MECHANICAL RELIABILITY ANALYSIS USING STRESS-STRENGTH INTERFERENCE MODEL AND ENGINEERING APPLICATION OF RELIABILITY: SYSTEM SAFETY ASSESSMENT

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ABSTRACT

MECHANICAL RELIABILITY ANALYSIS USING STRESS-STRENGTH INTERFERENCE MODEL AND ENGINEERING APPLICATION OF RELIABILITY: SYSTEM SAFETY ASSESSMENT

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Mechanical reliability is the main concern of this thesis. Reliability prediction of mechanical components is commonly performed via four different techniques which are Component Failure Data Analysis, Empirical Reliability Analysis, Stress-Strength Interference Model and Reliability Database and Handbook Usage. However, Stress-Strength Reliability Model, which has a great popularity in these reliability estimation methods in literature, is the main focus of this research.

Five methods are used in calculations modelled with Stress-Strength Interference. These are analytical methods, which are First Order Reliability Method (FORM), Second Order Reliability Method (SORM), and simulation methods, which are Monte Carlo Simulation (MCS), Importance Sampling (IS), Adaptive Kriging Monte Carlo Simulation (AK-MCS). Landing Gear Emergency Extension System is taken into account and analyzed in details. Venting valve, which is a part of this subsystem, is chosen as an example of mechanical component. While stress random variables of the venting valve are assumed to be uniformly distributed, strength random variables are considered as both normal and lognormal random variables in calculations. Five different reliability methods are performed in this manner and results are compared

extensively. Number of evaluation is also assessed as an indication of the strength of the method.

System safety assessment is an application area of reliability values. “SAE ARP 4761 Aerospace Recommended Practice” defines the process of safety assessment of the systems. A reliability requirement for venting valve is created in this manner and an example of validation is performed which shows the relation between system safety engineering and reliability engineering disciplines.

Keywords: Mechanical Reliability Engineering, System Safety Engineering, Probability of Failure, Reliability Prediction, Normal Distribution

ÖZ

GERİLİM-DAYANIM ÇATIŞMA MODELİ KULLANIMI İLE MEKANİK GÜVENİLİRLİK ANALİZİ VE GÜVENİLİRLİĞİN MÜHENDİSLİK UYGULAMASI: SİSTEM EMNİYETİ DEĞERLENDİRMESİ

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Mekanik güvenilirlik bu tezin üstünde en çok durulan ana meselesidir. Mekanik parçaların güvenilirlik tahmini yaygın olarak dört farklı teknik ile icra edilir. Bunlar Parça Hata Data Analizi, Deneysel Güvenilirlik Analizi, Gerilim-Dayanım Güvenilirlik Modeli ve Güvenilirlik Veri tabanı ve El kitabı Kullanımı olarak bilinir. Bununla birlikte, akademik kaynaklarda büyük bir popülerliğe sahip olan Gerilim-Dayanım Güvenilirlik Modeli bu araştırmanın ana odağıdır.

Gerilim-Dayanım Çatışma Modeli temel alınarak hesaplamalarda beş farklı metot kullanılmıştır. Bunlar analitik metotlar grubu altında Birinci Dereceden Güvenilirlik Modeli, İkinci Dereceden Güvenilirlik Modeli ve benzetim metotları olarak ise Monte-Carlo Simülasyonu, Önem Örneklemesi, Uyumlu Kriglenmiş Monte Carlo Simülasyonu uygulanmıştır. Örnek mekanik parça için İniş Takımı Acil Açılma Sistemi ele alınıp detaylı şekilde analiz edilmiştir. Bu alt sistemde tahliye valfi örnek olarak seçilmiştir. Tahliye valfinin gerilim rastgele değişkenleri tekdüze dağılımda varsayılmışken, dayanım rastgele değişkenleri normal ve lognormal dağılımda düşünülmüştür. Beş farklı güvenilirlik metodu bu minvalde icra edilip, sonuçlar geniş bir şekilde karşılaştırılmıştır. Sayısal işlem sayısı da en güçlü metodu belirlemede bir kıstas olarak değerlendirilmiştir.

Sistem emniyeti değerlendirmesi, güvenilirlik değerlerinin bir uygulama alanıdır. “SAE ARP 4761 Havacılık ve Uzay Öneri Pratiği” standardı, sistem emniyet değerlendirmesinin sürecini tanımlamaktadır. Bu minvalde, tahliye valfi için bir güvenilirlik gereksinimi oluşturulup, doğrulama eylemi örneği icra edilmiştir. Böylece sistem emniyeti mühendisliği ile güvenilirlik mühendisliği disiplinleri arasındaki ilişki gösterilmiştir.

Anahtar Kelimeler: Mekanik Güvenilirlik Mühendisliği, Sistem Emniyeti Mühendisliği, Hata Olasılığı, Güvenilirlik Tahmini, Normal Dağılım

To Mustafa (In the hope that he will be honored and granted, peace be upon him)

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CHAPTER 1

INTRODUCTION

The purpose of this study is to apply the techniques of mechanical components reliability discipline to some engineering problems. In this chapter, definition and brief information about mechanical reliability are performed. Literature survey about component reliability concept is also main consideration of this chapter. At the end, problem formulation and scope of this study is presented.

1.1. Mechanical Reliability Concept

In today's world, complicated and challenging designs are created to meet the needs and make life easier. As human life is more important than before, safety is the main issue in these designs. Cost is another issue as usual. The topic of reliability meets these requirements and clarifies some gray areas in the designs. Parameters such as material properties have uncertainty and designers should take this issue into account. Safety factors are generally used to overcome these problems. However, empirical safety factors may cause oversafe designs due to not being standardized and do not deal with the effect of different parameters on safety. Generally, influence of these parameters is considered as certain. The fact is that the effect is not uniform in deterministic designs. Reliability based probabilistic design is preferred to avoid these shortcomings and represents more economical solutions.

There are various definitions of reliability in literature. However, the most comprehensive one is presented by Kececioglu [1] as: Reliability is a conditional probability that equipment carries out purposed functions without failure under stresses due to environmental conditions in a given performance limits and specified time with pre-determined confidence level.

Reliability concept began to be taken into consideration after Second World War owing to increase in technical failures in large aircrafts and ships. Navy and Air Force of United States of America used some mathematical models in regard to failed electronic equipment. In order to understand reliability, failure rate which is defined as "Components failure frequency (Failure per unit time)" is proposed. The famous "Bath-Tub Curve" is introduced as in Figure 1.1 and it is assumed that constant failure rate is applicable for electronic equipment's service life. Nevertheless, in mechanical reliability field such efforts were performed in the early years of the 1960's [2]. In the beginning, constant failure rate assumption is also considered for mechanical parts, however in recent years with the advances in computers and analytical procedures, it is accepted that this assumption is no longer applicable for mechanical reliability calculations.

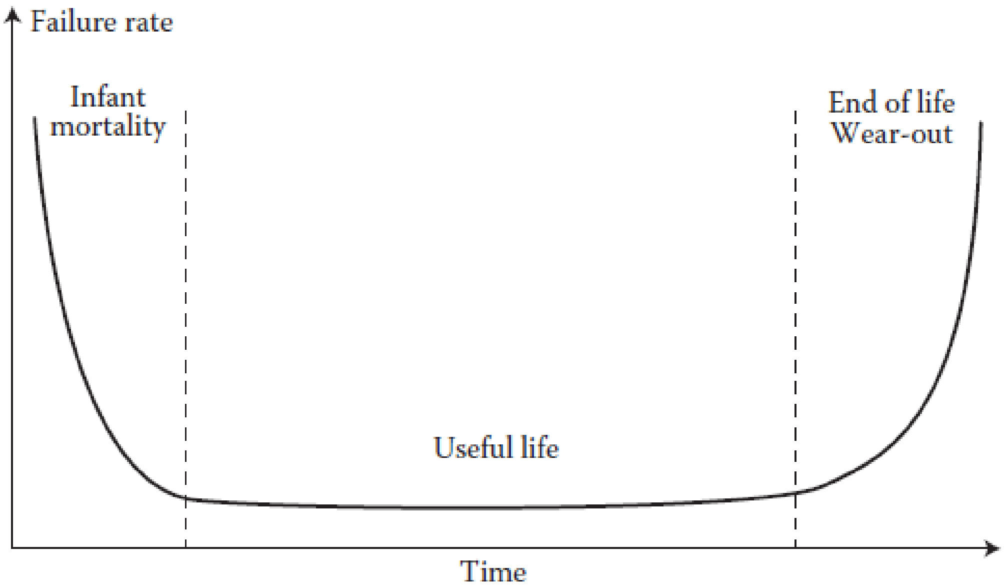


Figure 1.1. Bathtub Curve: Hypothetical Failure Rate versus Time [3]

In this classical bath-tub curve theory, there are three phase in the life of equipment: "Infant mortality" which is before service life period in which failure rate is very high due to manufacturing weaknesses. "Useful life" which is the second period in which constant failure rate assumption is considered and the last phase is "Wear-out" phase that component deterioration implies end of life [4].

In addition to Figure 1.1, Kececioglu presents mechanical component failure rate behavior in comparison to electronic equipment failure as shown in Figure 1.2.

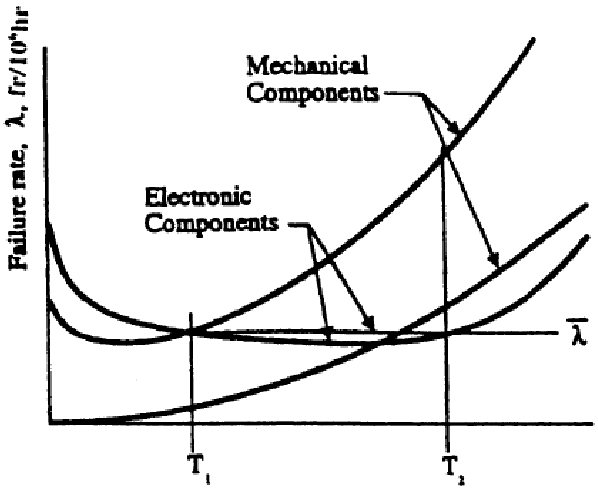


Figure 1.2. Bathtub Curves for both electronic and mechanical components [5]

In this graph, when the general failure rate behavior of mechanical components is analyzed, it can be seen that there are large differences between two types of components. In the first region, until time is equal to T_1 (Infant mortality region), failure rate becomes a concave shape due to material imperfections and manufacturing weaknesses. For the rest of life, failure rate of mechanical components increases drastically. This is mainly owing to environmental stress factors causing many failure modes such as buckling, wear, thermal shocks, corrosion and so on. However, there are relatively low numbers of failure mode for electronic equipment and due to low number of disturbance; an approximate constant failure frequency versus time could be possible.

Reliability of mechanical components is one of the most popular topics in literature especially in China, India, USA and Europe. However, due importance is not given in our country to this topic which is extremely important for aerospace and power-plant industries. Research in Turkey on this topic has a history of only ten years, but researchers around the world have been studying this topic for more than fifty years. One of the reasons behind this is that design of the highly reliable machines is relatively recent in Turkey. Nonetheless with changing priorities, this topic will come to forefront and more studies will be conducted.

Some of the common reliability prediction models for mechanical components are given in the next section. The literature is reviewed and some important studies regarding prediction techniques are presented.

1.2. Mechanical Reliability Prediction Techniques

Component failure data analysis, empirical reliability analysis, stress-strength reliability model and reliability database and handbook usage are the four main techniques in reliability prediction of mechanical components. In this section, these four models are introduced and their advantages/disadvantages are discussed briefly.

1.2.1. Component Failure Data Analysis

For the expensive mechanical components such as helicopter blades, jet motor blades and so on, their failure data are recorded regularly in a database system by big companies and military maintenance centers. These data can be used to obtain mean time to failure (MTTF) graphs over the years. Mean Time To Failure (MTTF) is the expected (mean) time to failure of unrepairable components or systems. An appropriate statistical distribution is obtained based on MTTF plots and the associated reliability approximation is performed. Weibull Distribution is generally the best fitted one and it is commonly used. However, the other statistical distributions such as Gamma Distribution, Bayesian Distribution can also fit the input data. Once distribution is decided and goodness of fit is tested, reliability characteristic of the specific component can be determined.

Field data is one of the best feedbacks for the design of the systems and optimization can be performed for the related component. Design can be improved based on these studies. Nevertheless, unavailability of data for some specific components due to unregularly records and deficiency in the time to failure data for the newly developed designs are the main two shortcomings of this method.

For the first reliability prediction method which is the component failure data analysis, relatively small number of studies are performed. Difficulties to find field data could be the main reason for it. However, Shahani and Babaei [6] made a helicopter blade reliability analysis based on some Iranian helicopters and found a parametric blade reliability equation which best fits the non-parametric data using Weibull distribution and concluded that helicopter blades show an increasing failure rate trend. In addition to this study, Keller, Giblin and Farnworth [7] analyzed components failure data for a fleet of heavy vehicles and they successfully fitted the data using Weibull distribution as well. As a result, Mean Time Between Failure (MTBF), which is the expected time between possible two consecutive failures for repairable components or systems, was calculated. Reliability curve fitting was also studied by Luxhoj and Shyur [8] for aging helicopter components. They proposed a neural network model which can more accurately fit the failure data in comparison to the standard curve fitting methods such as Weibull or Exponential distribution. However, the sample size was small in this study and thus more data was required for the generalization of the model.

All the researchers follow the same procedure in this prediction method. They collect the failure data for the related components and develop a statistical graph that describes the time to failure behavior. After performing the best fitting using an appropriate distribution, linear parameters which forms reliability of the mechanical component are determined. These results then give information about tendency of the component to failure in time.

1.2.2. Empirical Reliability Analysis

Empirical tests constitute the sources for this technique. Fatigue analysis of the components is performed by testing them according to the material type, the loading type and so on. Logarithmic results are acquired and based on this; characteristic life expectancy of the component can be calculated. Fatigue tests are commonly used in engineering and fatigue reliability based optimization provides good improvement in design. In this method, different test set-ups increases the application base to a variety of component and further several different combinations of components can be tested and analyzed. Nonetheless, logarithmic results could cause unavailability of correlation to failure rate since the reliability of the components is the main output of this analysis. For this reason, it is not preferable for the design group to make a completely new design.

There are relatively more techniques for analyses available in the literature in this topic and relatively more effort is needed since structural analysis and load effects on the mechanical component should be performed. This method is based on what is commonly known as "Fatigue Analysis of Components". In this technique, structural optimization is possible due to fact that random variable playing a role in the failure of the component can be detected and accordingly improvements can be employed. The sensitivity of random variables on reliability is one of the main considerations for this technique. An example can be found in the study by Li, Xie and Ding [9]. They predict the reliability of helicopter planetary gear train under equal and unequal load sharing analysis using fatigue test and develop a model for this type of gears. In addition, a reliability based fatigue life analysis methodology is developed by Li, Hu, Chandrashekhara, Du and Mishra [10] for a medium scale, horizontal axis hydrokinetic turbine blade. Effect of the uncertainties on fatigue life is also analyzed. Furthermore, Seddik, Sghaier, Atig and Fatallah [11] studied fatigue reliability of shot-peened metallic components through a probabilistic approach and improvement of shot-peening technique on these components are investigated.

In summary, it is important to note that in this type of reliability analysis, fundamental aim is not only calculating the fatigue reliability of the component but also proposing a model for the improvement of the component. Experimental analyses make this method more practical and new designs could be created based on these analyses. Nevertheless, much more effort and time are required for this reliability prediction method.

1.2.3. Stress-Strength Reliability Model

This reliability approach utilizes stress and strength statistical distribution associated with the material. Based on “Stress-Strength Interference Model”, probability of failure of the component can be calculated. In addition to this, engineering knowledge is required to characterize stress and strength behavior of the material. In this thesis, this technique is chosen for the prediction of reliability because with the accumulation of engineering knowledge, this technique is getting more and more popular in literature and attracting many scientists and engineers. Highly reliable mechanical components are desired to use in aerospace and nuclear power plant industries due to the cost and time efficiency in design. However, failure rate calculation is not applicable in this approach. Further detailed information in this model is given in Section 2.1.

Amongst all the prediction techniques, stress-strength reliability model is the most popular in literature. One of the reasons behind this popularity is that it represents the failure behavior of the mechanical components and margin of error in predicting the actual values is smaller in comparison to the other prediction techniques. Although fatigue reliability model is carried out under more conservative conditions, factor of safety should be used to avoid the inevitable differences between actual and laboratory environment for the design of mechanical components. On the other hand, stress-strength reliability method aims to avoid factor of safety and provides more realistic results. Some more techniques such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and tail modeling and so on are developed and generally Monte-Carlo Simulation (MCS) is used for verification of the results. Due to fact that verification of the results is possible, this method is preferred commonly in studies. More information about this prediction model is given in Section 2.3. Optimization can be performed based on these analyses and big companies improve their designs accordingly. In addition to this, new designs can be carried out based on stress-strength reliability model, but designers are unwilling to use the probabilistic design in place of the deterministic design. Acar and Haftka [12] mentioned this issue in their study and found that deterministic design causes over safe mechanical components. They proved that safety of an airplane can be increased with their proposed method without changing its weight. Each component has a different failure probability distribution and cannot be grouped easily. It is very important to determine the stress and strength model of the component and it takes good engineering experience. Prasad, Reddy, Srividya and Verma [13] used a stress-strength model for check valves utilized in nuclear power-plant systems. They predicted the reliability using SORM and compared with the database IAEA-TECDOC-478 to conclude that their approach gives reasonable outcomes. In addition to this, Wang [14] introduced a double integration model for the reliability of components and presented an illustration using pin to finally verify the outcomes with MCS. Similarly, a group of researchers in Bhabha Atomic Research Centre performed an investigation about decay heat removal system and proposed a methodology for reliability prediction of the system [15]. As the complexity of the problem is high, MCS is used to reach the results. Advanced studies in this area are integrated with finite element analysis for large components such as airplane wing, wind turbine blade and so on. However, it requires much more effort due to large number of simulations and researchers are reluctant to perform reliability analysis of such large structures [16]. An example related to this subject is studied by Kandemir [17]. Tail modeling is used and verified by MCS for the reliability estimation of wind turbine blade and it was concluded that tail modeling can predict reliability of horizontal axis wind turbine efficiently. Bayrak [18] contributed to this subject by proposing that Markov Chain Monte Carlo Method (MCMC) which is an advanced method of MCS gives better estimates than traditional tail modeling.

1.2.4. Reliability Database and Handbook Usage

There are some reliability databases and handbooks available in literature for reliability prediction of components. The most commonly used ones are mentioned below:

1.2.4.1. NPRD-95 (Nonelectric Parts Reliability Data)

This database is commonly used for aerospace industries due to its ease in usage and being inexpensive. It provides failure rates for a large number of mechanical and electro-mechanical components. Conformity to combination use with electronic components makes it attractive for newly developed designs owing to constant failure rate principle. However, this assumption does not provide sensible results for mechanical components. Ignoring cost increase and time consumption due to component replacement in maintenance, this data source is quite useful and cheap way for new designs. It is widely used and currently reliability analyses for designed components are carried out generally using this database.

1.2.4.2. NSWC-2011 Handbook of Reliability Prediction Procedures for Mechanical Equipment

This handbook provides reliability prediction guidelines for basic mechanical components by providing some equations and related graphs. An engineer can predict failure rate of a specific component by following the calculation procedure provided. It is a product of an extensive study involving the cooperation of the military, industry and academia and published by Naval Surface Warfare Center. Nevertheless, usage of this handbook can be complicated when material properties are not known in detail because the equations and graphs provided require some information about the component and these could not be available in some cases. It is mostly used by Naval and Aerospace Industry.

1.2.4.3. IAEA-TECDOC-478 Component Reliability Data for Use in Probabilistic Safety Assessment

In this data source, probability of failure for both mechanical and electro-mechanical items is available. It is prepared by International Atomic Energy Agency and a product of a comprehensive study. Repair time for individual component or equipment is given and this can help in the maintenance activities. It is generally used by Nuclear Power Plant Industry.

Database and Handbook Usage is not common in literature for reliability estimation of mechanical components because the common hypothesis of constant failure rate is not representing the actual behavior. Nonetheless, especially in aerospace and electronics industry, engineers widely use software tools such as ITEM Toolkit, Lambda Predict, and PTC Windchill Quality Solutions and so on for prediction of the reliability of components. This is owing to the fact that mechanical component reliability is not one of the main concerns of the companies, although it saves money and maintenance time in the long term. This policy is followed by the big companies and they make their designs in accordance with the advances on this topic and collaboration between the industry and academia is increasing day by day. It is important to note that these software packages are suitable for electronic components, but not for mechanical components in practice.

Some engineers contribute a few studies in this manner. Raze and a group of people from Eagle Company presented an example model contributing to NSWC Handbook for the reliability prediction of compressors [19]. Koç [20] used NPRD 95 in his study for the Mean Time Between Failures (MTBF) of some mechanical components of an unmanned aerial vehicle such as Main Wheel, Hydraulic Pump, and Fuel Tank to estimate its reliability. Shan, Huang and Yi [21] developed a software program based on NSWC-10 Handbook utilizing C# language for prediction of the reliability of mechanical standard components.

1.3. Problem Formulation and Scope of Study

The trend in literature for reliability estimation directs a reliability engineer in introducing the topic and providing the details about stress-strength reliability model because amongst all the techniques, it is the most realistic one and an engineer desires to obtain the actual value as close as possible.

Academic surveys about reliability of mechanical components are very limited and almost all studies are about structures such as bridges, trusses, frames and so on which are not applicable to standard mechanical components. The usual methods (FORM, SORM and MCS) for reliability prediction are also used in structures, however almost no research is performed for mechanical components. Some contribution to alleviate this deficiency is necessary for advances in this area and this can provide new ideas and broadens the horizons.

In this thesis, reliability of some mechanical components will be estimated using FORM, SORM and MCS techniques. System Safety Analysis is also introduced to carry out the engineering applications of the mechanical reliability concept. Main purpose of this thesis is to introduce the concept of mechanical reliability and contribute to this area using engineering knowhow. Electronic components such as battery, antennas, sensors and so on, and structure components such as bridges, trusses, frames and so on are not considered and they are beyond the scope of this thesis.

The thesis develops as follows: Chapter 2 presents fundamental terms of reliability including probability distributions used in this study and Stress Strength Interference Model. Reliability Estimation Methods based on this model are described theoretically and the information about methodology of reliability calculations is given in Chapter 3. First Order (FORM) and Second Order Reliability Methods (SORM) are termed as analytical methods, while Monte-Carlo Simulation (MCS), Importance Sampling (IS) and Adaptive Kriging Monte Carlo Simulation (AK-MCS) are termed as simulation methods which are all utilized in this study. Chapter 4 defines the limit state function of selected mechanical component and presents the results using five different techniques. Discussion and comparison of the results are also presented in this chapter. System Safety Analysis which is the engineering application of reliability is performed in Chapter 5. General discussions and conclusions are presented in Chapter 6, the last chapter of this thesis, and some future work suggestions are given.

CHAPTER 2

FUNDAMENTALS OF RELIABILITY

Basics of reliability, probability distributions and Stress-Strength Interference Model are described in this chapter. These terms are required in order to understand and clarify some the reliability concepts.

2.1. Basic Reliability Terms

In this part, some probability terms are briefly explained to refresh the reader's background. These basic terms are the random variables, the probability distribution function, the cumulative distribution function, the mean, the standard deviation and the coefficient of variation.

2.1.1. Random Variable

A random variable is a function that assigns real numbers to the outcomes ω in the sample space Ω of a random experiment. The range of a random variable consists of real numbers that are assigned to all possible outcomes of an experiment. Mathematically,

$$X : \Omega \longrightarrow \mathbb{R}, \tag{2.1}$$

$$X(\Omega) = \{X(\omega) \in \mathbb{R} \mid \omega \in \Omega\} . \tag{2.2}$$

It is categorized as two types, namely, continuous and discrete random variables. The range of continuous random variables consists of uncountable number of real numbers such as an interval. Length, time, pressure and stress are the examples of such variables. However, the range of discrete random variables consists of a countable number of real numbers either finite or infinite. Digital signals which only take 1 and 0 values can be given as an example. Experimental output of mechanical components are usually represented by continuous random variables. Hence, discrete random variables are out of scope in this study.

2.1.2. Probability Density Function (PDF)

It is a function that all possible values of the random variable is contained. Probability Density Function is denoted by $f_X(x)$ and has a unit of probability per value.

$$\int_{-\infty}^{\infty} f_X(x)dx = 1 \tag{2.3}$$

The area under PDF in the limits of a and b gives the probability value of the random variable occurrence in that area. Graphical representation is given in Figure 2.1;

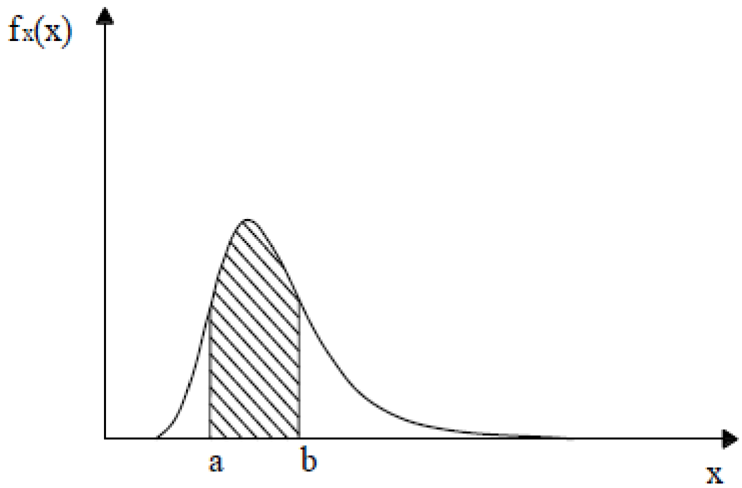


Figure 2.1. PDF of Random Variable X [22]

Continuous random variables X and Y are called independent if the following condition is satisfied;

$$f_{X,Y}(x,y) = f_X(x)f_Y(y) \quad for\ all\ x,y \tag{2.4}$$

The function $f_{X,Y}(x,y)$ is defined as the joint PDF that involves jointly continuous random variables X and Y.

2.1.3. Cumulative Distribution Function (CDF)

Cumulative Distribution Function, on the other hand, is defined as probability of a random variable between a limited value and all values before it and denoted by $F_X(x)$

$$F_X(b) = P(B \leq b) = \int_{-\infty}^{b} f_X(x)\, dx. \tag{2.5}$$

CDF can be used to determine the probability of random variable in a range

$$P(a \leq X \leq b) = \int_{a}^{b} f_X(x)\, dx = F_X(b) - F_X(a). \tag{2.6}$$

Another possible definition of independent continuous random variables X and Y in terms of CDF can be stated as;

$$F_{X,Y}(x, y) = F_X(x)F_Y(y) \quad for\ all\ x, y \tag{2.7}$$

2.1.4. Mean, Standard Deviation, Coefficient of Variation and Variance

Mean, Standard Deviation, Coefficient of Variation and Variance are some deterministic parameters characterizing the underlying probability distribution.

Mean is simply expected (average) value of a random variable over the sample space Ω and standard deviation is a measure of dispersion from the mean value. Mean and standard deviation are denoted by μ and σ, respectively. Namely,

$$\mu = \int x f_X(x) dx \equiv E\{X\} \tag{2.8}$$

where E is the average or expected value of X.

$$\sigma = \sqrt{\int (x-\mu)^2 f_X(x) dx} \equiv \sqrt{E\{(x-\mu)^2\}} \tag{2.9}$$

Coefficient of variation (CoV) is the ratio of the standard deviation to the mean and simply defined as measure of percent relative variability.

$$CoV = \frac{Mean\ (\mu)}{Standard\ Deviation\ (\sigma)} * 100\% \tag{2.10}$$

On the other hand, Variance is simply square of the standard deviation

$$\sigma^2 = V\{X\} = E\{X^2\} - \mu^2 \tag{2.11}$$

2.2. Continuous Probability Distributions

There are various continuous distributions which are commonly used in literature. Nevertheless, Normal (Gaussian) , Lognormal and Uniform Distributions are used to represent material properties and loads such as material dimensions, tensile strength, loading pressure and so on, more than the other types of distributions. Thus, only these three distributions are taken into account in this thesis.

2.2.1. Normal (Gaussian) Distribution

It is also known as Gaussian Distribution and commonly used in statistical and probabilistic calculations since the PDF associated with the independent random variables from experimental data or natural data shows great similarity with this distribution type. Time invariant stresses, material strength values and component dimensions are the examples whose distribution shows normal distribution behavior.

PDF of Normal Distribution is denoted by $N(\mu, \sigma)$ and given below;

$$f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \qquad where \quad -\infty < x < \infty \tag{2.12}$$

The graph of normal PDF is given in Figure 2.2 for $\mu = 0$ and various σ values.

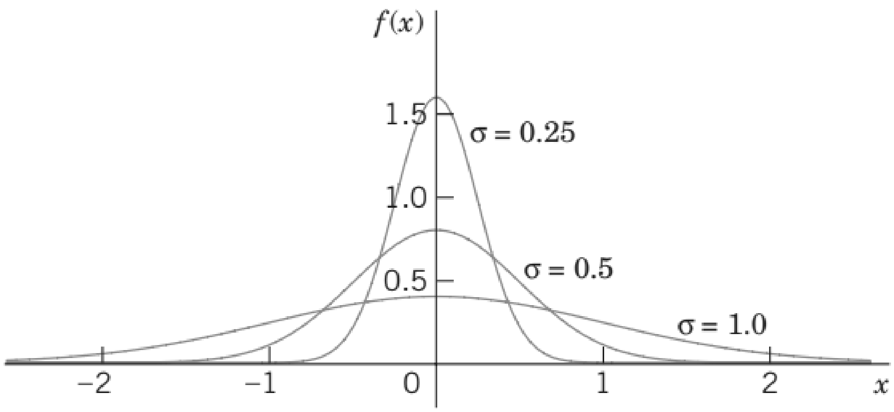


Figure 2.2. PDF Graph of Normal Distribution for μ=0 [23]

The associated CDF is given by;

$$F(b) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{b} \exp\left[\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx\right] \tag{2.13}$$

whose numerical values are tabulated in terms of the standard normal distribution table, namely $\Phi(Z){:}\, N(0,1)$ by using the conversion $Z = (X - \mu)/\sigma$.

In this table, the values are calculated using CDF of the standard normal distribution with μ=0 and σ=1 given by the following formula;

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left[-\frac{t^2}{2}\right] \, dt \tag{2.14}$$

2.2.2. Lognormal Distribution

A random variable should not take negative values physically in some engineering problems and normal distribution is not suitable theoretically in those problems. To overcome this, lognormal distribution is proposed. It is derived from the normal distribution and the associated random variable assumes only positive values. Fatigue life of metals and mass are the examples that suit the lognormal distribution behavior.

PDF of Lognormal Distribution is given by;

$$f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) \exp\left[-\frac{1}{2}\left(\frac{lnx-\mu}{\sigma}\right)^2\right] \ , \qquad where \qquad 0 \le x < \infty, \tag{2.15}$$

whose CDF is;

$$F(b) = \int_{-\infty}^{b} \left(\frac{1}{\sigma\sqrt{2\pi}}\right) \exp\left[-\frac{1}{2}\left(\frac{lnx-\mu}{\sigma}\right)^2\right] \ dx. \tag{2.16}$$

The graph of lognormal PDF is given in Figure 2.3 for $\mu = 0$ and various σ values.

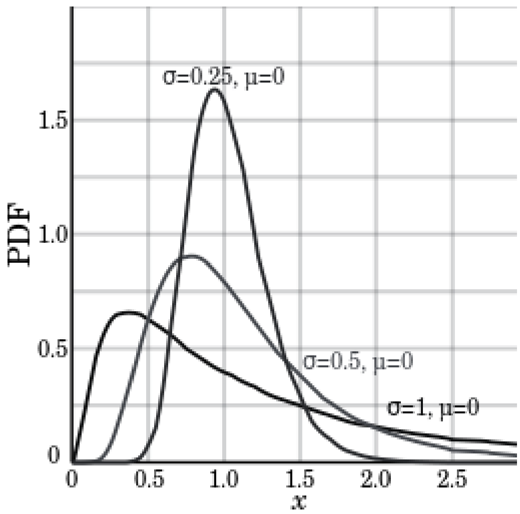


Figure 2.3. PDF Graph of Lognormal Distribution for μ=0 [24]

2.2.3. Continuous Uniform Distribution

When PDF assigns uniform values to a random variable varying in an interval between a and b, uniform distribution results.

PDF of Uniform distribution is given by;

$$f(x) = \begin{cases} \dfrac{1}{b-a} & for\ a \leq x \leq b \\ 0 & otherwise \end{cases} \tag{2.17}$$

In this equation, 'a' is called the location parameter and '$b - a$' is the range of the uniform random variable.

The graph of the uniform PDF is given in Figure 2.4.

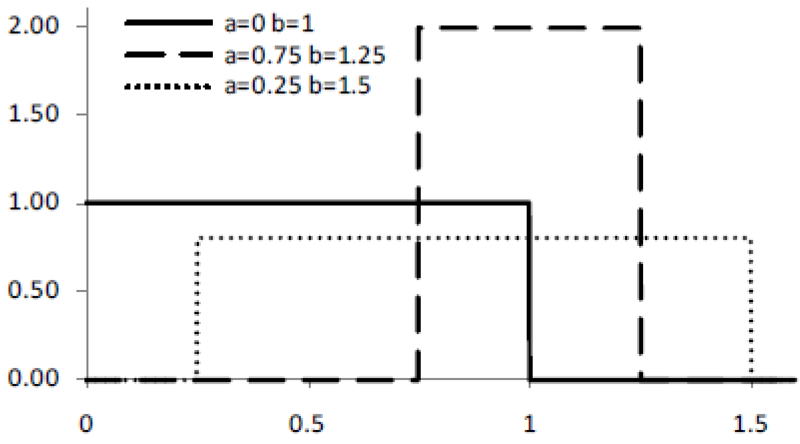


Figure 2.4. PDF Graph of Uniform Distribution [25]

Its CDF is;

$$F(x) = \begin{cases} \dfrac{x-a}{b-a}, & for\ a \leq x \leq b \\ 0 & for\ x < a \end{cases} \tag{2.18}$$

These three continuous distributions determine the scope of this study.

2.3. Stress-Strength Interference Model

Reliability of a mechanical component is determined based on Stress-Strength Interference model. For clarity and completeness, the definition of strength and stress is stated first. Strength is basically defined as the resistance to loads owing to external environment. Strength of similar mechanical components may vary due to material characteristics, geometric dimensions or production processes. Hence, variation of strength should be modeled using a probability distributions.

On the other hand, any factor which has the potency to create failure despite of material strength is called “Stress”. These factors are not necessarily limited to structural loads. Temperature, electric current and the environment also cause stress and the variation in stress may also be modeled by using a probability distribution.

If stress and strength PDFs are known for a component, reliability of the component can be defined as probability that strength exceeds stress. Namely,

$$R = P(X < S) \tag{2.19}$$

where the random variables S and X denote Strength and Stress, respectively.

This methodology is called "Stress-Strength Interference Model" and reliability of the mechanical component is determined based on this model. Figure 2.5 shows the interference area delimited by the stress and strength PDFs.

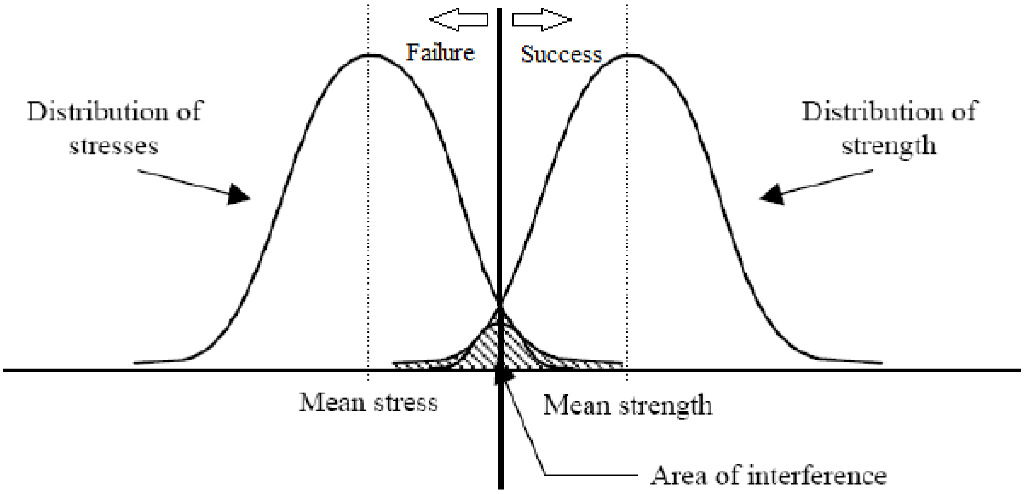


Figure 2.5. Stress-Strength Interference Model [26]

The interference area on the left shows the Probability of Failure (P_f) that is defined as the probability of the stress exceeding the strength. Thus, Reliability (R) is then defined in terms of the Probability of Failure as follows;

$$R = 1 - P_f = 1 - P(X \geq S) \tag{2.20}$$

Assume now that both stress and strength of a mechanical component exhibit normal distribution behavior;

$$f(x) = \left(\frac{1}{\sigma_x\sqrt{2\pi}}\right)\exp\left[-\frac{1}{2}\left(\frac{x - \mu_x}{\sigma_x}\right)^2\right], \qquad where \quad -\infty < X < \infty\,, \tag{2.21}$$

$$f(s) = \left(\frac{1}{\sigma_s\sqrt{2\pi}}\right)\exp\left[-\frac{1}{2}\left(\frac{s - \mu_s}{\sigma_s}\right)^2\right], \qquad where \quad -\infty < S < \infty\,, \tag{2.22}$$

where μ_x and σ_x are the mean and standard deviation of the stress and μ_s and σ_s are the mean and standard deviation of the strength, respectively.

Let Z denote limit state function or performance function described as;

$$Z = g(S, X) \tag{2.23}$$

such that

Z<0 is unsafe case

Z>0 is safe case

Z=0 is limit state

where Z is a random variable

$$Z = S - X \tag{2.24}$$

When S and X are independent random variables, the random variable Z is normally distributed;

$$f(z) = \left(\frac{1}{\sigma_z\sqrt{2\pi}}\right)\exp\left[-\frac{1}{2}\left(\frac{z-\mu_z}{\sigma_z}\right)^2\right], \qquad where \quad -\infty < z < \infty \tag{2.25}$$

where

$$\mu_z = \mu_s - \mu_x, \tag{2.26}$$

$$\sigma_z = (\sigma_x^2 + \sigma_s^2)^{\frac{1}{2}}. \tag{2.27}$$

The Reliability of Z is given by

$$R = P(z > 0) = \left(\frac{1}{\sigma_z\sqrt{2\pi}}\right)\int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{z-\mu_z}{\sigma_z}\right)^2\right] \, dz. \tag{2.28}$$

Let us define the random variable H as;

$$H = (Z - \mu_z)/\sigma_z. \tag{2.29}$$

Substitute μ_z and σ_z from Eqns. (2.26) and (2.27) respectively into Eq. (2.29), H becomes;

$$H = -\left[\frac{\mu_s - \mu_x}{(\sigma_x^2 + \sigma_s^2)^{\frac{1}{2}}}\right]. \tag{2.30}$$

Differentiate H in Eq.2.29 with respect to Z in order to write reliability function in terms of H;

$$\frac{dH}{dz} = \frac{1}{\sigma_z} \qquad or \qquad dz = \sigma_z * dH \tag{2.31}$$

After change of variables in Eq.2.28 from Z to H, the integration limits become,

$$H = \frac{0 - \mu_z}{\sigma_z}\ , \qquad for\ z = 0, \tag{2.32}$$

and

$$H = \infty\ , \qquad for\ z = \infty. \tag{2.33}$$

Hence, we get reliability function in terms H as follows;

$$R = \frac{1}{\sqrt{2\pi}} \int_{-\left[\frac{\mu_s - \mu_x}{(\sigma_x^2 + \sigma_s^2)^{\frac{1}{2}}}\right]}^{\infty} \exp\left[-\frac{H^2}{2}\right]\ dH = \frac{1}{\sqrt{2\pi}} \int_{-\beta}^{\infty} \exp\left[-\frac{H^2}{2}\right]\ dH \tag{2.34}$$

where $\beta = \left[(\mu_s - \mu_x)/(\sigma_x^2 + \sigma_s^2)^{\frac{1}{2}}\right]$. Probability of failure function then becomes;

$$P_f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} \exp\left[-\frac{H^2}{2}\right]\ dH. \tag{2.35}$$

Standard Normal Distribution Table could be used from Eq.2.14 to get;

$$P_f = P(Z \leq 0) = \Phi\left(\frac{0-(\mu_s - \mu_x)}{(\sigma_x^2 + \sigma_s^2)^{\frac{1}{2}}}\right) = \Phi\left(-\frac{\mu_z}{\sigma_z}\right) = \Phi(-\beta) \tag{2.36}$$

where β is defined as the reliability index.

Once the reliability index β is calculated, probability of failure can be determined using standardized normal distribution table as mentioned in Section 2.2.1. Then, Reliability of the mechanical component can be computed using Eq.2.20.

As illustrated above, if strength and stress probability distributions are known, it is easy to find reliability of the component using analytical calculations. Nevertheless, if strength or stress or both come associated with arbitrary probability distributions and as dependent or independent random variables, area of interference cannot be estimated so easily. Hence, some techniques have been developed to resolve this issue.

In this chapter, fundamentals of reliability concept are described, some probability distributions are explained and the Stress-Strength Interference Model, which is the fundamental in the study of finding reliability of a mechanical component, is stated.

Next chapter deals with the reliability methods based on Stress-Strength Interference Model and clarifies how to determine reliability of a mechanical component towards resolving the issue mentioned above.

CHAPTER 3

RELIABILITY ESTIMATION METHODS

Reliability estimation methods based on Stress-Strength Interference Model are presented in this chapter. It is a well-known fact that reliability is a part of probability theory, therefore reliability methods are also probability methods under the topic of stochastic processes in literature. Analytical and Simulation Methods are the main sections in this chapter.

3.1. Analytical Methods

First Order and Second Order Reliability Methods could be grouped as analytical methods. These methods are based on the first order and second order Taylor Series approximations respectively.

3.1.1. First Order Reliability Method (FORM)

This method was suggested by Hasofer and Lind [27] for standard normal variables. Firstly, it is important to transform normal or lognormal variables into standard normal variables in this technique as follows;

For normal distributions,

$$U = \frac{X - \mu_x}{\sigma_x}, \tag{3.1}$$

and for lognormal distributions,

$$U = \frac{\ln(X) - \mu_x^*}{\sigma_x^*} \; where \;\; \mu_x^* = ln\mu_x - \frac{{\sigma_x^*}^2}{2} \;\; and \;\; \sigma_x^* = \sqrt{\ln\left(1 + \frac{{\sigma_x}^2}{{\mu_x}^2}\right)}. \tag{3.2}$$

The linear state function as in the general form

$$g(X_1, X_2, \dots, X_n) = a_0 + a_1X_1 + \cdots + a_nX_n = a_0 + \sum_{i=1}^{n} a_iX_i \tag{3.3}$$

where X_i's are independent random variables, gives rise to the Reliability index;

$$\beta = \frac{a_0 + \sum_{i=1}^{n} a_i \mu_i}{\sqrt{\sum_{i=1}^{n}(a_i\sigma_i)^2}}. \tag{3.4}$$

In case of nonlinear state function, the method is totally different. It can be seen in Figure 3.1 that approximation of non-linear limit state function by its tangent plane (its linearization) from design point in normalized coordinate system gives the shortest distance from origin. This distance is seen to be the most probable failure point and could be used as an approximation for β. Note that Figure 3.1 is only for two random variables u_1 & u_2 case.

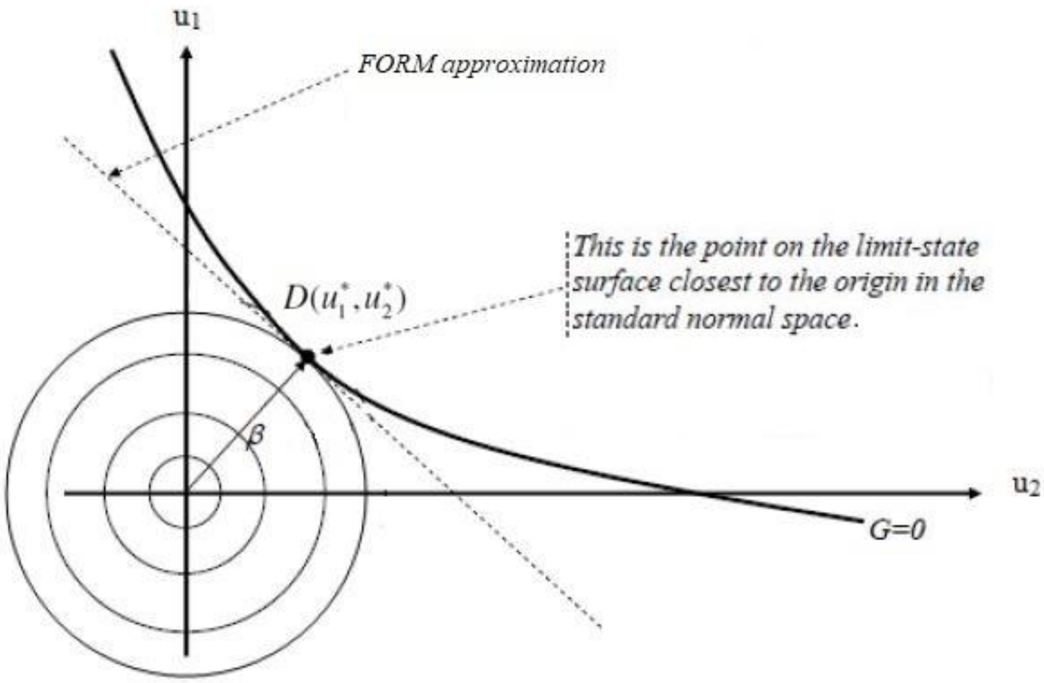


Figure 3.1. Non-linear Graphical Representation of FORM [30]

where D is the design point.

Minimization of OD is found using the following representation:

$$\min(\overline{OD}) = min\,\boldsymbol{u} = \sqrt{\boldsymbol{u}^T\boldsymbol{u}} \tag{3.5}$$

where T represents matrix transpose.

Boundary conditions are $g(\boldsymbol{x} = \boldsymbol{u}) = 0$.

Thus, from Lagrange Multiplier Method, reliability index for non-linear limit state function is as follows;

$$\beta_{H-L} = \frac{u_*^T \left(\frac{\partial g}{\partial u}\right)_*}{\left(\left(\frac{\partial g}{\partial u}\right)_*^T \left(\frac{\partial g}{\partial u}\right)_*\right)^{1/2}} = \frac{\sum_{i=1}^n u_i^* \left(\frac{\partial g}{\partial u_i}\right)_*}{\sqrt{\sum_{i=1}^n \left(\frac{\partial g}{\partial u_i}\right)_*^2}} \tag{3.6}$$

where $\left(\frac{\partial g}{\partial u}\right)_*$ is evaluated at point $D(u_1^*, u_2^*)$ or simply $\boldsymbol{u}_*$.

In the transformation coordinate system, design point could be written simply as;

$$u = -\ \alpha\beta_{H-L} \tag{3.7}$$

where

$$\alpha = \frac{\left(\frac{\partial g}{\partial u}\right)_*}{\left(\left(\frac{\partial g}{\partial u}\right)_*^T \left(\frac{\partial g}{\partial u}\right)_*\right)^{1/2}} = \frac{\left(\frac{\partial g}{\partial u_i}\right)_*}{\sqrt{\sum_{i=1}^n \left(\frac{\partial g}{\partial u_i}\right)_*^2}}. \tag{3.8}$$

In the original coordinate system, design coordinate becomes;

$$X_i^* = \mu_{X_i} -\ \alpha_i\beta_{H-L}\sigma_{X_i} \tag{3.9}$$

where i=1, 2 ,…, n for normal distribution random variables.

A basic algorithm is suggested by Rackwitz [29] to determine β_{H-L} and $\boldsymbol{u_i^*}$ as follows;

1. Write the performance function $g(X_i) = 0$ which includes basic random variables,
2. Take X_i^* which is generally the mean value of random variables as initial values of the design point and find u_i^* from Eqns.3.1 or 3.2,

Step 2 transforms random variables into standard normal variable and change the coordinate system into normalized coordinate system to get failure surface equation.

3. Calculate $\left(\frac{\partial g}{\partial u}\right)_*$ and α for the design point u_i^*,
4. Determine reliability index β_{H-L} from Eq.3.6 for the new design point,
5. Substitute the values from (3) and (4) into the Eq.3.9,
6. Put the new u_i^* values in terms of β_{H-L} into the limit state function $g(X_i) = 0$ and solve for β_{H-L},
7. Using β_{H-L} from Step 6 find u_i^* values from Eq.3.7,
8. Iterate the process from Step 3 to 7 until the β_{H-L} values in Step 6 and 4 are convergent.

Once β_{H-L} is computed, then reliability of the component can be found using the standard normal distribution tables. The Lognormal distribution case follows the same procedure above through the transformation process given in Eq.3.2.

Note that the shape of the limit state function (Figure 3.1) is significant as to being concave or convex. A good reliability approximation is possible in FORM when the shape of the failure surface is concave, however the approximation is poor when the shape is convex.

3.1.2. Second Order Reliability Method (SORM)

When non-linearity of the limit state function increases, the shape of the failure surface is likely to be convex. In this case, the joint PDF of the random variables decays slowly and a higher order approximation for the probability of failure is necessary. SORM is proposed for such a case. It is a second order refinement of the FORM to estimate probability of failure. Therefore, second order Taylor Series Expansion is used while constructing the appropriate formula.

Graphical representation for the comparison between FORM and SORM is shown in Figure 3.2.

As seen in the figure below, SORM provides a better approximation than FORM. This is due to fact that SORM approximation has higher order than FORM.

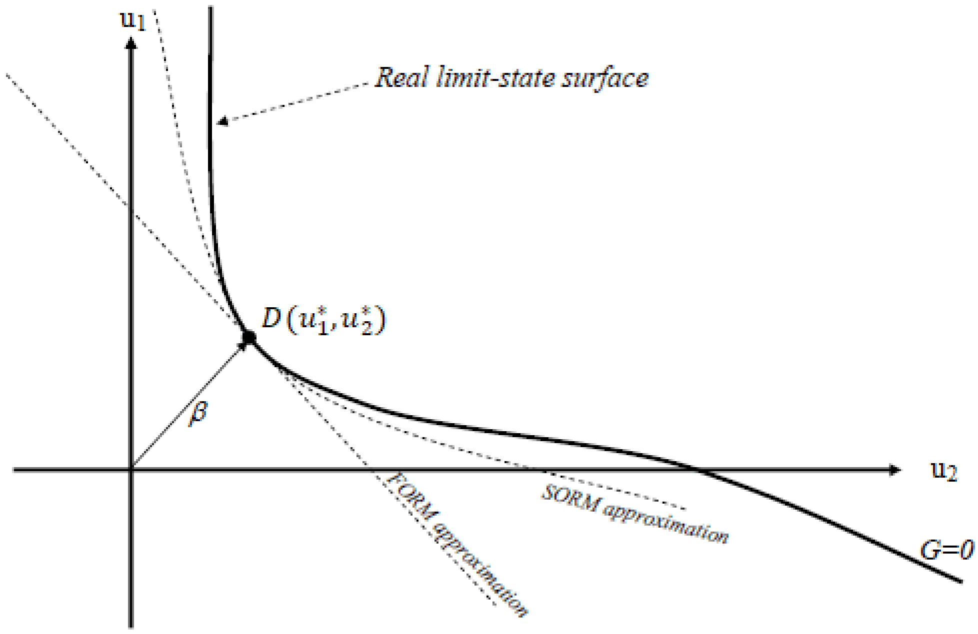


Figure 3.2. Graphical Comparison of FORM and SORM [30]

Different formulations are developed for the approximations higher than FORM. Breitung and Hohenbichler formulas are the most popular in literature for the SORM.

Probability of failure developed by Breitung [31] involves the following formula;

$$P_f = \Phi(-\beta_{H-L}) \prod_{i=1}^{n-1} (1 + \beta_{H-L} k_i)^{-1/2} \qquad for \;\; k_i < 1 \tag{3.10}$$

where k_i is the curvature of the performance function at design point and β is the reliability index of FORM.

Random variables $X_1, X_2, \ldots, X_n$ have usually some correlations and these correlations could be described in the covariance matrix C as follows:

$$C = \begin{bmatrix} {\sigma_{X_1}}^2 & \cdots & \rho_{1n}\sigma_{X_1}\sigma_{X_n} \\ \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_{X_1}\sigma_{X_n} & \cdots & {\sigma_{X_n}}^2 \end{bmatrix} \tag{3.11}$$

where $\sigma_{X1}, \sigma_{X2}, \ \ldots \ , \sigma_{Xn}$ are the standard deviations of the correlated variables and ρ_{ij} is the coefficient of correlation between the random variables X_i and X_j.

In reduced form, correlation coefficient matrix could be defined as;

$$C' = \begin{bmatrix} 1 & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & 1 \end{bmatrix}. \tag{3.12}$$

Y is defined as uncorrelated reduced normal variables and if X is transformed into Y, it could be said that the FORM can be usable. Then, performance function is stated in terms of Y using the following formula;

$$X = [{\sigma_X}^N][T]Y + {\mu_X}^N \tag{3.13}$$

where $[{\sigma_X}^N]$ is a diagonal matrix of the standard deviations which are equivalent, ${\mu_X}^N$is the vector of the equivalent means of the random variables and T denotes the transformation matrix whose columns are the eigenvectors of C'.

Determination of k_i in Eq.3.10 consists of two steps. First step is about rotating Y_i^* variables into $Y_i^{*\prime}$ with the help of rotation matrix R. In this way, last variable of R matrix coincides with unit gradient vector of the limit state at design point

$$Y_i^{*\prime} = RY_i^*. \tag{3.14}$$

R matrix is calculated in two steps. In the first step, R_0 matrix is constructed as follows:

$$R_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \cdots & \cdots & \alpha_n \end{bmatrix} \tag{3.15}$$

where $\alpha_1, \ldots \ , \alpha_n$ are the cosine directions of the unit gradient vector at design point.

In the second step, orthogonalization procedure of Gram-Schmidt is applied. Assume R matrix is the orthogonal matrix of R_0 such that *n*th rows of the two are the same. The other rows are determined as follows:

$$r_k = r_{0k} - \sum_{j=k+1}^{n} \frac{r_j r_{0k}^T}{r_j r_j^T} r_j \qquad for\ k = n-1, n-2, \dots ,1 \tag{3.16}$$

where $r_{01}, r_{02}, \ \dots \ , r_{0n}$ and $r_1, r_2, \ \dots \ , r_n$ are row vectors of R_0 and R respectively and $r_{0n} = r_n$.

After each row of R is orthonormalized i.e. $R^T R = I_n$, matrix A is calculated from the formula given below in the second step for k_i,

$$a_{ij} = \frac{(RFR^T)_{ij}}{|\nabla G(Y^*)|} \tag{3.17}$$

where a_{ij} denotes elements of matrix A and $\nabla G(U^*)$ represents the limit state surface normal vector and F denotes the second order matrix of the limit state surface in the standard normal coordinates which is given below:

$$F = \begin{bmatrix} \dfrac{\partial^2 g}{\partial u_1^2} & \dfrac{\partial^2 g}{\partial u_1 \partial u_2} & \cdots \\ \dfrac{\partial^2 g}{\partial u_1 \partial u_2} & \ddots & \vdots \\ \vdots & \cdots & \dfrac{\partial^2 g}{\partial u_n^2} \end{bmatrix}. \tag{3.18}$$

After determining matrix A, matrix B is constructed by deleting last row and last column of A. Eigenvalues of the matrix B are then the main curvature k_i. SORM of Breitung ends with calculating P_f using Eq.3.10.

Improved approximations of SORM is proposed by Hohenbichler [32] by the following formula:

$$P_f = \Phi(-\beta) \prod_{i=1}^{n-1} \left(1 + \frac{\varphi(-\beta_{H-L})}{\Phi(-\beta_{H-L})} k_i\right)^{-1/2} \qquad for\ \ k_i < 1 \tag{3.19}$$

where $\varphi(-\beta_{H-L})$ is the transformation of reliability index with respect to PDF and $\Phi(-\beta)$ is the transformation of reliability index with respect to CDF of the standard normal distribution.

3.2. Simulation Methods

Refinement of FORM into SORM is insufficient when non-linearity of the limit state function is high and shape of the limit state surface is highly convex. In this case, the analytical methods produce poor results and simulation techniques should be used instead. Monte Carlo Simulation (MCS) is one of the most commonly used methods in that sense. Importance Sampling (IS) and Adaptive Kriging Monte Carlo Simulation (AK-MCS) are other simulation methods which are improved versions of MCS in terms of the number of simulations required.

3.2.1. Monte Carlo Simulation Method (MCS)

This method is able to solve complicated problems and widely used when a problem description involves a lot of random variables for stress or strength or both. The logic simply follows from the trial and error approach. Any integral function could be solved and the result is accepted to be very close to the exact result if adequate number of trials are used.

Fundamental steps of MCS are as follows:

1. Create limit state function as $Z = g(X_1, X_1, X_2, \ldots, X_n)$ where probability distribution, mean and standard deviation of the random variables is known,
2. Using mean and standard deviation parameters, write the probability distribution functions as inputs,
3. Produce a predetermined number (N_{total}) of random variables for each parameter which shows the appropriate probability distribution behavior,
4. Try produced random variables into the performance function one by one,
5. Collect the output data for each trial as the number of failed group (N_{fail}) if the limit state function is $Z \leq 0$ at the end,

6. Find the probability of failure as follows:

$$P_f = N_{fail}/N_{\text{total}} \tag{3.20}$$

Confidence Interval is a necessary and supplementary matter for MCS after finding the probability of failure. MCS results are approximated values and Confidence Interval width increases the accuracy of the results by providing an interval for the estimation.

Confidence Intervals on P_f is determined by the following formula [33]:

$$P_f \in \left[P_f^- \equiv P_f + \sigma_{v,P_f}\Phi^{-1}(\alpha/2)\right] \; and \; \left[P_f^+ \equiv P_f + \sigma_{v,P_f}\Phi^{-1}(1-\alpha/2)\right] \tag{3.21}$$

where $\alpha \in [0,1]$ is confidence level and σ_{v,P_f} is the variance of P_f which is calculated as follows:

$$\sigma_{v,P_f}{}^2 = \frac{P_f(1-P_f)}{N_{\text{total}}} \tag{3.22}$$

Reliability index for MCS (β_{MCS}) is directly proportional to the upper and lower bounds of P_f, therefore following relation can be given for the upper and lower bounds as:

$$\beta_{MCS}{}^+ = -\Phi^{-1}\left(P_f^+\right) \; and \; \beta_{MCS}{}^- = -\Phi^{-1}\left(P_f^-\right). \tag{3.23}$$

Coefficient of Variation (CoV) is an important parameter whose smallness indicates the convergence of the MCS and defined by:

$$CoV = \frac{\sigma_{P_f}}{P_f} = \sqrt{\frac{(1-P_f)}{N_{\text{total}}P_f}} \tag{3.24}$$

Hence, it can be said that convergence rate increases with decreasing P_f and decreases with increasing N_{total}. For instance, to predict a $P_f = 10^{-4}$ with 5% accuracy, approximately $N_{\text{total}} = 4 * 10^6$ samples are needed.

Regardless of complexity of the problem, Monte Carlo Simulation is a very powerful problem solver. Nevertheless, slow convergence rate for low probability of failures is the main disadvantage of this method. Recently, some techniques have been developed to overcome this drawback and to reach satisfactory results with lower number of samples.

3.2.2. Importance Sampling Method (IS)

In this method, a combination of FORM and MCS methods is used due to fast convergence of FORM and robustness of MCS. Main idea is based on the reduction of Coefficient of Variation in general. This is achieved by populating sampling points into the most important region where probability of failure mainly occurs. Mathematically, auxiliary density function is created and it concentrates the overall PDF into an area of possibly failure occurring.

Various importance sampling techniques have been developed in this sense [34],[35]. However, the evolution of this method is completed and it takes its final form in [36].

In general, probability of failure is written in integral form as follows:

$$P_f = \int \dots \int_{g(X_1,X_2,\dots,X_n)} f_x(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \tag{3.25}$$

where n is the number of random variables.

This formula could be written by adding an indicator integrand function:

$$P_f = \int \dots \int I_g(x) f_x(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \tag{3.26}$$

where

$$I_g(x) = \begin{cases} 0 & if\ g(x) > 0 \\ 1 & if\ g(x) \le 0 \end{cases} \quad . \tag{3.27}$$

A new auxiliary density function is defined as $f_1(x)$ in order to get the samples in a concentrated and desired area. Then, P_f becomes:

$$P_{f,IS} = \int \left[I_g(x) \frac{f(x)}{f_1(x)} \right] f_1(x) dx. \tag{3.28}$$

In order to minimize CoV of $P_{f,IS}$ for improving the convergence of MCS, importance sampling function may be used as follows:

$$f_1(x) = \frac{\left| I_g(x) f(x) \right|}{\int \ldots \int \left| I_g(x) f(x) \right| dx}. \tag{3.29}$$

Standard normal distribution based importance sampling density function is developed by centering the design point. In this study, given samples in standard normal space $U = u_{(1)}, \ldots , u_{(n)}$ are correlated with the design point u_* found using FORM (Eq.3.7) to make an efficient sampling distribution;

$$f_1(u) = f(U - u_*). \tag{3.30}$$

Hence, estimation of $P_{f,IS}$ becomes:

$$P_{f,IS} = \frac{1}{N} e^{(-\beta_{H-L}{}^2/2)} \sum_{i=1}^{N} I_g(x_i) e^{(-u_{(k)} \cdot u_*)} \tag{3.31}$$

where k is the iteration number and N is the number of MCS.

Corresponding variance is found as follows:

$$\sigma_{v,P_{f,IS}}{}^2 = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{I_g(x_i) f(x_i)}{f_1(x_i)} - P_{f,IS} \right)^2. \tag{3.32}$$

Confidence interval procedure is the same as crude Monte Carlo Simulation. Therefore, Eqns. (3.21), (3.23) and (3.24) can be used to calculate the confidence bounds of $P_{f,IS}$, the coefficient of variation and the upper-lower bounds of the corresponding reliability index, respectively.

IS Method has some disadvantages as well. If the response surface function g have at least two different local concentrated regions for failure, a weakness of this method arises. However, this situation is not observed for non-periodic functions [36]. Secondly, the importance region should be chosen carefully, otherwise the approximation may mislead and results could be unreasonable. To overcome this problem, failure domain knowledge should be known before creating importance sample function.

3.2.3. Adaptive Kriging Monte Carlo Simulation Method (AK-MCS)

This method is first introduced by Echard [37] and could be described as modifying Monte Carlo Simulation with Kriging which is an active learning process. AK-MCS aims to reduce the large number of simulations in crude MCS. In order to do that a small number of experimental data is used as input and adaptive logic is implemented on the Monte Carlo population for all these data points. According to this logic, the best next point is chosen with the help of the learning function which focuses on the condensed points that cause probability of failure and the process continues until desired convergence criteria is reached. The main difference of this technique in comparison to Importance Sampling is that local regions are used to calculate global failure.

Before explaining the steps of this method, Kriging and Learning Function concepts should be explained.

Kriging Theory is a type of stochastic interpolation process assuming that limit state function $g(x)$ is treated as a realization of an output metamodel $\mathcal{M}^K(x)$. This metamodel is defined as follows;

$$\mathcal{M}^K(\boldsymbol{x}) = F(\boldsymbol{x}, \boldsymbol{\beta}) + z(\boldsymbol{x}) \tag{3.33}$$

where $F(\boldsymbol{x}, \boldsymbol{\beta})$ is the mean value of the response approximation and $z(\boldsymbol{x})$ is a Gaussian process modeling which has a zero mean.

$F(\boldsymbol{x}, \boldsymbol{\beta})$ denotes the tendency of Kriging model and consists of two terms;

$$F(x, \beta) = \boldsymbol{\beta}^T \boldsymbol{f}(\boldsymbol{x}) \tag{3.34}$$

where $\boldsymbol{\beta}^T = \{\beta_1, \beta_2, \ldots, \beta_k\}$ are the regression coefficients and $\boldsymbol{f}(\boldsymbol{x}) = \{\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{f}_2(\boldsymbol{x}), \ldots, \boldsymbol{f}_k(\boldsymbol{x})\}$ are the basis functions.

There are some Kriging tendency types in literature. However, ordinary Kriging type is used for AK-MCS method where tendency has an undefined constant value and basis function equals to 1. Namely,

$$\boldsymbol{\beta}^T \boldsymbol{f}(\boldsymbol{x}) = \beta_1 \text{ and } f_1(x) = 1. \tag{3.35}$$

The term z($\mathbf{x}$) of the main equation (Eq.3.33) is a zero mean Gaussian process. Covariance of z($\mathbf{x}$) can be described explicitly as:

$$z(\boldsymbol{x}) = \sigma_z^2 Z_0(\boldsymbol{x}, \boldsymbol{w}) \tag{3.36}$$

where σ_z^2 is the variance of the Gaussian process and $Z_0(\boldsymbol{x}, \boldsymbol{w})$ is called the Correlation Function with unit variance between two points $\boldsymbol{x}$ and $\boldsymbol{w}$ which represent correspondence between experimental points and new points in Kriging. There are some correlation function types, but the anisotropic-squared-exponential function (Anisotropic Gaussian Model) is chosen for AK-MCS method. Correlation function is given by:

$$Z_0(\boldsymbol{x}, \boldsymbol{w}) = \prod_{i=1}^{n} e^{[-\theta_i (x_i - w_i)^2]} \tag{3.37}$$

where x_i and w_i represent the coordinates of the points $\boldsymbol{x}$ and $\boldsymbol{w}$ respectively and n is the number of random variables. In addition to this, θ represents its hyperparameter and θ_i is described as a scalar which is inverse of the correlation length in i^{th} direction.

In this method, $\hat{\beta}$ in Eq.3.34 and $\hat{\sigma}_z^2$ in Eq.3.36 are approximated as [38]:

$$\hat{\beta} = (\mathbf{1}^T \mathbb{Z}_0{}^{-1} \mathbf{1})^{-1} \mathbf{1}^T \mathbb{Z}_0{}^{-1} \boldsymbol{Y} \tag{3.38}$$

$$\hat{\sigma}_z^2 = \frac{1}{p} (\boldsymbol{Y} - \beta \mathbf{1})^T \mathbb{Z}_0{}^{-1} (\boldsymbol{Y} - \beta \mathbf{1}) \tag{3.39}$$

where $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_p\}$ is the experimental data and $\boldsymbol{Y} = \{\boldsymbol{y}_1 = \mathcal{M}(\boldsymbol{x}_1), \boldsymbol{y}_2 = \mathcal{M}(\boldsymbol{x}_2), \dots, \boldsymbol{y}_p = \mathcal{M}(\boldsymbol{x}_p)\}$ is the corresponding response and $\mathbf{1}$ reperesents the vector whose elements are 1 with a length of p.

What is missing point in these equations are the hyperparameters θ_i. It is estimated by maximum likelihood estimation procedure over the matrix $\mathbb{Z}_0$ as follows:

$$\theta_i = arg\min_{\theta}(det\,\mathbb{Z}_0)^{\frac{1}{p}}\,\sigma_Z^2. \tag{3.40}$$

Approximated performance function of the given undefined point is determined by the formula due Matheron [39] that is also known as the Best Linear Unbiased Predictor (BLUP):

$$\hat{g}(\boldsymbol{x}) = \beta + \boldsymbol{r}(\boldsymbol{x})\mathbb{Z}_0{}^{-1}(\boldsymbol{Y} - \beta\mathbf{1}) \tag{3.41}$$

where $\boldsymbol{r}(\boldsymbol{x}) = \{Z_\theta(\boldsymbol{x}, \boldsymbol{x}_i)\}_{i=1,\dots,p}$.

The variance of the Kriging $\sigma_{\hat{g}}^2(\boldsymbol{x})$ is determined by the error between the performance function and its response. This error is the minimum of the mean square error between them:

$$\sigma_{\hat{g}}^2(\boldsymbol{x}) = \sigma_Z^2\left(1 + \boldsymbol{v}(\boldsymbol{x})^{\boldsymbol{T}}(\mathbf{1}^T\mathbb{Z}_0{}^{-1}\mathbf{1})^{-1}\boldsymbol{v}(\boldsymbol{x}) - \boldsymbol{r}(\boldsymbol{x})^{\boldsymbol{T}}\mathbb{Z}_0{}^{-1}\boldsymbol{r}(\boldsymbol{x})\right) \tag{3.42}$$

where $\boldsymbol{v}(\boldsymbol{x}) = \mathbf{1}^T\mathbb{Z}_0{}^{-1}\boldsymbol{r}(\boldsymbol{x}) - \mathbf{1}.$

Kriging variance at the experiment points $\boldsymbol{x_i}$ is null since the estimation of this approximation $\hat{g}(\boldsymbol{x}_i)$ over the $g(\boldsymbol{x}_i)$ is exactly the same at these points. As a result, it is easy to find the local variances analytically. This property is used mainly in reliability studies.

Secondly, Learning Function $L(\boldsymbol{x})$ should be clarified in order to understand AK-MCS steps better. The best next point $\boldsymbol{s}_*$ on the limit state function is appointed via this function where it is used for Active Learning Process as follows:

$$|\hat{g}(\boldsymbol{x})| - L(\boldsymbol{x})\sigma_{\hat{g}}(\boldsymbol{x}) = 0. \tag{3.43}$$

Based on the formula above, it can be said that Learning Function of AK-MCS is simply the division of mean to the standard deviation of $\hat{g}(\boldsymbol{x})$. Therefore, local probability of failure becomes:

$$P_{f_{local}} = \Phi\big(-L(\boldsymbol{x})\big). \tag{3.44}$$

The best next point $\boldsymbol{s}_*$ is chosen from a sample set created via Monte Carlo Sampling $\boldsymbol{S} = \{\boldsymbol{s}_1, \boldsymbol{s}_2, \dots, \ \boldsymbol{s}_N\}$. The only parameter in this choice is maximizing $P_{f_{local}}$.

Learning Criteria or stopping function is proposed by Schobi et al. [40] as follows:

$$\frac{\hat{P}_f^+ - \hat{P}_f^-}{\hat{P}_f^0} \leq \epsilon_{\widehat{P_f}} \tag{3.45}$$

where $\epsilon_{\widehat{P_f}} = 10\%$ and probability of failures are determined as:

$$\hat{P}_f^0 = \mathrm{P}(|\hat{g}(\boldsymbol{x})| \leq 0), \tag{3.46}$$

$$\hat{P}_f^+ = \mathrm{P}(|\hat{g}(\boldsymbol{x})| - k\sigma_{\hat{g}}(\boldsymbol{x}) \leq 0), \tag{3.47}$$

$$\hat{P}_f^- = \mathrm{P}(|\hat{g}(\boldsymbol{x})| + k\sigma_{\hat{g}}(\boldsymbol{x}) \leq 0). \tag{3.48}$$

Here $k = \Phi^{-1}(1 - \alpha/2)$ with $\alpha \in [0,1]$ is the confidence level.

AK-MCS algorithm based on Kriging and Learning Function has the following steps:

1. Create a small number of experimental data $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \ \boldsymbol{x}_p\}$ and corresponding responses $\boldsymbol{Y} = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \dots, \ \boldsymbol{y}_p\}$.
2. Determine a Kriging model $\hat{g}(\boldsymbol{x})$ from these input and output locally.
3. Generate a large Monte Carlo Sampling $\boldsymbol{S} = \{\boldsymbol{s}_1, \boldsymbol{s}_2, \dots, \ \boldsymbol{s}_N\}$ for the candidate of the best next point and determine corresponding responses from Kriging metamodel $\hat{g}_s = \{\hat{g}_{s_1}, \hat{g}_{s_2}, \dots, \ \hat{g}_{s_N}\}$.
4. Select the best next point $\boldsymbol{s}_*$ using learning function and add the experimental data $\boldsymbol{X}$.

5. Apply Stopping Function in Eq.3.45.
6. If convergence is obtained move to step 8. Otherwise, proceed to Step 7.
7. Update the former experimental design with $\boldsymbol{s}_*$ and its response $g(\boldsymbol{s}_*)$. Go back to Step 2 and repeat this cycle until stopping criteria is met.
8. Calculate the probability of failure using MCS with Kriging metamodel performance function $\hat{g}(\boldsymbol{x})$.

Reliability results can be obtained by evaluating a small number of experimental data in AK-MCS method. Hence, computation time is reduced and results are obtained with more efficiently. Among all the reliability methods mentioned, AK-MCS is the most effective and practical one in mechanical reliability context. Next chapter is concerned with to the application of these methods.

CHAPTER 4

RELIABILITY ANALYSIS OF MECHANICAL COMPONENTS

In this chapter, some mechanical components of the helicopter are taken into account. After describing the component, problem definition is established and the limit state function of the component is constructed. Reliability estimations based on the methods mentioned in the previous chapter are performed. Reliability results and related graphs are presented and discussed.

4.1. Landing Gear Emergency Extension System

Landing Gear System is extensive at the bottom surface of the aircrafts. In early designs of the rotorcrafts, a pair of skid is used in landing operations. With the development of the helicopter designs, swiveling wheels came into prominence. In addition to this, retractable landing gears became more popular due to its elimination of the drag force as speed of the aircrafts are increased in today's technology. A typical retractable landing gear system is shown in Figure 4.1.

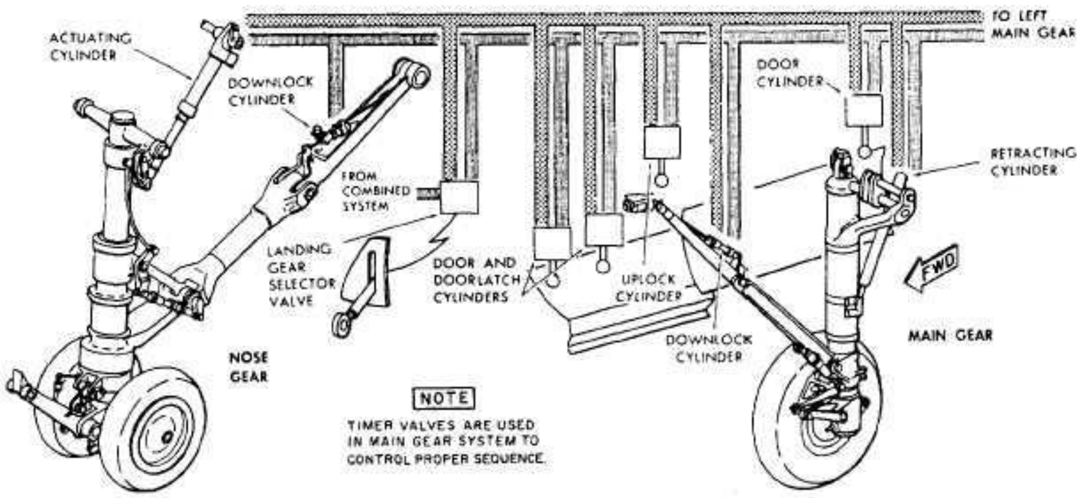


Figure 4.1. Landing Gear System Representation [41]

Landing Gear System is actuated by hydraulic fluid power. However, in case the main hydraulic power system fails, Emergency Extension System is activated for the extension of the Nose Landing Gear (NLG) and Main Landing Gears (MLG). Emergency Extension System performs the extension of the landing gear with the power of nitrogen pressure in the helicopters. Air pressure and mechanical free fall are some types of emergency extension as well. However, in this study only nitrogen storage bottle system is analyzed. This system includes a nitrogen bottle, a nitrogen selector valve, a shuttle valve also known as landing gear selector valve, mechanical handle for the activation and related hydraulic pipework. A general nitrogen bottle emergency extension system is shown in Figure 4.2.

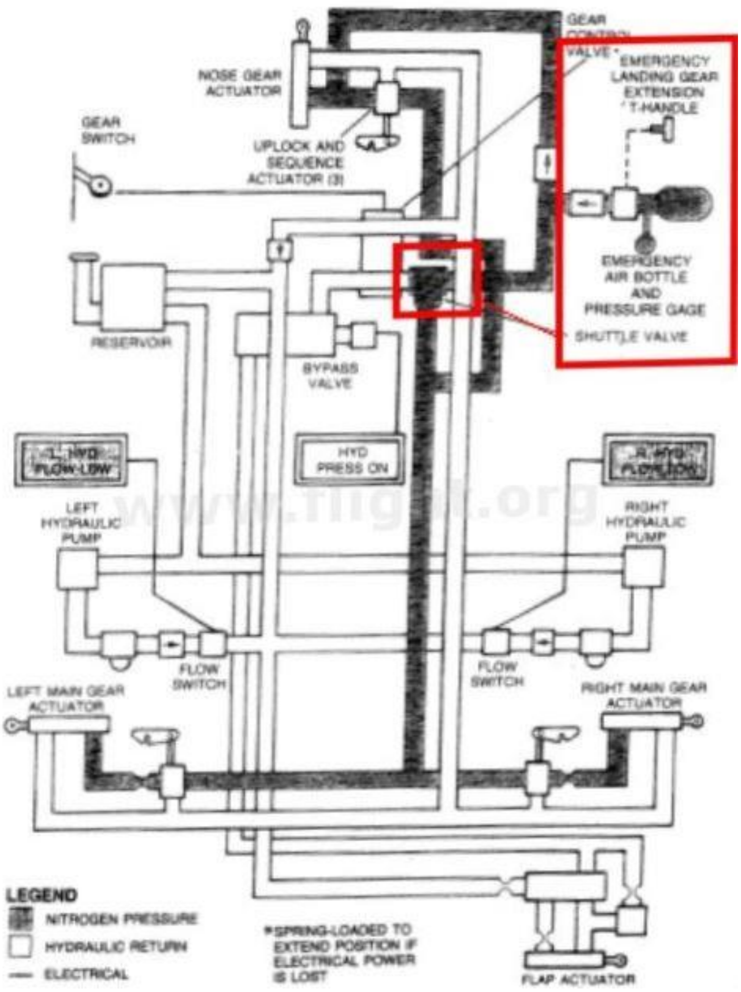


Figure 4.2. Nitrogen Bottle Emergency Extension System [42]

Working Principle of Emergency Extension System is that a nitrogen bottle stores required nitrogen pressure for the extension of Landing Gears and a nitrogen selector valve activates the Landing Gear Selector Valve for extension of the gears when required. Charging manifold manages the nitrogen bottle pressure with the help of a check valve. See Figure 4.3 for working principle of the sub-system.

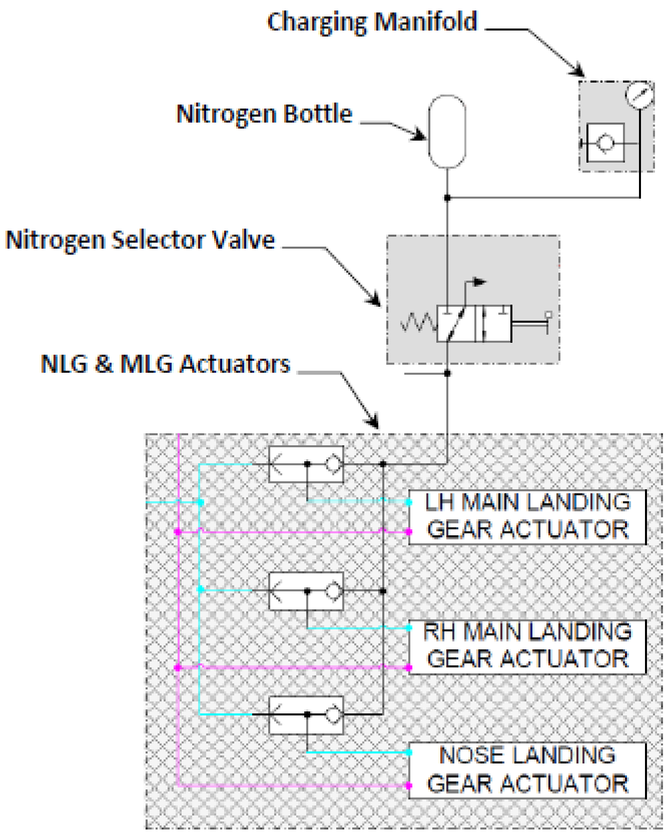


Figure 4.3. LG Emergency Extension Sub-system working principle

Landing Gear Selector Valve should be described to understand the system clearly. It allows normal operations (Extension/Retraction) of the landing gears via hydraulic power and composed of a spool-sleeve valve and a venting valve.

A solenoid controls the spool/sleeve valve which has 3 positions and 4 ways:

- Neutral position: Landing gears are connected to return.

- ➢ Position 1: Landing gears extension chamber are connected to hydraulic pressure.
- ➢ Position 2: Landing gears retraction chamber are connected to hydraulic pressure.

The oil flows to return line of the hydraulic system from the landing gear actuators' retraction chamber with the help of the venting valve.

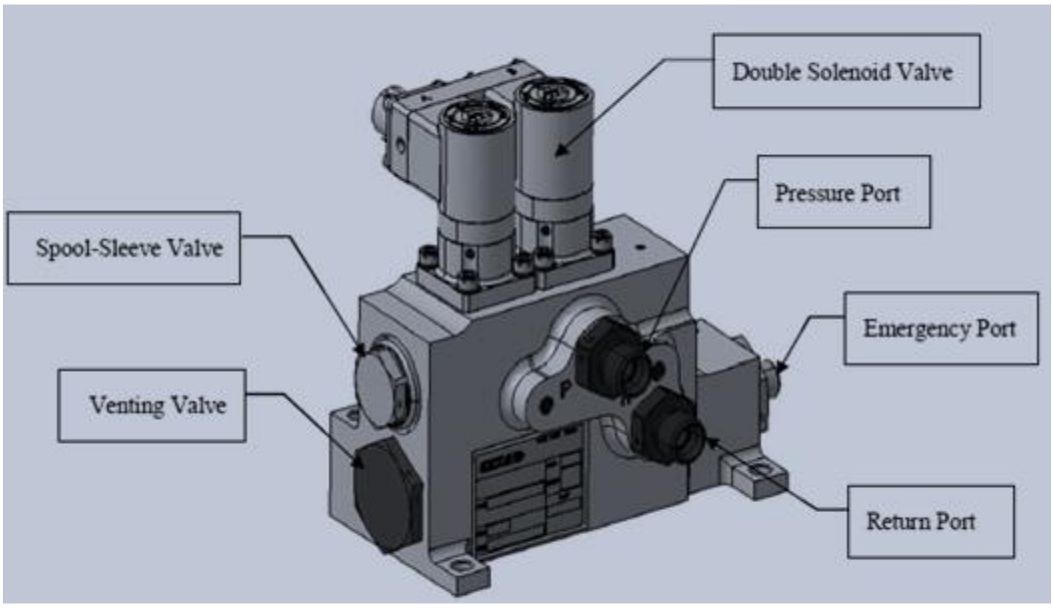


Figure 4.4. LG Selector Valve Isometric View

Nitrogen pressure is used for the nitrogen selector valve activation. In this way, landing gears' oil supply is cut down so that retraction of landing gears is prevented. Meanwhile, return line is connected to the line which is coming from the retraction chambers. This system enables the oil which is gathered in this chamber to be returned to the reservoir. Consequently, deployment movement is not affected.

LG selector valve' lines are described below and shown in Figure 4.5.

- ➢ Emergency (E): Emergency (Nitrogen) Line
- ➢ Pressure (P): Main Pressure Line
- ➢ Return (R): Return Line
- ➢ Supply (S1): LG Extension Line

➢ Supply (S2): LG Retraction Line

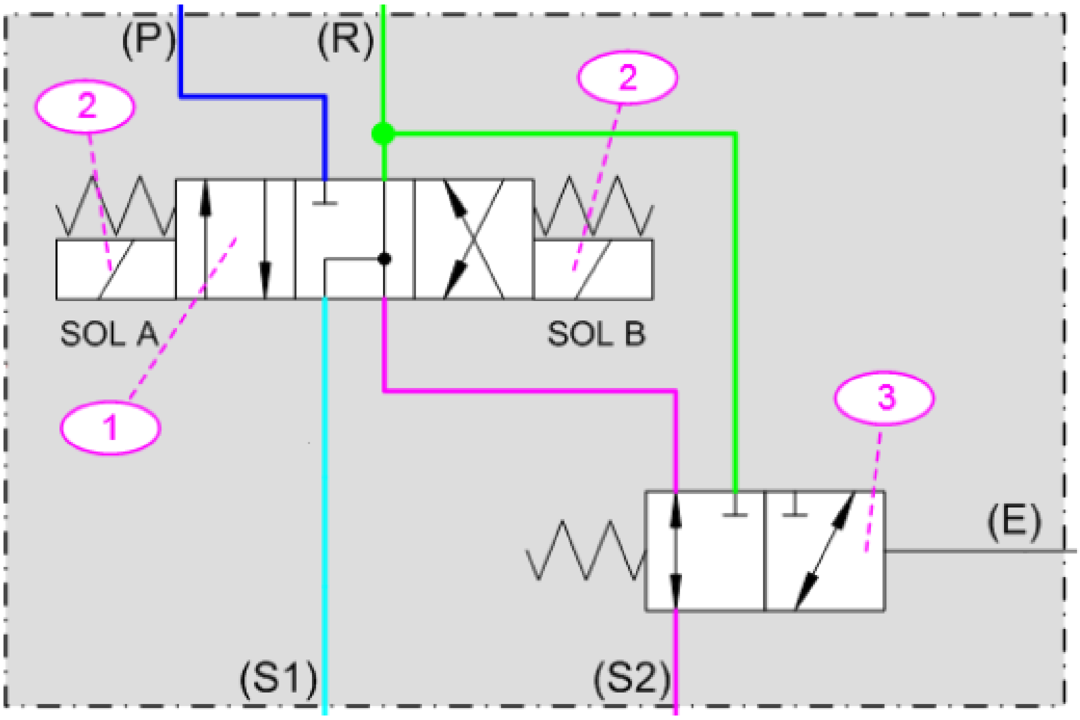


Figure 4.5. LG Emergency Extension Sub-System Architecture

In the architecture above, the valves numbered as 1, 2, and 3 represent Spool-Sleeve Valve, Double-Solenoid Valve and Venting Valve, respectively. In this study, Venting Valve is assessed for the mechanical reliability estimations.

4.2. Venting Valve

In case of an emergency activation, venting valve transports oil which is gathered in the landing gears to the return line. The oil pressure supply is simultaneously cut down in order to prevent landing gears from retraction.

Once the pressure from the emergency line arrives at the emergency port of the LG Selector Valve, the S2 port is connected to the return line. Venting Valve consists of two springs, two poppets, a ball, a housing and some holes for related hydraulic lines. Detailed figure is shown Figure 4.6.

When the emergency pressure arrives at LG Selector Valve piston, the venting valve connects oil chamber of the landing gears to the return line. This function is provided by the emergency piston which pushes the retaining ball that keeps the LG retraction chamber from the return line. See Figure 4.7. for details.

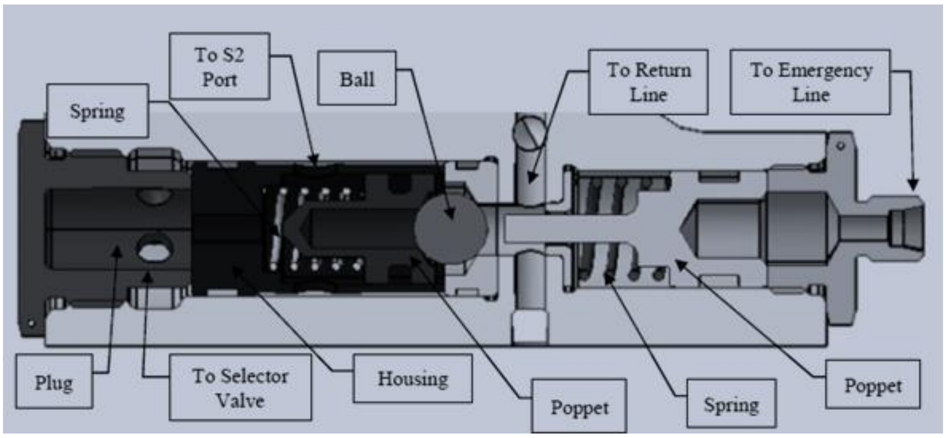


Figure 4.6. Details of Venting Valve

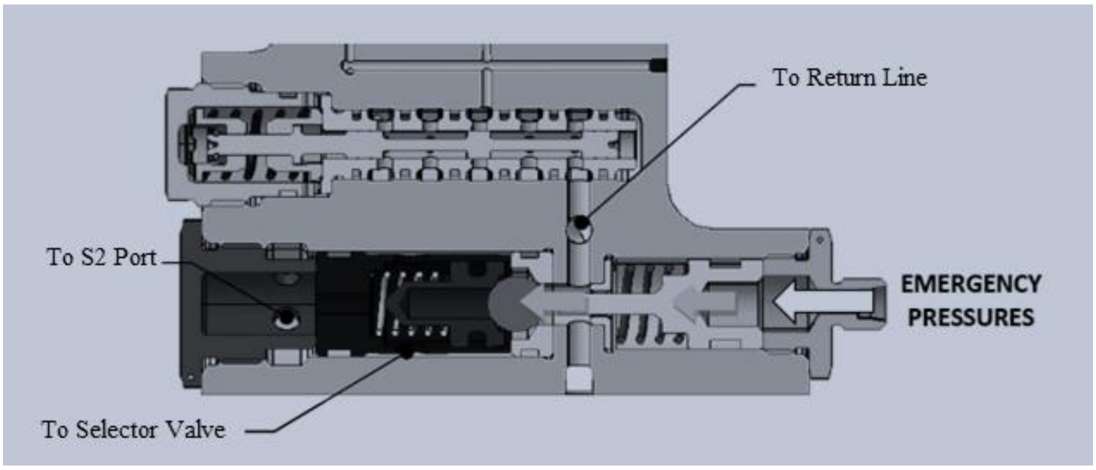


Figure 4.7. Details of Venting Valve Mechanism

4.3. Problem Definition and Limit State Function for Venting Valve

Reliability of the venting valve can be found by analyzing failure modes instead of using general reliability database. When the failure modes of the Venting Valve is taken into account, dominant failure mode is determined as stuck-closed in the guidance of Mechanistic Model Approach [28]. The main cause of this failure mode is due to not sending enough fluid pressure to venting valve from nitrogen bottle for the extension of landing gears according to engineering judgement.

The minimum load provided for the movement of the emergency piston needs to overcome the load provided by the supply pressure to the retaining ball and the springs.

Limit state condition shown below needs to be fulfilled for the lowest pressure applied to the emergency port, which corresponds to the pre-charge pressure of the nitrogen bottle:

$$p_{emergency} * A_{piston} > p_{supply} * A_{ball} + F_{springs}. \tag{4.1}$$

The areas needed for Eq.4.1 are shown in Figure 4.8. The diameter values are chosen according to the maximum loads which compress the springs fully.

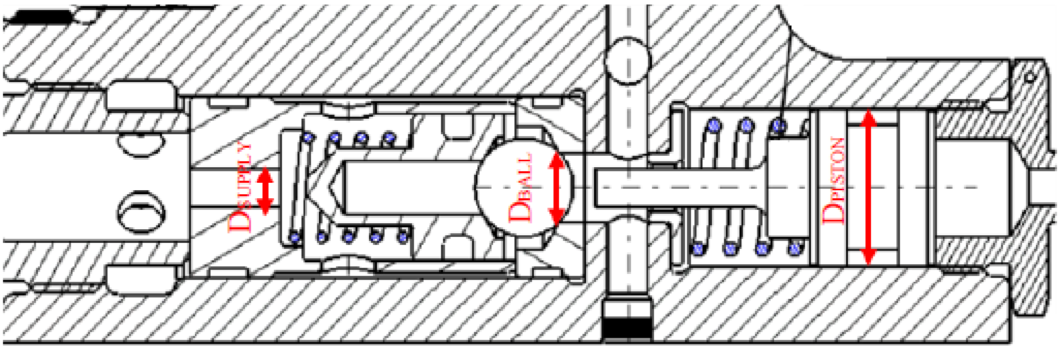


Figure 4.8. Areas used for calculation for Venting Valve

After some arrangements, limit condition of the venting valve for the failure mode of stuck-closed, when required, becomes:

$$p_{emergency} > p_{supply}\left(\frac{D_{ball}}{D_{piston}}\right)^2 + \frac{4F_{springs}}{\pi D_{piston}{}^2}. \tag{4.2}$$

Then, Limit State Function or Performance Function becomes:

$$g(x) = p_{emergency} - p_{supply}\left(\frac{D_{ball}}{D_{piston}}\right)^2 + \frac{4F_{springs}}{\pi D_{piston}{}^2} \leq 0. \tag{4.3}$$

Therefore, probability of failure for the venting valve is calculated as:

$$P_f = P\left(p_{emergency} \le p_{supply}\left(\frac{D_{ball}}{D_{piston}}\right)^2 + \frac{4F_{springs}}{\pi {D_{piston}}^2}\right). \tag{4.4}$$

In this function, “Stress” variable is $p_{emergency}$ and the right-hand side of the inequality is “Strength” variable according to Stress-Strength Interference Model.

After finding the probability of failure, the reliability of the component is simply subtraction of this value from total probability which is 1.

4.4. Reliability Estimations for Venting Valve

In this study, reliability predictions are obtained by simulation analysis. Matlab version R2018a is used for simulations and UQLab which is a dispatch of Matlab program is used for graphics and reliability results [44].

Results are found using five different reliability methods mentioned in Chapter 3 and related graphs are presented for the venting valve. It is important to note that Confidence Interval (CI) is taken as $\pm5\%$ in MCS, IS and AK-MCS methods. For these three simulation methods, Coefficient of Variation (CoV) is also considered as 1%. Number of simulations in MCS is determined via trial-and-error approach according to probability of failure result (Eq.3.21).

Random variables which model the limit state equations are determined from experimental data which are arranged and interpreted from primitive data to statistically meaningful values. In addition to this, engineering knowledge and experience provide support on determining the random variables.

Two distribution types which are Normal and Log-Normal Distributions are considered for the “Strength” variables. This is due to fact that normal distribution may become meaningless for fluidic values and material properties when negative. This problem is overcome when lognormal probability distribution is used because the values in this distribution type are always positive. Investigation of the difference between the two approaches makes this study distinctive. For “Stress” variable, only uniform distribution type is used for reliability predictions.

4.4.1. Normal Distribution Strength Variables for Venting Valve

Statistical random variables for venting valve is given in Table 4.1. Coefficient of Variation is taken as 1% because it is a general approach for determining standard deviation of the experimental data. For material properties and fluidic values, this value is quite reasonable.

Table 4.1. *Random variables for normal distribution strength of venting valve*

Variables	Mean	Coefficient of Variation (COV)	Distribution Type
p_{supply}(MPa)	20	0.01	Normal
D_{ball}(mm)	81	0.01	Normal
D_{piston}(mm)	350	0.01	Normal
$F_{springs}$(N)	223	0.01	Normal
$p_{emergency}$(MPa)	1.2	0.01	Uniform

4.4.1.1. Result for First Order Reliability Method (FORM)

Reliability results of FORM are given in Table 4.2:

Table 4.2. *FORM results of venting valve for normal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}FORM}$	2.6190e-04
Hasofer-Lind Reliability Index	β_{HL}	3.4683
Number of Evaluations	N_{FORM}	195

Relation between number of iterations and Hasofer-Lind reliability index is presented below in Figure 4.9. Convergence rate can also be interpreted using this graph.

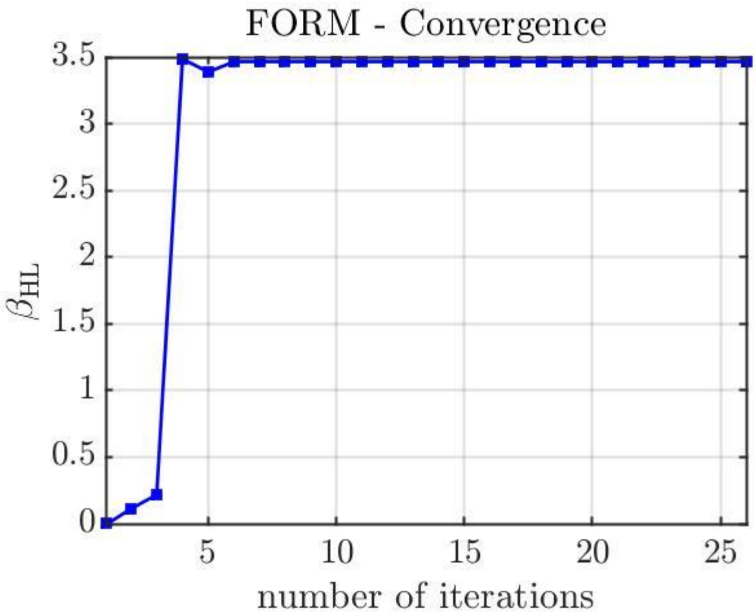


Figure 4.9. FORM reliability index vs number of iterations of venting valve for normal distribution strength

Reliability of venting valve using FORM for normal distribution strength values is 0.999738.

4.4.1.2. Results for Second Order Reliability Method (SORM)

Two types of SORM, which are proposed by Breitung and Hohenbichler, are presented. Reliability results of SORM are given in Table 4.3:

Table 4.3. *SORM results of venting valve for normal distribution strength*

Description	Symbol	Value
Probability of Failure (Breitung)	$P_{f\text{-SORM-Breitung}}$	1.9115e-04
Probability of Failure (Hohenbichler)	$P_{f\text{-SORM-Hohenbichler}}$	1.8794e-04
Number of Evaluations	N_{FORM}	252

Reliability of venting valve using SORM-Breitung for normal distribution strength values is 0.999809. On the other hand, Reliability of venting valve using SORM-Hohenbichler for normal distribution strength values is 0.999812.

4.4.1.3. Result for Monte Carlo Simulation (MCS) Method

Reliability results of MCS are given in Table 4.4. Note that the number of evaluations is determined using Eq.3.41.

Table 4.4. *MCS results of venting valve for normal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}MCS}$	2.1400e-04
Reliability Index	β_{MCS}	3.5222
Number of Evaluations	N_{MCS}	2.0e+06

Relation between the reliability index and the number of evaluations is presented in Figure 4.10. Convergence rate can also be interpreted using this graph. Note that Convergence Interval (CI) is $\pm 5\%$ in Figure 4.10.

Reliability of venting valve using MCS for normal distribution strength values is 0.999786.

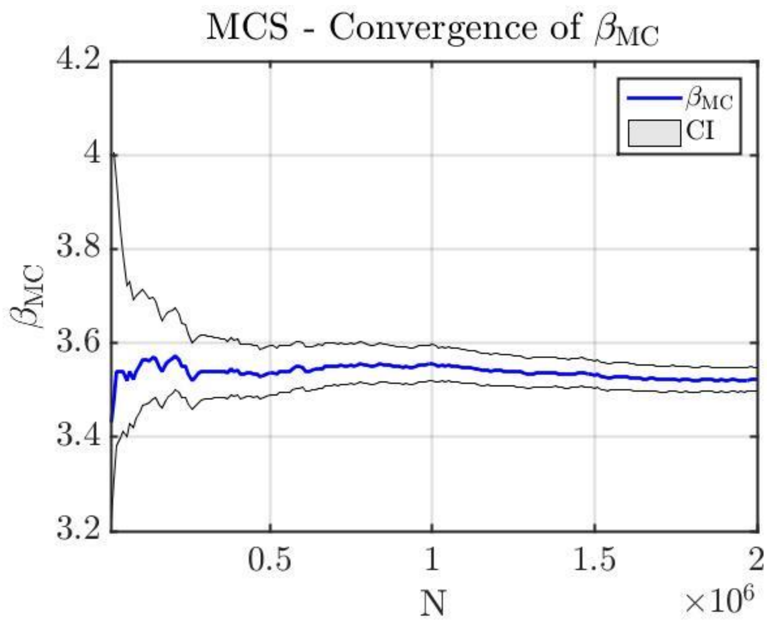


Figure 4.10. MCS reliability index convergence of venting valve for normal distribution strength

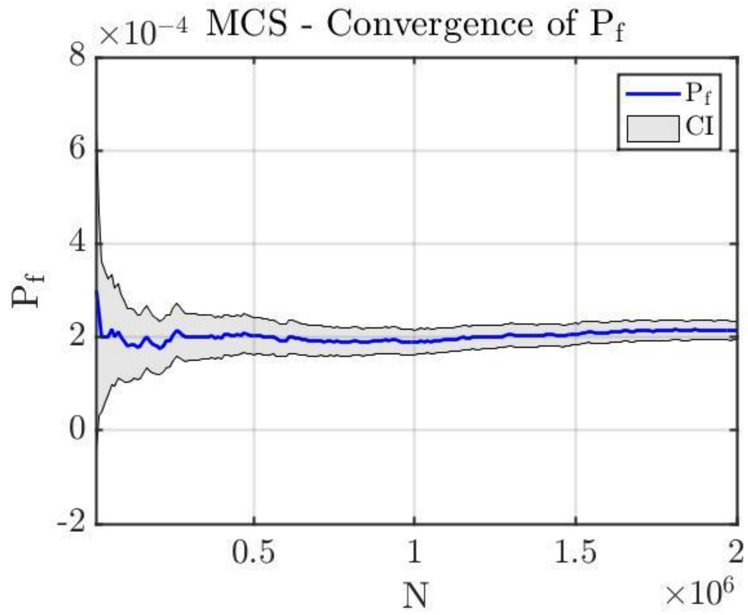


Figure 4.11. MCS probability of failure convergence of venting valve for normal distribution strength

Relation between probability of failure and number of evaluations is presented in Figure 4.11. Using Figure 4.11, convergence rate of probability of failure can also be determined. Note that Convergence Interval (CI) is ±5% in Figure 4.11.

4.4.1.4. Result for Importance Sampling (IS) Method

Reliability results of IS are given in Table 4.5.

Table 4.5. *IS results of venting valve for normal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}IS}$	2.0030e-04
Reliability Index	$\beta_{\text{-}IS}$	3.5397
Number of Evaluations	N_{IS}	1195

Reliability of venting valve using IS for normal distribution strength values is 0.999800.

Relation between probability of failure and the number of evaluations is presented in Figure 4.12. Using Figure 4.12, convergence rate of probability of failure can also be determined. Note that Convergence Interval (CI) is ±5% in Figure 4.12.

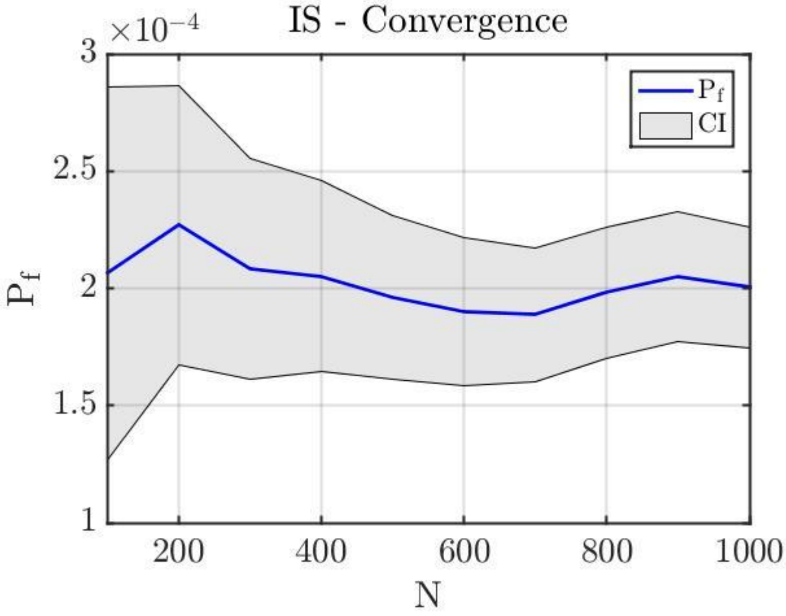


Figure 4.12. IS probability of failure convergence of venting valve for normal distribution strength

4.4.1.5. Result for Adaptive Kriging Monte Carlo Simulation Method (AK-MCS)

Reliability results of AK-MCS are given in Table 4.6.

Table 4.6. *AK-MCS results of venting valve for normal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}IS}$	1.4000e-04
Reliability Index	$\beta_{\text{-}IS}$	3.6331
Number of Evaluations	N_{IS}	23

Reliability of venting valve using AK-MCS for normal distribution strength values is 0.999860.

Relation between probability of failure and number of evaluations is presented in Figure 4.13. Accordingly, convergence rate of probability of failure can also be determined. Note that Convergence Interval (CI) is ±5% in Figure 4.13.

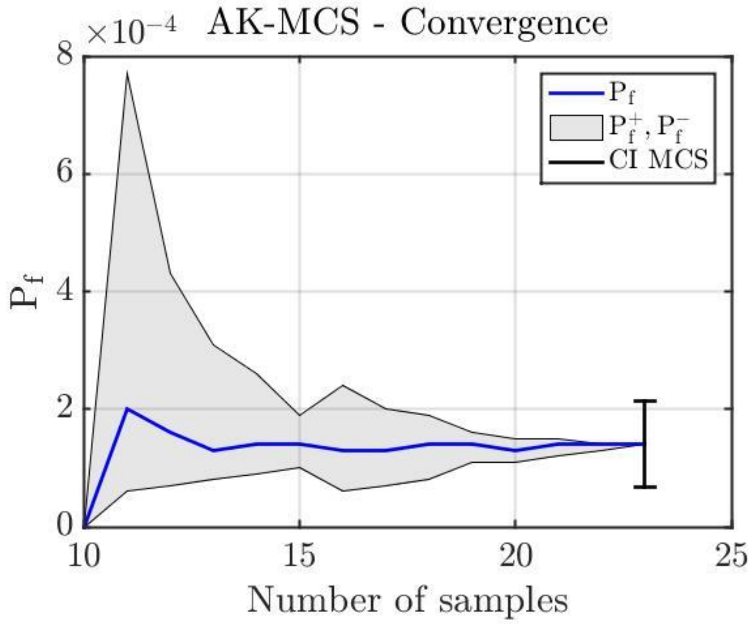


Figure 4.13. AK-MCS probability of failure convergence of venting valve for normal distribution strength

4.4.2. Log-Normal Distribution Strength Variables for Venting Valve

Statistical random variables for venting valve is given in Table 4.7. Coefficient of Variation is taken as 1% since it is a general approach for determining standard deviation of the experimental data. For material properties and fluidic values, this value is quite reasonable.

Table 4.7. *Random variables for lognormal distribution strength of venting valve*

Variables	Mean	Coefficient of Variation (COV)	Distribution Type
p_{supply}(MPa)	20	0.01	Log-Normal
D_{ball}(mm)	81	0.01	Log-Normal
D_{piston}(mm)	350	0.01	Log-Normal

$F_{springs}$(N)	223	0.01	Log-Normal
$p_{emergency}$(MPa)	1.2	0.01	Uniform

4.4.2.1. Result for First Order Reliability Method (FORM)

Reliability results of FORM are given in Table 4.8:

Table 4.8. *FORM results of venting valve for lognormal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}FORM}$	2.6091e-04
Hasofer-Lind Reliability Index	β_{HL}	3.4683
Number of Evaluations	N_{FORM}	209

Relation between number of iterations and Hasofer-Lind reliability index is presented in Figure 4.14. Accordingly, convergence rate can also be determined.

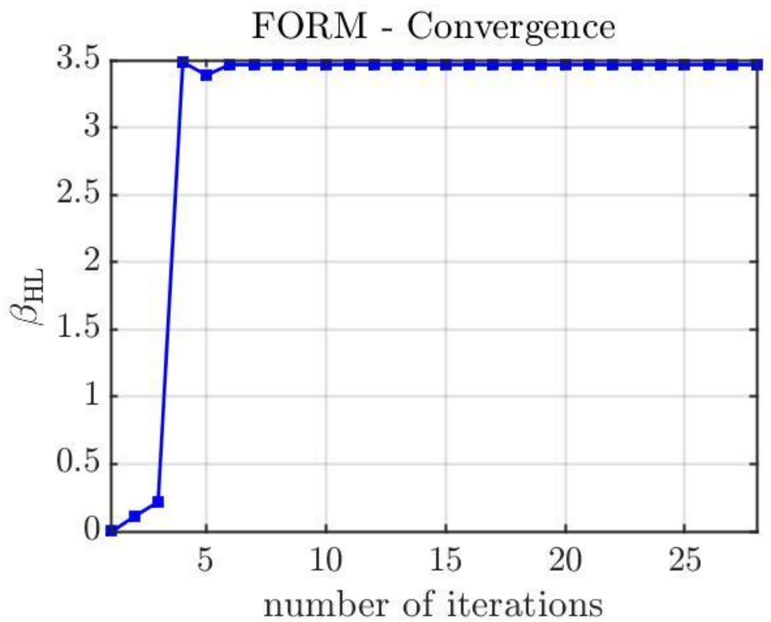


Figure 4.14. FORM reliability index vs number of iterations of venting valve for lognormal distribution strength

Reliability of venting valve using FORM for lognormal distribution strength values is 0.999739.

4.4.2.2. Results for Second Order Reliability Method (SORM)

Two types of SORM, which are proposed by Breitung and Hohenbichler, are presented. Reliability results of SORM are given in Table 4.9:

Table 4.9. *SORM results of venting valve for lognormal distribution strength*

Description	Symbol	Value
Probability of Failure (Breitung)	$P_{f\text{-SORM-Breitung}}$	1.9131e-04
Probability of Failure (Hohenbichler)	$P_{f\text{-SORM-Hohenbichler}}$	1.8816e-04
Number of Evaluations	N_{FORM}	266

Reliability of venting valve using SORM-Breitung for lognormal distribution strength values is 0.999809, while Reliability of venting valve using SORM-Hohenbichler for lognormal distribution strength values is 0.999812.

4.4.2.3. Result for Monte Carlo Simulation (MCS) Method

Reliability results of MCS are given in Table 4.10. Note that the number of evaluations is determined using Eq.3.41.

Table 4.10. *MCS results of venting valve for lognormal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-MCS}}$	2.1400e-04
Reliability Index	β_{MCS}	3.5222
Number of Evaluations	N_{MCS}	2.0e+06

Relation between reliability index and number of evaluations is presented in Figure 4.15. Accordingly, convergence rate of reliability index can also be determined. Note that Convergence Interval (CI) is $\pm 5\%$ in Figure 4.15.

Reliability of venting valve using MCS for lognormal distribution strength values is 0.999786.

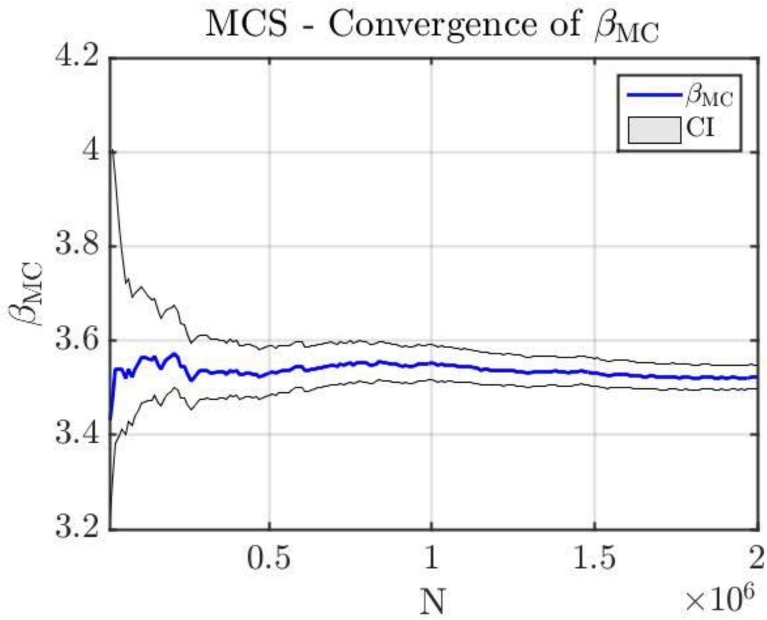


Figure 4.15. MCS reliability index convergence of venting valve for lognormal distribution strength

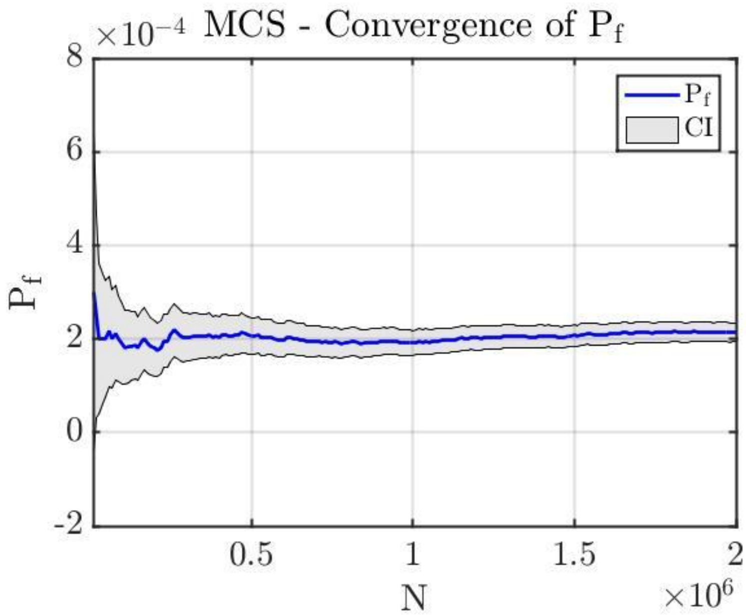


Figure 4.16. MCS probability of failure convergence of venting valve for lognormal distribution strength

Relation between probability of failure and number of evaluations is presented in Figure 4.16. Accordingly, convergence rate of probability of failure can also be determined. Note that Convergence Interval (CI) is $\pm$5% in Figure 4.16.

4.4.2.4. Result for Importance Sampling (IS) Method

Reliability results of IS are given in Table 4.11.

Table 4.11. *IS results of venting valve for lognormal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}IS}$	2.0528e-04
Reliability Index	$\beta_{\text{-}IS}$	3.5332
Number of Evaluations	N_{IS}	1209

Reliability of venting valve using IS for lognormal distribution strength values is 0.999795.

Relation between probability of failure and number of evaluations is presented in Figure 4.17. The convergence rate of probability of failure can also be determined. Note that Convergence Interval (CI) is ±5% in Figure 4.17.

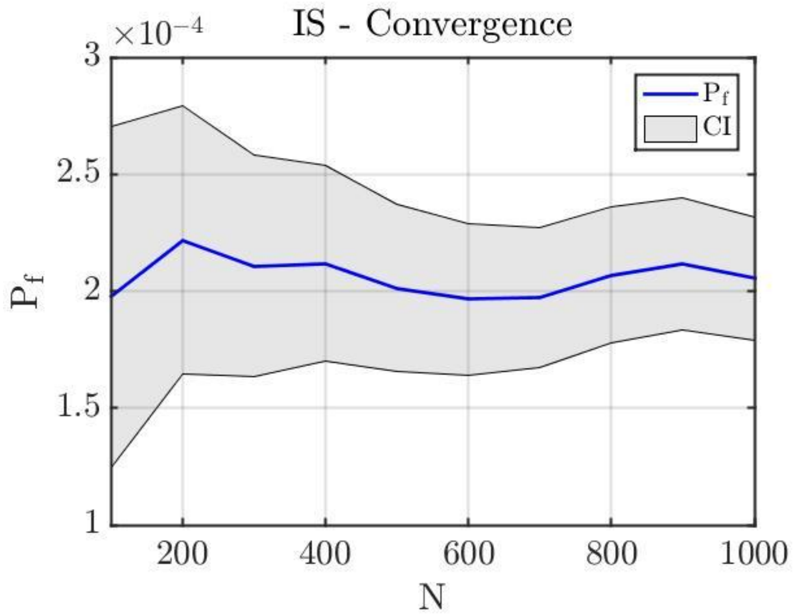


Figure 4.17. IS probability of failure convergence of venting valve for lognormal distribution strength

4.4.2.5. Result for Adaptive Kriging Monte Carlo Simulation Method (AK-MCS)

Reliability results of AK-MCS are given in Table 4.12.

Table 4.12. *AK-MCS results of venting valve for lognormal distribution strength*

Description	Symbol	Value
Probability of Failure	$P_{f\text{-}IS}$	1.3000e-04
Reliability Index	$\beta_{\text{-}IS}$	3.6522
Number of Evaluations	N_{IS}	22

Reliability of venting valve using AK-MCS for lognormal distribution strength values is 0.999870.

Relation between probability of failure and number of evaluations is presented in Figure 4.18. Accordingly, the convergence rate of probability of failure can also be determined. Note that Convergence Interval (CI) is ±5% in Figure 4.18.

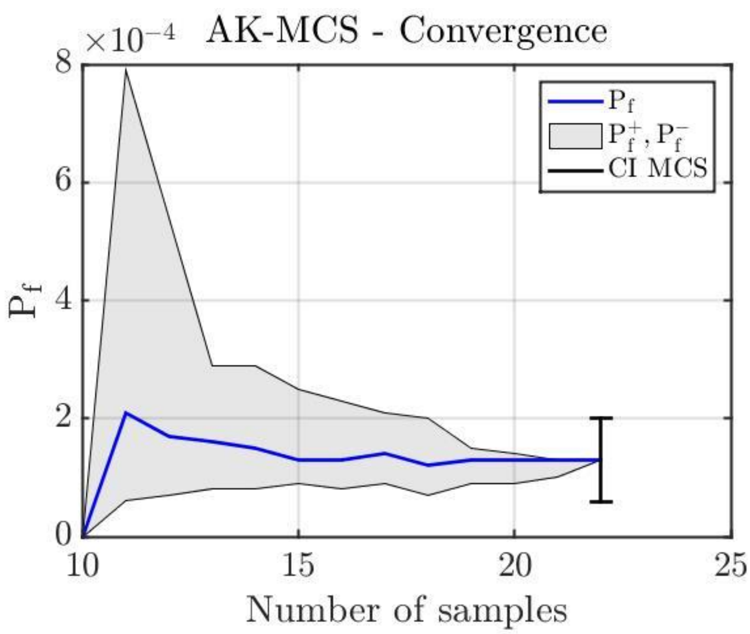


Figure 4.18. AK-MCS probability of failure convergence of venting valve for lognormal distribution strength

4.5. Reliability Results and Comparisons

For a stress-strength interference problem, two different distribution approaches are used and five different methods are employed in the first approach, normal distribution strength and uniform distribution stress data are considered. In the second approach, strength values are considered to behave like lognormal probability distribution and all the other assumptions are kept the same.

Reliability results are accepted to be exact when a required value (Eq.3.24) of simulation is performed using Monte Carlo Simulation. Therefore, reliability results of the other methods are compared with MCS results.

According to Eq.3.24, approximately two million evaluations should be carried out if probability of failure value is to be about 2E-4 and Coefficient of Variation is taken to be 1%. In Table 4.13, comparison of the reliability results using five different methods for normal distribution strength and uniform distribution stress is presented.

Table 4.13. *Comparison of the reliability results of venting valve for normal distribution strength*

Reliability Method	Probability of Failure	Number of Evaluations	Difference of P_f from MCS Result (%)
FORM	2.6190e-04	195	22.4
$SORM_{Breitung}$	1.8794e-04	252	-12.2
$SORM_{Hohenbichler}$	1.9115e-04	252	-10.7
MCS	2.1400e-04	2.0e+06	N/A
IS	2.0030e-04	1195	-6.4
AK-MCS	1.4000e-04	23	-34.6

Among all reliability prediction techniques, the closest approximated P_f value to MCS is obtained using Importance Sampling. About 6% difference is observed. For the other methods, Adaptive Kriging Monte Carlo Simulation Method has maximum difference in probability of failure result compared to MCS. Nevertheless, all five methods are proven to be good approximation techniques and they do not have much significant difference in comparison to the actual result.

When number of evaluations are taken into account, AK-MCS method has the fewest number (23) and it is the most reasonable and affordable result for engineers. This method is the most developed one in reliability owing to being experimental and easily applicable to all mechanical components.

Comparison of the reliability results of the five different method for lognormal distribution strength and uniform distribution stress is shown in Table 4.14. It is important to note that all the other assumptions are the same and these results are more realistic for nature of materials due to dealing with only positive values in lognormal probability distribution.

Table 4.14. *Comparison of the reliability results of venting valve for lognormal distribution strength*

Reliability Method	Probability of Failure	Number of Evaluations	Difference of P_f from MCS Result (%)
FORM	2.6091e-04	209	21.9
$SORM_{Breitung}$	1.8816e-04	266	-12.1
$SORM_{Hohenbichler}$	1.9131e-04	266	-10.6
MCS	2.1400e-04	2.0e+06	N/A
IS	2.0528e-04	1209	-4.1
AK-MCS	1.3000e-04	22	-39.3

There is no significant difference in reliability results when strength values has a tendency of lognormal probability distribution. IS has a closer value in this case and AK-MCS has lower number of evaluations. According to Table 4.14, although a general convergence trend to MCS is observed, AK-MCS reliability result does not follow this trend. This shows the weak side of this method, however this situation do not change the importance of this technique because reaching an approximated reliability result in 22 evaluations makes this study valuable and efficient.

Next chapter considers the engineering applications of these reliability values. In aircraft industry, safety assessment needs these values for certification issues based on some aeronautical standards. The topic of Reliability makes sense when safety assessment is performed. These are essential in the aircraft design especially for new designs of products with high added value. In this way, reliability results find important use in an engineering area.

CHAPTER 5

ENGINEERING APPLICATION OF RELIABILITY: SYSTEM SAFETY ASSESSMENT

Reliability Analysis is an indivisible part of System Safety Assessment. These two engineering disciplines are related to each other and commonly used in designs of the products with high added value. In this chapter, System Safety Assessment is explained in detail and a small example is performed to understand the safety process and to show the engineering application of the reliability results.

5.1. Description of Safety Assessment Process

Safety Assessment Process in aircrafts consists of three main titles which are Functional Hazard Assessment (FHA), Preliminary System Safety Assessment (PSSA) and System Safety Assessment (SSA). These processes are dependent on each other and sequential.

FHA is performed after aircraft or system functions are defined. These functions are defined based on the functional capability of the aircraft. In this process, potential functional failures are identified and their criticality are classified according to operational phases (Flight phase, Stationary phase) and detection methods (Pilot Detection, System Detection or Hidden Failure).

Criticality of the functional failures determines the reliability allocation of these failures. This allocation is performed in PSSA process as a safety requirement. Fault Tree Analysis (FTA) helps in this matter. In FTA, all sources (mechanical, electrical and so on) of top-failures are investigated according to system architecture. FTA process continues until all the details are considered and written down. Reliability allocation of each equipment or component is done and PSSA process then ends.

SSA process consists of verification of the reliability allocations by reaching target failure probability of the top-failure conditions [45]. In the previous chapter, failure modes and effects are considered and probability of failure value for venting valve is computed. Reliability values of the components or systems can be predicted in various ways as explained in Chapter 1.

Thus, the safety process can be summarized partially as described above. However, this procedure does not explain the whole safety process, it only gives some ideas about the engineering application of the reliability values.

In this study, a small example of the safety process is performed. Before the assessment, some definitions and descriptions from the standards should be clarified. Note that writing of standards cannot be changed or rewritten since these may cause incomprehensibility or incoherency.

5.2. Determination of Failure Condition Criticality

During FHA, failure conditions of functions are determined. "Classification of these functional failures is determined according to their effects on aircraft. These effects are categorized in AC 29.1309 as follows [47]:

A. CATASTROPHIC: Failure Conditions which would result in multiple fatalities to occupants, fatalities or incapacitation to the flight crew, or result in loss of rotorcraft.

NOTE: The safety objectives associated with Catastrophic Failure Conditions may be satisfied by demonstrating that:

- No single failure will result in a Catastrophic Failure Condition; and
- Each Catastrophic Failure Condition is Extremely Improbable

B. HAZARDOUS: Failure Conditions which would reduce the capability of the rotorcraft or the ability of the crew to cope with adverse operating conditions to the extent that there would be:

- A large reduction in safety margins or functional capabilities;

- Physical distress or excessive workloads such that the flight crew's ability is impaired to where they could not be relied on to perform their tasks accurately or completely.
- Possible serious or fatal injury to a passenger or a cabin crew member, excluding the flight crew

NOTE: Hazardous failure conditions can include events that are manageable by the crew by use of proper procedures which, if not implemented correctly or in a timely manner, may result in a Catastrophic Event.

C. MAJOR: Failure conditions which would reduce the capability of the rotorcraft or the ability of the crew to cope with adverse operating conditions to the extent that there would be, for example, a significant increase in crew workload or in conditions impairing crew efficiency, physical distress to occupants, possibly including injuries, or physical discomfort to the flight crew.

D. MINOR: Failure conditions which would not significantly reduce rotorcraft safety, and which would involve crew actions that are well within their capabilities. Minor failure conditions may include, for example, a slight reduction in safety margins or functional capabilities, a slight increase in crew workload, such as, routine flight plan changes or some physical discomfort to occupants.

E. NO SAFETY EFFECT: Failure Conditions that would have no effect on safety; for example, Failure Conditions that would not affect the operational capability of the rotorcraft or increase crew workload, however, could result in an inconvenience to the occupants, excluding the flight crew."

5.2.1. Safety Classifications and Probability Levels

"The following safety objectives, which are based on the AC 29.1309 to ensure an acceptable safety level for equipment and systems as installed on platform, are applicable on Failure Conditions at System Level [47]:

- "Failure Conditions with No Effect on safety may be more frequent than Reasonably Probable.

- Minor Failure Conditions may be Reasonably Probable.
- Major Failure Conditions must be no more frequent than Remote.
- Hazardous/Severe-Major Failure Conditions must be no more frequent than Extremely Remote.
- Catastrophic Failure Conditions must be Extremely Improbable."

5.2.2. Probability of Failure Levels

"For the transport category rotorcrafts, failure probability levels are defined in AC 29.1309 as follows [47]:

A. **EXTREMELY IMPROBABLE:** Extremely improbable events are so unlikely that they need not be considered to ever occur, unless engineering judgment would require their consideration. A probability on the order of $1x10^{-9}$ per flight hour or less is assigned to this classification.
B. **EXTREMELY REMOTE:** Extremely remote events are not expected to occur during the total operational life of a random single rotorcraft of a particular type, but may occur a few times during the total operational life of all rotorcraft of a particular type, that are based on a probability on the order of between $1x10^{-7}$ per flight hour and $1x10^{-9}$ per flight hour.
C. **REMOTE:** Remote events are expected to occur a few times during the total operational life of a random single rotorcraft of a particular type, but may occur several times during the total operational life of a number of rotorcraft of a particular type, that are based on a probability on the order of between $1x10^{-5}$ per flight hour and $1x10^{-7}$ per flight hour.
D. **REASONABLY PROBABLE:** Reasonably probable events may be expected to occur several times during the operational life of each rotorcraft that are based on a probability on the order of between $1x10^{-3}$ per flight hour and $1x10^{-5}$ per flight hour.

E. FREQUENT: Frequent events may be expected to occur often during the operational life of each rotorcraft that are based on a probability on the order of $1x10^{-3}$ per flight hour or greater."

Failure condition classifications and probability level of failures are summarized in Figure 5.1. Verification of the failure condition probability requirement is decided accordingly.

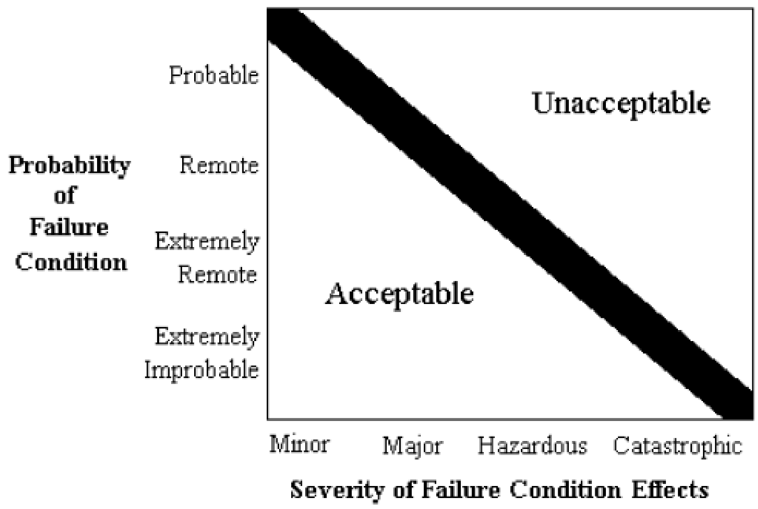


Figure 5.1. Failure Condition Severity vs Probability of Failure Condition [47]

5.3. Fault Tree Analysis (FTA)

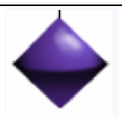
According to definition in SAE ARP 4761, a Fault Tree Analysis (FTA) is a deductive failure analysis which focuses on one particular event and provides a method for determining causes of this event. In other words, a Fault Tree Analysis is a "top-down" system evaluation procedure in which a qualitative model for a particular event is formed and then evaluated. [45]

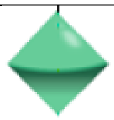
FTAs consist of two kinds of symbols namely, logic and event. AND-gates and OR-gates, which are used as main logic symbols, are the Boolean logic.

AND-gate is used when the undesired top level event can only occur when all the next lower conditions are true. The OR-gate is used when the undesired event can occur if any one or more of the next lower conditions are true.

Most commonly used event symbols include a rectangle, triangle, circle, and diamond as given in Table 5.1.

Table 5.1. *Symbolic Items in Fault Tree Analysis*

Symbol	**Definition**
	**AND Gate:** The AND gate is used to indicate that the output occurs if and only if all the input events occur. There must be at least two inputs to an AND gate.
	**OR Gate:** The OR gate is used to indicate that the output occurs if and only if at least one of the input events occurs. There must be at least two inputs to an OR gate.
	**Transfer Gate:** The Transfer gate is used to link logic in separate areas of a fault tree. When a Transfer gate is selected for insertion, a Transfer In gate is inserted in the fault tree. The Transfer In gate is then linked to a Transfer Out gate, which represents the top gate of another fault tree. There are two primary uses of Transfer gates: • An entire fault tree may not fit on a single sheet of paper. Transfer gates can be used to organize various portions of a large fault tree on separate sheets of paper. The same fault tree logic may be used in different places in a fault tree. Transfer gates can be used to define this logic once and use it wherever necessary.
	**Remarks Gate:** The Remarks gate is used for the entry of comments. A Remarks gate has no calculation data associated with it and therefore has no effect on calculations. However, the tree branch may continue after a Remarks gate.
	**Undeveloped Event:** The undeveloped event is used when further resolution of the event is not necessary for proper evaluation of the

	fault tree or when the information necessary for developing the event is not currently available.
	**Repeated Undeveloped Event:** The repeated undeveloped event is a single event that affects multiple portions of a fault tree.
	**Basic Event:** A basic event represents the lowest level, or terminating event, in a fault tree. A basic event has no inputs. It simply represents the occurrence of an event in the system being analyzed. A basic event is either a component-level event that is not further resolved or an external event. Basic events are the most commonly used primary events in FTA.

In this study, PTC Windchill Quality Solutions 11.0 Enterprise Edition is used for constructing Fault Trees and performing related calculations.

5.4. Safety Assessment of Landing Gear Emergency Extension System

Functional hazard analysis is performed first. Function Name is determined as "Provide Landing Gears Extraction" and Functional Failure Condition is "Loss of Landing Gears Extraction". Failure condition is classified as "Major" since helicopter is capable of performing landing operation with its body without landing gears. This most critical failure condition in this function occurs in the flight phase of "Landing". There is no need to consider other flight phases since this function will not be used in flight except landing.

Failure classification is assessed as "Major" since pilot workload is needed and when the other actions are considered in landing operation, significant increase in pilot workload will likely to be observed. Physical distress to passengers due to small probability of injuries and physical discomfort to pilot are the other causes of "Major" criticality as described in AC 29.1309.

FHA example of this failure condition is shown in Table 5.2:

Table 5.2. *FHA example of Landing Gear Extraction Function*

Failure Condition	**Operational Phase**	**Effect of Failure Condition on Aircraft/Crew**	**Severity**
Loss of Landing Gears Extraction	Landing	All landing gears cannot be extracted. Helicopter will be landed on fuselage. Since pilot will be aware of situation, landing will be performed carefully. Significant reduction in safety margins Crew detection: Pilot will detect failure on landing gear system status indications. Crew action: Pilot will perform landing to a suitable landing area.	Major

When the other functional failures are considered, “Partial Loss of Landing Gears Extraction (Loss of One or Two Landing Gear)” is also another factor. However, severity of these failure conditions is not more than “Major” and repercussions are the same as “Total Loss of Landing Gears”. If the failure condition in Table 5.2 is verified, then verification of “Partial Loss Landing Gears” is also completed.

“Inadvertent Extension of Landing Gears” is another situation that possibly occurs in flight. In a case of inadvertent landing gear extension, helicopter can be controlled by pilot. Repercussions may cause mission or flight cancellation and physical discomfort to passenger. Nevertheless, criticality of this failure condition is considered as “Minor” as defined in AC 29.1309.

These two failure conditions and repercussions are shown in Table 5.3. According to SAE-ARP 4761, justification of “Minor” failure conditions can be provided by design and engineering appraisals and there is no need to show probability of failure justification [45]. Hence, verification of probability of failure requirement of “Loss of Landing Gears Extraction” will be enough for this work.

Table 5.3. *FHA example of Landing Gear Extraction Function (Other FC's)*

Failure Condition	**Operational Phase**	**Effect of Failure Condition on Aircraft/Crew**	**Severity**
Partial Loss of Landing Gears Extraction (Loss of One or Two Landing Gear)	Landing	One or two landing gears cannot be extracted. Significant reduction in safety margins and significant increase in pilot workload. Crew detection: Pilot will detect failure on landing gear system status indications. Crew action: Pilot will perform landing to a suitable landing area.	Major
Inadvertent extension of main/nose landing gears	Flight	Main and/or nose landing gear is extracted inadvertently. Helicopter performance will be decreased. However, helicopter could be controlled by the pilot. Crew detection: Pilot can be aware of situation from displays and sound of extension. Crew action: Mission/flight can be cancelled.	Minor

Second stage is to create FTA and allocate the probability of failure for each equipment or subsystem. Note that for Major failure conditions, probability of failure should be less than 1×10^{-5} per flight hour according to AC 29.1309. Flight hour is taken as 1 hour as an assumption.

In Figure 5.2, PSSA process is performed. Probability allocations are distributed according to FTA symbols and to probability theory described in the previous section [45].

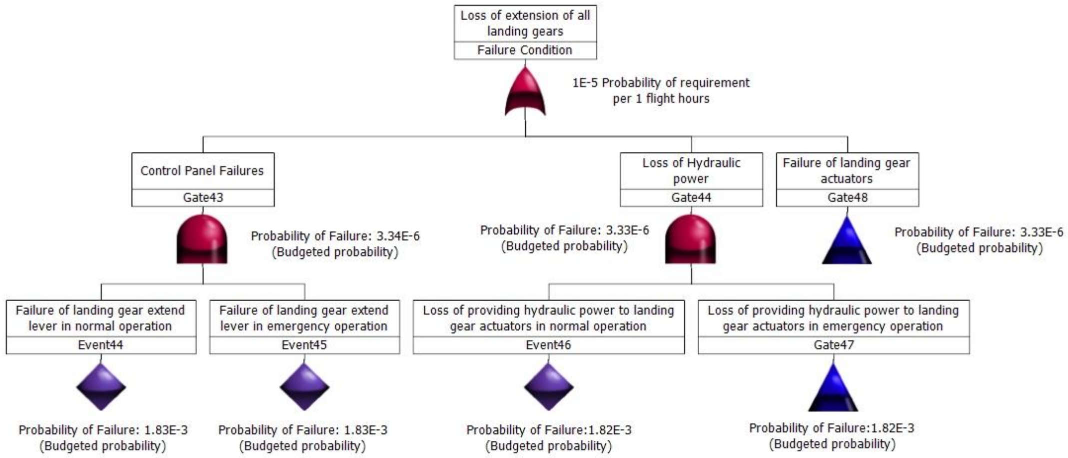


Figure 5.2. PSSA example in Fault Tree Analysis (FTA) part 1

Main causes of this failure are collected under three titles which are Control Panel Failures, Landing Gear Actuators Failures and Hydraulic Power Failures according to the system architecture. Landing Gear Actuator Failures can simply be divided into three main titles in AND-Gate as shown in Figure 5.3:

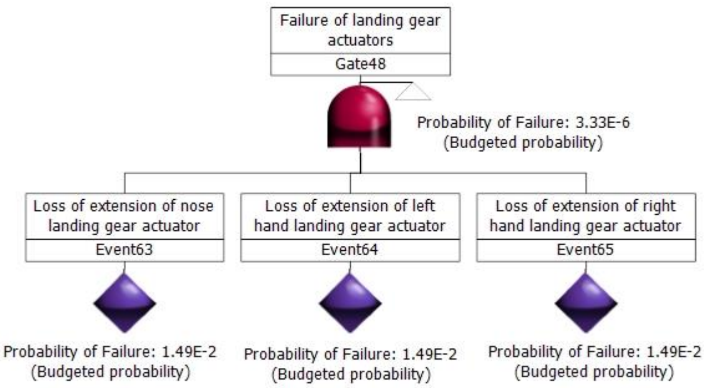


Figure 5.3. PSSA example in Fault Tree Analysis (FTA) part 2

Control Panel and Hydraulic Power Failures have two sub-failures for normal and emergency operations because landing gear design has redundancy in control mechanism and hydraulic control.

“Gate 47” is the main consideration for this study since it includes venting valve failure which causes “Loss of emergency operation capability” as shown in Figure 5.4.

Engineering application starts at this point and reliability values of the venting valve can be used. Leakage and failure of nitrogen selector valve, leakage of charging manifold and nitrogen bottle leakage are the other causes of the top-failures. Nonetheless, these causes are beyond the scope of this thesis.

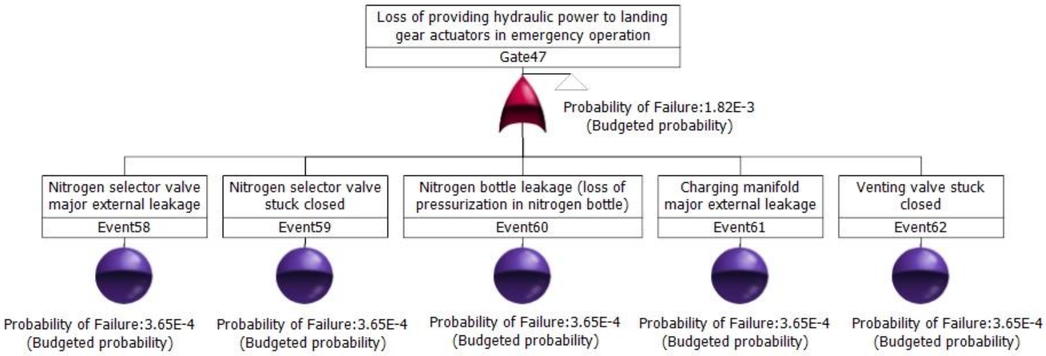


Figure 5.4. PSSA example in Fault Tree Analysis (FTA) part 3

All the reasons of extraction failure of landing gears are considered and shown in the FTA's. In Figure 5.4, budgeted probability of venting valve failure (Event 62) is 3.65×10^{-4}. This requires that probability of failure of venting valve should be less than this value.

If the reliability results that are calculated in Chapter 4, are considered, two distribution types and five different reliability results, a total of ten results for venting valve are all below this probability requirement. However, the exact value is accepted to be MCS result of log-normal strength input data which is 2.14×10^{-4} because input data is compatible with nature of materials and number of evaluations are accepted to be enough. Therefore, this value will be used and SSA process ends by putting all probability of failure values of the rest of the items to the FTA as shown in Figures 5.5-5.7.

It is important to note that venting valve engineering application is proposed in this chapter and the safety requirement for venting valve is one of the main focuses in this thesis. The other probability of failure results come from technical engineering determinations which may or may not be related to the Stress-Strength Interference

Model. The other reliability prediction techniques are used especially for electrical equipments in control panel failures.

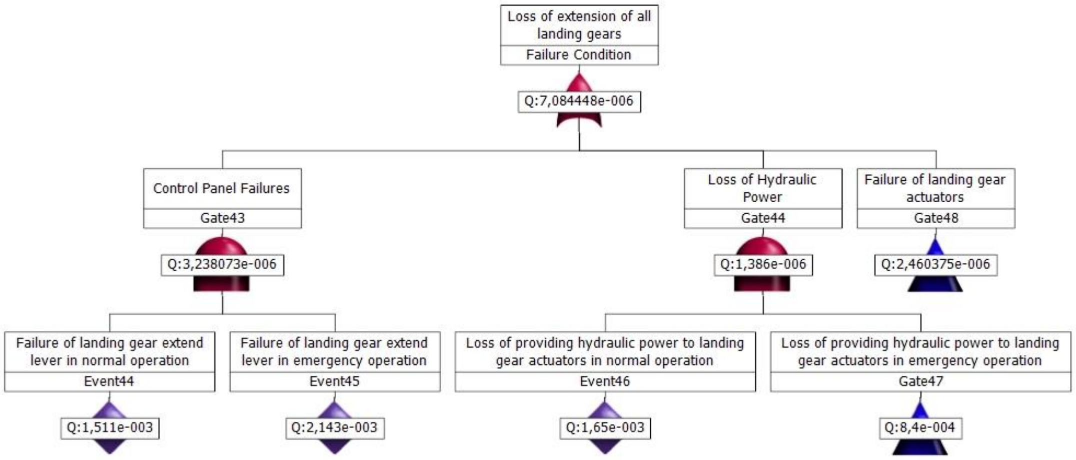


Figure 5.5. SSA example in Fault Tree Analysis (FTA) part 1

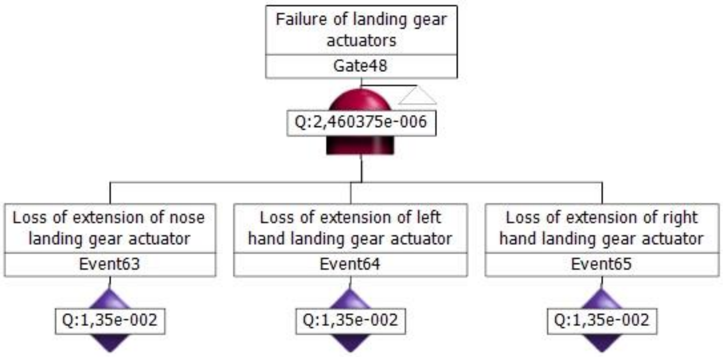


Figure 5.6. SSA example in Fault Tree Analysis (FTA) part 2

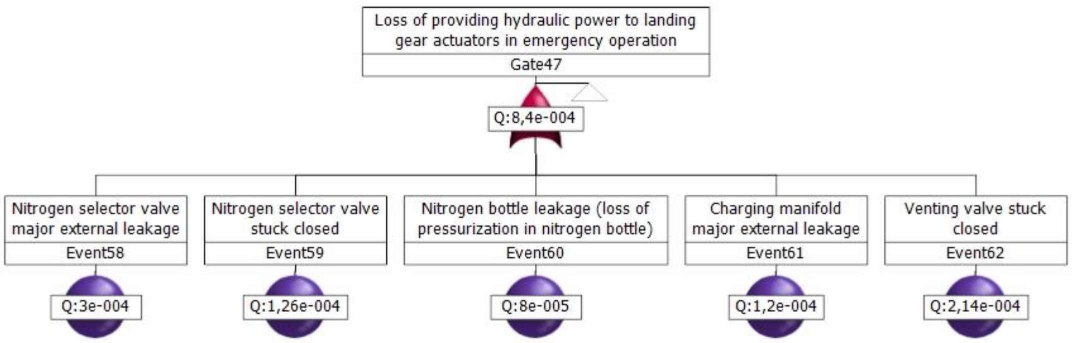


Figure 5.7. SSA example in Fault Tree Analysis (FTA) part 3

When all the reliability predictions of the components or equipment are added, the main probability target which is less than $1x10^{-5}$ for a Major failure condition is satisfied with the value of $7.084x10^{-6}$. Then the verification of functional failure is completed. In other words, this failure condition risk is under control with an acceptable level when the System Safety Engineering approach is considered. This inference is also valid for venting valve since probability requirement of less than $3.65x10^{-4}$ is achieved with a result of $2.14x10^{-4}$.

In this chapter, a brief illustration of engineering application of reliability is described. System Safety Process is explained and FHA, PSSA and SSA processes are performed on the example of the emergency extraction landing gear system. The reliability value determined in Chapter 4 is used and the verification for the probability requirement of venting valve is completed. Top-failure condition is shown to be on an acceptable risk level.

Next chapter includes conclusions and future work. All the arguments, methods and results are summarized and some concluding remarks for this study is made in an objective manner.

CHAPTER 6

DISCUSSIONS AND CONCLUSIONS

6.1. Discussions

In this study, Mechanical Reliability concept is taken into account extensively. This concept should not confused with Electronic Equipment Reliability because of the differences between electronic equipments and mechanical components, that are having constant failure rate and increasing failure rate, respectively, due to the environmental effects and material properties. For the prediction of reliability, four different techniques are commonly used that are Component Failure Data Analysis, Empirical Reliability Analysis, Stress-Strength Reliability Model and Reliability Database and Handbook Usage. Amongst all the prediction techniques, Stress-Strength Interference Model is chosen for this study because this method focuses on the reliability results and these results are time independent. In other words, “Time” is not considered and it is beyond the scope of this thesis. In this way, disadvantages associated with the property of increasing failure rate is partially avoided and stability of the results is achieved.

In the application of Stress-Strength Interference Model, five methods are introduced which are First Order Reliability Method (FORM), Second Order Reliability Method (SORM) as analytical methods and Monte Carlo Simulation (MCS), Importance Sampling (IS), Adaptive Kriging Monte Carlo Simulation (AK-MCS) as simulation techniques. The example is chosen from Landing Gear Emergency Extension System which is used extensively in aircrafts. In fact, significant importance is given to the extension of the landing gears in the design of the helicopters. Complicated hydraulic design solutions for helicopters always think of the emergency conditions. This subsystem includes a venting valve which is important in the extension of landing gears in an emergency condition. This study attempts combining a mechanical component with Stress-Strength Interference Model as an illustration.

Stress random variable for venting valve is considered to be having uniform probability distribution. On the other hand, normal and lognormal probability distribution types are used for strength random variables. Results are obtained by using UQLab which is a dispatch of MATLAB. MCS result is considered to be the exact because it is a rule of thumb that if enough number of evaluations or simulations are performed, MCS convergences to an accurate value.

When the reliability results are compared, negligible differences (About 10^-5) are observed for the same method in using different strength random variable distributions. Therefore, it can safely be said that normal distribution and lognormal distribution may be used interchangeably. Moreover, data set that obeys the normal distribution can be used as input even though normal distribution assumes negative values. Number of evaluations does not change significantly and takes about ten more evaluations when switched from normal to lognormal distributions.

If MCS method is accepted to produce the reference reliability result, then IS method produces the closest value (About %5 error) with the least number of evaluations (about 1200). FORM results have about %22 error in both distribution cases, while AK-MCS method produces around %35-39 difference in the two distribution cases. SORM of Breitung and Hohenbichler, which are improved versions of FORM, produces somewhat better results with %10 error that is about %10.65 error for SORM-Hohenbichler and about %12.15 error for SORM-Breitung. In summary, IS method is a good approximation technique for these types of problems and one may be misled if only AK-MCS method is used.

Number of evaluations is another important factor in reliability computations. AK-MCS shows the best performance in this manner, around 22-23 evaluations in this study. This performance is expected because this method is applicable to experimental studies and reliability results can be obtained in little time. Thus, this technique should be chosen for reliability prediction of mechanical components due to being easy to implement and a time saver. However, one more method should also be used to crosscheck and assess the outcome. Other methods cannot compete with this evaluation number. FORM has 195 evaluations for normal distribution case and 209 evaluations for lognormal distribution case. Although SORM shows some improvement in reliability results, number of evaluations is higher than FORM. In fact, 252 evaluations are needed for normal distribution case with SORM-Breitung and SORM-Hohenbichler, while 266 evaluations are needed for lognormal distribution case instead. Even though IS method shows a great performance in getting close to the reference value, the number of evaluations does not support this performance and almost 1200 evaluations are needed for venting valve problem. MCS method requires 2 million evaluations and this is a very time consuming and costly.

After determining the reliability value for the venting valve, system safety assessment for emergency extension sub-system is performed in a small scale as an engineering application in accordance with the process defined in “SAE ARP 4761 Aerospace Recommended Practice”. Reliability requirement for venting valve is constructed via Fault Tree Analysis and the design is verified by probability of failure result. A small level system safety process is performed and a failure condition with the criticality of Major is shown to be in a probabilistic range of acceptable risk. In other words, requirement of AC 1309.29, which is “Major Failure Condition must be no more frequent than Remote”, is verified.

This study is performed as an illustration of safety and reliability engineering disciplines. All discussions and conclusions are geared towards contributing to these disciplines in the framework of Mechanical Reliability.

It should be stressed that reliability based designs have great advantage over deterministic ones. For example, Emergency Nitrogen Pressure of Venting Valve is 1.2 MPa in this study and it is shown that this pressure value is enough for safety and reliability requirements. Nevertheless, deterministic design overdetermines this pressure to be 12 MPa for performing this function. This is due to fact that deterministic designs include safety factors and some extra precautions for uncertainties. Overall, this causes an over safe design. If an engineer also estimates material behaviors and environmental conditions of a mechanical component or an electronic equipment to a high degree of precision, then one could predict the probability of failure of this component or equipment to a high degree. Therefore, design of the products performed based on reliability avoids the consideration of unnecessary safety factors. This results in optimized and highly reliable products with cost saving features.

6.2. Future Work Recommendations

Adaptive Kriging Monte Carlo Simulation (AK-MCS) method shows a good performance in number of evaluations, but poor performance in reliability results. If some improvement in this method is achieved, AK-MCS will lead amongst the reliability prediction techniques. This could be done by changing “Learning Function” of this method. More complex learning functions may create better solutions.

Subset Simulation Method which is another reliability method could be used for reliability estimation of mechanical components using Stress-Strength Interference Model. Mechanical Reliability Prediction Techniques mentioned in Chapter 1 could be used for venting valve for comparison.

This study is performed for a sub-system of a helicopter, namely, Landing Gear Emergency Extension System. This sub-system is not only for rotorcrafts but also for the others having landing gear such as Commercial Airplanes or Unmanned Aerial Vehicle and so on. Stress-Strength Interference Model could be used extensively for these to estimate reliability of all mechanical components of their systems.

Probability distributions could be determined based on the research and development activities and the other distribution types such as Weibull Distribution, Exponential Distribution and so on may be used for stress or strength random variables in accordance with the outcomes of these activities. In this way, specific probability based designs could be created and extensively used.

Time-independent model is chosen for estimation in this study. As an extension of this study, “Time” could be considered as a third dimension. However, it would not be an easy task because material behavior over time for mechanical components has not been determined until recently. Sudarsanam [43] provides a model in his thesis on this topic. He deals with the tensile strength of the materials and uses a degradation formula for time behavior. We expect that simple theoretical degradation formula may not interpret the behavior of the material strength in time because each material should have a specific time degradation behavior and using a general formula is not reasonable in that sense. Therefore, future studies should concentrate on and contribute to the relation between time and reliability for mechanical components in Stress-Strength Interference Model.

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