TIME AND STATE DEPENDENT PARAMETERIZED MODEL REFERENCE ADAPTIVE CONTROL

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
AEROSPACE ENGINEERING

AUGUST 2019

Approval of the thesis:

TIME AND STATE DEPENDENT PARAMETERIZED MODEL REFERENCE ADAPTIVE CONTROL

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ABSTRACT

TIME AND STATE DEPENDENT PARAMETERIZED MODEL REFERENCE ADAPTIVE CONTROL

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August 2019, 142 pages

Unknown external disturbances or nonlinear dynamics, could affect both the stability and the performance of the of the air vehicles adversely. Unavoidable structure of this reality occurrance in real life applications, led the researches to design adaptive control which could eliminate the deficiencies of the nominal controller. So that, the main aim of the total controller, to satisfy the robustness and performance of the controller could be established.

Model following controller, and the model reference adaptive controllers are the ones which determine a reference model, and satisfy the system model responses track the reference model. The performance of the model reference adaptive controller depends on the success in prediction of the uncertainties mentioned above.

The prediction is done, by the multiplication of basis functions and adaptive weight coefficients. The convergence of the predicted weights to their true values means accurate uncertainty prediction. Concurrent adaptive learning, eliminates PE restriction for parameter convergence, enhances the performance of the model reference adaptive controller. Data storage in concurrent adaptive learning is based on singular value maximizing.

Chebsyhev polynomials and Fourier series are used as time dependent basis functions, and state dependent function for the state dependent basis functions, in uncertainty prediction. The adaptive weight update law is defined for both types of uncertainties, depending on the Lyapunov stability theorem. The time and state dependent model reference adaptive controller gives the best reference model tracking results, compared to the ones which use solely time or state dependent basis functions, and the related weight update laws.

Keywords: Adaptive Learning, Time State Dependent Uncertainty Parametrization, Fourier Series, Chebsyhev polynomials, Concurrent Adaptive Learning

ÖZ

ZAMAN VE DURUM DEĞİŞKENLİ PARAMETRİZE EDİLMİŞ MODEL REFERANS ADAPTİF KONTROL

Okumuş, Zeynep
Yüksek Lisans, Havacılık ve Uzay Mühendisliği
Tez Danışmanı: Dr. Öğr. Üyesi Ali Türker Kutay

Ağustos 2019, 142 sayfa

Bilinmeyen dışsal rahatsızlıklar veya linear olmayan dinamikler, bir hava aracının hem kararlılık hem de performansını olumsuz etkileyebilir. Gerçek hava koşullarında oluşması engellenemez gerçeğin bu yapısı, araştırmacıları normal kontrolcünün eksiklerini gidermek için adaptif kontrolcü tasarlamaya yöneltmiştir. Böylece asıl amacı gürbüzlük ve performans sağlamak olan toplam kontrolcü sağlanmış olacaktır.

Model takip kontrolcü ve model referans adaptif kontrolcü, referans model belirler ve sistem modelinin referans modeli takip etmesi sağlanır. Model referans adaptif kontrolcünün pefromansı, yukarıda bahsedilen rahatsızlıkları giderebilmesiyle ölçülür.

Belirsizlik tahmini, taban fonksiyonu ile adaptif ağırlık katsayısının çarpımıyla elde edilir. Tahmin edilen ağırlıkların doğru değerlerine yaklaşması, belirsizlik tahmininin iyi yapıldığını gösterir. Eşzamanlı adaptif öğrenme, değişken yaklaşması için sürekli uyarma kriterini kaldırarak, model referans adaptif kontrolcünün performansını artırır. Eşzamanlı adaptif öğrenmede, bilgi depolaması tek değer maksimizesine dayanır.

Belirsizlik tahmininde, Chebsyhev polinomları, ve Fourier serileri, zaman değişkenli taban fonksiyonu olarak, ve durum değişkenli fonksiyon durum dğişkenli taban fonksiyonu olarak kullanılmıştır. Adaptif ağırlık güncelleme kanunu, Lyapunov kararlılık kanununa dayanarak, her iki belirsizlik türü için belirlenmiştir, Durum ve zaman değişkenli parametrize edilmiş model referans adaptif kontrolcü, sadece zaman veya durum değişkenli taban fonksiyonu ve ilgili ağırlık güncelleme kanununu kullanan kontrolcülere göre daha iyi referans model takip performansı göstermiştir.

Anahtar Kelimeler: Adaptif Kontrolcü, Zaman ve Durum Değişkenli Belirsizlik Parametrizesi, Fourier Serisi, Chebsyhev Polinomları, Eşzamanlı Adaptif Öğrenme

To my family

ACKNOWLEDGEMENTS

I would like to express the deepest appreciation to my supervisor Asst. Prof. Dr. Ali Türker Kutay, for his guidance and interest for this study. His enthusiasm, support, satisfaction of freedom, patience, faith in me and encouragement has led me to make and finalize this thesis. I wish to express my sincere thanks to my committee members, Asst. Prof. Dr. Ali Türker Kutay, Prof. Dr. Ozan Tekinalp, Assoc. Prof. Dr. İlkay Yavrucuk, Prof. Dr. Mehmet Önder Efe and Prof. Dr. Metin Uymaz Salamcı.

I would like to thank to all METU Aerospace Engineering faculty members, assistants, for their education throughout my both undergraduate and graduate education life. I would like to thank to anyone who has positive contribution in my life.

I would like to thank, owe more than everything, the endless love for my parents, Süheyla Okumuş and Fuat Okumuş. Their love and support is invaluable.

I would like to thank my brother Halit Okumuş, for his love and faith in me throughout my life.

I also thank my dear brother Emre Okumuş, for his invaluable love and faith in me throughout my life.

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LIST OF ABBREVIATIONS

ABBREVIATIONS	
CCAL	: Concurrent Adaptive Learning
CP	: Chebyshev Polynomials
CPBMRAC	: Chebyshev Polynomials Based Model Reference Adaptive Control
CP TD CCAL MRAC	: Chebyshev Polynomials Time Dependent Concurrent Adaptive Learning Model Reference Adaptive Control
FS	: Fourier Series
FSBMRAC	: Fourier Series Based Model Reference Adaptive Control
FS TD CCAL MRAC	: Fourier Series Time Dependent Concurrent Adaptive Learning Model Reference Adaptive Control
FS TSD CCAL MRAC	: Fourier Series Time State Dependent Concurrent Adaptive Learning Model Reference Adaptive Control
MFC	: Model Following Controller
MIMO	: Multi Input Multi Output
MRAC	: Model Reference Adaptive Controller
NN	: Neural Networks
PE	: Persistent of Excitation
SVM	: Singular Value Maximizing
WCC	: Weight Converge Condition

CHAPTER 1

INTRODUCTION

1.1. Motivation

The uncertainties and unmodelled dynamics appearing in both the system model and its environment, causes nonlinearity and time varying structure in the flying vehicle. The need for control system to eliminate mentioned uncertainties arises. An adaptive autopilot system design for a slender delta wing air vehicle is the purpose of this study. A control system should satisfy two main characteristics. During the control system design procedure, the criteria should be satisfied such that the system is both stable and performs properly. Stability, in other words boundedness of the signal vectors has to be satisfied, since the control actuation system has physical limitations. Performance, in other words tracking the reference model accurately in an acceptable time interval has to be satisfied, since the aim of the controller is to eliminate the difference between the outputs of the reference model and the plant. Output error convergence to zero causes the reference model tracking accurately. Throughout the process, the system performance and robustness should be calibrated within the bandwith of the control system.

Unexpected disturbance like gust or turbulence, and unknown dynamics of the system like nonlinearity or unmodelled dynamics could cause threat to the system for both being stable and performing properly. Adaptive control has emerged in order to eliminate such unexpected disturbance or unknown dynamics effects on the system being stable and performing properly. The system is regulated by the control system by applying control input. Disturbance to the system could be categorized in two classes, such as matched disturbance and dismatched disturbance. Matched disturbance means that disturbance enters to the system at the control input application point. Dismatched disturbance is the one which cannot be observed in the control input application point. In this study, matched disturbance will be studied. By the application of adaptive control input to the system where the matched disturbance enters to the system, the unexpected disturbance could be removed. Adaptive control input, namely u_{ad} is composed by the multiplication of constant adaptive weights and the basis function. The basis function is the first step for prediction of the unknown disturbance structure. Since it is difficult to determine the disturbance thoroughly by the approximation of basis function at first step, the adaptive control input, u_{ad} should be updated by multiplying the basis function with the constant adaptive weights. So the second step for prediction of the unknown disturbance structure is, determination of constant adaptive weight coefficients. The adaptive weight coefficients are updated at each time step during the simulation. Convergence or being restricted in a domain region of the adaptive weight coefficients means that, the unknown disturbance structure is predicted approximately. Elimination of the disturbance process gets through two steps. First the disturbance structure should be predicted approximately, second by the application of the control input that disturbance should be removed. By the adaptive control input structure composition process as mentioned, the first step, namely disturbance structure prediction is being done. Since the disturbance is matched, by the application of control input, the second step, namely removal of the disturbance is fulfilled. Since the adaptive weight coefficients of the basis function converges to their true values or remain in a limited domain region throughout the simulation, it is a time dependent process to remove a disturbance when it enters to the system. Control input signals determine the amount of control surface deflection needed to apply to the air vehicle which has been exposed to the unknown disturbance. As mentioned, both the nonlinearities and unmodelled dynamics of the system and the physical environment could be included in the unknown disturbance definition. The magnitude of the adaptive control surface deflection, namely u_{ad}, should be limited in a domain region, since the control surface deflection has physical limits. The control system being stable is determined so. The disturbance elimination is done by the composition and then application of the adaptive control input. Adaptive control input structure is constructed during the simulation, by the update of the adaptive weight coefficients. The time passed through the process and the approximation level of the disturbance, are success criteria for this time dependent process. As time passed shortens and the approximation level of the disturbance increases, the disturbance determination and elimination gets better. This determines the performance of the control system.

So, since in the real flight conditions, modelling nonlinearities and unmodelled parts of air vehicles thoroughly is not possible, it is important to design control systems. Adaptive control is a special working area of control system design. It is dedicated to remove unexpected disturbances or unmodelled parts of the system, which is not included in the design process of the air vehicle. The capability of adaptive control system to meet the deficient modeling of air vehicles in real flight conditions, has caught attention by so many researches. Many theses and articles have been written about adaptive autopilot design. Making a contribution to the adaptive autopilot design has been the major motivation for this thesis work.

1.2. Literature Review

Automatic control, namely adaptive control is a widely studied area since it performs in real flight conditions. Highly agile missiles, fighter aircrafts, or other autonomous air vehicles operate in highly dynamic air conditions. Like the case given in (Go & Ramnath, 2008), air performance capabilities are enhanced in modern fighter aircraft for the purpose of air superiority. Nonlinear flight conditions like highly complex dynamics, such as high angle of attack wing rock motion, or constant amplitude and definite frequency determined sustained lateral oscillations could be required for the purpose mentioned. In (Aditya, 2015), the complex interactions between the aerodynamics, structural dynamics and propulsion system of an air breathing hypersonic vehicle is identified by the direct adaptive control model designed. As stated in (Kasnakoğlu, 2016), (Ka, Dworak, & Jaroszewski, 2013), (F.A. Faruqi & Vu, 2002), (Farhan A Faruqi, 1990), MIMO, multi input multi output methods are usually preferred for missiles, helicopters, and multirotor vehicles where dynamical couplings are dominant. In (Noriega, 2016), generic adaptive autopilot is designed for general aviation aircraft for use in several makes and models to remove internal and external disturbances. Adaptive autopilot has evolved to eliminate such unmodelled nonlinearities and disturbances.

The controller parameters are adjusted in MRAC, which is model reference adaptive controller, with the aim of satisfying the plant system behaves as the reference model by yielding the same output to the application of the same reference input. (Korul, Tosun, & Isik, n.d.), (Burak, 2016). The adaptive law of weight update uses the difference between the outputs of the reference model and the plant. The decrease in the error means that the uncertainty is predicted approximately. The adaptive law of weight update is constructed by the principle of Lyapunov Stability Theorem. Tracking error and adaptive weight convergence is guaranteed by the weight update law. (Calise, Sharma, & Corban, 2008).

As stated in (I.M. Y. Mareels, 1988), (Boyd & Sastry, 1986), PE, persistent of excitation, and sufficient richness is introduced in the adaptive control context to guarantee the exponential convergence of adaptive algorithms. PE, persistent of excitation is a restriction criteria put for reference input command entering to the system. Adaptive weight convergence, without diverging is the main aim of the PE criteria application, so uncertainty prediction will be done accurately. Uncertainty prediction is a time process. So, especially if, system state dependent approximation function is used for uncertainty parametrization, then it will be better to excite the reference command input, for better learning the uncertainty entering to the system. This will enhance exponentially bounded transient performance of the system. But, it will put a restriction on the reference input command, which is hard to monitor or control. On the other end, if time dependent periodic FS; Fourier series or CP, Chebyshev polynomials is used as the approximation function, then since the value of the approximation function will change at each time step, it will learn the uncertainty without putting a restriction on the reference command input. To sum up, PE condition, firstly guarantees robustness of the control system, by violating adaptive weight divergence. Secondly, satisfies uncertainty elimination, by supplying adaptive weight convergence to the real values. By the way, FS, Fourier series expansion method usage as the approximation function, firstly guarantees robustness of the control system, by updating adaptive weights according to the Lyapunov stability theorem. Secondly, satisfies uncertainty elimination, by supplying that the adaptive weights converge to the actual values or weights stay in a limited range. As stated in (Chowdhary, 2010), (Girish Chowdhary, Tansel Yücelen, Maximillian Mühlegg, 2014), (Maximilian, M,Chowdhary G, n.d.), (Chowdhary & Johnson, 2011), (Quindlen, Chowdhary, & How, 2015), CCAL, concurrent adaptive learning can guarantee convergence of the error in tracking output to zero and error in weight prediction to zero, by satisfying the linear independence of the stored data; also eliminates both the requirement of the system states excitation persistently and the PE restriction criteria on the reference command input. Especially if the approximation function used to predict the uncertainty is state dependent, then using CCAL, concurrent adaptive learning algorithm in the adaptive weight update process would be better, rather than PE, persistent of excitation, restriction on the reference command input. CCAL, will use recorded data with the current data at the same time step of the simulation, so adaptive weight convergence will be satisfied as the simulation works. The key point in the CCAL algorithm, is to increase the quality of the adaptive weight update law, by controlling the recorded data in terms of its spectral properties. It will record the current data on the recorded data, only if the new data will increase the minimum value of the singular value of the recorded data. This method is named as SVM, singular value maximizing.

System state dependent uncertainty parametrization, and time dependent uncertainty parametrization are two ways for prediction of uncertainty. In (Qu, 2003), (Asadi & Shandiz, 2017), (Polycarpou & Mears, 1998), there are examples of state dependent uncertainty parametrization, and in (Choon-Ki Ahn, Beom-Soo Kim, 2015), (Haddad, Hayakawa, & Stasko, 2010), (Tyukin, Prokhorov, Member, & Leeuwen, 2007), examples of time dependent uncertainty parametrization could be found. Since uncertainty elimination is done in two basic steps. Firstly, uncertainty parametrization, and then in order to eliminate uncertainty effect on the system, secondly, adaptive weight update law should be determined. Adaptive weight update law could also be categorized according to whether the uncertainty parametrization is done dependent on the system states or dependent on time.

1.3. Contribution of This Thesis

The need for operation in wide spectrum air conditions of agile aircrafts, autonomous vehicles and missiles, has required the emergency of adaptive control in 1950s. The main idea of the adaptive control is to regulate the system behaviour according to the changing environment and system dynamics. Change in environment and system dynamics results in unmodelled nonlinearities or unknown linear parts of the system. Unmodelled nonlinearities, namely disturbances, could be predicted with the usage of proper basis functions multiplied with constant adaptive weights. Unknown linear parts of the system, namely plant model, could be determined or predicted if the system model is known exactly or not. Adaptive control could be classified as direct adaptive controller and indirect adaptive controller according to the need for plant model parameter estimation. In direct adaptive controller, the plant is known but the disturbances are predicted with the usage of adaptive elements. In indirect adaptive controller, plant model parameter estimation is done online and then controller parameters are computed based on the estimated plant model. One of the most famous type of direct adaptive controller is MRAC. MRAC is constructed to regulate the system behaviour according to the changing environment and system dynamics. In order to do this, first a reference model is specified, and then the adaptive control satisfies the system to react as the specified model. The name of the controller is MFC, with the inclusion of adaptive elements it is named as MRAC. In this thesis also, MRAC is used as the adaptive controller. The common point between the MFC and

MRAC, is having reference model. The two controllers differ such that, MRAC has adaptive control parameters. Control parameters need to be adjusted during the flight, since having extremely nonlinear dynamics results in change in aircraft dynamics during the flight. As the adaptive term implies, adaptive control parameters namely, the weights of the network are adjusted continuously by the adaptation law. In the case that, the difference between the outputs of the model and the plant converges to zero, the desirable response of the system could be achieved. The adaptation law is derived according to the term which is the difference between the outputs of the model and the plant. The derivation is done according to the Lyapunov stability theorem. To sum up, agile aircrafts have extremely nonlinear dynamics during flight. Nonlinearities which is not modelled or linearities of the system which is not known could exist in both environment and system model. Nonlinearities which is not modelled appearing in the environment could be defined as disturbances, and linearities of the system which is not known could be defined as uncertainty in plant model. To make the controller stable and satisfy transient performance characteristics, disturbances acting on and uncertainties of the system model should be eliminated. The need for the elimination of the existence of uncertainties in both the environment and plant models, caused the design of adaptive controller. Indirect adaptive controller is designed to predict both the disturbances appearing in the environment, and the uncertainties or unknown dynamics in plant model. Direct adaptive controller is designed to predict only the disturbances appearing in the environment, the plant model dynamics is assumed to be known. As an example MRAC could be given, which uses adaptive controller parameters. Adaptive controller parameters are updated during the simulation. So, during the design process of the adaptive controller, parametrization of the uncertainty and update law of the adaptive weights are two basic steps. As mentioned, during the update law of the adaptive weights Lyapunov stability theorem is taken as the basis. Uncertainty parametrization is done by approximation functions depending on whether the uncertainty is structured or unstructured. In the case that uncertainty could be parametrized with the usage of known functions depending on either the system states or time, it could be defined as structured uncertainty. On the other hand, if uncertainty could only be approximated with the usage of basis functions multiplied by constant adaptive weight coefficients, then it could be defined as unstructured uncertainty. Basis functions used in prediction could also be dependent on system states or time. Since unstructured uncertainty needs prediction algorithm, rather than the structured uncertainty, many approximation methods have been studied in the adaptive controller design study area. NN, neural networks, for instance are used as universal approximators in order to predict unstructured uncertainties. As an advantage, the functions being continuous and integrable, and defined on a compact domain and to within any tolerance makes the prediction of uncertainty process more accurate. As a disadvantage, tuning work of the structure and parameters makes NN usage as the approximator function harder. Since tuning of the structure and parameters is needed, NN's are also system state dependent usually, as in the case in (Blumel, Hughes, & White, 2000), (Suresh, Omkar, Mani, & Sundararajan, 2008).

In this thesis, it is proposed that, both uncertainty parametrization and adaptive weight update law should be done according to the dependency of the uncertainty on the time or system state variables, without paying attention to the structure of the uncertainty whether structured or unstructured. If the uncertainty entering to the system externally, i.e. disturbance is structured, then the basis function should be in terms of the variable, which determines the type of the structure for the uncertainty. If the dependent variable for the structured uncertainty is time, then the basis function should also depend on time, if system state variables, vice versa. Uncertainty parametrization step should be done depending on the variable type. Adaptive weight update law should also be done accordingly. If the uncertainty entering to the system externally, i.e. disturbance is unstructured but dependent on time variable, then the basis function should be selected as Fourier series based basis function, or Chebyshev polynomials of the first kind, which is dependent on the time. Adaptive weight update law should also be done accordingly. If both time dependent unstructured uncertainty and state variable dependent structured uncertainty enters to the system, then as the uncertainty parametrization method, linear combination of time dependent approximation basis functions, like Fourier series and state variable dependent approximation basis functions should be constructed. Also adaptive weight update law should be applied to each kind of approximation function separately.

1.4. Thesis Structure

In chapter 1, motivation, literature review, thesis contribution, and thesis structure is given.

In chapter 2, MFC is given. Sample system model, design of model following controller, and simulation example are given.

In chapter 3, MRAC is given. Design process for model reference adaptive control, simulation with MRAC for the challenging case, concurrent adaptive learning added MRAC, law of update for concurrent learning, and algorithm of the data selection procedure for the stored history stack is given.

In chapter 4, time and state dependent parameterized MRAC is given. Time dependent uncertainty parametrization, Chebyshev polynomials based MRAC, Fourier series based MRAC, Fourier series based MRAC design using a sample system model, weight update law for time dependent uncertainty parametrization, state dependent uncertainty parametrization, weight update law for state dependent uncertainty parametrization, time and state dependent combined uncertainty parametrization, weight update law for time and state dependent combined uncertainty parametrization, stability proof is given.

In the fifth chapter 5, simulation results is given. Comparison of MRAC and FS TSD CCAL MRAC, comparison of FSBMRAC and FS TSD CCAL MRAC, comparison of FS TD CCAL MRAC and FS TSD CCAL MRAC, comparison of all controllers, max disturbance elimination, FS TD CCAL MRAC (Disturbation factor=2), FS TSD CCAL MRAC (Disturbation factor=2&100) is given. Also, comparison of disturbance and control input, concurrent adaptive learning effect is given.

In the sixth chapter 6, conclusion is given.

CHAPTER 2

MODEL FOLLOWING CONTROL

Model following control is a method of modern control theory, wherein the plant behaves as the reference model in terms of output, to obtain the response desired. The desired response output of a dynamical system can be determined by setting response of the system model in steady state conditions and in transient conditions. (Fujio & Ishida, 2016). The model-following system has been studied by several researchers. (S & Y, 2016), (Sato, 2009), (Inoue et al., 2015)

In this chapter, firstly definition of a sample system model is done which is a basis for the upcoming study. Secondly, the structure of the MFC is presented and the design is given.

2.1. Sample of System Model

A simple system model is defined in (Gezer, 2014), for the reason of discussing the methods for controller design given in this study. The number of the states of the system is two. The input directly affects the first state, and the second state is obtained by taking the integral of the first state. The system model is mathematically defined as,

$$\begin{aligned} \dot{x}_1(t) &= u(t) + \delta(x(t), t), & x_1(0) &= x_{10}, \ t \in \mathrm{R}_+ \\ \dot{x}_2(t) &= x_1(t), & x_2(0) &= x_{20}, \ t \in \mathrm{R}_+ \end{aligned} \tag{2.1}$$

The equations model the rolling motion for a slender delta wing exposed to the wing rock dynamics defined in (Yucelen & Johnson, 2012). Roll rate, the first state, is symbolized by $x_1(t) \in \mathrm{R}$, and the roll angle for the slender delta wing, the second state, is by $x_2(t) \in$ R, input by the control system on the system model is by, $u(t) \in$ R. The state space model of the 2.1 is in the form of,

$$\dot{x}\,(t) = Ax(t) + B(u(t) + \delta(x(t), t)) \tag{2.2}$$

where the system matrix is symbolized by A= $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and the input matrix is by B = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and the state vector is by $x\ (t) \in$ R^{2x1}. The matched disturbance $\delta(x(t), t))$ on the system is formed by $\delta_{ex}(t)$ which is the external disturbance, and $\delta_{wr}(x(t))$ which is the dynamics of the wing rock.

$\delta_{ex}(t)$, the external disturbance is a random disturbance dependent on time. The system is subjected to this disturbance in order to represent unexpected random events occurring in the air during flight, like wind, or gust.

$$\delta(t, x(t)) = \ \delta_{ex}(t) + \ \delta_{wr}(x(t)) \tag{2.3}$$

The dynamics of wing rock given in (Yucelen & Johnson, 2012) as,

$$\delta_{wr}\big(x(t)\big) = \alpha_1 x_2 + \alpha_2 x_1 + \ \alpha_3 |x_2| x_1 + \alpha_4 |x_1| x_1 + \ \alpha_5 {x_1}^3 \tag{2.4}$$

The dynamics of wing rock motion is modelled in the equation (2.4), by the multiplication of constant aerodynamic coefficients and the nonlinear functions depending on the system states. The constant aerodynamic coefficient values are, $\alpha_1 = 0.1414$, $\alpha_2 = 0.5504$, $\alpha_3 = -0.0624$, $\alpha_4 = 0095$, and $\alpha_5 = 0.0215$ as given in (Yucelen & Johnson, 2012).

The vector form for the wing rock dynamics equation could be shown as follows,

$$\delta_{wr}\big(x(t)\big) = [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5] \begin{bmatrix} x_2 \\ x_1 \\ |x_2|x_1 \\ |x_1|x_1 \\ {x_1}^3 \end{bmatrix} \tag{2.5}$$

$$= \alpha\, f\ (x(t))$$

with $\alpha \epsilon\ R^{1x5}$ and $f\big(x(t)\big)\epsilon\ R^{5x1}$.

Up to this point, first the linear system model of which structure is known is formed as the system model. Secondly, totally random external disturbance of which structure is unknown and dependent on time is added to the system externally. Thirdly, the wing rock dynamics which is multiplication of unknown constant aerodynamics coefficients and system states dependent known functions is added to the system externally.

Control input is applied on the system by the usage of actuator mechanism So, it could also be modelled. Control actuator system is the dynamic model which determines the relationship between the commanded and actuated control input. In this study, linear differential equation having the degree of two is used to model the actuator dynamics. The actuator dynamics representation is,

$$\dot{x_c} = A_c x_c(t) + B_c u_c(t)\ . \tag{2.6}$$

where the state vector for control actuator system is defined as x_c (t)ϵ R^{2x1}, and the commanded input on the system is defined as u_c (t) ϵ R. $x_c(t) = \begin{bmatrix} u\ (t) \\ \dot{u}\ (t) \end{bmatrix}$, is the state vector which is set up with the control input actuated $u(t)$ and its derivative $\dot{u}\ (t)$. The system matrix for the control actuator is, $A = \begin{bmatrix} 0 & 1 \\ -\omega_c^2 & -2\zeta_c\omega_c \end{bmatrix}$, and the input matrix is $B = \begin{bmatrix} 0 \\ \omega_c^2 \end{bmatrix}$. ω_c, is the control actuator system natural frequency and ζ_c, is the damping ratio. The selected values for the control actuator system model is as $\omega_c = 50\ rad/s$ and $\zeta_c = 0.7$.

The combined state matrix for system and control actuator model is,

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{u}(t) \\ \ddot{u}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_c^2 & -2\zeta_c\omega_c \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ u(t) \\ \dot{u}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_c^2 \end{bmatrix} u_c(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta(x(t), t) \tag{2.7}$$

The compact form of the combined system state could be represented mathematically as,

$$\dot{x_p}(t) = A_p x_p(t) + B_p u_c(t) + B_d \delta(t, x(t)) \tag{2.8}$$

where $A_p \epsilon\ R^{4x4}$ is the system matrix belonging to the plant, $B_p \epsilon\ R^{4x1}$ is the input matrix to the plant, and $B_d \epsilon\ R^{4x1}$ is the input matrix representing the disturbance.

The compound system in (2.8) could be examined whether controllable or not by determining the rank belonging to the controllability matrix,

$$C = \left[B_p\ \ A_p B_p\ \ A_p^2 B_p\ \ A_p^3 B_p\right] \tag{2.9}$$

The controllability matrix $C \epsilon\ R^{4x4}$ being full rank, means that the controllability of the compound system is available.

2.2. Model Following Control Design

The system dynamics for the system model and for the control actuator is obtained. The reference model determination is the second step in designing MFC. ω_n which is the natural frequency desired and ζ_n which is the damping ratio, are the criteria parameters to determine the reference model. The system transient response exhibiting these criteria parameters, form the dynamics desired for the reference model.

The reference model could be constructed by using second order linear differential equation and with the usage of natural frequency desired and the damping ratio. On the other hand, it could also be constructed by placing the poles belonging to the system model eigenvalues to the desired pole locations as long as the system model is controllable. This method is named as full state feedback control. The formula belonging to Ackermann is used in the calculation of the gains in this process. (Ogata, 2002). The number of the states for the plant in (2.8) is four. Rate of roll angle, roll angle, control input actuated, and derivative of control input actuated are the states, respectively. In model following design process, the eigenvalue positions of only the rigid body motion model dynamics will be changed. The original locations of the poles for the control actuator will be sustained. So two desired criteria for the system model should be determined.

The desired locations of the eigenvalues of the rigid body dynamics, as the desired criteria, are determined by using the roots belonging to the characteristic equation representing the desirable dynamics,

$$s^2 + 2\zeta_n\omega_n s + \omega_n^2 = 0 \tag{2.10}$$

The roots of the characteristic equation are, $\lambda_1 = -\zeta_n\omega_n + \sqrt{\zeta_n^2\omega_n^2 - \omega_n^2}$ and $\lambda_2 = -\zeta_n\omega_n - \sqrt{\zeta_n^2\omega_n^2 - \omega_n^2}$. The roots of the characteristic equation for the control actuator system are $\lambda_3 = -\zeta_c\omega_c + \sqrt{\zeta_c^2\omega_c^2 - \omega_c^2}$, $\lambda_4 = -\zeta_c\omega_c - \sqrt{\zeta_c^2\omega_c^2 - \omega_c^2}$. The desirable eigenvalues belonging to the reference model are determined by the roots of the two characteristic equation. To sum up, the reference model characteristic equation should be as,

$$(s - \lambda_1)(s - \lambda_2)(s - \lambda_3)(s - \lambda_4) = 0 \tag{2.11}$$

As mentioned above the formula belonging to Ackermann is used in the calculation of the gains for a system which is available for the feedback of the full states. The system model having the same characteristic equation with the specified model is the aim. The Ackermann's formula for a system model having a degree of four could be as,

$$K_r = [0\ 0\ 0\ 1]\ C\ (A_p - \lambda_1\ I_{4x4})(A_p - \lambda_2\ I_{4x4})(A_p - \lambda_3\ I_{4x4})(A_p - \lambda_4\ I_{4x4}) \tag{2.12}$$

The controllability matrix in (2.9) is represented as matrix C, and the identity matrix as I_{4x4}. The gain vector calculated as $K_r \in \mathrm{R}^{1x4}$ is the vector representing the gain of feedback to acquire the dynamics of the specified model. The representation for the reference model could be given as,

$$\begin{aligned} \dot{x_r}(t) &= \left(A_p - B_p K_r\right) x_r(t) + B_p K_r \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} r(t) \\ &= A_r x_r(t) + B_r r(t) \end{aligned} \tag{2.13}$$

$A_r \in \mathrm{R}^{4x4}$ is the system matrix belonging to the reference model, and $B_r \in \mathrm{R}^{4x1}$ is the input matrix. $x_r(t) \in \mathrm{R}^{4x1}$ is the state vector belonging to the reference model, and $r(t) \in \mathrm{R}$ for the reference command.

The natural frequency and the damping ratio desired for the reference model are selected as, $\omega_n = 0.4\ rad/s$, $\zeta_n = 0.707$. The formula belonging to Ackermann is used to calculate the gains for full state feedback controller to acquire the reference model, as $K_r = [0.57\ 0.16\ 0\ 0\ ]$. Since the states belonging to the control actuator system are chosen to sustain the responses for the open loop case, the eigenvalues representing these states are hold at their initial locations. So the last two gains of the gain vector is zero.

The combination of the equation (2.8) for the plant dynamics, and the equation (2.13) for the reference model can be shown as,

$$\dot{x_{pr}}(t) = \begin{bmatrix} \dot{x_{pr}}(t) \\ \dot{x_r}(t) \end{bmatrix} = \begin{bmatrix} A_p & 0_{4x4} \\ 0_{4x4} & A_r \end{bmatrix} x_{pr}(t) + \begin{bmatrix} B_p \\ 0_{4x1} \end{bmatrix} u_c(t) + \begin{bmatrix} 0_{4x1} \\ B_r \end{bmatrix} r(t) \quad (2.14)$$

The combined equation (2.14) state vector is $x_{pr} \in$ R^{8x1}. The rewritten form of the more compact form of this equation can be shown,

$$\dot{x_{pr}}(t) = A_{pr} x_{pr}(t) + B_{pru} u_c(t) + B_{prr} r(t) \quad (2.15)$$

$A_{pr} \in$ R^{8x8}, is the system matrix belonging to the compound system, $B_{pru} \in$ R^{8x1}, is the matrix for input by the control system on the system model and $B_{prr} \in$ R^{8x1} is the reference command input matrix.

As an addition of integral state, the integral of the difference between the reference roll angle, and the plant roll angle is taken. The calculated integral state is as,

$$\dot{x_\iota}(t) = \begin{bmatrix} 0 & -1 & 0_{1x3} & 1 & 0_{1x2} \end{bmatrix} x_{pr} = x_{r2} - x_2 \quad (2.16)$$

The integral state could be defined as,

$$x_i(t) = \int_{t=0}^{t} (x_{r2} - x_2)\, dt \quad (2.17)$$

The roll angle of the system (2.1) is represented as the state x_2 in (2.17), and the roll angle of the reference model is represented as the state x_{r2} in (2.13).

The form of the combined equation (2.14) added the integral state is given as,

$$\dot{x_s}(t) = \begin{bmatrix} \dot{x_{pr}} \\ \dot{x_\iota} \end{bmatrix} = \begin{bmatrix} & & A_{pr} & & & 0_{8x1} \\ 0 & -1 & 0_{1x3} & 1 & 0_{1x2} & 0 \end{bmatrix} \begin{bmatrix} x_{pr} \\ x_i \end{bmatrix} + \begin{bmatrix} B_{pru} \\ 0 \end{bmatrix} u_c(t) + \begin{bmatrix} B_{prr} \\ 0 \end{bmatrix} r(t) \quad (2.18)$$

The design of the MFC is based on this mathematical representation which is the open-loop equation belonging to the total system. The compactly rewritten form of the equation is,

$$\dot{x_s} = A_0 x_s + B_{0u} u_c(t) + B_{0r} r(t) \tag{2.19}$$

where, the state vector is $x_s \epsilon\ \mathrm{R}^{9x1}$, the system matrix is $A_0 \epsilon\ \mathrm{R}^{9x9}$, the matrix for input by the control system on the system model is $B_{0u} \epsilon\ \mathrm{R}^{9x1}$, and the input matrix for reference command in the open loop system equation is $B_{0r} \epsilon\ \mathrm{R}^{9x1}$.

The open loop system formulized at (2.19), The states are, rate of roll angle $x_1(t)$, roll angle $x_2(t)$, control input actuated $u(t)$, derivative of control input actuated $\dot{u}\ (t)$, rate of reference roll angle $x_{r1}(t)$, reference roll angle x_{r2}(t), reference control input actuated $x_{r3}(t)$, derivative of the reference control input actuated $x_{r4}(t)$, and the integral of the difference $x_i(t)$. Since the measurability of the real states is available, also all states belonging to the open-loop system equation could be get for feedback.

The optimality is chosen as the calculation method for the gain of feedback belonging to the MFC. The determined cost function is minimized during the optimization process. The performance vector in quadratic form is the basis for the cost function. The chosen system states and control inputs are combined linearly in order to construct the performance vector. The selection of the performance vector is as,

$$z\ (t) = [\ \ (x_{r1} - x_1) \quad (x_{r2} - x_2) \quad (x_{r3} - u) \quad (x_{r4} - \dot{u}) \quad x_i \quad u_c\ ]^T \tag{2.20}$$

where $z \in \mathrm{R}^{6x1}$. The selected weights are used to penalize the linearly combined states in performance vector.

$Q_z = diag\left(\left[\ Q_{x1} \quad Q_{x2} \quad Q_u \quad Q_{\dot{u}} \quad Q_{x_i} \quad Q_{u_c}\right]\right) \in \mathrm{R}^{6x6}$, is the weight matrix. For the aim of determining the optimal controller gain, the cost function which is to be minimized is,

$$J = \int_{t=0}^{\infty} (z^T(t) Q_z z(t)) dt \tag{2.21}$$

The cost function given in (2.21), can also be expressed as the linear quadratic regulation function is as,

$$J = \int_{t=0}^{\infty} \left(x_s^T(t) W_x x_s(t) \; + \; u_c^T(t) W_u u_c(t) \; + \; 2x_s^T(t) W_{xu} u_c(t) \right) dt \tag{2.22}$$

$z(t)$, which is the performance vector is linear combination of system states as given,

$$z(t) = C_z x_s(t) + D_z u_c(t) \tag{2.23}$$

where $C_z = \begin{bmatrix} -I_{4x4} & I_{4x4} & 0_{4x1} \\ 0_{1x4} & 0_{1x4} & 1 \\ 0_{1x4} & 0_{1x4} & 0 \end{bmatrix} \in \mathrm{R}^{6x9}$ is the selector matrix from the system states, and $D_z = \begin{bmatrix} 0_{5x1} \\ 1 \end{bmatrix} \in \mathrm{R}^{6x1}$ is the selector matrix for the system commanded control input. By inserting (2.23) into (2.21), the rewritten form of the cost function is get as,

$$J = \int_{t=0}^{\infty} \left(\left(C_z x_s(t) \; + \; D_z u_c(t) \right)^T Q_z \left(C_z x_s(t) \; + \; D_z u_c(t) \right) \right) dt \tag{2.24}$$

which yields,

$$J = \int_{t=0}^{\infty} ((x_s^T(t) C_z^T Q_z + u_c^T(t) D_z^T Q_z)(C_z x_s(t) + D_z u_c(t))) dt \tag{2.25}$$

If the terms in (2.25) are collected as,

$$J = \int_{t=0}^{\infty} (x_s^T(t) C_z^T Q_z C_z x_s(t) + u_c^T(t) D_z^T Q_z D_z u_c(t) + 2x_s^T(t) C_z^T Q_z D_z u_c(t)) dt \tag{2.26}$$

If the equations in (2.22), and (2.26) are compared term by term, the weight matrices could be related with the equations as,

$$W_x = C_z^T Q_z C_z \quad W_u = D_z^T Q_z D_z \quad W_{xu} = C_z^T Q_z D_z \tag{2.27}$$

where $W_x \epsilon \ R^{9x9}$ *,* $W_u \epsilon \ R$ *and* $W_{xu} \epsilon \ R^{9x1}$ *.*

With the usage of the minimization function which is solved optimally, the input by the control system on the system model is calculated as,

$$u_c(t) = -K \, x_s(t) \tag{2.28}$$

The total gain $K \epsilon \ \mathrm{R}^{1x9}$ for the MFC is given in the equation (2.28). The linear quadratic regulation method is used in the computation of the ultimate gain of the controller K. As the procedure of this method, the Riccati equation is solved firstly,

$$A_0^T X + X A_0 - (X B_{0u} + W_{xu}) W_u^{-1} (B_{0u}^T X + W_{xu}^T) + W_x \tag{2.29}$$

is solved. The solution of the Riccati equation is, $X \epsilon \ \mathrm{R}^{9x9}$. Then, the calculation of the gain of the controller is done by the equation,

$$K = W_u^{-1} (B_{0u}^T X + N^T) \tag{2.30}$$

In the end, the design is concluded.

The equation for the closed loop of the MFC, is determined by substituting the commanded input of the control system on the system model with (2.28) as,

$$\begin{aligned} \dot{x_s}(t) &= A_0 x_s(t) + B_{0u}(-K x_s(t)) + B_{0r} r(t) \\ &= (A_0 - B_{0u} K) x_s(t) + B_{or} r(t) \\ &= A_{cl} x_s(t) + B_{cl} r(t) \end{aligned} \tag{2.31}$$

The system matrix for the closed loop is $A_{cl} \epsilon\ \mathrm{R}^{9x9}$, and the closed loop input matrix is $B_{cl} \epsilon\ \mathrm{R}^{9x1}$.

Figure 2.1 gives the block diagram for the MFC. As seen, the reference command directly drives the reference model, and to compute the control input commanded u_c the compound state vector is multiplied by the gain of the controller K.

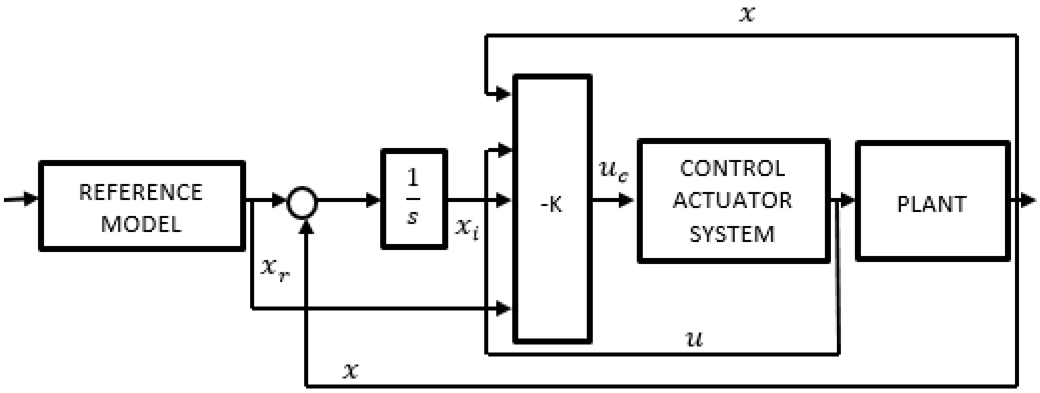

Figure 2.1 Model Following Control

The weight matrix given in (2.21), used in the calculation of the cost function to calculate the optimal controller gain is selected as, (Gezer, 2014)

$$Q_z = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.32}$$

The resulting controller gain of the MFC is, (Gezer, 2014a)

$$K = [20.9 \quad 37.7 \quad 5.6 \quad 1 \ -20.9 \ \ -37.7 \ \ -5.6 \ \ -1 \ \ -31.6] \tag{2.33}$$

The computation for the gain of the controller K, concludes the MFC design for the sample system model.

2.3. Simulation Example

The equation (2.7) gives the model for the plant belonging to the slender delta wing combined with the actuator model and the disturbance input. The system block diagram subjected to the external disturbance, dynamics of wing rock, and measurement noise inputs is given in Figure 2.2.

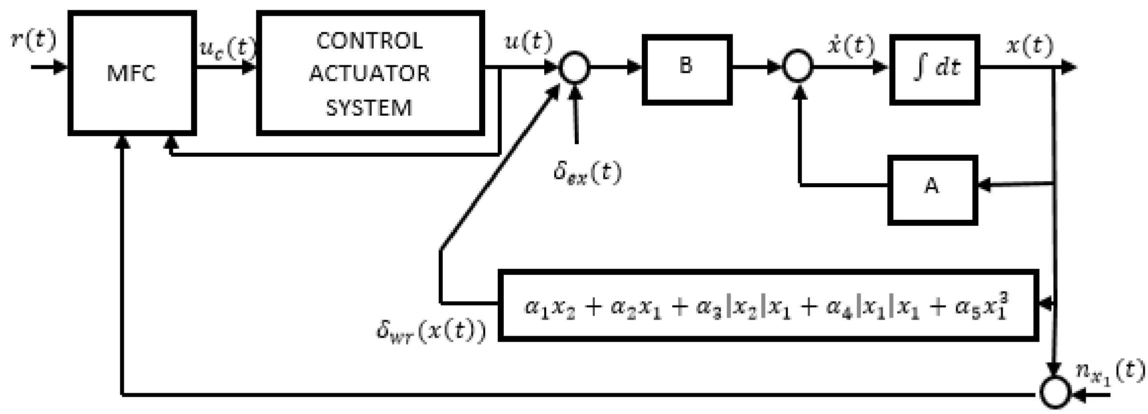

Figure 2.2 MFC with dynamics of wing rock, external disturbance and measurement noise

For the MFC controller performance observation, two types of reference command step is applied. Step command is the first one and sine wave is the second one. The step command applied to the MFC controlled system, is a step sequence as given in Figure 2.3, the sine wave command applied to the MFC controlled system is as given in Figure 2.4.

As mentioned in (2.1), two parts constitute the disturbance applying on the system Random external disturbance is the first part of the disturbance. The possibility of gust subjection upon the delta wing is modelled with this input of external disturbance. The random external disturbance is function of time only and makes peak around -10 degrees and +10 degrees. This means that in case of that amount of gust occurrance upon the system, to remove the unexpected disturbance effect upon the system, the controller should apply exact opposite amount of control surface deflection command. Figure 2.5 shows the random external disturbance.

Wing rock dynamics is the second part of the disturbance applying upon the system. The angle of roll and rates of roll for the delta wing could be affected by the wing rock dynamics which produces external moment upon the system The mathematical relation expressing the disturbance of the wing rock $\delta_{wr}(x(t))$ in terms of the system states is given in (2.4) as,

$$\delta_{wr}\big(x(t)\big) = \alpha_1 x_2 + \alpha_2 x_1 + \alpha_3 |x_2| x_1 + \alpha_4 |x_1| x_1 + \alpha_5 x_1^3 \tag{2.34}$$

Figure 2.6 shows the wing rock dynamics applying upon the system.

It is modelled by the assumption that the noise disturbing the evaluation of the roll rate of the system $n_{x1}(t)$ is a Gaussian distributed random signal which has zero mean and $10^{-4}\ rad/s$ variance, as shown in Figure 2.7.

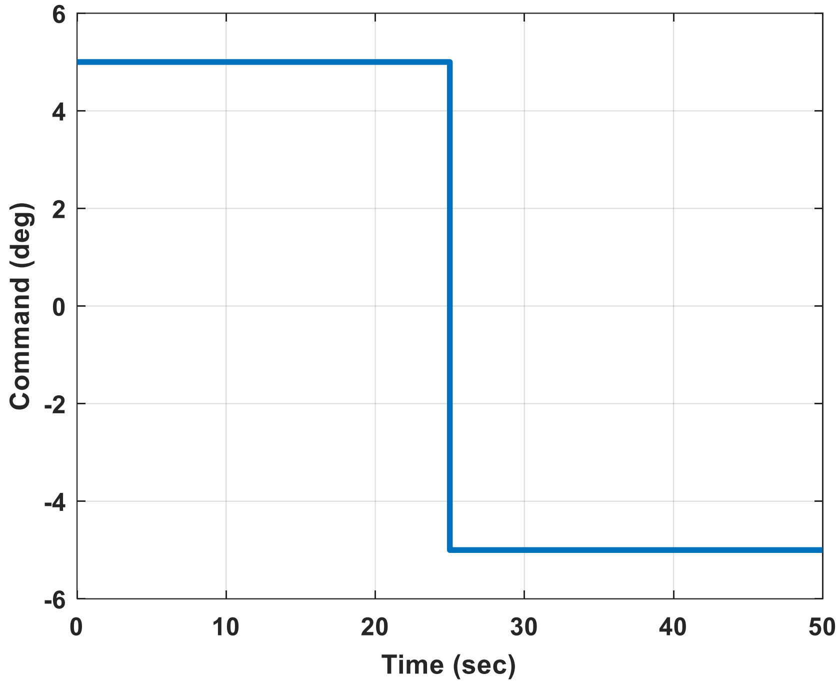

Figure 2.3 Step Command Input

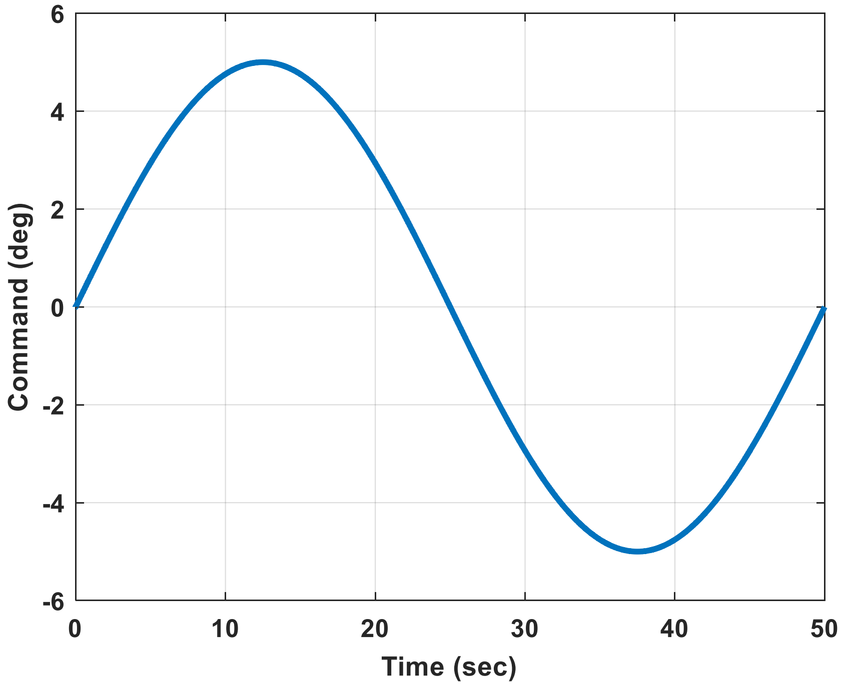

Figure 2.4 Sine Wave Command Input

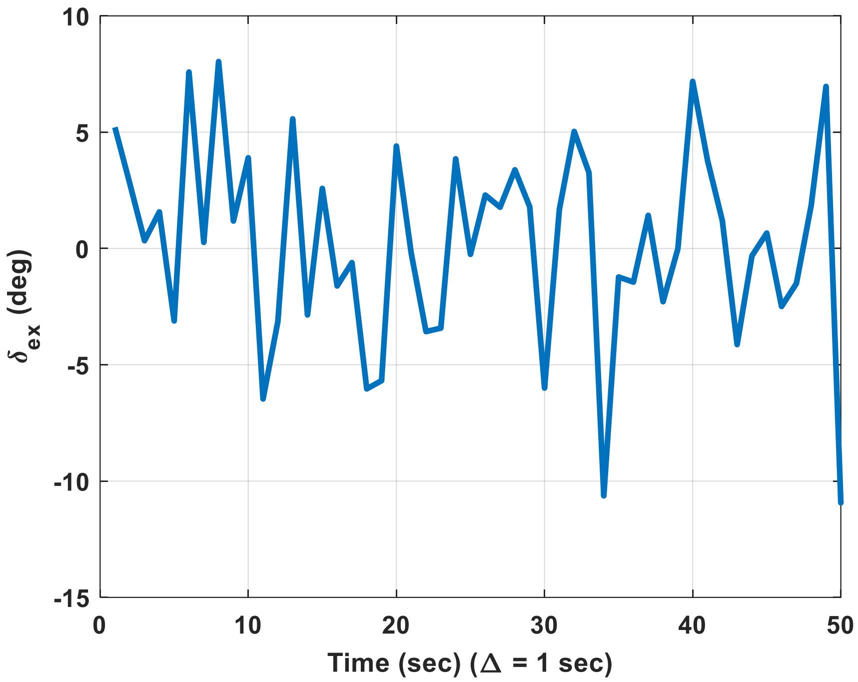

Figure 2.5. Random External Disturbance vs time

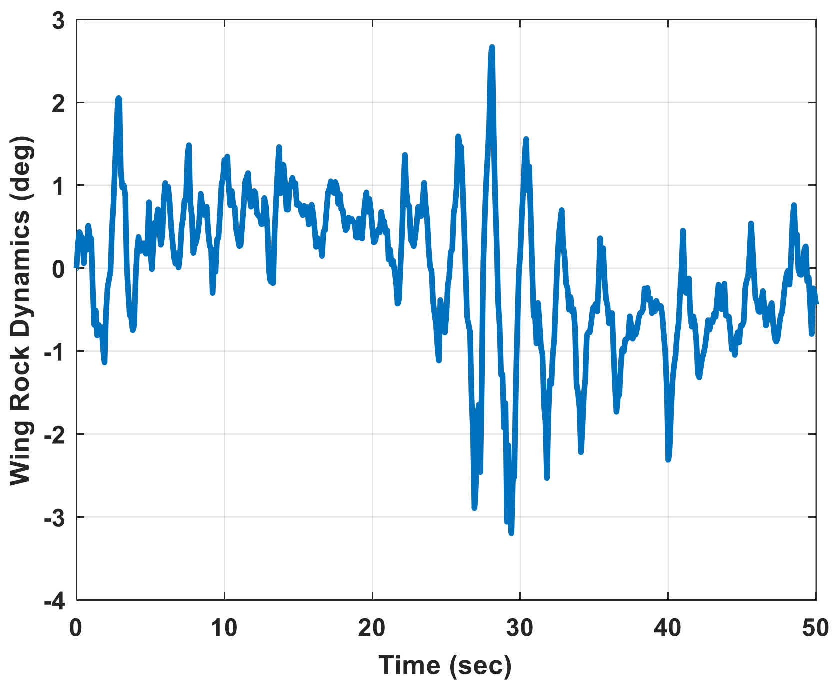

Figure 2.6. Wing Rock Dynamics vs time

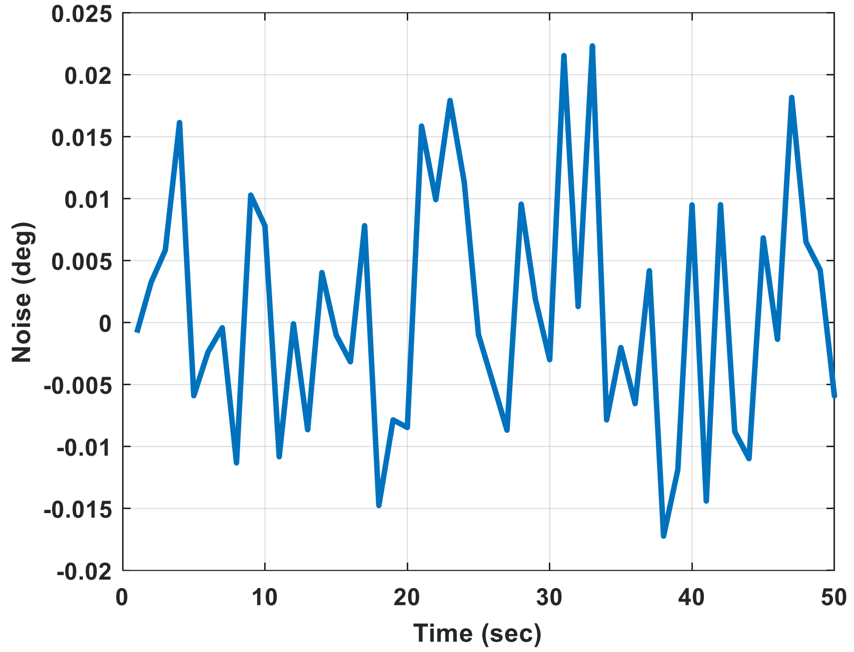

Figure 2.7 Noise added to Roll Turn Rate vs time

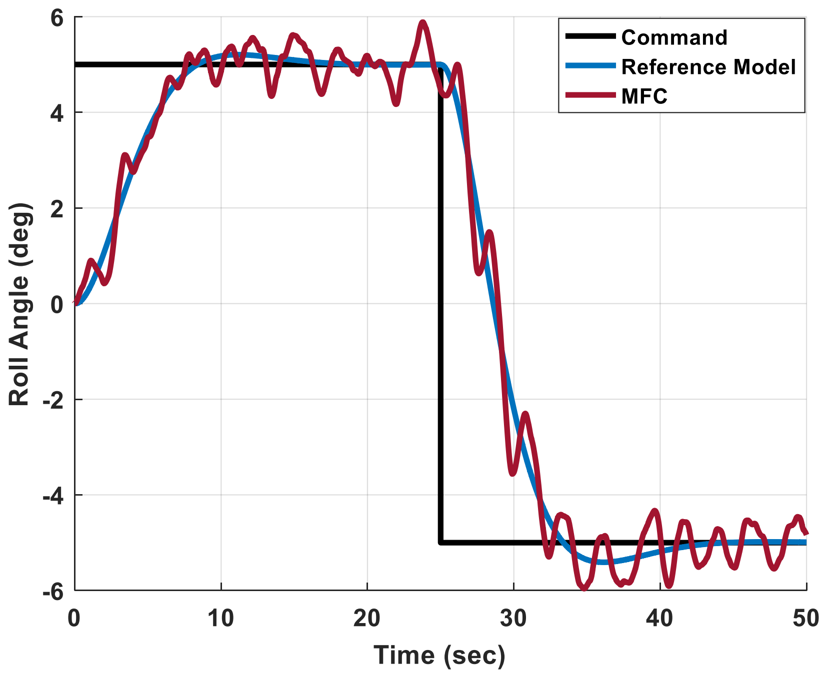

Figure 2.8 MFC Response to Step Command Input Under the Effect of Wing Rock Dynamics, Random External Disturbance and Noise

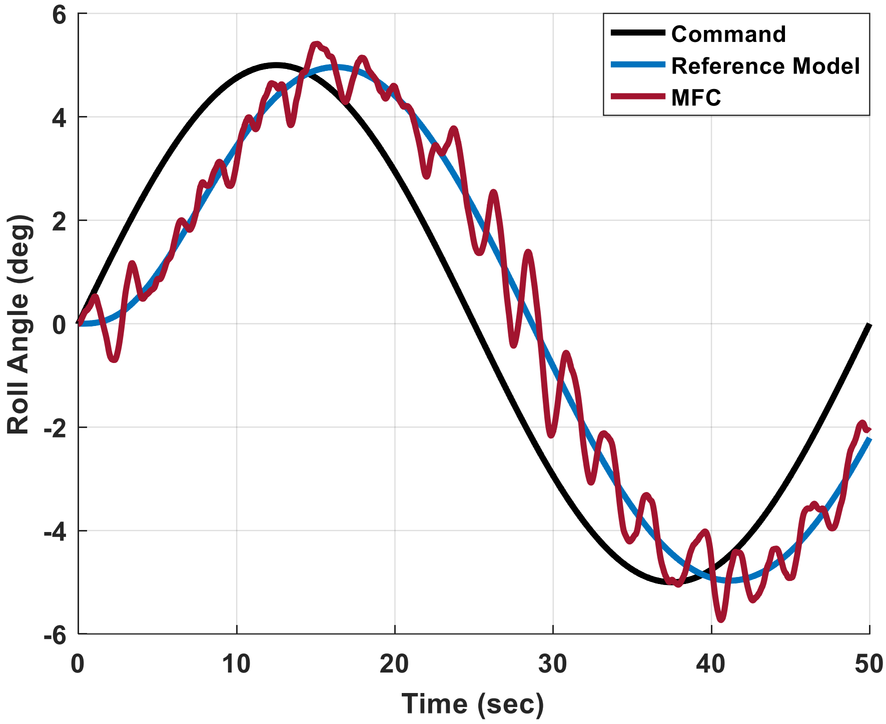

Figure 2.9 MFC Response to Sine wave Command Input Under the Effect of Wing Rock Dynamics, Random External Disturbance and Noise

Figure 2.8 shows that, the reply of the MFC controlled system being exposed to the dynamics of wing rock, random external disturbance and noise, is fluctuating around the reference model response. It could be acceptable since not diverging from the reference model response, however needs to be further enhanced. The random external disturbance used in MFC, has 20 variance and simulated at 0.1 time step. Figure 2.4 shows the command which is sine wave applied to the MFC controlled system. The reply of the MFC controlled system is seen in Figure 2.9, which is exposed to the random external disturbance, dynamics of wing rock, and noise, and subjected to the command input which is sine wave that is given in Figure 2.4. As the case in Figure 2.8 the system could pursue the reference model response with the impact of MFC, without diverging from the path. The shift between the reference roll angle and commanded sine wave is a result of designed controller. Though the shift, with the impact of MFC, the behaviour of the model belonging to the system resembles to the behaviour of the reference model as a response to the reference input, which could eliminate the effects of random external disturbance, dynamics of wing rock, and noise. Though, since the fluctuations seen in Figure 2.9, it could be said that MFC should be further developed to eliminate the disturbances occurring on the system model.

CHAPTER 3

MODEL REFERENCE ADAPTIVE CONTROL

MRAC, model reference adaptive controller, is one of the most utilized adaptive controller type used for modern control applications. Samples of MRAC design could be seen in (Jain & M.J, 2013), (Eugene Lavretsky, n.d.), (Stepanyan & Krishnakumar, 2014), (Liu, Tao, & Joshi, 2010), (Arabi, Yucelen, & Gruenwald, 2018).

As in the MFC, model following controller, MRAC, also uses the reference model, as to be a precedent for the system model. The term which is the difference between the outputs of the reference model and the plant model is aimed to converge to zero as much as possible. For the aim of excluding the difference between the outputs of the models, due to the uncertainties in the system model and environmental disturbances, adaptive elements are determined. The adaptive elements are updated according to the law of weight update during the simulation.

In the primary MRAC, the parametrization of the uncertainty is done using the known functions dependent on the system variables. In this manner, it is assumed that the uncertainty is structured and the parametrization is done using the system variables.

This section is formed by four sections. MRAC design method is given in 3.1, simulation with MRAC for the case challenging is given in 3.2, concurrent learning model reference adaptive control method and the update law of weight regarding with the proposed method is given in 3.3 and 3.4, algorithm of data selection for the stored data memory during the concurrent adaptive learning is given in 3.4.1.

3.1. Model Reference Adaptive Control

The mathematical representation corresponding the MRAC applied upon a general system model is given in this section. Consider the general mathematical representation for the system model as the one given in (2.1) as,

$$\dot{x}(t) = Ax(t) + B[u(t) + \Delta(t)] \tag{3.1}$$

$x(t) \in \mathrm{R}^n$ is the vector representing the state and $u \in \mathrm{R}$ is the input of the control system. It is assumed that the general system has single input, and $u \in \mathrm{R}$ is the input of the control system. $\Delta(t) \in \mathrm{R}$ is the external disturbance upon the system. The structure of the disturbance is assumed to be in the form of,

$$\Delta(t) = W\beta(x(t)) \tag{3.2}$$

For the disturbance estimation in (3.2), $\beta(x(t))$ symbolizes the parametrization of the uncertainty. In MRAC, in this section, the parametrization of the uncertainty is done as being dependent on the system states. The constant ideal weights are represented by W, which are the weights of the corresponding parametrization.

$u(t)$ which is the input of the control system is computed by the algebraic subtraction of the adaptive controller from the nominal controller as,

$$\mathrm{u}(t) = u_n(t) - u_{ad}(t) \tag{3.3}$$

with nominal control input $u_n(t) \in \mathrm{R}$, and for adaptive control input $u_{ad}(t) \in \mathrm{R}$.

In the calculation of the nominal input $u_n(t)$, a full-state feedback method is used. The full-state feedback controller is,

$$u_n(t) = -K_r x(t) + K_r H r(t) \tag{3.4}$$

$K_r \in R^{1xn}$ is the controller gain. $r(t) \in R$, is the reference command input to the system. $H \in R^{nx1}$ is the reference input matrix.

Since the aim is to satisfy that the system model which is closed loop controlled with the nominal input behaves as the specified model, the gain of the controller K_r is calculated for the sake. In other words, the reply of the closed loop system controlled with nominal input without any disturbances, is equal to the reference model response which is desired.

Hence, the system equation for the specified model used for the MRAC is as,

$$\dot{x_r}(t) = (A - BK_r)\, x_r(t) + BK_r H r(t)$$
$$\dot{x_r}(t) = A_r x_r(t) + B_r r(t) \tag{3.5}$$

$x_r(t) \in R^{nx1}$ which is the specified model state, has the same dimension with the system state. Reference model system matrix $A_r = A - BK_r \in R^{nxn}$, and reference model input matrix $B_r = BK_r H \in R^{nx1}$, are as seen from the specified model equation.

In order to exclude the disturbance from the system, the adaptive controller is designed. The purpose of the adaptive controller is to exclude the disturbance upon the system. So, it is expected that the adaptive input and the disturbance have the same form of structure, so as the adaptive input could cancel the disturbance. The disturbance is assumed to have structured uncertainty, such that it is parametrized as multiplication of known function and the weight coefficients. The known function depends on system states, and the weight coefficients is stated by a vector which parameterizes each component of the known function. Since as mentioned the adaptive input should be in the same form of disturbance structure, it should also be in the form of multiplication of weight coefficients with the known function which depends on system states. In this case, since the weights are not known by the controller, it is assumed that the ideal weight coefficients exists and they should be estimated. The formulation for the adaptive controller is,

$$u_{ad}(t) = \widehat{W}(t)\ \beta(x(t)) \tag{3.6}$$

The dimension of the predicted weights $\widehat{W}(t)$ is the same with the ideal weights and adaptive controller updates these weights at each step during the simulation.

The update law of the weights used in MRAC is,

$$\dot{\widehat{W}}(t) = \Gamma\beta(t)e(t)^T PB \tag{3.7}$$

The learning rate for the update law of the weights is represented by Γ, in (3.7). The learning rate determines the structure of the design. The increase in the value of Γ, causes the adaptation mechanism try to update the weight coefficient faster. As a result of this, it estimates the disturbance on the system faster. The decrease in the value of Γ, causes the stiffness of learning the disturbance mechanism get to be tougher. This results, increase in the time duration in learning the disturbance structure. So, the design selection multiplier Γ, is defined as the rate of learning for the adaptation.

The term of the difference between the outputs of the reference model and the plant model is represented as e as given in (3.7). The mathematical relation for the calculation of e is given as,

$$e(t) = \mathrm{x(t)} - \mathrm{x_r}(t) \tag{3.8}$$

where $e(t) \in \mathrm{R}^{nx1}$. This term also drives the adaptation mechanism. As the term of the difference between the outputs of the reference model and the plant model decreases, e converges to zero neighbourhood. This means that weight coefficient estimation of the ideal weights has become successful in the uncertainty parametrization process. On the other hand, the increase in e, drives the weight update law in the way to increase the coefficients in order to estimate the disturbance more faster.

The constant matrix P in (3.7) is computed by using the equation of Lyapunov which is,

$$A_r^T P + P A_r + R = 0 \tag{3.9}$$

As shown in (3.9), in the Lyapunov equation, A_r, system matrix of the reference model is used. The matrix R is a symmetric matrix with all positive eigenvalues, which is a design selection matrix. Any positive definite matrix could be selected to manage the MRAC adaptation mechanism.

The stability proof of Lyapunov theorem is omitted in this section. A simple explanation for the proof of stability of the basic MRAC, in which parametrization of the uncertainty is done by using familiar functions that depend on the states of the system can be found in (Yucelen, 2012).

The block diagram for the MRAC is shown, in Figure 3.1.

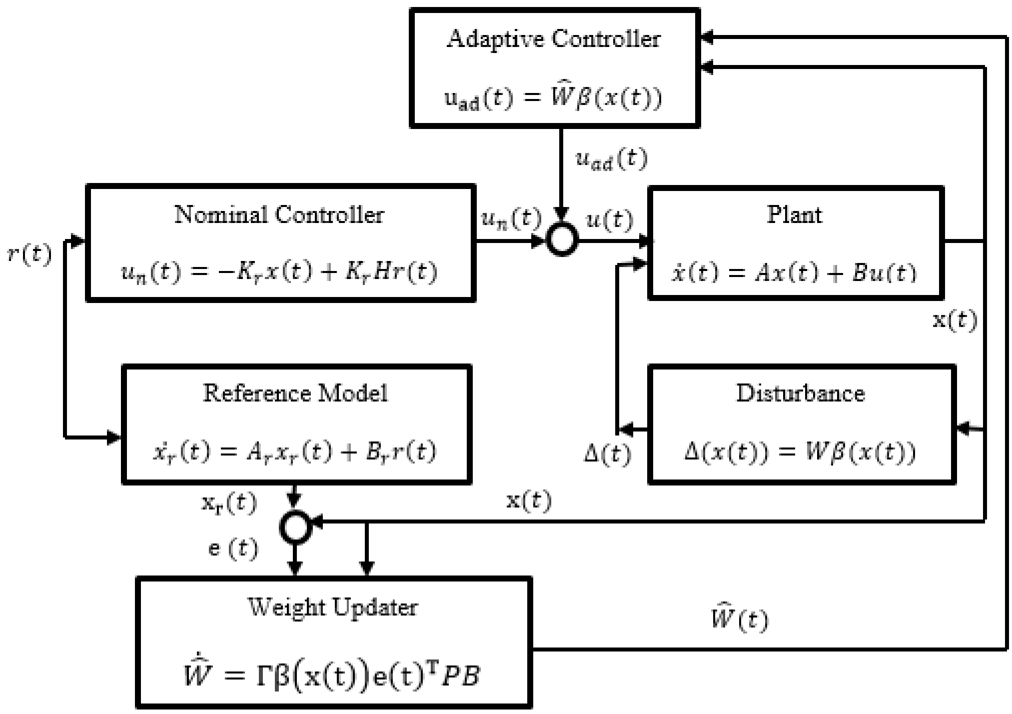

Figure 3.1 MRAC Block Diagram

As seen from the figure, the update law of the weight is driven by the term of difference between the outputs of the reference model and the plant model. The adaptive control input is also driven by update law of weight, and updated with the new weight concurrently. For the aim of excluding the disturbance applying upon the system, the algebraic difference of the nominal control input and the adaptive control input is get. It is assumed that the uncertainty parametrization is known, and constructed with the known functions that depends on the system states. So, both the update law of weight and the adaptive control input uses the states of the system as the feedback.

The general overview of the MRAC is given in this section. The case in which the defined sample system model exposed to challenging disturbance, and controlled with MRAC is given in the upcoming section.

3.2. Simulation with MRAC for the Challenging Case

The achievement of the MRAC method under the case that is challenging, such that external disturbance, dynamics of wing rock and noise on the rate of roll angle for the slender delta wing is examined in this section. Wing rock dynamics is function of system states, as given in (2.4). Since as the parametrization of the uncertainty, MRAC uses known functions that depend on system states, wing rock dynamics equation is a good example as the basis function construction. The wing rock dynamics equation is constructed by the multiplication of constant ideal weights and known function of system states, while the parametrization function in MRAC is also constructed by the same function that depend on system states. Determination of the function coefficients will be done in the sections related with the update law of the weight.

The equation of the dynamics of wing rock for the defined delta wing is,

$$\begin{aligned} \delta_{wr}(t) &= W\beta(x(t)) \\ &= \alpha_1 x_2 + \alpha_2 x_1 + \alpha_3 |x_2| x_1 + \alpha_4 |x_1| x_1 + \alpha_5 x_1^3 \end{aligned} \quad (3.10)$$

The vector form of the function $\beta(x(t))$ in (3.10) that depends on the states of the system is,

$$\beta(x(t)) = \begin{bmatrix} x_2 \\ x_1 \\ |x_2|x_1 \\ |x_1|x_1 \\ x_1^3 \end{bmatrix} \quad (3.11)$$

The vector of the ideal weights of which elements match the elements in the system state function vector is

$$W = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \quad \alpha_5] \quad (3.12)$$

The mathematical value for the ideal weights is taken from (Yucelen & Johnson, 2012), as given 2.1.

$$W = [\alpha_1 \;\; \alpha_2 \quad \alpha_3 \quad \alpha_4 \;\; \alpha_5] \quad (3.13)$$

$$= [0.1414 \quad 0.5504 \quad -0.0624 \quad 0.0095 \quad 0.0215]$$

The system equation with the dynamics of wing rock is,

$$\dot{x}(t) = Ax(t) + B(u(t) + W\beta(x(t)) + \delta_{ex}(t)) \quad (3.14)$$

The system matrix A, and the input matrix B corresponding the roll dynamics of a slender delta wing are,

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (3.15)$$

$\omega_n = 0.4\, rad/s$ and $\zeta_n = 0.707$ are the selected values of the natural frequency desired and the damping ratio for the reference model. The formula belonging to

Ackermann is used to calculate the specified model and the gains of the nominal controller K_r as given in (2.12). The gains are,

$$K_r = [0.57\ 0.16] \tag{3.16}$$

The state space equation for the reference model is,

$$\begin{aligned} \dot{x_r}(t) &= \left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [0.57 \quad 0.16]\right) x_r(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [0.57 \quad 0.16] \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \\ \dot{x_r}(t) &= \begin{bmatrix} -0.57 & -0.16 \\ 1 & 0 \end{bmatrix} x_r(t) + \begin{bmatrix} 0.16 \\ 0 \end{bmatrix} r(t) \end{aligned} \tag{3.17}$$

So the compact form of the specified model is,

$$\dot{x_r}(t) = A_r x_r(t) + B_r r(t) \tag{3.18}$$

The nominal controller used in the MRAC is,

$$u_n(t) = -K_r x(t) + K_r H r(t) \tag{3.19}$$

where the selection matrix is,

$$H = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3.20}$$

The nominal controller numerical representation is,

$$u_n(t) = -[0.57 \quad 0.16] x(t) + 0.16 r(t) \tag{3.21}$$

The adaptive control input for the MRAC is computed with,

$$u_{ad}(t) = \widehat{W}(t)\, \beta(x(t)) \tag{3.22}$$

It is assumed that the vector of parametrization $\beta(x(t))$ is known. The representation of the vector with the dimension is $\beta\big(x(t)\big) \in \mathrm{R}^{5x1}$. The representation of the parametrization weightings vector with the dimension is $W \in \mathrm{R}^{1x5}$. The law of the weight update could be used in the computation of the estimated weights. The law of the weight update is as,

$$\dot{\widehat{W}}(t) = \Gamma\beta(t)e(t)^T PB \tag{3.23}$$

The representation of the learning rate Γ with the dimension in the law of weight update is $\Gamma \in \mathrm{R}^{5x5}$ and is chosen as a design selection (Gezer, 2014b). The learning rate for the MRAC design in this study is chosen as,

$$\Gamma = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \tag{3.24}$$

The selection of learning rate is a design criteria, since it effects the weight update rule directly. As the rate of learning rises more than the required value, then robustness of the system model declines. The further rise, could also cause instability of the system under defined disturbances. On the other hand, the decrease in learning rate leads the adaptation to be insensitive to the disturbances.

The Lyapunov equation given in (3.9) is used in the calculation of the matrix P in (3.23). Another design selection in the Lyapunov equation is the R matrix used. The selected matrix is as, (Gezer, 2014b)

$$R = \begin{bmatrix} 1000 & 0 \\ 0 & 0.01 \end{bmatrix} \tag{3.25}$$

The selection of the R matrix effects the sensitivity level of the adaptation upon the term of error. The term of error is defined as the difference between the outputs of the reference model and the plant system. Each diagonal element in the R matrix effects the corresponding element of the error vector. The states of the delta slender wing are rate of roll and angle of roll respectively. By regulating the elements on the diagonal of the R matrix, the magnitude of the effect and the type of the state to effect the adaptation law could be determined. As a design selection, in this study, it is determined as the value in (3.25).

For examining the challenging case, two sets of inputs are used. These inputs are stair step command shown in Figure 2.3, and the sine wave command shown in Figure 2.4.

The simulations are done including $\delta_{ex}(t)$ which is the random external disturbance shown in Figure 3.4, $\delta_{wr}(x(t))$ which is dynamics of wing rock defined in (3.10), and the noise in roll rate measurement defined in 2.3.

The block diagram of the MRAC controlled system is given in Figure 3.. The control actuator system is not shown in the figure. It is also not included to the design of the controller. However, the control actuator system with the properties defined in 2.1, is used in the simulations.

Since the measurement of the states is assumed to be fast enough, the measurement dynamics is neglected.

Figure 3.7 gives the reply of the MRAC exposed to the given disturbances upon the step command input. Figure 3.8 gives the reply of the MRAC exposed to the given disturbances upon the sine wave command input.

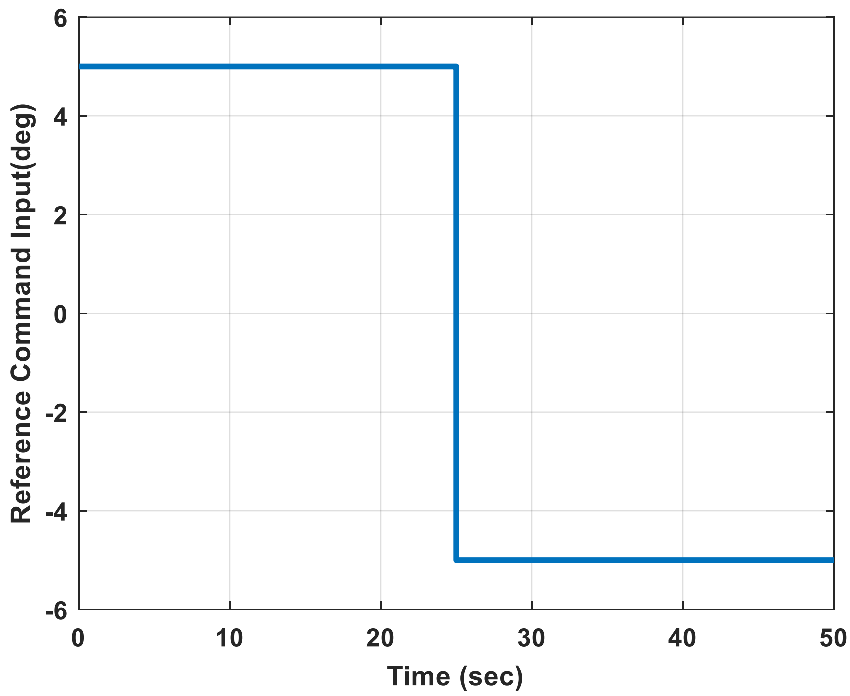

Figure 3.2. Reference Command Input vs time

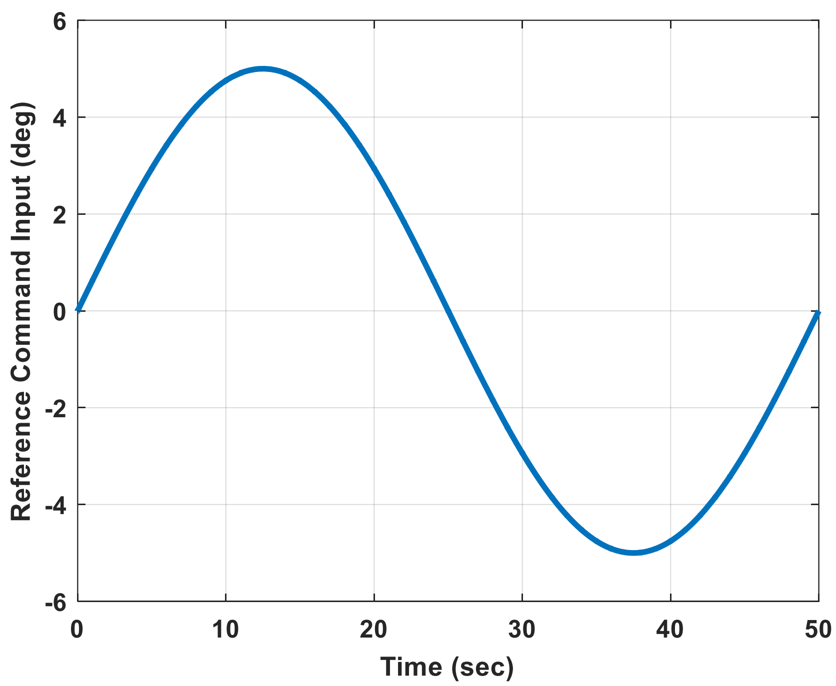

Figure 3.3. Reference Command Input vs time

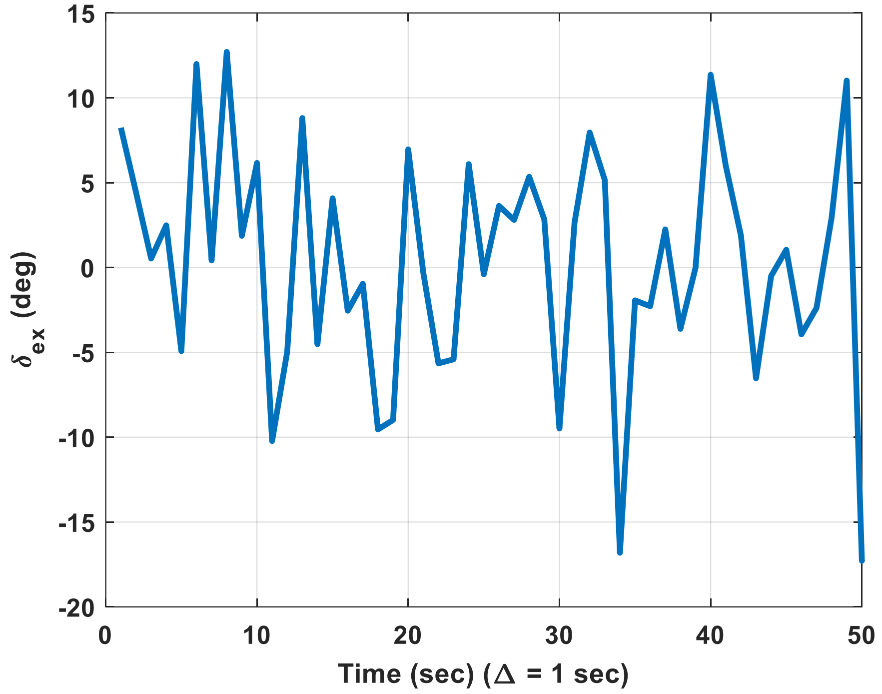

Figure 3.4. Random External Disturbance vs time

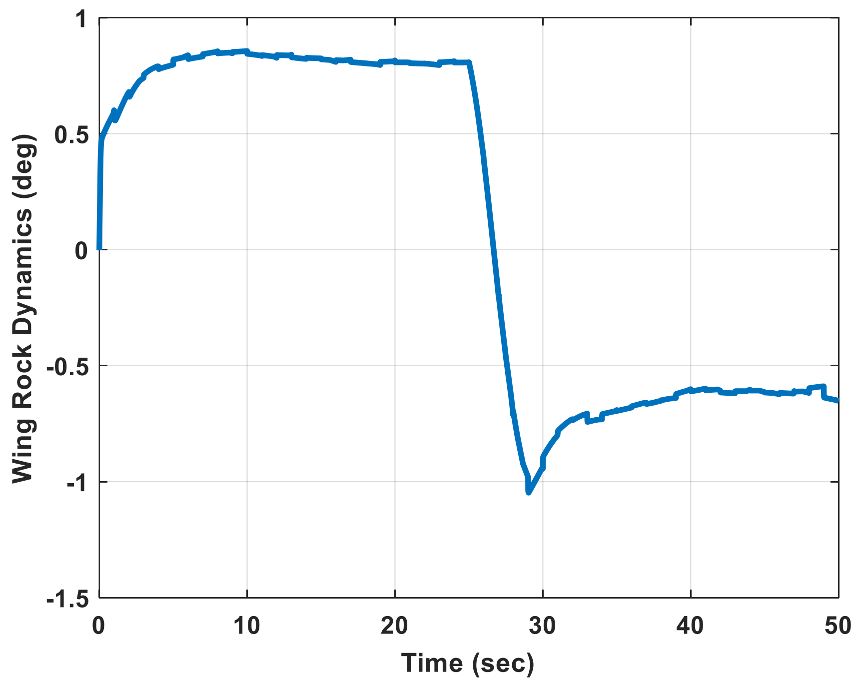

Figure 3.5. Wing Rock Dynamics vs time

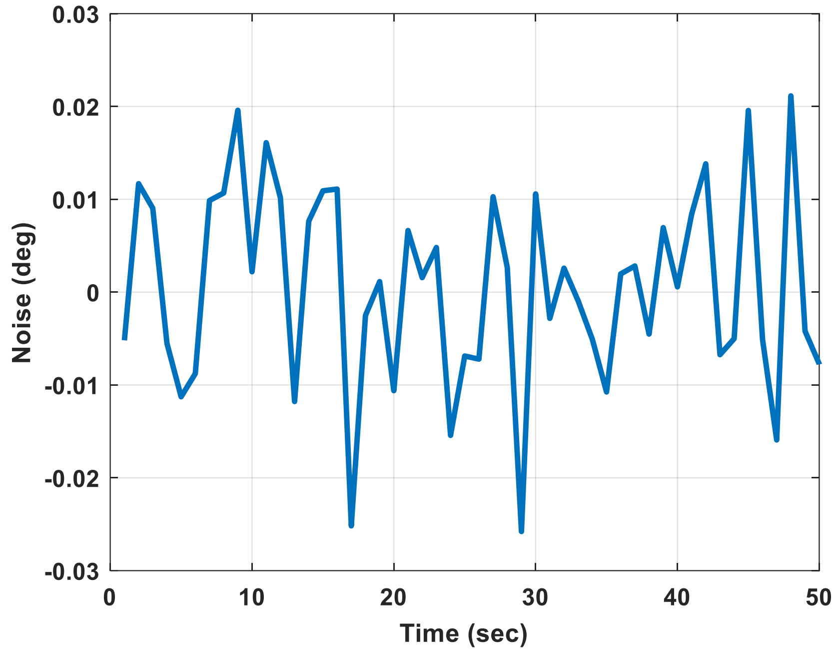

Figure 3.6.Noise added to Roll Turn Rate vs time

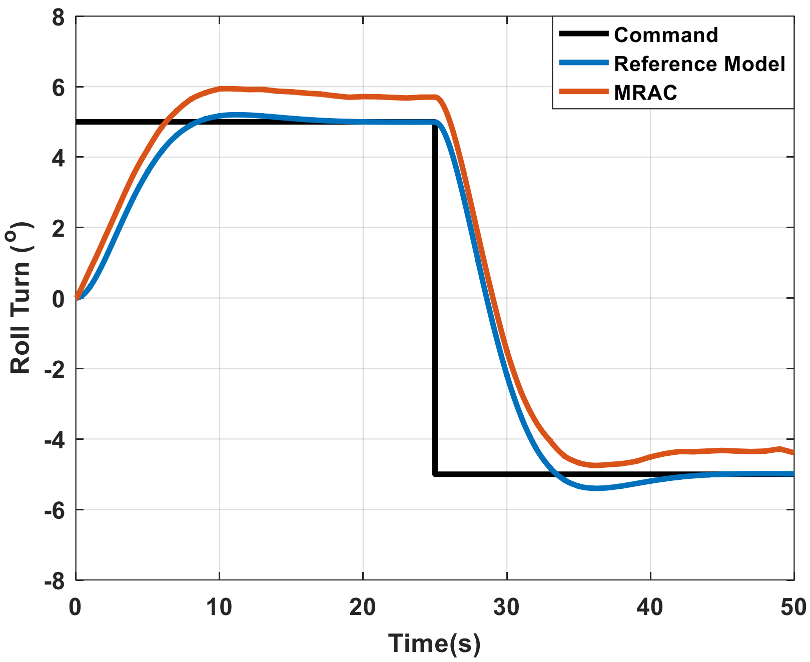

Figure 3.7. MRAC Response to Step Command Input under the Effect of Dynamics of Wing Rock, Random External Disturbance and Noise

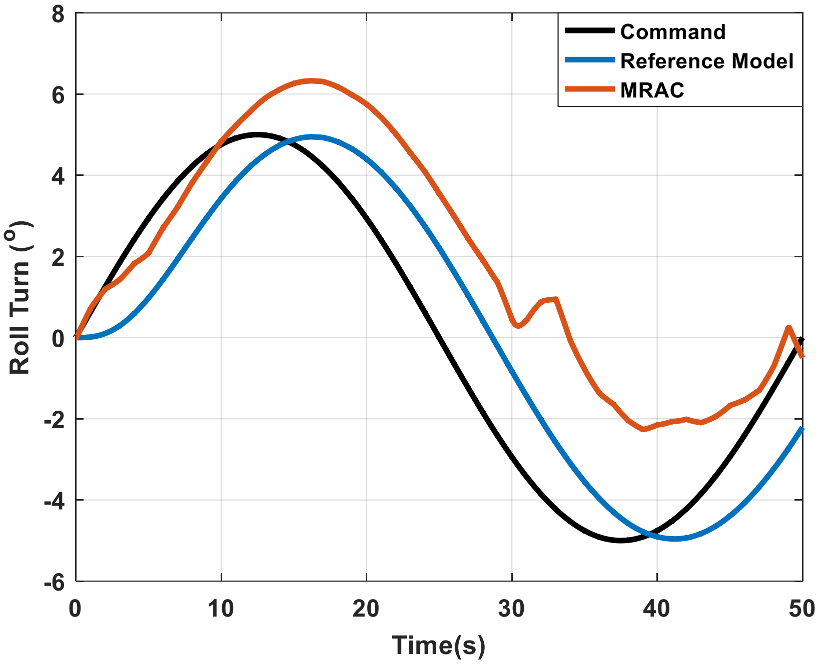

Figure 3.8. MRAC Response to Sine Wave Command Input under the Effect of Dynamics of Wing Rock, Random External Disturbance and Noise

As it is seen from both Figure 3.7, and Figure 3.8, the response of the slender delta wing controlled with MRAC, exposed to the effect of random external disturbance, dynamics of wing rock and noise, is far away from the reference model response. So, it is not acceptable behaviour in terms of performance of the controller, though could be reasonable in terms of stability since the response is not diverging under the effects of mentioned disturbances. The reason of poor performance of the MRAC in this study is proposed that, the uncertainty parametrization in MRAC has not been done in terms of the structure of the uncertainty. In fact, the divergence is caused from the inclusion of the unexpected random external disturbance which depends on time. But in MRAC, the parametrization of the uncertainty is done in terms of the system variables. In this thesis, it is proposed that each uncertainty should be predicted in terms of its own structure. Such that if it is in terms of time, then the uncertainty parametrization should also be done likewise, if it is in terms of state variables, vice versa. The examples will be given in the following sections in detail.

3.3. Concurrent Learning Model Reference Adaptive Control

The superiority of a control system is its capability to capture the system plant model accurately. However the coupled dynamics occuring due to the aerodynamic interactions of the body parts of the air vehicle, unmodelled dynamics like the measurement dynamics, nonlinearities, could cause unknown plant dynamics. Additionally, from the environmental perspective, the events occuring in the open air is unpredictable, like the turbulences, or measurement noise, or any other unexpected external disturbances. So, both the plant and the environmental dynamics is prone to deficiencies in terms of modelling. Adaptive controller or the adaptive element inclusion to the basic controller has emerged in order to eliminate this gap in modelling, and lead the controller to do its mission thoroughly, like preserving that the system model is stable and performs properly.

Depending on the case whether the parametrization of the uncertainty could linearly be done with the known basis functions or not, the uncertainties could be structured or unstructured. In the MRAC, if the uncertainty is structured, then, the adaptive element is also constructed by the same parametrization function. If the uncertainty is unstructured, but is continuous in a restricted domain, then multi layer Neural Networks or other polynomial approximation techniques are used as the well known regression techniques.

The general representation of a structured or unstructured uncertainty is defined as,

$$\Delta(x(t)) = W^T f(x(t)) \tag{3.26}$$

where a constant matrix of unknown gain is denoted by W, and the basis function used to parameterize the uncertainty is denoted by $f(x(t))$. In (3.26), the uncertainty parametrization $f(x(t))$ could be either in terms of time only, or in terms of system state at first degree, and time in second degree. If it is in terms of system state at first degree, it is defined as in terms of system state, in this study.

If the uncertainty addition into the system is through the control channel, then the uncertainty is defined as matched. So, the adaptive control input could cancel the uncertainty, in case of appropriate adaptive control input determination. In MRAC, appropriate adaptive control input determination is done by the same parametrization function used in uncertainty parametrization, as given in (3.27)

$$\mathrm{u_{ad}}(t) = W^T f(x(t)) \tag{3.27}$$

where $f(x(t))$ denotes the basis function used to parametrization of the uncertainty. W denotes the weight matrix that needs to be updated according to Lyapunov stability theorem at each time step of the simulation for the weight convergence. Convergence of the weights or remaining in a compact domain means that the uncertainty could be predicted approximately.

In MRAC, the main issue is to satisfy asymptotic command tracking, such that the system responses in the same way with the reference model upon the reference command input despite the uncertainties. This could be in case when the weights do not converge. On the other hand, the weights could also converge, while the command is tracked also. For the convergence of the weights, the necessary criteria that is to be satisfied is PE, meaning persistency of excitation. PE, puts restriction on the reference command input, such that reference command input causes all the system states to be excited. So, the convergence of the weights and thus uncertainty parametrization would be successful. PE condition, both satisfies weight convergence and also guarantees the boundedness of the parameters. However it is difficult to put criteria on the input to the system which is hard to predict and monitor. It is stated in (Eugene Lavretsky, n.d.) that in linear systems, there is no need of PE for weight convergence, if the input is summation of sinusoids with different frequencies. However Chowdhary (Öveç, 2016), declares that in nonlinear systems, as a proposal, PE condition could be loosen, by the addition of concurrent learning algorithm to the classical MRAC scheme. In classical MRAC applications, the adaptive control input is calculated at one step of the simulation and assumed to cancel the disturbance at each time step of the simulation. On the other hand, the convergence of the weight is not guaranteed in this case. In order to solve this, many learning algorithms have evolved. These algorithms work to form the adaptive control input iteratively to improve tracking accuracy. So, online adaptive control schemes are used by these algorithms to learn the disturbance, and some of studies could be found in (Pearlmutter, 1990). The algorithm of concurrent learning, is based on usage of the previous and the current data together to adapt the law of control weights. The main advantage of this algorithm is that, if the recorded data is valuable enough to express the disturbance, then without requiring PE condition, the convergence of the weights to their actual values or remaining in a restricted region is available. The convergence of weight is desired since it guarantees the performance in time that is transient be bounded exponentially and the convergence of error exponentially, so both the system behaves as the reference model, and the signals do not diverge.

$\phi(t)$, is a vector signal that is bounded is exciting persistently if for all $t > t_0$ there exists $T > 0$ and $\gamma > 0$ such that, (Öveç, 2016)

$$\int_{t}^{t+T} \phi(\tau)\phi^T(\tau)d\tau \geq \gamma I \tag{3.28}$$

where $\int_{t}^{t+T} \phi(\tau)\phi^T(\tau)d\tau \in R^{mxm}$.

This condition means that if the exogeneous reference input contains as many spectral lines as the number of unknown parameters, then the states of the plant are excited persistently. In concurrent adaptive learning, this excitation is satisfied without the need of putting restriction on the reference input command, since uses the previous and current data at the same time. The history stack is named for the past data. It is expected that as many elements $\phi_k \in \mathrm{R}^m$ which are linearly independent as the dimension of the basis of the uncertainty should be placed in the history stack. If $Z =$

$[\phi_1 \quad \phi_2 \quad \cdots \quad \phi_p]$ denotes the history stack, then as the rank condition $rank(Z) = m.$ should be satisfied. It is ensured that by the effect of WCC, the accumulated data is valuably sufficient to form an appropriate basis for the linearly parametrized uncertainty.

The main transcedence of WCC to PE condition is its easiness in practical application. Calculation the rank of a matrix online is straight while PE condition is hard to confirm in most cases. For linear systems, in the case that summation of sinusoids with different frequencies is chosen to form the exogeneous reference command, the PE conditions are satisfied. A single frequency could satisfy that gains of adaptive controller converge to their corresponding actual values exponentially (Öveç, 2016). However, for nonlinear systems this is difficult to apply. Since in real life cases mostly, the exogeneous input could not be known before priorily, online assesment of the PE condition is almost impossible. Additionaly, in real applications, excitation the states persistently is not desirable due to adverse effects for instance fuel limitation or inessential stress loads application. However, the WCC condition could be satisfied in a limited time period without further exciatation effort.

3.4. Concurrent Learning Weight Update Law

The law of concurrent weight update is based on adding an augmentation term that is recorded data based to the basic law of weight update, given in (3.23).

The recorded and current data are used at the same time, in concurrent learning adaptive control, to ensure tracking the reference exponentially and convergence of parameter without the need of excitation of the states persistently. (Chowdhary, 2010), (Burak, 2016).

Adaptive law of the concurrent learning has the representation form,

$$\dot{\widehat{W}}(t) = \Gamma\left(\beta\big(x(t)\big)e(t)^T PB + \sum_{j=1}^{p} \beta\big(x_j\big)\epsilon_j^T(t) \right) \tag{3.29}$$

where j represents the recorded data point at time t_j, and

$$\epsilon_j(t) = \Delta(\mathrm{x_j}) - \widehat{W}^T(t)\beta(x_j) \tag{3.30}$$

In order to evaluate (3.30), for the j^{th} data point, $\beta(x_j)$ and $\Delta(x_j)$ are required. In a history stack, namely Z, the basis vector $\beta(x_j) \in \mathrm{R}^s$ is stored.

$$Z = [\beta(\mathrm{x}_1), \beta(\mathrm{x}_2), \beta(\mathrm{x}_3), \ldots, \beta(\mathrm{x_p})] \in \mathrm{R}^{sxp} \tag{3.31}$$

If Z in (3.31) represents the history stack, then $rank(Z) = s$. Meaning that, the stored data stack includes as many linearly independent columns as the dimension of the basis vector.

In other words, the number of the basis vectors p stored in Z must be at least the dimension of the basis vectors s, i.e. $p \geq s$. Besides the basis vector $\beta(x_j)$, the related model error $\Delta(x_j)$ needs to be defined for the evaluation of (3.29). (Burak, 2016)

In case B has full column rank, by utilizing left pseudo inverse of B, $\Delta(x_j)$ can be observed from (2.2),

$$\Delta(x_j) = (B^T B)^{-1} B^T [\dot{x}_j - \mathrm{Ax_j} - Bu_j] \tag{3.32}$$

A, B, x_j and u_j are known, and only the estimation of $\dot{x}$ is required, for the aim of estimating the uncertainty of system. The measurement of $\dot{x}$ explicitly is assumed to be accessible in this thesis, so the estimation of $\dot{x}_j$ is available.

3.4.1. Data Selection Algorithm for the History Stack

It is stated in (3.3) that concurrent adaptive learning guarantees convergence of weight without the necessity of the PE restriction. Concurrent adaptive learning does this, with the usage of such an algorithm that, it stores the upcoming data in the history stack, as long as it includes the same number of linearly independent parameters with the basis of the uncertainty. So, the spectral features of the data should be taken into account, during the fill of the history stack. To satisfy the rank condition given in 3.4, an algorithm should be constructed in order to avoid useless storage of the data, such that the new coming data should be stored only if it is dissimilar from the last recorded data point. If it were possible to store all the data, then such an algorithm construction would be needless, but the limited capacity of the hardware restricts the real life applications. In (Chowdhary & Johnson, 2014), the comparison of the functioning of several data point selection algorithms, namely cyclic history-stack, static history-stack, and singular value maximizing are given. Comparing them, singular value maximizing approach is the one that provides the fastest parameter convergence. The concept behind the singular value maximizing method is that, the minimum eigenvalue of η determines the rate of convergence of weight directly proportionally. (Burak, 2016) It is formulated as,

$$\sigma_{min}\,(Z) = [\lambda_{min}\,(ZZ^T)]^{1/2} = \,[\lambda_{min}\,(\eta)]^{1/2} \tag{3.33}$$

It is inferred that, the convergence rate is dependent on the minimum eigenvalue λ_{min} of the symmetric matrix $\Omega = \sum_{j=1}^{p} \beta_j \beta_j^T$ of which proof is given in detail in (Chowdhary & Johnson, 2014). Based on that, for the data selection, at any j^{th} time step, it is aimed to maximize the minimum eigenvalue λ_{min} of the history stack $Z_j = [\,\beta_1, \beta_2, \dots, \beta_p]$. In the expressions, the index of the last stored point is denoted by $p \in \mathrm{N}$, the associated stored data stack column that is recorded at the j^{th} time step is represented by β_j, where the entire stored data stack at the same time instant is denoted by Z_j. The uppermost allowable value of p, which is the uppermost stack span is denoted by $\bar{p}$, and $\bar{p} \geq s$ must be satisfied for the convergence of the weights, where s is the rank of the basis of uncertainty.(Öveç, 2016).

The SVM algorithm adds any sufficiently different point to the stack until the stack is full. Once the stack is full, the algorithm overwrites only if the upcoming data increases the minimum eigenvalue of the symmetric matrix Ω (and so Z_j) when it is replaced with one of the existing points. In order to assure whether the upcoming data is sufficiently different from the existing data, the following rank condition is checked

$$rank([Z\ \beta]\ ) > rank\ (Z) \tag{3.34}$$

The algorithm of SVM is applied when this rank condition is satisfied. Hence, maximization of the minimum singular value of the stored data stack with rich terms is aimed during the stack construction. For saving data points, the SVM algorithm is used in concurrent adaptive learning algorithm. The SVM algorithm flowchart is given as,

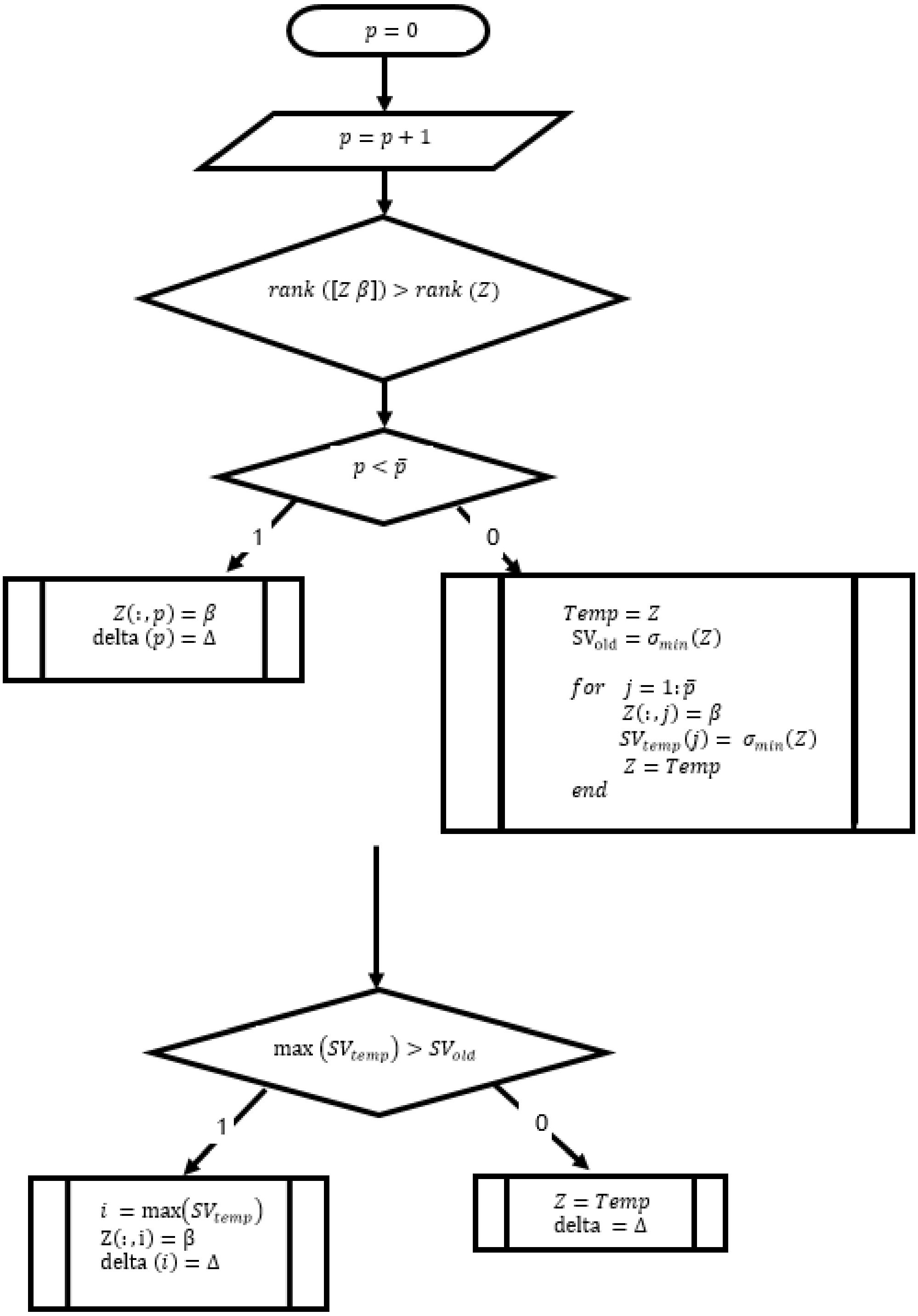

Figure 3.9. Singular Value Maximizing Algorithm Flow Chart

CHAPTER 4

TIME AND STATE DEPENDENT PARAMETERIZED MODEL REFERENCE ADAPTIVE CONTROL

4.1. Time Dependent Uncertainty Parametrization

Uncertainty parametrization could be done in terms of time variable, such that the basis function β given in (4.1), as the same function given in 3,

$$u_{ad}(t) = \widehat{W}(t)\,\beta_t(t) \tag{4.1}$$

could be constructed in terms of time variable. Time dependent uncertainty parametrization examples are given in 4.1.1, and 4.1.2, as Chebyshev polynomials based MRAC and Fourier series based MRAC, respectively. Weight update law for time dependent uncertainty parametrization is given in 4.1.3.

4.1.1. Chebyshev Polynomials Based Model Reference Adaptive Control

If the exact expression between the input and the output structures is not known, then the uncertainty is defined as unstructured uncertainty. In case the uncertainty is defined inside a restricted domain region and is continuous, then universal function approximators like sigmoidal neural networks or radial basis functions can be used as the approximation functions. Neural networks require substantial amount of effort. So, as an alternative and easier way, polynomial approximation is the other general technique to model the uncertainties. Function approximation could not give satisfactory results, since without considering the degree of the polynomial, the estimation is not ensured to converge to the true function. In theory, the precision of approximation is directly proportional with the degree of polynomial. However, rising the degree of the approximating polynomial beyond a limit, could produce overparametrization.

Usage of Chebyshev polynomial of which terms are orthogonal as the basis functions for the approximation of the unstructured uncertainty is a better way for elevation of standard polynomial approximation, as stated in (Öveç, 2016). The main advantage of an orthogonal function is its better approximation capability compared to a regular polynomial function. An available method for approximating functions with a series of terms that are linearly independent is provided by orthogonal polynomial basis function which satisfies faster convergence. In this section, Chebyshev polynomials will be used as the approximating basis function in the case of unstructured uncertainty

Chebyshev polynomials has an important function in the approximation theory, by forming a series of orthogonal polynomials. The polynomials are defined for $x\epsilon\,[-1\ 1]$. In this thesis, $\cos(t)$ is used instead of x variable.

$$\begin{gathered} T_0(x) = 1 \\ T_1(x) = x \\ T_2(x) = 2x^2 - 1 \\ T_3(x) = 4x^3 - 3x \\ \vdots \end{gathered} \tag{4.2}$$

The explicit formulation for Chebyshev polynomials is defined as,

$$T_n(x) = \cos(narccos(x)) \tag{4.3}$$

The Chebyshev polynomials could also be represented by the following recurrence relation for $n \geq 1$,

$$T_{n+1} = 2xT_n - T_{n-1} \tag{4.4}$$

The orthogonality property could also be defined for the defined interval $x \epsilon\ [-1\ 1]$ multiplied by the factor $\sqrt{(1-x^2)^{-1}}$ as,

$$\int_{-1}^{1} \frac{\mathrm{T_i}(x) T_j(x)}{\sqrt{1-x^2}}\, dx = \begin{cases} 0 & i \neq j \\ \frac{\pi}{2} & i = j \neq 0 \\ \pi & i = j = 0 \end{cases} \tag{4.5}$$

The advantage of the orthogonality property is that, the orthogonal terms are linearly independent, and so could cover the uncertainty space more accurately. This causes that basis polynomials of which terms are orthogonal converge faster with less terms compared to the regular polynomials.

The unstructured uncertainty could be defined as an arbitrary function $f(t)$. The function $f(t)$ could be approximated using Chebsyhev polynomials as,

$$f(t) = \sum_{i=1}^{N} w_i \beta_i(t) \tag{4.6}$$

where $\beta = [\beta_1\ \ \beta_2\ \ ...\ \ \beta_N]$ is the basis function formed by Chebsyhev polynomials array up to degree of $N-1$. The coefficients of the Chebsyhev polynomials are denoted by $W = [w_1\ \ w_2\ \ ...\ \ w_N]$, and are updated with the following update law of weight.

$$\dot{\widehat{W}}(t) = \Gamma \beta(t) e(t)^T P B \tag{4.7}$$

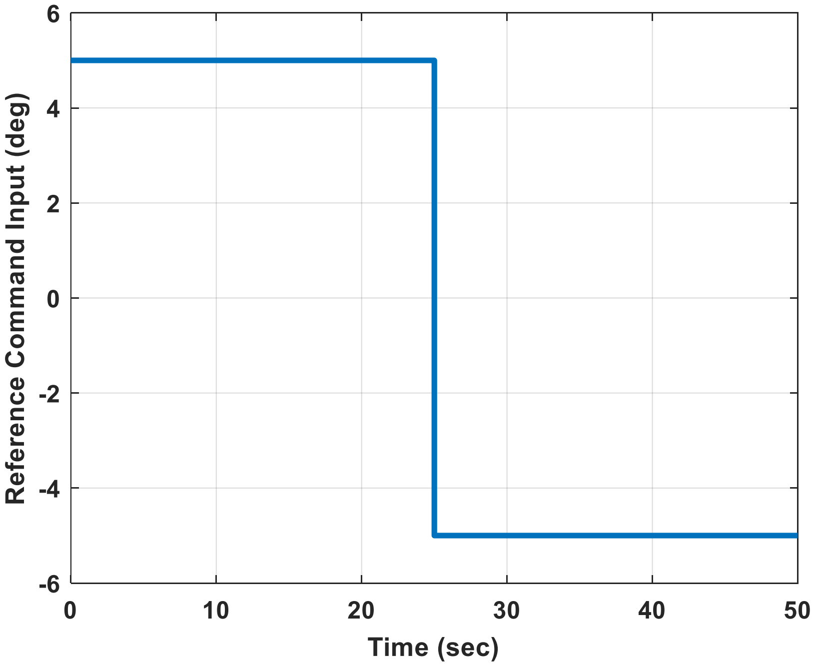

Figure 4.1. Reference Command Input vs time

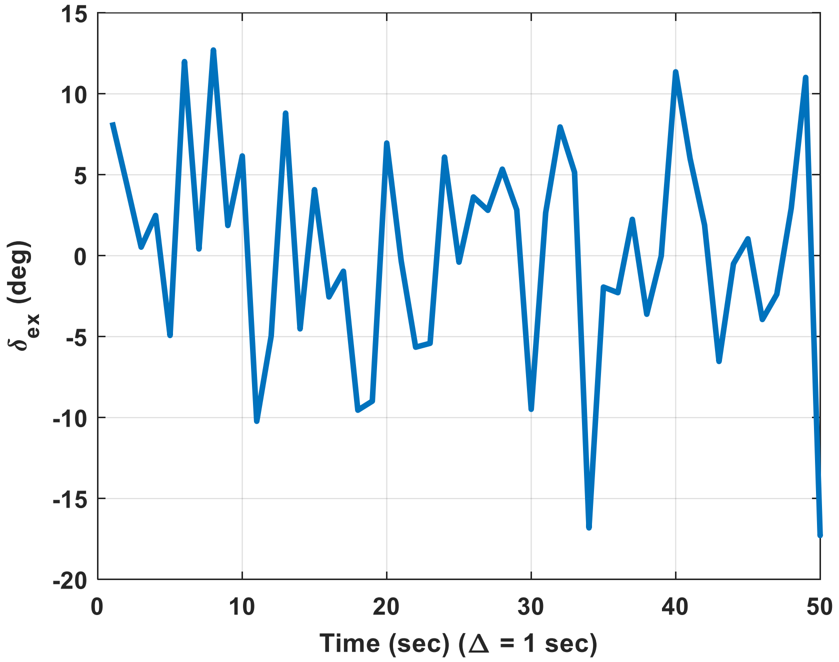

Figure 4.2. External Disturbance vs time

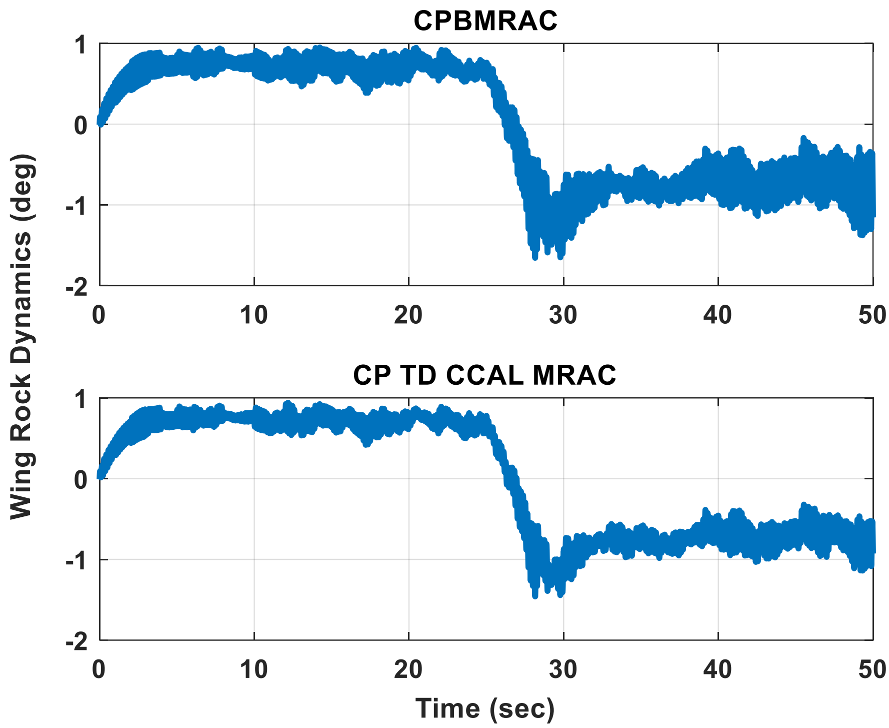

Figure 4.3. Wing Rock Dynamics vs time

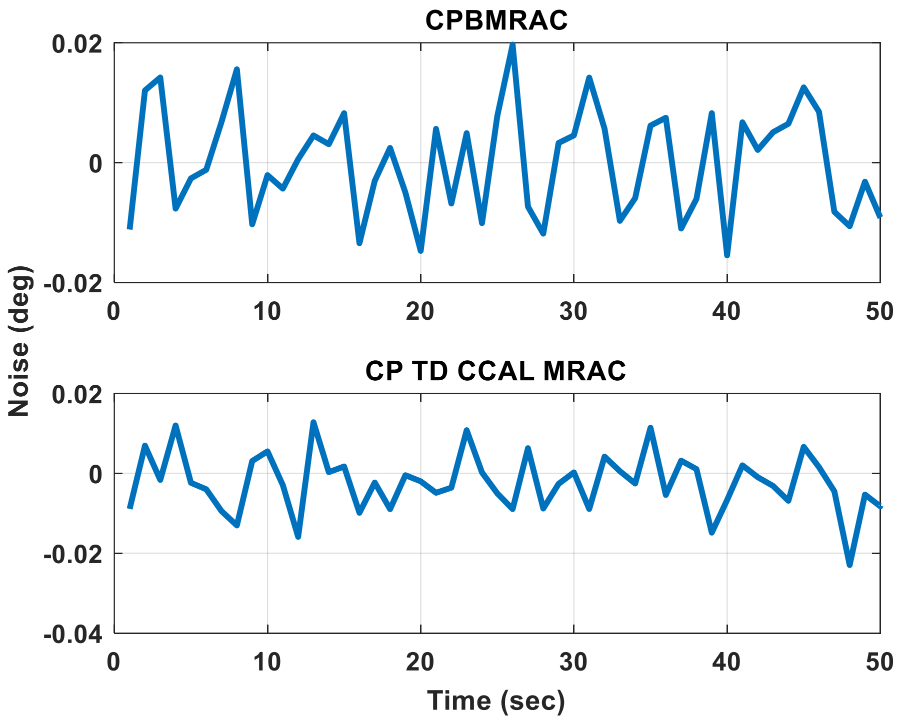

Figure 4.4. Noise added to Roll Turn Rate vs time

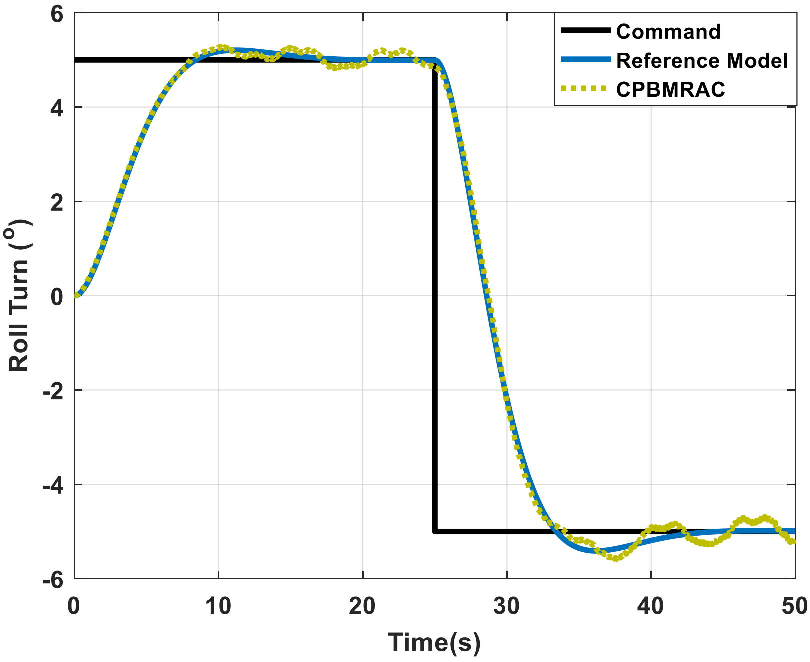

Figure 4.5. Chebsyhev Polynomials Based Model Reference Adaptive Control

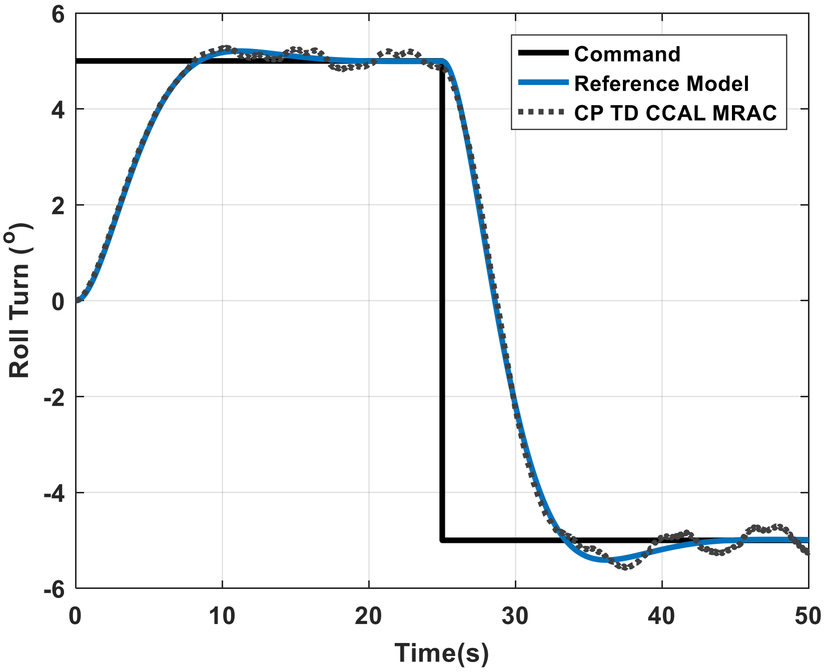

Figure 4.6. Chebsyhev Polynomials Based Time Dependent Concurrent Adaptive Learning Model Reference Adaptive Control

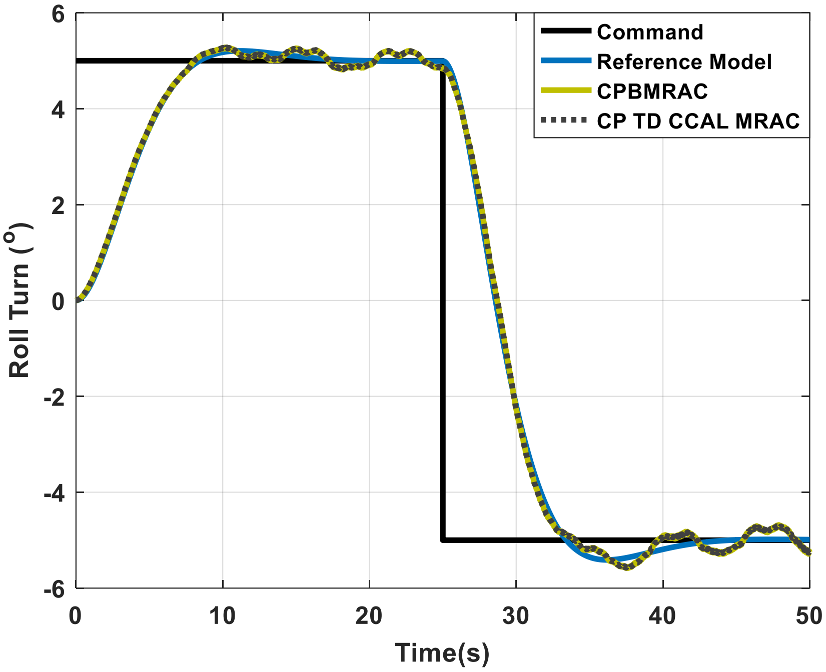

Figure 4.7. Comparison of Chebsyhev Polynomials Based Uncertainty Parametrization Methods

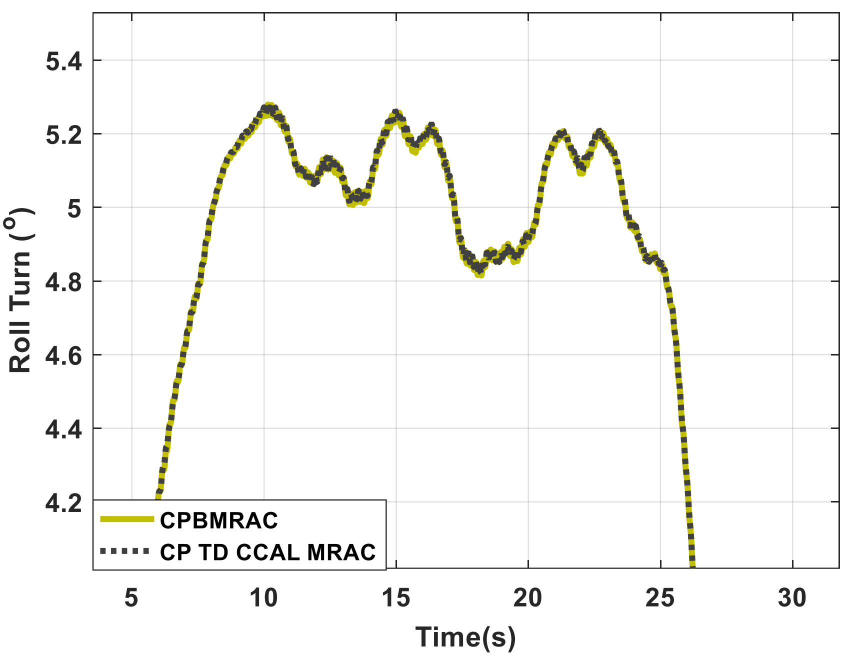

Figure 4.8. Zoomed part of the Figure 4.7

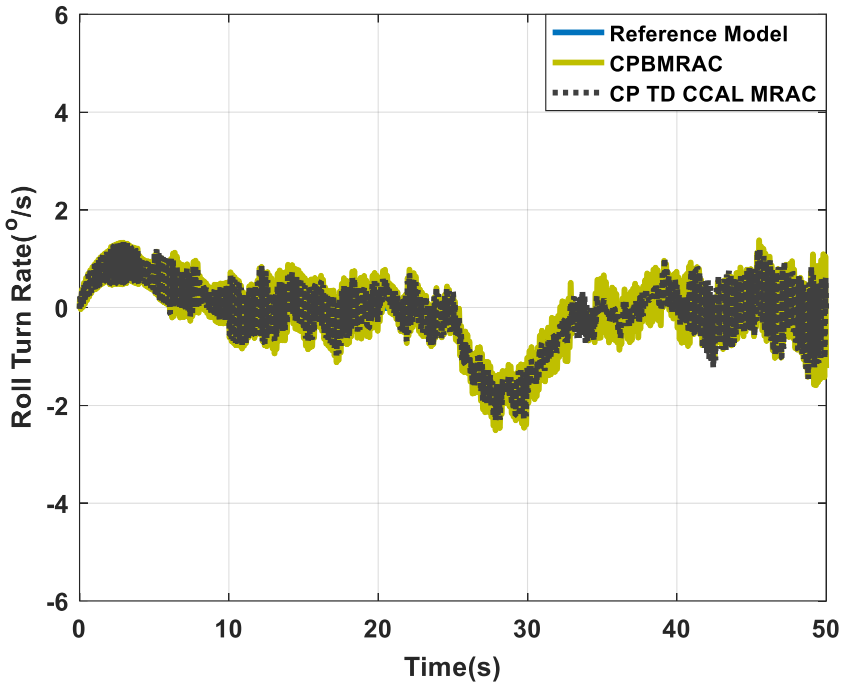

Figure 4.9 Comparison of Chebsyhev Polynomials Based Uncertainty Parametrization Methods

The study is done with the first 20 terms of the Chebsyhev polynomials of the first kind, which is given in (4.2). In Figure 4.5, Chebsyhev polynomials based MRAC graph, and in Figure 4.6, Chebsyhev polynomials based time dependent concurrent adaptive learning model reference adaptive control are given. Since they seem to be giving the same results, in Figure 4.7, comparison of the two methods has been done, and both of them seem to track the reference model closely exposed to the external disturbance, dynamics of wing rock and noise, stimulated with reference command. The fluctuations could be due to the $\cos(t)$ term in the Chebsyhev polynomials parameterization. In Figure 4.8, zoomed part of the Figure 4.7 is given, and it is seen that CP TD CCAL MRAC, gives less fluctuated results than the CPBMRAC. It could be due to the reason that, the usage of the recorded data in concurrent adaptive learning leads to faster learning of the uncertainty, than the solely MRAC. Though, since the uncertainty parametrization is time dependent, as the basis function is formed with Chebsyhev polynomials, the system states are perturbed at each time step of the simulation, and this made the similar benefit of the concurrent adaptive learning of which the key point is to eliminate the PE requirement, for the convergence of the weights, and so learning of the uncertainty. This situation led that both of the methods cause to learn the uncertainty and make the system model track the reference model closely. In Figure 4.9, the roll turn rate of the delta wing, the first system state x_1 of (2.1) is inspected. As seen, again both the adaptive learning methods give similar results, and cause the system model track the reference model closely. Again CP TD CCAL MRAC, gives less fluctuated results than the CPBMRAC, due to its data storage capability which leads to faster learning of the uncertainty and so does the elimination from the system model.

4.1.2. Fourier Series Based Model Reference Adaptive Control

Fourier series could be used in the representation of any periodic function. (Gezer, 2014) Simple periodic functions usage in the formation of that function means the representation of a function by Fourier series. Sine and cosine functions are these periodic functions. Any periodic function is formed by summing up these simple sine and cosine functions multiplied by a fixed weighing factor.

The periodic disturbances could also be estimated by Fourier series. Meaning that, Fourier series could be used in the parametrization of the uncertainty on the system. The trigonometric functions forming the Fourier series, sine and cosine functions, are kept as the variable part of the adaptive input, and the weighing factors are used to predict the disturbance acting on the system and cancel it. The proof of the stability for Fourier series based MRAC is given in (Gezer, 2014a) in detail.

If the function $f(t)$ repeats itself in a period, then it is named as periodic. The repetitive structure of a function $f(t)$ could be shown as,

$$f(t) = \mathrm{f(t + T)} \tag{4.8}$$

In fact any function operating in a limited time duration could be accepted as a periodic function, so the statement that only repetitive functions are periodic could be relaxed. For instance, assume that the function $f(t) = t$ operating in a limited time duration and so is not a periodic function, such that, $f(t) = t \quad \forall\, t \in [0, \mathrm{t_f}]$. Then, the function $f(t)$ could be represented with a Fourier series expansion which has enough long period and series length. The representation for the Fourier series expansion is shown as,

$$f(t) \simeq \mathrm{F(t)} = \mathrm{a_0} + \sum_{k=1}^{N} a_k \cos\left(k \frac{2\pi}{T} t\right) + b_k \sin\left(k \frac{2\pi}{T} t\right) \tag{4.9}$$

The Fourier series coefficients are represented by the coefficients a_0, a_k and b_k. The coefficient number is represented by index k, and the series length by N. The coefficients of the Fourier series a_0, a_k and b_k could be computed by the equations given,

$$\mathrm{a_0} = \frac{1}{T} \int_{-T/2}^{\mathrm{T}/2} f(t)\, dt \tag{4.10}$$

$$a_k = \frac{2}{T} \int_{-T/2}^{\mathrm{T}/2} f(t) \cos(k \frac{2\pi}{T} t)\, dt \tag{4.11}$$

$$b_k = \frac{2}{T} \int_{-T/2}^{\mathrm{T}/2} f(t) \sin(k \frac{2\pi}{T} t)\, dt \tag{4.12}$$

The calculated coefficients are scalar constant for the given function $f(t)$.

4.1.2.1. Fourier Series Based MRAC Design for Sample System Model

The Fourier series based MRAC design is done for the slender delta wing system model given in (2.1),

$$\dot{x}(\mathrm{t}) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(u(t) + \delta_{wr}\big(x(t)\big) + \delta_{ex}(t) \right) \tag{4.13}$$

The same desired characteristic criteria are selected, for the reference model design.

The desired natural frequency and damping ratio for the reference model are selected as, $\omega_n = 0.4\, rad/s$ and $\zeta_n = 0.707$. The reference model has the same form as (3.5),

The nominal controller of the plant is the same with the selected specified model in other words reference model's controller. The gains of the nominal controller is computed by using the formula belonging to Ackermann. The computed gains for the system to have the desirable closed loop response characteristics are, (Gezer, 2014b).

$$K_r = [0.57 \quad 0.16] \tag{4.14}$$

The nominal controller representation is,

$$u_n = -K_r x(t) + K_r H r(t) \tag{4.15}$$

where $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Fourier series is used in the formation of adaptive control element. Combination of sine and cosine terms with different frequencies is used in the formation of the Fourier series based basis function. The basis function vector for a series length of N,

$$\beta(t) = \begin{bmatrix} 1 \\ \cos(1\frac{2\pi}{T}t) \\ \cos(2\frac{2\pi}{T}t) \\ \cdots \\ \cos(N\frac{2\pi}{T}t) \\ \sin(1\frac{2\pi}{T}t) \\ \sin(2\frac{2\pi}{T}t) \\ \cdots \\ \sin(N\frac{2\pi}{T}t) \end{bmatrix} \tag{4.16}$$

The basis function has the dimension of $2N + 1$. This dimension is formed by N terms of cosine functions and N terms of sine functions. and 1 term for bias.

For the design of the controller for delta wing system, the series length of the Fourier series is selected as $N = 10$. (Gezer, 2014b). Therefore, the basis function has the dimension of 21.

The Fourier series period is chosen as $T = 200\ sec$, that is 3 times longer than the operation time of the simulation, which is $50\ sec$ (Gezer, 2014b).

So, both the series length N, and the period T should be determined for the construction of the Fourier series. Then, the parameters of the adaptive controller could be selected.

The learning rate for the update law of weight is chosen as,

$$\Gamma = 2\ 10^{-1} I_{21x21} \tag{4.17}$$

The dimension of the identity matrix I_{21x21} is 21x21.

The selection of R, the design selection matrix for the Lyapunov equation is as, (Gezer, 2014b)

$$\mathrm{R} = \begin{bmatrix} 1000 & 0 \\ 0 & 0.01 \end{bmatrix} \tag{4.18}$$

The same selected matrix R used in MRAC design which is given in 3 is used. The adaptive control input is calculated as,

$$\mathrm{u_{ad}} = \widehat{W}(t)\beta(t) \tag{4.19}$$

The estimation of the ideal weights of the Fourier series, is represented as the weighing vector $\widehat{W}(t)$.

This vector could be shown as,

$$\widehat{W}(t) = [\,\widehat{a_0} \quad \widehat{a_1} \quad \widehat{a_2} \quad \ldots \quad \widehat{a_{10}} \quad \widehat{b_1} \quad \widehat{b_2} \ldots \quad \widehat{b_{10}}\,] \tag{4.20}$$

The update law of weight for MRAC,

$$\dot{\widehat{W}}(t) = \Gamma\beta(t)e(t)^T PB \tag{4.21}$$

The block diagram representing the Fourier series based MRAC is given in Figure 4.10.

MRAC includes the external disturbance, the dynamics of wing rock and measurement noise in the rate of roll angle.

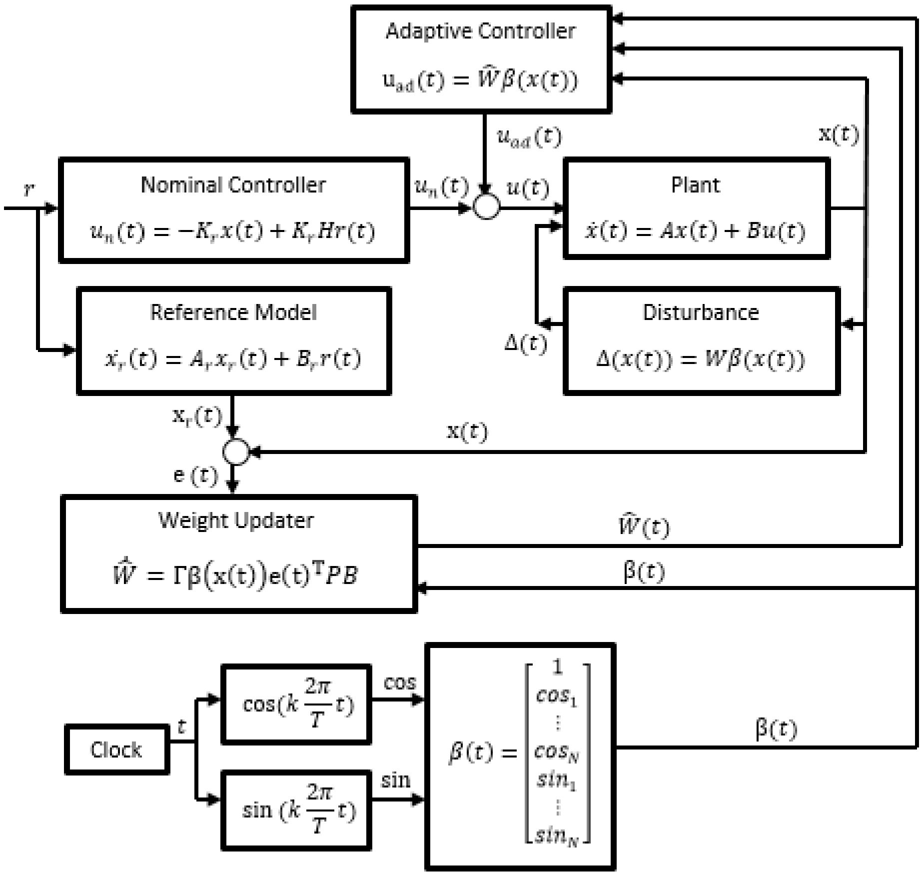

Figure 4.10. Fourier Series Based MRAC Block Diagram

As in the section of 4.1.1, where the Chebyshev polynomials based adaptive learning methods, CPBMRAC and CP TD CCAL MRAC, are compared through Figure 4.1 and Figure 4.9, in this section also Fourier series based adaptive learning methods, FSBMRAC and FS TD CCAL MRAC, are compared, through Figure 4.15 and Figure 4.19. As seen in Figure 4.15, and Figure 4.16, the Fourier series based adaptive learning satisfies more closer reference model tracking than the Chebyshev polynomials based adaptive learning results. This could be an indicator that, Fourier series has a faster learning capability than the Chebyshev polynomials, under the case given in this study, due to its periodic structure. Although the dynamics of wing rock, external disturbance, and the noise applying upon the system model are not periodic disturbances, the Fourier series could learn the uncertainty with its periodic structure, by exciting the system states all the time.

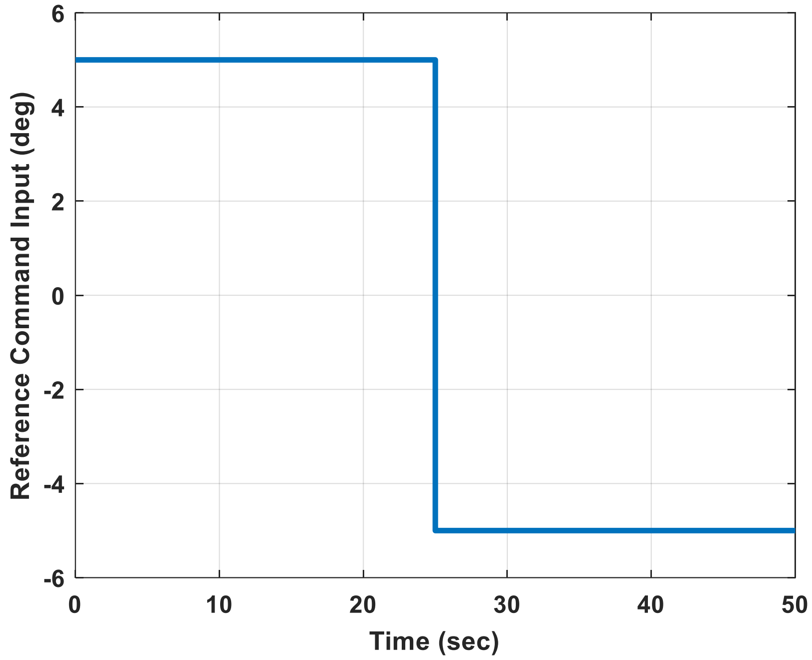

Figure 4.11. Reference Command Input vs time

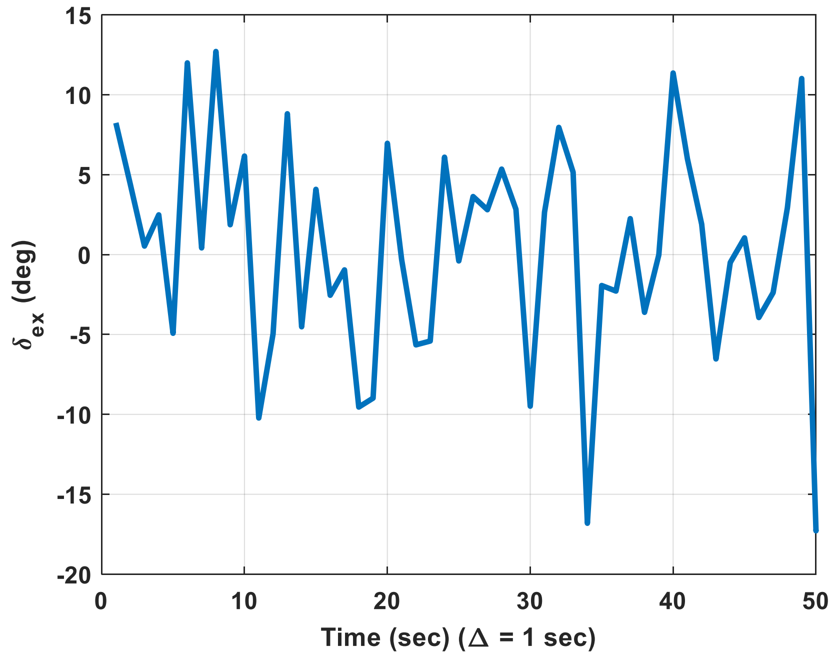

Figure 4.12. External Disturbance vs time

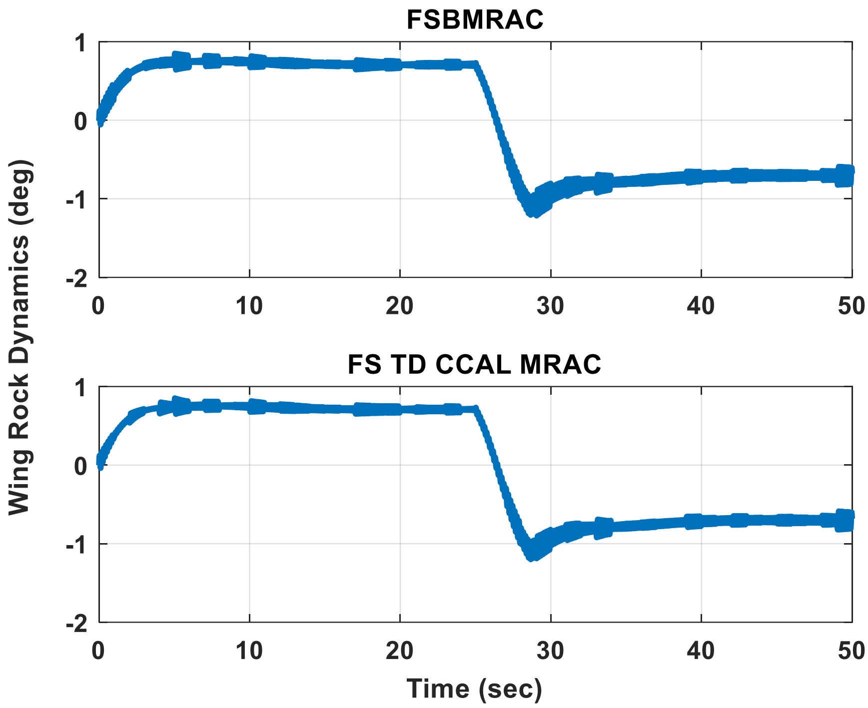

Figure 4.13. Wing Rock Dynamics vs time

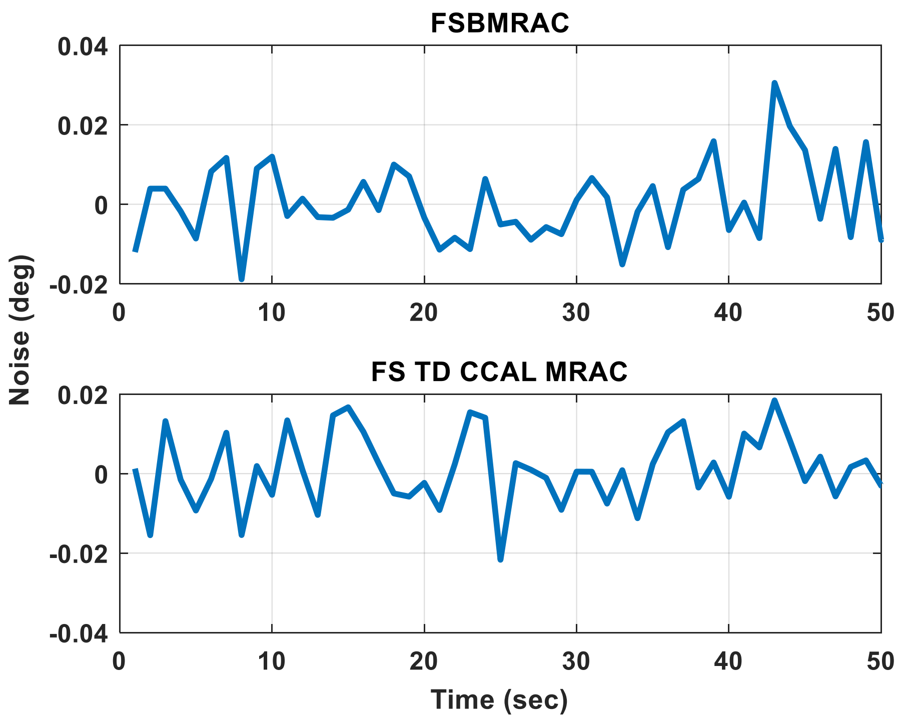

Figure 4.14. Noise vs time

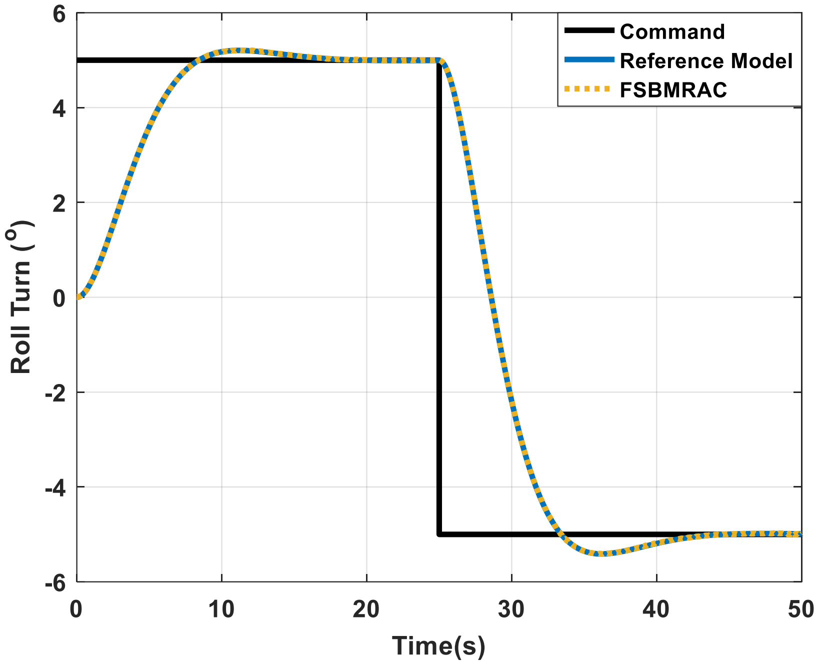

Figure 4.15. Fourier Series Based Model Reference Adaptive Control

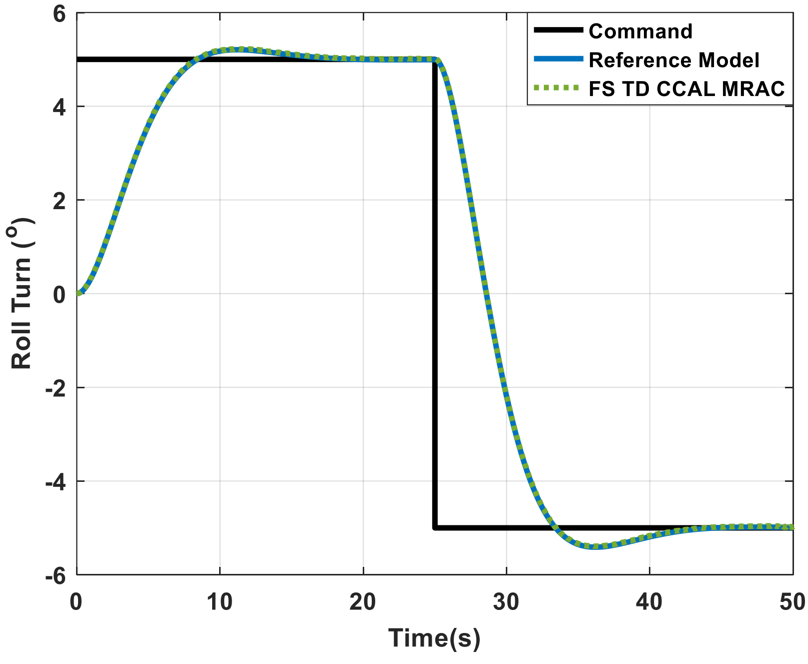

Figure 4.16 Fourier Series Based Time Dependent Concurrent Adaptive Learning Model Reference Adaptive Control

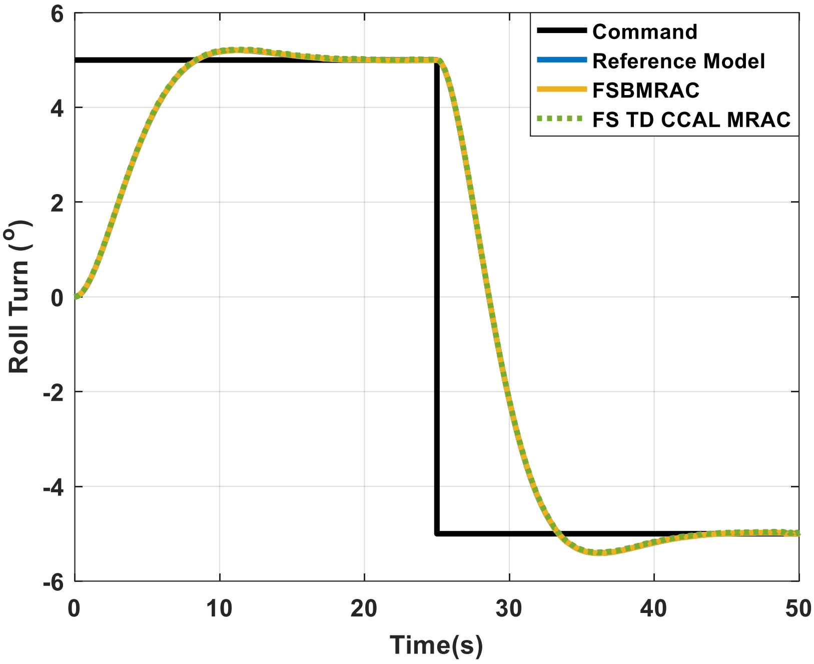

Figure 4.17. Comparison of Fourier Series Based Uncertainty Parametrization Methods

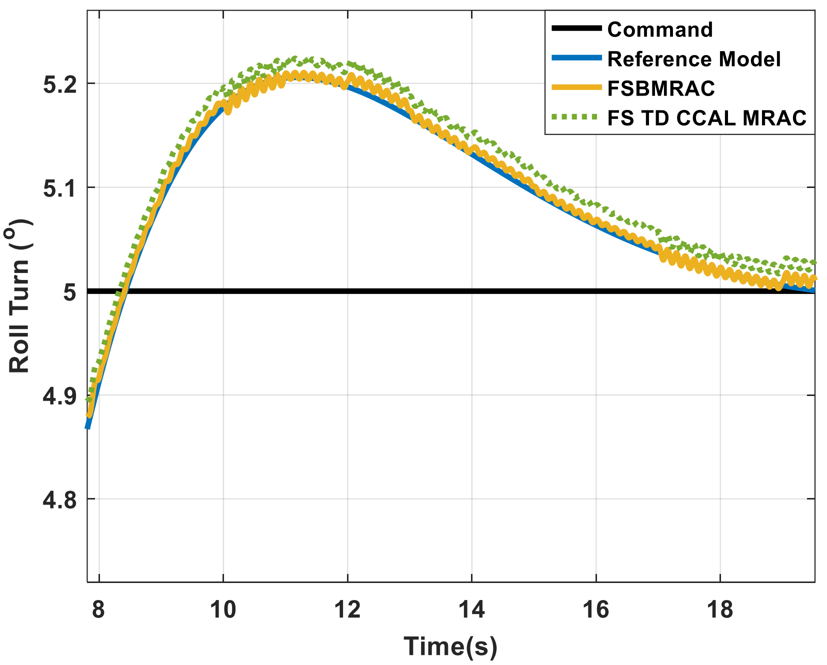

Figure 4.18. Zoomed part of the Figure 4.17

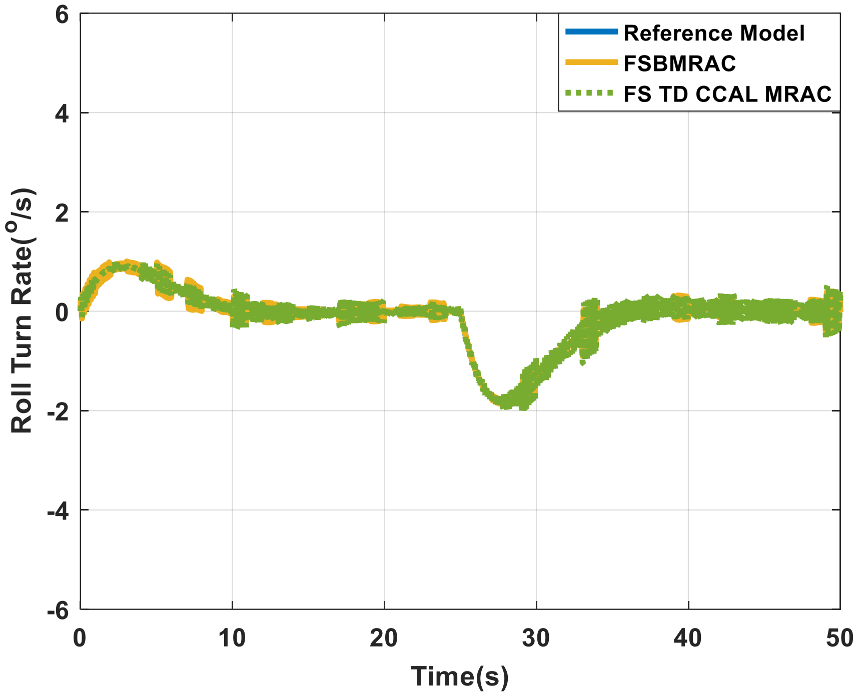

Figure 4.19. Comparison of Fourier Series Based Uncertainty Parametrization Methods

Both FSBMRAC and FS TD CCAL MRAC, could lead the system model track the reference model closely as seen in Figure 4.17 and Figure 4.18. In case of Chebyshev polynomials usage as the basis function, it is seen through Figure 4.7 and Figure 4.8, that concurrent adaptive learning caused less fluctuations than the solely model reference adaptive control due to its data storage capability. However, in the case of Fourier series usage as the basis function, it is seen thorough Figure 4.17 and Figure 4.18, solely FSBMRAC, could give closer reference model tracking even than the FS TD CCAL MRAC, due to the faster learning capability of the Fourier series. In Figure 4.19, the roll turn rate of the delta wing, the first system state x_1 of (2.1) is inspected. Again, it is seen that model reference adaptive control of which basis function is Fourier series, could also be used instead of concurrent adaptive learning method.

4.1.3. Weight Update Law for Time Dependent Uncertainty Parametrization

The uncertainty parametrization is whether it is structured or unstructured could be done with the basis functions. Adaptive control input is then constructed with the multiplication of the basis function, such that β, and the weighing factor, W. Since the uncertainty is approximated with the adaptive input, then by the application of the adaptive input in the control input, the uncertainty could be removed from the system model. The weighing factor in the adaptive control is updated at each time step of the simulation. The Lyapunov stability theorem dependent law, is used as the law of weight update. The law of weight update for the time dependent uncertainty parametrization used in this study is given in (4.22).

$$\dot{\widehat{W}}(t) = \Gamma\left(\beta_t(t)e(t)^T PB + \sum_{j=1}^{p} \beta_t(t)\epsilon_j^T(t) \right) \tag{4.22}$$

4.2. State Dependent Uncertainty Parametrization

Uncertainty parametrization could also be done in terms of system state variable, such that the basis function β given in (4.23)

$$\beta_x(x(t)) = \begin{bmatrix} x_2 \\ x_1 \\ |x_2|x_1 \\ |x_1|x_1 \\ x_1^3 \end{bmatrix} \tag{4.23}$$

which is the same basis function used in the wing rock dynamics equation for the defined delta wing, in (3.10). So, the uncertainty parametrization $f(x(t))$ could be either in terms of time only, or in terms of system state at first degree, and time in second degree. If it is in terms of system state at first degree, it is defined as in terms of system state, in this study.

Law of weight update for the system state dependent uncertainty parametrization is given in 4.2.1.

4.2.1. Weight Update Law for State Dependent Uncertainty Parametrization

The law of weight update for the state dependent uncertainty parametrization used in this study is given in (4.24), of which detailed derivation is given in (Yucelen & Haddad, 2012).

$$\dot{\widehat{W}}(t) = \Gamma\left(\beta_x\big(x(t)\big)e(t)^T PB - \beta_x\big(x(t)\big)\beta_x\big(x(t)\big)^T \widehat{W} B^T B\right) \tag{4.24}$$

4.3. Combination of Time and State Dependent Uncertainty Parametrization

In 4.1, time dependent uncertainty parametrization is given, and in 0 state dependent uncertainty parametrization is given. In this section, as the contribution of this thesis, combination of time and state dependent uncertainty parametrization is defined, as the part of the adaptive element, to be used in the simulations. In this thesis, it is proposed that, whether structured or unstructured, each disturbance should be predicted, in the terms of itself variable. Such that, if the uncertainty is in terms of time, then the basis function should also be constructed as in terms of time variable. If the uncertainty is in terms of state variable, then the parametrization should also be in terms of state variable. In case both time dependent and state dependent disturbance enters to the system, separately, then each of them should be predicted with the basis function in terms of its variable. Then the adaptive control input, should be calculated as the summation of each one. For instance for the system model given in 2.3, the external disturbance applying upon the system is in terms of time variable, and the dynamics of wing rock is in terms of state variable. So, each should be predicted separately. In this section, Fourier series is used for time dependent uncertainty parametrization as given in (4.25), and state variable dependent basis function as given in (4.26).

$$\beta_t(t) = \begin{bmatrix} 1 \\ \cos(1\frac{2\pi}{T}t) \\ \cos(2\frac{2\pi}{T}t) \\ \dots \\ \cos(N\frac{2\pi}{T}t) \\ \sin(1\frac{2\pi}{T}t) \\ \sin(2\frac{2\pi}{T}t) \\ \dots \\ \sin(N\frac{2\pi}{T}t) \end{bmatrix} \tag{4.25}$$

where N, the series length is taken as 10; and T the Fourier series period is taken as $200\ sec$.

$$\beta_x(x(t)) = \begin{bmatrix} x_2 \\ x_1 \\ |x_2|x_1 \\ |x_1|x_1 \\ x_1^3 \end{bmatrix} \tag{4.26}$$

where the roll turn rate is x_1, and the roll turn of the slender delta wing is x_2 as given in 2.1.

4.3.1. Weight Update Law for Combination of Time and State Dependent Uncertainty Parametrization

In 4.1.3, weight update law for time dependent uncertainty parametrization, and in 4.2.1, weight update law for state dependent uncertainty parametrization is given. In this section, weight update law in case of combination of time and state dependent uncertainty parametrization occurrance is given, as the proposal of this thesis. In case both time dependent and state dependent disturbance enters to the system, separately, then the law of weight update for the adaptive element should be constructed separately for each of them. In this section, (4.27) is the equation of the law of weight update for the time dependent uncertainty parametrization, and (4.28) is the equation of the law of weight update for the state dependent uncertainty parametrization.

$$\dot{\widehat{W}}(t) = \Gamma\left(\beta_t(t)e(t)^T PB + \sum_{j=1}^{p} \beta_t(t)\epsilon_j^T(t) \right) \tag{4.27}$$

$$\dot{\widehat{W}}(t) = \Gamma\left(\beta_x\big(x(t)\big)e(t)^T PB - \beta_x\big(x(t)\big)\beta_x\big(x(t)\big)^T \widehat{W} B^T B \right) \tag{4.28}$$

So, the adaptive control input for the combination of time and state dependent uncertainty parametrization is as given in (4.29),

$$u_{ad}(t) = \widehat{W}(t)\, \beta(x(t)) \tag{4.29}$$

where,

$$u_{ad}(t) = u_{ad1}(t) + u_{ad2}(t) \tag{4.30}$$

Assume that u_{ad1} eliminates the time dependent uncertainty parametrized disturbance, and u_{ad2} eliminates the state dependent uncertainty parametrized disturbance. Then,

$$u_{ad1}(t) = \widehat{W}_1 \beta_t(t) \tag{4.31}$$

where,

$$\dot{\widehat{W}}_1(t) = \Gamma\left(\beta_t(t)e(t)^T PB + \sum_{j=1}^{p} \beta_t(t)\epsilon_j^T(t) \right) \tag{4.32}$$

$$\beta_t(t) = [1 \quad \cos(1\frac{2\pi}{T}) \quad \cdots \quad \cos(N\frac{2\pi}{T}) \quad \sin(1\frac{2\pi}{T}) \quad \cdots \quad \sin(N\frac{2\pi}{T}) \;] \quad (4.33)$$

$$u_{ad2}(t) = \widehat{W}_2 \beta_x(x(t)) \quad (4.34)$$

where,

$$\dot{\widehat{W}}_2(t) = \Gamma\left(\beta_x\big(x(t)\big)e(t)^T PB - \beta_x\big(x(t)\big)\beta_x\big(x(t)\big)^T \widehat{W} B^T B\right) \quad (4.35)$$

$$\beta_x\big(x(t)\big) = [x_2 \quad x_1 \quad |x_2|x_1 \quad |x_1|x_1 \quad x_1^3] \quad (4.36)$$

4.4. Stability Proof

The stability proof is done by using (Yucelen & Johnson, 2012) and (Yucelen & Haddad, 2012). The adaptive control for uncertainty suppression section is used for the second reference. We begin by presenting a simple formulation of the adaptive control problem without loss of generality (Yucelen & Johnson, 2012). Specifically, consider the nonlinear uncertain dynamical system given by,

$$\dot{x}(t) = Ax(t) + B[u(t) + \delta(x,t)], \qquad x(0) = x_0, \qquad t \in R_+ \quad (4.37)$$

where $x(t) \in R^n$ is the state vector available for feedback, $u(t) \in R^m$ is the control input restricted to the class of admissible controls consisting of measurable functions, $\delta\colon R^n \rightarrow R^m$ is an uncertainty, $A \in R^{nxn}$ is a known system matrix, and $B \in R^{nxm}$ is a known control input matrix such that $\det(B^T B) \neq 0$ and the pair (A, B) is controllable.

Assumption 1. The uncertainty in (4.37) is parametrized as,

$$\delta(x) = W_1^T \beta_t(t) + W_2^T \beta_x(x(t)) + \epsilon,\ x \in R^n \quad (4.38)$$

where $W_1 \in R^{pxm}, W_2 \in R^{sxm}$ are unknown weight matrixes, and $\beta_t: R^n \rightarrow R^p$, $\beta_x: R^n \rightarrow R^s$ are known basis functions of the form, $\beta_t(t) = \left[\beta_1(t), \beta_2(t), \beta_3(t), \dots, \beta_p(t)\right]^T$,

$\beta_x\big(x(t)\big) = [\beta_{x1}(x(t)), \beta_{x2}(x(t)), \beta_{x3}(x(t)), \dots, \beta_{xs}(x(t))]^T$ respectively.

It is assumed that the approximation error ϵ, is bounded and the following inequality is hold $||\epsilon|| \leq \epsilon_{b1}$ where $\epsilon_{b1} \in R$ is known positive constant. (Patre, 2009)

Next, consider the ideal reference system capturing a desired closed loop dynamical system performance given by,

$$\dot{x_r}(t) = A_r x_r(t) + B_r r(t), \qquad x_r(0) = x_{r0}, \qquad t \in R_+ \tag{4.39}$$

where $x_r(t) \in R^n$ is the state reference vector, $r(t) \in R^m$ is a bounded command for tracking (or $r(t) \equiv 0$ $for\ stabilization$), $A_r \in R^{nxn}$ is the Hurwitz reference system matrix, and $B_r \in R^{nxm}$is the command input matrix. The objective of the adaptive control problem is to construct a feedback control law $u(t)$ such that the state vector $x(t)$, asymptotically follows the reference state vector $x_r(t)$ subject to Assumption 1.

For the purpose of solving the adaptive control problem, consider the feedback control law given by,

$$u(t) = u_n(t) + u_{ad}(t) \tag{4.40}$$

where $u_n(t)$ and $u_{ad}(t)$ are the nominal feedback control law and the adaptive feedback control law, respectively. Let the nominal control law be given by,

$$u_n(t) = K_1 x(t) + K_2 r(t) \tag{4.41}$$

where, $K_1 \in R^{mxn}$ and $K_2 \in R^{mxm}$are the nominal feedback and the nominal feedforward gains, respectively, such that $A_r = A + BK_1$, $B_r = BK_2$, and $\det(K_2) \neq 0$ hold. Now, using (4.40), (4.41) in (4.37) with the Assumption 1 yields,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B[K_1x(t) + K_2r(t) + u_{ad}(t) + \delta(x(t))] \\ &= [A + BK_1]x(t) + BK_2r(t) + B\left[u_{ad} + W_1^T\beta_t(t) + W_2^T\beta_x\big(x(t)\big) + \epsilon\right] \\ &= A_rx(t) + B_rr(t) + B\left[u_{ad}(t) + W_1^T\beta_t(t) + W_2^T\beta_x\big(x(t)\big) + \epsilon\right]\end{aligned} \tag{4.42}$$

Next, let the adaptive feedback control law be given by,

$$u_{ad}(t) = -\widehat{W}_1{}^T(t)\beta_t(t) - \widehat{W}_2{}^T(t)\beta_x(x(t)) \tag{4.43}$$

where $\widehat{W}_1 \in R^{pxm}, \widehat{W}_2 \in R^{sxm}$ are the estimates of W_1, W_2 respectively, satisfying the weight update laws,

$$\dot{\widehat{W}}_1(t) = \Gamma\beta_t(t)e^T(t)PB, \quad \widehat{W}_1(0) = \widehat{W}_{10}, \quad t \in R_+ \tag{4.44}$$

$$\begin{aligned}\dot{\widehat{W}}_2(t) = \Gamma\beta_x\big(x(t)\big)e^T(t)PB - \Gamma\beta_x\big(x(t)\big)\beta_x\big(x(t)\big)^T\widehat{W}_2B^TB, \\ \widehat{W}_2(0) = \widehat{W}_{20}, \quad t \in R_+\end{aligned} \tag{4.45}$$

where $\Gamma \in R_+^{sxs} \cap S^{sxs}$ is the learning rate matrix, $e(t) \triangleq x(t) - x_r(t)$ is the system error state vector, and $P \in R_+^{nxn} \cap S^{nxn}$ is a solution of the Lyapunov equation,

$$0 = A_r^TP + PA_r + R \tag{4.46}$$

where $R \in R_+^{nxn} \cap S^{nxn}$ can be viewed as an additional learning rate. Since A_r is Hurwitz, it follows from converse Lyapunov theory that there exists a unique P satisfying for a given R.

Now, using (4.43) in (4.42) yields,

$$\dot{x}(t) = A_r x(t) + B_r r(t) + B[\,-\widehat{W}_1{}^T(t)\beta_t(t) - \widehat{W}_2{}^T(t)\beta_x\big(x(t)\big) \qquad (4.47)$$
$$+W_1^T(t)\beta_t(t) + W_2^T(t)\beta_x\big(x(t)\big) + \epsilon]$$
$$\dot{x}(t) = A_r x(t) + B_r r(t) - B\widetilde{W}_1{}^T(t)\beta_t(t) - B\widetilde{W}_2{}^T\beta_x\big(x(t)\big) + \epsilon$$

and the system error dynamics is given by using (4.39) and as

$$\dot{e}(t) = -A_r x_r(t) - B_r r(t) +$$
$$+A_r x(t) + B_r r(t) - B\widetilde{W}_1{}^T(t)\beta_t(t) - B\widetilde{W}_2{}^T\beta_x\big(x(t)\big) + \epsilon \qquad (4.48)$$
$$= A_r e(t) - B\widetilde{W}_1{}^T(t)\beta_t(t) - B\widetilde{W}_2{}^T(t)\beta_x\big(x(t)\big) + \epsilon,$$
$$e(0) = e_0, \quad t \in R_+$$

where $\widetilde{W}(t) \triangleq \widehat{W}(t) - W$, and $e_0 \triangleq x_0 - x_{r0}$.

The weight update law given by (4.44) and (4.45), can be derived using Lyapunov analysis by considering the Lyapunov function candidate,

$$V\big(e, \widetilde{W}_1, \widetilde{W}_2\big) = e^T P e + tr\, \widetilde{W}_1{}^T \Gamma^{-1} \widetilde{W}_1 + tr\, \widetilde{W}_2{}^T \Gamma^{-1} \widetilde{W}_2 \qquad (4.49)$$

Note that $V(0,0,0) = 0$ and $V\big(e, \widetilde{W}_1, \widetilde{W}_2\big) > 0$ for all $\big(e, \widetilde{W}_1, \widetilde{W}_2\big) \neq (0,0,0)$. Now, differentiating (4.49) yields step by step,

First take the derivative of the first term $e^T P e$ of (4.49) by the chain rule,

$$\frac{d}{dt}(e^T P e) = \dot{e}^T P e + e^T P \dot{e}$$
$$= \Big(A_r e(t) - B\widetilde{W}_1{}^T(t)\beta_t(t) - B\widetilde{W}_2{}^T(t)\beta_x\big(x(t)\big) + \epsilon\Big)^T P e(t) +$$
$$+ e^T(t) P \Big(A_r e(t) - B\widetilde{W}_1{}^T(t)\beta_t(t) - B\widetilde{W}_2{}^T(t)\beta_x\big(x(t)\big) + \epsilon\Big)$$
$$= e^T(t) A_r^T P e(t) - B\widetilde{W}_1(t)\beta_t(t) P e(t) - B\widetilde{W}_2(t)\beta_x(x(t)) P e(t) \qquad (4.50)$$
$$+ e^T(t) P A_r e(t) - e^T(t) P B\widetilde{W}_1{}^T(t)\beta_t(t) - e^T(t) P B\widetilde{W}_2{}^T(t)\beta_x\big(x(t)\big) + \epsilon$$
$$= e^T(t)(A_r^T P + P A_r) e(t) - 2e^T(t) P B\widetilde{W}_1{}^T(t)\beta_t(t) - 2e^T(t) P B\widetilde{W}_2{}^T(t)\beta_x\big(x(t)\big) + \epsilon$$
$$= -e^T(t) R e(t) - 2e^T(t) P B\widetilde{W}_1{}^T(t)\beta_t(t) - 2e^T(t) P B\widetilde{W}_2{}^T(t)\beta_x\big(x(t)\big) + \epsilon$$

Second take the derivative of the second term $tr\ \widetilde{W}_1{}^T\Gamma^{-1}\widetilde{W}_1$ of (4.49) by the chain rule, (it is assumed that $\dot{\widetilde{W}} = \dot{\widehat{W}}$)

$$\begin{aligned}\frac{d}{dt}(tr\widetilde{W}_1{}^T\Gamma^{-1}\widetilde{W}_1) = \frac{d}{dt}(tr(\widehat{W}_1 - W_1)^T\Gamma^{-1}(\widehat{W}_1 - W_1))\\ = tr\dot{\widehat{W}}_1{}^T(t)\Gamma^{-1}\widetilde{W}_1(t) + tr\widetilde{W}_1{}^T(\mathrm{t})\Gamma^{-1}\dot{\widehat{W}}_1(t)\\ = tr\widetilde{W}_1{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_1(t) + tr\widetilde{W}_1{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_1(t)\\ = 2tr\widetilde{W}_1{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_1(t)\end{aligned} \tag{4.51}$$

Third take the derivative of the third term $tr\ \widetilde{W}_2{}^T\Gamma^{-1}\widetilde{W}_2$ of (4.49) by the chain rule,

$$\begin{aligned}\frac{d}{dt}(tr\widetilde{W}_2{}^T\Gamma^{-1}\widetilde{W}_2) = \frac{d}{dt}(tr(\widehat{W}_2 - W_2)^T\Gamma^{-1}(\widehat{W}_2 - W_2))\\ = tr\dot{\widehat{W}}_2{}^T(t)\Gamma^{-1}\widetilde{W}_2(t) + tr\widetilde{W}_2{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_2(t)\\ = tr\widetilde{W}_2{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_2(t) + tr\widetilde{W}_2{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_2(t)\\ = 2tr\widetilde{W}_2{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_2(t)\end{aligned} \tag{4.52}$$

By adding (4.50), (4.51), and (4.52), the differentiation of (4.49) is get as given in (4.53),

$$\begin{aligned}\dot{V}(e,\widetilde{W}_1,\widetilde{W}_2) = -e^T(t)Re(t) - 2e^T(t)PB\widetilde{W}_1{}^T(t)\beta_t(t) - 2e^T(t)PB\widetilde{W}_2{}^T(t)\beta_x(x(t)) +\\ +2tr\widetilde{W}_1{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_1(t) + 2tr\widetilde{W}_2{}^T(t)\Gamma^{-1}\dot{\widehat{W}}_2(t) + \epsilon\end{aligned} \tag{4.53}$$

where using (4.44) and (4.45) in (4.53) results in,

$$\begin{aligned}\dot{V}(e,\widetilde{W}_1,\widetilde{W}_2) = -e^T(t)Re(t) - 2e^T(t)PB\widetilde{W}_1{}^T(t)\beta_t(t) - 2e^T(t)PB\widetilde{W}_2{}^T(t)\beta_x(x(t)) +\\ +2tr\widetilde{W}_1{}^T(t)\Gamma^{-1}(\Gamma\beta_t(t)e^T(t)PB)\\ +2tr\widetilde{W}_2{}^T(t)\Gamma^{-1}(\Gamma\beta_x(x(t))e^T(t)PB - \Gamma\beta_x(x(t))\beta_x^T(x(t))\widehat{W}_2\,B^TB) + \epsilon\end{aligned} \tag{4.54}$$

$$\dot{V}(e,\widetilde{W}_1,\widetilde{W}_2) = -e^T(t)Re(t) - 2e^T(t)PB\widetilde{W}_1{}^T(t)\beta_t(t) - 2e^T(t)PB\widetilde{W}_2{}^T(t)\beta_x(x(t)) + \\ +2tr\widetilde{W}_1{}^T(t)\beta_t(t)e^T(t)PB \\ +2tr\widetilde{W}_2{}^T(t)\beta_x(x(t))e^T(t)PB - 2tr\,\widetilde{W}_2{}^T(t)\beta_x(x(t))\beta_x^T(x(t))\widehat{W}_2B^TB + \epsilon \tag{4.55}$$

$$\dot{V}(e,\widetilde{W}_1,\widetilde{W}_2) = -e^T(t)Re(t) \\ -2tr\widetilde{W}_2{}^T(t)\beta_x(x(t))\beta_x^T(x(t))\widehat{W}_2B^TB + \epsilon \tag{4.56}$$

$$\dot{V}(e,\widetilde{W}_1,\widetilde{W}_2) = -e^T(t)Re(t) \\ -2tr\widetilde{W}_2{}^T(t)\beta_x(x(t))\beta_x^T(x(t))\widetilde{W}_2(t)B^TB \\ -2tr\widetilde{W}_2{}^T(t)\beta_x(x(t))\beta_x^T(x(t))W_2(t)B^TB + \epsilon \tag{4.57}$$

$$\dot{V}(e,\widetilde{W}_1,\widetilde{W}_2) = -e^T(t)Re(t) \\ -2\beta_x^T(x(t))\widetilde{W}_2(t)(B^TB)^{\frac{1}{2}}(B^TB)^{\frac{1}{2}}\widetilde{W}_2{}^T(t)\beta_x(x(t)) \\ -2\beta_x^T(x(t))W_2(B^TB)^{\frac{1}{2}}(B^TB)^{\frac{1}{2}}\widetilde{W}_2{}^T(t)\beta_x(x(t)) + \epsilon \tag{4.58}$$

$$\leq -\lambda_{min}(R)\|e(t)\|_2^2 - 2\gamma\left\|(B^TB)^{\frac{1}{2}}\widetilde{W}_2{}^T(t)\beta_x(x(t))\right\|_2^2 \\ +2\gamma\left\|(B^TB)^{\frac{1}{2}}\widetilde{W}_2{}^T(t)\beta_x(x(t))\right\|_2\left\|(B^TB)^{\frac{1}{2}}W_2^T\beta_x(x(t))\right\|_2 + \epsilon_{b1}, t \geq 0 \tag{4.59}$$

Furthermore, using Young's inequality in the last term of (4.59) gives, (Yucelen & Haddad, 2012)

$$2\gamma\left\|(B^TB)^{\frac{1}{2}}\widetilde{W}_2{}^T(t)\beta_x(x(t))\right\|_2\left\|(B^TB)^{\frac{1}{2}}W_2^T(t)\beta_x(x(t))\right\|_2 \\ \leq \sigma\gamma\left\|(B^TB)^{\frac{1}{2}}W_2^T(t)\beta_x(x(t))\right\|_2^2 + \frac{\gamma}{\sigma}\left\|(B^TB)^{\frac{1}{2}}\widetilde{W}_2{}^T(t)\beta_x(x(t))\right\|_2^2, t \geq 0, \sigma > 0. \tag{4.60}$$

Next, setting $\sigma = \frac{1}{2}$, yields

$$\dot{V}\left(e(t),\widetilde{W}_1(t),\widetilde{W}_2(t)\right) \tag{4.61}$$

$$\leq -\lambda_{min}(R)\|e(t)\|_2^2 + \frac{\gamma}{2}\left\|(B^TB)^{\frac{1}{2}}W_2^T(t)\beta_x\big(x(t)\big)\right\|_2^2 + \epsilon_{b1}\,,\, t \geq 0$$

Since $\left\|\beta_x\big(x(t)\big)\right\|_2 \leq l_{\beta x0} + l_{\beta x1}\|x\|_2 \leq l_{\beta x0} + l_{\beta x1}\|e\|_2 + l_{\beta x1}\|x_r\|_2 \leq \bar{l}_{\beta x0} + l_{\beta x1}\|e\|_2$ and $\|\beta_x(x(t))\|_2^2 \leq \bar{l}_{\beta x0}^2 + 2\bar{l}_{\beta x0}l_{\beta x1}\|e\|_2 + l_{\beta x1}^2\|e\|_2^2$, it follows that

$$\frac{\gamma}{2}\left\|(B^TB)^{\frac{1}{2}}W_2^T\beta_x(x(t))\right\|_2^2 \leq \frac{\gamma}{2}\left\|(B^TB)^{\frac{1}{2}}\right\|_F^2\omega^2\left(\bar{l}_{\beta x0}^2 + 2\bar{l}_{\beta x0}l_{\beta x1}\|e\|_2 + l_{\beta x1}^2\|e\|_2^2\right) \tag{4.62}$$

Substituting (4.62) into (4.61) yields,

$$\begin{aligned}\dot{V}\left(e(t),\widetilde{W}_1(t),\widetilde{W}_2(t)\right) &\leq -c_1\|e(t)\|_2^2 + c_2\|e(t)\|_2 + c_3 \\ &= -\left[\sqrt{c_1}\|e(t)\|_2 - \frac{c_2}{2\sqrt{c_1}}\right]^2 + \frac{c_2^2}{4c_1} + c_3 + \epsilon_{b1}\,, \qquad t \geq 0\end{aligned} \tag{4.63}$$

Let v be given as,

$$v \triangleq \left[\frac{c_2^2}{4c_1^2} + \frac{c_3}{c_1}\right]^{\frac{1}{2}} + \frac{c_2}{2c_1} \tag{4.64}$$

where, $c_1 \triangleq \lambda_{min}(R) - \frac{1}{2}\gamma\left\|(B^TB)^{\frac{1}{2}}\right\|_F^2\omega^2 l_{\beta x1}^2 > 0$, $c_2 \triangleq \gamma\left\|(B^TB)^{\frac{1}{2}}\right\|_F^2\omega^2\bar{l}_{\beta x0}l_{\beta x1}$, $c_3 \triangleq \frac{1}{2}\gamma\left\|(B^TB)^{\frac{1}{2}}\right\|_F^2\omega^2 l_{\beta x0}^2$, $\bar{l}_{\beta x0} \triangleq l_{\beta x0} + l_{\beta x1}x_r^*$, and $\|x_r(t)\| \leq x_r^*$, $t \geq 0$. and recall that $\left\|\widetilde{W}_2(t)\right\|_F \leq \widetilde{\omega}_{max}, t \geq 0$. Now, for $\|e\|_2 \geq v$, it follows that $\dot{V}\left(e(t),\widetilde{W}_1(t),\widetilde{W}_2(t)\right) \leq 0$ for all $\left(e(t),\widetilde{W}_1(t),\widetilde{W}_2(t)\right) \in D_e \backslash D_r$ and $t \geq 0$, where

$$D_e \triangleq \left\{\left(e,\widetilde{W}_1(t),\widetilde{W}_2(t)\right) \in R^n\ x\ R^{pxm}\ xR^{sxm}\right\} : x \in R^n \tag{4.65}$$

$$D_r \triangleq \left\{\left(e,\widetilde{W}_1(t),\widetilde{W}_2(t)\right) \in R^n\ x\ R^{pxm}\ xR^{sxm}\right\} : \|e\|_2 \leq v \tag{4.66}$$

Finally, define

$D_\alpha \triangleq \left\{\left(e, \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \in R^n \; x \; R^{pxm} \; xR^{sxm}\right\} : V\left(e, \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \leq \alpha$, where α is the maximum value such that $D_\alpha \subseteq D_e$, and define $D_\beta \triangleq \left\{\left(e, \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \in R^n \; x \; R^{pxm} \; xR^{sxm}\right\} : V\left(e, \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \leq \beta$, where $\beta > \hat{\beta}(\mu) = \mu^2 = \lambda_{max}(P)v^2 + \lambda_{max}(\Gamma^{-1})\widetilde{\omega}^2{}_{max}$.

To show ultimate boundedness of the closed loop system (4.44) ($\dot{\widehat{W}}_1$), (4.45) ($\dot{\widehat{W}}_2$), (4.48) ($\dot{e}(t)$), note that $D_\beta \subset D_\alpha$, since the approximation $\Delta(x) = W^T\beta(x)$, $x \in R^n$ holds in R^n. Now, since $\dot{V}\left(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \leq 0, t \geq 0$, for all $(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)) \in D_e \backslash D_r$ and $D_r \subset D_\alpha$, it follows that D_α is positively invariant. Hence, if $(e(0), \widetilde{W}_1(0), \widetilde{W}_2(0)) \in D_\alpha$, then it follows from Corollary 4.4 of (*H. K. Khalil, Nonlinear Systems, Upper Saddle River, NJ: Prentice Hall, 1996*, n.d.) that the solution $\left(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)\right), t \geq 0$, to (4.44) ($\dot{\widehat{W}}_1$), (4.45) ($\dot{\widehat{W}}_2$), (4.48) ($\dot{e}(t)$) is ultimately bounded with respect to $(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t))$ with ultimate bound given by $\hat{\alpha}^{-1}(\beta) = \sqrt{\beta}$, which yields

$$\epsilon > [\lambda_{max}(P)v^2 + \lambda_{max}(\Gamma^{-1})\widetilde{\omega}^2{}_{max}]^{\frac{1}{2}} \tag{4.67}$$

Over the interval $t \in [0, T)$, $\dot{V}\left(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \leq 0$ since $\left(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \in D_e \backslash D_r$. This implies that

$$V\left(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \leq V(e(0), \widetilde{W}_1(0), \widetilde{W}_2(0)) \quad t \in [0, \mathrm{T}) \tag{4.68}$$

Using the inequalities $\lambda_{min}(P)\|e\|_2^2 \leq V\left(e, \widetilde{W}_1, \widetilde{W}_2\right)$ and $V\left(e(0), \widetilde{W}_1(0), \widetilde{W}_2(0)\right) = tr\widetilde{W}^T(0)\Gamma^{-1}\widetilde{W}(0) \leq \|\Gamma^{-1}\|_F \left\|\widetilde{W}(0)\right\|_F^2$ in (4.68) gives

$$\left\|\widetilde{W}(t)\right\|_F \leq \left\|\widetilde{W}(0)\right\|_F \left[\frac{\|\Gamma^{-1}\|_F}{\lambda_{min}(\Gamma^{-1})}\right]^{\frac{1}{2}} \tag{4.69}$$

This completes the proof.

To sum up,

$$\dot{V}\left(e(t), \widetilde{W}_1(t), \widetilde{W}_2(t)\right) \leq 0, t \geq 0 \tag{4.70}$$

which guarantees that the system error state vector $e(t)$ and the weight error $\widetilde{W}$(t) are Lyapunov stable, and hence are bounded for all $t \in R_+$. Since $\beta(x(t))$ is bounded for all $t \in R_+$, it follows from (4.48) that $\dot{e}$(t) is bounded, and hence $\ddot{V}(e(t), \widetilde{W}(t))$ is bounded for all $t \in R_+$. Now, it follows from Barbalat's lemma that,

$$\lim_{t \to \infty} \dot{V}\ (e(t), \widetilde{W}(\mathrm{t})) = 0, \tag{4.71}$$

which consequently shows that $e(t) \to 0$ as $t \ \to \infty$.

For the case when the nonlinear uncertain dynamical system given by (4.37) includes bounded exogenous disturbances, measurement noise, and/or the uncertainty in (4.37) cannot be perfectly parameterized, then Assumption 1 can be relaxed by considering

$$\delta(t, x) = W(t)^T \beta(x(t)) + \epsilon(t, x), \ \ x \ \in \ D_x \tag{4.72}$$

where $W(t) \in R^{sxm}$ is an unknown time-varying weight matrix satisfying $\|W(t)\|_F \leq \omega$ and $\|\dot{W}(t)\|_F \leq \dot{\omega}$ with $\omega \in \mathrm{R}_+$ and $\dot{\omega} \in \mathrm{R}_+$ being unknown scalars, $\beta: D_x \to R^s$ is a known basis function of the form $\beta(x) = \left[1, \beta_1(x), \beta_2(x), \ldots, \beta_{(s-1)}(x)\right]^T$, $\epsilon: R_+ \, x \, D_x \ \to R^m$ is the system modeling error satisfying $\|\epsilon(t, x)\|_2 \leq \epsilon$ with $\epsilon \in R_+$ being an unknown scalar, and D_x is a compact subset of R^n. In this case, the weight update law given by (4.44) can be replaced by

$$\dot{\widehat{W}}(t) = \Gamma Proj\left[\widehat{W}(t), \beta\big(x(t)\big)e^T(t)PB\right]$$
$$\widehat{W}(0) = \widehat{W}_0, \qquad t \epsilon\, R_+ \tag{4.73}$$

with $\Gamma = \gamma I_s,\ \ \gamma\ \in\ R_+,$ to guarantee the uniform boundedness of the system error state vector $e(t)$ and the weight error $\widetilde{W}(t)$, where $Proj$ denotes the projection operator.

Remark 1. Even though the above analysis shows that the state vector $x(t)$ asymptotically converges to the reference state vector $x_r(t)$ (in steady-state), $x(t)$ can be far different from $x_r(t)$ in transient time. High-gain learning rates can be used in (4.44) in order to achieve fast adaptation and to minimize the distance between $x(t)$ and $x_r(t)$ in transient time. However, update law with high-gain learning rates possibly yield to high-frequency oscillations especially during the transient system response resulting in system instability for real applications.

CHAPTER 5

SIMULATION RESULTS

In this thesis, it is proposed that in model reference adaptive control, each uncertainty parametrization should be constructed in terms of the uncertainty parametrization type that it is aimed to predict. In other words, the time dependent uncertainty should be predicted with the time dependent parametrized adaptive element, and the state dependent uncertainty should be predicted also, with the state dependent parametrized adaptive element. Then, the summation of these elements should be formed to be the adaptive control input of the MRAC. To reach this result, MFC, MRAC, CPBMRAC, CP TD CCAL MRAC, FSBMRAC, FS TD CCAL MRAC, FS TSD CCAL MRAC are studied. The system model given in 2.1, is presumed to be subjected to the reference command input given in Figure 2.3, and under the effect of external disturbance which is random given in Figure 2.5, and wing rock dynamics of which is given in (2.4), and noise is added to the roll turn rate as given in Figure 2.2. The results of the simulations are given in this section, sequentially. In 5.1, comparison of MRAC and FS TSD CCAL MRAC, in 5.2 comparison of FSBMRAC; and FS TSD CCAL MRAC, in 5.3 comparison of FS TD CCAL MRAC and FS TSD CCAL MRAC, in 5.4 comparison of all controllers, in 5.5 max disturbance elimination, in subparts 5.5.1 FS TD CCAL MRAC in case of multiplication term of external disturbance and wing rock dynamics is 2, in 5.5.2 FS TSD CCAL MRAC in case of multiplication term of external disturbance and wing rock dynamics is 2, and in 5.5.3 FS TSD CCAL MRAC in case of multiplication term of external disturbance and wing rock dynamics is 100 is given.

5.1. Comparison of MRAC and FS TSD CCAL MRAC

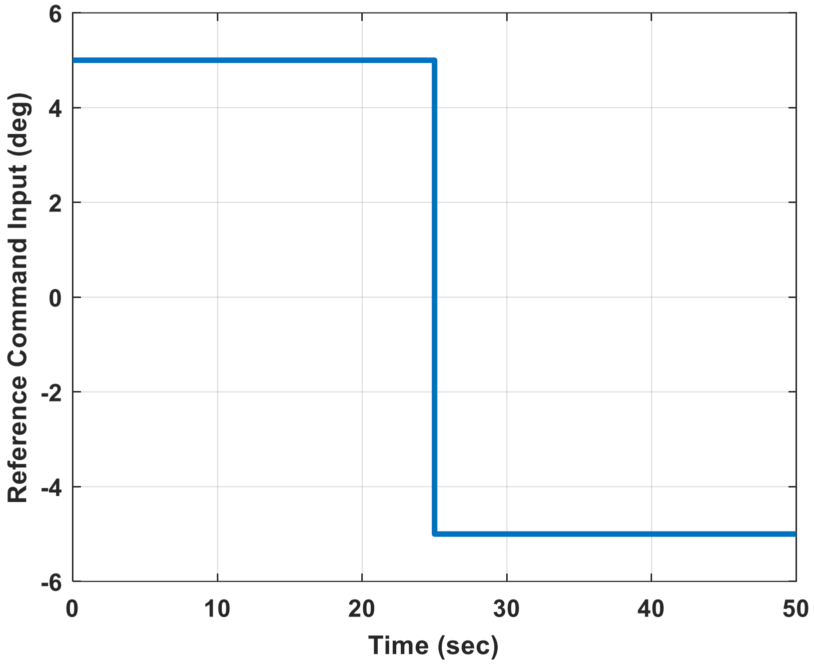

Figure 5.1. Reference Command Input vs time

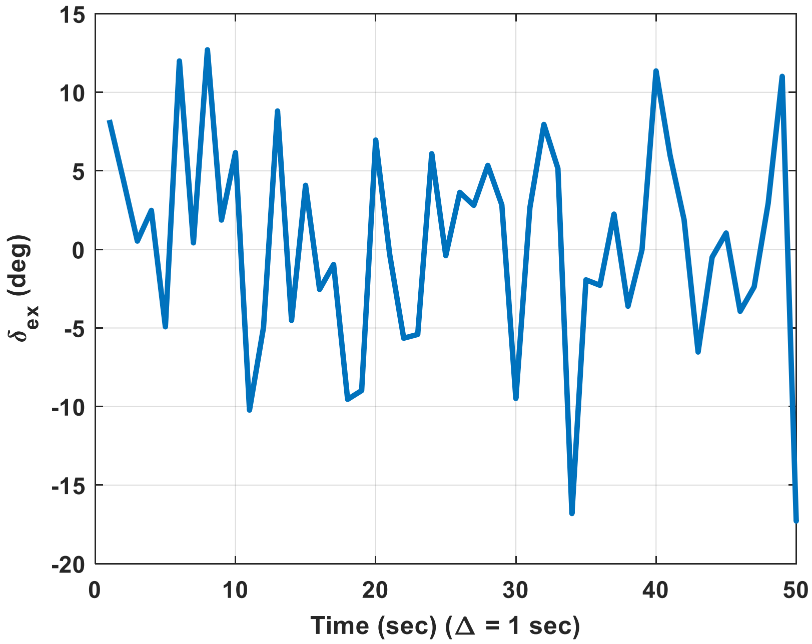

Figure 5.2. External Disturbance vs time

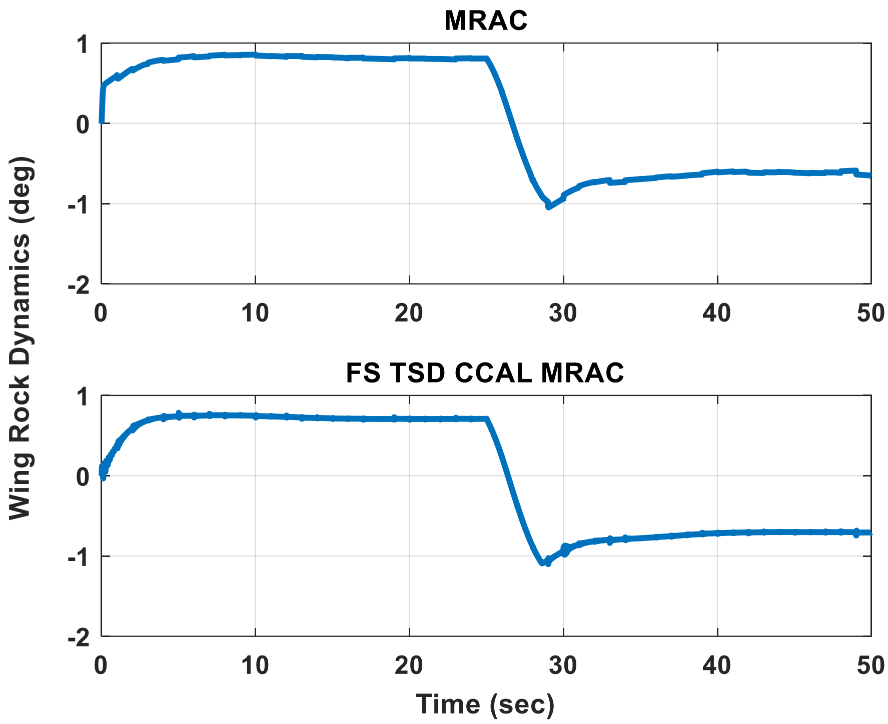

Figure 5.3. Wing Rock Disturbance vs time

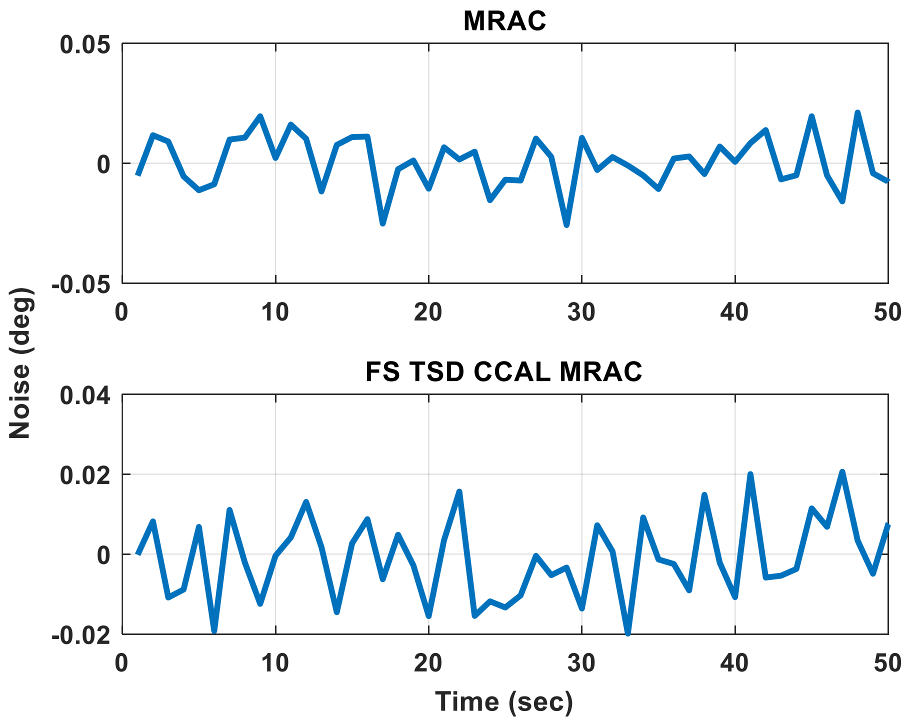

Figure 5.4. Noise vs time

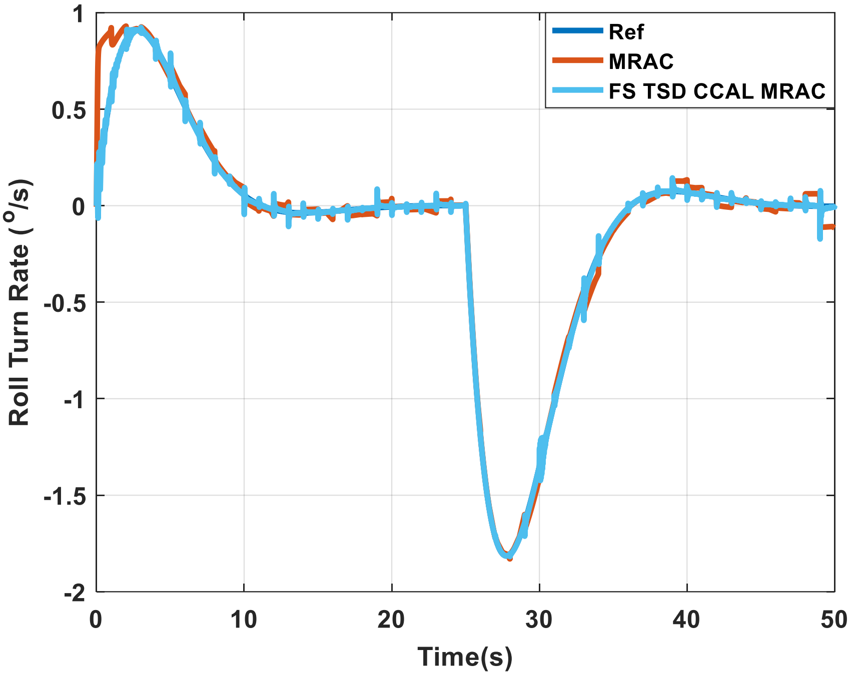

Figure 5.5.Roll Turn Rate vs time

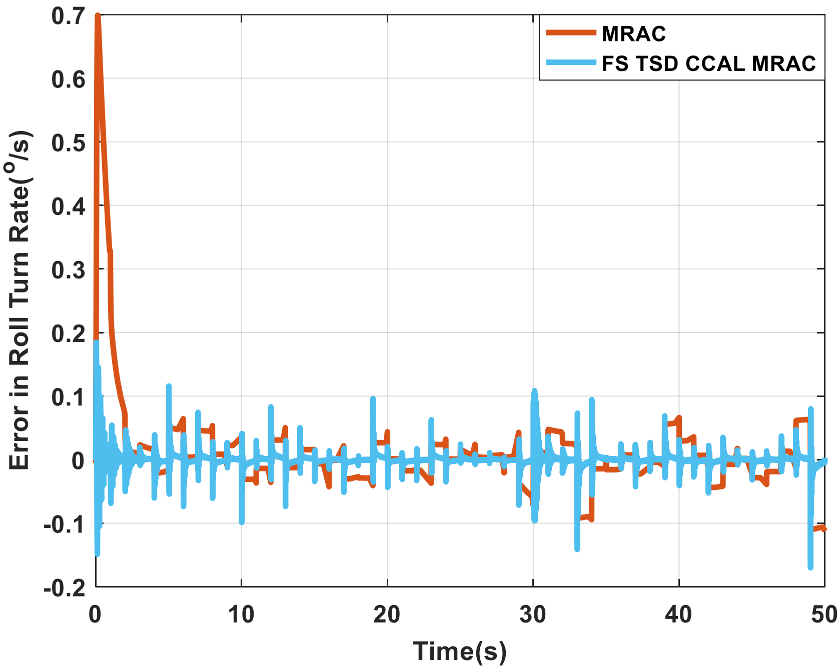

Figure 5.6. Error in Roll Turn Rate vs time

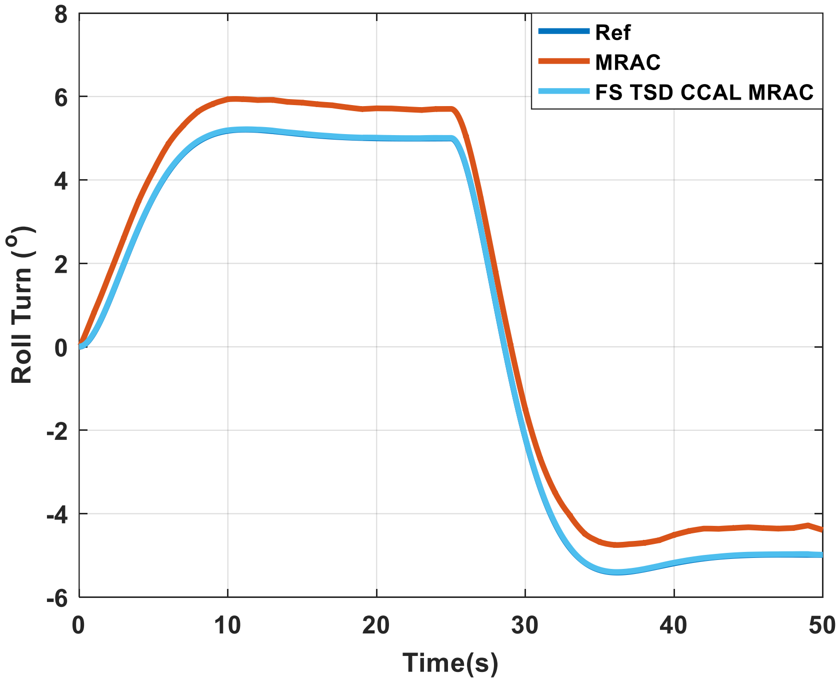

Figure 5.7. Roll Turn vs time

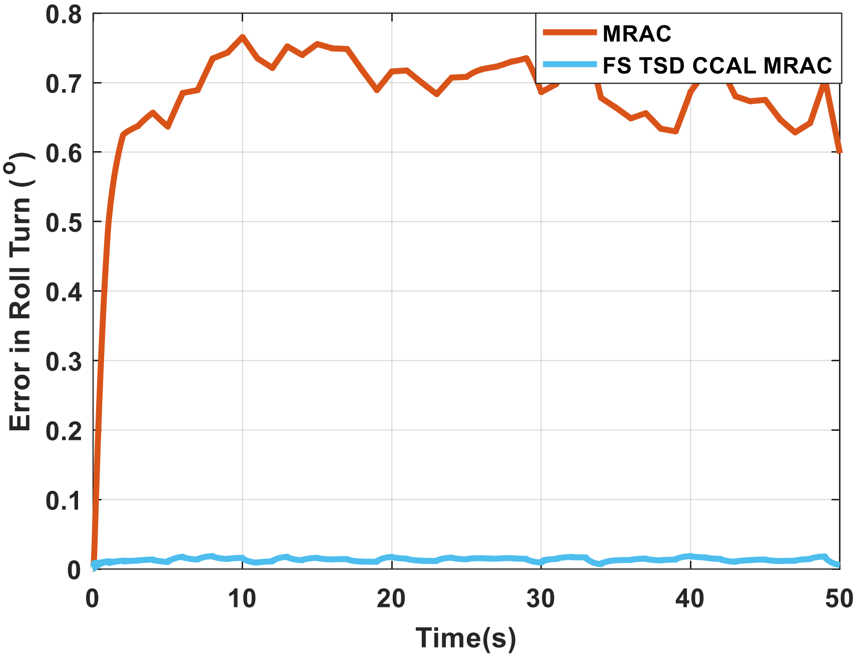

Figure 5.8 Error in Roll Turn vs time

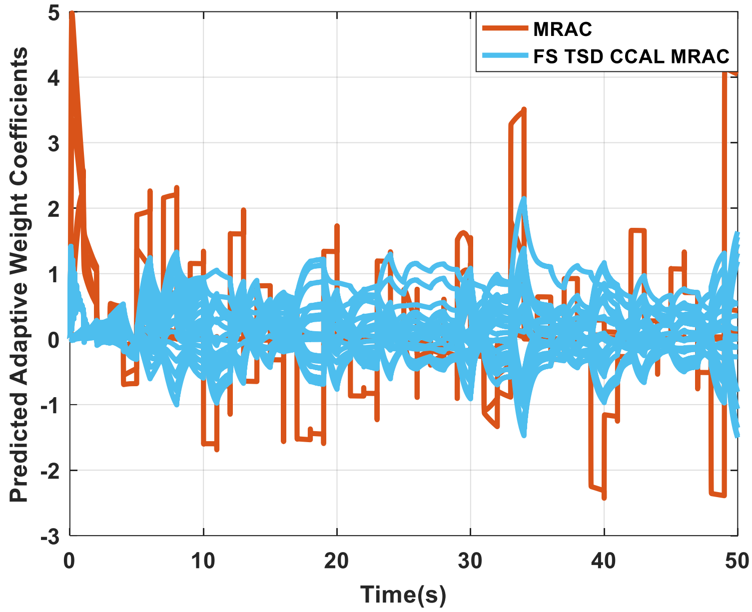

Figure 5.9. Predicted Adaptive Weight Coefficients vs time

In this section comparison of MRAC and FS TD CCAL MRAC is done. As seen from Figure 5.5, and Figure 5.6, roll turn rate, the response of the state variable x_1 of the system model, slender delta wing, of which model equation is given in 2.1, subjected to the reference command input given in Figure 2.3, and under the disturbance conditions given in Figure 2.2, is drawn. It is seen that MRAC gives some fluctuated results compared to the FS TD CCAL MRAC, while closing the reference model response. This could be due to its lack of prediction capability of the time dependent random external disturbance, due to the state dependent uncertainty parametrization structure of the MRAC. As seen from Figure 5.7, and Figure 5.8, roll turn, the reply of the state variable x_2 of the system model, is drawn. The shift in the response of the MRAC from the reference model is more obvious in these figures, which is an indication that the roll turn state x_2 is more prone to the disturbances, since it is the direct integral of the x_1, as given in (2.1), the disturbance effects are accumulated. As seen in Figure 5.9, the predicted adaptive weights stay in a more compact region, in FS TD CCAL MRAC than the MRAC, which is an indication of better uncertainty parametrization.

5.2. Comparison of FSBMRAC and FS TSD CCAL MRAC

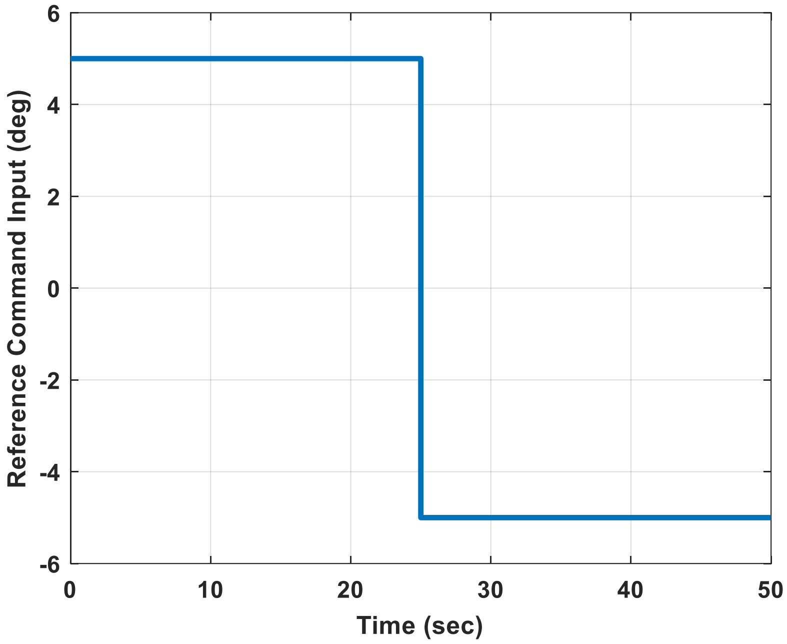

Figure 5.10. Reference Command Input vs time

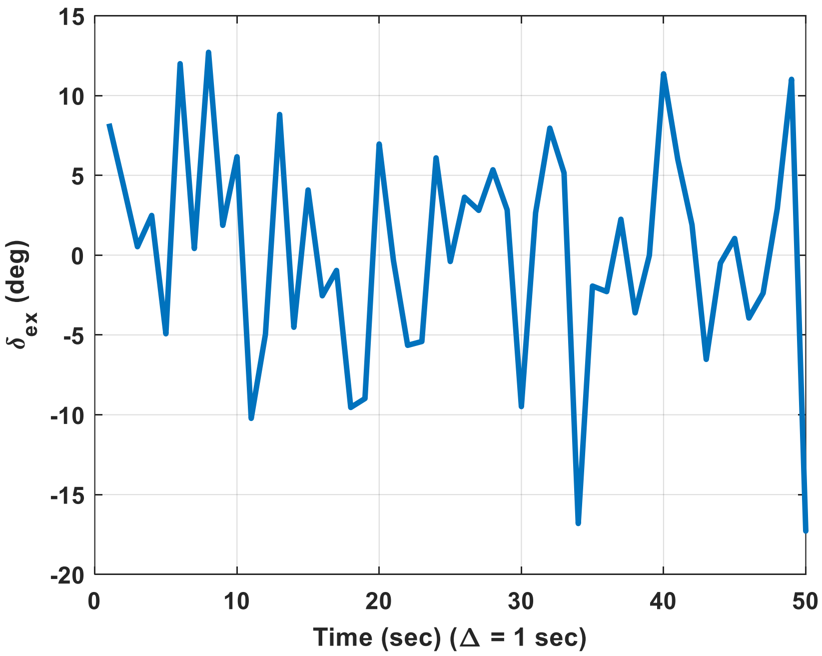

Figure 5.11. External Disturbance vs time

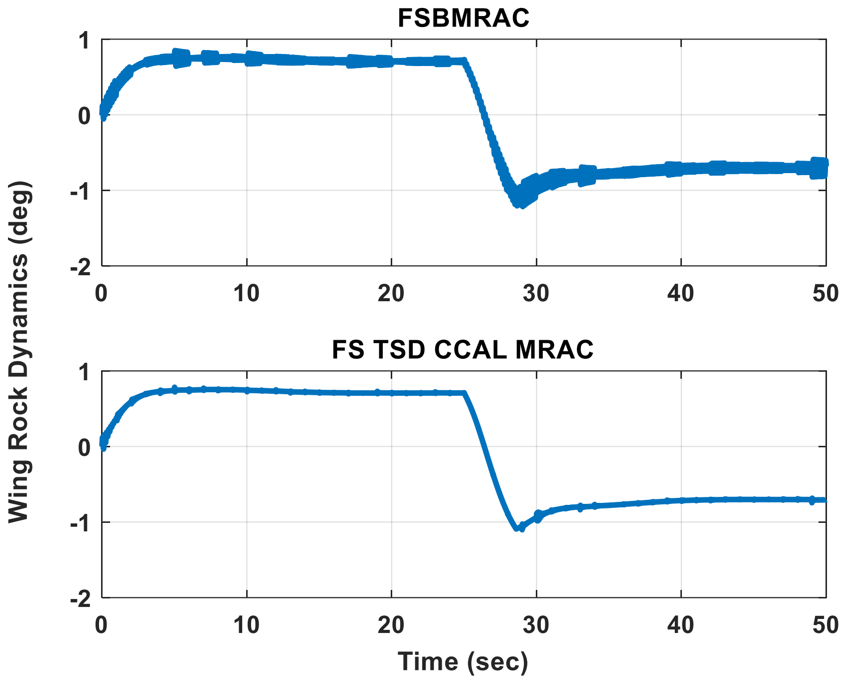

Figure 5.12. Wing Rock Dynamics vs time

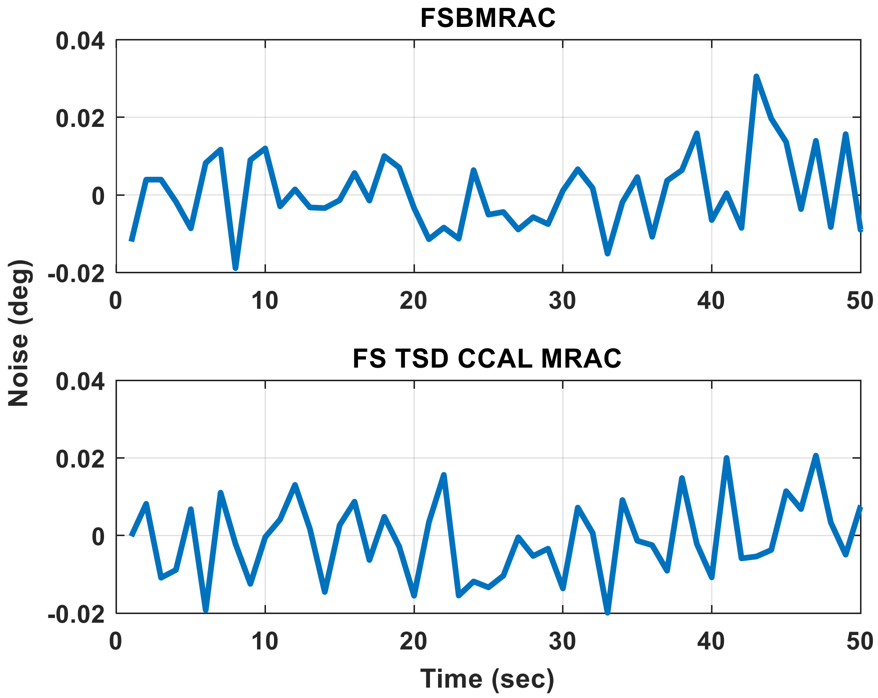

Figure 5.13. Noise vs time

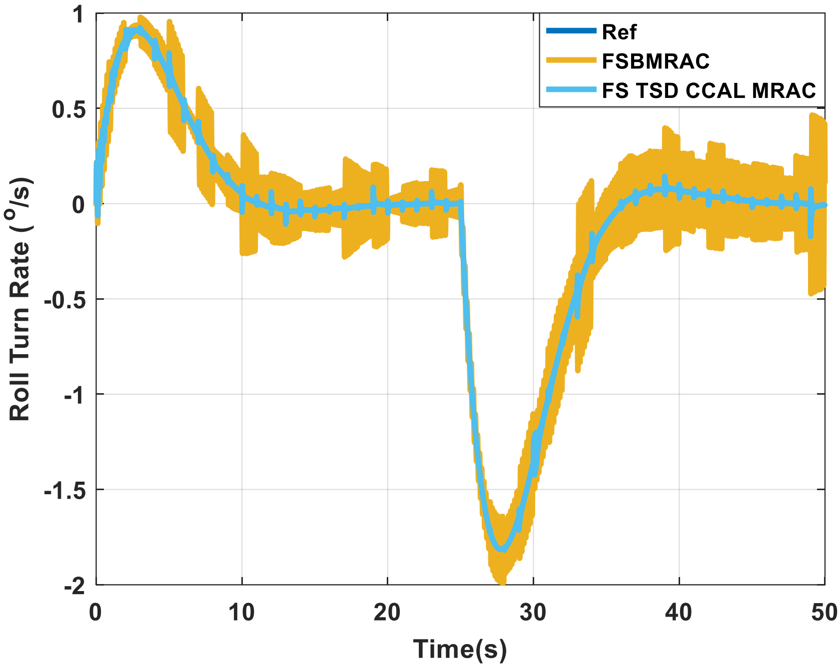

Figure 5.14. Roll Turn Rate vs time

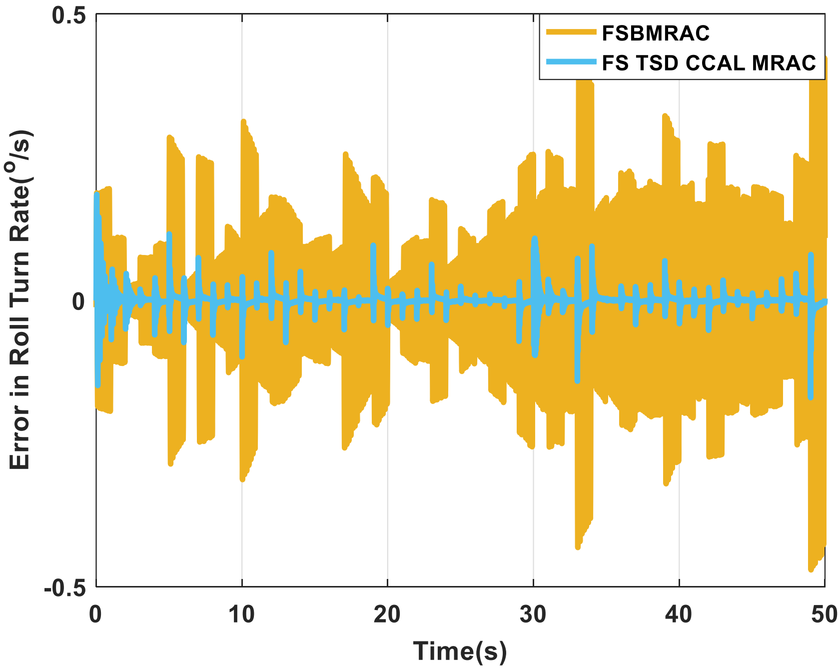

Figure 5.15. Error in Roll Turn Rate vs time

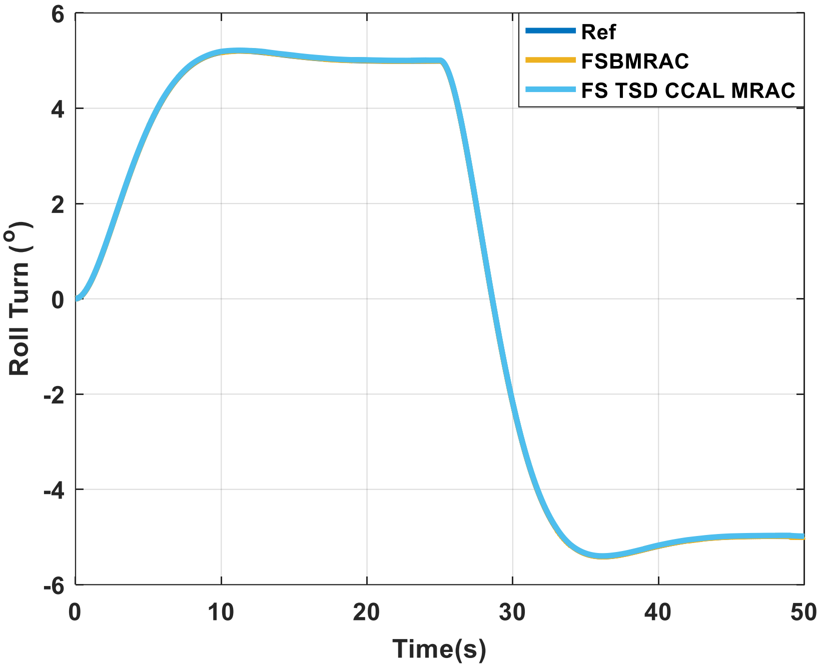

Figure 5.16. Roll Turn vs time

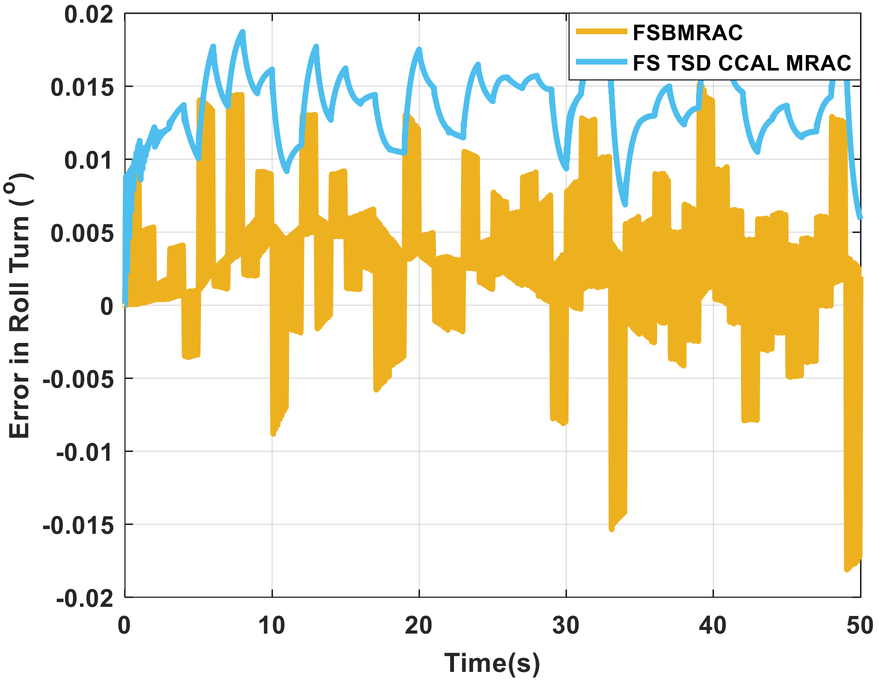

Figure 5.17. Error in Roll Turn vs time

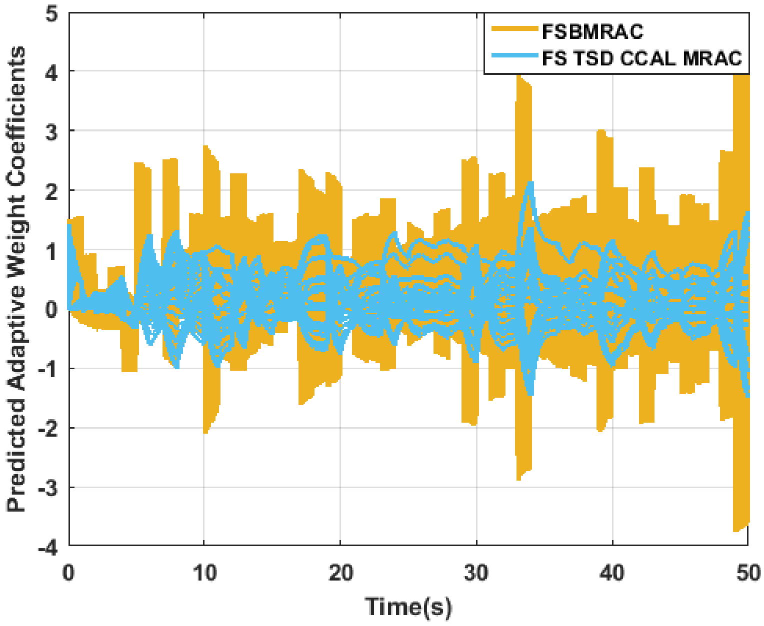

Figure 5.18. Predicted Adaptive Weight Coefficients vs time

In this section comparison of FSBMRAC and FS TSD CCAL MRAC is done. As seen from Figure 5.14, and Figure 5.15, that, roll turn rate, the response of the state variable x_1 of the system model, is drawn. Though the higher uncertainty capability of the FSBMRAC, due to the periodic structure of the Fourier series, the FS TSD CCAL MRAC, gives close reference model tracking than the FSBMRAC. The periodic structure of the Fourier series, could eliminate the need for the concurrent adaptive learning, as could be seen in Figure 4.17, such that the persistent excitation of the learning mechanism of the adaptive control could show similar results with the concurrent adaptive learning though its valuable data storage capability which could enhance uncertainty prediction. Though the superior capability of the FSBMRAC, as seen from Figure 5.16, and Figure 5.17, the slight difference between the results of FSBMRAC and FS TSD CCAL MRAC, could be explained as the lack of prediction capability of the state dependent structured uncertainty the wing rock dynamics, due to the solely time dependent uncertainty parametrization structure of the FSBMRAC. As seen from Figure 5.18, the predicted adaptive weights stay in a more compact region, in FS TSD CCAL MRAC than the FSBMARC, which is an indication of better uncertainty parametrization.

5.3. Comparison of FS TD CCAL MRAC and FS TSD CCAL MRAC

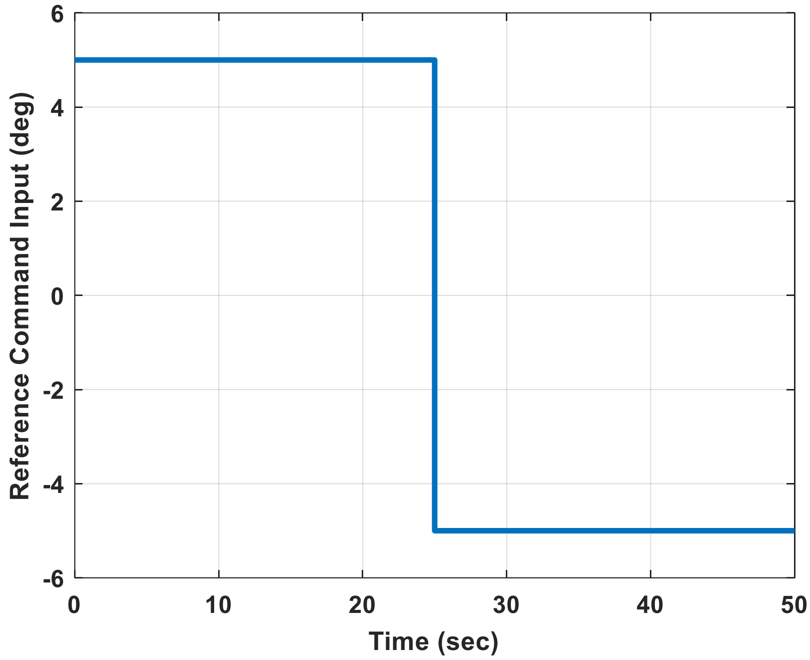

Figure 5.19. Reference Command vs time

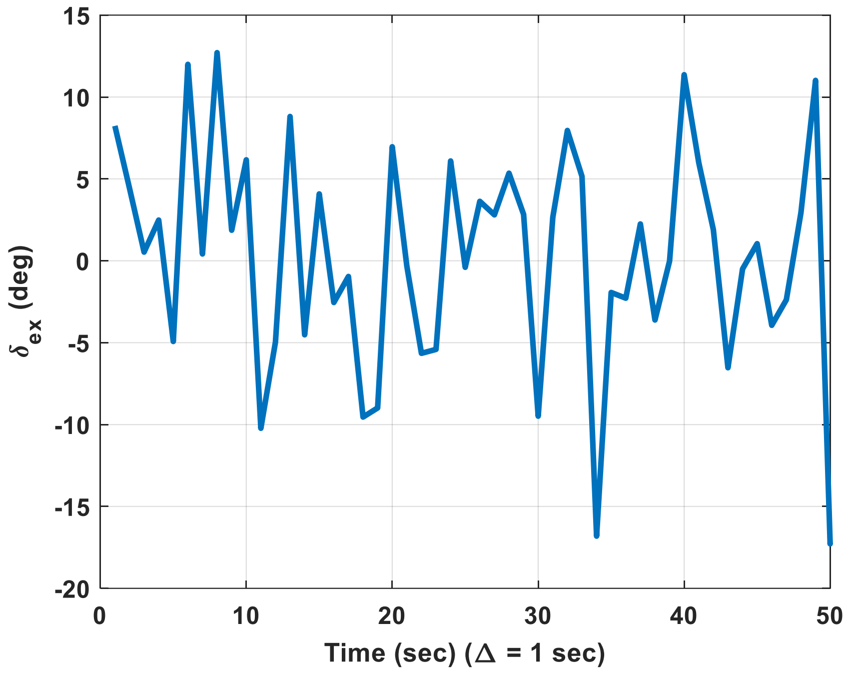

Figure 5.20. External Disturbance vs time

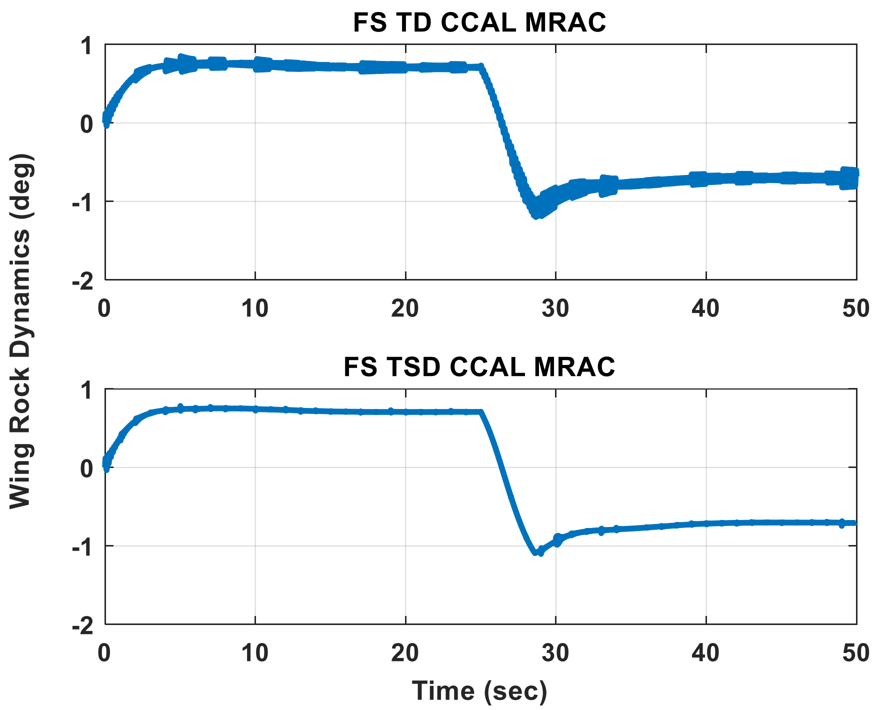

Figure 5.21. Wing Rock Dynamics vs time

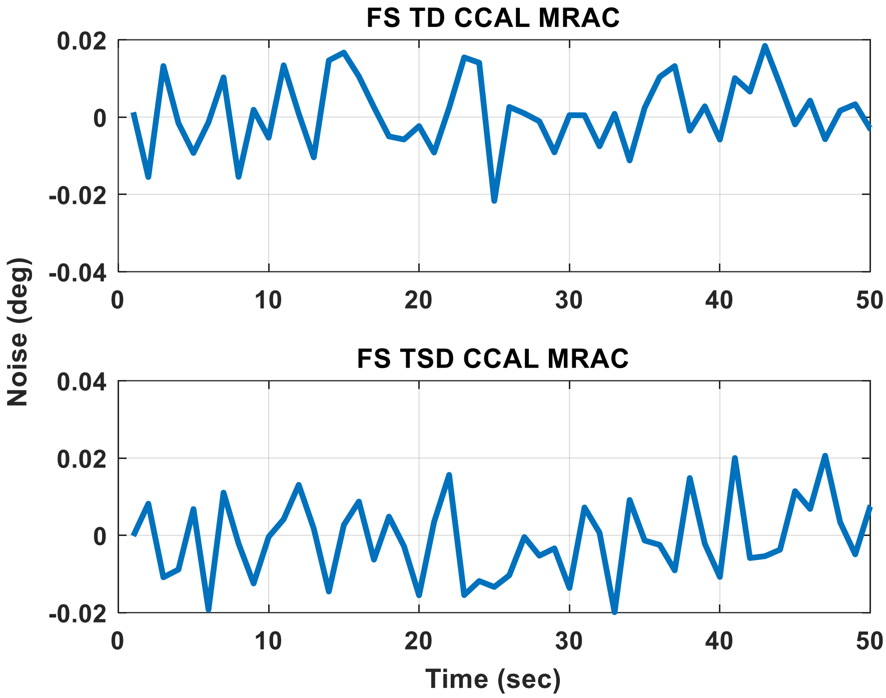

Figure 5.22. Noise vs time

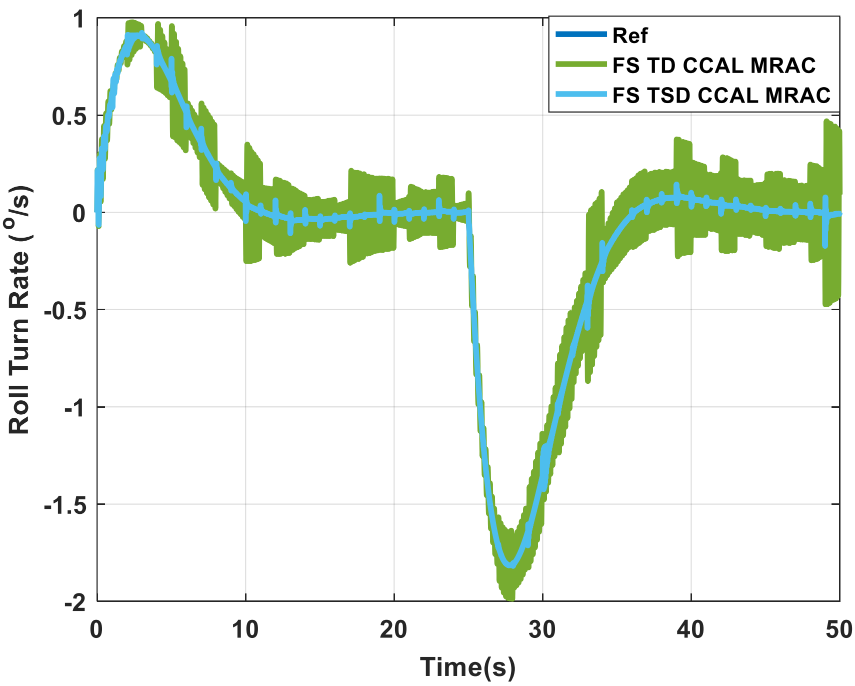

Figure 5.23. Roll Turn Rate vs time

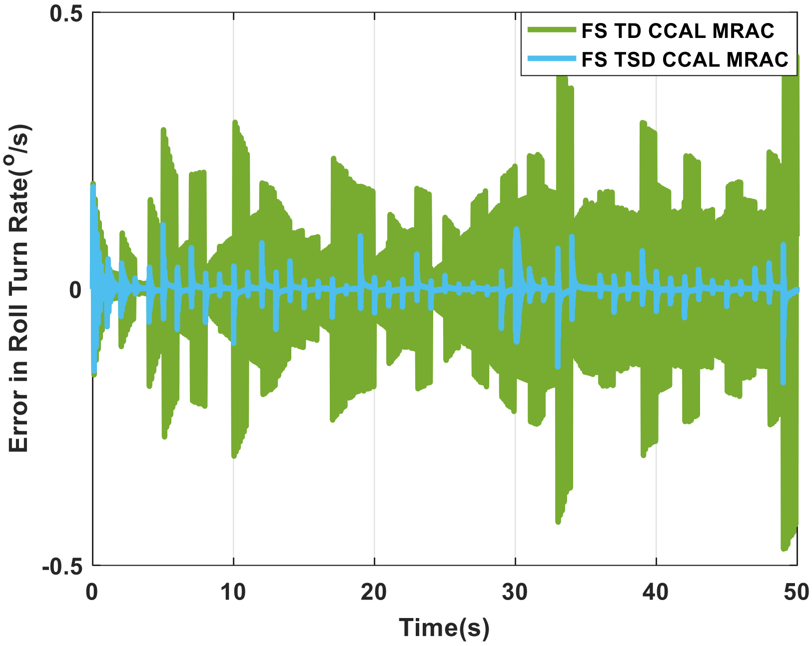

Figure 5.24. Error in Roll Turn Rate vs time

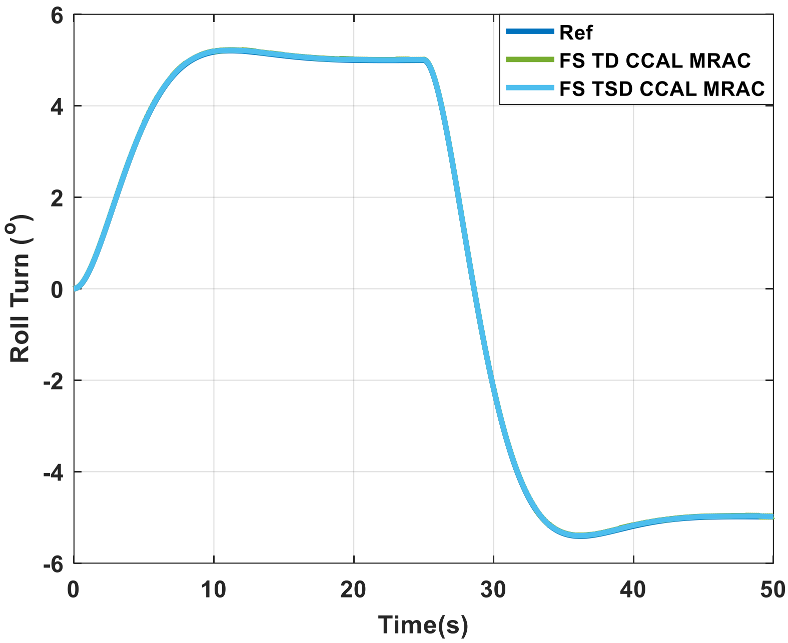

Figure 5.25. Roll turn vs time

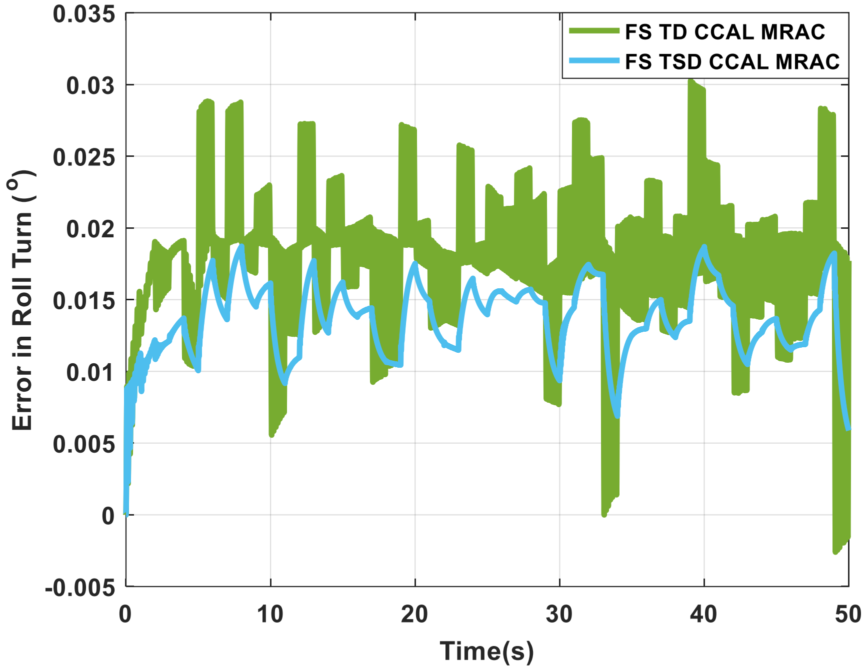

Figure 5.26. Error in Roll Turn vs time

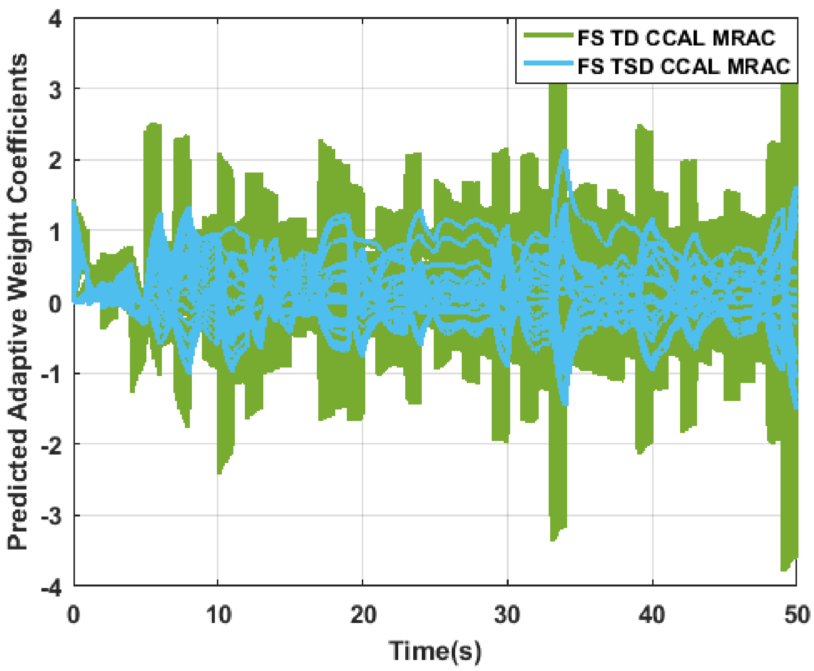

Figure 5.27. Predicted Adaptive Weight Coefficients

In this section comparison of FS TD CCAL MRAC and FS TSD CCAL MRAC is done. Figure 5.23, and Figure 5.24, shows similar pattern as the results shown in Figure 5.14, and Figure 5.15, which are the responses of the roll turn rate, x_1, of the system model. Similarly, Figure 5.25 and Figure 5.26. , shows similar pattern as the results shown in Figure 5.16, and Figure 5.17, which are the responses of the roll turn, x_2, of the system model. The progress in uncertainty prediction success, through from the FSBMRAC to FS TSD CCAL MRAC, is similar to, through from the FS TD CCAL MRAC to FS TSD CCAL MRAC. As seen in Figure 4.17, FSBMRAC gives similar results to the FS TD CCAL MRAC, due to its periodic structure. Though, from the figures through Figure 5.23 to Figure 5.27, FS TSD CCAL MRAC gives closer reference model tracking than the FS TD CCAL MRAC, since it could predict both state dependent structured and time dependent random unstructured external disturbance, due to having uncertainty parametrization structure for both of them separately. As seen in Figure 5.27, the predicted adaptive weights stay in a more compact region, in FS TSD CCAL MRAC than the FS TD CCAL MRAC, which is an indication of better uncertainty parametrization.

5.4. Comparison of all Controllers

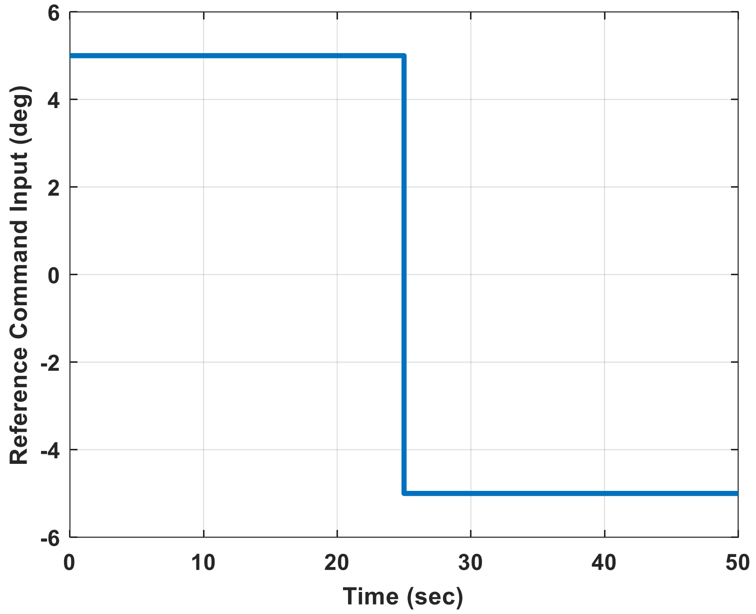

Figure 5.28. Reference Command Input vs time

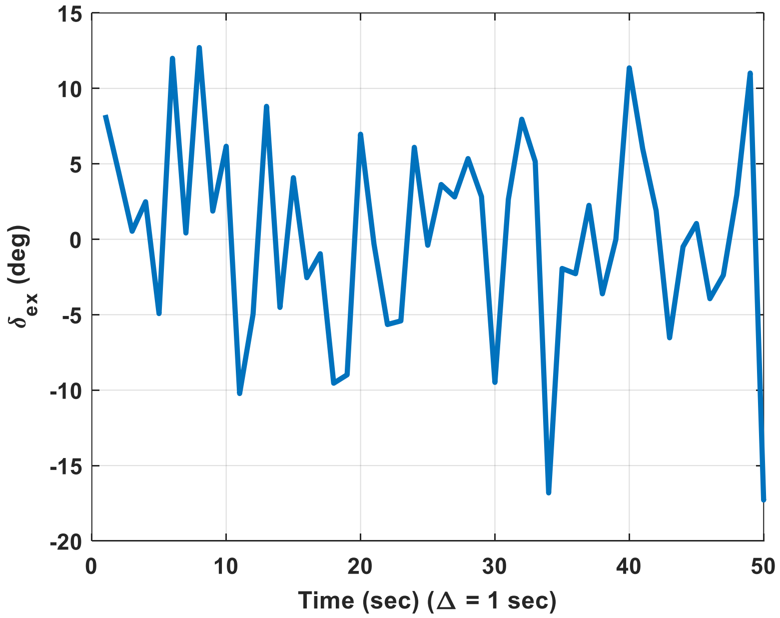

Figure 5.29. External Disturbance vs time

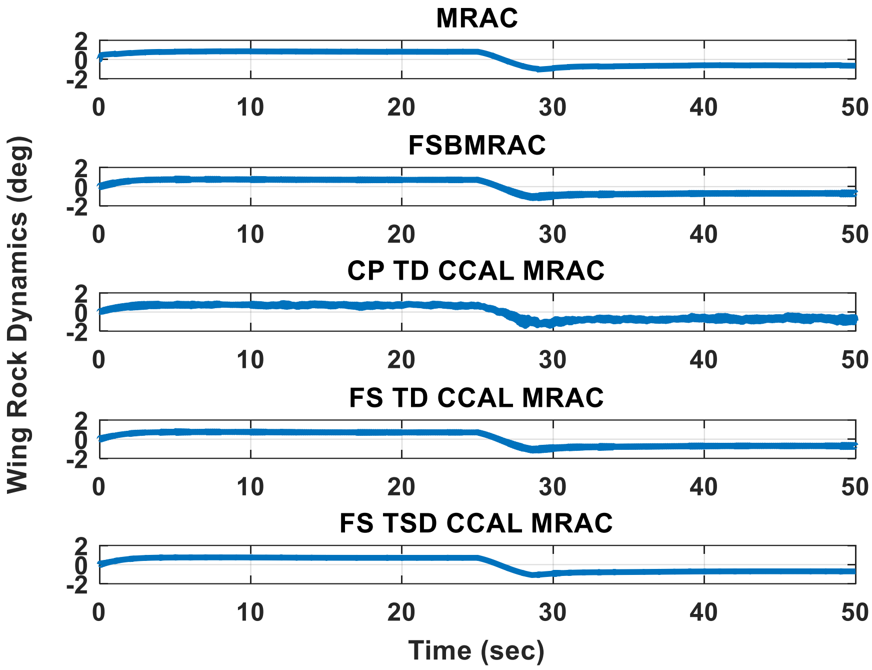

Figure 5.30. Wing Rock Dynamics vs time

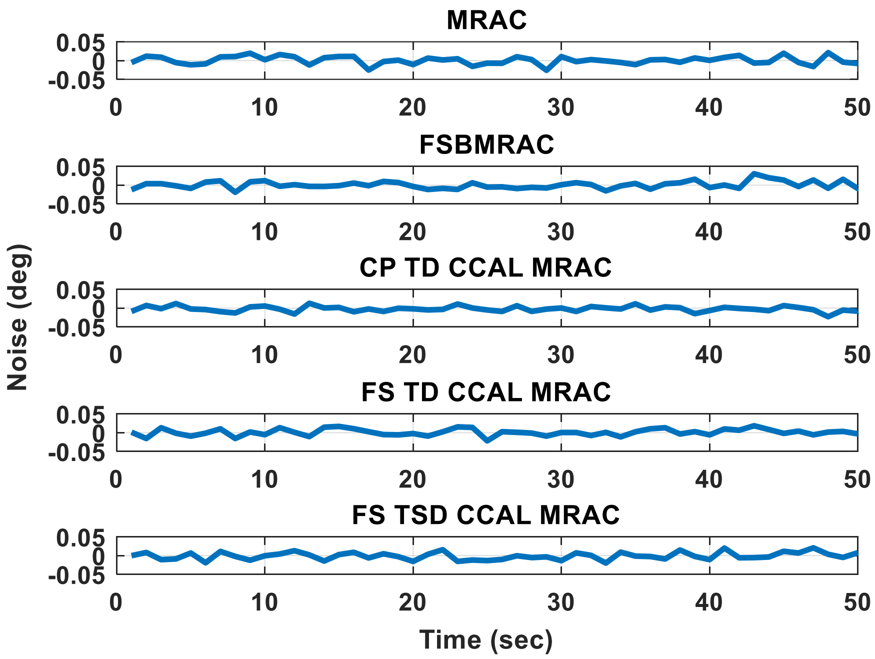

Figure 5.31. Noise vs time

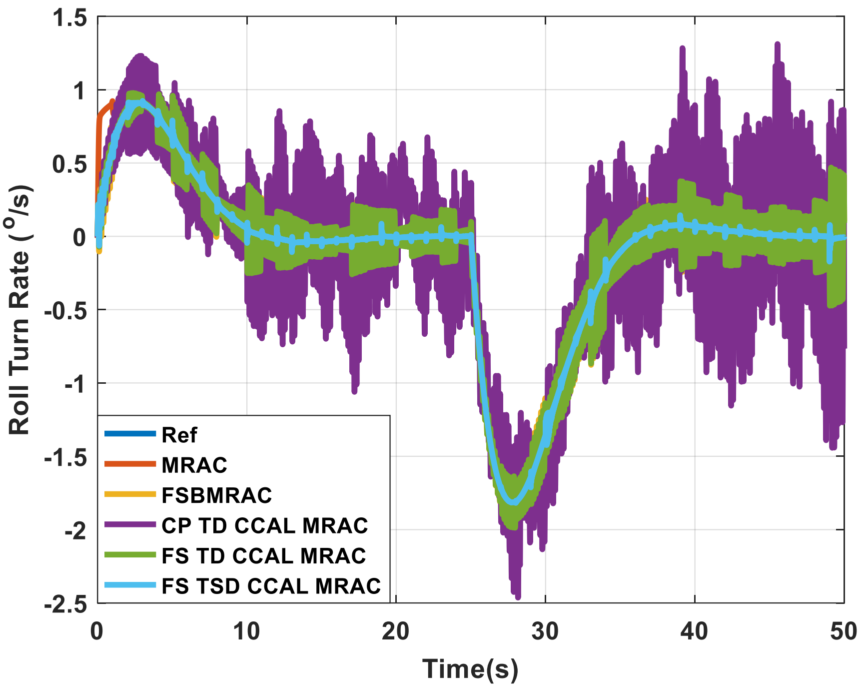

Figure 5.32. Roll Turn Rate vs time

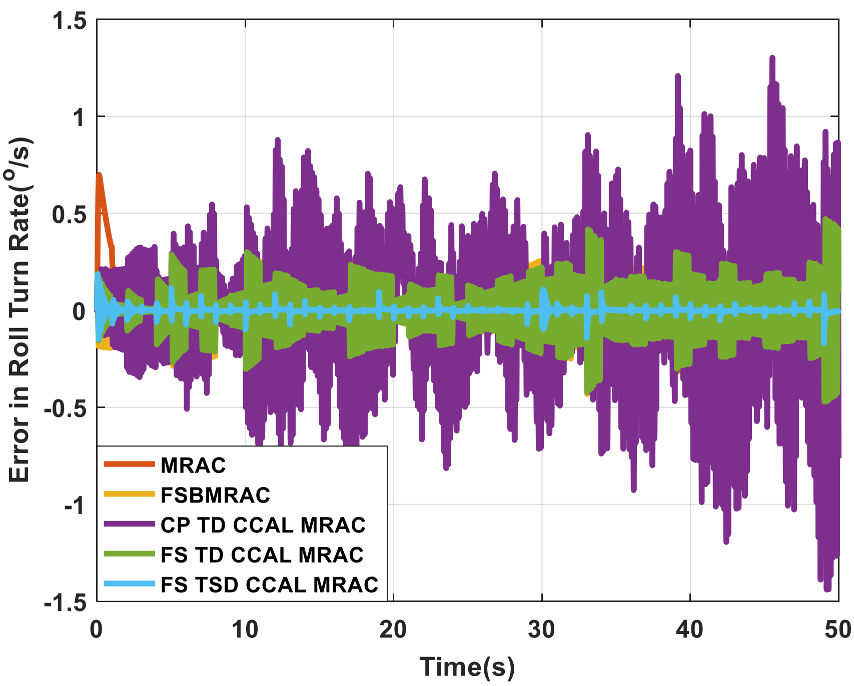

Figure 5.33. Error in Roll Turn Rate

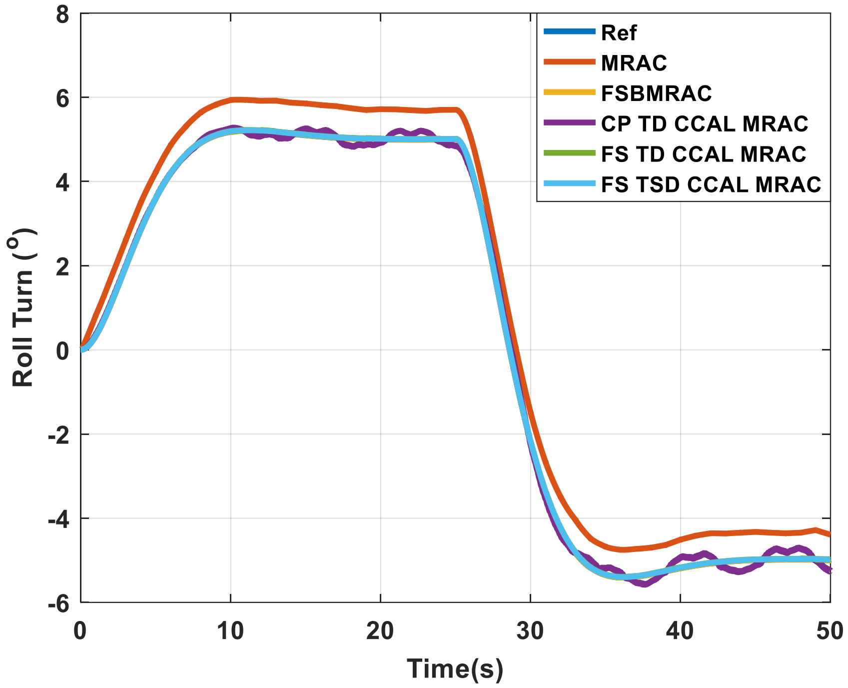

Figure 5.34. Roll Turn vs time

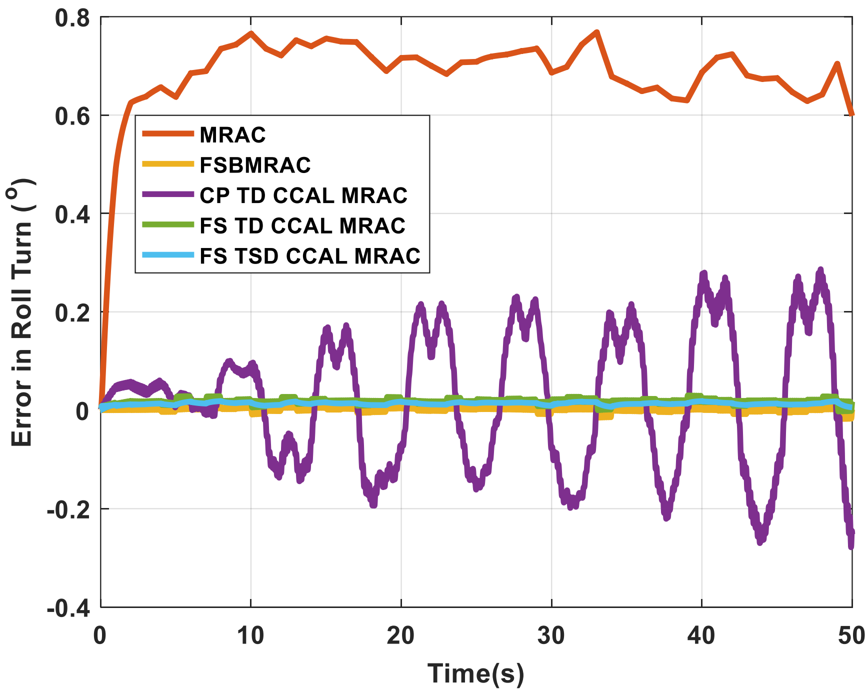

Figure 5.35. Error in Roll Turn vs time

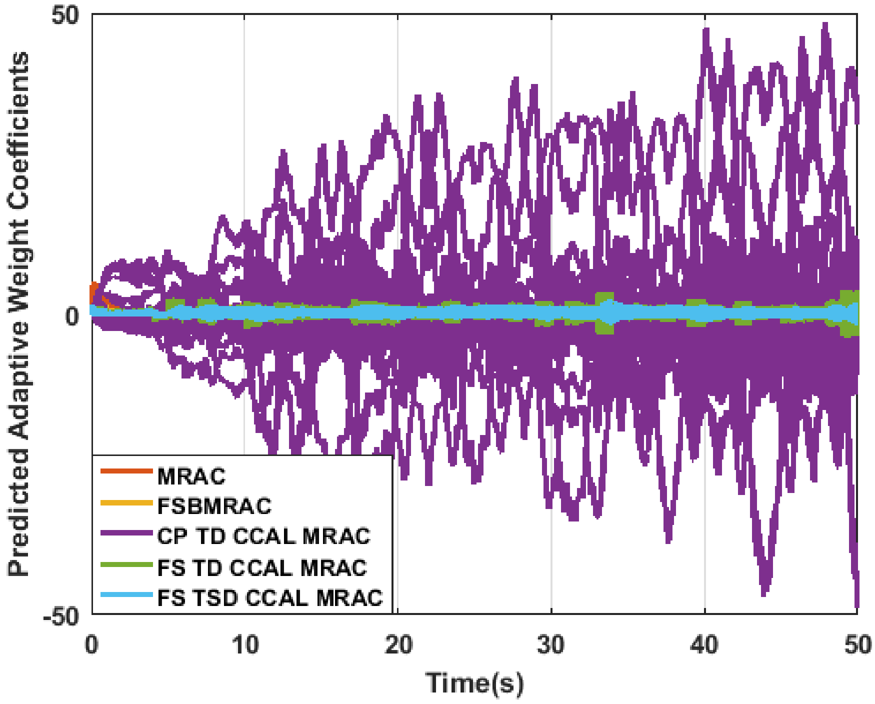

Figure 5.36. Predicted Adaptive Weight Coefficients vs time

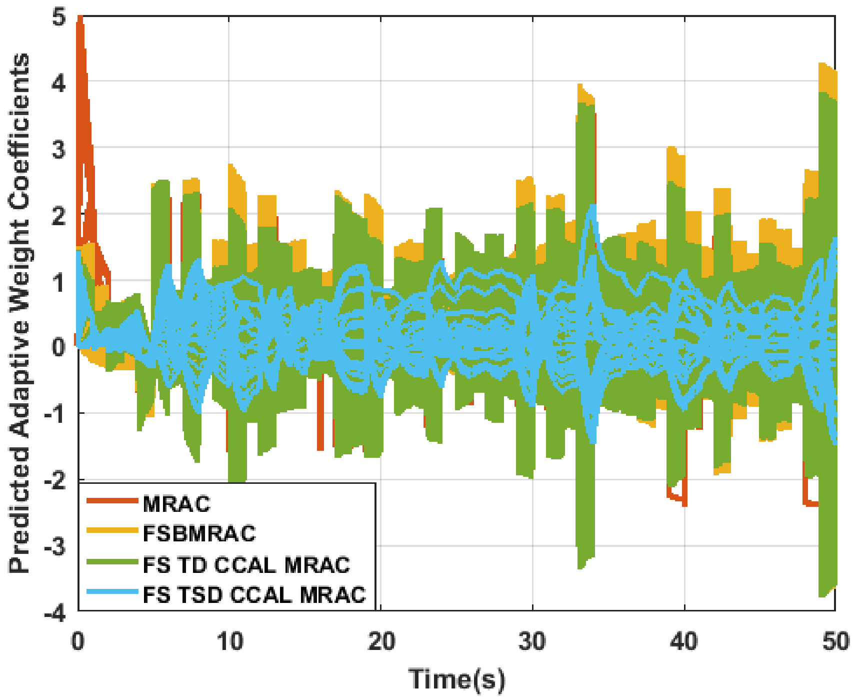

Figure 5.37. Predicted Adaptive Weight Coefficients vs time

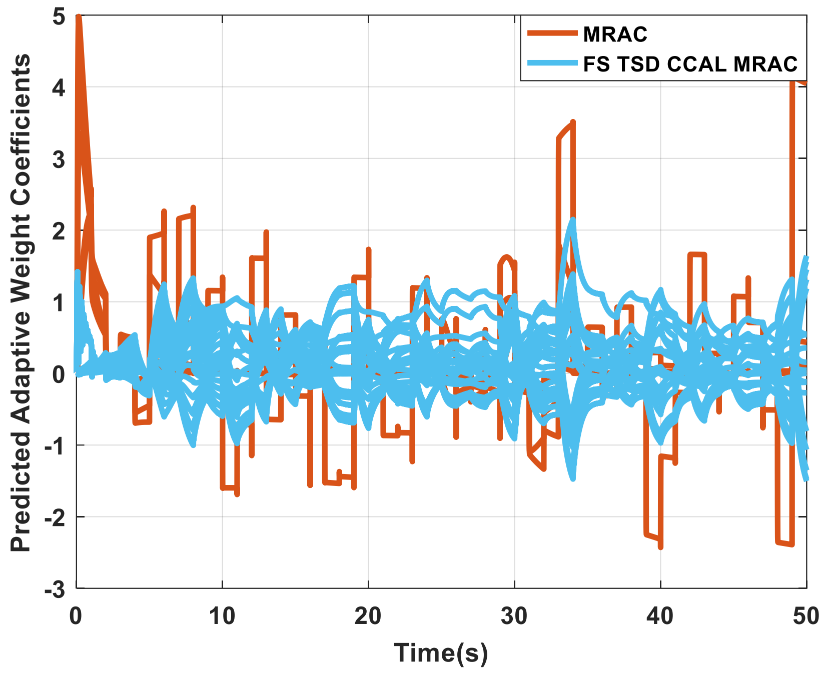

Figure 5.38. Predicted Adaptive Weight Coefficients vs time

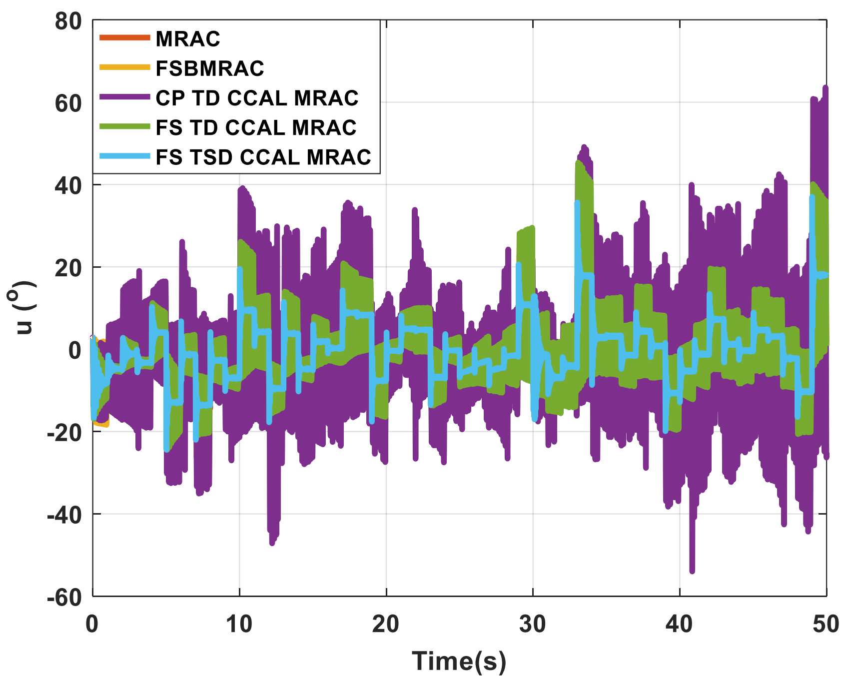

Figure 5.39. Comparison of Control Surface Deflection vs time for all Controllers

It is seen from Figure 5.32, that as proposed in this thesis, the best method at tracking the reference model, is FS TSD CCAL MRAC, since each uncertainty is parametrized in its own dependent variable. By the way, FSBMRAC gives as close results as FS TD CCAL MRAC, as was the case in Figure 4.17. This could be explained such that, concurrent adaptive learning expedites learning of the uncertainty and elimination, by storing valuable data. By the way, there should be similar property in Fourier series characteristic, so that both of them show close tracking performance. In Fourier series based MRAC, the parametrization part of the uncertainty in the adaptive element is time dependent, not dependent on system states. So, the part of the adaptive element that is related with parametrization of uncertainty, which is constructed with the periodic elements, is persistently excited, in order to learn the uncertainty. The learning mechanism of the adaptive control is active, so that it is prone to learn any disturbance acting on the system and eliminate it. Chebyshev polynomial based MRAC, however is not as successful as the Fourier series based MRAC, as seen in Figure 5.32. This could be due to its structure being more suitable for state dependent parameterization, though is preferred in this study, taking the advantage of the series structure of the Chebyshev polynomial, to use in the prediction of unstructured random external time dependent disturbance. CP TD CCAL MRAC shows more fluctuations diverging from the reference model, than the FS TD CCAL MRAC, due to the $\cos(t)$ term used in Chebyshev polynomial parameterization in this study. Since both of them are used as time dependent uncertainty parametrization methods, they are expected to predict especially time dependent external random disturbance. However, it is seen that Fourier series is better than the Chebyshev polynomials in this case. It could also be due to the periodic structure of the Fourier series to be good at time dependent uncertainty prediction. Though the Chebyshev polynomials are constructed with the linearly independent terms, to be good at uncertainty prediction, periodic structure could be better way. Since it is time dependent, and is persistently excited at each time step of the simulation. Figure 5.32 shows that, the MRAC is slightly fluctuating around the reference model, not seen from the figure though. Also, in Figure 5.34 MRAC is shifted around the reference model, compared to the other controllers. It could be due to the reason, that since the uncertainty parametrization part of the MRAC is system state dependent, it is successful at prediction of the state dependent uncertainty, such that the wing rock dynamics. However, it is not good at prediction of time dependent random external disturbance, which is unstructured disturbance case in this study. So, MRAC could predict the structured state dependent uncertainty, due to state dependent uncertainty parametrization part of the MRAC. However, it could not predict the unstructured time dependent random external disturbance, which is the key point of this study. Since the random external disturbance is unstructured and random, it is more realistic and chaotic case for this study than the structured wing rock dynamics. So, this could be the reason why MRAC diverges from the reference model more than the other controllers.

As seen from Figure 5.33, and Figure 5.35, also FS TSD CCAL MRAC, gives the least error, in tracking the reference model response. As seen from Figure 5.36, Figure 5.37, Figure 5.38, the predicted adaptive weights stay in a more compact region, in FS TSD CCAL MRAC than the other controllers, which is an indication of better uncertainty parametrization.

In Figure 5.39, the comparison of control surface deflection for all controllers is given. Since it is better to have a requirement for lesser control surface deflection, resulting in less energy consumption, and delayed material fatigue, it is seen that FS TSD CCAL MRAC gives better control input deflection values than the other controllers.

5.5. Max Disturbance Elimination

In this section, max disturbance elimination capabilities of the controllers are examined. The aim of the study in this section is, to force the limits of the controller system, under the excessive disturbance case. In 5.5.1, the response of the FS TD CCAL MRAC is studied, under the case where, both the dynamics of wing rock and the random external disturbance is multiplied by 2. In 5.5.2, the response of the FS TSD CCAL MRAC is studied, under the case where, both the dynamics of wing rock and the random external disturbance is multiplied by 2. In 5.5.3, the response of the FS TSD CCAL MRAC is studied, under the case where, both the dynamics of wing rock and the random external disturbance is multiplied by 100. Of course, since it is assumed that the uncertainty acting to the system is matched, all the disturbance is entering to the system through the control channel, which means that both the random external disturbance and the wing rock dynamics, could be thought as the increase in the aileron command input to the system. So, they are also in the terms of degrees. In the max disturbance case, speculative disturbances are given upon the system without considering the real physical limits of the control actuator system. The excessive disturbances, could be thought as unexpected excessive nonlinearities, which is hard to occur in real life.

5.5.1. FS TD CCAL MRAC (Disturbation Factor=2)

In this section, the response of the FS TD CCAL MRAC is examined in case of multiplication term of external disturbance and wing rock=2. If random external disturbance acts to the system as given in Figure 5.40, and the wing rock dynamics as given in Figure 5.41, and the noise added to the roll turn rate as given in Figure 5.42, the results of the system response are get as given through the Figure 5.43, to the Figure 5.47. In case of increase in the multiplication term, the system model becomes unstable, and the response of the system states could go to infinity. This is the indicator that, the FS TD CCAL MRAC, could eliminate up to the disturbances given through the Figure 5.40 to the Figure 5.42. Though the divergence is seen in Figure 5.44, the reference model tracking still continues at Figure 5.45, and Figure 5.46. In Figure 5.47, the predicted adaptive weight coefficients also diverge.

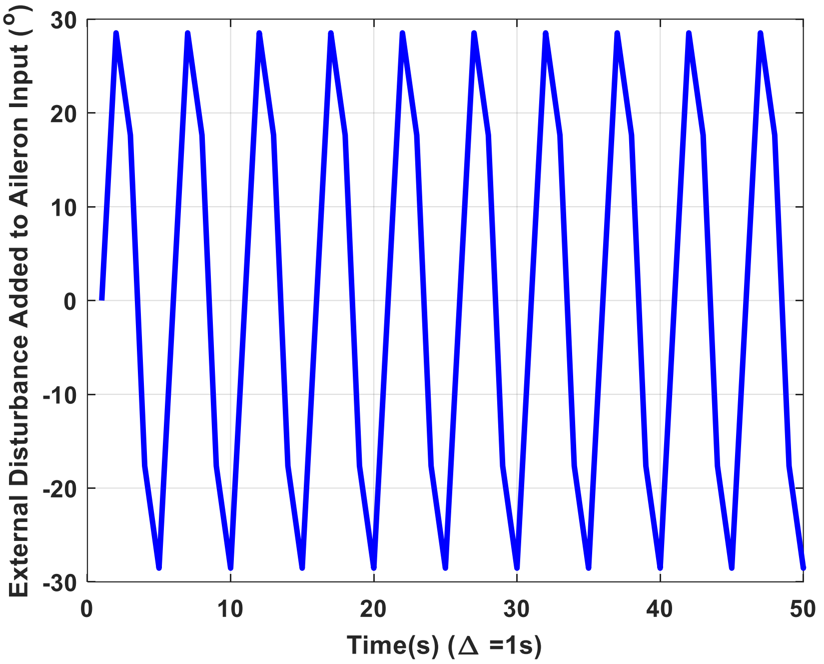

Figure 5.40. External Disturbance Added to Aileron Input vs time

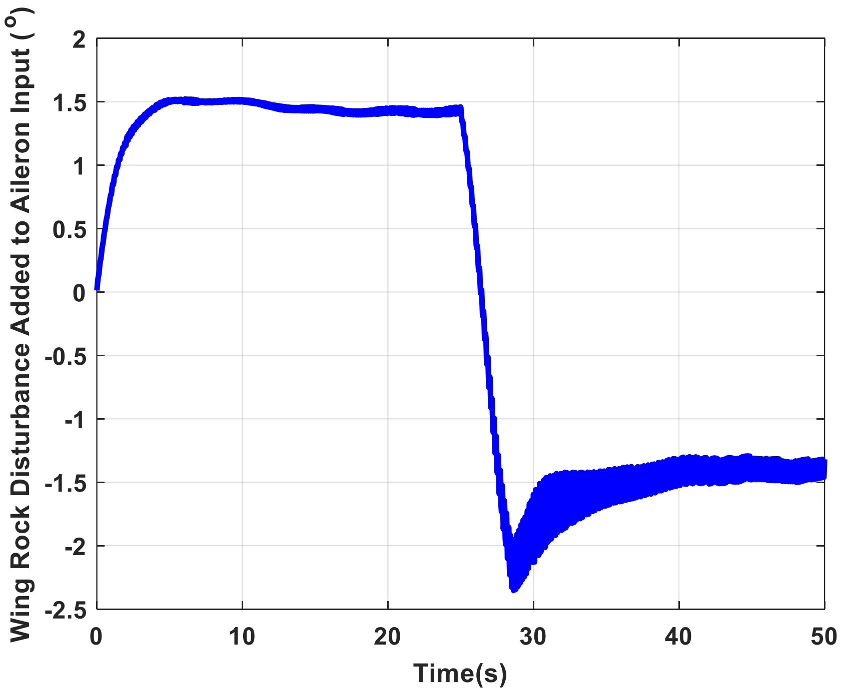

Figure 5.41. Wing Rock Disturbance Added to Aileron Input vs time

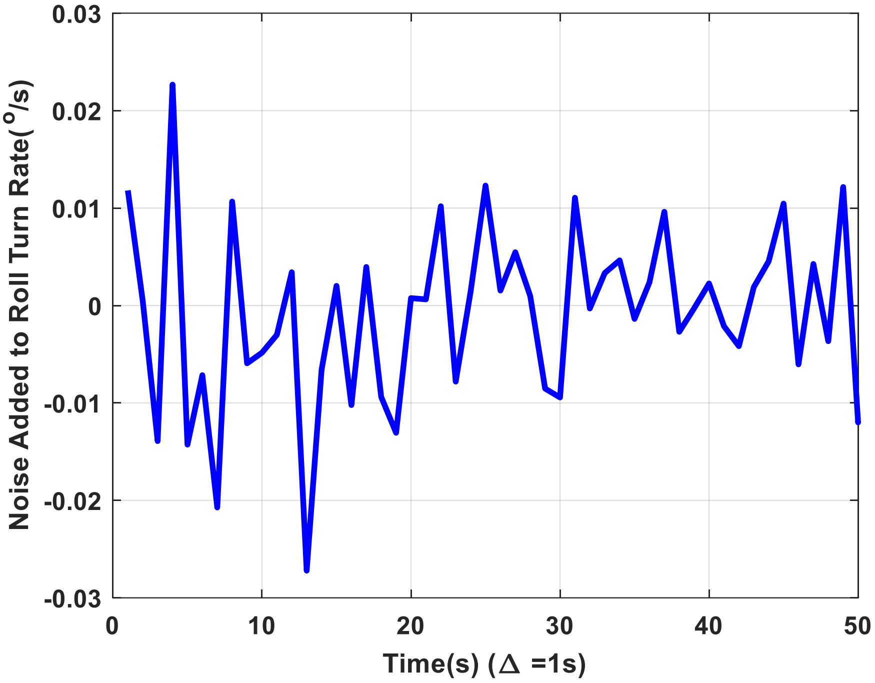

Figure 5.42. Noise Added to Roll Turn Rate vs time

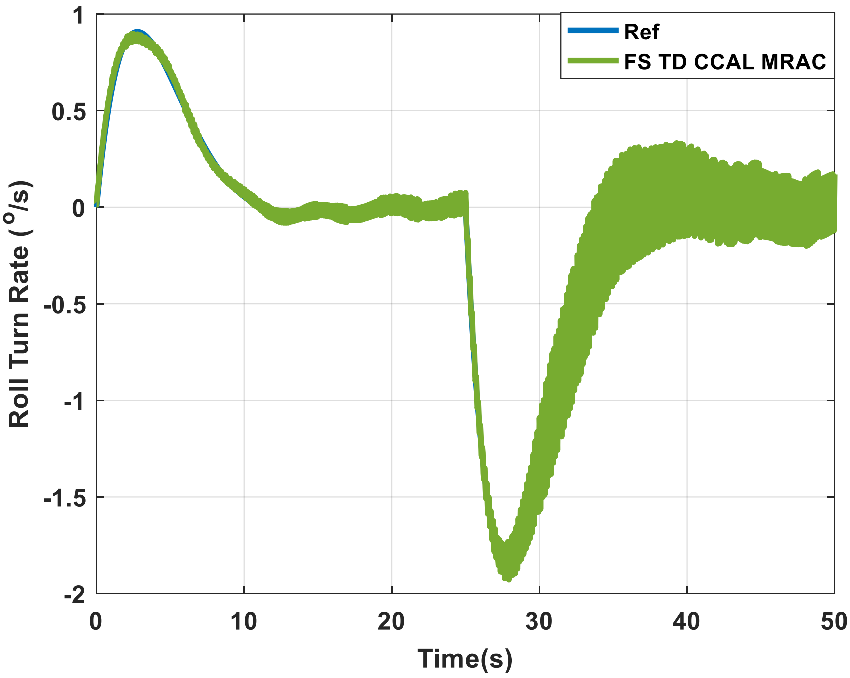

Figure 5.43. Roll Turn Rate vs time

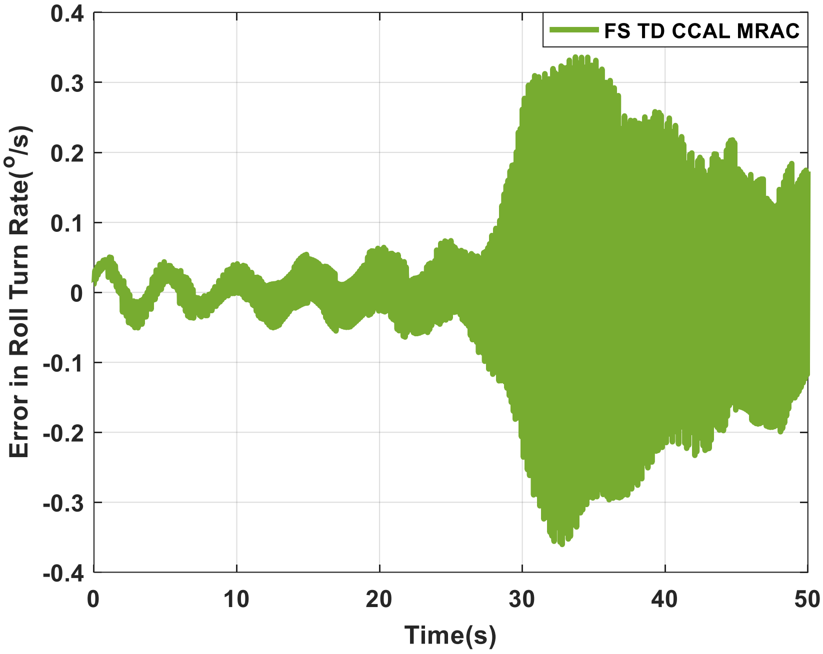

Figure 5.44. Error in Roll Turn Rate vs time

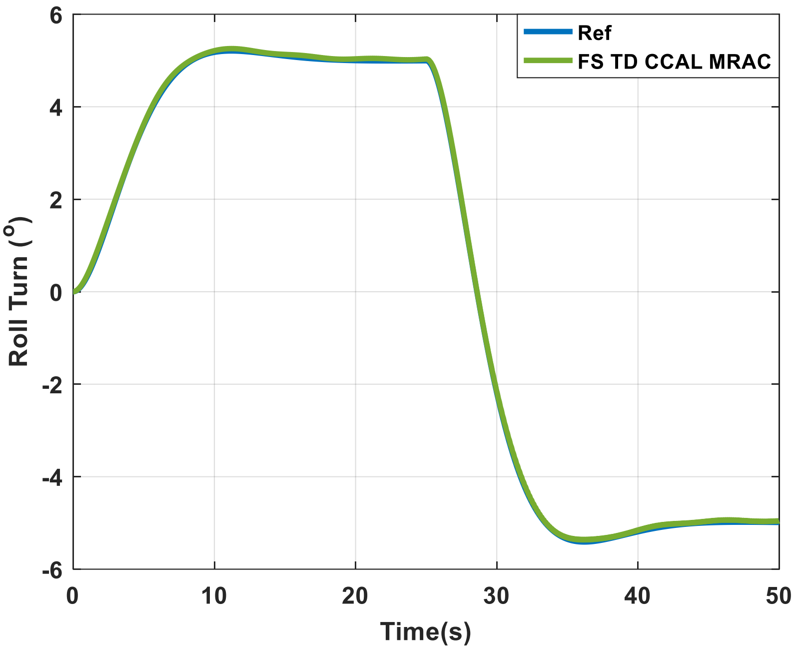

Figure 5.45. Roll Turn vs time

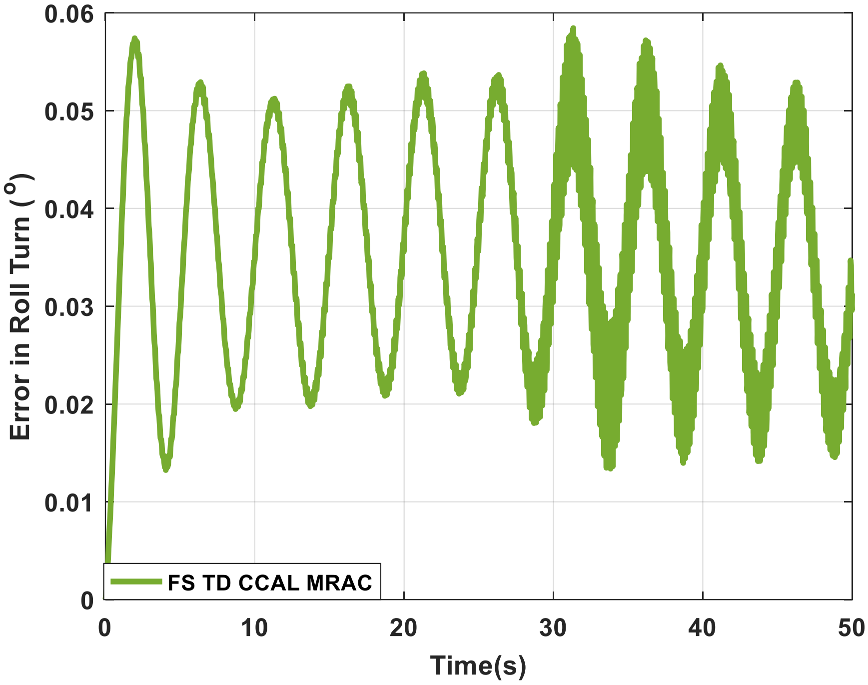

Figure 5.46. Error in Roll Turn vs time

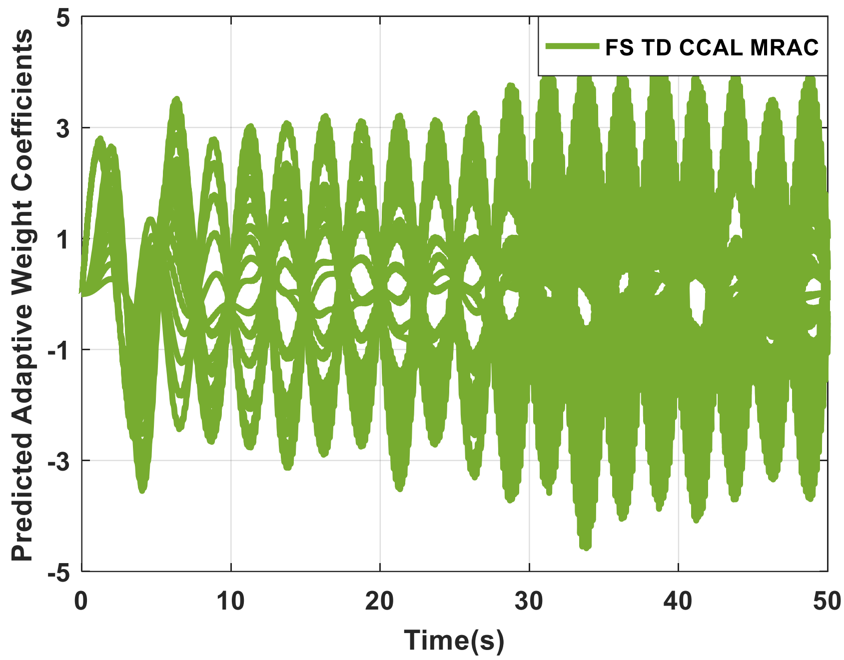

Figure 5.47. Predicted Adaptive Weight Coefficients vs time

5.5.2. FS TSD CCAL MRAC (Disturbation Factor=2)

In this section, the response of the FS TSD CCAL MRAC is examined in case of multiplication term of external disturbance and wing rock=2. In case of external disturbance which is random acting to the system as given in Figure 5.48, and the dynamics of wing rock as given Figure 5.49, and the noise added to the roll turn rate as given in Figure 5.50, the results of the system response are get as given through the Figure 5.51 to the Figure 5.55. The reason for choosing the disturbation factor as 2, is to compare the results of FS TSD CCAL MRAC with the results of the FS TD CCAL MRAC under the same disturbation factor conditions given in 5.5.1. It is seen that in Figure 5.51, Figure 5.52, Figure 5.53, and Figure 5.54 the fluctuations are less than the results compared to, Figure 5.43, Figure 5.44, Figure 5.45, and Figure 5.46. Also the predicted adaptive weight coefficients remain in a narrower region in Figure 5.55, compared to Figure 5.47.

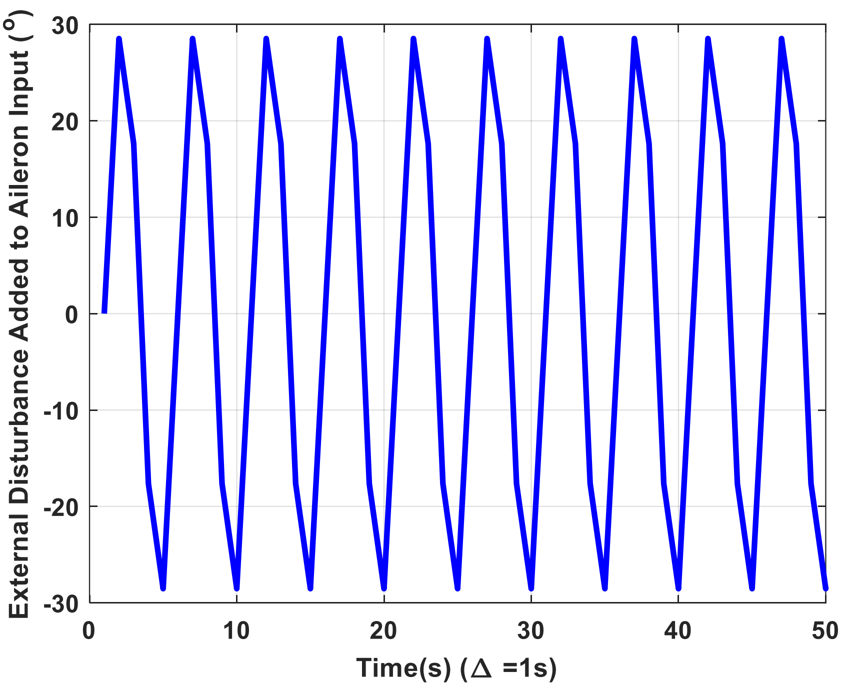

Figure 5.48. External Disturbance vs time

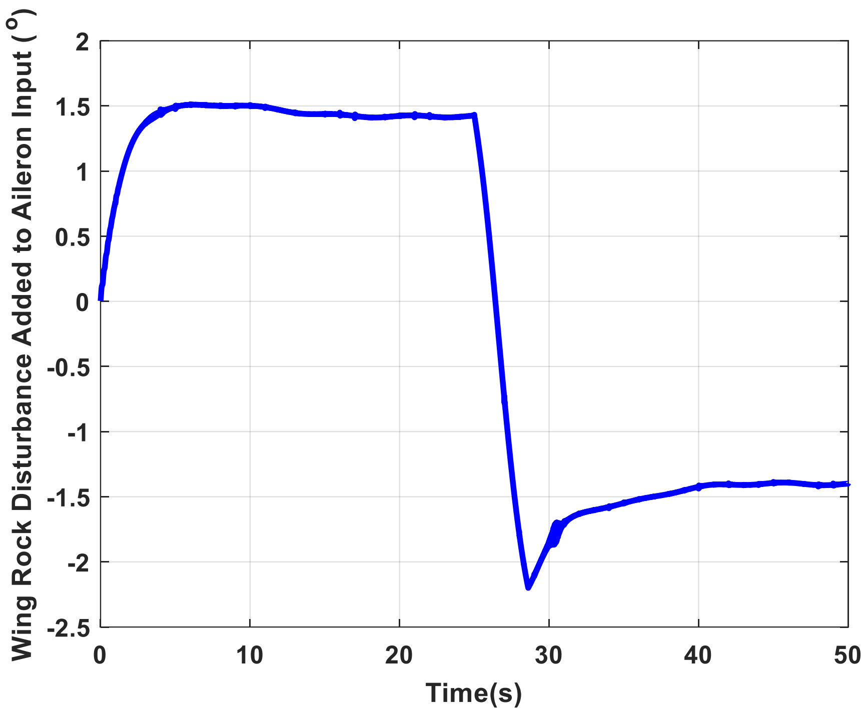

Figure 5.49. Wing Rock Disturbance Added to Aileron Input vs time

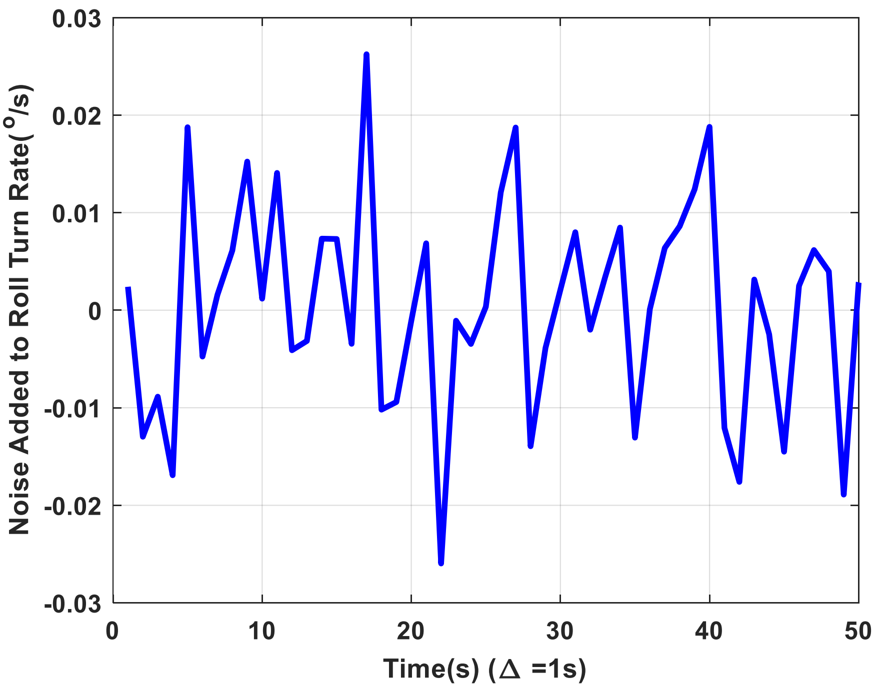

Figure 5.50. Noise vs time

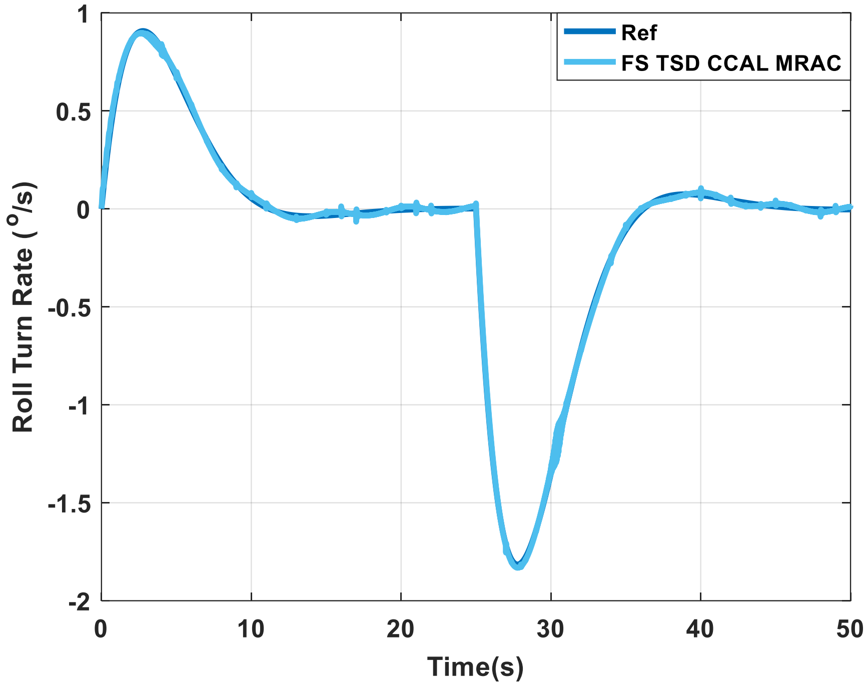

Figure 5.51. Roll Turn Rate vs time

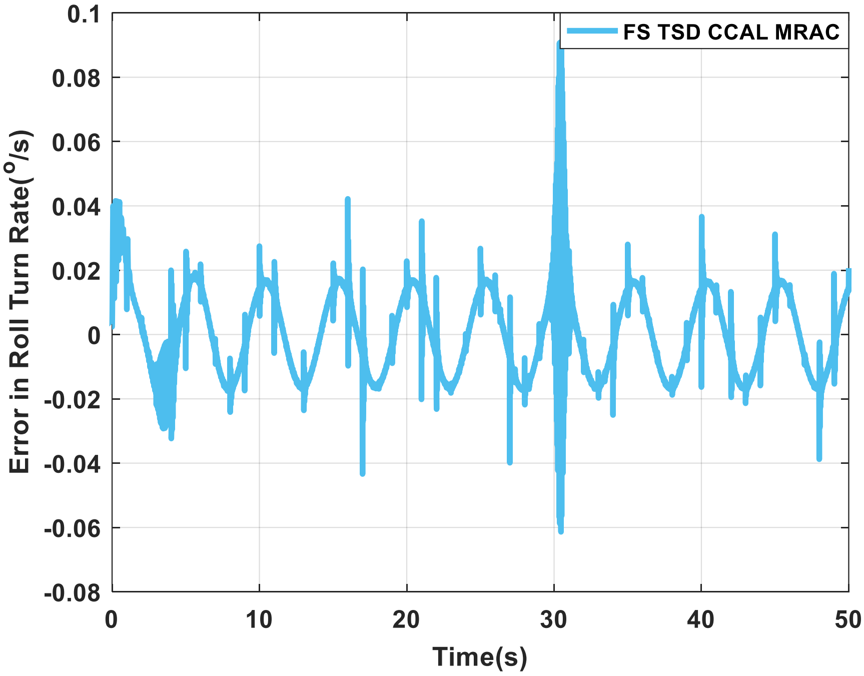

Figure 5.52. Error in Roll Turn Rate vs time

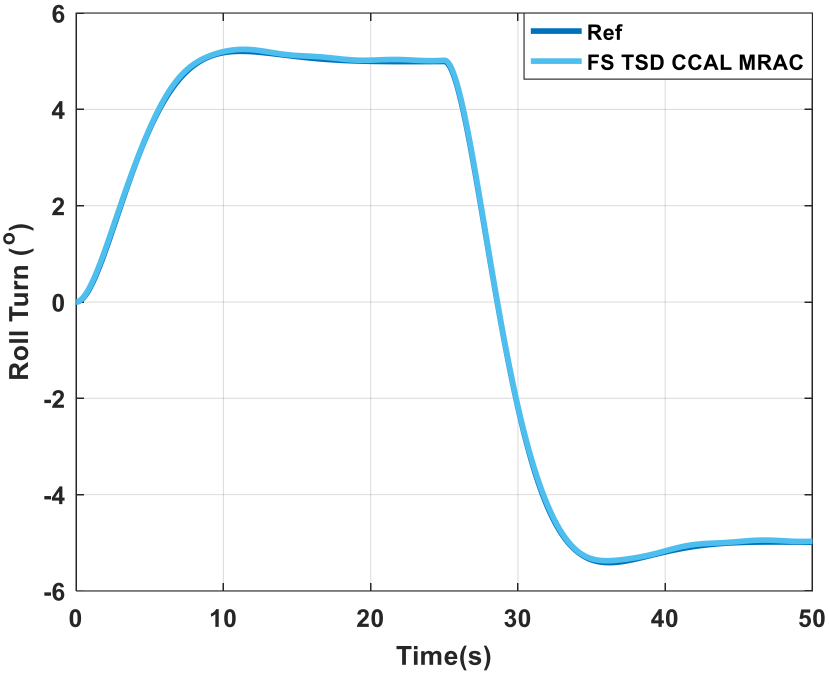

Figure 5.53. Roll Turn vs time

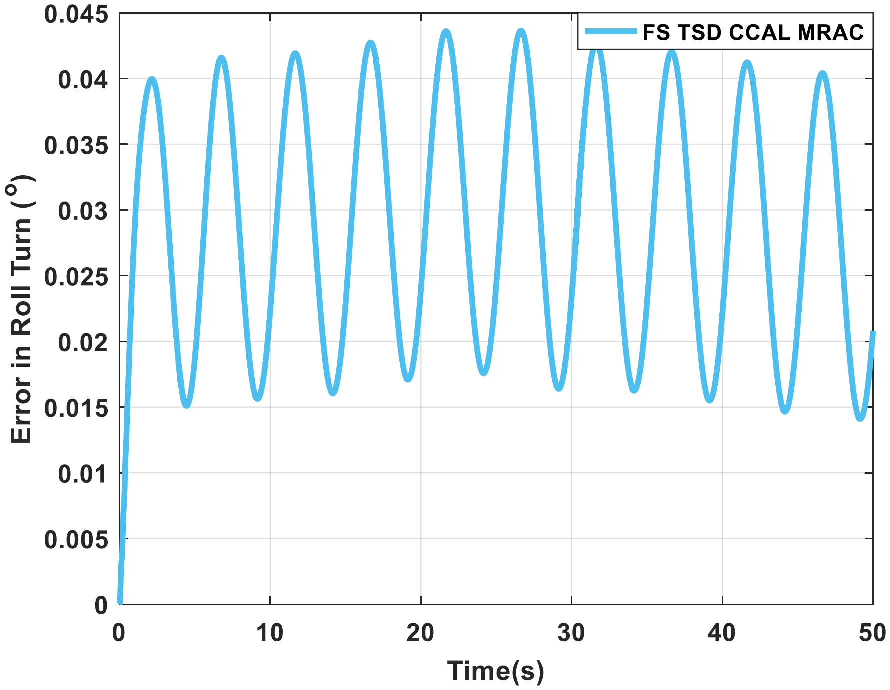

Figure 5.54. Error in roll turn vs time

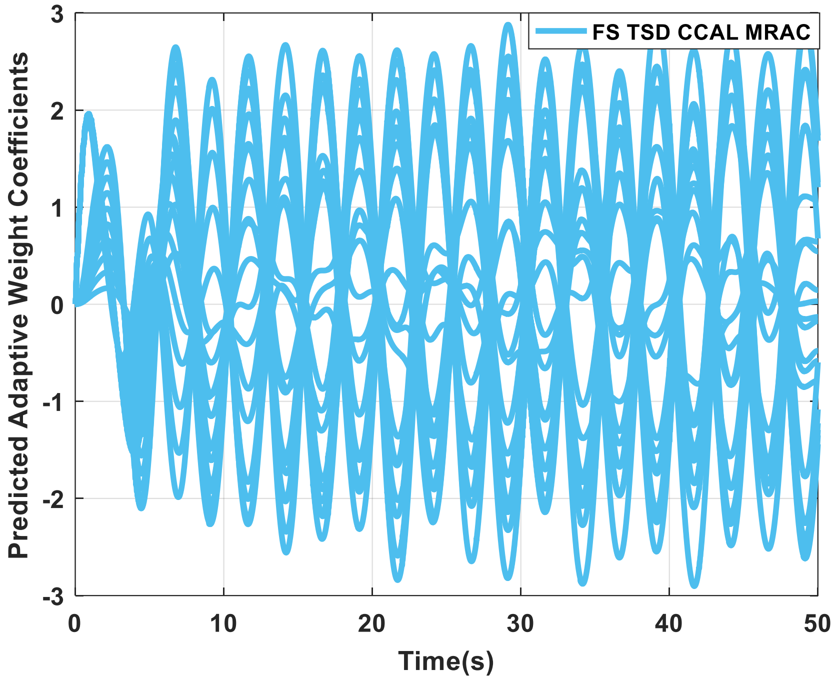

Figure 5.55. Predicted Adaptive Weight Coefficients vs time

5.5.3. FS TSD CCAL MRAC (Disturbation Factor=100)

In this section, the response of the FS TSD CCAL MRAC is examined in case of multiplication term of external disturbance and wing rock=100. If random external disturbance acts to the system as given in Figure 5.56, and the wing rock dynamics as given in Figure 5.57, and the noise added to the roll turn rate as given in Figure 5.58, the results of the system response are get as given through the Figure 5.59, to the Figure 5.63. The multiplication term being 100 is a good indicator that, the FS TSD CCAL MRAC is good at prediction of the uncertainties acting on the system, and removing them. It should not be thought as an unrealistic case of having 100 multiplied external disturbance and dynamics of wing rock acting on the system. Since both the disturbance and the control input enters through the same channel to the system, the adaptive control input could eliminate the disturbance according to its own prediction, at the same simulation step without permitting it enter to the system. So, since it is an algebraic operation, 100 times disturbance could be eliminated by the application of the adaptive control input of magnitude -100 times the disturbance magnitudewise. The key point is the success of the adaptive element to predict the disturbance thoroughly. As seen in this study, in FS TSD CCAL MRAC, time dependent random unstructured uncertainty could be predicted by the time dependent uncertainty parameterized adaptive element, and system state dependent structured uncertainty, that is wing rock dynamics, could be predicted by the state dependent uncertainty parameterized adaptive element. So, whatever the magnitude of the disturbance applying on the system is, it could be predicted by its own adaptive element. As soon as the prediction mechanism works, at the same time step, the disturbance could be eliminated by the adaptive control input. So, it will be as if no disturbance has entered to the system, since the disturbance is predicted, and eliminated, at the same time step of the simulation, as the disturbance enters to the system. In Figure 5.59, and Figure 5.61, the fluctuations around the reference model are seen, but still there is no divergence. In Figure 5.60, and Figure 5.62, also the error figures are given, so that it is acceptable as a good tracking performance. In Figure 5.63, predicted adaptive weights stay in a more compact region. Weight convergence or staying in a compact region, means that the uncertainty is predicted approximately.

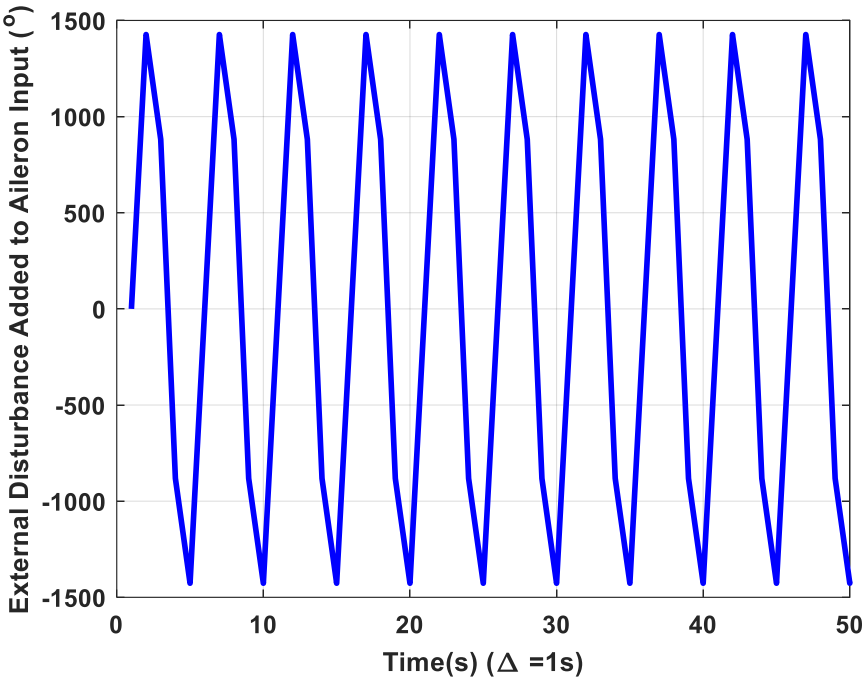

Figure 5.56. External Disturbance Added to Aileron Input vs time

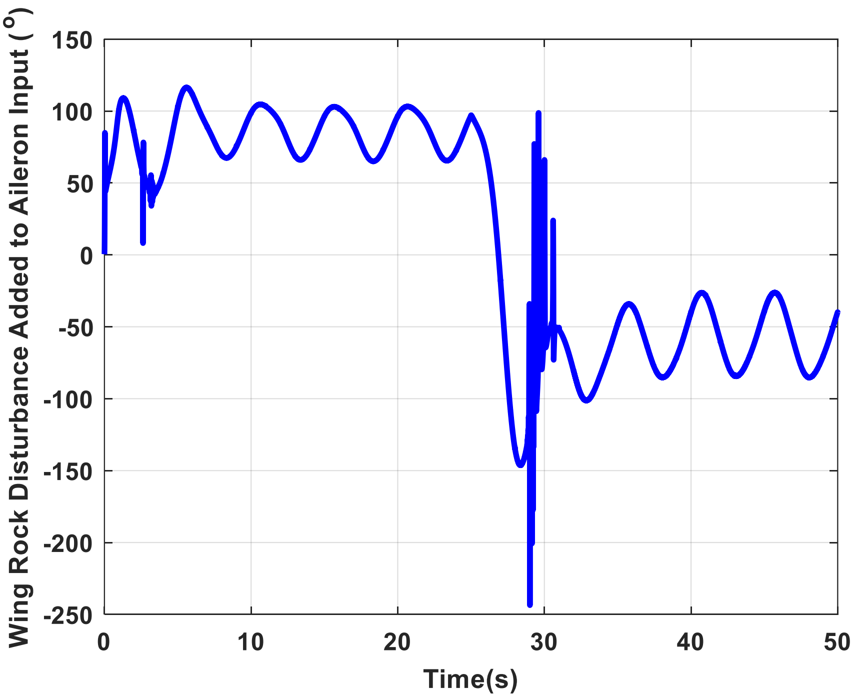

Figure 5.57. Wing Rock Disturbance Added to Aileron Input vs time

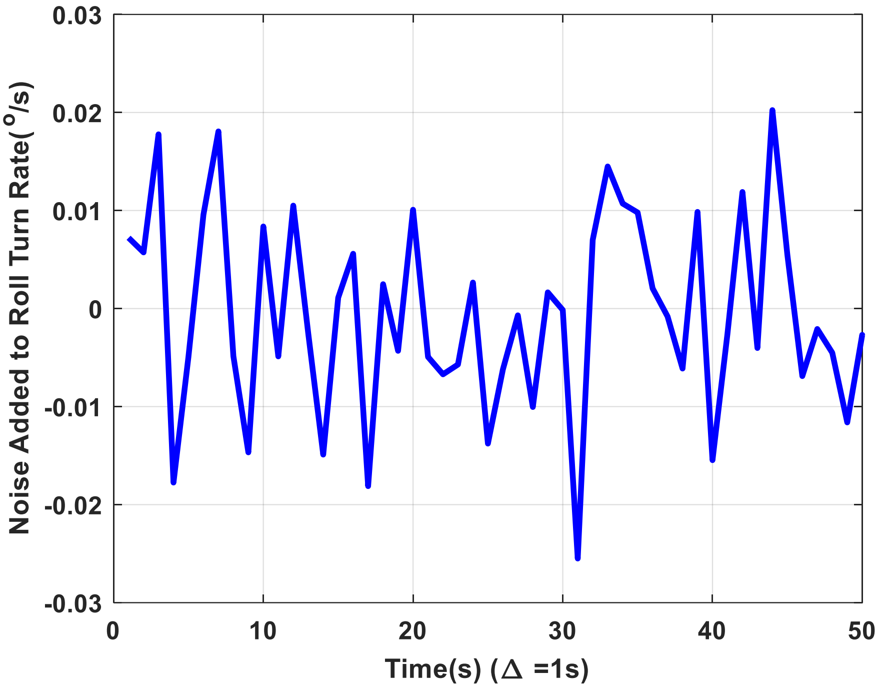

Figure 5.58. Noise Added to Roll Turn Rate vs time

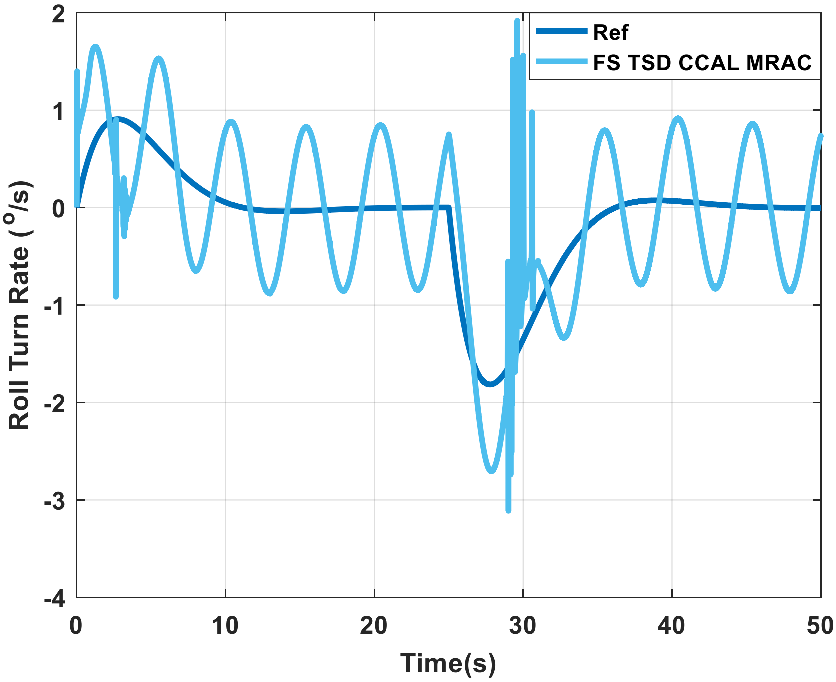

Figure 5.59. Roll Turn Rate vs time

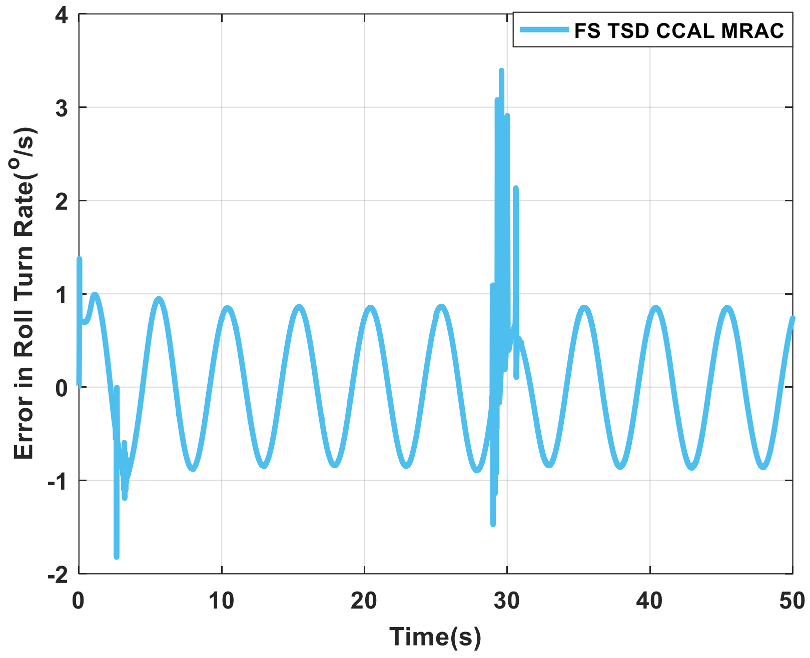

Figure 5.60. Error in Roll Turn Rate vs time

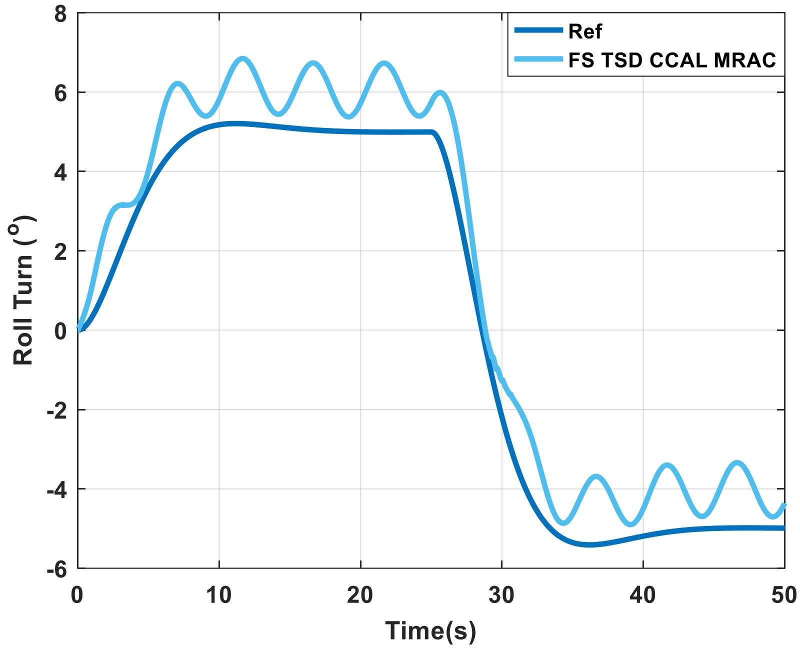

Figure 5.61. Roll Turn vs time

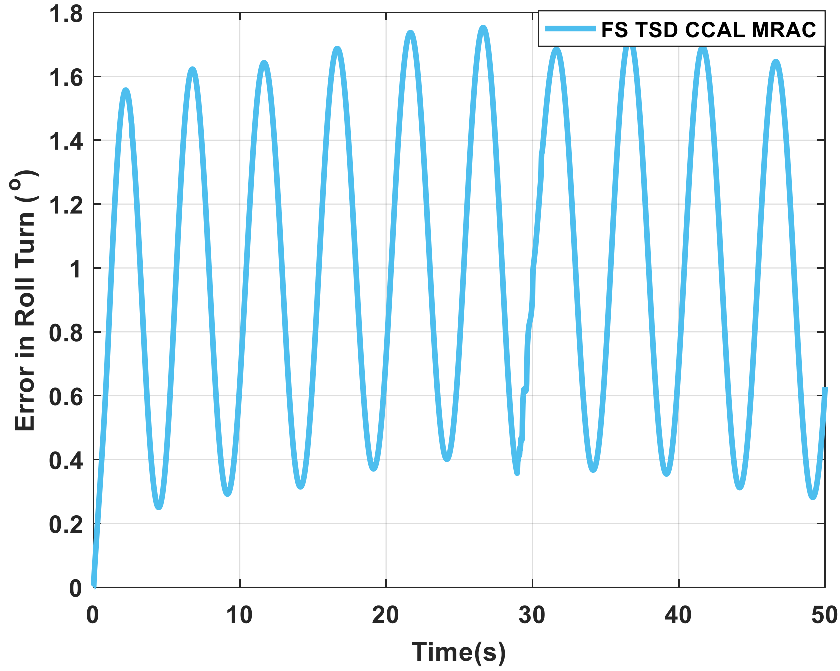

Figure 5.62. Error in Roll Turn vs time

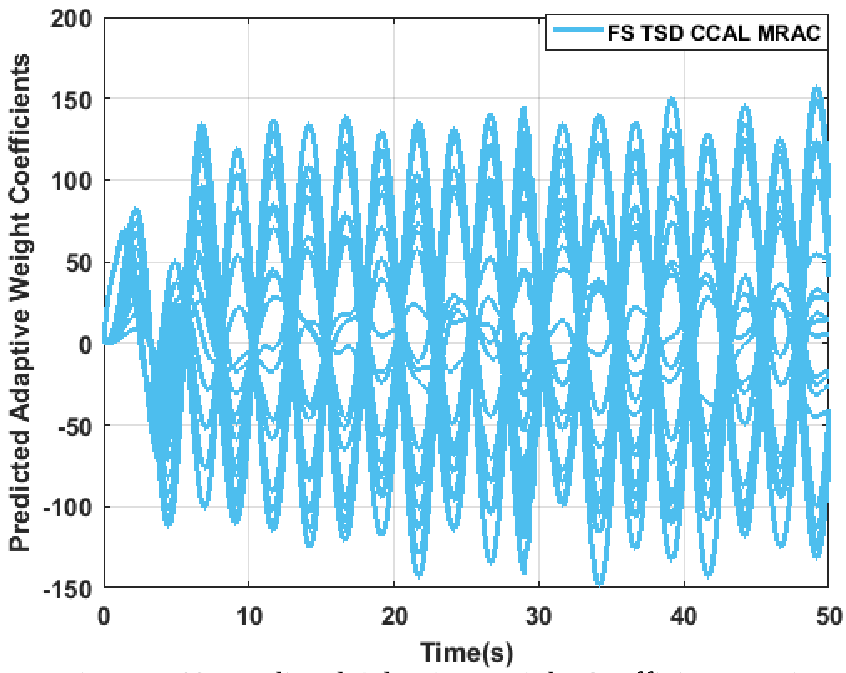

Figure 5.63. Predicted Adaptive Weight Coefficients vs time

5.6. Comparison of Disturbance and Control Input

In this section, the results of FS TSD CCAL MRAC is studied. It is assumed in this thesis that each disturbance should be predicted in its own dependent variable, and then the adaptive input control should be determined separately, depending on the disturbance variable. In this section the graphs of u_{ad}, u_{adt}, and u_{adx} are examined, separately. The disturbances, time dependent random external disturbance, state dependent dynamics of wing rock, and noise are given. It is studied whether the time dependent adaptive control input eliminates the time dependent disturbances, and the state dependent adaptive control input eliminates the state dependent disturbances. External disturbance is given in Figure 5.64, wing rock dynamics is given in Figure 5.65, noise is given in Figure 5.66. Figure 5.67 shows that the time dependent random external disturbance given in Figure 5.64 could be removed up to a large extent. Figure 5.68 shows the state dependent adaptive control input fluctuates around [-20 20] degrees, and could remove the state dependent disturbances given in, Figure 5.65 and Figure 5.66, which is around [-1 1] degrees. Figure 5.69, gives the total u_{ad} input which is algebraic summation of , u_{adt}, and u_{adx}, is also around [-20 20] degrees. Figure 5.70 gives u_n which is normal input graph, it is around [-3 3] degrees. Figure 5.71, gives the total control input u, which is algebraic subtraction of u_{ad} from the u_n, is also around [-20 20] degrees.

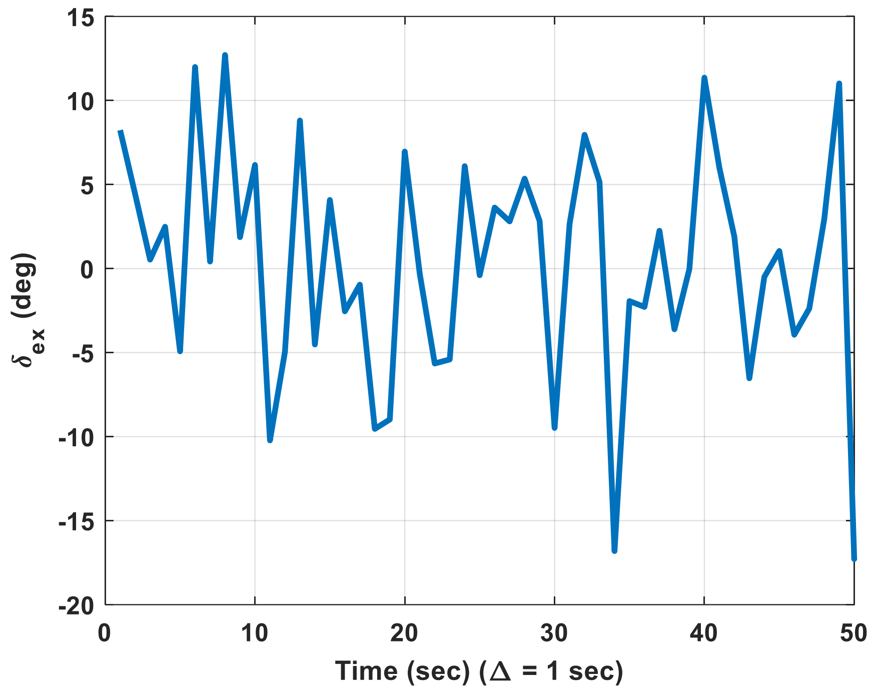

Figure 5.64. External Disturbance vs time

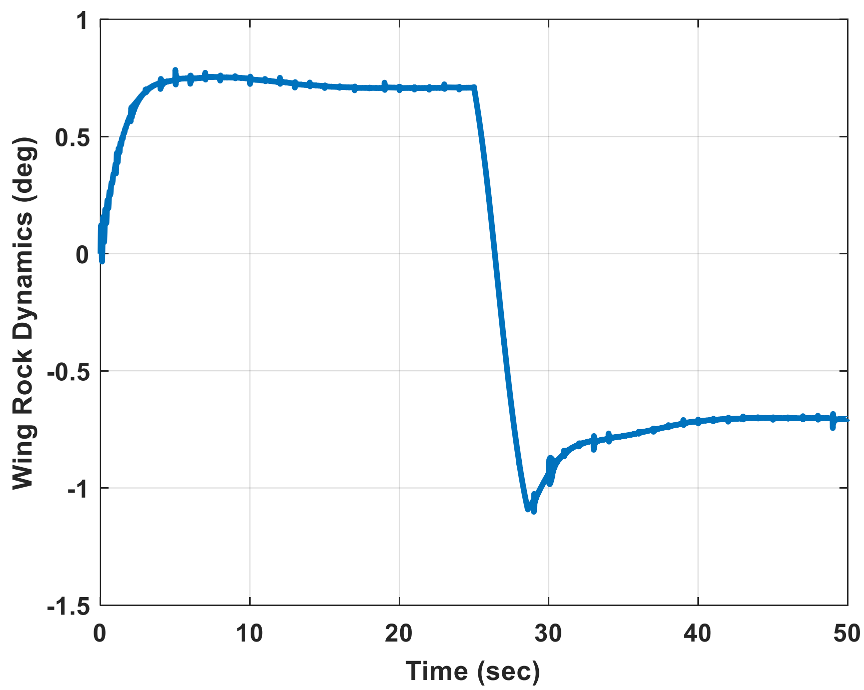

Figure 5.65. Wing Rock Dynamics vs time

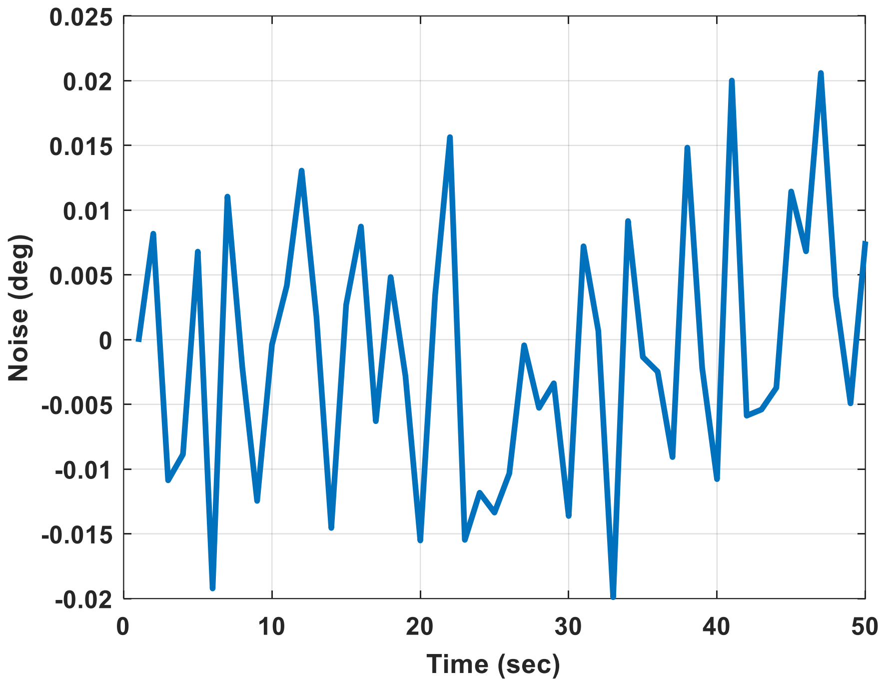

Figure 5.66. Noise vs time

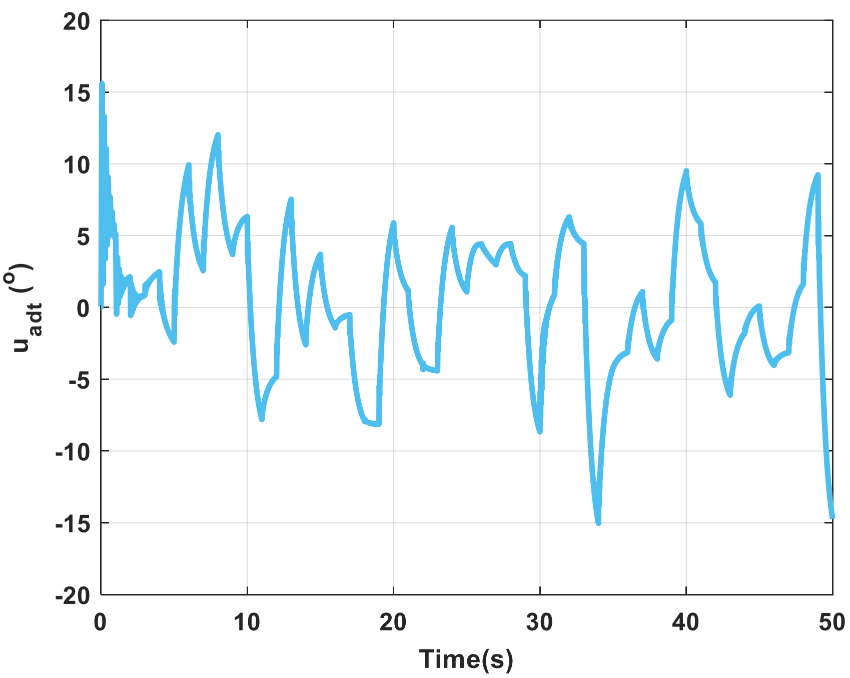

Figure 5.67. u_{adt} vs time

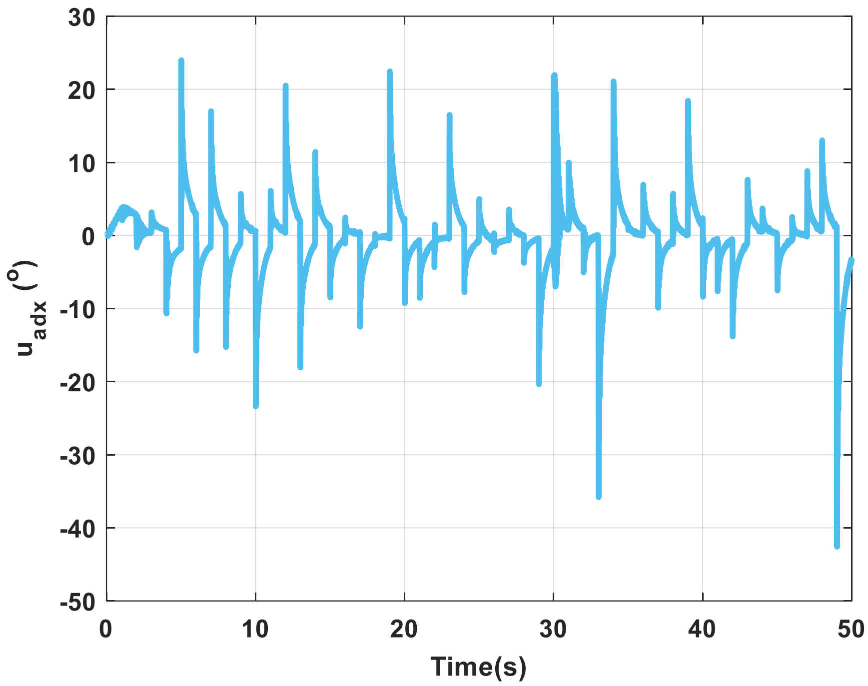

Figure 5.68. u_{adx} vs time

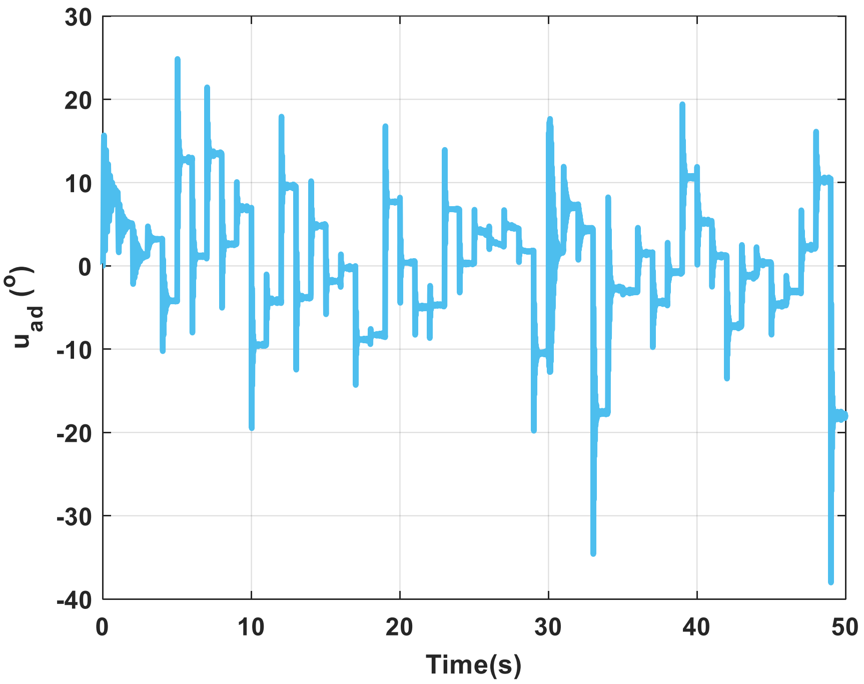

Figure 5.69. u_{ad} vs time

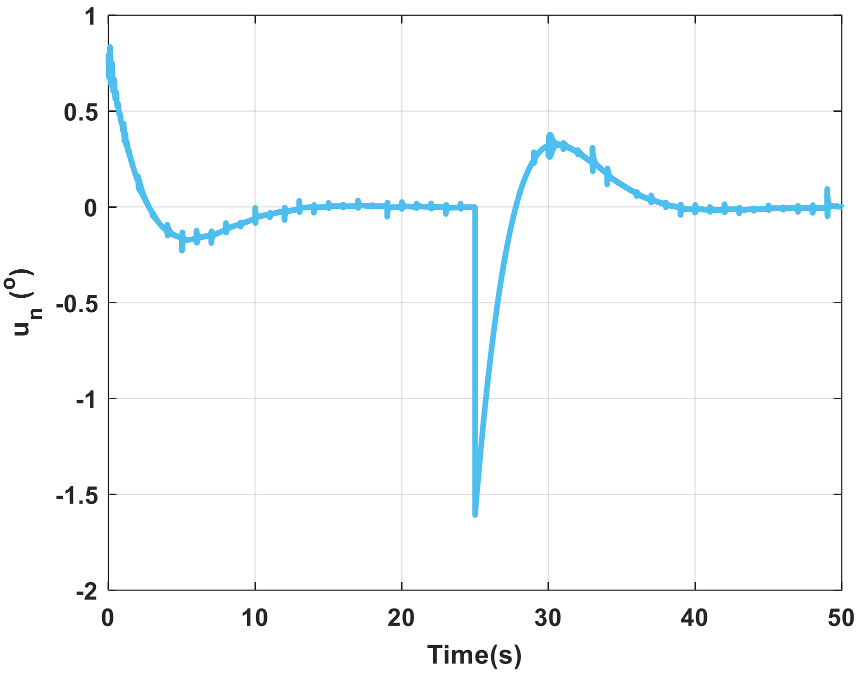

Figure 5.70. u_n vs time

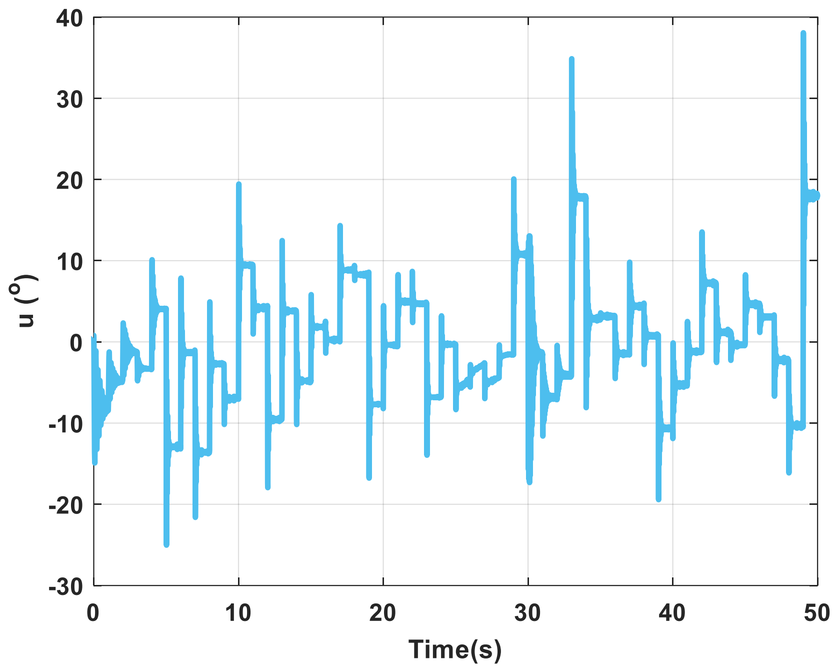

Figure 5.71. u vs time

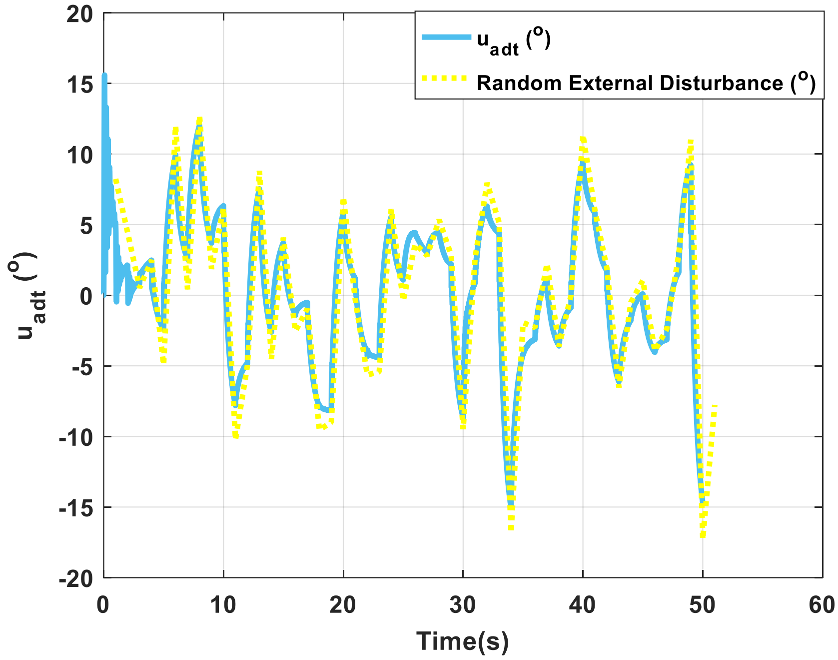

Figure 5.72. Comparison of Time Dependent Adaptive Control Input and Random External Disturbance

It is seen in Figure 5.72, that time dependent adaptive control input u_{adt} could estimate the time dependent random external disturbance, successfully.

5.7. Concurrent Adaptive Learning Effect

In this section the effect of concurrent adaptive learning on the MRAC is studied. In Figure 5.73, and Figure 5.74, the comparison of u_{adt} and random external disturbance is given. FS TSD CCAL MRAC model is used and, concurrent adaptive learning is closed and opened, to see the effect on the MRAC. In Figure 5.75, and Figure 5.76, the same results are given under the effect of 2 times multiplied random external disturbance.

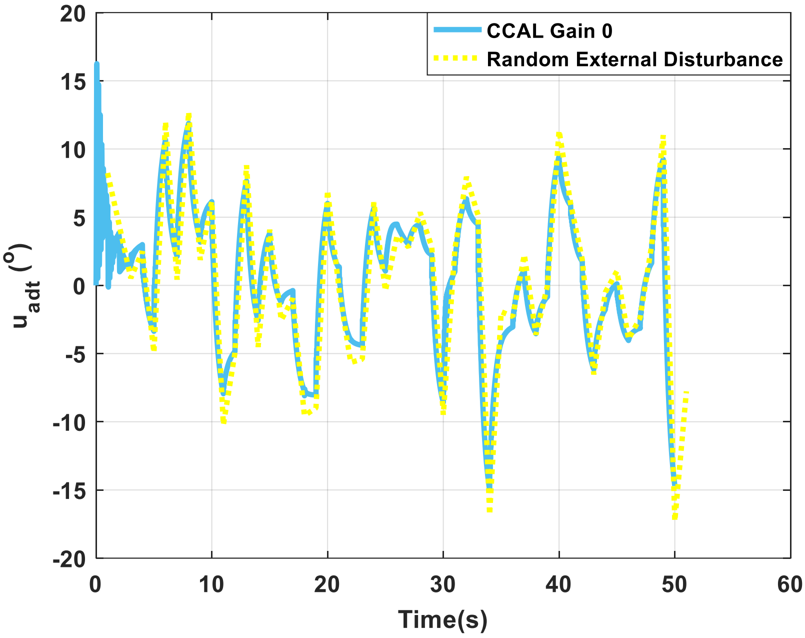

Figure 5.73. Comparison of Time Dependent Adaptive Control Input and Random External Disturbance when Concurrent Adaptive Learning is Closed

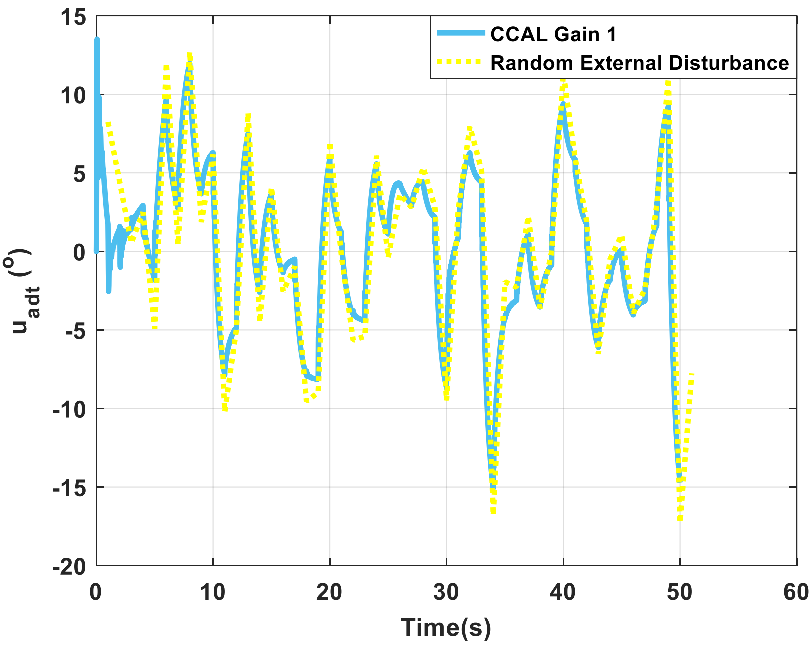

Figure 5.74. Comparison of Time Dependent Adaptive Control Input and Random External Disturbance when Concurrent Adaptive Learning is Open

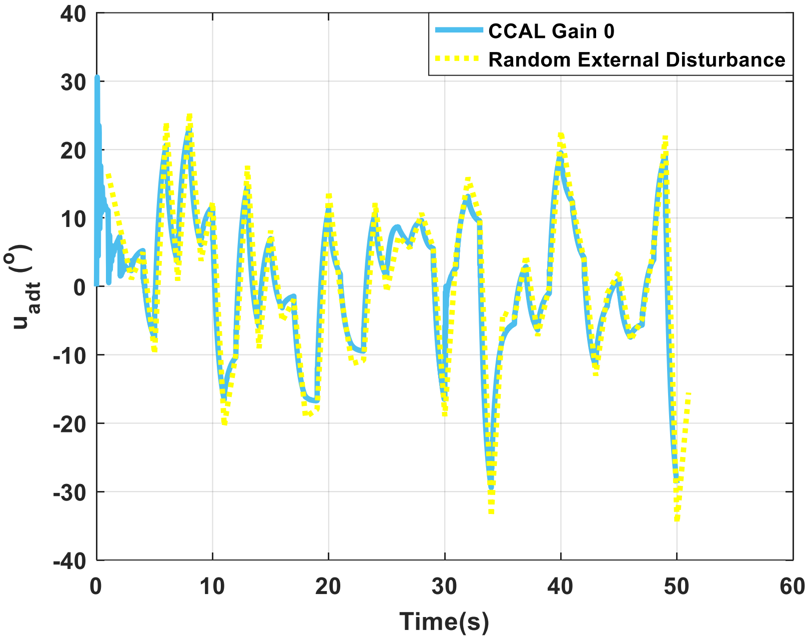

Figure 5.75. Comparison of Time Dependent Adaptive Control Input and Random External Disturbance when Concurrent Adaptive Learning is Closed

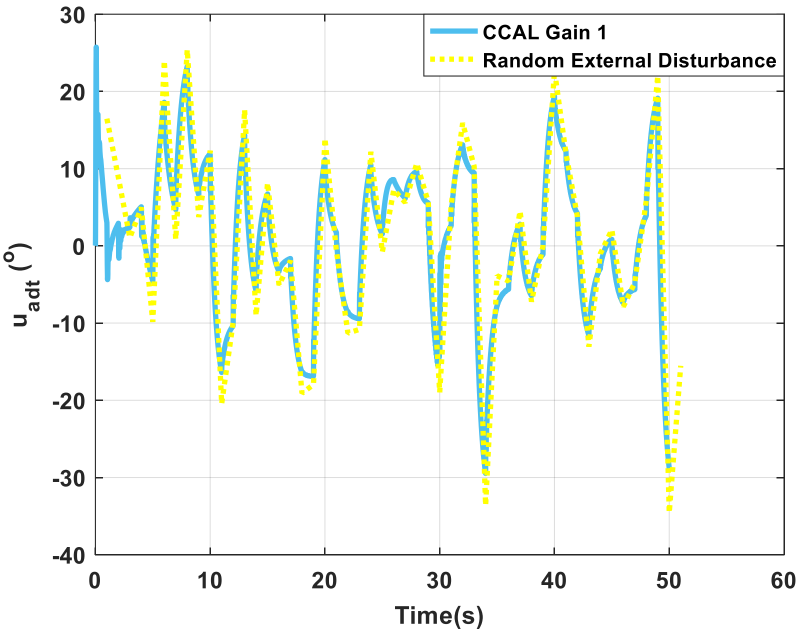

Figure 5.76. Comparison of Time Dependent Adaptive Control Input and Random External Disturbance when Concurrent Adaptive Learning is Open

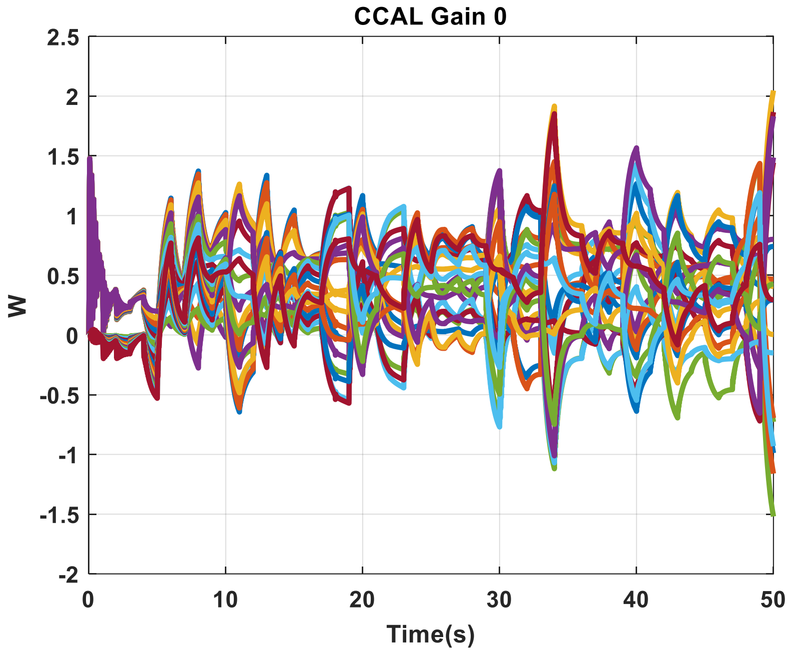

Figure 5.77. Adaptive Weight Coefficients when Concurrent Adaptive Learning is Closed

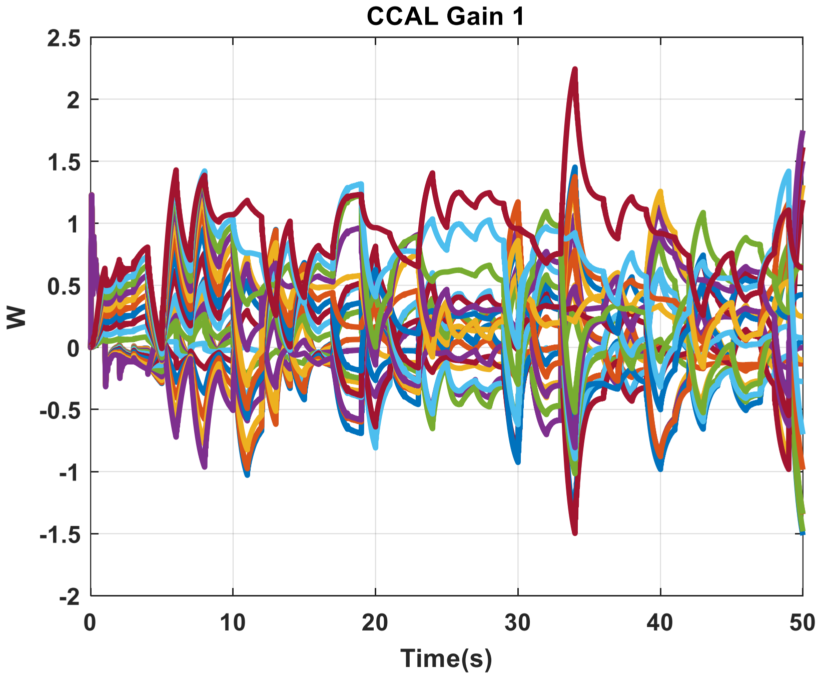

Figure 5.78. Adaptive Weight Coefficients when Concurrent Adaptive Learning is Open

It is seen that, although the FS TSD CCAL MRAC could predict and eliminate the time dependent random external disturbance successfully, whether the CCAL algorithm is opened or closed, there is a positive effect of CCAL algorithm. That is, by the application of CCAL algorithm, the control actuator mechanism needs less deflection angle, for the elimination of the disturbance, at the beginning of the simulation, which is good for reducing energy consumption and delaying material fatigue. In Figure 5.73, u_{adt} is above 15 [deg] at the beginning of the simulation while in Figure 5.74, u_{adt} is below 15 [deg], by the application of CCAL algorithm. The time dependent random external disturbance is multiplied by 2, and again the same pattern is seen at the simulation results, such that, in Figure 5.75, u_{adt} is above 30 [deg] at the beginning of the simulation while in Figure 5.76, u_{adt} is below 30 [deg], by the application of CCAL algorithm. Though the external disturbance is random and time dependent, not periodic, the Fourier series is good at prediction of the uncertainty, with its periodic structure. This could be explained as, though the structure of the Fourier series is periodic, but the disturbance is random, the calculated control input could track the reference command, at a global perspective. Since the main aim is to track the reference command at a global perspective, Fourier series usage is acceptable. In the Figure 5.77, and Figure 5.78, when the CCAL is closed and opened, adaptive weights remain in a bounded region. So, both the weights remain bounded and the reference command tracking is satisfied. Also, though it seems as if in Figure 5.77, and Figure 5.78, the CCAL usage effect on weight converge is similar with the nonused case, as mentioned above, the CCAL usage lessens the control actuator deflection angle, as a positive effect.

CHAPTER 6

CONCLUSION

In this thesis, a novel approach for the model reference adaptive control, is given, of which is time and state dependent parameterized MRAC. In this method, it is proposed that, the parameterization part of the MRAC should be done according to the type of the uncertainty, whether it is structured or unstructured. If the acting disturbance is defined in terms of time variable, then the adaptive part should also be constructed in terms of time variable. If the acting disturbance is defined however in terms of system state variable, then the adaptive part should be constructed in terms of the state variable, too. The adaptive weight coefficient update laws should also be constructed in the same manner. In the end, for the determination of the adaptive control input, each type of the adaptive control input should be summed. It is proposed that, the separate adaptive control determination causes to predict the related uncertainty, more approximately. In this thesis, the unstructured random external time dependent uncertainty, and structured system state dependent wing rock dynamics, and also noise added to the state could be predicted with the new, time and state dependent parameterized model reference adaptive control, and could be removed from the system, successfully.

In the control design process, as the system model a slender delta wing is used. The reference model is determined during the model following control design. The reference model states are determined by the pole placement method, and then by using the formula belonging to Ackermann, the full state feedback gains are calculated. By using the calculated gains, the nominal control input is determined. By the application of the nominal control input, the response of the system model in closed loop structure, is resembled to the transient response of the specified in other words reference model. In the MRAC, for the determination of the nominal control input, the same full state feedback gains are used as in the model following controller. The adaptive control input is determined by the multiplication of the basis function which is state dependent, and the update law of weight, which relies on Lyapunov stability theorem.

In the time and state dependent parameterized model reference adaptive control, first time dependent parameterized model reference adaptive control is given. Time dependent series are used as the basis function, like the Chebsyhev polynomials, and the Fourier series, respectively. First the Chebsyhev polynomials based MRAC is given. Though the structure of the Chebsyhev polynomials is more suitable for state dependent parameterization, it is used in this study as the time dependent parameterization, by taking the cosine of the time variable as the variable. The reason for that usage is that, since the random external disturbance is unstructured and time dependent in this study, the series structure of the Chebsyhev polynomials is used by differing the dependent variable as the cosine of the time variable. Secondly, the Fourier series based MRAC is given. It is seen that Fourier series based MRAC gives more close reference model tracking, due to its periodic structure, than the Chebsyhev polynomials based model reference adaptive control, though its forming from the linearly independent terms structure. This could be an indicator that, Chebsyhev polynomials is better at prediction of state dependent uncertainty parametrization, rather than time dependent uncertainty parametrization. To improve the MRAC, concurrent adaptive learning algorithm is added to the MRAC. The key advantage of the concurrent adaptive learning is the exclusion of the PE requirement for the convergence of the adaptive weights. This is done by storing only the valuable data, which enhances the uncertainty prediction, at last. The storage of the predicted uncertainty is done according to the singular value maximizing algorithm. Secondly state dependent parameterized model reference adaptive control is given. State dependent basis function is used as the prediction of the uncertainty. In the end, time and state dependent parameterized model reference adaptive control is designed, which uses both time and state dependent basis functions to predict the related uncertainties. In the end, the adaptive control inputs are summed, to eliminate the total disturbances from the system model. The results of the designed controllers are compared to determine which one is better at uncertainty prediction and elimination. MFC, MRAC, CPBMRAC, CP TD CCAL MRAC, FSBMRAC, FS TD CCAL MRAC, FS TSD CCAL MRAC are studied. It is seen that, Fourier series based time and state dependent concurrent adaptive learning model reference adaptive control gives best reference model tracking, for the conditions given in this study. This led to the dissertation of each uncertainty should be predicted by its own uncertainty parameterization method.

The designed controller's robustness is tested by the Lyapunov stability theorem and the performance of the designed controller is also viewed by the graphs which compares the responses of the specified model states and the plant model states on the same graph.

To sum up, in this study prediction of the unstructured time dependent random external disturbance and structured state dependent wing rock dynamics, is studied. Also, state dependent noise addition to the roll turn rate is included as the uncertainty case. Since the random external disturbance is unstructured and random, it is more realistic and chaotic case for this study than the structured wing rock dynamics. So, as a future study, the response of the time and state dependent parametrized model reference adaptive control, to the structured time dependent external disturbance, and unstructured random state dependent disturbance could be studied.

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