibrium durability level will also be higher. We know that maintenance and operating cost is incurred by the customers and the cost per use is decreasing in durability. Further, a higher maintenance cost (m) negatively influences customer's willingness-to-pay, and usage decision after purchase. From the numerical results, it can be argued that the firm reacts to this change in a way to compensate the loss of customer utility so that she can purchase the product. Note that the two pieces of the function reflects the two market coverage cases. Now, we will look into the effect of manufacturing cost.

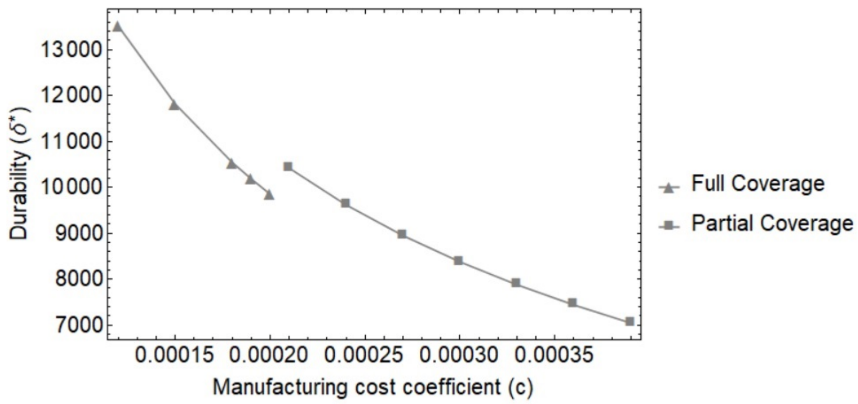


Figure 4.4: Effect of manufacturing cost coefficient on durability under selling (M= 100, θ_H = 12, θ_L = 9, m = 30000, β = 0.3)

Figure 4.4 shows that when manufacturing cost coefficient is higher for the product, the firm will prefer to produce products with lower durability. To balance the increase in the manufacturing cost caused by the increase in c, the firm will lower the product durability. It can be observed that this behaviour is the same for both market coverage cases: selling to the high segment only and selling to both segments. In Figure 4.4, it can be detected when the firm sells only to the high segment, decreasing pattern is the same but the durability level is higher at each point.

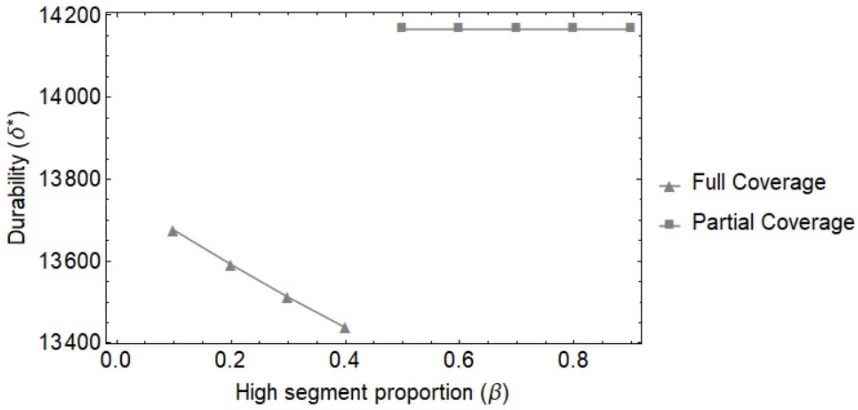


Figure 4.5: Effect of high segment proportion on durability under selling (M = $100, \theta_H$ = $12, \theta_L$ = $9, m$ = $30000, c$ = 0.00012)

In the Figure 4.5, we can see that higher β values causes a decrease in the firm's durability choice in a similar way to that we observed with customer heterogeneity. Until the firm decides to cover the market partially, increase in the population of the high segment customers will lead to lower durability levels. Once the firm decides that partial coverage is more profitable and sells to the high segment only, raise in the high segment population will not influence the durability decision.

Under full market coverage, the firm's pricing will be bound by the low segment's willingness to pay θ_L. Thus, an increase in θ_H or β, will increase the aggregate usage level without any change in the margin. Under these cases, the firm reacts by lowering durability, to save from the manufacturing cost and to increase the quantity manufactured at the same time. When only the high segment is served (partial coverage), however, θ_H has an impact on the firm's pricing, and any escalation on this parameter increases durability.

Summarizing our numerical observations, under the selling business model, equilibrium durability level is decreasing in customer heterogeneity ($\theta_H - \theta_L$) and maintenance&operating cost coefficient (m) while it is increasing in manufacturing cost coefficient (c).

CHAPTER 5

ANALYTICAL COMPARISONS

In this chapter, we compare the two business models in terms of profitability and environmental impact for a given durability level. This comparison enables us to see the profitability results without the effect of durability decision. Numerical analyses are conducted to discuss the analytical findings. To put the two business models on equal footing, throughout the analysis we will assume $\alpha = 1$.

Corollary 5.0.1. *Profits of selling and servicizing business models can be summarized with two tables below:*

Table 5.1: Servicizing Profit

The firm serves to	$\boldsymbol{\Pi^*_{serv}}$	$\boldsymbol{\beta}$	$\boldsymbol{m(\delta) + c\delta}$
Both segments	$M(\frac{\theta_{avg}-m(\delta)-c\delta}{2})^2$		$\big(0\ ,\ \theta_L - (\theta_H - \theta_L)\big]$
		$(0,\ \beta_V]$	$\big(\theta_L - (\theta_H - \theta_L)\ ,\ \theta_L\big]$
High segment	$M\beta(\frac{\theta_H-c\delta-m(\delta)}{2})^2$	$(\beta_V, 1)$	$\big(\theta_L - (\theta_H - \theta_L)\ ,\ \theta_L\big]$
			$\big[\theta_L,\ \theta_H\big)$
No segment	0		$\big[\theta_H,\ \infty\big)$

where $\beta_V = (\frac{\theta_L-m(\delta)-c\delta}{\theta_H-\theta_L})^2$

Table 5.2: Selling Profit

The firm sells to	Π^*_{sell}	β	$m(\delta) + c\alpha\delta$
Both segments	$M\frac{(\theta_L - m(\delta) - 2\alpha c\delta)(\theta_{avg} - m(\delta))}{2\alpha}$	$(0,\ \beta_L]$	$\big(0\ ,\ \theta_L - c\alpha\delta\big]$
High segment	$M\beta\frac{(\theta_H - m(\delta) - 2\alpha c\delta)(\theta_H - m(\delta))}{2\alpha}$	$(\beta_L,\ 1)$	$\big(0\ ,\ \theta_L - c\alpha\delta\big]$
			$\big(\theta_L - c\alpha\delta\ ,\ \theta_H - c\alpha\delta\big)$
No segment	0		$\big[\theta_H - c\alpha\delta\ ,\ \infty\big)$

where $\beta_L = \frac{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\alpha\delta)}{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2\alpha c\delta) + (\theta_H - m(\delta))(\theta_H - \theta_L)}$

5.1 Profitability

Corollary 5.0.1 shows how market coverage and profitability of the two business models change as market and product parameters differ. For $\alpha = 1$, we observe that thresholds for marginal cost of usage $(m(\delta) + c\delta)$ are lower in a selling business model, i.e., $\theta_L - c\delta < \theta_L$ and $\theta_H - c\delta < \theta_H$. It shows that the selling firm is more cost-sensitive compared to the servicizing firm. We know that an increase in any product related cost leads to an increase in price/fee. Thus, the selling firm's lower cost threshold might be a result of greater price sensitivity for customers in a selling business model. In other words, it might be suggested that under a selling business model, customers are more inclined to choose remaining inactive in case of a price inflation.

On the other hand, servicizing can remain profitable for a wider cost interval. It indicates that there exists a cost level where servicizing is the only profitable alternative out of the two. Therefore, it may serve as a major financial advantage for servicizing when product costs are high.

In order to compare profitability of the two business models analytically, summary results in Table 5.1 and 5.2 enables us to define comparison intervals of Π_{serv} and Π_{sell} in terms of β and $m(\delta) + c\delta$. To determine these intervals, we need to know the order of threshold levels for β and $m(\delta) + c\delta$. So, we need to compare $\{\beta_L, \beta_V\}$ for the threshold levels of β, and we need to compare $\{(\theta_L - (\theta_H - \theta_L)),\ (\theta_L - c\delta),\ \theta_L,\ (\theta_H - c\delta),\ \theta_H\}$ for the threshold levels of $m(\delta) + c\delta$ to find the order among these values.

Lemma 5.1.1. *If* $m(\delta) + c\delta \leq \theta_H - c\delta$, *then* $\beta_L < \beta_V$.

$$\beta_L = \frac{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\alpha\delta)}{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2\alpha c\delta) + (\theta_H - m(\delta))(\theta_H - \theta_L)} < \left(\frac{\theta_L - m(\delta) - c\delta}{\theta_H - \theta_L}\right)^2 = \beta_V$$

Lemma 5.1.1 shows that as the high segment population increases, compared to a servicizing firm, a selling firm will be more inclined to serve high segment customers only. It could be evaluated as a consequence of pay-per-use pricing system. A servicizing firm, by managing the tradeoff between the fees and the total usage, utilizes the demand of the low segment customers more effectively. Therefore, it will keep serving to the whole market, unless the high segment population reaches a considerable level.

Next, we can focus on the comprehensive profitability comparison between servicizing and selling business models for a given durability level considering the entire market parameters β, $m(\delta) + c\delta$ and $\theta_H - \theta_L$.

Proposition 5.1.2. *For a given durability level, comparison of firm profits in a selling and servicizing business models can be summarized as below:*

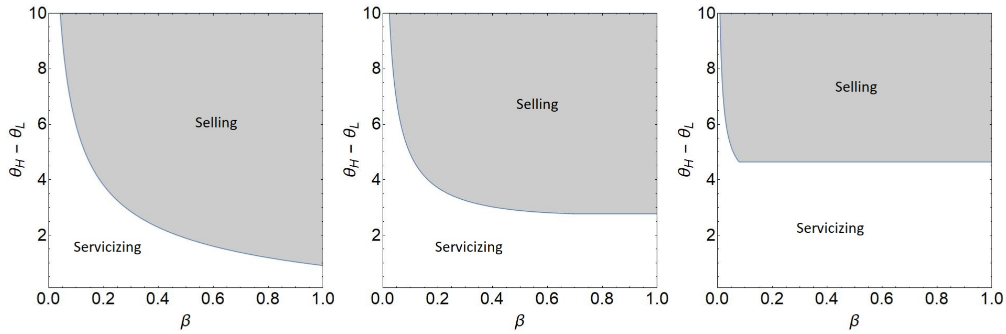
Table 5.3: Comparison of profitability for a given durability level

Conditions						Result
	β^*_{LV1}	β^*_{LV2}	β	$\theta_H - \theta_L$	$m(\delta)+c\delta$	
1	$(0,\beta_L)$		$(0,\ \beta_{LV1}]$	$(0,\infty)$	$\big(0\ ,\ \theta_L - c\delta\big]$	
2	$[\beta_L,\infty)$		$(0,\beta_L]$	$(0,\infty)$	$\big(0\ ,\ \theta_L - c\delta\big]$	
3		$(-\infty,\beta_L)$	$[\beta_L,1)$	$(0,c\delta]$	$\big(0\ ,\ \theta_L - c\delta\big]$	
4		$[\beta_L,1)$	$[\beta_{LV2},1)$	$(0,c\delta]$	$\big(0\ ,\ \theta_L - c\delta\big]$	
5		$(0,1)$	$[\beta_{LV2},1)$	$(0,\frac{c\delta}{2}]$	$\big(\theta_L - c\delta\ ,\ \theta_H - c\delta\big]\cap(0,T]$	
6		$(0,1)$	$[\beta_{LV2},1)$	$(\frac{c\delta}{2},c\delta)$	$\big(\theta_L - c\delta\ ,\ \theta_L-(\theta_H-\theta_L)\big]\cap(0,T]$	
7		$(0,\beta_V)$	$[\beta_{LV2},\beta_V]$	$(\frac{c\delta}{2},c\delta)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_H - c\delta\big]\cap(0,T]$	
8			$[\beta_V,1)$	$(\frac{\sqrt{2}c\delta}{2},c\delta)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_H-\sqrt{2}c\delta\big]$	
9			$[\beta_V,1)$	$(c\delta,\sqrt{2}c\delta)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_H-\sqrt{2}c\delta\big]$	$\Pi_{sell} \geq \Pi_{serv}$
10			$[\beta_V,1)$	$(\sqrt{2}c\delta,\infty)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_L\big]$	
11				$(\sqrt{2}c\delta,\infty)$	$\big(\theta_L,\ \theta_H-\sqrt{2}c\delta\big]$	
12		$(0,\beta_L)$	$[\beta_L,1)$	$[c\delta,\infty)$	$\big(0\ ,\ \theta_L-(\theta_H-\theta_L)\big]$	
13		$[\beta_L,1)$	$[\beta_{LV2},1)$	$[c\delta,\infty)$	$\big(0\ ,\ \theta_L-(\theta_H-\theta_L)\big]$	
14		$(0,\beta_L]$	$[\beta_L,\beta_V]$	$[c\delta,\infty)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta\big]$	
15		(β_L,β_V)	$[\beta_{LV2},\beta_V)$	$[c\delta,\infty)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta\big]$	
16		$(0,\beta_V]$	$[\beta_{LV2},\beta_V)$	$[c\delta,\infty)$	$\big(\theta_L-c\delta\ ,\ \theta_L\big]$	
17	$(-\infty,0]$		$(0,\beta_L)$	$(0,\infty)$	$\big(0\ ,\ \theta_L - c\delta\big]$	
18	$(0,\beta_L)$		(β_{LV1},β_L)	$(0,\infty)$	$\big(0\ ,\ \theta_L - c\delta\big]$	
19		$[\beta_L,1)$	$[\beta_L,\beta_{LV2})$	$(0,c\delta]$	$\big(0\ ,\ \theta_L - c\delta\big]$	
20		$[1,\infty)$	$[\beta_L,1)$	$(0,c\delta]$	$\big(0\ ,\ \theta_L - c\delta\big]$	
21		$(0,1)$	$(0,\beta_{LV2})$	$(0,\frac{c\delta}{2}]$	$\big(\theta_L - c\delta\ ,\ \theta_H - c\delta\big]\cap(0,T]$	
22		$[1,\infty)$		$(0,\frac{c\delta}{2}]$	$\big(\theta_L - c\delta\ ,\ \theta_H - c\delta\big]\cap(0,T]$	
23		$(0,1)$	$(0,\beta_{LV2})$	$(\frac{c\delta}{2},c\delta)$	$\big(\theta_L - c\delta\ ,\ \theta_L-(\theta_H-\theta_L)\big)\big]\cap(0,T]$	
24		$[1,\infty)$		$(\frac{c\delta}{2},c\delta)$	$\big(\theta_L - c\delta\ ,\ \theta_L-(\theta_H-\theta_L)\big]\cap(0,T]$	
25		$(0,\beta_V)$	$(0,\beta_{LV2})$	$(\frac{c\delta}{2},c\delta)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_H - c\delta\big]\cap(0,T]$	
26		$[\beta_V,1)$	$(0,\beta_V)$	$(\frac{c\delta}{2},c\delta)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_H - c\delta\big]\cap(0,T]$	
27			$[\beta_V,1)$	$(\frac{c\delta}{2},\frac{\sqrt{2}c\delta}{2})$	$\big(\theta_L-(\theta_H-\theta_L),\theta_H - c\delta\big]$	
28			$[\beta_V,1)$	$(\frac{\sqrt{2}c\delta}{2},c\delta)$	$\big(\theta_H-\sqrt{2}c\delta,\theta_H - c\delta\big]$	$\Pi_{sell} < \Pi_{serv}$
29			$[\beta_V,1)$	$(c\delta,\sqrt{2}c\delta)$	$\big(\theta_H-\sqrt{2}c\delta,\theta_L\big]$	
30				$(c\delta,\sqrt{2}c\delta)$	$\big(\theta_L,\ \theta_H - c\delta\big]$	
31				$(\sqrt{2}c\delta,\infty)$	$\big(\theta_H-\sqrt{2}c\delta,\theta_H - c\delta\big]$	
32		$[\beta_L,1)$	(β_L,β_{LV2})	$[c\delta,\infty)$	$\big(0\ ,\ \theta_L-(\theta_H-\theta_L)\big]$	
33		(β_L,β_V)	(β_L,β_{LV2})	$[c\delta,\infty)$	$\big(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta\big]$	
34		$(0,\beta_V]$	$(0,\beta_{LV2})$	$[c\delta,\infty)$	$\big(\theta_L-c\delta\ ,\ \theta_L\big]$	
35		$[\beta_V,1)$	$(0,\beta_V)$	$[c\delta,\infty)$	$\big(\theta_L-c\delta\ ,\ \theta_L\big]$	
36				$(0,\frac{c\delta}{2}]$	$\big(\theta_L - c\delta\ ,\ \theta_H - c\delta\big]\cap(T,\infty)$	
37				$(\frac{c\delta}{2},c\delta]$	$\big(\theta_L - c\delta\ ,\ \theta_L-(\theta_H-\theta_L)\big]\cap(T,\infty)$	
38			$(0,\beta_V)$	$(\frac{c\delta}{2},c\delta]$	$\big(\theta_L-(\theta_H-\theta_L)\ ,\ \theta_L\big]\cap(T,\infty)$	
39					$\big(\theta_H-c\delta\ ,\ \theta_H\big)$	

where $T = \frac{(\theta_H-\theta_L)^2+\theta_L^2+m(\delta)c\delta}{2\theta_L-m(\delta)}$

$^*\beta_{LV1}$ and β_{LV2} and their expanded conditions are given in Appendix L

Proposition 5.1.2 puts forward three main factors of profitability comparison between selling and servicizing business models for a given durability level. These are: marginal cost of usage $(m(\delta)+c\delta)$, proportion of high segment customers β and valuation difference of customers $(\theta_H - \theta_L)$. This proposition also gives the detailed conditions on the profitability comparison of the two business models for a given durability.



(a) m = 20000, c = 0.00025 (b) m = 25000, c = 0.00030 (c) m = 30000, c = 0.00035

Figure 5.1: Profitability comparisons under different consumer heterogeneity, high segment proportion and marginal usage cases ($\delta = 12000,\ \theta_L = 8, M = 100$)

Based on the proposition 5.1.2, figure 5.1 identifies the relationship between the firm's business model choice and the factors of this choice. Subfigures are organized in a way that the marginal cost of usage $(m(\delta) + c\delta)$ increases from left to right. First, it can be deduced that as marginal cost of usage $m(\delta) + c\delta$ increases the firm is more likely to choose servicizing business model. When marginal cost of usage is low, it is more profitable for the firm to sell its products. This result might be related with the fact that servicizing offers a more affordable method for customers. When the product is costly, purchasing may not be financially viable for many customers. Under a selling business model, customers not only pay a high purchasing price, but also pay the maintenance and operating expenses as they use the product. Customers may reduce costs by using less; however, they still need to pay the purchasing price as a prior action. Rational customers will not find it worthwhile to purchase the product when they anticipate low usage level. Therefore, selling does not offer an affordable way to obtain utility for low usage. On the other hand, servicizing enables customers to purchase the utility even for a unit of usage. When the cost of usage is high, servicizing might have the advantage of generating more profit by reaching many customers.

Second, the figure 5.1 show that increasing consumer valuation heterogeneity, by increasing the high end θ_H, would lead to the choice of selling business model. Also, it can be seen that when the market is highly populated by low segment customers, i.e., β is considerably low, profitable region for selling is smaller. These results indicate that parallel to the common belief, the market dominance of high segment customers promotes selling business model compared to servicizing. This dominance could be caused by large customer heterogeneity $(\theta_H - \theta_L)$ or by the immense population of the high segment customers β. Under this condition, an important part of the population is willing to use a lot, and pay accordingly. The firm reacts to this situation by choosing selling strategy.

5.2 Environmental Impact

Environmental impact can be modeled by considering two phases of a product life cycle: use phase and manufacturing phase. During the use phase, the product damages the environment due to electricity, water, fuel or other natural resources' consumption. In the manufacturing phase, besides the resource consumption for manufacturing purposes, activities of logistics also cause a damaging effect on the environment. These two factors can be combined and modeled for per period as $E = e_u\Omega + e_mQ$ where e_u is the environmental impact per use and e_m is the environmental impact per unit produced. Implementing $Q = \frac{\Omega}{\delta}$, environmental impact per period is defined as $E = e_u\Omega + e_m\frac{\Omega}{\delta}$.

Proposition 5.2.1. *When both business models are profitable, i.e. when $m(\delta) + c\delta < \theta_H - c\delta$, environmental impact and aggregate usage comparison can be summarized as below:*

Table 5.4: Environmental Impact and Aggregate Usage Comparison for a Given Durability Level

Result	**Conditions**		
	$\boldsymbol{\beta}$	$\boldsymbol{m(\delta)+c\delta}$	$\boldsymbol{\theta_H-\theta_L}$
$E_{sell} > E_{serv}$, $\Omega_{sell} > \Omega_{serv}$	$(0,\beta_L]$	$(0,\theta_L-c\delta]$	$(0,\infty)$
	$(\beta_L,1)\cap(\beta_E,1)$	$(0,\theta_L-c\delta]$	$(0,c\delta]$
	$(\beta_E,1)$	$(\theta_L-c\delta,\theta_H-c\delta]$	$(0,\frac{c\delta}{2}]$
	$(\beta_E,1)$	$(\theta_L-c\delta,\theta_L-(\theta_H-\theta_L)]$	$(\frac{c\delta}{2},c\delta]$
	$(0,\beta_V]\cap(\beta_E,1)$	$(\theta_L-(\theta_H-\theta_L),\theta_H-c\delta]$	$(\frac{c\delta}{2},c\delta]$
	$(\beta_L,1)\cap(\beta_E,1)$	$(0,\theta_L-(\theta_H-\theta_L)]$	$[c\delta,\infty)$
	$(\beta_L,\beta_V]\cap(\beta_E,1)$	$(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta]$	$[c\delta,\infty)$
	$(0,\beta_V]\cap(\beta_E,1)$	$(\theta_L-c\delta,\theta_L]$	$[c\delta,\infty)$
	$(\beta_V,1)$	$(\theta_L-(\theta_H-\theta_L),\theta_H-c\delta]$	$(\frac{c\delta}{2},c\delta]$
	$(\beta_V,1)$	$(\theta_L-(\theta_H-\theta_L),\theta_L]$	$[c\delta,\infty)$
	$(0,1)$	$(\theta_L,\theta_H-c\delta]$	$[c\delta,\infty)$
$E_{sell} \le E_{serv}$, $\Omega_{sell} \le \Omega_{serv}$	$(\beta_L,1)\cap(0,\beta_E]$	$(0,\theta_L-c\delta]$	$(0,c\delta]$
	$(0,\beta_E]$	$(\theta_L-c\delta,\theta_H-c\delta]$	$(0,\frac{c\delta}{2}]$
	$(0,\beta_E]$	$(\theta_L-c\delta,\theta_L-(\theta_H-\theta_L)]$	$(\frac{c\delta}{2},c\delta]$
	$(0,\beta_V]\cap(0,\beta_E]$	$(\theta_L-(\theta_H-\theta_L),\theta_H-c\delta]$	$(\frac{c\delta}{2},c\delta]$
	$(\beta_L,1)\cap(0,\beta_E]$	$(0,\theta_L-(\theta_H-\theta_L)]$	$[c\delta,\infty)$
	$(\beta_L,\beta_V]\cap(0,\beta_E]$	$(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta]$	$[c\delta,\infty)$
	$(0,\beta_V]\cap(0,\beta_E]$	$(\theta_L-c\delta,\theta_L]$	$[c\delta,\infty)$

where $\beta_E = \frac{\theta_L-m(\delta)-c\delta}{\theta_H+\theta_L-2m(\delta)}$

For a given durability level, when both business models serve the same size of customer population, selling causes a greater environmental impact and aggregate usage level.

For a given durability level, environmental performance of the two business models depend on the aggregate usage levels. Thus, Proposition 5.2.1 gives the comparison results for both environmental impact and aggregate usage.

This result shows that servicizing can cause a greater environmental impact only when it leads to higher market coverage compared to selling. In this case, high segment customer proportion shows a significant impact on this comparison. We see that once the high segment proportion β exceeds β_E, servicizing is the environmentally superior option in all cases. This also indicates that compared to servicizing, selling leads to greater total usage when high segment population increases in the market.

It should be noted that above results hold when the two business models are profitable. When product related costs exceed the threshold, i.e., when $m(\delta) + c\delta \geq \theta_H - c\delta$, selling is no longer profitable. Within the interval of $\theta_H > m(\delta) + c\delta \geq \theta_H - c\delta$, only servicizing is profitable. Thus, within this interval only servicizing can cause usage and environmental impact.

CHAPTER 6

NUMERICAL EXPERIMENTS

In this chapter, we conduct numerical experiments for the comparison of selling and servicizing business models with respect to profitability, durability and the environmental impact. For this purpose, we investigate the effect of different parameter changes in an extensive range. First, we focus on the durability choices under the two business models. Next, profitability comparison is conducted considering the firm's optimal durability choice. Finally, environmental impact differences of the two business models are analyzed and conditions for both better profitability and environmental performance are determined. For simplicity, it is assumed that $\alpha = 1$ throughout this chapter.

6.1 Durability Decisions

Recall that under both business models, the firm first determines the durability. Then, purchasing price or service fee levels are decided. Durability decision determines the usage capacity of the product. Therefore, it indicates how long the products are going to last in the market. Under a selling business model, revenue is proportional with the manufacturing volume. However, under a servicizing business model, revenue is proportional with the total usage. Thus, it is believed that the selling firm, in order to manufacture and sell more, would want customers to replenish their products more frequently while the servicizing firm would try to extend the product life and decrease the manufacturing volume to save the costs. By numerical experiments, we investigate whether servicizing in fact leads to more durable products.

We conduct numerical analysis by comparing optimal durability levels of selling and servicizing strategies. By enumerating the parameters within extensive range of values, profit maximizing durability decisions are obtained. Lower and upper bound of parameter values are as follows:

Table 6.1: Parameter Values of Numerical Experiments for the Durability Choice

m	c	θ_L	θ_L	β
$[1000, 100000]$	$[0.00002, 0.002]$	$[5, 12]$	$[5.5, 22]$	$[0.02, 1)$

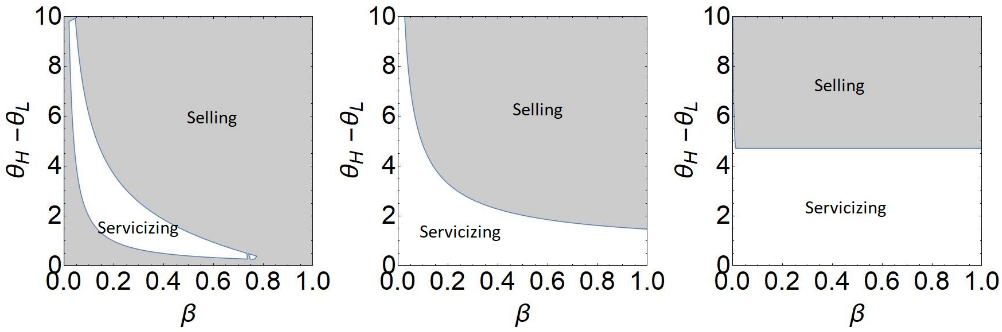
Taking market population as M = 100, 90,000 instances are evaluated. When servicizing business model has positive profit, all of the instances show that $\delta^*_{serv} > \delta^*_{sell}$.

This result verifies expectations regarding the durability decision under the servicizing business model. In fact, to the extent of our analysis range, we couldn't find a condition under which selling offers more durable products. When we investigated whether selling could be the only viable option for the manufacturer, we observed that servicizing was also profitable whenever selling was.(see threshold levels in Table 5.1 and 5.2).

6.2 Profitability

As an alternative business model, servicizing is promising for firms only if it is more profitable compared to selling. Since it is more affordable to use a product under a servicizing business model, it might generate more firm profit by reaching more customers. On the other hand, since each unit of usage is charged under servicizing, it causes a decrease in individual usage level which might lead to consumer aversion. The effect of market and product parameters on the profitability comparison of the two business model is controversial. In this section, we try to identify conditions under which servicizing is more profitable.

Figure 6.1 shows the relationship between market & product parameters and business model choice of the firm. For each business model, dominant regions with respect to profitability are indicated in the figure. For the profitability comparison of the two business models, three main factors can be analyzed: marginal cost of usage



(a) m = 30000, c = 0.00025 (b) m = 35000, c = 0.00030 (c) m = 45000, c = 0.00040

Figure 6.1: More profitable business model ($\theta_L = 9, M = 100$)

$(m(\delta) + c\delta)$, customer valuation difference ($\theta_H - \theta_L$) and the high segment customer proportion β.

To begin with, it can be highlighted that as in the analysis conducted with given durability (see Figure 5.1), an increase in the marginal cost of usage favors servicizing. When the components of marginal cost of usage is analyzed separately, it is seen that the effect of the changes in m and c are very similar (see Appendix N). Here, we note that servicizing is more likely to be chosen when the product is costly to produce or costly to operate. This finding is consistent with what we observe in the industry since the most prominent servicizing examples are for high cost products. For instance, cars are servicized by major manufacturers like Daimler (Car2Go) and BMW (DriveNow). On the other hand, relatively low segment car manufacturers like Volkswagen and Peugeot has stopped their servicizing applications (Harrup, 2016), (Bosteels, 2016).

Other factors for the profitability comparison of the two business models are customer heterogeneity or customer valuation gap $(\theta_H - \theta_L)$ and the high segment proportion β. Our numerical analysis shows that, a market with more heterogeneous customers would favor the selling business model. Similarly, as the high segment population increases, the firm will be more likely to choose selling. The two factors together determine the dominance of the high segment customers in the market. If usage need of high segment customers θ_H or the high segment proportion β is high, then the high segment has a great significance in the market. When the high segment is significant in the market, consumers put more value on owning a product rather than purchasing a service charged by each use. In this case, selling is the profitable alternative due to high prices and usage levels achieved.

Figure 6.1 also shows that the significance of β decreases as marginal cost of usage $(m(\delta) + c\delta)$ increases. We observe that for low cost products, the high segment proportion β and customer heterogeneity $(\theta_H - \theta_L)$ together determines the business model choice. However, for high cost products, regardless of the high segment proportion β, the firm prefers servicizing when customer heterogeneity $(\theta_H - \theta_L)$ is low, and prefers selling when $(\theta_H - \theta_L)$ is high. Since profitability of selling is more sensitive to cost increases, after a certain level of cost, selling is only profitable when there are customers with relatively high usage needs in the market; i.e., θ_H is high.

6.3 Environmental Impact

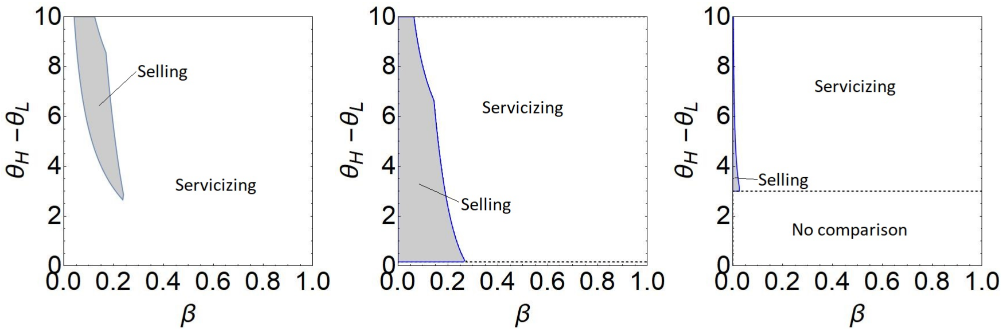
We model the environmental impact considering two phases of a product lifecycle: use phase and production phase [1]. In the use phase, typically, environmental impact is induced by aggregate usage. In the production phase, environmental impact grows when aggregate usage increases since it raises the production level. Durability has a diminishing effect on the production phase environmental impact since it decreases the per period production level.

Since servicizing tends to produce higher durability, it may be argued that it has a lower environmental impact due to the production phase. However, servicizing may cause greater overall impact due to higher aggregate usage it induces. Under a servicizing business model, while pricing each unit of use can decrease individual usage levels; compared to selling, servicizing can offer products in a more affordable way, reaching more customers and leading to increased aggregate usage.

We analyze the environmental impact differences of the two business models for the intervals of different market and product parameter values. For environmental impact parameters, we first use $\frac{e_m}{e_u} = 1000$, later we check the robustness of our results for an

[1] Disposal phase is not explicitly included here, but could be taken as a component of the production phase impact since end-of-life products are equal to the quantity produced on the average.

extensive range of $\frac{e_m}{e_u}$ values.



(a) m = 30000, c = 0.00025 (b) m = 35000, c = 0.00030 (c) m = 45000, c = 0.00040

Figure 6.2: Environmentally superior business model ($\theta_L = 9, M = 100$)

Figure 6.2 reflects conditions of environmental superiority for each business model. Environmental comparison is not meaningful unless both business models are profitable. Thus, it is specified in the figure as a region split by a dashed line when at least one of the business models is not profitable [2]. As it is expected, we see that profitability reduces and comparable regions get smaller as product related costs increase. Factors of environmental performance comparison can now be analyzed under comparable regions.

First, it can be seen that as marginal cost of usage increases, the region for servicizing first gets smaller; but when the marginal cost of usage is high enough, servicizing environmentally dominates selling in almost all levels of $(\theta_H - \theta_L)$ and β. It is seen that when marginal cost of usage is in an intermediate level, selling has the biggest region of environmental superiority. Effects of m and c can be seen separately in Appendix N.

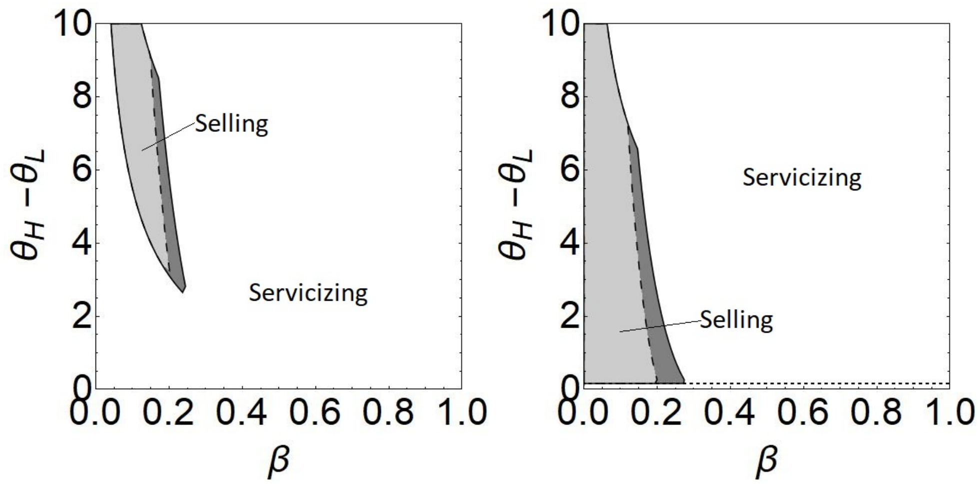
Numerical experiments also show that the high segment proportion β has a significant effect on the environmental comparison of the two alternatives. We observe that selling may be environmentally superior to servicizing only when the high segment population is low. In fact, for the low cost products, selling is environmentally preferable only for a small range of β on the left side of the figures. However, for the low cost products, superiority of selling cannot be maintained at extremely low values of

[2] When marginal cost of usage exceeds a certain level, the business is not profitable. We know that the threshold for selling is lower ($\theta_H - c\delta < \theta_H$). Figures show nonprofitable regions for selling

β.

Customer heterogeneity also has an effect on the environmental comparison. When marginal cost of usage $(m(\delta) + c\delta)$ increases, starting from selling, the business models become unprofitable under a certain level of high segment valuation θ_H, and consequently customer heterogeneity. Therefore, along with marginal cost of usage, $(\theta_H - \theta_L)$ determines the threshold for the environmentally comparable region. Figure 6.2 shows that when the costs are low, high levels of usage valuation θ_H (and consequently $\theta_H - \theta_L$) is required for the superiority of selling. When the costs are high, lower customer heterogeneity provides more opportunities for selling's superiority.

We also analyze the effect of the ratio of environmental coefficients $\frac{e_m}{e_u}$. We used $\frac{e_m}{e_u} = 1000$ for the above analysis. After conducting numerical experiments with different levels of $\frac{e_m}{e_u}$, we observe that this ratio has limited effect on the environmental comparison.

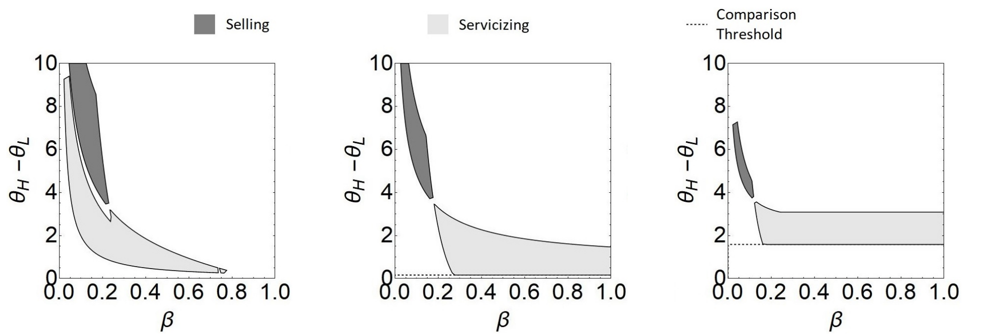


(a) m = 30000, c = 0.00025 (b) m = 35000, c = 0.00030

Figure 6.3: Effect of $\frac{e_m}{e_u}$ ratio $(\theta_L = 9, M = 100)$

In Figure 6.3, in both subfigures dark shaded region shows the change of selling's environmental superiority from $\frac{e_m}{e_u} = 0.00001$ to $\frac{e_m}{e_u} = 100000$. As $\frac{e_m}{e_u}$ increases, from $\frac{e_m}{e_u} = 0.00001$ to $\frac{e_m}{e_u} = 100000$, we see that in both subfigures, the region for selling's superiority shrinks while the region for servicizing's superiority slightly gets bigger by the amount of dark shaded regions. In other words, as environmental impact due to production gets higher, servicizing's relative environmental performance increases. However, considering the scale of the change, $\frac{e_m}{e_u}$ does not seem to have a major impact on the environmental comparison of the two business models.

Through numerical analysis, we have presented the factors of the comparison regarding profitability and environmental performance. Next, we investigate cases where a business model is both economically and environmentally superior.



(a) m = 30000, c = 0.00025 (b) m = 35000, c = 0.00030 (c) m = 40000, c = 0.00035

Figure 6.4: Both environmentally and profitably preferable conditions ($\theta_L = 9, M = 100$)

Figure 6.4 shows the conditions under which a business model dominates on both economic and environmental performance. We observe that as marginal cost of usage ($m(\delta) + c\delta$) increases, the region where selling is preferred gets smaller while the region for servicizing does not show a major change.

Evaluating figure 6.4 shows that selling's equilibrium conditions are mostly for high customer heterogeneity ($\theta_H - \theta_L$) and low proportion of the high segment β. For servicizing, equilibrium tendencies differ with marginal cost of usage level. For relatively low levels of ($m(\delta) + c\delta$), servicizing can be a better choice when customer heterogeneity ($\theta_H - \theta_L$) or high segment proportion β is low. It implies that for low cost products, servicizing can be a better alternative only when the high segment customers are not too dominant in the market. We also see that when marginal cost of usage is high enough, servicizing cannot be a better alternative in a market with high customer valuation gap ($\theta_H - \theta_L$) and with low population of the high segment customers β.

CHAPTER 7

CONCLUSIONS

In this thesis, we study the comparison of servicizing and selling business models in terms of durability, profitability and environmental impact. As a potentially greener alternative, servicizing has not been studied in detail. Thus, servicizing's environmental superiority over selling is still controversial. To study the comparison of the two alternatives, we consider a monopolist firm that either sells or servicizes its durable product. We use a game theoretical setting, where consumers react to the firm's durability and price (or service fee) decisions by choosing their optimal usage levels. Maintenance and operating cost is incurred by the firm under the servicizing business model while it is incurred by customers under the selling business model. We model durability as the maximum level of usage a product can endure. This aspect of the model provides a way to capture the relation between customers's usage level and the product lifetime.

Our findings for a given durability level suggest that servicizing is not the more profitable option when market is dominated by high segment customers either by means of population or by means of product valuation. We analytically show that for a given durability level, servicizing remains profitable for higher levels of product related costs. In other words, when the product is very costly to manufacture or operate, such that selling is no longer profitable, servicizing might be the financially viable option. Also, analytical findings suggest that when both business models serve the same size of customer population, selling causes a greater environmental impact and aggregate usage level.

On the durability aspect of the comparison, our numerical experiments show that servicizing leads to products of higher durability. Since the servicizing business model does not shift the ownership to customers and dictates only the sale of the functionality/service, it is expected to produce higher durability. Results of this study supports this promise. As a factor of environmental impact, increased durability may also explain our findings in environmental comparison through numerical experiments.

We show that environmental superiority of servicizing hinges on three factors: product related costs, i.e., maintenance & operating cost and manufacturing cost, customer heterogeneity, and the high segment customer proportion. Our numerical findings show that for high cost products, servicizing has better environmental performance compared to selling. When the product related costs are at intermediate level, servicizing is environmentally superior unless the high segment customer population is low. When the product costs are low, customer heterogeneity and the high segment proportion determine the environmental superiority. This study also shows that environmental impact differences due to use and manufacturing have a limited effect on this comparison. Considering all factors, we see that in most cases servicizing is the environmentally preferable choice. Environmental impact of servicizing is higher only for a narrow range of conditions. These findings support the arguments that servicizing might be a greener alternative.

Observing the same three factors, we define the conditions where each business model is both more profitable and more environmentally friendly. This thesis shows that except for the low cost products, servicizing is more profitable and greener when customer heterogeneity is low and more than a small proportion of the market is high segment. We show that when the costs are low enough, servicizing cannot be both more profitable and the environmentally preferable alternative. Also, when product costs are high enough, selling cannot be both more profitable and environmentally better.

Since servicizing implies firm's ownership of the product, it may enable numerous customers to use the same product. This can lead to a decrease in total production volume. However, in our model, pooling does not have an impact on the total production volume. Durability model of this thesis implies that total production volume decreases when multiple customers use the product at the same time like sharing a journey in the same car. Otherwise, consecutive usage of the same product will only cause quicker depreciation without causing a decrease in total production volume.

This study can be extended to include hybrid models where the firm offers both servicizing and selling. It might be important to see how consumers behave when they have options to purchase the service or the product. Durability choices of the firm might then differ for selling and servicizing. Another extension might be studying this comparison under a competitive market. Customer and firm behaviour might change under a competitive setting which may lead to different profitability and environmental impact consequences for both selling and servicizing.

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APPENDICES

A. Proof of Lemma 3.2.1

Customer type i maximizes his utility $U_i(q_i, f, \delta) = \theta_i q_i - \frac{q_i^2}{2} - q_i f$ while determining his usage amount. Then, FOC and SOC derivations are as follows:

$$\frac{dU_i(q_i, f, \delta)}{dq_i} = \theta_i - q_i - f$$
$$\frac{d^2U_i(q_i, f, \delta)}{dq_i^2} = -1 < 0$$

Since $U_i(q_i, f, \delta)$ is a concave function, we can derive the optimal usage amount q_i from the FOC above. Since the feasible interval of usage amount is the non-negative range of real numbers, we conclude that $q_i^*(\delta, f) = (\theta_i - f)^+$.

B. Proof of Lemma 3.2.3

Assume that q_{max} represents the maximum possible usage amount of a product in a single period (generally due to time limit). For example, a washing machine spending 2 hours for a single wash, can take a maximum of 12 loads a day.

In a servicizing business model, it is possible that more than one customer is served with a single product. Assuming perfect product utilization; i.e., each product is used q_{max} amount per period, the firm needs $\frac{\Omega}{q_{max}}$ active units to meet the demand. The firm does not produce new products until the lifetime of the ones in the market ends. Since each product is designed to last for δ amount of usage, lifetime of the products in the market is $\frac{\delta}{q_{max}}$ periods. If we use Little's Law, $L = \lambda W$, below definitions can be used:

L = $\frac{\Omega}{q_{max}}$: number of products in the system(market) at any time
W = $\frac{\delta}{q_{max}}$: the average time a product spent in the system (average durability period)
λ : average effective production rate(units / period)

It can be seen that by definition $\lambda = Q$. Therefore, Q can be derived as:

$$Q = \frac{L}{W} = \frac{\Omega}{q_{max}} / \frac{\delta}{q_{max}}$$
$$Q = \frac{\Omega}{\delta}$$

C. Proof of Proposition 3.2.4

Recall that profit function of the servicizing firm is defined as:

$$\Pi_{serv} = \begin{cases} M(\theta_{avg} - f)(f - \delta c - m(\delta)) & \text{if } 0 < f < \theta_L \\ \beta M(\theta_H - f)(f - \delta c - m(\delta)) & \text{if } \theta_L \leq f < \theta_H \\ 0 & \text{if } \theta_H \leq f \end{cases}$$

As a piecewise function the sub-functions of Π_{serv} can be defined as:

$$\Pi^1_{serv} = M(\theta_{avg} - f)(f - \delta c - m(\delta))$$
$$\Pi^2_{serv} = \beta M(\theta_H - f)(f - \delta c - m(\delta))$$
$$\Pi^3_{serv} = 0$$

Lemma C.1. Π_{serv} *is continuous over* $f \in [0, \infty]$

Proof of Lemma C.1. We know that as polynomial functions, Π^1_{serv} and Π^2_{serv} are continuous in f.

Π_{serv} is also continuous over $f \in [0, \infty]$ since $\lim_{f \to \theta_L}(\Pi^1_{serv}) = \lim_{f \to \theta_L}(\Pi^2_{serv})$ and $\lim_{f \to \theta_H}(\Pi^2_{serv}) = 0$ as shown below:

$$\lim_{f \to \theta_L} \Pi^1_{serv} \stackrel{?}{=} \lim_{f \to \theta_L} \Pi^2_{serv}$$
$$M(\theta_{avg} - \theta_L)(\theta_L - \delta c - m(\delta)) \stackrel{?}{=} \beta M(\theta_H - \theta_L)(\theta_L - \delta c - m(\delta))$$
$$M(\theta_{avg} - \theta_L) \stackrel{?}{=} \beta M(\theta_H - \theta_L)$$
$$\beta M(\theta_H - \theta_L) = \beta M(\theta_H - \theta_L)$$
$$\lim_{f \to \theta_H} \Pi^2_{serv} \stackrel{?}{=} \Pi^3_{serv}$$
$$\lim_{f \to \theta_H} \beta M(\theta_H - f)(f - \delta c - m(\delta)) \stackrel{?}{=} 0$$
$$\beta M(\theta_H - \theta_H)(\theta_H - \delta c - m(\delta)) = 0$$

■

Lemma C.2. *When $f < \theta_L$, Π_{serv} is concave in f.*

Proof of Lemma C.2. In this interval $\Pi_{serv} = \Pi^1_{serv}$. Thus, it is sufficient to show that $\frac{d^2\Pi^1_{serv}}{df^2} < 0$.

$$\frac{d^2\Pi^1_{serv}}{df^2} = \frac{d^2}{df^2}(M(\theta_{avg} - f)(f - \delta c - m(\delta))) \overset{?}{<} 0$$
$$\frac{d(M(-(f - \delta c - m(\delta) + \theta_{avg} - f)))}{f} = -2M < 0$$

■

Lemma C.3. *When $\theta_L \le f < \theta_H$, Π_{serv} is concave in f.*

Proof of Lemma C.3. In this interval $\Pi_{serv} = \Pi^2_{serv}$. Thus, it is sufficient to show that $\frac{d^2\Pi^2_{serv}}{df^2} < 0$.

$$\frac{d^2\Pi^2_{serv}}{df^2} = \frac{d^2}{df^2}(\beta M(\theta_H - f)(f - \delta c - m(\delta))) \overset{?}{<} 0$$
$$\frac{d(\beta M(-(f - \delta c - m(\delta)) + \theta_H - f))}{df} = -2\beta M < 0$$

■

Since concavity is proven, unconstrained maximizer of Π^1_{serv}, that we denote by f', is calculated by solving $\frac{\Pi^1_{serv}}{df} = 0$.

$$M(m(\delta) - 2f' + c\delta + \beta\theta_H + \theta_L - \beta\theta_L) = 0$$
$$M(m(\delta) - 2f' + c\delta + \theta_{avg}) = 0$$
$$\frac{\left(m(\delta) + c\delta + \theta_{avg}\right)}{2} = f'$$

Similarly, since concavity is proven, the unconstrained maximizer of Π^2_{serv} ,that we denote by f'' is calculated by solving $\frac{d\Pi^2_{serv}}{df} = 0$.

$$\beta M(m(\delta) - 2f'' + c\delta + \theta_H) = 0$$
$$\frac{\left(m(\delta) + c\delta + \theta_H\right)}{2} = f''$$

Note that f'' is always greater than f' due to the assumptions $\theta_H > \theta_L$ and $\beta < 1$:

$$f'' - f' = \frac{\theta_H + m(\delta) + c\delta}{2} - \frac{\theta_{avg} + m(\delta) + c\delta}{2} = \frac{(\theta_H - \theta_L)(1 - \beta)}{2} > 0$$

Profit maximizer service fee, f^*, can be derived by analyzing all possible cases of f' and f'' with respect to the boundaries of pieces in Π_{serv}, i.e. θ_L, θ_H. There are five possible cases of f' and f'':

$$\begin{aligned}
\text{Case 1}: &\quad 0 \leq f', f'' \leq \theta_L \\
\text{Case 2}: &\quad 0 \leq f' \leq \theta_L < f'' \leq \theta_H \\
\text{Case 3}: &\quad 0 < \theta_L < f', f'' \leq \theta_H \\
\text{Case 4}: &\quad 0 \leq f' \leq \theta_L, \theta_H < f'' \\
\text{Case 5}: &\quad 0 < \theta_L < f', \theta_H \leq f''
\end{aligned}$$

Throughout this proof we use $K = m(\delta) + c\delta$ as a shortcut notation.

Case 1:

When $f', f'' < \theta_L$, profit maximizing fee $f^* = f'$. Since Π_{serv} is continuous and Π^2_{serv} is concave, $\Pi^1_{serv}(f') \geq \Pi^1_{serv}(\theta_L) = \Pi^2_{serv}(\theta_L) \geq \Pi^2_{serv}(f)\ \forall f \in [\theta_L, \theta_H)$. Conditions that imply $f', f'' < \theta_L$ can be simplified as below:

$f' \leq \theta_L$:

$$\begin{aligned}
\frac{\theta_{avg} + K}{2} &\leq \theta_L \\
(1-\beta)\theta_L + \beta\theta_H + K &\leq 2\theta_L \\
\beta(\theta_H - \theta_L) &\leq \theta_L - K \\
K &\leq \theta_L - \beta(\theta_H - \theta_L)
\end{aligned}$$

which can be translated into $m(\delta) + c\delta \leq \theta_L - \beta(\theta_H - \theta_L)$.

$f'' \leq \theta_L$:

$$\begin{aligned}
\frac{\theta_H + K}{2} &\leq \theta_L \\
K &\leq \theta_L - (\theta_H - \theta_L)
\end{aligned}$$

which can be translated into $m(\delta) + c\delta \leq \theta_L - (\theta_H - \theta_L)$.

Note that when the second condition holds the first one automatically satisfied. Therefore, $f^* = f' = \frac{m(\delta)+c\delta+\theta_{avg}}{2}$ when $m(\delta) + c\delta \leq \theta_L - (\theta_H - \theta_L)$. This corresponds to a part of the interval in the first condition of Proposition 3.2.4

Case 2:

When $f' \leq \theta_L < f'' \leq \theta_H$, profit maximizing fee f^* is determined based on the comparison of $\Pi^1_{serv}(f')$ and $\Pi^2_{serv}(f'')$. This comparison is valid when

$$\begin{array}{ll} f' \leq \theta_L & \theta_L < f'' \leq \theta_H \\ K \leq \theta_L - \beta(\theta_H - \theta_L) & \theta_L < \frac{\theta_H + K}{2} \leq \theta_H \\ & \theta_L - (\theta_H - \theta_L) < K \leq \theta_H \end{array}$$

Combination of the two conditions can be simplified as:

$$\theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta < \theta_L - \beta(\theta_H - \theta_L) < \theta_H$$

The two functions to be compared can be defined as below:

$$\begin{aligned} \Pi^1_{serv}(f') &= M(\frac{\theta_{avg} - m(\delta) - c\delta}{2})^2 \\ \Pi^2_{serv}(f'') &= M\beta(\frac{\theta_H - c\delta - m(\delta)}{2})^2 \end{aligned}$$

In order to compare the two functions we first derive $\Pi^1_{serv}(f') - \Pi^2_{serv}(f'')$:

$$= \frac{(1-\beta)M\big((c\delta + m(\delta))^2 - \beta\theta_H^2 - 2(c\delta + m(\delta) - \beta\theta_H)\theta_L + (1-\beta)\theta_L^2\big)}{4}$$

Substituting $K = m(\delta) + c\delta$, we search for the threshold level in terms of $m(\delta) + c\delta$:

$$\Pi^1_{serv}(f') - \Pi^2_{serv}(f'') = \frac{(1-\beta)M\big(K^2 - \beta\theta_H^2 - 2(K - \beta\theta_H)\theta_L + (1-\beta)\theta_L^2\big)}{4}$$

It is seen that $\Pi^1_{serv}(f') - \Pi^2_{serv}(f'')$ is convex in $m(\delta) + c\delta$ since,

$$\frac{d^2(\Pi^1_{serv}(f') - \Pi^2_{serv}(f''))}{dK^2} = M\frac{(1-\beta)}{2} > 0$$

To show break-even points for the comparison of $\Pi^1_{serv}(f')$ and $\Pi^2_{serv}(f'')$, we derive the roots of $K = m(\delta) + c\delta$ for $\Pi^1_{serv}(f') - \Pi^2_{serv}(f'') = 0$. The two solutions are:

$$\begin{aligned} m(\delta) + c\delta &= \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \\ m(\delta) + c\delta &= \theta_L + \sqrt{\beta}(\theta_H - \theta_L) \end{aligned}$$

Note that the second root $m(\delta) + c\delta = \theta_L + \sqrt{\beta}(\theta_H - \theta_L)$ is not in the feasible region for Case 2. Therefore only the first root serves as a break-even point. Considering the convexity of $\Pi^1_{serv}(f') - \Pi^2_{serv}(f'')$, below results are obtained for the feasible interval of Case 2:

$$\Pi^1_{serv}(f') > \Pi^2_{serv}(f'') \quad \text{for} \quad \theta_L - (\theta_H - \theta_L) \ < m(\delta) + c\delta \ < \ \theta_L - \sqrt{\beta}(\theta_H - \theta_L)$$
$$\Pi^1_{serv}(f') > \Pi^2_{serv}(f'') \quad \text{for} \quad \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \ \leq m(\delta) + c\delta \ < \ \theta_L - \beta(\theta_H - \theta_L)$$

Above derivations show the profit maximizing fee when $f' < \theta_L < f'' < \theta_H$ is as below:

$$f^* = f' = \frac{\left(m(\delta) + c\delta + \theta_{avg}\right)}{2} \quad \text{for} \quad \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta < \theta_L - \sqrt{\beta}(\theta_H - \theta_L)$$
$$f^* = f'' = \frac{\left(m(\delta) + c\delta + \theta_H\right)}{2} \quad \text{for} \quad \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \leq m(\delta) + c\delta < \theta_L - \beta(\theta_H - \theta_L)$$

This explains a part of the second condition and together with Case 1 completes the proof of the first condition in Proposition 3.2.4

Case 3:

When $\theta_L < f', f'' < \theta_H$, profit maximizing fee is $f^* = f''$ .Since Π^1_{serv} and Π^2_{serv} are both concave in f, $\Pi^1_{serv}(\theta_L) = \Pi^2_{serv}(\theta_L) < \Pi^2_{serv}(f) \ \forall f \in (\theta_L, \theta_H)$. The conditions can be simplified as below:

$\theta_L < f'$:

$$m(\delta) + c\delta > \theta_L - \beta(\theta_H - \theta_L)$$

$\theta_L < f''$:

$$m(\delta) + c\delta > \theta_L - (\theta_H - \theta_L)$$

$f', f'' < \theta_H$:

$$\frac{\theta_H + m(\delta) + c\delta}{2} < \theta_H \quad \text{which implies} \quad m(\delta) + c\delta < \theta_H$$

Aggregating the conditions would give the corresponding interval of $m(\delta) + c\delta$ as:

$$\theta_L - \beta(\theta_H - \theta_L) < m(\delta) + c\delta < \theta_H$$

Therefore, the profit maximizing fee when $\theta_L < f', f'' < \theta_H$ can be shown as:

$$f^* = f'' = \frac{\left(m(\delta) + c\delta + \theta_H\right)}{2} \quad \text{for} \quad \theta_L - \beta(\theta_H - \theta_L) < m(\delta) + c\delta < \theta_H$$

Together with Case 2, this case completes the proof of the second condition of Proposition 3.2.4.

Case 4:

When $\theta_H < f''$ and $0 < f' < \theta_L$, this case arises when

$$\begin{aligned} \theta_H &< f'' = \frac{\theta_H + m(\delta) + c\delta}{2} \\ \theta_H &< m(\delta) + c\delta \end{aligned}$$

$f' < \theta_L$ can also be rewritten as:

$$\beta < \frac{\theta_L - m(\delta) - c\delta}{\theta_H - \theta_L}$$

Since $0 < \beta$ and $\theta_H > \theta_L$ both conditions cannot be satisfied together. Then, this case is not feasible.

Case 5:

When $\theta_H < f''$ and $\theta_L < f'$, profit maximizing fee $f^* = [\theta_H, 0)$. Note that Π_{serv} is continuous, both Π^1_{serv} and Π^2_{serv} are concave in f, and $\Pi^3_{serv} = 0$. In this case, it can be stated that $\Pi^1_{serv}(f) = \Pi^1_{serv}(\theta_L) = \Pi^2_{serv}(\theta_L) < \Pi^2_{serv}(f) \leq \Pi^2_{serv}(\theta_H) = 0 \ \forall \ f$.

Conditions of $\theta_H < f''$ and $\theta_L < f'$, can be simplified as:

$$\theta_H < m(\delta) + c\delta \quad \text{and} \quad \theta_L - \beta(\theta_H - \theta_L) < m(\delta) + c\delta$$

Since $\theta_L - \beta(\theta_H - \theta_L) < m(\delta) + c\delta$ is satisfied when $\theta_H < m(\delta) + c\delta$, it can be concluded that for $\theta_H < m(\delta) + c\delta$, $f^* = [\theta_H, 0)$. This case completes the proof of the third condition in the proposition.

To summarize, optimal service fee decision can be written as follows:

$$f^* = \begin{cases} \frac{\theta_{avg} + m(\delta) + c\delta}{2} & \text{if} \quad m(\delta) + c\delta \leq \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \text{ (Case 1 and Case 2)} \\ \frac{\theta_H + m(\delta) + c\delta}{2} & \text{if} \quad \theta_L - \sqrt{\beta}(\theta_H - \theta_L) < m(\delta) + c\delta < \theta_H \text{ (Case 2 and Case 3)} \\ [\theta_H, \infty) & \text{if} \quad m(\delta) + c\delta \geq \theta_H \text{ (Case 5)} \end{cases}$$

D. Proof of Proposition 3.2.6

To consolidate all terms of durability(δ) together, we first define a term $D(\delta)$, as $D(\delta) = m(\delta) + c\delta = \frac{m}{\delta} + c\delta$. To define the range of $D(\delta)$, we first try to find out the behaviour of the function by analyzing first and second derivatives with respect to δ.

$$\frac{d^2(D(\delta))}{d\delta^2} = \frac{d^2(\frac{m}{\delta} + c\delta)}{d\delta^2} = 2\frac{m}{\delta^3} > 0$$

Second order derivative shows that $D(\delta)$ is always convex in δ. Convexity indicates that a real solution of $\frac{d(D(\delta))}{d\delta} = 0$ will minimize $D(\delta)$.

$$\begin{aligned}\frac{d(D(\delta))}{d\delta} = \frac{d(\frac{m}{\delta} + c\delta)}{d\delta} &= 0 \\ -\frac{m}{\delta^2} + c &= 0 \\ \delta &= \sqrt{\frac{m}{c}}\end{aligned}$$

Using this solution, we can deduce that $D(\sqrt{\frac{m}{c}})$ is the minimum point of $D(\delta)$. Thus, we can define the lower limit as $D(\sqrt{\frac{m}{c}}) = 2\sqrt{mc}$.

Next, we look for the limits with respect to the extremes of the domain $\delta \in (0, \infty)$.

$$\lim_{\delta\to 0}(\frac{m}{\delta} + c\delta) = \lim_{\delta\to 0}\frac{m}{\delta} + \lim_{\delta\to 0} c\delta = \infty$$

$$\lim_{\delta\to \infty}(\frac{m}{\delta} + c\delta) = \lim_{\delta\to \infty}\frac{m}{\delta} + \lim_{\delta\to \infty} c\delta = \infty$$

Observing that as a maximum point, $D(\delta)$ converges to infinity, and minimum point of $D(\delta)$ is $2\sqrt{mc}$, it can be concluded that $D(\delta) \in [2\sqrt{mc}, \infty)$.

Now, using $D(\delta)$, profit function Π^*_{serv} can be rewritten as below:

$$\Pi^*_{serv} = \begin{cases} \frac{M(\theta_{avg} - D(\delta))^2}{4} & \text{if } 0 < D(\delta) \leq \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \\ \frac{M\beta(\theta_H - D(\delta))^2}{4} & \text{if } \theta_L - \sqrt{\beta}(\theta_H - \theta_L) < D(\delta) < \theta_H \\ 0 & \text{if } \theta_H \leq D(\delta) \end{cases}$$

Lemma D.1. *Π^*_{serv} is a decreasing and continuous function in $D(\delta)$.*

$$\begin{aligned}\lim_{D(\delta)\to\theta_L - \sqrt{\beta}(\theta_H - \theta_L)}\left(\frac{M(\theta_{avg} - D(\delta))^2}{4}\right) &=? \lim_{D(\delta)\to\theta_L - \sqrt{\beta}(\theta_H - \theta_L)}\left(\frac{M\beta(\theta_H - D(\delta))^2}{4}\right) \\ \frac{M(\theta_{avg} - (\theta_L - \sqrt{\beta}(\theta_H - \theta_L)))^2}{4} &=? \frac{M\beta(\theta_H - (\theta_L - \sqrt{\beta}(\theta_H - \theta_L)))^2}{4} \\ (\sqrt{\beta}(\sqrt{\beta} + 1)(\theta_H - \theta_L))^2 &= \beta((\sqrt{\beta} + 1)(\theta_H - \theta_L))^2\end{aligned}$$

$$\begin{aligned}\lim_{D(\delta)\to\theta_H}\left(\frac{M\beta(\theta_H - D(\delta))^2}{4}\right) &=? \ 0 \\ \frac{M\beta(\theta_H - \theta_H)^2}{4} &= 0\end{aligned}$$

From above equalities, we see that the function is continuous. Now, we can look into the first order derivatives of subfunctions to capture decreasing nature of Π^*_{serv}.

$$\frac{d(\frac{M(\theta_{avg}-D(\delta))^2}{4})}{dD(\delta)} <? \ 0$$

$$-M\frac{\theta_{avg}-D(\delta)}{2} <? \ 0$$

Since the domain for $\frac{M(\theta_{avg}-D(\delta))^2}{4}$ *is* $D(\delta) \in (0, \theta_L - \sqrt{\beta}(\theta_H - \theta_L)]$, $(\theta_{avg} - D(\delta))$ *is a positive term. As a result, the first order derivative for the first subfunction is negative.*

$$\frac{d(\frac{M\beta(\theta_H-D(\delta))^2}{4})}{dD(\delta)} <? \ 0$$

$$-M\beta\frac{\theta_H-D(\delta)}{2} <? \ 0$$

Since the domain for $\frac{M\beta(\theta_H-D(\delta))^2}{4}$ *is* $D(\delta) \in (\theta_L - \sqrt{\beta}(\theta_H - \theta_L), \theta_H)$, $(\theta_H - D(\delta))$ *is a positive term. As a result, the first order derivative for the second subfunction is also negative. This proves that* Π^*_{serv} *is a decreasing and continuous function in its domain.*

Lemma proves that as $D(\delta)$ increases, Π^*_{serv} decreases when $D(\delta) < \theta_H$. That being the case, the maximum value of Π^*_{serv} can be achieved when $D(\delta)$ is minimum. Since the minimum value for $D(\delta)$ is $D(\sqrt{\frac{m}{c}})$, for $D(\delta) < \theta_H$, $\delta = \sqrt{\frac{m}{c}}$ is the profit maximizing durability level. When $D(\delta) \geq \theta_H$, durability level does not have an effect on profit since the profit is always zero. Then, δ may take any value on its domain.

$$\delta^* = \begin{cases} \sqrt{\frac{m}{c}} & \text{if} \quad 2\sqrt{mc} < \theta_H \\ (0, \infty) & \text{if} \quad 2\sqrt{mc} \geq \theta_H \end{cases}$$

E. Proof of Lemma 4.2.1

Customer type i maximizes his per-period utility $U_i = \theta_i q_i - \frac{1}{2}q_i^2 - q_i - m(\delta)$ to determine his usage level.

$$FOC : \frac{dU_i(q_i, \delta)}{dq_i} = \theta_i - q_i - m(\delta)$$

$$SOC : \frac{d^2U_i(q_i, \delta)}{dq_i^2} = -1 < 0$$

By concavity, the first order condition above is sufficient to guarantee optimality. Thus, $q_i^* = (\theta_i - m(\delta))^+$.

F. Proof of Lemma 4.2.2

Implementing optimal usage levels of q_i^*, per period gross utility of a consumer can be characterized as

$$\begin{aligned} U_i^* &= \theta_i(\theta_i - m(\delta)) - \frac{(\theta_i - m(\delta))^2}{2} - (\theta_i - m(\delta))m(\delta) \\ &= \frac{\theta_i^2}{2} - \theta_i m(\delta) + \frac{m(\delta)^2}{2} = \frac{(\theta_i - m(\delta))^2}{2} \end{aligned}$$

Then, purchasing condition $U_i^* \frac{\alpha\delta}{q_i^*} \geq p$ can be simplified as:

$$\Big(\frac{(\theta_i - m(\delta))^2}{2}\Big) \cdot \Big(\frac{\alpha\delta}{\theta_i - m(\delta)}\Big) = \frac{\alpha\delta(\theta_i - m(\delta))}{2} \geq p$$

G. Proof of Lemma 4.2.4

We can use Little's Law to calculate the number of products manufactured per-period in the steady state. Here, we define the number of products in the market (in use) as L, the average number of periods each product spends in the system (average durability period) as W, and average production rate (units / period) as λ. Note that, then we have $\lambda = Q$. In order to find Q, we can evaluate the market for two customer segments separately. For the low segment, $L_L = (1 - \beta)M$ and for the high segment, $L_H = \beta M$ since each product is dedicated to one customer. The average time a product endures can be calculated by considering two customer segments' usage levels,

$$W_L = \frac{\delta}{q_L}, \qquad W_H = \frac{\delta}{q_H}$$

Using Little's Law we can formulate the production rate as below: When the firm sells to both customer segments, we need to sum production rates needed for the two segments:

$$\begin{aligned} Q = \lambda &= \frac{L_L}{W_L} + \frac{L_H}{W_L} \\ Q = \lambda &= \frac{(1-\beta)Mq_L}{\delta} + \frac{\beta M q_H}{\delta} \\ Q = \lambda &= \frac{\Omega}{\delta} \end{aligned}$$

When the firm sells to high segment,

$$Q = \lambda = \frac{L_H}{W_L} = \frac{\beta M q_H}{\delta} = \frac{\Omega}{\delta}$$

It can be seen that for both market coverage cases that aggregate usage is defined, $Q = \frac{\Omega}{\delta}$, thus proving the Lemma.

H. Proof of Proposition 4.2.5

Lemma H.1. *When $m(\delta) < \theta_{avg} = \beta\theta_H + (1-\beta)\theta_L$, Π^1_{sell} and Π^2_{sell} are continuous increasing functions of p in their respective feasible intervals.*

Proof of Lemma H.1. Both Π^1_{sell} and Π^2_{sell} are in the form of linear polynomial functions where coefficients of p are $\frac{M(\theta_{avg}-m(\delta))}{\delta}$ and $\frac{M\beta(\theta_H-m(\delta))}{\delta}$ respectively.

When price is in the interval of $\frac{\delta(\theta_L-m(\delta))}{2} \geq p > 0$, positive aggregate usage means:

$$\Omega = (\theta_{avg} - m(\delta))M > 0$$

That means coefficient of p, i.e. $\frac{M(\theta_{avg}-m(\delta))}{\delta}$ is also positive, and Π^1_{sell} is increasing in p.

When price is in the interval of $\frac{\alpha\delta(\theta_H-m(\delta))}{2} \geq p > \frac{\alpha\delta(\theta_L-m(\delta))}{2}$, positive aggregate usage means:

$$\Omega = \beta(\theta_H - m(\delta))M > 0$$

That means coefficient of p, i.e. $\frac{M\beta(\theta_H-m(\delta))}{\delta}$ is also positive, and Π^2_{sell} is increasing in p.

■

Based on the structure of Π_{sell} and the previous lemma, we can characterize Π_{sell} with respect to $m(\delta)$ as follows:

	Π^1_{sell}	Π^2_{sell}
$m(\delta) < \theta_L$	increasing	increasing
$\theta_L \leq m(\delta) < \theta_{avg}$	not valid	increasing
$\theta_{avg} \leq m(\delta) < \theta_H$	not valid	increasing
$\theta_H \leq m(\delta)$	not valid	not valid

The "not valid" expression represents cases where $\Pi^{1(2)}_{sell}$ would never arise for any $p \geq 0$. When Π^1_{sell} and(or) Π^2_{sell} are increasing, they are maximized at the upper bound p of their feasible range; i.e., at

$$p' = \frac{\alpha\delta(\theta_L - m(\delta))}{2} \quad and \quad p'' = \frac{\alpha\delta(\theta_H - m(\delta))}{2} \quad \text{respectively.}$$

The profit functions' maximum values are then:

$$\begin{aligned}\Pi^1_{sell}(p') &= M\frac{(\theta_{avg} - m(\delta))\left(\frac{\delta(\theta_L - m(\delta)}{2\alpha} - c\delta^2\right)}{\delta} \\ &= M\frac{(\theta_L - m(\delta) - 2\alpha c\delta)(\theta_{avg} - m(\delta))}{2\alpha}\end{aligned}$$

$$\begin{aligned}\Pi^2_{sell}(p'') &= M\frac{\beta(\theta_H - m(\delta))\left(\frac{\delta(\theta_H - m(\delta)}{2\alpha} - c\delta^2\right)}{\delta} \\ &= M\beta\frac{(\theta_H - m(\delta) - 2\alpha c\delta)(\theta_H - m(\delta))}{2\alpha}\end{aligned}$$

When $m(\delta) < \theta_L$, Π^1_{sell} and Π^2_{sell} are both valid. Then, we must compare $\Pi^1_{sell}(p')$, $\Pi^2_{sell}(p'')$ and $\Pi^3_{sell} = 0$ to find the profit maximizing price. Here, both $\Pi^1_{sell}(p')$ and $\Pi^2_{sell}(p'')$ are positive and need to be compared when $\theta_L > m(\delta) + 2c\alpha\delta$.

$\Pi^1_{sell}(p') \leq 0$ and $\Pi^2_{sell}(p'') > 0 = \Pi^3_{sell}$ if $\theta_L \leq m(\delta) + 2c\alpha\delta < \theta_H$ and

$\Pi^1_{sell}(p') < 0$ and $\Pi^2_{sell}(p'') \leq 0 = \Pi^3_{sell}$ if $\theta_H < m(\delta) + 2c\alpha\delta < \theta_H$

When $\theta_L \leq m(\delta) < \theta_H$, we compare only Π^2_{sell} versus $\Pi^3_{sell} = 0$ since Π^1_{sell} is not valid. In this range, we have,

$\Pi^2_{sell}(p'') > 0 = \Pi^3_{sell}$ if $\theta_H > m(\delta) + 2c\alpha\delta$ and

$\Pi^2_{sell}(p'') \leq 0 = \Pi^3_{sell}$ if $\theta_H \leq m(\delta) + 2c\alpha\delta < \theta_H$

When $\theta_H < m(\delta)$, we have the only valid case, $\Pi^3_{sell} = 0$.

When $m(\delta)+c\alpha\delta < \theta_L - m(\delta) - c\alpha\delta$, where both $\Pi^1_{sell}(p')$ and $\Pi^2_{sell}(p'')$ are positive, optimal price is determined by comparing $\Pi^1_{sell}(p')$ and $\Pi^2_{sell}(p'')$. When we analyze how $\Pi^1_{sell}(p') - \Pi^2_{sell}(p'')$ changes with respect to β, it is seen that the function is decreasing at a constant rate.

$$\frac{d(\Pi^1_{sell}(p') - \Pi^2_{sell}(p''))}{d\beta} < 0$$
$$M\frac{\left(\frac{\delta(\theta_L - m(\delta))}{2\alpha} - c\delta^2\right)(\theta_H - \theta_L)}{\delta} - M\frac{\left(\frac{\delta(\theta_H - m(\delta))}{2\alpha} - c\delta^2\right)(\theta_H - m(\delta))}{\delta} < 0$$
$$-M\frac{m(\delta)^2 + \theta_H^2 - 2\alpha c\delta(\theta_L - m(\delta)) - \theta_H\theta_L + \theta_L^2 - m(\delta)(\theta_H + \theta_L)}{2\alpha} < 0$$
$$-M\frac{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2\alpha c\delta) + (\theta_H - m(\delta))(\theta_H - \theta_L) + 2\theta_L m(\delta)}{2\alpha} < 0$$

Solving for $\Pi^1_{sell}(p') - \Pi^2_{sell}(p'') = 0$ shows that the threshold level of β is when

$$\beta = \beta_L = \frac{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\alpha\delta)}{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2\alpha c\delta) + (\theta_H - m(\delta))(\theta_H - \theta_L)} < 1$$

Threshold level β_L shows that:

$$\begin{aligned} \Pi^1_{sell}(p') \geq \Pi^2_{sell}(p'') \quad &\text{if} \quad 0 < \beta \leq \beta_L \\ \Pi^1_{sell}(p') < \Pi^2_{sell}(p'') \quad &\text{if} \quad 0 < \beta_L < \beta \end{aligned}$$

We can combine all of our findings above as follows:

$$p^* = \begin{cases} p' = \frac{\alpha\delta(\theta_L - m(\delta))}{2} & \text{if } \theta_L > m(\delta) + 2c\alpha\delta \text{ and } \beta \leq \beta_L \\ p'' = \frac{\alpha\delta(\theta_H - m(\delta))}{2} & \text{if } \theta_L > m(\delta) + 2c\alpha\delta \text{ and } \beta > \beta_L \text{ or } \theta_H > m(\delta) + 2c\alpha\delta \geq \theta_L \\ \left[\frac{\alpha\delta(\theta_H - m(\delta))}{2}, \infty\right) & \text{if } m(\delta) + 2c\alpha\delta \geq \theta_H \end{cases}$$

I. Proof of Corollary 5.0.1

For servicizing business model, piecewise conditions of the profit function are the conditions of optimal service fee as in Proposition 3.2.4:

$$f^* = \begin{cases} \frac{\theta_{avg} + m(\delta) + c\delta}{2} & \text{if} \quad m(\delta) + c\delta \leq \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \\ \frac{\theta_H + m(\delta) + c\delta}{2} & \text{if} \quad \theta_L - \sqrt{\beta}(\theta_H - \theta_L) < m(\delta) + c\delta < \theta_H \\ [\theta_H, \infty) & \text{if} \quad m(\delta) + c\delta \geq \theta_H \end{cases}$$

We can rewrite $m(\delta) + c\delta \le \theta_L - \sqrt{\beta}(\theta_H - \theta_L)$ as:

$$\begin{aligned} m(\delta) + c\delta &\le \theta_L - \sqrt{\beta}(\theta_H - \theta_L) \\ \sqrt{\beta}(\theta_H - \theta_L) &\le \theta_L - m(\delta) - c\delta \\ \beta &\le (\frac{\theta_L - m(\delta) - c\delta}{\theta_H - \theta_L})^2 \end{aligned}$$

This shows that we can use a threshold level $\beta = \beta_V = (\frac{\theta_L - m(\delta) - c\delta}{\theta_H - \theta_L})^2$. We know that this threshold level is defined for the domain $\beta \in (0, 1)$. Therefore, it is functional between $m(\delta) + c\delta \in (\theta_L - (\theta_H - \theta_L), \theta_L)$ where

$$\begin{aligned} \lim_{\beta \to 1} \theta_L - \sqrt{\beta}(\theta_H - \theta_L) =& \theta_L - (\theta_H - \theta_L) \\ \lim_{\beta \to 0} \theta_L - \sqrt{\beta}(\theta_H - \theta_L) =& \theta_L \end{aligned}$$

Then, servicizing profit function can be written with conditions in terms of β and $m(\delta) + c\delta$ as below:

$$\Pi^*_{serv} = \begin{cases} M(\frac{\theta_{avg} - m(\delta) - c\delta}{2})^2 & \text{if} \quad m(\delta) + c\delta \le \theta_L - (\theta_H - \theta_L) \\ M(\frac{\theta_{avg} - m(\delta) - c\delta}{2})^2 & \text{if} \quad \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta \le \theta_L \text{ and } \beta \le \beta_V \\ M\beta(\frac{\theta_H - m(\delta) - c\delta}{2})^2 & \text{if} \quad \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta \le \theta_L \text{ and } \beta > \beta_V \\ M\beta(\frac{\theta_H - m(\delta) - c\delta}{2})^2 & \text{if} \quad \theta_L \le m(\delta) + c\delta < \theta_H \\ 0 & \quad \theta_H \le m(\delta) + c\delta \end{cases}$$

For the selling business model, the piecewise conditions for the profit function are the conditions in Proposition 4.2.5.

J. Proof of Lemma 5.1.1

Rewriting $\beta_L < \beta_V$ would give:

$$\frac{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta)}{(\theta_H - \theta_L)^2 + \theta_H\theta_L + m(\delta)^2 - 2c\delta(\theta_L - m(\delta)) - m(\delta)(\theta_H + \theta_L)} <? \frac{(\theta_L - m(\delta) - c\delta)(\theta_L - m(\delta) - c\delta)}{(\theta_H - \theta_L)^2}$$

Here, we made the substitution of $A = \theta_L - m(\delta) - c\delta$

$$\begin{aligned} \frac{(A + c\delta)(A - c\delta)}{(\theta_H - \theta_L)^2 + \theta_H\theta_L + m(\delta)^2 - 2c\delta(\theta_L - m(\delta)) - m(\delta)(\theta_H + \theta_L)} &<? \frac{A^2}{(\theta_H - \theta_L)^2} \\ \frac{A^2 - (c\delta)^2}{(\theta_H - \theta_L)^2 + (\theta_L - m(\delta))(\theta_H - m(\delta) - 2c\delta)} &< \frac{A^2}{(\theta_H - \theta_L)^2} \end{aligned}$$

The denominator of the expression in the left is greater while its nominator is smaller when $m(\delta) + c\delta \leq \theta_H - c\delta$. Thus, $\beta_L < \beta_V$ when $m(\delta) + c\delta \leq \theta_H - c\delta$.

K. Proof of Proposition 5.1.2

To determine the comparison intervals we need to know the order of threshold levels for β and $m(\delta) + c\delta$. We showed previously that $\beta_L < \beta_V$ in the comparable region, i.e. when $m(\delta) + c\delta < \theta_H - c\delta$. It can be observed that level of $\theta_H - \theta_L$ defines the order of the threshold values for $m(\delta) + c\delta$.

When $\theta_H - \theta_L \geq c\delta$;

$$0 < \theta_L - (\theta_H - \theta_L) \leq \theta_L - c\delta < \theta_L \leq \theta_H - c\delta < \theta_H$$

When $\frac{c\delta}{2} \leq \theta_H - \theta_L < c\delta$;

$$0 < \theta_L - c\delta < \theta_L - (\theta_H - \theta_L) \leq \theta_H - c\delta < \theta_L < \theta_H$$

When $\theta_H - \theta_L < \frac{c\delta}{2}$

$$0 < \theta_L - c\delta < \theta_H - c\delta < \theta_L - (\theta_H - \theta_L) < \theta_L < \theta_H$$

Therefore, comparison intervals of $m(\delta) + c\delta$ differs as the magnitude of $\theta_H - \theta_L$ changes. By combining Table 5.1 and Table 5.2 below table can be synthesized to show resulting comparison intervals for different market parameters .

Table K.1: Comparison Intervals of Selling and Servicizing Profit Functions

Comparison	$\boldsymbol{\beta}$	$\boldsymbol{m(\delta)+c\delta}$	$\boldsymbol{\theta_H-\theta_L}$
Π^1_{serv} vs Π^1_{sell}	$(0,\beta_L]$	$(0,\theta_L-c\delta]$	$(0,\infty)$
Π^1_{serv} vs Π^2_{sell}	$(\beta_L,1)$	$(0,\theta_L-c\delta]$	$(0,c\delta]$
		$(\theta_L-c\delta,\theta_H-c\delta]$	$(0,\frac{c\delta}{2}]$
		$(\theta_L-c\delta,\theta_L-(\theta_H-\theta_L)]$	$(\frac{c\delta}{2},c\delta]$
	$(0,\beta_V]$	$(\theta_L-(\theta_H-\theta_L),\theta_H-c\delta]$	$(\frac{c\delta}{2},c\delta]$
	$(\beta_L,1)$	$(0,\theta_L-(\theta_H-\theta_L)]$	$[c\delta,\infty)$
	$(\beta_L,\beta_V]$	$(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta]$	$[c\delta,\infty)$
	$(0,\beta_V]$	$(\theta_L-c\delta,\theta_L]$	$[c\delta,\infty)$
Π^2_{serv} vs Π^2_{sell}	$(\beta_V,1)$	$(\theta_L-(\theta_H-\theta_L),\theta_H-c\delta]$	$(\frac{c\delta}{2},c\delta]$
	$(\beta_V,1)$	$(\theta_L-(\theta_H-\theta_L),\theta_L]$	$[c\delta,\infty)$
		$(\theta_L,\theta_H-c\delta]$	$[c\delta,\infty)$
Π_{serv} vs 0		$(\theta_H-c\delta,\theta_H)$	

Note that each result in the proposition is based on the comparison defined above. As it can be seen in the table above, there are three types of comparison of the two functions: (Π^1_{sell} vs Π^1_{serv}), (Π^2_{sell} vs Π^1_{serv}) and (Π^2_{sell} vs Π^2_{serv}). Now, we will analyze each comparison and how they correspond to each result in the proposition.

Results numbered as 1,2, 17 and 18 are due to the comparison of Π^1_{serv} and Π^1_{sell}. For the analysis we use the information of how $\Pi^1_{sell}-\Pi^1_{serv}$ behaves as a function of β.

Lemma K.1. $\Pi^1_{sell}-\Pi^1_{serv}$ *is concave with respect to* β *since*

$$\frac{d^2(\Pi^1_{sell}-\Pi^1_{serv})}{d\beta^2}=-M\frac{(\theta_H-\theta_L)^2}{2}<0$$

There are two roots of $\Pi^1_{sell}-\Pi^1_{serv}=0$

$$\beta^1_{LV1}=\frac{-\sqrt{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}-c\delta}{(\theta_H-\theta_L)}$$

$$\beta^2_{LV1}=\frac{-\sqrt{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}+c\delta}{\theta_H-\theta_L}$$

Since $(\theta_L - m(\delta) - 2c\delta) \geq 0$ where Π^1_{sell} and Π^1_{serv} are compared, the terms in the square root cannot be negative. Thus, $\beta^1_{LV1}, \beta^2_{LV1} \in \mathbb{R}$ in the proposed results. We also know that $\beta^1_{LV1} < 0$ in the comparison interval since $(\theta_L - m(\delta) - 2c\delta) \geq 0$, which is equivalent to $m(\delta) + c\delta \leq \theta_L - c\delta$. We denote $\beta_{LV1} = \beta^2_{LV1}$ as the threshold level of β in $\Pi^1_{sell}, \Pi^1_{serv}$ comparison. Below conditions summarize the basis for results 1,2, 17 and 18 in the proposition.

$$\begin{aligned} \Pi^1_{serv} > \Pi^1_{sell} \quad &\text{if} \quad \beta < \beta_{LV1} \\ \Pi^1_{serv} \leq \Pi^1_{sell} \quad &\text{if} \quad \beta_{LV1} \leq \beta \end{aligned}$$

Since when $m(\delta) + c\delta \leq \theta_L - c\delta$, it is possible for β_{LV1} to take any value. Results for different intervals of β_{LV1} is indicated in results 1,2, 17 and 18 in the proposition.

Results of 3-7, 12-16, 19-26 and 32-38 are due to the comparison of Π^1_{serv} and Π^2_{sell}. For the analysis we use the information of how $\Pi^2_{sell} - \Pi^1_{serv}$ behaves as a function of β.

Lemma K.2. $\Pi^2_{sell} - \Pi^1_{serv}$ *is concave with respect to* β *since*

$$\frac{d^2(\Pi^2_{sell} - \Pi^1_{serv})}{d\beta^2} = -M\frac{(\theta_H - \theta_L)^2}{2} < 0$$

There are two roots of $\Pi^2_{sell} - \Pi^1_{serv} = 0$

$$\begin{aligned} \beta^1_{LV2} = &\frac{(\theta_H - \theta_L)^2 + \theta_H\theta_L - m(\delta)(\theta_H + \theta_L - m(\delta) - 2c\delta) - c\delta(\theta_H + \theta_L)}{(\theta_H - \theta_L)^2} \\ &- \frac{\sqrt{(\theta_H - m(\delta))(\theta_H - m(\delta) - 2c\delta)[(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta) + (\theta_H - \theta_L)^2]}}{(\theta_H - \theta_L)^2} \end{aligned}$$

$$\begin{aligned} \beta^2_{LV2} = &\frac{(\theta_H - \theta_L)^2 + \theta_H\theta_L - m(\delta)(\theta_H + \theta_L - m(\delta) - 2c\delta) - c\delta(\theta_H + \theta_L)}{(\theta_H - \theta_L)^2} \\ &+ \frac{\sqrt{(\theta_H - m(\delta))(\theta_H - m(\delta) - 2c\delta)[(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta) + (\theta_H - \theta_L)^2]}}{(\theta_H - \theta_L)^2} \end{aligned}$$

Lemma K.3. *If* $\frac{(\theta_H - \theta_L)^2 + \theta_L^2 + m(\delta)c\delta}{2\theta_L - m(\delta)} < m(\delta) + c\delta$*, then* $\beta^1_{LV2}, \beta^2_{LV2} \notin \mathbb{R}$ *and* $\Pi^1_{serv} > \Pi^2_{sell}$.

Proof of Lemma K.3. The roots β^1_{LV2} and β^2_{LV2} are imaginary if the expression in the square root is negative, i.e., if $(\theta_H - m(\delta))(\theta_H - m(\delta) - 2c\delta)[(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta) + (\theta_H - \theta_L)^2] < 0$. We know that in the comparable interval $(\theta_H - m(\delta))$ and

$(\theta_H - m(\delta) - 2c\delta)$ are positive. Then the values of β^1_{LV2} and β^2_{LV2} can be imaginary only if below condition is satisfied.

$$(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta) + (\theta_H - \theta_L)^2 < 0$$
$$(\theta_H - \theta_L)^2 + \theta_L^2 + m(\delta)c\delta - (2\theta_L - m(\delta))(m(\delta) + c\delta) < 0$$
$$\frac{(\theta_H - \theta_L)^2 + \theta_L^2 + m(\delta)c\delta}{2\theta_L - m(\delta)} < m(\delta) + c\delta$$

■

Since $\Pi^2_{sell} - \Pi^1_{serv}$ is concave and $\lim_{\beta\to-\infty}(\Pi^2_{sell} - \Pi^1_{serv}) = \lim_{\beta\to\infty}(\Pi^2_{sell} - \Pi^1_{serv}) = -\infty$, when there are no real roots of $\Pi^2_{sell} - \Pi^1_{serv} = 0$, i.e. when $\frac{(\theta_H-\theta_L)^2+\theta_L^2+m(\delta)c\delta}{2\theta_L-m(\delta)} < m(\delta) + c\delta$, Π^1_{serv} is always greater than Π^2_{sell}. Results of 5-7, 21-26 and 36-38 correspond to $\beta^1_{LV2}, \beta^2_{LV2} \notin \mathbb{R}$

Note that the condition for imaginary $\beta^1_{LV2}, \beta^2_{LV2}$ cannot hold when $m(\delta) + c\delta \leq \theta_L - c\delta$, i.e. $0 \leq \theta_L - m(\delta) - 2c\delta$. The condition cannot hold also when $\theta_H - \theta_L \geq c\delta$. It can be shown by substituting $A = \theta_L - m(\delta) - c\delta$ in the condition:

$$(A + c\delta)(A - c\delta) + (\theta_H - \theta_L)^2 < 0$$
$$A^2 - (c\delta)^2 + (\theta_H - \theta_L)^2 < 0$$
$$(\theta_H - \theta_L)^2 < (c\delta)^2 - A^2$$

Thus, it is seen that if $(\theta_H - \theta_L) \in [c\delta, \infty)$ or $(m(\delta) + c\delta) \in (0, \theta_L - c\delta]$, then $\beta^1_{LV2}, \beta^2_{LV2} \in \mathbb{R}$. Therefore, we know that $\beta^1_{LV2}, \beta^2_{LV2} \in \mathbb{R}$ for the results of 3-4, 12-16, 19-20 and 32-35, since these results are derived when $m(\delta) + c\delta \leq \theta_L - c\delta$ or $\theta_H - \theta_L \geq c\delta$ in Table 1.6.

When $\beta^1_{LV2}, \beta^2_{LV2} \in \mathbb{R}$, i.e. $\frac{(\theta_H-\theta_L)^2+\theta_L^2+m(\delta)c\delta}{2\theta_L-m(\delta)} \geq m(\delta) + c\delta$, due to the concavity, below conditions summarize the basis for results 3-4, 12-16, 19-20 and 32-35 in the proposition.

$$\Pi^1_{serv} > \Pi^2_{sell} \quad \text{if} \quad \beta < \beta^1_{LV2} \quad \text{or} \quad \beta^2_{LV2} < \beta$$
$$\Pi^1_{serv} \leq \Pi^1_{sell} \quad \text{if} \quad \beta^1_{LV2} \leq \beta \leq \beta^2_{LV2}$$

$\beta^2_{LV2} \in \mathbb{R}$ is always out of the defined comparison intervals of β as indicated below: We denote $\beta_{LV2} = \beta^1_{LV2}$, as the threshold level of β for $\Pi^2_{sell}, \Pi^1_{serv}$ comparison. While different intervals of β^1_{LV2} are demonstrated as β_{LV2} in the proposition, β^2_{LV2} levels

Table K.2: Intervals of β^2_{LV2} for the defined conditions

$\boldsymbol{\beta^2_{LV2}}$	$\boldsymbol{\beta}$	$\boldsymbol{m(\delta)+c\delta}$	$\boldsymbol{\theta_H-\theta_L}$
$[1,\infty)$	$(\beta_L,1)$	$(0,\theta_L-c\delta]$	$(0,c\delta]$
$[1,\infty)$	$(0,1)$	$(\theta_L-c\delta,\theta_H-c\delta]$	$(0,\frac{c\delta}{2}]$
$[1,\infty)$	$(0,1)$	$(\theta_L-c\delta,\theta_L-(\theta_H-\theta_L)]$	$(\frac{c\delta}{2},c\delta]$
$[\beta_V,\infty)$	$(0,\beta_V]$	$(\theta_L-(\theta_H-\theta_L),\theta_H-c\delta]$	$(\frac{c\delta}{2},c\delta]$
$[1,\infty)$	$(\beta_L,1)$	$(0,\theta_L-(\theta_H-\theta_L)]$	$[c\delta,\infty)$
$[\beta_V,\infty)$	$(\beta_L,\beta_V]$	$(\theta_L-(\theta_H-\theta_L),\theta_L-c\delta]$	$[c\delta,\infty)$
$[\beta_V,\infty)$	$(0,\beta_V]$	$(\theta_L-c\delta,\theta_L]$	$[c\delta,\infty)$

do not take place in the results since they are indirectly indicated by other conditions in the proposition.

Results of 8-11 and 27-31 are due to the comparison of Π^2_{sell} and Π^2_{serv}. For the analysis we use the information of how $\Pi^2_{sell}-\Pi^2_{serv}$ behaves as a function of θ_H.

Lemma K.4. *($\Pi^2_{sell}-\Pi^2_{serv}$) is a convex function in θ_H*

$$\frac{d^2(\Pi^2_{sell}-\Pi^2_{serv})}{\theta_H^2}=\frac{\beta M}{2}>0$$

Solving $\Pi^2_{sell}-\Pi^2_{serv}=0$ will give the break-even point(s) for the comparison.

$$\begin{aligned}
\Pi^2_{sell}-\Pi^2_{serv}&=0\\
M\beta\frac{(\theta_H-m(\delta)-2\alpha c\delta)(\theta_H-m(\delta))}{2}-M\beta(\frac{\theta_H-c\delta-m(\delta)}{2})^2&=0\\
M\beta\frac{(\theta_H-m(\delta))^2-2c\delta(\theta_H-m(\delta))-(c\delta)^2}{4}&=0\\
M\beta\frac{(\theta_H-m(\delta)-c\delta)^2-(\sqrt{2}c\delta)^2}{4}&=0\\
M\beta\frac{(\theta_H-m(\delta)-(1+\sqrt{2})c\delta)(\theta_H-m(\delta)-(1-\sqrt{2})c\delta)}{4}&=0
\end{aligned}$$

Note that the roots are independent of the level of β:

$$(1)\quad \theta_H=m(\delta)+c\delta-\sqrt{2}c\delta$$

$$(2)\quad \theta_H=m(\delta)+c\delta+\sqrt{2}c\delta$$

Both Π^2_{sell} and Π^2_{serv} are defined when $m(\delta)+c\delta<\theta_H-c\delta$. Therefore (1) is out of the feasible region for the comparison. Considering the convexity of $(\Pi^2_{sell}-\Pi^2_{serv})$, defined θ_H levels imply,

$$\Pi^2_{serv} < \Pi^2_{sell} \quad \text{if} \quad \theta_H > m(\delta) + c\delta + \sqrt{2}c\delta$$
$$\Pi^2_{serv} \geq \Pi^2_{sell} \quad \text{if} \quad m(\delta) + 2c\delta \leq \theta_H \leq m(\delta) + c\delta + \sqrt{2}c\delta$$

This result can be rewritten in terms of $m(\delta) + c\delta$ as follows:

$$\Pi^2_{serv} < \Pi^2_{sell} \quad \text{if} \quad m(\delta) + c\delta < \theta_H - \sqrt{2}c\delta$$
$$\Pi^2_{serv} \geq \Pi^2_{sell} \quad \text{if} \quad \theta_H - \sqrt{2}c\delta \leq m(\delta) + c\delta \leq \theta_H - c\delta$$

For the first row for Π^2_{sell} vs Π^2_{serv} in the comparison table,i.e. when $\beta \in (\beta_V, 1)$, $(m(\delta)+c\delta) \in (\theta_L-(\theta_H-\theta_L), \theta_H-c\delta]$ and $(\theta_H-\theta_L) \in (\frac{c\delta}{2}, c\delta]$, $m(\delta)+c\delta = \theta_H-\sqrt{2}c\delta$ is a break-even point if it lies within the defined region of comparison:

$$\theta_L - (\theta_H - \theta_L) < \theta_H - \sqrt{2}c\delta \leq \theta_H - c\delta$$
$$\frac{\sqrt{2}c\delta}{2} < \theta_H - \theta_L$$

Therefore, we can divide the interval of $(\theta_H - \theta_L)$ into two regions where $m(\delta) + c\delta = \theta_H - \sqrt{2}c\delta$ is a break-even point in $(\frac{\sqrt{2}c\delta}{2}, c\delta)$, and it is not a break-even point in $[\frac{c\delta}{2}, \frac{\sqrt{2}c\delta}{2}]$. It implies below conditions:

$$\Pi^2_{serv} > \Pi^2_{sell} \quad \text{if } \beta_V \leq \beta, \quad \frac{c\delta}{2} \leq \theta_H - \theta_L \leq \frac{\sqrt{2}c\delta}{2} \quad , \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta \leq \theta_H - c\delta$$
$$\Pi^2_{serv} > \Pi^2_{sell} \quad \text{if } \beta_V \leq \beta, \quad \frac{\sqrt{2}c\delta}{2} < \theta_H - \theta_L \leq c\delta \quad , \theta_H - \sqrt{2}c\delta < m(\delta) + c\delta \leq \theta_H - c\delta$$
$$\Pi^2_{serv} \leq \Pi^2_{sell} \quad \text{if } \beta_V \leq \beta, \quad \frac{\sqrt{2}c\delta}{2} < \theta_H - \theta_L \leq c\delta \quad , \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta \leq \theta_H - \sqrt{2}c\delta$$

This explains results numbered as 8, 27 and 28 in the table.

For the second row for Π^2_{sell} vs Π^2_{serv} in the comparison table, i.e., when $\beta \in (\beta_V, 1)$, $(m(\delta) + c\delta) \in (\theta_L - (\theta_H - \theta_L), \theta_L]$ and $(\theta_H - \theta_L) \in (c\delta, \infty)$, $m(\delta) + c\delta = \theta_H - \sqrt{2}c\delta$ is a break-even point if it lies within the defined region of comparison:

$$\theta_L - (\theta_H - \theta_L) < \theta_H - \sqrt{2}c\delta \leq \theta_L$$
$$\frac{\sqrt{2}c\delta}{2} < \theta_H - \theta_L \leq \sqrt{2}c\delta$$

Therefore, we can divide the interval of $(\theta_H - \theta_L)$ into two regions where $m(\delta) + c\delta = \theta_H - \sqrt{2}c\delta$ is a break-even point in $(c\delta, \sqrt{2}c\delta)$, and it is not a break-even point in $[\sqrt{2}c\delta, \infty)$. It implies below conditions:

$$\Pi^2_{serv} \leq \Pi^2_{sell} \quad \text{if } \beta_V \leq \beta, \quad \sqrt{2}c\delta \leq \theta_H - \theta_L \quad , \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta \leq \theta_L$$
$$\Pi^2_{serv} > \Pi^2_{sell} \quad \text{if } \beta_V \leq \beta, \quad c\delta < \theta_H - \theta_L \leq \sqrt{2}c\delta \quad , \theta_H - \sqrt{2}c\delta < m(\delta) + c\delta \leq \theta_L$$
$$\Pi^2_{serv} \leq \Pi^2_{sell} \quad \text{if } \beta_V \leq \beta, \quad c\delta < \theta_H - \theta_L \leq \sqrt{2}c\delta \quad , \theta_L - (\theta_H - \theta_L) < m(\delta) + c\delta \leq \theta_H - \sqrt{2}c\delta$$

This explains results 9, 10 and 29 in the table.

When $\beta \in (0,1)$, $(m(\delta) + c\delta) \in (\theta_L, \theta_H - c\delta]$ and $(\theta_H - \theta_L) \in (c\delta, \infty)$, $m(\delta) + c\delta = \theta_H - \sqrt{2}c\delta$ is a break-even point if it lies within the defined region of comparison:

$$\theta_L < \theta_H - \sqrt{2}c\delta \leq \theta_H - c\delta$$
$$\sqrt{2}c\delta < \theta_H - \theta_L$$

Therefore, we can divide the interval of $(\theta_H - \theta_L)$ into two regions where $m(\delta) + c\delta = \theta_H - \sqrt{2}c\delta$ is a break-even point in $(\sqrt{2}c\delta, \infty)$, and it is not a break-even point in $(c\delta, \sqrt{2}c\delta]$. It implies below conditions:

$$\begin{aligned}
&\Pi^2_{serv} > \Pi^2_{sell} \quad \text{if} \quad c\delta < \theta_H - \theta_L \leq \sqrt{2}c\delta \quad , \theta_L < m(\delta) + c\delta \leq \theta_H - c\delta \\
&\Pi^2_{serv} > \Pi^2_{sell} \quad \text{if} \quad \sqrt{2}c\delta < \theta_H - \theta_L \quad , \theta_H - \sqrt{2}c\delta < m(\delta) + c\delta \leq \theta_H - c\delta \\
&\Pi^2_{serv} \leq \Pi^2_{sell} \quad \text{if} \quad \sqrt{2}c\delta < \theta_H - \theta_L \quad , \theta_L < m(\delta) + c\delta \leq \theta_H - \sqrt{2}c\delta
\end{aligned}$$

This explains results 11, 30 and 31 in the proposition.

Result of 39 is due the comparison of Π_{serv} subfunctions with 0 when $m(\delta) + c\delta > \theta_H - c\delta$. Since from the formulation we know that Π^1_{serv} and Π^2_{serv} are always positive, these comparisons always imply the choice for servicizing.

Note that the proposition 5.1.2 does not cover uninteresting cases where $\Pi_{serv} = \Pi_{sell} = 0$. When marginal cost of usage is too high, i.e., $m(\delta) + c\delta > \theta_H$, the market is not profitable for any of the two business models.

L. Expanded Conditions of β_{LV1} and β_{LV2}

Using below definitons of β_{LV1} and β_{LV2}, related conditions in Proposition 5.1.2 can be seen in Table L.3.

$$\beta_{LV1} = \frac{-\sqrt{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta)} + c\delta}{\theta_H - \theta_L}$$
$$\beta_{LV2} = \frac{(\theta_H - \theta_L)^2 + \theta_H\theta_L - m(\delta)(\theta_H + \theta_L - m(\delta) - 2c\delta) - c\delta(\theta_H + \theta_L)}{(\theta_H - \theta_L)^2} - \frac{\sqrt{(\theta_H - m(\delta))(\theta_H - m(\delta) - 2c\delta)[(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta) + (\theta_H - \theta_L)^2]}}{(\theta_H - \theta_L)^2}$$

Table L.3: Expanded Conditions on β_{LV1} and β_{LV2}

Conditions	Expanded Conditions
$\beta_{LV1} \leq 0$	$-c\delta + \sqrt{(\theta_L - m(\delta))(\theta_L - m(\delta) - 2c\delta)} \leq 0$
$0 < \beta_{LV1} \leq \beta_L$	$0 < \frac{-c\delta+\sqrt{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}}{\theta_H-\theta_L} \leq \frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)}$
$\beta_L < \beta_{LV1}$	$\frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)} < \frac{-c\delta+\sqrt{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}}{\theta_H-\theta_L}$
$\beta_{LV2} < \beta_L$	$\frac{(\theta_H-\theta_L)^2+\theta_H\theta_L-m(\delta)(\theta_H+\theta_L-m(\delta)-2c\delta)-c\delta(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2} - \frac{\sqrt{(\theta_H-m(\delta))(\theta_H-m(\delta)-2c\delta)[(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-\theta_L)^2]}}{(\theta_H-\theta_L)^2} < \frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)}$
$0 < \beta_{LV2} < \beta_V$	$0 < \frac{(\theta_H-\theta_L)^2+\theta_H\theta_L-m(\delta)(\theta_H+\theta_L-m(\delta)-2c\delta)-c\delta(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2} - \frac{\sqrt{(\theta_H-m(\delta))(\theta_H-m(\delta)-2c\delta)[(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-\theta_L)^2]}}{(\theta_H-\theta_L)^2} < (\frac{\theta_L-m(\delta)-c\delta}{\theta_H-\theta_L})^2$
$\beta_L < \beta_{LV2} < \beta_V$	$\frac{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)}{(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-m(\delta))(\theta_H-\theta_L)} < \frac{(\theta_H-\theta_L)^2+\theta_H\theta_L-m(\delta)(\theta_H+\theta_L-m(\delta)-2c\delta)-c\delta(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2} - \frac{\sqrt{(\theta_H-m(\delta))(\theta_H-m(\delta)-2c\delta)[(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-\theta_L)^2]}}{(\theta_H-\theta_L)^2} < (\frac{\theta_L-m(\delta)-c\delta}{\theta_H-\theta_L})^2$
$\beta_V < \beta_{LV2} < 1$	$(\frac{\theta_L-m(\delta)-c\delta}{\theta_H-\theta_L})^2 < \frac{(\theta_H-\theta_L)^2+\theta_H\theta_L-m(\delta)(\theta_H+\theta_L-m(\delta)-2c\delta)-c\delta(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2} - \frac{\sqrt{(\theta_H-m(\delta))(\theta_H-m(\delta)-2c\delta)[(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-\theta_L)^2]}}{(\theta_H-\theta_L)^2} < 1$
$1 < \beta_{LV2}$	$1 < \frac{(\theta_H-\theta_L)^2+\theta_H\theta_L-m(\delta)(\theta_H+\theta_L-m(\delta)-2c\delta)-c\delta(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2} - \frac{\sqrt{(\theta_H-m(\delta))(\theta_H-m(\delta)-2c\delta)[(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-\theta_L)^2]}}{(\theta_H-\theta_L)^2}$
$0 < \beta_{LV2} < 1$	$0 < \frac{(\theta_H-\theta_L)^2+\theta_H\theta_L-m(\delta)(\theta_H+\theta_L-m(\delta)-2c\delta)-c\delta(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2} - \frac{\sqrt{(\theta_H-m(\delta))(\theta_H-m(\delta)-2c\delta)[(\theta_L-m(\delta))(\theta_L-m(\delta)-2c\delta)+(\theta_H-\theta_L)^2]}}{(\theta_H-\theta_L)^2} < 1$

M. Proof of Proposition 5.2.1

Environmental impact of servicizing and selling business models are formulated as follows:

$$E_{serv} = e_u\Omega_{serv} + e_m\frac{\Omega_{serv}}{\delta} = \Omega_{serv}(e_u + \frac{e_m}{\delta})$$
$$E_{sell} = e_u\Omega_{sell} + e_m\frac{\Omega_{sell}}{\delta} = \Omega_{sell}(e_u + \frac{e_m}{\delta})$$

It is seen that when δ is given, the environmental impact comparison depends on the comparison of Ω_{serv} vs Ω_{sell}. To compare these two piecewise functions, we refer Appendix K for the comparison intervals derived for profitability comparison. Since comparison intervals are the same for aggregate usage, we can derive below table from the comparison intervals in Appendix K. Note that we do not include the comparison of Ω_{serv} vs $\Omega_{sell} = 0$ where selling is no longer profitable. We compare environmental impacts when both business models are profitable.

Table M.4: Comparison Intervals for Selling and Servicizing Aggregate Usage Levels

Comparison	$\boldsymbol{\beta}$	$\boldsymbol{m(\delta) + c\delta}$	$\boldsymbol{\theta_H - \theta_L}$
Ω^1_{serv} vs Ω^1_{sell}	$(0, \beta_L]$	$(0, \theta_L - c\delta]$	$(0, \infty)$
Ω^1_{serv} vs Ω^2_{sell}	$(\beta_L, 1)$	$(0, \theta_L - c\delta]$	$(0, c\delta]$
		$(\theta_L - c\delta, \theta_H - c\delta]$	$(0, \frac{c\delta}{2}]$
		$(\theta_L - c\delta, \theta_L - (\theta_H - \theta_L)]$	$(\frac{c\delta}{2}, c\delta]$
	$(0, \beta_V]$	$(\theta_L - (\theta_H - \theta_L), \theta_H - c\delta]$	$(\frac{c\delta}{2}, c\delta]$
	$(\beta_L, 1)$	$(0, \theta_L - (\theta_H - \theta_L)]$	$[c\delta, \infty)$
	$(\beta_L, \beta_V]$	$(\theta_L - (\theta_H - \theta_L), \theta_L - c\delta]$	$[c\delta, \infty)$
	$(0, \beta_V]$	$(\theta_L - c\delta, \theta_L]$	$[c\delta, \infty)$
Ω^2_{serv} vs Ω^2_{sell}	$(\beta_V, 1)$	$(\theta_L - (\theta_H - \theta_L), \theta_H - c\delta]$	$(\frac{c\delta}{2}, c\delta]$
	$(\beta_V, 1)$	$(\theta_L - (\theta_H - \theta_L), \theta_L]$	$[c\delta, \infty)$
		$(\theta_L, \theta_H - c\delta]$	$[c\delta, \infty)$

Table shows that possible comparison cases are: (1) Ω^1_{sell} vs Ω^1_{serv}, (2) Ω^2_{sell} vs Ω^1_{serv} and (3) Ω^2_{sell} vs Ω^2_{serv}. It should be noted that the first and the third cases represents full market coverage and partial market coverage cases for both business models. Thus, in those cases, the same population size is served under both business models.

For the first comparison we test $\Omega^1_{sell} > \Omega^1_{serv}$:

$$\begin{aligned} M(\theta_{avg} - m(\delta)) &\overset{?}{>} M\frac{\theta_{avg} - m(\delta) - c\delta}{2} \\ \theta_{avg} - m(\delta) &> -c\delta \end{aligned}$$

We know $\theta_{avg} - m(\delta)$ is positive for $\Omega^1_{sell},\ \Omega^1_{serv} > 0$. Thus, above condition holds and $\Omega^1_{sell} > \Omega^1_{serv}$.

For the second comparison we test $\Omega^2_{sell} > \Omega^1_{serv}$:

$$\begin{aligned} M\beta(\theta_H - m(\delta)) &\overset{?}{>} M\frac{\theta_{avg} - m(\delta) - c\delta}{2} \\ 2\beta(\theta_H - m(\delta)) &\overset{?}{>} \theta_L + \beta(\theta_H - \theta_L) - m(\delta) - c\delta \\ \beta(\theta_H + \theta_L - 2m(\delta)) &\overset{?}{>} \theta_L - m(\delta) - c\delta \\ \beta &\overset{?}{>} \frac{(\theta_L - m(\delta) - c\delta)}{(\theta_H + \theta_L - 2m(\delta))} \end{aligned}$$

Thus, for the comparison intervals of Ω^2_{sell} vs Ω^1_{serv} in Table M.4, $E_{sell} > E_{serv}$ when $\beta > \frac{(\theta_L - m(\delta) - c\delta)}{(\theta_H + \theta_L - 2m(\delta))}$ and $E_{sell} \leq E_{serv}$ when $\beta \leq \frac{(\theta_L - m(\delta) - c\delta)}{(\theta_H + \theta_L - 2m(\delta))}$.

For the third comparison we test $\Omega^2_{sell} > \Omega^2_{serv}$:

$$\begin{aligned} M\beta(\theta_H - m(\delta)) &\overset{?}{>} M\beta\frac{\theta_H - m(\delta) - c\delta}{2} \\ (\theta_H - m(\delta)) &\overset{?}{>} \theta_L + \beta(\theta_H - \theta_L) - m(\delta) - c\delta \\ \theta_H - m(\delta) &> -c\delta \end{aligned}$$

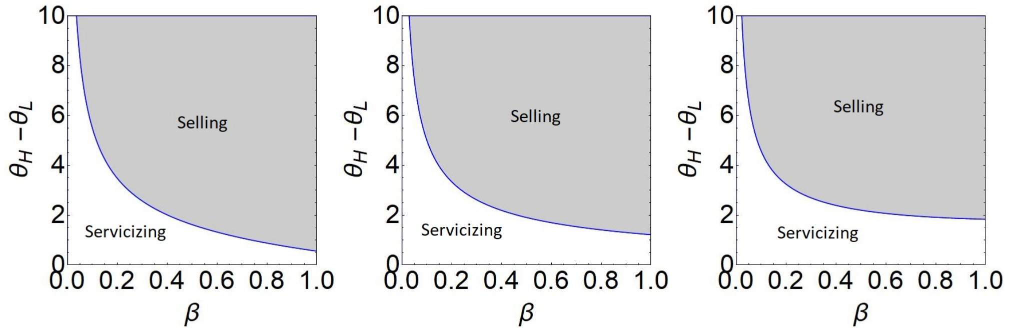
We know $\theta_H - m(\delta)$ is positive for $\Omega^2_{sell}, \Omega^2_{serv} > 0$. Thus, above condition holds and $\Omega^2_{sell} > \Omega^2_{serv}$.

Combining all three cases together with Table M.4, we obtain following results:

Result	Conditions		
	β	**$m(\delta) + c\delta$**	**$\theta_H - \theta_L$**
$E_{sell} > E_{serv}$, $\Omega_{sell} > \Omega_{serv}$	$(0, \beta_L]$	$(0, \theta_L - c\delta]$	$(0, \infty)$
	$(\beta_L, 1) \cap (\beta_E, 1)$	$(0, \theta_L - c\delta]$	$(0, c\delta]$
	$(\beta_E, 1)$	$(\theta_L - c\delta, \theta_H - c\delta]$	$(0, \frac{c\delta}{2}]$
	$(\beta_E, 1)$	$(\theta_L - c\delta, \theta_L - (\theta_H - \theta_L)]$	$(\frac{c\delta}{2}, c\delta]$
	$(0, \beta_V] \cap (\beta_E, 1)$	$(\theta_L - (\theta_H - \theta_L), \theta_H - c\delta]$	$(\frac{c\delta}{2}, c\delta]$
	$(\beta_L, 1) \cap (\beta_E, 1)$	$(0, \theta_L - (\theta_H - \theta_L)]$	$[c\delta, \infty)$
	$(\beta_L, \beta_V] \cap (\beta_E, 1)$	$(\theta_L - (\theta_H - \theta_L), \theta_L - c\delta]$	$[c\delta, \infty)$
	$(0, \beta_V] \cap (\beta_E, 1)$	$(\theta_L - c\delta, \theta_L]$	$[c\delta, \infty)$
	$(\beta_V, 1)$	$(\theta_L - (\theta_H - \theta_L), \theta_H - c\delta]$	$(\frac{c\delta}{2}, c\delta]$
	$(\beta_V, 1)$	$(\theta_L - (\theta_H - \theta_L), \theta_L]$	$[c\delta, \infty)$
	$(0, 1)$	$(\theta_L, \theta_H - c\delta]$	$[c\delta, \infty)$
$E_{sell} \leq E_{serv}$, $\Omega_{sell} \leq \Omega_{serv}$	$(\beta_L, 1) \cap (0, \beta_E]$	$(0, \theta_L - c\delta]$	$(0, c\delta]$
	$(0, \beta_E]$	$(\theta_L - c\delta, \theta_H - c\delta]$	$(0, \frac{c\delta}{2}]$
	$(0, \beta_E]$	$(\theta_L - c\delta, \theta_L - (\theta_H - \theta_L)]$	$(\frac{c\delta}{2}, c\delta]$
	$(0, \beta_V] \cap (0, \beta_E]$	$(\theta_L - (\theta_H - \theta_L), \theta_H - c\delta]$	$(\frac{c\delta}{2}, c\delta]$
	$(\beta_L, 1) \cap (0, \beta_E]$	$(0, \theta_L - (\theta_H - \theta_L)]$	$[c\delta, \infty)$
	$(\beta_L, \beta_V] \cap (0, \beta_E]$	$(\theta_L - (\theta_H - \theta_L), \theta_L - c\delta]$	$[c\delta, \infty)$
	$(0, \beta_V] \cap (0, \beta_E]$	$(\theta_L - c\delta, \theta_L]$	$[c\delta, \infty)$

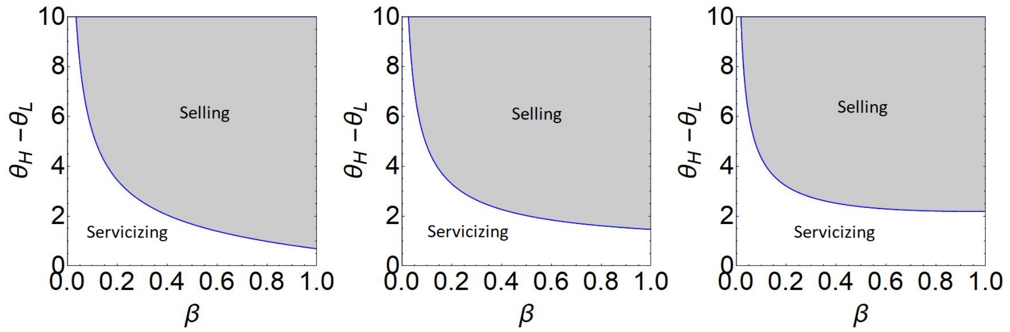
where $\beta_E = \frac{\theta_L - m(\delta) - c\delta}{\theta_H + \theta_L - 2m(\delta)}$

N. Separate Effects of m and c in Numerical Experiments



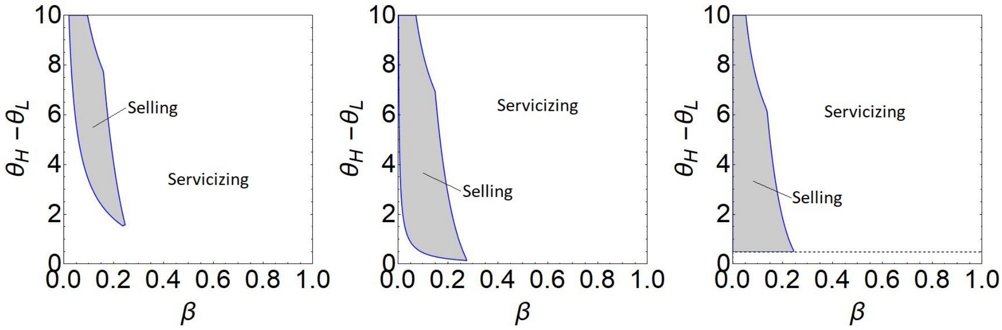
(a) m = 35000 (b) m = 40000 (c) m = 45000

Figure N.1: Effect of m in profitability comparison ($\theta_L = 9, M = 100, c = 0.00025$)



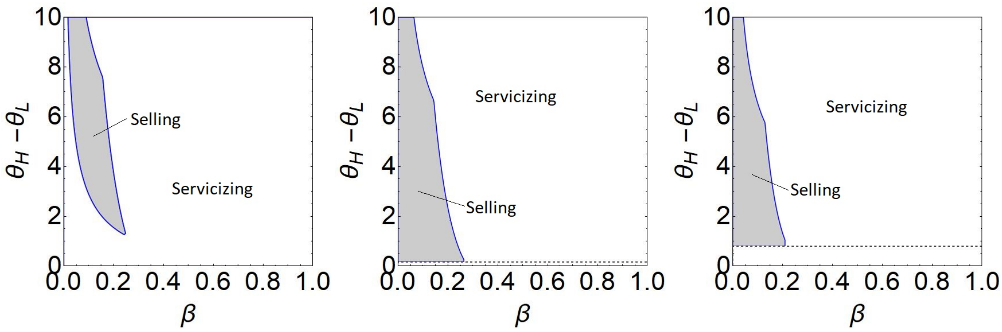
(a) c = 0.00030 (b) c = 0.00035 (c) c = 0.00040

Figure N.2: Effect of c in profitability comparison ($\theta_L = 9, M = 100, m = 30000$)



(a) m = 35000 (b) m = 40000 (c) m = 45000

Figure N.3: Effect of m in environmental comparison ($\theta_L = 9, M = 100, c = 0.00025$)



(a) c = 0.00030 (b) c = 0.00035 (c) c = 0.00040

Figure N.4: Effect of c in environmental comparison ($\theta_L = 9, M = 100, m = 30000$)