PROSPECTIVE TEACHERS' INTERPRETATION OF MULTIPLICATIVE SITUATIONS WITH FRACTIONS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

PROSPECTIVE TEACHERS' INTERPRETATION OF MULTIPLICATIVE SITUATIONS WITH FRACTIONS

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This study examined middle school prospective teachers' interpretations of multiplicative situations–multiplication and division–when solving problems with fractions. Multiplication situations with fractions are at the heart of middle school mathematics, so learning and teaching this concept is crucial. To improve learning and teaching these concepts, there is a need to educate middle school prospective teachers and design education programs for middle school prospective teachers to support their reasoning. Through coursework which emphasizes the importance of fraction-as-number and various interpretation of multiplicative situations with fractions with various curriculum materials, prospective teachers would be more competent in operating with fractions.

In this study, middle school prospective teachers' solutions were examined through a perspective that connects multiplication and division into a coherent framework. The analytical framework of this study placed emphasis on multiplication, partitive division, and quotitive division situations. The data were collected from 13 middle school prospective teachers' final exam problems completed as part of a content course at a large university in the Southeastern United States. Findings revealed that (a) prospective teachers used strategies involving multiplicative situations after completing a two-semester sequence of mathematics content courses on fraction tasks (b) this instructional approach supported the development of an understanding of multiplicative operations with fractions and understanding of the meaning of multiplication and division for middle school prospective teachers, and (c) when allowed to choose methods prospective teachers used the partitive division (PDS) more often than the quotitive division (QDS) and multiplication situation (MS) correctly and appropriately.

Keywords: Multiplication situation, Partitive division situation, Quotitive division situation, Middle school prospective teachers

ÖZ

ORTAOKUL ÖĞRETMEN ADAYLARININ KESİRLER İLE ÇARPIMSAL DURUMLARI (ÇARPMA VE BÖLME) YORUMLAMALARI

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Bu çalışma, ortaokul öğretmen adaylarının kesirler ile ilgili problemleri çözerken, çarpımsal durumları (çarpma ve bölme) yorumlamalarını incelemiştir. Kesirler ile çarpma ve bölme işlemleri, ortaokul matematiğinin merkezinde yer aldığından, bu kavramı öğrenmek ve öğretmek çok önemlidir. Kesirler kavramını ve kesirlerle çarpma işlemlerini öğrenmeyi ve öğretmeyi geliştirmek için, orta okul öğretmen adaylarının eğitime ve onların bu konu üzerinde fikir yürütmelerini destekleyecek eğitim programlarının tasarlanmasına ihtiyaç vardır.

Sayı olarak kesirlerin önemini vurgulayan dersler ve çeşitli müfredat materyalleri ile kesirler içeren çarpımsal durumların çeşitli yorumlamaları sayesinde, öğretmen adayları kesirler ile çalışma konusunda daha yetkin olacaktır. Bu çalışmada, ortaokul öğretmen adaylarının çözümleri, çarpma ve bölmeyi mantıklı ve tutarlı bir çerçeveye bağlayan bir bakış açısıyla incelenmiştir. Bu çalışmanın analitik çerçevesi, çarpma, parçalamalı bölme (partitive division) ve gruplamalı bölme (quotitive division) modellerine vurgu yaptı. Veriler, Güneydoğu Amerika Birleşik Devletleri'ndeki büyük bir üniversitede bir ders kapsamında tamamlanan final sınavı problemleri kullanılarak 13 ortaokul öğretmen adaylarından toplandı. Bulgular, (a) öğretmen adaylarının, iki dönemlik bir matematik dersleri dizisini kesir görevleri üzerine tamamladıktan sonra, çoklayıcı durumları içeren stratejileri kullandıklarını (b) bu öğretim yaklaşımı, öğretmen adaylarının kesirler ile çarpma işlemlerinin anlamasını ve çarpımsal durumlarının yorumlarını anlamalarını desteklemiştir ve (c) öğretmen adaylarının kullanacakları modeli kendilerinin seçmesi sağlandığında öğretmen adaylarının parcalamali bölme modellerini gruplamalı bölme ve çarpım durum modellerinden daha doğru ve uygun bir şekilde kullandığı görülmüştür.

Anahtar Kelimeler: Kesirler, Çarpımsal durumlar, Parçalamalı bölme modeli, Gruplamalı bölme modeli, Ortaokul öğretmen adayları

I dedicate this work to my parents for their endless love and support.

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CHAPTER 1

INTRODUCTION

"Division of fractions is often considered the most mechanical and least understood topic."

Dina Tirosh (2000)

Multiplication and division with fractions are among the most challenging topics in the middle school mathematics (Tchoshanov, 2011). More studies appear to be warranted on the multiplication and division with fractions as these concepts continue to be complicated for prospective teachers (Tirosh, 2000; Simon 1993) . For example, many prospective teachers exhibit difficulty in understanding division problems with fractions. As reported by Isik and Kar (2012), some common difficulties of prospective teachers for division with fractions are as follows

> Fraction-as-number, unit confusion, assigning natural number interpretations to fractions, problems using ratio proportions, being unable to establish part-whole relationships, dividing by the denominator of the divisor, using multiplication instead of division, and increasing errors by inverting and multiplying the divisor fraction (p. 2-7).

The progress of developing knowledge for fractions starts in elementary school and continues to middle school. The development of fractional concepts in the middle school helps students to progress from additive to multiplicative reasoning. Multiplicative reasoning is major topic of middle school (e.g., Greer, 1992; Lamon, 2007) and a basic to multiplication, division, fractions, ratio, proportions, and so on (e.g., Vergnaud, 1983). It also requires them to foster knowledge for proportional understanding, and comprehend the relationship between fractions, ratios, and proportions (Sowder et al., 1998).

When fostering the required knowledge from fundamental level of understanding through advanced level of understanding of the fractions and mathematical operations with fractions, teachers play an essential role. To help students, middle school teachers need to understand how students' reason when they are dealing with fractions (Sowder et al., 1998). To provide students with in-depth understanding of fractional concepts and help them enhance their understanding and reasoning for fractional relationships and their operations, middle school teachers should have comprehensive college experiences with a procedural and conceptual understanding of fractions (Ball, 1993; Siebert, 2002; Taber, 2002). Unfortunately, most of the prospective teachers have been traditionally educated, and they might not be well prepared to teach these concepts.

For example, in a study by Ball (1990), when prospective teachers asked to create word problems or the meaning of fraction multiplication and fraction division situations, only 26% of them were able create a word problem that shows the meaning of the division problem appropriately. Similarly, in a different study, when prospective teachers were asked to make explanations for the meaning of fraction arithmetic in the given situation, most of prospective teachers could not succeed to explain the meaning and go beyond invert and multiply algorithm (Li & Kulm, 2008).

Many mathematics concepts like fractions and ratios or multiplication and division have generally been treated as discrete and unconnected mathematical concepts. However, according to Vergnaud, ratios are proportional relationships that are part of the multiplicative conceptual field (as cited in Beckmann & Izsák, 2015). The multiplicative conceptual field is "a web of interrelated ideas that also include multiplication and division, fractions, linear functions, and more" (Beckmann & Izsák, 2015, p,18). Students development of a fundamental knowledge for understanding ratio and rate and the connection of ratio and rate to multiplication and division before the middle school is vital so that they can be prepared to understand fractions and fraction arithmetic (National Council of Teachers of Mathematics [NCTM], 2001).

Most curriculum materials, with a few exceptions, do not provide content that explores the similarities and differences between multiplication and division, fractions and ratios, or fractions and rates, and so on in a manner that is accessible to learners (Namkung, Fuchs, & Koziol, 2018). Accordingly, it should not be shocking when prospective teachers, who were not given the opportunity to develop and create the required knowledge, make the same errors as middle school students. Additionally, it is essential to be aware of how slowly learners can develop these ideas (Lamon, 1995; Mack, 1995; Thompson, 1994). Thus, a learner may not be expected to develop a full knowledge for fractions in early ages and it can last longer.

In many curriculum textbooks for prospective teachers, multiplication and division are taught as simple extensions of addition and subtraction. Definition of multiplication has been taught as repeated addition (CCSSI, 2010); however, this approach can cause some issues for students. Students think that adding positive numbers make them to have a larger number than addends and that lead students think that product is always greater than factors of the multiplication operation. Also, it is not easy for students to think on the repeated addition when multiplying two fractions; for instance, when students are asked $\frac{1}{7} \times \frac{4}{5}$, they might have difficulties to think about the repeated addition. Similarly, for division, division is repeated subtraction and fair sharing is widely used (CCSSI, 2010), but this also might cause some issues. For instance, when students are sharing 20 pencils equally for 5 people, it is straightforward, but this sharing might be challenging when they need to use fractions. It is the fact that these are not the only ways to teach multiplication and division, and there are alternative ways to represent multiplication and division.

Many studies have shown that prospective teachers do not have a good comprehension of multiplication and division operations or rational numbers (Ball, 1990; Borko et al., 1992; Depaepe, 2015; Graeber et al., 1989; Olanoff, Lo, & Tobias, 2014; Post et al., 1988; Simon, 1993; Tirosh, 2000). For instance, according to Ball (1990), some prospective teachers did not answer division problems correctly, and very few of them provided appropriate mathematical explanations for the underlying principles and meanings. According to Ma (1999), the combination of multiplicative operations and fractions is a challenging concept for prospective teachers because multiplicative operations with fractions connects two difficult concepts and pushes them to develop mutually procedural and conceptual understanding.

Multiplication and division can be more complex than originally thought, a fact argued by Greer (1992) and Hiebert & Behr (1988) since multiplication and division are closely related, and interpretations of multiplication and division is sometimes not as accurate as it needs to be for multiplicative situations (Vergnaud, 1983).

1.1. Problem Statement and Rationale

Multiplicative situations are psychologically complex (Greer, 1992; Hiebert & Behr, 1988) and have often been treated in middle school textbooks as discrete and unconnected (Sowder, Armstrong, Lamon, Simon, Sowder, and & Thompson, 1998). Fractions are a fundamental foundational skill for future mathematics success (The National Mathematics Advisory Panel NMAP, 2008). Multiplicative situations with fractions, the most intricate operation with the most complicated numbers, can be viewed as a concept at the peak of arithmetic (Ma, 1999). Research showed that a considerable number of middle school prospective teachers had difficulty with multiplicative situations (Graeber, Tirosh, and & Glover, 1989).

While much has been written about fractions, there is an absence of literature which explicitly addresses how prospective teachers interpret multiplicative situation on fractions (Bradshaw, Izsák, Templin, & Jacobson, 2014). Thus, this study investigated middle school prospective teachers' interpretations of multiplicative situations on a fraction task which had five sub-questions. The data were collected from the final paper-and-pencil final exam for a mathematics content course offered to prospective middle school mathematics teachers at a state university in southeastern United States.

Therefore, the purpose of this study was to explore middle school prospective teachers' performance of interpretation of multiplicative situations on paper-and-pencil test items about fraction on a word problem. Accordingly, the present study examined the following research questions:

1. What interpretations do middle school prospective teachers make with fraction problems that involve multiplicative situations?
2. To what extent did middle school prospective teachers make explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods?

CHAPTER 2

LITERATURE REVIEW

This chapter reviews the literature on fractions and fractions arithmetic. The chapter first discusses key terms used in the literature and then summarizes reports of fractions, prospective teachers and fractions, conceptual and procedural knowledge for multiplicative situations with fractions, prospective teachers' pedagogical content knowledge for multiplicative situations with fractions, and multiplicative situations interpretations.

2.1. Concept of the Fraction and Prospective Teachers' Understanding Fractions

NCTM (2000) states that students should develop a deep understanding for fractions in the middle school to develop their skills to use fractions in problem solving. Fractions are relational representations that can be perceived as continuous or discrete quantities and are a challenging concept. A fraction is composed of a numerator and a denominator such as $\frac{a}{b}$ where a is numerator and b is denominator. For example, to represent the situation of 3 slices of a brownie which has 8 slices, we can use the figure below.

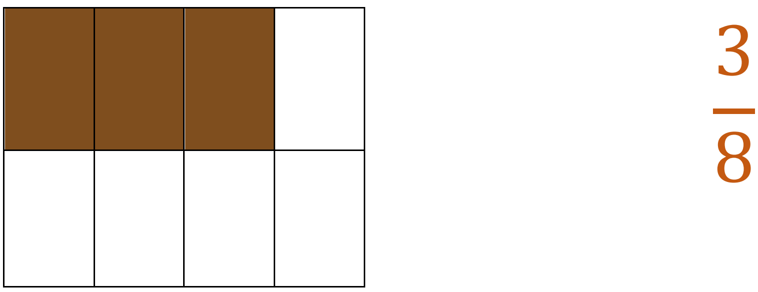


Figure 1. Fraction representation

When working with fractions, some operations can be required, and division operation is one of them. The general rule of division operation with fractions can be represented as $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$ where a, b, c, and d $\in Z$ where b, c, and d $\neq 0$ when we consider division operation with two fractions. This is a procedural definition for fractions. Conceptual definition of fractions should include both general principles about fractions, as well as knowledge of which principles underlie procedures for operating on them by connecting fractions to division, multiplying a whole number or a fraction by a fraction, and understanding multiplication as scaling in preparation for ratios and proportional relationships, and extending understanding of division to divide unit fractions by whole numbers and to divide whole numbers by unit fractions. Fraction arithmetic is a vital part of middle school mathematics. For example, in the Common Core State Standards Initiative ([CCSSI], 2010), it is stated that fraction arithmetic is a major topic of middle school, and teaching this concept starts with fraction addition and subtraction. Instruction of fraction addition and subtraction begins with the operations with common denominators and then includes operations with unequal denominators, and then students learn fraction multiplication and division. Students start to build a fundamental knowledge with the operations and applications to ratios, rates, and proportions in the fourth grade and then this gradually helps them be prepared to arithmetic operations with fractions.

According to The National Assessment of Educational Progress reports (2001), fractions can be accepted as "exceedingly difficult for children to master" (p. 5). According to NMAP (2008), at least 40 percent of middle school students have struggled with fractions and 50 percent of middle and high school students' challenges with elementary level fraction content. This finding indicates that fractions are considered a fundamental foundational skill for "successful participation in the contemporary American workforce" (NMAP, 2008, p. 3–11) to be able to accomplish daily activities (e.g., modifying recipes, ordering supplies) and to make decisions for assessing risks for medical treatments (Reyna & Brainerd, 2008; Subramaniam & Verma, 2009).

Siegler and Pyke (2013) conducted a study with sixth and eighth graders who were given 16 fraction arithmetic problems, four for each of the four arithmetic operations. Siegler and Pyke (2013) reported that the students' performance was better on fraction addition and subtraction as opposed to fraction multiplication and division. According to Bailey Hoard, Nugent, and Geary (2015), these results are not universal since they found that Chinese 6th grade students had a better performance on these 16 fraction arithmetic questions, approximately 90% of problems solved correctly; however, these results are an indicator for the U.S. children's fraction arithmetic performance (Bailey et al., 2012; Siegler, Thompson, & Schneider, 2011).

In some U.S. textbooks, fraction division has far less instruction than other arithmetic operations. For example, in Everyday Mathematics (2002) which has been the commonly used middle school curriculum text books and workbooks, there are 250 fraction multiplication questions, whereas there are 54 fraction division questions (Son & Senk, 2010). In Saxon Math (Hake & Saxon, 2003) textbooks, there are more questions for the fraction multiplication (n=122) than fraction division (n=56). Although fraction division is the least mastered arithmetic operation for students and teachers in the U.S., it is interesting that these very traditional textbooks do not include enough questions for fraction division (Siegler & Lortie-Forgues, in press; Siegler & Pyke, 2013). However, in Korean mathematics textbooks, there are 239 fraction multiplication question and 440 fraction division questions. It is the fact that 440 fraction division questions are 8 times higher than 54 and there are more questions for fraction division in Korean textbooks as opposed to U.S. textbooks (Son & Senk, 2010).

The NCTM (1989, 2000, 2006) provides a guide for mathematics curriculum development in the U.S. and advocates that fractional content integrates understanding of fractions as part of the number line, understanding of the relationship of fractions to whole numbers, and proficiency and fluency with addition, subtraction, multiplication, and especially with division of fractions. By considering the NCTM standards, NMAP (2008) proposed that students should be fluent in identifying and representing fractions by the end of grade 4, comparing magnitudes of fractions and adding and subtracting of fractions by the end of grade 5, multiplication and division of fractions by the end of grade 6, and all operations with positive and negative fractions by the end of grade 7.

Research in the literature showed that the concept of the division with fractions is challenging not only for students (Carpenter, Lindquist, Brown, Kouba, Silver, & Swafford, 1988) but also for prospective teachers (Ball, 1990; Simon, 1993). The findings of various studies (Ball, 1990; Simon, 1993) also revealed that providing a variety of problem-solving situations when teaching this concept, encouraging students to use multiple representations and to develop an understanding of different interpretations of divisions is momentous. There should be more emphasis on understanding and conceptualize division with fractions which would ultimately assist aspiring teachers and their future students. Prospective teachers and in-service teachers should understand division situations that utilize various numbers types (e.g., whole numbers, fractions, and decimals), combinations of number sizes (e.g., a smaller number divided by a more substantial number and vice versa), and contextual settings (e.g., continuous as well as discrete settings) (Izsák, Lobato, Orrill, & Jacobson, 2011). To spotlight the importance of understanding division with fractions, more studies are needed because more knowledgeable teachers are necessary to teach such multifaceted concepts of mathematics.

Both prospective teachers and in-service teachers should be treated as active learners since they build their understanding for mathematical concepts (Putnam & Borko, 1997) to scaffold their students' learning by using their skillset and integrating it into new situations. Therefore, research about prospective teachers' knowledge to teach division of fractions is essential to increase students' understanding of the concept since prospective teachers should "develop a sound and deep understanding of mathematics knowledge for teaching to build their confidence for classroom instruction" (Li & Kulm, 2008, p.833).

Lee Shulman (1986) stated that "Those who can, do. Those who understand, teach." (p.14). As educators in colleges or schools, we need to critique that whether or not we do teach what we know well and do not teach what we do not know enough. We need to think what prospective teachers will do when they find themselves in the position of having to teach a fundamental topic like division with fractions that they struggle. I highlight that when prospective teachers are provided learning and teaching opportunities to conceptualize the mathematical concept of a division with fractions, they will speak more confidently in their classroom when serving as teachers in the future.

2.1.Conceptual and Procedural Knowledge of Prospective Teachers for Fractions

According to the NMAP (2008), development of conceptual and procedural knowledge is essential in a mastery of division with fractions to create links between discrete pieces of knowledge. To scaffold students for learning fractions and division with fractions in the elementary and middle schools, effective instructional practices are essential. This is possible with well-educated prospective teachers who will teach their future students with an explicit and systematic instruction including step-by-step explanations.

The psychological complexity of fractions can be overlooked because of its operational simplicity. The operational aspect of learning fractions and multiplicative situations with fractions requires procedural knowledge. It is the fact that "…procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols" (Hiebert & Lefevre, 1986, p. 7) and "…procedural knowledge consists of rules or procedures for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbols" (Hiebert & Lefevre, 1986, p. 8). Furthermore, the psychologically complex component of fraction and multiplicative situations with fractions requires conceptual knowledge. Conceptual knowledge is "…a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network" (Hiebert & Lefevre, 1986, pp.3-4).

In tasks about multiplicative situations with fractions, connecting aspects of procedural knowledge (i.e., formulations, definitions, and mathematical operations) to properties of conceptual knowledge (i.e., linking the all pieces of information) are particularly important for academic achievement in middle school mathematics. Numerous studies have reported prospective teachers and in-service teachers have conceptual struggles with different aspects of fractions (e.g., Ball, 1990; Borko et al., 1992; Izsák, 2008; Izsák, Jacobson, de Araujo, & Orrill, 2012; Ma, 1999; Sowder, Philipp, Armstrong, & Schappelle, 1998; Tirosh & Graeber, 1990). Research has determined that teachers' mathematical knowledge for teaching has an important impact on student learning (Hill, Rowan & Ball, 2005). Teachers' both procedural and conceptual approaches have a crucial impact on students' outcomes and especially teachers' conceptual approach is so critical for students (Watson, Beswick, Brown, & Callingham, 2007; Cheeseman, 2007; Cooper, Baturo, & Grant, 2006). According to Van de Walle, Karp, and Bay-Williams (2004), when students are working on the multiplicative operations with fractions and when their answer should be more than being able to do a procedure.

Knowing this concept means that students can opine about the examples or situations for the division with fractions. It also means that students can use various strategies to solve problems, estimate an answer, represent the circumstances, and make a meaningful explanation about what happens in the multiplicative operations with fractions (Sinicrope, Mick, & Kolb, 2002; Yim, 2010). Groff (1996) explored that students often have difficulties to remember their prior experiences with fractions and how to use their prior experiences with fractions for new situations. There has been a rising acknowledgment over the past several decades about students' having been taught fraction arithmetic by memorizing steps and numerical procedures instead of learning these conceptually (Bradshaw et al., 2014). The lack of instructional support for students' conceptual understanding of fraction arithmetic results in students who are prone to make errors, since they can overlook the steps or generalize inappropriately (National Research Council, 2001).

Although most teachers can use algorithms to calculate the correct quotient of two fractions, many studies have reported that many prospective teachers and in-service teachers have struggled with meanings for fraction division (e.g., Ball, 1990; Ma, 1999; Ponte & Chapman, 2008; Tirosh & Graeber, 1990). It is the fact that teachers are guide to help students learn fractions (McDiarmid and Wilson 1991); however, research showed that teachers have difficulties for the meaning of multiplicative situations (Azim 1995; Borko et al. 1992; Post et al. 1991; Simon and Blume 1994; Tirosh 2000). Thus, there is an urgent need to help prospective teachers and in-service teachers to develop a conceptual understanding for meaning for operations of fractions so they can teach students why computation methods like “keep, change, flip” or “invert and multiply” make sense (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Tzur & Timmerman, 1997). For example, when students have been taught to divide fractions by memorizing to “keep, change, and flip” or “invert and multiply,” they are usually confused about whether they should invert, keep, or change the divisor or the dividend. As an example, a student can struggle to solve $\frac{3}{7} \div \frac{2}{9}$ by computing $\frac{3}{7} \times \frac{9}{2}$ or $\frac{7}{3} \times \frac{2}{9}$. Many prospective teachers have struggled with the fraction arithmetic, especially multiplication and division in middle grades. One of their main difficulties for fraction arithmetic is to identify the word problems that call for multiplication or division (Kaasila, Pehkonen, & Hellinen, 2010; Ma, 1999).

Kilic (2013) conducted a study in Turkey and proposed that prospective teachers have difficulties to understand meaning multiplicative situations with fractions, and Luo, Lo, and Leu (2011) stated that there is a need to prepare prospective teachers to make a better sense with both conceptual and procedural understanding on fraction multiplication or division operations. Research showed that prospective teachers have difficulties with fraction multiplication (Isık, 2011; Luo 2009) and fraction division (Isık, 2011; Rizvi 2004). In studies, no prospective teachers could create word problems for fractions with division (Rivzi, 2004) and for fraction multiplication (Luo, 2009). Revzi (2004) also found a remarkably interesting fact that prospective teachers in this research recognized that they have never been asked to create word problems on multiplicative situations with fractions during their all education lives. Also, this kind of results make researchers to ruminate about reasons prospective teachers’ inadequacy of conceptual understanding on the fraction multiplication and division concept. In this vein, Chapman (2012) stated that prospective teachers adequacy on the mathematical knowledge for a specific concepts is based on their post experiences. Unfortunately, prospective teachers do not have enough experiences (Tobias, Serow, & Schmude, 2010).

2.2. Pedagogical Content Knowledge (PCK)

Pedagogical content knowledge (PCK) of teachers is related to a specific mathematics domain, such as proportional reasoning (Watson, Callingham, & Donne, 2008), fractions (Watson, Beswick, & Brown, 2006), or so on. Shulman (1986) said that PCK provides teachers the ways to represent the mathematical content to students by using the most powerful appropriate tools in an efficient way. When a teacher who has a prominent level PCK, this teacher can have the enough knowledge to select the best tools amongst various options according to the grades and skills of the students according to the specific mathematical topic. Then students can understand the topic meaningfully that is possible when students are able to connect the ideas and connect to their previous knowledge (Hiebert & Carpenter, 1992) and teachers are the guides to support students to make meaningful connections. Therefore, prospective teachers' knowledge for the content, noticing the crucial nuances of the content of mathematics and practicing ways they can teach their students mean a lot for their students. If prospective teachers recognize what they struggle on when learning, they can recognize their needs and recoup the missing points.

2.3. Multiplicative Situations Interpretations

Research has shown that prospective teachers and teachers struggle to interpret multiplicative situations (e.g., Graeber & Tirsoh, 1988; Harel & Behr, 1995; Harel et. al.1994; Izsák & Jacobson, 2015; Tirsoh & Graeber, 1990). Fischbein, Deri, Nello, and Marino (1985) proposed that mathematical operations are "attached to the primitive behavioral models which have an impact on the choice of an operation" (p. 3). Even after students learned far beyond their primitive models for the mathematical operations, they keep being influenced by primitive models (Simon, 1993). Fischbein et al. (1985) stated that the primitive model of multiplication is repeated addition and the primitive models of division is partitive (fair share) and quotative (measurement). Simon (1993) also stated that partitive division could be defined "an object or collection of objects is divided into a number of equal fragments or sub collections" and with quotitive division the purpose is to determine "how many times a given quantity is contained in another quantity" (p. 235). For instance, according to Izsák Jacobson, de Araujo, and Orrill (2012) "if each car can take 5 people and 20 people want to go on a field trip, how many cars are needed (how many 5s are in 20)?" (p. 391) and this is a quotitive situation. Also, "if 20 people want to go on a field trip and there are 5 cars, how many people should get into each car (how many people in each of 5 cars)?" (p. 391) and this is a partitive situation.

In other words, the division operation v ÷ w where v and/or w are fractions can be thought of as "How many w's are there in one v?". Ölmez (2014) also defines the operation v ÷ w as v items divided into w groups. In this division operation, v ÷ w, we should decide whether "we are looking for the number of groups (how many groups) or for the size of each group (how many in each group)" (p. 6). A response would have two interpretations. It can be interpreted as how many groups division when v items are divided by w (i.e., v ÷ w) in each group to find the number of groups. In contrast, it can be interpreted as how many in each group division when v items are shared by w groups equally to find the number of units in one group.

For example, on the one hand, when each brownie of an 8-slice brownie is sliced in half and each slice are placed on a plate (i.e., 8 ÷ 1/2), we would need 16 plates (the number of groups) (see Figure 2).

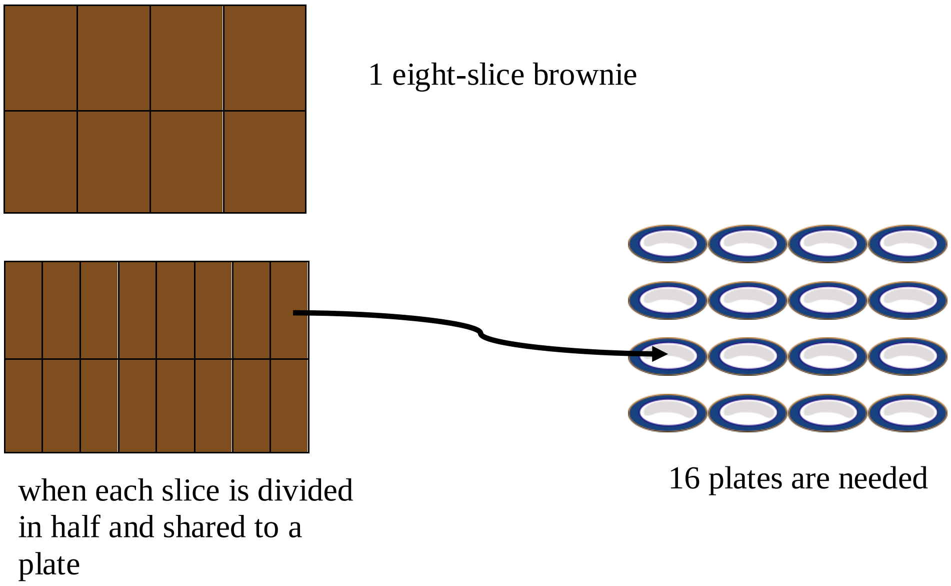


Figure 2. How-many-group division

On the other hand, if we are looking for how many slices in a whole group if there are 8 slices in half of a group (i.e., 8 ÷ 1/2), there becomes 16 slices on the plate (the number of units in 1 group) (see Figure 3).

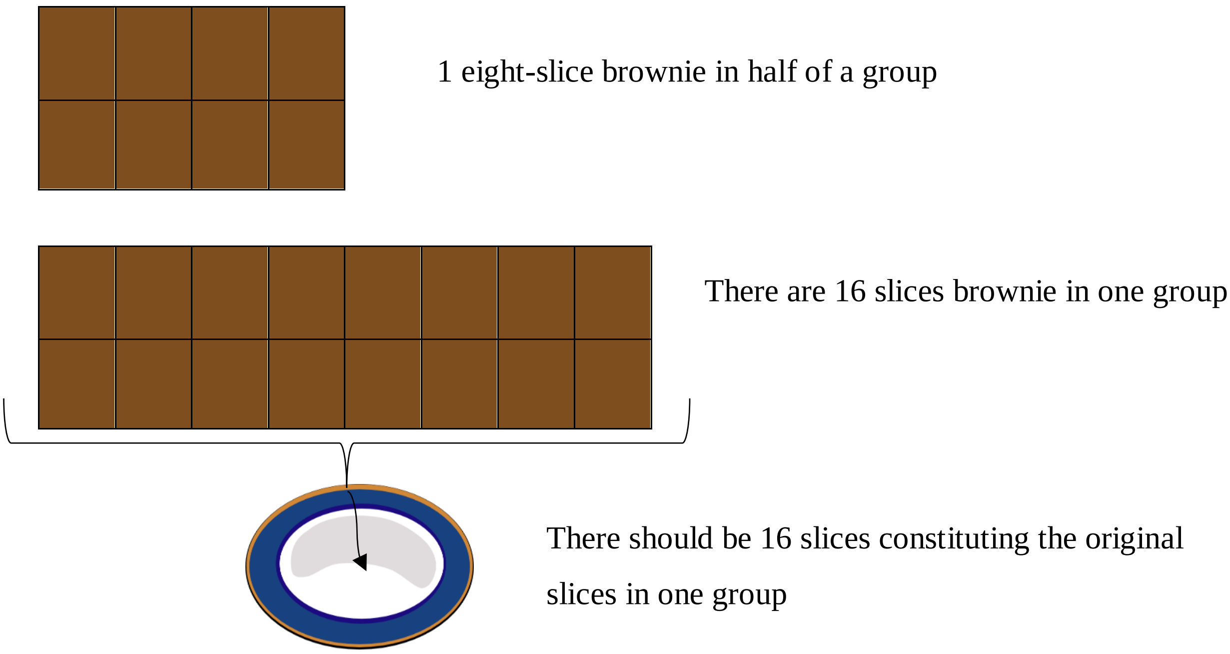
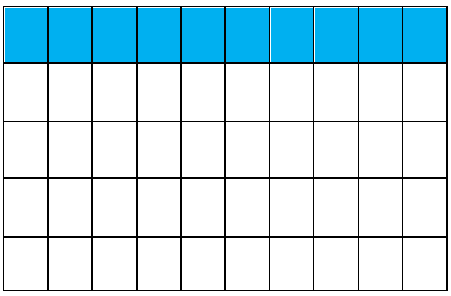
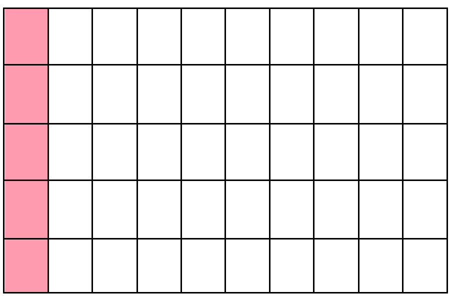


Figure 3. How-many-in-one-group division

Merely having an interpretation for multiplication and division (quotitive or partitive) is not always sufficient to complete a fraction task which requires some multiplicative operation when numbers are embedded in problem situations (Izsák, Lobato, Orrill, & Jacobson, 2011). There is a need to be able to understand the quantities to which numbers refer and to show units and groups in an equation. For example, for the problem: *A chocolate factory uses* $\frac{1}{10}$ *of a bag of cacao in each batch of chocolate. The factory used* $\frac{1}{5}$ *of a bag of cacao yesterday. How many batches of chocolates did the factory make?*, there is a need to decide what the number of groups, number of units, product amount, and the what the question is asking for. First, the whole amount is $\frac{1}{5}$ and that is product amount, the amount in each batch is $\frac{1}{10}$ that is the number of the units, and the question is asking number of the batches that is the number of the groups. Therefore, considering the equation in the form of (the number of units) • (the number of groups)= (product amount), our equation is $\frac{1}{10} \cdot$ $(\text{the number of groups}) = \frac{1}{5}$. Thus, the number of groups is $\frac{1}{5} \div \frac{1}{10} = \frac{1}{5} \cdot 10 = 2 =$ $\text{the number of groups}$ that gives the number of the batches and concludes the solution (see Figure 4).



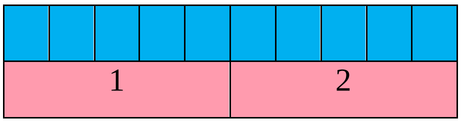
The shaded area represents $\frac{1}{5}$



The shaded area represents $\frac{1}{10}$

Question: How many groups of pink shaded area are there in blue shaded area?

Answer: There will be two groups of $\frac{1}{10}$ in $\frac{1}{5}$



$$\frac{1}{5} \div \frac{1}{10} = \frac{1}{5} \cdot 10 = 2$$

Figure 4. Pictorial solution for example question

Understanding various interpretations of multiplicative situations with fractions and conceptual underpinnings of this concept is crucial since making reasoning for these foundations takes the understanding division with fractions beyond memorization of the rules. There has been a rising acknowledgment over the past several decades that students have been taught fraction arithmetic by memorizing steps and numerical procedures instead of learning these mathematical concepts conceptually (Murray, Olivier, & De Beer, 1999).

2.1. Analytical Framework

In this study, the analytical framework developed by Izsák, Jacobson, and Orrill (2014) was used to explore the prospective teachers' reasoning about the fractions. Beckmann and Izsák (2015) formulized the multiplicative situations equation representing multiplicative structure in the form of N•M= P., where the multiplicand (N) is the number of the units in each whole group, the multiplier (M) is the number of the groups, and the product amount (P) is the total number of units in all the groups. A key feature is consistently writing multiplicand and the multiplier in the same order. Following Beckmann and Izsák, in this study, multiplication expressions are considered in the form of "multiplicand • multiplier" (N•M). I used this analytical framework in my study because it provides a distinct way to compare different types of interpretations by using N•M= P equation. According to Izsák et al. (2011), interpretation of partitive division requires answering the question "how many objects (or units) are in each group when A objects (or units) are separated into B groups" (p. 17). In contrast, interpretation of quotitive division situations requires answering the question "how many groups are formed when A objects (or units) are separated into groups of B objects (or units)" (p. 17). According to both Izsák, et al. (2011) and Ma (1999), teachers have a better performance with whole numbers with partitive division (i.e., fair sharing). They also proposed that there are very few teachers who can recognize multiplicative situations. To better equip teachers to better understand operations, more research exploring pre-service and in-service teachers' understanding of multiplicative relationships is needed.

Multiplication/Division	N • M = P (# of units in each/one whole group) • (#of groups) = (# of units in M group)
Multiplication Situations (MS)	N• M= ☐ where P is the unknown
Partitive Division Situations (PDS)	☐ • M= P where N is the unknown
Quotitive Division Situations (QDS)	N • ☐= P where M is the unknown

Figure 5. The analytical framework used in the study (adapted from Izsák et al., 2011, p. 19)

Figure 5 shows that in an equation in the form of N•M = P,

- when N and M are known and P is unknown, it is a multiplication situation.
- when M and P are known, it is partitive, fair sharing, or how many units in 1 group situation.
- when N and P are known, it is quotitive, measurement, or how many groups situation.

Figure 6 shows three sample questions from my data and for each question, sample solutions and their interpretations using the analytical framework (see Figure 5). According to Figure 6, first question is MS, the second one is PDS, and third question is QDS.

#	Problem	Solution	Analytical Framework	Interpretation
1	If $\frac{3}{4}$of a gallon ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound ice cream?	$\frac{3}{4}$ is number of units in 1 group that is N, $\frac{2}{3}$ is the number of the group that is M, and the number of units in $\frac{2}{3}$ group that is product amount (P) is unknown. That is $\frac{3}{4} \times \frac{2}{3} = \mathrm{P}$ so $\mathrm{P} = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$.	N• M= ☐ where P is the unknown	MS
2	If $\frac{2}{3}$of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?	$\frac{2}{3}$ is number of groups, the number of the units in 1 group is unknown that is N, and $\frac{3}{4}$ is the number of the units in M group that is product amount. That is $\mathrm{N} \times \frac{2}{3} = \frac{3}{4}$, so $\mathrm{N} = \frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$.	☐• M= P where P is the unknown	PDS
3	If $\frac{3}{4}$of a gallon ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon ice cream weigh?	$\frac{3}{4}$ is number of units in 1 group, the number of the group is unknown that is M, and $\frac{2}{3}$ is the number of the units in $\frac{3}{4}$ group that is product amount. That is $\frac{3}{4} \times \mathrm{M} = \frac{2}{3}$, so $\mathrm{M} = \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.	N • ☐ = P where N is the unknown	QDS

Figure 6. Sample questions with their solutions and interpretations utilizing the analytical framework

CHAPTER 3

METHOD

The purpose of this qualitative case study was to investigate middle school prospective teachers' performance on paper-and-pencil test items about fractions by concentrating on their use of the multiplicative situation interpretations. I investigated to what extent middle school prospective teachers make explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods. In this chapter, first I will discuss the research method. Then I will describe the data collection and analysis procedures followed.

3.1. Research Design

The purpose of this study was to explore middle school prospective teachers' performance of interpretation of multiplicative situations on paper-and-pencil test items about fraction on a word problem. The focus was on their use of the equation in the form of N • M= P by determining the number of units and the number of groups in a fraction word problem. I examined how they interpreted the multiplicative situation in the fraction task regarding quantities (i.e., units and groups) with an equation considering N • M= P, various notations, and specific meanings for multiplication and division (i.e., MS, PDS, and QDS).

To address the research questions: (1) What interpretations do middle school prospective teachers make with fraction problems that involve multiplicative situations?; (2) To what extent did middle school prospective teachers make explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods?, qualitative methods were utilized to examine pre-service teachers' solutions.

Qualitative research methods were used to discover the meanings that participants created in context or an activity (Wolcott, 2009). When reviewing prospective teachers written work qualitatively, I analyzed features of solutions to determine what they employed. I also provided frequency tables to reveal additional information and help clarify the primarily qualitative study. With qualitative approach, "the researcher builds a complex, holistic picture, analyzes words, reports detailed views of informants, and conducts the study in a natural setting" (Creswell, 2008, p. 15).

This qualitative case study explored the following question: (a) *What interpretations do middle school prospective teachers make with fraction problems that involve multiplicative situations?* (b) *To what extent did middle school prospective teachers make explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods?* Yin (2003, 2009) proposed that in a case study, participant behavior manipulation is not possible. This methodology can be accepted as "a phenomenon...bounded by a certain context…in effect [it is] your unit of analysis" (Miles & Huberman, 1994, p. 25). In this study, the prospective teachers' classroom represents the unit of analysis. In the data collection and data analysis part, I explain the qualitative data collection and analysis procedure.

3.2. Participants and Data Collection

Data for the present study were collected from 13 junior middle school prospective mathematics teachers at a large, public university in the Southeastern United States during the Spring 2017 semester of a two-semester mathematics content course. The first semester focused on numbers and operations, and the second semester focused on topics related to fraction division, ratio, proportional relationships, and algebra. Both courses emphasized the meaning of multiplicative situations and included multiplication, division, and fractions. These courses were designed to help prospective teachers develop practices outlined in the Common Core State Standards (CCSS) for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

These practices are “make sense of problems and persevere in solving them; reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; model with mathematics; use appropriate tools strategically; attend to precision; look for and make use of structure; and look for and express regularity in repeated reasoning” (CCSS, 2010). Throughout the course, a textbook “Mathematics for Elementary Teachers with Activities” (Beckmann, 2014) was used. It was a routine practice in these courses that the prospective teachers worked in groups during class and work on tasks collaboratively interpreting the multiplicative situation in the fraction task regarding quantities considering N•M= P formulization, using various notations, and specific meanings for multiplication and division; however, they individually completed homework assignments and examinations.

The timelines below summarize the topics covered in the first and second semester courses. As shown in Figure 7, number operations were included mainly in the first course and algebra was included in the second course.

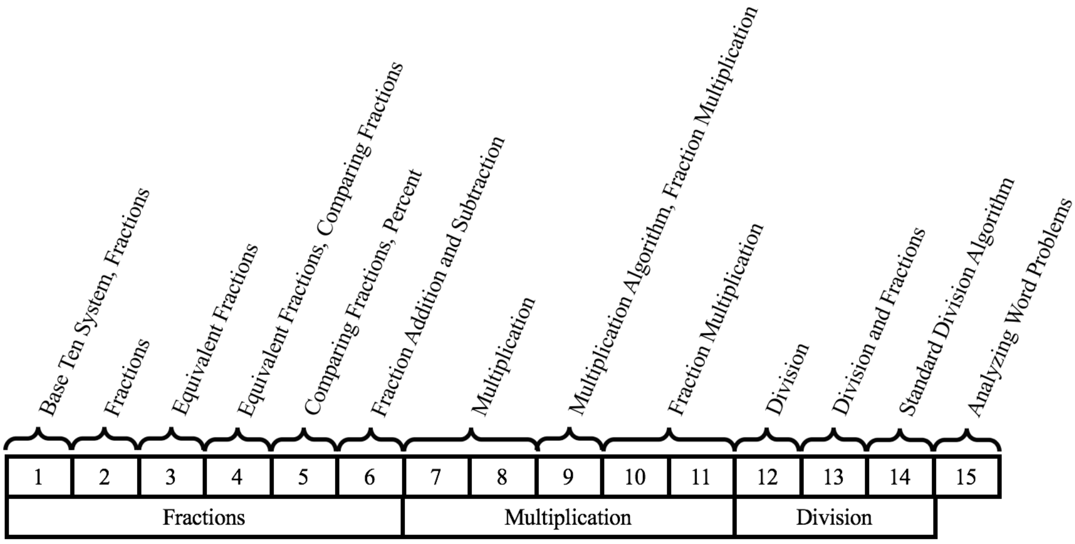


Figure 7. Timeline of the First Course in Fall 2017

As shown in Figure 8, the second course addressed fraction division, ratio and proportional relationships, statistics, probability, and number theory.

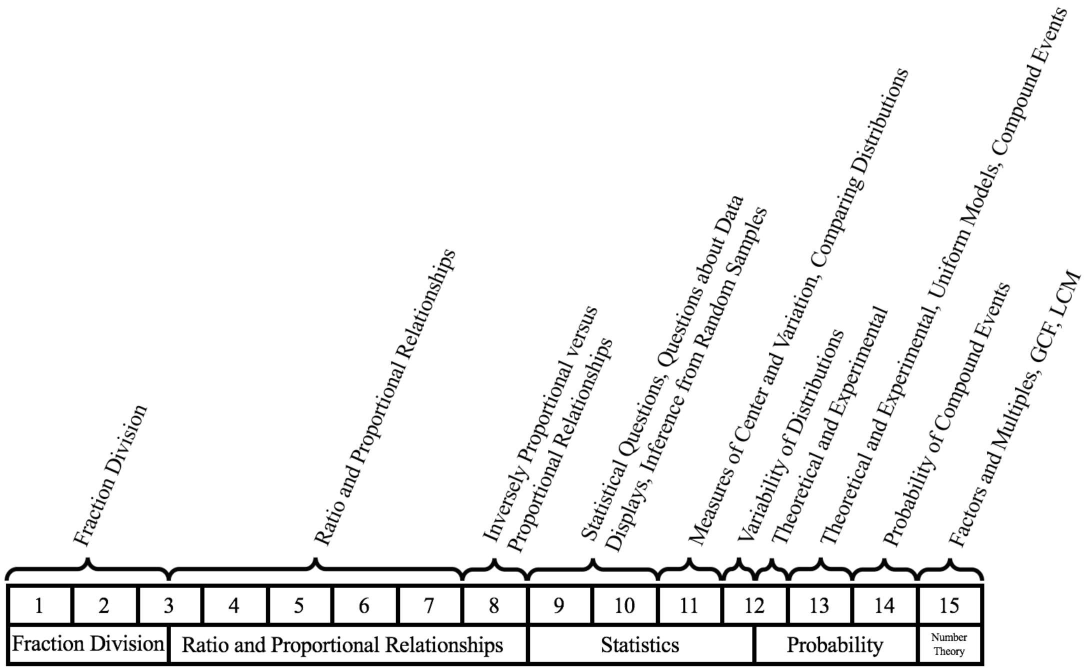


Figure 8. Timeline of the Second Course in Spring 2018

Instructor of this course who worked with prospective teachers for years and allows students think, question, and look for ways to go more deeply into the material. In the classroom, prospective teachers collaborate with peers and share and generate knowledge with their groupmates first and classmates later, so prospective teachers grapple with the ideas together in the groups when working on in class activities. Instructor of this course is interested in prospective teachers' reasoning about ratio and proportional relationships and associated multiplicative ideas. In the class, the instructor aimed to figure out what is harder for the teachers, what is easier, and how they think about those tightly intertwined ideas. The instructor of this course thinks that there are two types of ratios and proportional relationships that fit with the two types of division. There is a "how many in each group" type of division and a "how many groups" type of division, and there is a parallel situation for ratios and proportional relationships. Therefore, the instructor decided to explore how prospective teachers understand different definitions or versions of ratios and proportional relationships. This a two-semester mathematics content course was designed by considering those purposes. This course helped students make sense of problems and persevere in solving, reasoning, modeling, arguing, and critiquing them and develop a fundamental knowledge for fractions, fraction arithmetic, ratio and proportional relationships, statistics, probability, and number theory.

In this study, I selected a question which comprised of five sub-questions from the second semester's final exam to analyze (see Figure 9) since this question allowed the middle grades prospective teachers to choose their solutions for MS, PDS, or QDS, as opposed to items that directed them to use a particular one.

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

Figure 9. The task items used for data collection

For each sub question, structure of the task and answer key is provided in Figure 9.

Task item	Usual Potential Answers	Usual Quantities	Writing Equations	Situation Interpretations
a	For the equation N• M = P where N=? M= 2/3, and P= 3/4.	N is number of base units in 1 group, M is the number of groups, and P is the number of base units in 2/3 group.	This question is ?•2/3= 3/4	The operation is 3/4÷2/3 which is asking how many units in one group. Thus, it is how-many-units-in-one-group division or PDS problem.
b	For the equation N• M = P where N=?, M= 3/4, and P= 2/3.	N is number of base units in 1 group, M is the number of groups, and P is the number of base units in 3/4 group.	This question is ?•3/4= 2/3	The operation is 2/3÷3/4 which is asking how many units in one group. Thus, it is how-many-units-in-one-group division or PDS problem.
c	For the equation N• M = P where N=3/4, M=2/3, and P=?	N is number of base units in 1 group, M is the number of groups, and P is the number of base units in 2/3 group.	This question is 3/4•2/3= P	The operation is 3/4•2/3 which is asking for multiplication operation. Thus, it is MS problem.
d	For the equation N• M = P where N=3/4, M=?, and P= 2/3.	N is number of base units in 1 group, M is the number of groups, and P is the number of base units in ? group.	This question is 3/4•M= 2/3	The operation is 2/3÷3/4 which is asking how many groups. Thus, it is how-many-group division or QDS problem.
e	For the equation N• M= P where N=3/4, M=2/3, and P=?	N is number of base units in 1 group, M is the number of groups, and P is the number of base units in 2/3 group.	This question is 3/4•2/3= P	The operation is 3/4•2/3 which is asking for multiplication operation. Thus, it is MS problem.

Figure 9. Structure of the task and answer key

Data analysis was guided by grounded theory methodology. Charmaz (2006) defined grounded theory as methods that "consist of systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct theories 'grounded' in the data themselves" (p. 2). The purpose in grounded theory is for researcher allow what is relevant to one's research question to emerge from the data. Hutchinson (1988) focusses the relevance of grounded theory method to education, noting that grounded theory allows for the explanation of the everyday world of teachers and students. With "a focus on lived experience, patterns of experience and judging and appraising the experience", grounded theory provides a way to "study the richness and diversity of human experience" (Hutchinson, 1988, p. 127).

Creativity is crucial in the grounded theory since researcher can "break through assumptions and create new order out of the old" (p. 27). Being creative helps naming categories. Grounded theory follows a three-stage process to coding the data. Open coding involves identifying concepts. Data were compared and sorted according to themes. In axial coding stage, the researcher looks for relationships between categories by comparing with each other. Another stage in the coding is the selective coding. In selective coding the core category in which the final analysis was done is selected. The words and codes were analyzed and grouped together according to common theme (Charmaz, 2006). Grounded theory provided a framework to analyze data through sensitizing the concepts.

In this study, I will use Figure 10 as a coding schema. It has four categories as category 0, category 1, category 2, and category 3. Having categories was helpful because it provided me to have a strong organizational structure is paramount to see how each solution is different than another solution and to understand if prospective teachers solved problems correctly, completely or if they included any information about multiplication and division, units and groups, and multiplicative situation interpretations.. The hierarchy between categories allowed me to identify each solution for each prospective teacher.

Drawing on the analytical framework, prospective teachers' solutions were classified under category 0, category 1, category 2, and category 3 (see Figure 10).

Those categories were prepared according to steps for grounded theory as open coding. In a solution, I am looking at three main things: (a) if prospective teachers use multiplication or division correctly (i.e., use of the multiplication sign × or •, and mentioning multiplication in a statement), (b) if prospective teachers included any explicit and correct information about the number of units and groups, and (c) if prospective teachers interpreted the multiplicative situation correctly. The category 0 included prospective teachers' solutions that consist of use of incorrect, incomplete, or not included multiplication or division operations, have incorrect, incomplete, or not included information about the units and groups, and have incorrect, incomplete, or not included interpretations of multiplicative situations altogether. The category 1 comprised of prospective teachers' solutions that include incorrect, not completed, or not included information for two of a, b, and c. The category 2 comprised of prospective teachers' solutions that include only one incorrect, incomplete, or not included from a, b, and c. The category 3 comprised of prospective teachers' solutions that includes correct solutions for a, b, and c altogether.

3.3. Ethical Considerations

In order for research to be trusted four constructs need to be developed: credibility, transferability, dependability, and confirmability (Marshall & Rossman, 2006). In this study credibility is addressed by analyzing the data from the classroom assessments. Credibility is also addressed by analyst triangulation because two different researchers have reviewed the analysis and the results were carefully compared. Transferability is demonstrated when researchers "argue that findings will be useful to others in similar situations, with similar research questions or questions of practice" (Marshall & Rossman, 2006, p. 201). Confirmability in this research was addressed through adhering to internal consistency of the data in relation to findings, interpretations, and recommendations. To address confirmability, I situated the study in the research literature. Two raters independently worked on the data by looking at the criteria tables for categories and criteria for each interpretation (which have been created and developed by me). This resulted in an interrater reliability (Cohen's Kappa) of between .87-.95 on individual questions.

In this study, all of the ethical guidelines of the University's Institutional Review Board (IRB) were followed. In order to protect the participants' privacy, all of the student's exam questions were coded using pseudonyms. The data were password-protected, and the computer was locked when not in use. All of the participants were made aware that I would be careful to protect their confidentiality and privacy. As is customary, they were also offered the opportunity to withdraw from the study at any time without a penalty in the course.

3.4. Data Analysis

In the qualitative data analysis, "a code is a researcher-generated construct that symbolizes or translate data and thus attributes interpreted meaning to each individual datum for later purposes of pattern detection, categorization, assertion, or proposition development, theory-building, and other analytic processes" (Saldana, 2017, p. 1). In this study, after data collection, I used open coding. I read data word-by-word initially. Then I broke the data up into their component parts or properties, looked for tacit assumptions, or explicated implicit actions and meanings; crystallized the significant of the points, and compared data with data. In this process, I checked if prospective teachers included multiplicative situations (multiplication and division) and how they represented multiplication and division operations, if they included any information about groups and base units in their response, if they included multiplicative situation interpretation explicitly or implicitly by considering definition and distinctions of interpretations. After detecting all of these, I compared 13 prospective teachers' responses for five sub-questions on the final exam of the content course by considering my open coding. With axial coding stage, based on my comparison of prospective teachers' responses, I decided to have three categories. I looked at relationships among categories that allowed making connections among prospective teachers' responses. Table 1 shows structure of the task and potential answers. The prospective teachers can have different solution methods.

Table 1. *Coding Schema*

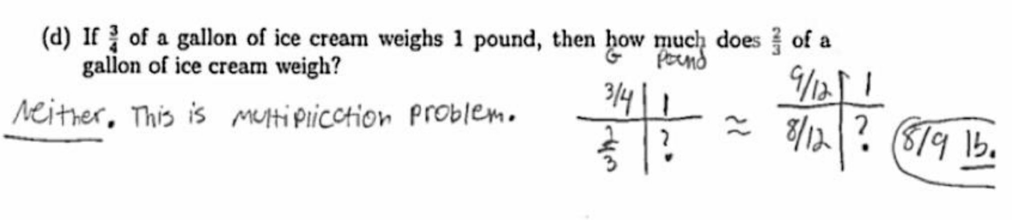
In my analysis, I checked if prospective teachers: **a) Include division or multiplication** o Indicator for multiplication: ▪ Use of the multiplication sign × or • (e.g., N•M=P) ▪ Mention multiplication in a statement o Indicator for division: ▪ Use of division operation sign ÷ ▪ Mention division in a statement ▪ Use the multiplication with a missing factor (e.g., N • ? = P) ▪ **b) Include quantities and number of units and number of groups** o Showing the number of units and groups for M, N and P correctly and this can be explicit or implicit ▪ e.g., for question d, N is the number of units that is 3/4 and M is the number of groups and that is unknown when P is the number of units in M group and that is 2/3. That is an explicit demonstration of quantities and number of units and number of groups. ▪ e.g., for question c, ¾ is pint ice-cream in 1 group, 2/3 is group, and ? is the pint ice cream in 2/3 group. That is not an explicit demonstration of quantities and number of units and number of groups, but it is still correct and this will be enough for my coding. ▪ **c) Include multiplicative situation interpretations** o Indicators for Multiplicative Situation Interpretations ▪ when M and P are known and N is unknown: how-many-in-one-groups division or partitive division ▪ when N and P are known: how-many-groups division or quotitive division ▪ when M and N are known and P is unknown: multiplication situation or neither (i.e., neither means it is not PDS or QDS)

Table 1 (continued).

Criteria for Category 0	Criteria for Category 1
• If prospective teachers have incorrect solutions for a, b, and c altogether, • If prospective teachers have incomplete solutions for a, b, and c altogether, • If prospective teachers do not include a, b, and c altogether in their solutions, their response for the question is categorized in category 0	• If prospective teachers have incorrect solution for two of a, b, and c, • If prospective teachers have incomplete solutions for two of a, b, and c, • If prospective teachers do not mention two of a, b, and c their response for the question is categorized in category 1
Criteria for Category 2	**Criteria for Category 3**
• If prospective teachers have incorrect solution for one of a, b, and c, • If prospective teachers have incomplete solutions for one of a, b, and c, • If prospective teachers do not mention one of a, b, and c their response for the question is categorized in category 2	• If prospective teachers have correct solutions for a, b, and c altogether their response for the question is categorized in category 3

By using coding schema, I provide some examples of the use of coding schema for each category from Figure 10a through Figure 10d. PT 6's solution for d (see Figure 10a) was coded as category 0 since PT 6 used multiplication rather than division, did not include any information about the number of units and groups, and interpreted this situation as MS instead of PDS. PT 5 for question b (see Figure 10b) had similar issues for the number of units and groups and interpretation, but PT 5 used division correctly, so this solution was in category 1. Also, PT 12 for the solution of a (see Figure 10c), correctly used multiplication, and interpreted situation as MS, but unfortunately, did not included any information about the number of groups and units, so I coded this solution as category 2. Unlike these solutions, PT 7 used multiplication correctly, included information about units and groups, and interpreted the situation correctly, so it is in category 3 (see Figure 10d).

PT's Solution


(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

Neither. This is multipliccation problem.

$\begin{array}{c|c} 3/4 & 1 \\ \hline \frac{2}{3} & ? \end{array} \approx \begin{array}{c|c} 9/12 & 1 \\ \hline 8/12 & ? \end{array}$ 8/9 lb.

Explanation

Category 0:

a) Multiplication/Division: *Incorrect*
- PT 6 used the multiplication, but for this question PT 6 should use division.

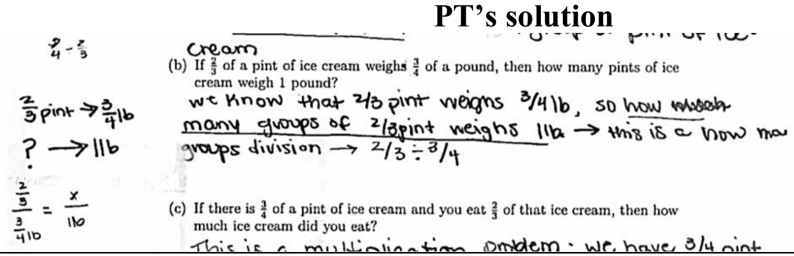
b) Groups and base units: *Not Included*
- PT 6 did not include information about groups and base units.

c) Multiplicative situation interpretation: *Incorrect*

PT 6 interpreted this situation as MS not QDS.

Figure 10a. Samples of PTs' solutions for category 0

PT's solution


(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

we know that 2/3 pint weighs 3/4 lb, so how much many groups of 2/3 pint weighs 1lb → this is a how ma groups division → $2/3 \div 3/4$

$\frac{2}{3}$ pint → $\frac{3}{4}$ lb

? → 1 lb

$\frac{\frac{2}{3}}{\frac{3}{4}\text{lb}} = \frac{x}{1\text{lb}}$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

Explanation

Category 1:

a) Multiplication/Division: *Correct*
- PT 5 used the division operation sign.

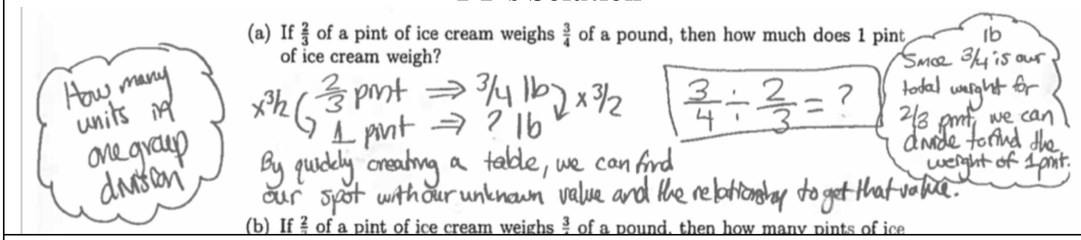
b) Groups and base units: *Not Included*
- PT 5 did not include information about groups and base units.

c) Multiplicative situation interpretation: *Incorrect*

PT 5 interpreted this situation as QDS not PDS.

Figure 10b. Samples of PTs' solutions for category 1

PT's Solution


(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

How many units in one group division

$\times\frac{3}{2}$ ($\frac{2}{3}$ pint ⇒ $\frac{3}{4}$ lb) $\times\frac{3}{2}$
1 pint ⇒ ? lb

$\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$

By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.

lb
Since 3/4 is our total weight for 2/3 pint, we can divide to find the weight of 1 pint.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice

Explanation

Category 2:

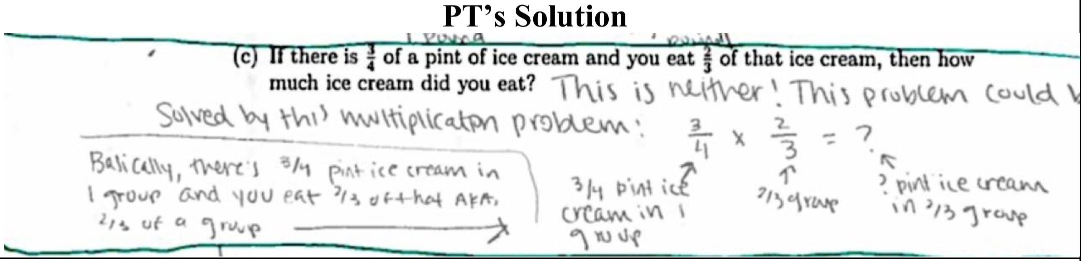
a) Multiplication/Division: *Correct*
- PT 12 used the division operation sign.

b) Groups and base units: *Not Included*
- PT 12 did not include information about groups and base units.

c) Multiplicative situation interpretation: *Correct*
- PT 12 interpreted this situation as PDS.

Figure 10c. Samples of PTs' solutions for category 2

PT's Solution


(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is neither! This problem could b

Solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$

Basically, there's 3/4 pint ice cream in 1 group and you eat 2/3 of that AKA, 2/3 of a group

3/4 pint ice cream in 1 group
2/3 group
? pint ice cream in 2/3 group

Explanation

Category 3:

a) Multiplication/Division: *Correct*
- PT 7 used the multiplication operation sign.

b) Groups and base units: *Correct*
- PT 7 included information about groups and base units correctly.

c) Multiplicative situation interpretation: *Correct*
- PT 7 interpreted this situation as MS.

Figure 10d. Samples of PTs' solutions for category 3

CHAPTER 4

RESULTS

Middle grades prospective teachers who completed the two-semester sequence of content courses emphasizing topics related to ratio, proportional relationships, fraction division, algebra, and the meaning of multiplication were able to interpret the multiplicative operations appropriately. When working on a task that allowed them to decide their own choice for multiplication situation (MS), partitive division situation (PDS), and quotitive division situation (QDS). Results showed that prospective teachers used strategies involving multiplicative situations after completing a two-semester sequence of mathematics content courses on fraction tasks. This instructional approach supported the development of an understanding of multiplicative operations with fractions and understanding of the meaning of multiplication and division for middle school prospective teachers.

In Table 2, I used color codes for each PST's solutions for each sub questions. The purpose of this table is to be able to explicitly and specifically show the extent middle school prospective teachers made explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods and interpretations middle school prospective teachers made with fraction problems that involve multiplicative situations. More details for each prospective teacher for each question were included in appendices (see Appendices A-M). Table 1 shows each prospective teachers' performance for every sub-question. For example, prospective teacher 1 (PT 1) used division operation sign for the solution of b correctly and did not include information about units and groups in her solution. Also, PT 1 interpreted the situation as PDS correctly and explicitly. Thus, PT 1 is in the category 2 considering the coding schema. According to table below, green shows these three criteria that use of multiplication and/or division, use of groups and base units, and division interpretations are completely correct, red shows that prospective teachers' solutions for each sub question is wrong, yellow shows that in prospective teachers responses for each sub question the criteria was not included, and blue shows that in prospective teachers' responses these three criteria were included but the response was incomplete.

The study showed that six prospective teachers (PT 1, PT 2, PT 4, PT 7, PT 8, and PT 13) used multiplicative situation operation signs correctly and appropriately for each question, four prospective teachers (PT 2, PT 7, PT 8, and PT 13) included the necessary information for the number of the groups and base units correctly and appropriately for each question, seven prospective teachers (PT 1, PT 2, PT 3, PT 4, PT 7, PT 8, and PT 13) interpreted multiplicative situations correctly and appropriately for each question. Three prospective teachers (PT 6, PT 9, and PT 11) used multiplicative situation operation signs in four out of five sub-questions correctly and appropriately, three prospective teachers (PT 4, PT 9, and PT 11) included the necessary information for the number of the groups and base units in four out of five sub-questions correctly and appropriately, and four prospective teachers (PT 1, PT 4, PT 5, PT 6, and PT 9) interpreted multiplicative situations in four out of five sub-questions correctly and appropriately.

Table 2. *Prospective teachers' solutions for the five sub questions with color codes*

PTs	**Criteria**	**a**	**b**	**c**	**d**	**e**
PT 1 (NB)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 2	Cat 3	Cat 3	Cat 2

Table 2 (continued).

PT 2 (MB)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 3	Cat 3	Cat 3
PT 3 (AC)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 1	Cat 2	Cat 3
PT 4 (EE)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group (unit is wrong, but group is correct)	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 3	Cat 2	Cat 3

Table 2 (continued).

PT 5 (JG)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 2	Cat 1	Cat 1	Cat 2	Cat 2
PT 6 (JH)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 2	Cat 3	Cat 3	Cat 0	Cat 2
PT 7 (LH)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 3	Cat 3	Cat 3

Table 2 (continued).

PT 8 (CH)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 3	Cat 3	Cat 3
PT 9 (CJ)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 0	Cat 2	Cat 3
PT 10 (CM)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 1	Cat 1	Cat 1	Cat 1	Cat 0

Table 2 (continued).

PT 11 (SP)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 0	Cat 1	Cat 3	Cat 2
PT 12 (AS)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 2	Cat 1	Cat 0	Cat 0	Cat 3
PT 13 (WY)	Multiplication/ Division	Use of ÷	Use of ÷	Use of ×	Use of ÷	Use of ×
	Group/Base Unit	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group	Mention unit and group
	Multiplicative Situations Interpretation	Interpret it as PDS or how-many-units-in-one-group	Interpret it as PDS or how-many-units-in-one-group	Interpret it as MS or neither	Interpret it as QDS or haw-many-groups	Interpret it as MS or neither
Category (Cat)		Cat 3	Cat 3	Cat 3	Cat 3	Cat 3

Color	Meaning
	Done correctly
	Done incorrectly
	Not Included
	Included but incomplete

Also, I provide a different version of the Table 1 for more clarification without using colors. In the next table PTs' correct responses are showed with "+", incorrect responses are showed with "X", incomplete solutions are showed with "-", not included solutions showed with "N". According to Table 2, 8% of 65 solutions is categorized in Category 0 (5 out of 65 solutions), 15% of 65 solutions is categorized in Category 1 (10 out of 65 solutions), 17% of 65 solutions is categorized in Category 2 (11 out of 65 solutions), and 60% of 65 solutions is categorized in Category 3 (39 out of 65 solutions) (see Table 2). This shows that instruction focusing on definitions multiplicative interpretations of the situations during a mathematics content course was helpful for prospective teachers to help their learning conceptually multiplicative operations with fractions and operate correctly and explicitly what they learned.

According to analysis showed by Table 1 and 2, all solutions categorized in categories 1-3, so there is no solution was categorized as category 0 for the question a. However, PT 11's solution for b was categorized in category 0 since this prospective teacher used division operation sign incorrectly, included the number of groups and units incorrectly, and interpreted situations incorrectly. Also, solutions of PT 9 and PT 12 were categorized in category 0 since PT 9 did solve whole problem incorrectly similar to PT 11 and PT 12 did incomplete arithmetical operation for situation, did not include any information about the number of units and groups, and interpreted situations incorrectly.

Solutions of PT 6 and PT 12 were categorized in category 0 since they both did not use operation sign correctly, did not include any information about the number of units and groups, and did not interpret correctly (see Table 3).

Table 3. *Prospective teachers' solutions for the five sub-questions*

	a				b				c				d				e				
Criteria	C1	C2	C3	Cat	C1	C2	C3	Cat	C1	C2	C3	Cat	C1	C2	C3	Cat	C1	C2	C3	Cat	
PT 1	+	+	+	3	+	-	+	2	+	+	+	3	+	+	+	3	+	-	+	2	
PT 2	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	
PT 3	+	+	+	3	+	+	+	3	N	N	+	1	X	+	+	2	+	+	+	3	
PT 4	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	
PT 5	+	N	+	2	+	N	X	1	-	N	+	1	+	N	+	2	+	N	+	2	
PT 6	+	N	+	2	+	+	+	3	+	+	+	3	X	N	X	0	+	N	+	2	
PT 7	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	
PT 8	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	
PT 9	+	+	+	3	+	+	+	3	X	X	X	0	+	+	X	2	+	+	+	3	
PT 10	X	N	+	1	X	N	+	1	+	N	N	1	X	X	+	1	X	X	X	0	
PT 11	+	+	+	3	X	X	X	0	+	N	N	1	+	+	+	3	+	+	N	2	
PT 12	+	N	+	2	+	N	X	1	+	N	+	2	-	N	X	0	+	+	+	3	
PT 13	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	+	+	+	3	
Frequencies of Categories in each question				a				b				c				d				e	Total
Cat 0				0				1				1				2				1	5
Cat 1				1				3				4				1				0	10
Cat 2				3				1				1				3				4	11
Cat 3				9				8				7				7				8	39
Total				13				13				13				13				13	65

According to analysis when prospective teachers could choose methods, they used the PDS more often than the QDS and MS correctly and appropriately. Table 3 shows counts for solution classifications to the fraction problem and its five sub questions. Recall that the task asked for three interpretations as MS, PDS, or QDS. The counts in Table 4 shows that solutions were provided by 13 middle grades prospective teachers and each of them solved five sub-questions question a to question e. 65 solutions were obtained entirely. 26 of these items would be expected to include MS, 26 of these items would be expected to include PDS, and 13 of these items would be expected to include QDS. However, according to prospective teachers' solutions 77% (20 out of 26) of solutions included MS interpretation, 88% (23 out of 26) of solutions included PDS (19 out of 26) , and 77% (10 out of 13) of solutions included QDS correctly.

Table 4. *Frequency of correct interpretation for each task item (n = 13)*

Sub-questions	**Correct interpretation for each sub-question**	**MS**	**PDS**	**QDS**
a	PDS	0	13	0
b	PDS	0	10	0
c	MS	9	0	0
d	QDS	0	0	10
e	MS	11	0	0
Total #		20/26	23/26	10/13
Percentage		77%	88%	77%

Table 5 shows the frequency of each interpretation for each student and task item. According to this table, 8% of 65 solutions were categorized in Category 0, 17% of 65 solutions were categorized in Category 1, 18% of 65 solutions were categorized in Category 2, and 57% of 65 solutions were categorized in Category 3. Those results show that more than half of the students correctly and explicitly used division or multiplication in their solutions including both the number of groups and units, and interpreted situations correctly and explicitly for the situations (see Table 5).

Table 5. *Frequency of each interpretation as MS, PDS, and QDS for each student and each task item*

	a	b	c	d	e		
	PDS	PDS	MS	QDS	MS	Total #	Percentage
Category 0		PT 11	PT 9 PT 12	PT 6 PT 12	PT 10	5	8%
Category 1	PT 10	PT 5 PT 10 PT 12	PT 3 PT 5 PT 6 PT 10 PT 11	PT 10		11	17%

Table 5 (continued).

	a	b	c	d	e		
Category 2	PT 5 PT 6 PT 12	PT 1		PT 3 PT 4 PT 5 PT 9	PT 1 PT 5 PT 6 PT 11	12	18%
Category 3	PT 1 PT 2 PT 3 PT 4 PT 7 PT 8 PT 9 PT 11 PT 13	PT 2 PT 3 PT 4 PT 6 PT 7 PT 8 PT 9 PT 13	PT 1 PT 2 PT 4 PT 7 PT 8 PT 13	PT 1 PT 2 PT 7 PT 8 PT 11 PT 13	PT 2 PT 3 PT 4 PT 7 PT 8 PT 9 PT 12 PT 13	37	57%

Table 6 shows counts for solution interpretations of 65 solutions for each category. The counts in Table 5 shows that 12% of 26 solution items which requires MS placed in Category 0, 19% of 26 solution items which requires MS placed in Category 1, 15% of 26 solution items which requires MS placed in Category 2, and 54% of 26 solution items which requires MS placed in Category 3. According to this result, more than half of 26 solutions which requires MS were interpreted correctly.

Another result concerning Table 5 shows that 4% of 26 solution items which needs PDS placed in Category 0, 15% of 26 solution items which needs PDS placed in Category 1, 15% of 26 solution items which requires PDS placed in Category 2, and 65% of 26 items which requires PDS placed in Category 3. Like MS, more than half of 26 solutions which requires PDS were interpreted correctly.

Furthermore, the other result according to Table 5 demonstrates that 15% of 13 solution items which requires QDS placed in Category 0, 15% of 13 solution items which requires QDS placed in Category 1, 31% of 13 solution items which requires QDS placed in Category 2, and 46% of 13 solution items which requires QDS placed in Category 3. In contrast to the first two results according to the Table 5, less than half of 13 solutions which requires QDS were interpreted correctly. Also, when the percentages in each category were considered, approximately half of prospective teachers completely can correctly solve each problem by showing arithmetical operations and including information quantitates and interpretations of situations. Instruction that support learners in developing conceptual understanding by extending their knowledge of multiplicative situations from the context of whole numbers and exploratory evidence indicates that mathematics instruction of this form can have a positive impact on student learning (Poon 2014). These results reveal that through instruction engaging learners to develop a knowledge for multiplicative situation interpretations with fractions help prospective teachers learn the interpretations with conceptual understanding.

Table 6. *Frequency of each interpretation as MS, PDS, and QDS for each category*

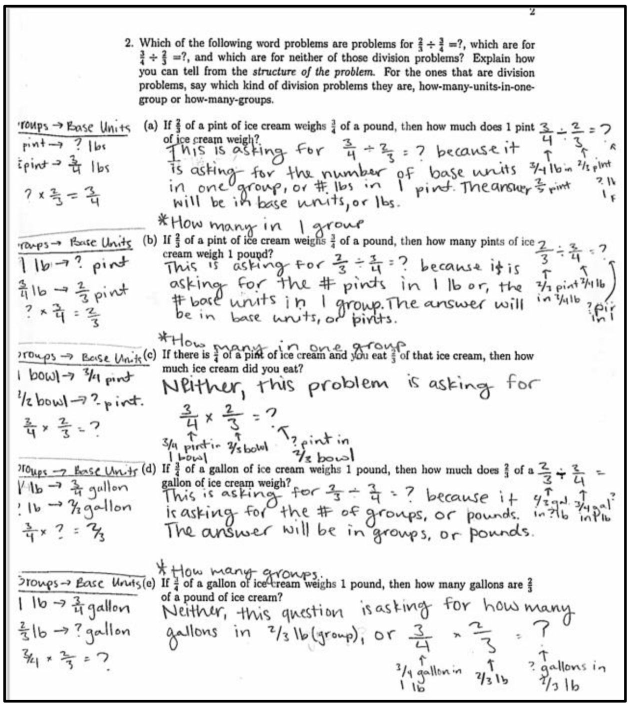
	Category 0		Category 1		Category 2		Category 3	
	#	%	#	%	#	%	#	%
MS	3/26	12	5/26	19	4/26	15	14/26	54
PDS	1/26	4	4/26	15	4/26	15	17/26	65
QDS	2/13	15	1/13	15	4/13	31	6/13	46

In the next section, I will provide at least one example from prospective teacher solutions for each interpretation and categories. I selected those examples that is related to the title of the example randomly from the appendix that shows more detailed explanation for each solution of each prospective teachers.

4.1. Some Examples from Prospective Teachers' Solutions for Multiplicative Situation Interpretations

4.1.1. Prospective Teachers' Interpretations of Multiplication Situations (MS)

There are 65 responses in total from 13 middle school prospective teachers. Prospective teachers interpreted 77% of multiplication situations in the gives task items. According to results, only 6 of 26 solutions requires MS prospective teachers included incorrect interpretations, so 20 out of 26 solutions requires MS were interpreted correctly by prospective teachers. For example, PT 2 interpreted the given situation as MS when N and M are known, and P is unknown. The PT 2's MS interpretations for c and e were categorized in Category 3 (see Table 3 and 4) because PT 2 correctly and appropriately interpreted the situation in c and e as MS by sing multiplication sign and mentioning this situation as "neither" that means this is not a PDS or QDS. The solutions also included an equation which mainly has appropriate values for N, M, and P as it is seen in the task item c and e (see Figure 11).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2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

Groups → Base Units
pint → ? lbs
$\frac{2}{3}$ pint → $\frac{3}{4}$ lbs
$? \times \frac{2}{3} = \frac{3}{4}$

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
This is asking for $\frac{3}{4} \div \frac{2}{3} = ?$ because it is asking for the number of base units in one group, or # lbs in 1 pint. The answer will be in base units, or lbs.
*How many in 1 group
$\frac{3}{4} \div \frac{2}{3} = ?$
3/4 lb in 2/3 pint
2/3 pint
? lb in 1 p

Groups → Base Units
1 lb → ? pint
$\frac{3}{4}$ lb → $\frac{2}{3}$ pint
$? \times \frac{3}{4} = \frac{2}{3}$

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # pints in 1 lb or, the # base units in 1 group. The answer will be in base units, or pints.
*How many in one group
$\frac{2}{3} \div \frac{3}{4} = ?$
2/3 pint in 3/4 lb
3/4 lb
? pint in 1

Groups → Base Units
1 bowl → 3/4 pint
1/2 bowl → ? pint.
$\frac{3}{4} \times \frac{2}{3} = ?$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
Neither, this problem is asking for
$\frac{3}{4} \times \frac{2}{3} = ?$
3/4 pint in 1 bowl
2/3 bowl
? pint in 2/3 bowl

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
? lb → 2/3 gallon
$\frac{3}{4} \times ? = 2/3$

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # of groups, or pounds. The answer will be in groups, or pounds.
*How many groups.
$\frac{2}{3} \div \frac{3}{4} =$
2/3 gal in ? lb
3/4 gal in 1 lb

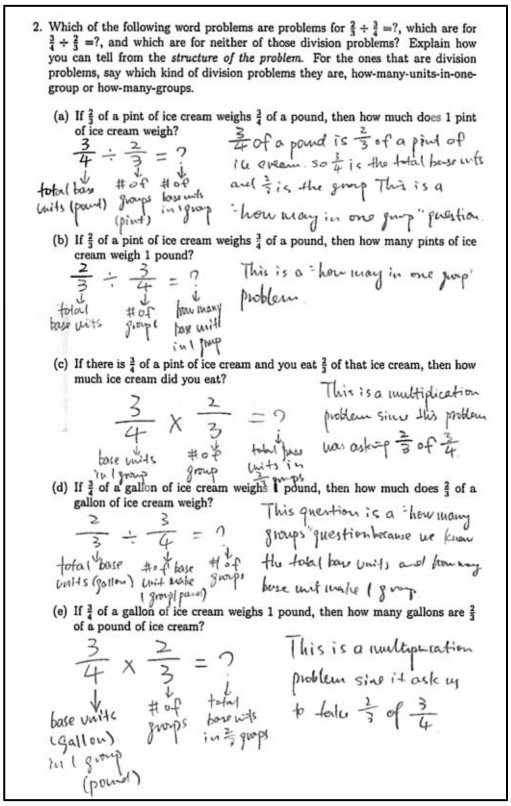
Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
$\frac{2}{3}$ lb → ? gallon
$3/4 \times \frac{2}{3} = ?$

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Neither, this question is asking for how many gallons in 2/3 lb (group), or $\frac{3}{4} \times \frac{2}{3} = ?$
3/4 gallon in 1 lb
2/3 lb
? gallons in 2/3 lb

Figure 11. PT 2's solution

4.1.2. Prospective Teachers' Interpretations of Partitive Division Situations (PDS)

Prospective teachers interpreted 88% of partitive division situations in the gives task items. Three solution requires PDS were incorrect, so 23 out of 26 solutions included correct interpretation for PDS. For example, PT 13 interpreted the given situation as PDS when M and P are known and N is unknown. PT 13's PDS interpretations for a and b were categorized in Category 3 (see Table 2). PT 13 mentioned the division is partitive or how-many-units-in-one-group. The solutions included an equation which mainly included appropriate values for N, M, and P (i.e., N is unknown, M is 2/3, and P is 3/4 or N is unknown, M is 3/4, and P is 2/3) as it is seen in the task item a and b. The PT 13 correctly and appropriately interpreted the situations in c and e as MS by mentioning it is how-many-units-in-one-group division problem (see Figure 12).



2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{3}{4} \div \frac{2}{3} = ?$

total base units (pound) | # of groups (pint) | # of base units in 1 group

$\frac{3}{4}$ of a pound is $\frac{2}{3}$ of a pint of ice cream. so $\frac{3}{4}$ is the total base units and $\frac{2}{3}$ is the group This is a "how many in one group" question.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\frac{2}{3} \div \frac{3}{4} = ?$

total base units | # of groups | how many base unit in 1 group

This is a "how may in one group" problem

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

$\frac{3}{4} \times \frac{2}{3} = ?$

base units in 1 group | # of group | total base units in $\frac{2}{3}$ groups

This is a multiplication problem since this problem was asking $\frac{2}{3}$ of $\frac{3}{4}$

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{2}{3} \div \frac{3}{4} = ?$

total base units (gallon) | # of base unit make 1 group (pound) | # of groups

This question is a "how many groups" question because we know the total base units and how many base unit make 1 group

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

$\frac{3}{4} \times \frac{2}{3} = ?$

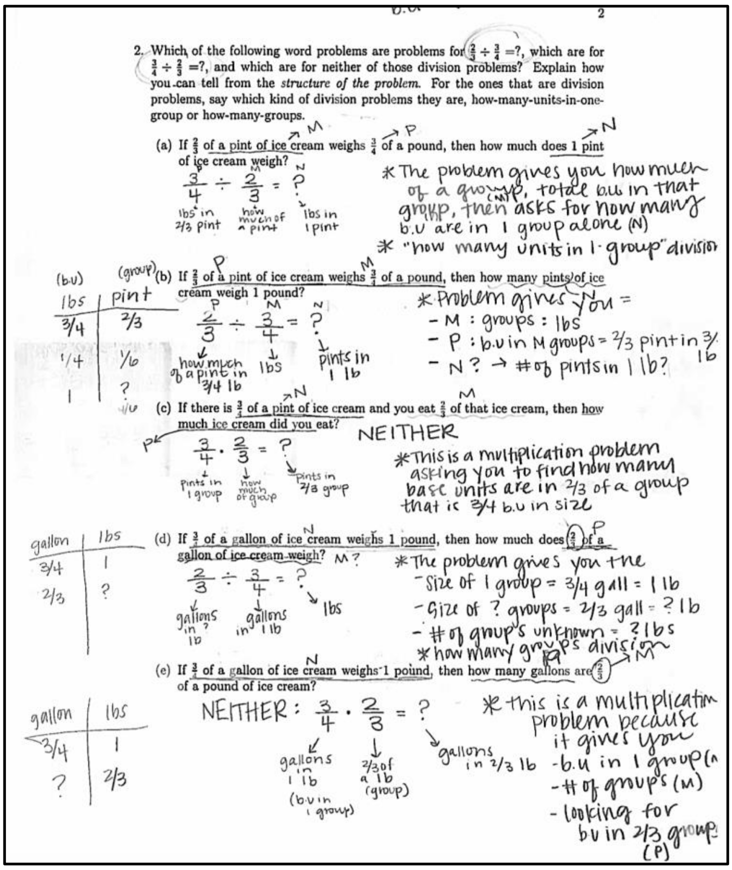
base units (gallon) in 1 group (pound) | # of groups | total base units in $\frac{2}{3}$ groups

This is a multiplication problem since it ask us to take $\frac{2}{3}$ of $\frac{3}{4}$

Figure 12. PT 13's solution

4.1.3. Prospective Teachers' Interpretations of Quotitive Division Situations (QDS)

Prospective teachers interpreted 77% of QDS in the gives task items. Three of 13 solutions require QDS were interpreted incorrectly by prospective teachers, so 10 out of 13 solutions were correctly interpreted. For example, PT 7 interpreted the given situation as QDS when N and P are known, the division is quotitive or how-many-groups. PT 7's QDS interpretations for d were categorized in Category 3. The solutions included an equation which mainly included appropriate values for N, M, and P (i.e., N is 3/4, M is unknown, and P is 2/3) as it is seen in the task item a and b. The PT 4 correctly and appropriately interpreted the situations in d as QDS by mentioning this is how-many-groups division problem (see Figure 13).


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem.* For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{3}{4} \div \frac{2}{3} = ?$

lbs in 2/3 pint — how much of a pint — lbs in 1 pint

* The problem gives you how much of a group, total b.u in that group, then asks for how many b.u are in 1 group alone (N)

* "how many units in 1-group" division

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

(b.u) lbs	(group) pint
3/4	2/3
1/4	1/6
1	?

$\frac{2}{3} \div \frac{3}{4} = ?$

how much of a pint in 3/4 lb — lbs — pints in 1 lb

* Problem gives you =
- M : groups : lbs
- P : b.u in M groups = 2/3 pint in 3/4 lb
- N ? → # of pints in 1 lb?

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

NEITHER

$\frac{3}{4} \cdot \frac{2}{3} = ?$

pints in 1 group — how much of group — pints in 2/3 group

* This is a multiplication problem asking you to find how many base units are in 2/3 of a group that is 3/4 b.u in size

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

gallon	lbs
3/4	1
2/3	?

$\frac{2}{3} \div \frac{3}{4} = ?$

gallons in ? lb — gallons in 1 lb — lbs

* The problem gives you the
- Size of 1 group = 3/4 gall = 1 lb
- Size of ? groups = 2/3 gall = ? lb
- # of group's unknown = ? lbs

* how many groups division

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

gallon	lbs
3/4	1
?	2/3

NEITHER: $\frac{3}{4} \cdot \frac{2}{3} = ?$

gallons in 1 lb (b.u in 1 group) — 2/3 of a lb (group) — gallons in 2/3 lb

* this is a multiplication problem because it gives you
- b.u in 1 group (N)
- # of groups (M)
- looking for b.u in 2/3 group (P)

Figure 13. PT 4's solution

In the tables, in the appendix, I present each PST's work for five sub questions. In these tables, for each question, prospective teachers' use of multiplication and/or division, group and base units, and multiplicative situation interpretations have been reported. Also, in the following part, I provide some analysis examples for category 0-3.

4.2.1. Prospective teachers' solutions with respect to Category 0

Table 1 and 2 have been useful to see how students have done for each sub question with color codes. For example, solution of PT 11 for the sub question b was categorized in category 0 since PT 11 did not perform well on the sub question and instead of division used multiplication, did not correctly include information for the number of units and groups and interpreted as MS rather than PDS (see Figure 14).

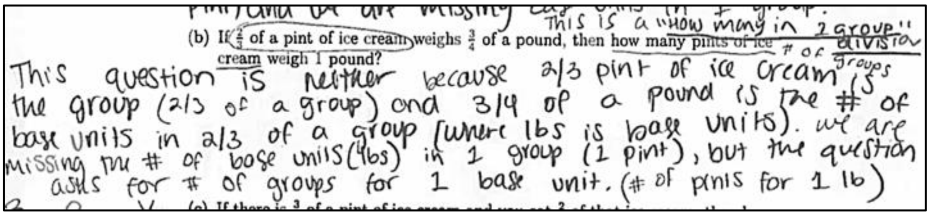

This is a "How many in 1 group" # of groups division
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). we are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)

Figure 14. PT 11's solution for b

In the figure 15, PT 9 for c did not solve problem correctly and did not include any information about units and groups and did not interpret this situation correctly (see Figure 15). PT 9 used subtraction instead of multiplication that shows PT 9 did not get if she needs to think multiplicative situations.

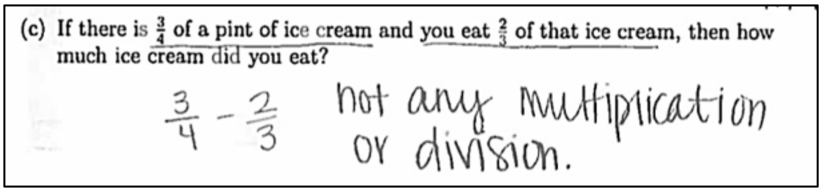
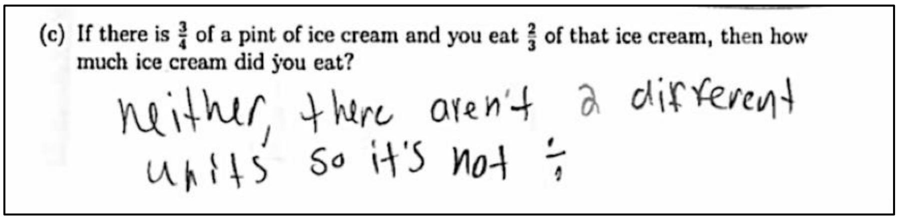

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
$\frac{3}{4} - \frac{2}{3}$ not any multiplication or division.

Figure 15. PT 9's solution for c

4.2.2. Prospective teachers' solutions with respect to Category 1

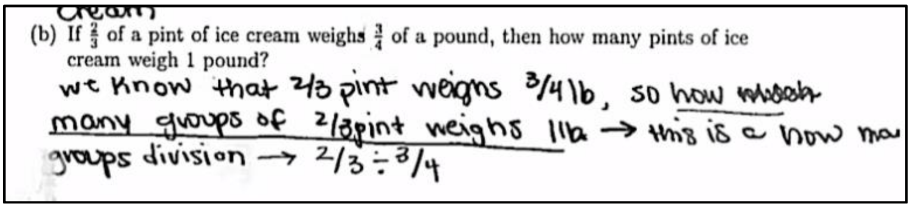
The solution of PT 3 for the sub question c was categorized in category 1 since PT 3 did not perform well on the sub question. Although PT 3 mentioned "neither", PT 3 did not use the multiplication operation sign and did not solve the problem (see Figure 16).


(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

neither, there aren't a different units so it's not ÷

Figure 16. PT 3's solution for c

PT 5 solved the problem by using division, but PT 5 did not include information about groups and base units explicitly and correctly and did not include correct information about the interpretation about the partitive situation correctly (see Figure 17).

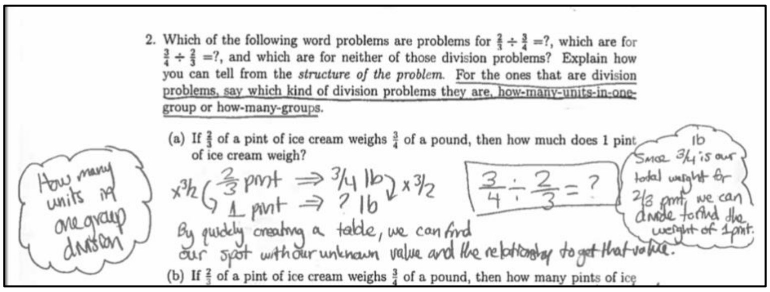

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

we know that 2/3 pint weighs 3/4 lb, so how ~~much~~ many groups of 2/3 pint weighs 1 lb → this is a how ma groups division → 2/3 ÷ 3/4

Figure 17. PT 5's solution for b

4.2.3. Prospective teachers' solutions with respect to Category 2

The solution of PT 12 for a was categorized in category 2 since PT 12 used the division correctly, did not include groups and base units in the solution, and interpreted situation correctly as how many units in one group division", so it was PDS (see Figure 18). That is why, PT 12's first and last criteria are green and second one is yellow (see Table 2).



2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

How many units in one group division

$\times\frac{3}{2}\left(\begin{array}{l}\frac{2}{3}\text{ pint} \Rightarrow \frac{3}{4}\text{ lb}\\ 1\text{ pint} \Rightarrow ?\text{ lb}\end{array}\right)\times\frac{3}{2}$ $\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$

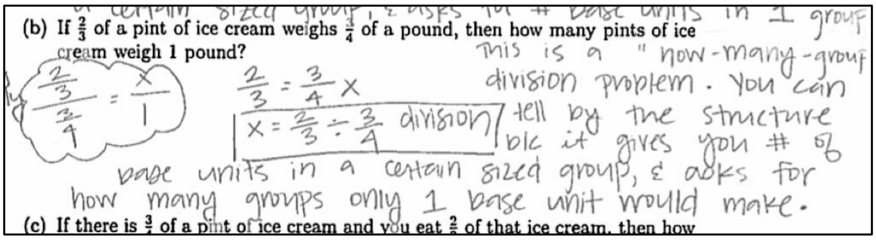
By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.

Since 3/4 is our total weight for 2/3 pint, we can divide to find the weight of 1 pint. lb

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice

Figure 18. PT 12's solution for a

The solution of PT 1 for b was categorized in category 2 (see Appendix A and Table 2) since PT 1 used multiplication operation and included information about group and base units and interpreted the situation as QDS, but it should be PDS (see Figure 19). That is why first and last criteria were shown with green, while the second one was shown with red (see Table 2).



(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\dfrac{\frac{2}{3}}{\frac{3}{4}} = \dfrac{x}{1}$

$\frac{2}{3} = \frac{3}{4}x$

$\boxed{x = \frac{2}{3} \div \frac{3}{4}\text{ division}}$

This is a "how-many-group division problem. You can tell by the structure b/c it gives you # of base units in a certain sized group, & asks for how many groups only 1 base unit would make.

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how

Figure 19. PT 1's solution for b

4.2.4. Prospective teachers' solutions with respect to Category 3

The solution of PT 9 for a was categorized in category 3 (see Appendix I and Table 2) since PT 9 used multiplication operation and included group and base units and interpreted the situation as PDS (see Figure 20). That is why all three criteria were shown with green (see Table 2).

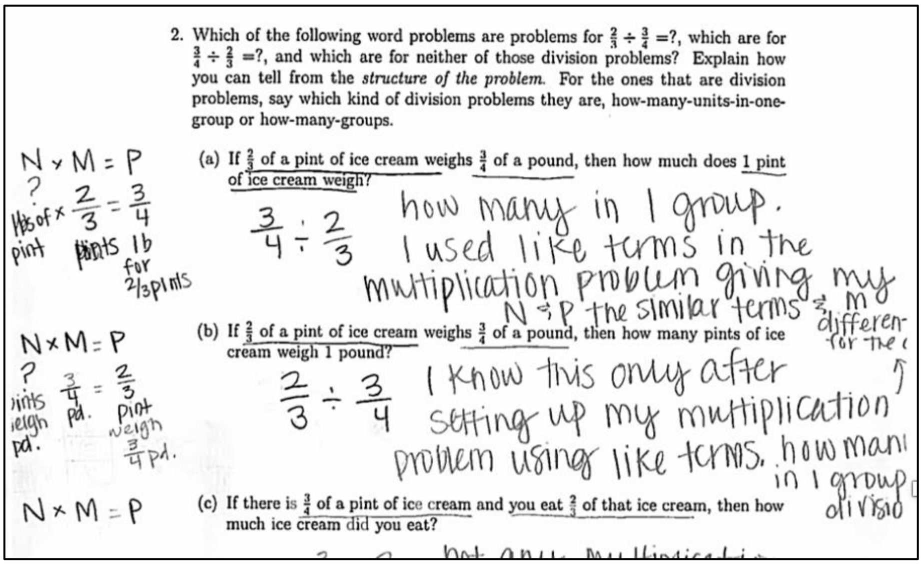


Figure 20. PT 9's solution for a

The solution of PT 2 for b was categorized in category 3 (see Appendix B and Table 2) since PT 2 used multiplication operation and included group and base units and interpreted the situation as PDS (see Figure 21). That is why all three criteria were shown with green (see Table 2).

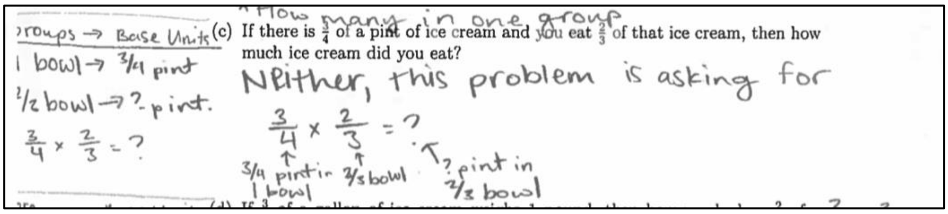


Figure 21. PT 2's solution for b

The solution of PT 7 for c was categorized in category 3 (see Appendix G and Table 2) since PT 7 used multiplication operation and included group and base units and interpreted the situation as MS (see Figure 22). That is why all three criteria were shown with green (see Table 2).

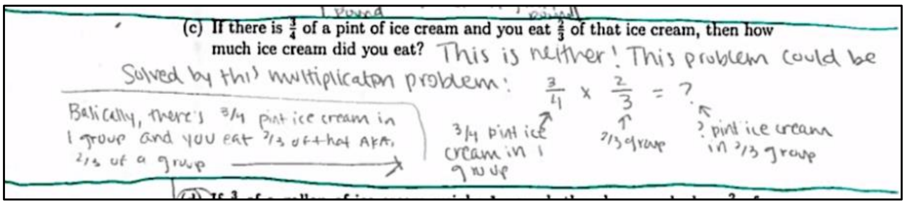


Figure 22. PT 7's solution for c

The solution of PT 8 for d was categorized in category 3 (see Appendix H and Table 2) since PT 8 used multiplication operation and included group and base units and interpreted the situation as QDS (see Figure 23). That is why all three criteria were shown with green (see Table 2).

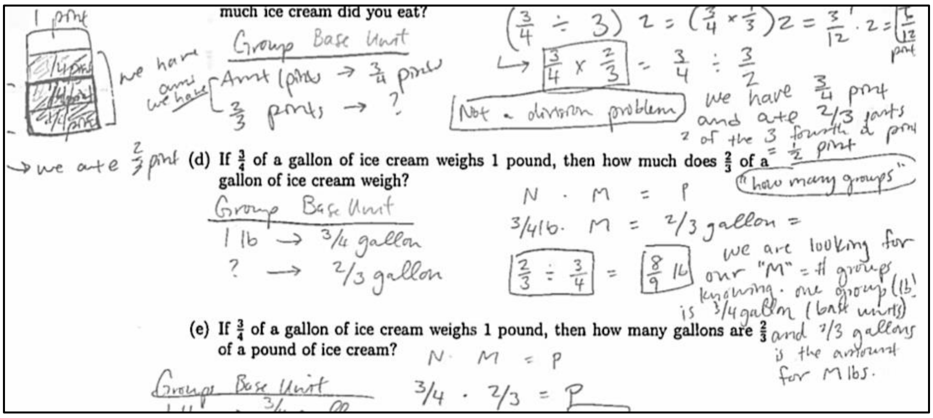


Figure 23. PT 8's solution for d

The solution of PT 13 for e was categorized in category 3 (see Appendix M and Table 2) since PT 13 used multiplication operation and included group and base units and interpreted the situation as MS (see Figure 24). That is why all three criteria were shown with green (see Table 2). As well as these examples, in the next part, solutions of PT 1- 13 with details were provided with all details.

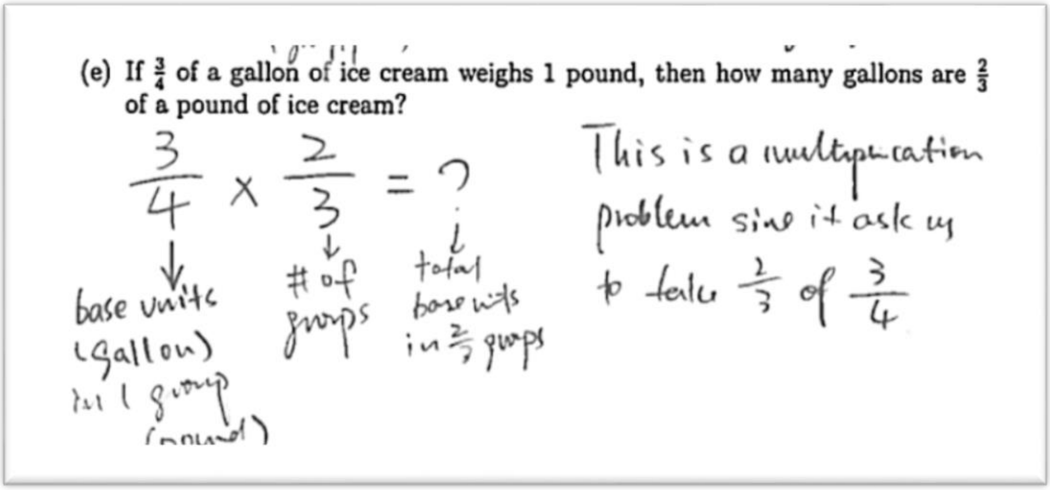


Figure 24. PT 13's solution for e

CHAPTER 5

DISCUSSION AND CONCLUSION

Results of this study revealed that the prospective teachers in this study correctly interpreted 77% of multiplication, 88% of partitive division, and 77% of quotitive division situations. 8% of 65 solutions is categorized in Category 0, 15% of 65 solutions is categorized in Category 1, 17% of 65 solutions is categorized in Category 2, and 60% of 65 solutions is categorized in Category 3.

Multiplicative operations with fractions are at the heart of middle grades mathematics, so learning and teaching this concept is crucial. Kursav (2017) proposed that when topics related to fractions, fraction division, fraction multiplication, ratio, proportional relationships, and algebra were emphasized in a course, middle grades prospective teachers were able to interpret multiplicative situations appropriately. Prospective teachers understanding and awareness of fractions arithmetic and interpretations of the situations as an essential skill for mathematics and beyond was limited. The small but representative selection of prospective teachers provided some evidence of prospective teachers' developing knowledge for fractions and fraction arithmetic considering quantities and multiplicative situation interpretations throughout the content course. This study revealed that through coursework which emphasized interpreting fraction, multiplicative situations prospective teachers were more competent in operating with fractions.

This study investigated what interpretations middle school prospective teachers make with fraction problems that involve multiplicative situations and to what extent middle school prospective teachers make explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods.

Similar to the finding of Beckmann et al. (2005) who reported that when prospective teachers were given questions that did not specify the use of division they still have used division and incorporated the meaning of division, prospective teachers in this study used multiplication and division in 53 responses of 65 total responses for sub questions. In the present study, the use of division was not specified since the aim was to investigate whether or not the prospective teachers could use the meaning of division without any direction.

The instructional approach to topics in the multiplicative conceptual field appeared to support the development of middle grades prospective teachers' understanding of multiplicative operations with fractions. This approach also supports prospective teachers' understanding of the meaning of multiplication and division and the use of each interpretation's (i.e., MS, PDS, and QDS) features. Izsák et al. (2015), this framework offers prospective teachers an approach to thinking about multiplicative operations in fraction problems.

To address my research question about what interpretations middle school prospective teachers, make with fraction problems that involve multiplicative situations, I can say that in this study, prospective teachers used the PDS more often than QDS and MS correctly and appropriately. This result represents the first determination regarding prospective teachers' tendency when choosing which appropriate interpretation to work with. According to Fischbein et al. (1985) "Arithmetical operations were assumed to remain attached to primitive behavioral models that influence tacitly the choice of an operation even after the learner has had solid formal-algorithmic training" (p.3). Although in 65 solutions, PDS (88%) is used more often than QDS (77%), the percentage of use of QDS is not low. This result is consistent with a considerable amount of research that prospective teachers attempted to use the PDS rather than the QDS, and prospective teachers found QDS more difficult than PDS (e.g., Greer 1992; Nillas 2003). One of the main reasons of finding QDS more difficult is that learners' intuitive model for division is the partitive model. However, when the divisor is less than one, PDS may not be adequate. It is obvious that prospective teachers in this study worked on fraction problems and so there was the need to QDS.   Thus, for this case, the quotitive model is also helpful like PDS. Fischbein et al. (1985) stated that quotitive interpretation was not influential until around age 14 or 15. Research has showed that learners have difficulties about division and those stem from a lacking view of division (Fischbein, Deri, Nello, & Marino, 1985; Tirosh & Graeber, 1990).

To address my research questions about the extent middle school prospective teachers make explicit use of specific features from the instruction including the use of equations and quantitative meanings for multiplication and division in their solution methods, I investigated how prospective teachers used groups and units in their solutions and how they used multiplication and division in their solutions. I categorized prospective teachers' solutions according to their performance on each question considering their use of multiplication and division, units and groups, and interpretations, 8% of 65 solutions are categorized in Category 0, 15% of 65 solutions are categorized in Category 1, 17% of 65 solutions are categorized in Category 2, and 60% of 65 solutions are categorized in Category 3. This result indicates that more than half of solutions of prospective teachers is categorized in category 3, so those solutions included completely correct answers for use of multiplication and division, use of units and groups, and multiplicative situation interpretations. Not only solutions in category 3, but also some other solutions in category 2 and category 1 included some correct responses for use of multiplication and division, use of units and groups, and multiplicative situation interpretations (see Table 1).

All those in mind, I can conclude that prospective teachers can solve multiplicative situation problems when provided with a learning environment that encourages them to draw on their intuitive thinking strategies and knowledge of multiplicative situations within a designed content course similar to in this study. Given an opportunity to experience a range of structures for multiplicative situations provides a solid basis for learners' developing a well conceptual understanding of the concept.

5.1. Limitations and Implications for Future Research

As with many qualitative studies, this study is also not without limitations. First, the sample in this study is only representative of one mathematics education program at one university and cannot be generalized to a broader context of prospective teacher education programs. Additionally, the only data source used in this study were the exam results. Future studies can include additional data sources such as individual focus group interviews and classroom observations.

While the sample size of the study is small, more participants are needed in more classes for future work. Also, studies including interviews are required to further understand prospective teachers' interpretations of multiplicative situations with fractions. According to Thanheiser et al. (2014), fractions as numbers is an area that needs to be investigated genuinely further and that is why there is a need a study which includes interviews and bigger data sets.

Implications for this study can include theoretical, methodological, and pedagogical considerations. Although the final exam question consisted of only one main question, there were five distinct sub questions that asked the PST's to perform variety of multiplicative situation interpretations. In future research, different versions of these questions/tasks can be conducted. This research should also shed light to teacher education programs in their curriculum interventions for prospective middle level mathematics teachers specifically for factions with multiplication and division concepts. Additional research should be conducted in different teacher education programs to compare and inform different programs

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APPENDICES

M. PT 1's solutions for the five sub questions

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<td>PT 1
NB</td>
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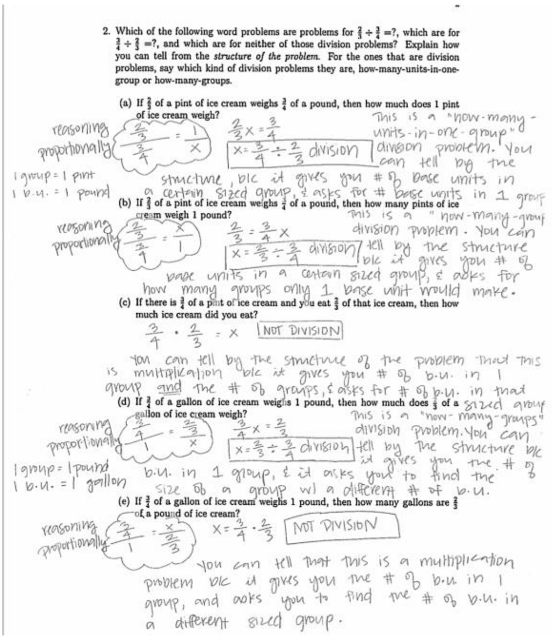

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

reasoning proportionally $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{1}{x}$ $\frac{2}{3}x = \frac{3}{4}$ $\boxed{x = \frac{3}{4} \div \frac{2}{3} \text{ division}}$

1 group = 1 pint
1 b.u. = 1 pound

This is a "how-many-units-in-one-group" division problem. You can tell by the structure, b/c it gives you # of base units in a certain sized group, & asks for # base units in 1 group

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

reasoning proportionally $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{x}{1}$ $\frac{2}{3} = \frac{3}{4}x$ $\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$

This is a "how-many-group division problem. You can tell by the structure b/c it gives you # of base units in a certain sized group, & asks for how many groups only 1 base unit would make.

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

$\frac{3}{4} \cdot \frac{2}{3} = x$ $\boxed{\text{NOT DIVISION}}$

You can tell by the structure of the problem that this is multiplication b/c it gives you # of b.u. in 1 group and the # of groups, & asks for # of b.u. in that sized group

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{\frac{2}{3}}{x}$ $\frac{3}{4}x = \frac{2}{3}$ $\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$

1 group = 1 pound
1 b.u. = 1 gallon

This is a "how-many-groups" division problem. You can tell by the structure b/c it gives you the # of b.u. in 1 group, & it asks you to find the size of a group w/ a different # of b.u.

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{x}{\frac{2}{3}}$ $x = \frac{3}{4} \cdot \frac{2}{3}$ $\boxed{\text{NOT DIVISION}}$

You can tell that this is a multiplication problem b/c it gives you the # of b.u. in 1 group, and asks you to find the # of b.u. in a different sized group.
</td>
<td>
2.a.

- Multiplication/Division
  - PT 1 used the division operation sign ÷ $$\frac{3}{4} \div \frac{2}{3}$$
- Group and Base units
  - PT 1 included information about groups and base units explicitly and correctly.
- Multiplicative situation interpretation
  - PT1 mentioned this situation as "how-many-units-in-one-group division problem"
  - Thus, PT1 interpreted this situation as PDS.
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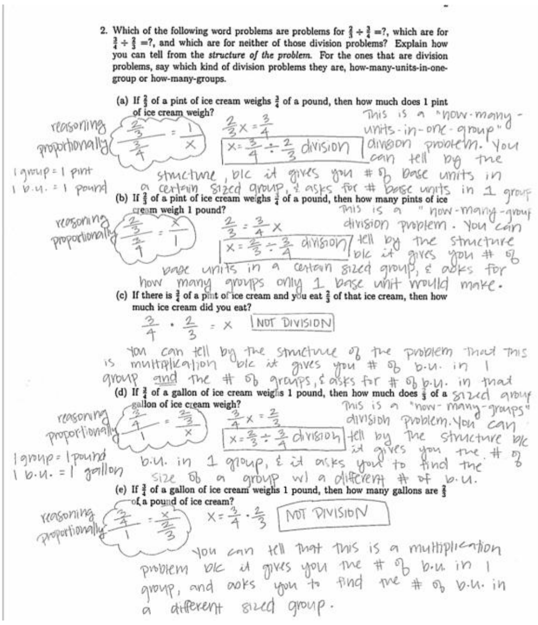
<td>
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(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
reasoning proportionally $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{1}{x}$
$\frac{2}{3}x = \frac{3}{4}$
$\boxed{x = \frac{3}{4} \div \frac{2}{3} \text{ division}}$
This is a "how-many-units-in-one-group" division problem. You can tell by the structure, b/c it gives you # of base units in a certain sized group, & asks for # base units in 1 group
1 group = 1 pint
1 b.u. = 1 pound
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
reasoning proportionally $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{x}{1}$
$\frac{2}{3} = \frac{3}{4}x$
$\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$
This is a "how-many-group division problem. You can tell by the structure b/c it gives you # of base units in a certain sized group, & asks for how many groups only 1 base unit would make.
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
$\frac{3}{4} \cdot \frac{2}{3} = x$ $\boxed{\text{NOT DIVISION}}$
You can tell by the structure of the problem that this is multiplication b/c it gives you # of b.u. in 1 group and the # of groups, & asks for # of b.u. in that sized group
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{\frac{2}{3}}{x}$
$\frac{3}{4}x = \frac{2}{3}$
$\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$
This is a "how-many-groups" division problem. You can tell by the structure b/c it gives you the # of b.u. in 1 group, & it asks you to find the size of a group w/ a different # of b.u.
1 group = 1 pound
1 b.u. = 1 gallon
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{x}{\frac{2}{3}}$
$x = \frac{3}{4} \cdot \frac{2}{3}$ $\boxed{\text{NOT DIVISION}}$
You can tell that this is a multiplication problem b/c it gives you the # of b.u. in 1 group, and asks you to find the # of b.u. in a different sized group.</td>
<td>2.b.
• Multiplication/Division
  o PT 1 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
• Group and Base units
  o PT 1 included some information about groups and base units, but it is not complete and explicit since PT 1 said that “it gives you # of base units and asks how many groups only 1 base unit would make”. PT 1 did not mention which one is base unit or group and the number of the groups and base units.
• Multiplicative situation interpretation
  o PT 1 mentioned this situation as “how-many-groups division problem”
  o Thus, PT 1 interpreted this situation as QDS, but it should be PDS</td>
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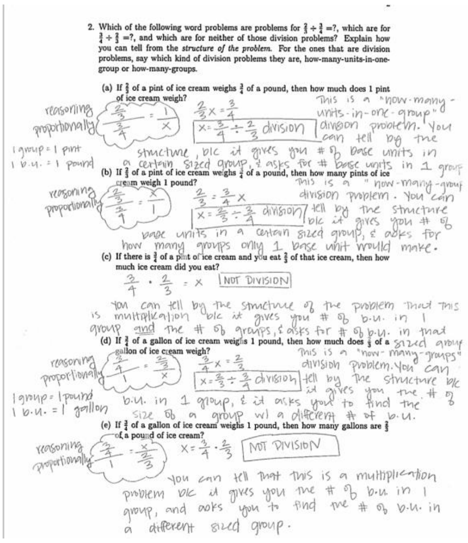

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem.* For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

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$\frac{2}{3}x = \frac{3}{4}$

$\boxed{x = \frac{3}{4} \div \frac{2}{3} \text{ division}}$

This is a "how-many-units-in-one-group" division problem. You can tell by the structure, b/c it gives you # of base units in a certain sized group, & asks for # base units in 1 group

1 group = 1 pint
1 b.u. = 1 pound

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

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$\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$

This is a "how-many-group" division problem. You can tell by the structure b/c it gives you # of base units in a certain sized group, & asks for how many groups only 1 base unit would make.

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$\frac{3}{4} \cdot \frac{2}{3} = x$ $\boxed{\text{NOT DIVISION}}$

You can tell by the structure of the problem that this is multiplication b/c it gives you # of b.u. in 1 group <u>and</u> the # of groups, & asks for # of b.u. in that sized group

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{\frac{2}{3}}{x}$

$\frac{3}{4}x = \frac{2}{3}$

$\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$

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$x = \frac{3}{4} \cdot \frac{2}{3}$ $\boxed{\text{NOT DIVISION}}$

You can tell that this is a multiplication problem b/c it gives you the # of b.u. in 1 group, and asks you to find the # of b.u. in a different sized group.
</td>
<td>
2.c.

- Multiplication/Division
  - PT 1 used the multiplication operation sign × $\frac{3}{4} \times \frac{2}{3}$
- Group and Base units
  - PT 1 included information about groups and base units explicitly and correctly.
- Multiplicative situation interpretation
  - PT interpreted this situation as MS
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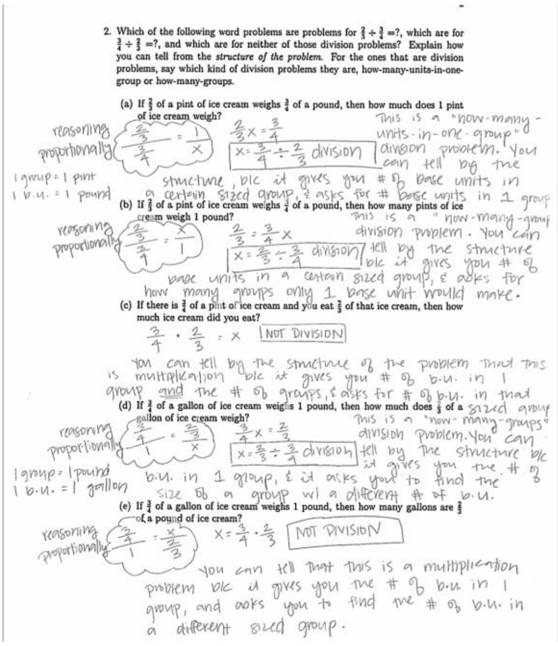

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

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$x = \frac{3}{4} \cdot \frac{2}{3}$ $\boxed{\text{NOT DIVISION}}$

You can tell that this is a multiplication problem b/c it gives you the # of b.u. in 1 group, and asks you to find the # of b.u. in a different sized group.
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2.d.

Multiplication/Division
PT 1 used the division operation sign ÷
$\frac{2}{3} \div \frac{3}{4}$

Group and Base units
PT 1 included information about groups and base units explicitly and correctly.

Multiplicative situation interpretation
PT mentioned "how-many-groups division problem"
PT interpreted it as QDS.

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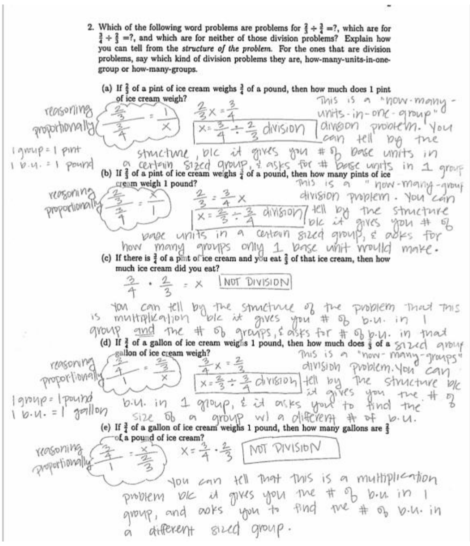

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

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$\frac{3}{4} \cdot \frac{2}{3} = x$ $\boxed{\text{NOT DIVISION}}$
You can tell by the structure of the problem that this is multiplication b/c it gives you # of b.u. in 1 group <u>and</u> the # of groups, & asks for # of b.u. in that sized group

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{\frac{2}{3}}{x}$
$\frac{3}{4}x = \frac{2}{3}$
$\boxed{x = \frac{2}{3} \div \frac{3}{4} \text{ division}}$
This is a "how-many-groups" division problem. You can tell by the structure b/c it gives you the # of b.u. in 1 group, & it asks you to find the size of a group w/ a different # of b.u.
1 group = 1 pound
1 b.u. = 1 gallon

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
reasoning proportionally $\frac{\frac{3}{4}}{1} = \frac{x}{\frac{2}{3}}$
$x = \frac{3}{4} \cdot \frac{2}{3}$ $\boxed{\text{NOT DIVISION}}$
You can tell that this is a multiplication problem b/c it gives you the # of b.u. in 1 group, and asks you to find the # of b.u. in a different sized group.
</td>
<td>
2.e.

Multiplication/Division
PT 1 used multiplication with multiplication operation sign ×
$$\frac{3}{4} \times \frac{2}{3}$$
Group and Base units
PT 1 included information about groups and base units but it is not complete and explicit since PT 1 said that “it gives you both # of bu in 1 group and asks you to find # bu in a different sized group”. In PT 1’s statement it is not clear which one is the number of base units or which one is the number of groups and not clear with the number of the groups with “different sized groups”.
Multiplicative situation interpretation
PT 1 interpreted this situation as MS.

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</table>

N. PT 2's solutions for the five sub questions

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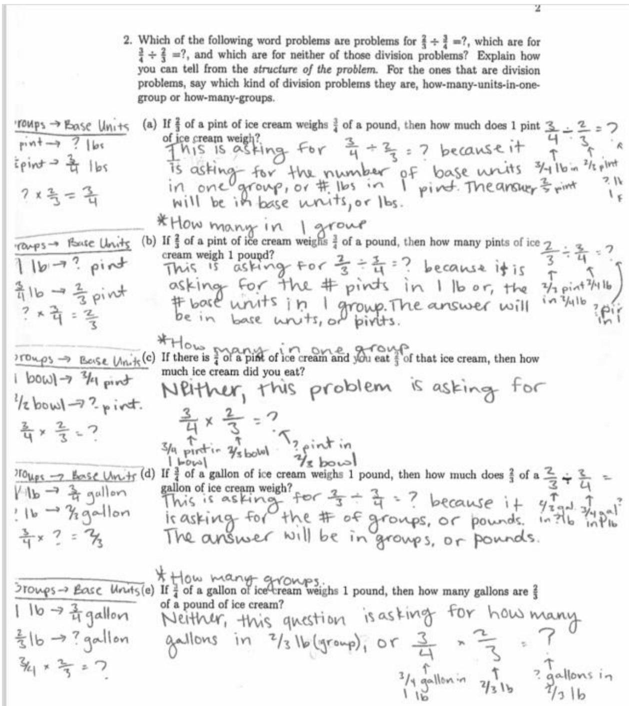
<td>PT 2
MB</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

Groups → Base Units
pint → ? lbs
$\frac{2}{3}$ pint → $\frac{3}{4}$ lbs
$? \times \frac{2}{3} = \frac{3}{4}$

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{3}{4} \div \frac{2}{3} = ?$
3/4 lb in 2/3 pint
? lb 1 p

This is asking for $\frac{3}{4} \div \frac{2}{3} = ?$ because it is asking for the number of base units in one group, or # lbs in 1 pint. The answer will be in base units, or lbs.

*How many in 1 group

Groups → Base Units
1 lb → ? pint
$\frac{3}{4}$ lb → $\frac{2}{3}$ pint
$? \times \frac{3}{4} = \frac{2}{3}$

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\frac{2}{3} \div \frac{3}{4} = ?$
2/3 pint in 3/4 lb
3/4 lb
? pint in 1

This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # pints in 1 lb or, the # base units in 1 group. The answer will be in base units, or pints.

*How many in one group

Groups → Base Units
1 bowl → 3/4 pint
1/2 bowl → ? pint.
$\frac{3}{4} \times \frac{2}{3} = ?$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

Neither, this problem is asking for

$\frac{3}{4} \times \frac{2}{3} = ?$
3/4 pint in 1 bowl
2/3 bowl
? pint in 2/3 bowl

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
? lb → 2/3 gallon
$\frac{3}{4} \times ? = 2/3$

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{2}{3} \div \frac{3}{4} =$
2/3 gal. in ? lb
3/4 gal in 1 lb

This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # of groups, or pounds. The answer will be in groups, or pounds.

*How many groups.

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
$\frac{2}{3}$ lb → ? gallon
$3/4 \times \frac{2}{3} = ?$

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

Neither, this question is asking for how many gallons in 2/3 lb (group), or $\frac{3}{4} \times \frac{2}{3} = ?$
3/4 gallon in 1 lb
2/3 lb
? gallons in 2/3 lb
</td>
<td>
2.a.

- Multiplication/Division
  - PT 2 used the multiplication with a missing factor $?\times \frac{2}{3} = \frac{3}{4}$
  - then PT 2 used the division operation sign ÷ $\frac{3}{4} \div \frac{2}{3}$
- Group and Base units
  - PT 2 included information about the group and units explicitly and correctly.
  - Groups→ Base units 1 pint → ? lbs. 2/3 pint → 3/4 lbs.
- Multiplicative situation interpretation
  - PT 2 mentioned that this situation is "how many in one group"
  - PT 2 identified this situation as PDS.
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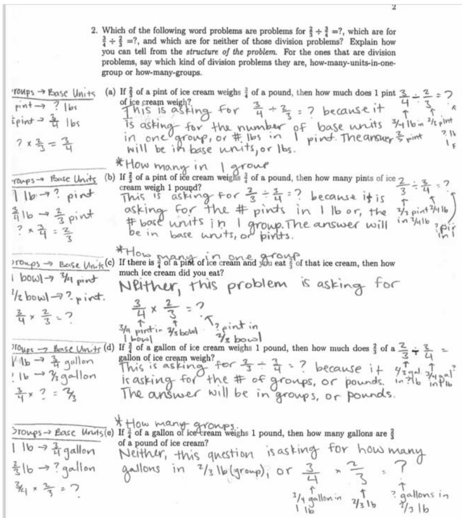
B (continued).

<table>
<tr>
<td>PT 2
MB</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

Groups → Base Units
pint → ? lbs
$\frac{2}{3}$ pint → $\frac{3}{4}$ lbs
$? \times \frac{2}{3} = \frac{3}{4}$
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3} = ?$
This is asking for $\frac{3}{4} \div \frac{2}{3} = ?$ because it is asking for the number of base units in one group, or # lbs in 1 pint. The answer will be in base units, or lbs.
*How many in 1 group

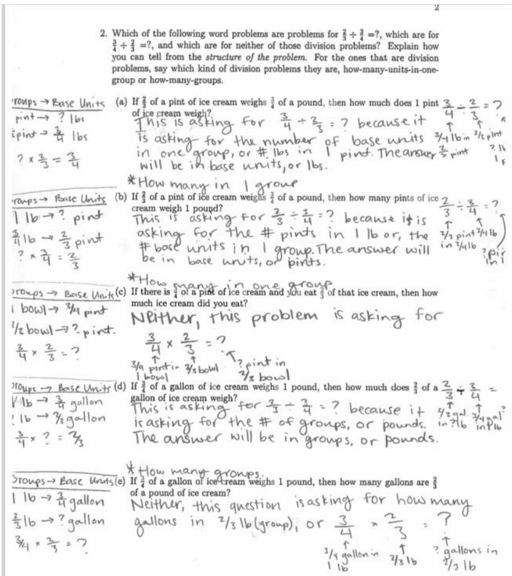
Groups → Base Units
1 lb → ? pint
$\frac{3}{4}$ lb → $\frac{2}{3}$ pint
$? \times \frac{3}{4} = \frac{2}{3}$
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3} \div \frac{3}{4} = ?$
This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # pints in 1 lb or, the # base units in 1 group. The answer will be in base units, or pints.
*How many in one group

Groups → Base Units
1 bowl → 3/4 pint
1/2 bowl → ? pint.
$\frac{3}{4} \times \frac{2}{3} = ?$
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
Neither, this problem is asking for
$\frac{3}{4} \times \frac{2}{3} = ?$
3/4 pint in 1 bowl, 2/3 bowl, ? pint in 2/3 bowl

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
? lb → 2/3 gallon
$\frac{3}{4} \times ? = 2/3$
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{2}{3} \div \frac{3}{4} =$
This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # of groups, or pounds. The answer will be in groups, or pounds.
*How many groups.

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
$\frac{2}{3}$ lb → ? gallon
$3/4 \times \frac{2}{3} = ?$
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Neither, this question is asking for how many gallons in 2/3 lb (group), or $\frac{3}{4} \times \frac{2}{3} = ?$
3/4 gallon in 1 lb, 2/3 lb, ? gallons in 2/3 lb</td>
<td>2.b.
Multiplication/Division
PT 2 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
Group and Base units
PT 2 included information about groups and units explicitly and correctly.
Groups→ Base units
pint → ? lbs.
It is asking for # pints in 1 lb. or the # base units in 1 group. The answer will be base units or pints.
Multiplicative situation interpretation
PT 2 mentioned this situation as “how many in one group”
Thus, PT 2 interpreted this situation as PDS.</td>
</tr>
</table>

B (continued).

PT 2
MB
	
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	This is asking for $\frac{3}{4} \div \frac{2}{3} = ?$ because it is asking for the number of base units in one group, or # lbs in 1 pint. The answer will be in base units, or lbs.
	*How many in 1 group
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # pints in 1 lb or, the # base units in 1 group. The answer will be in base units, or pints.
	*How many in one group
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	Neither, this problem is asking for $\frac{3}{4} \times \frac{2}{3} = ?$
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # of groups, or pounds. The answer will be in groups, or pounds.
	*How many groups.
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	Neither, this question is asking for how many gallons in 2/3 lb (group), or $\frac{3}{4} \times \frac{2}{3} = ?$
		2.c.
		• Multiplication/Division
		  ○ PT 2 used the multiplication with a multiplication operation sign x
		    $\frac{3}{4} \times \frac{2}{3} = ?$ where $\frac{3}{4}$ pint in one bowl, $\frac{2}{3}$ bowl, and ? is ? pint in $\frac{2}{3}$ bowl
		• Group and Base units
		  ○ PT 2 included information about the number of groups and units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 2 identified this situation as multiplication situation as MS.

B (continued).

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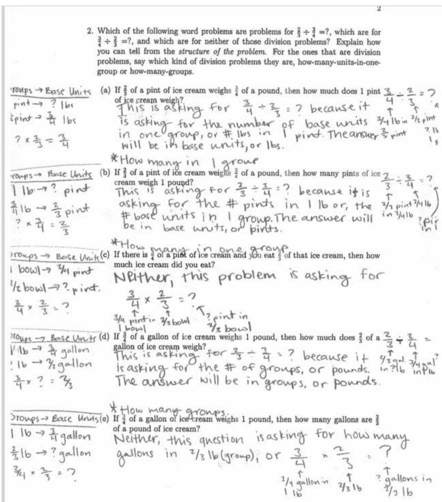
MB</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

Groups → Base Units
1 pint → ? lbs
$\frac{2}{3}$ pint → $\frac{3}{4}$ lbs
$? \times \frac{2}{3} = \frac{3}{4}$
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? $\frac{3}{4} \div \frac{2}{3} = ?$
This is asking for $\frac{3}{4} \div \frac{2}{3} = ?$ because it is asking for the number of base units in one group, or # lbs in 1 pint. The answer will be in base units, or lbs.
3/4 lb in 2/3 pint
*How many in 1 group

Groups → Base Units
1 lb → ? pint
$\frac{3}{4}$ lb → $\frac{2}{3}$ pint
$? \times \frac{3}{4} = \frac{2}{3}$
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? $\frac{2}{3} \div \frac{3}{4} = ?$
This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # pints in 1 lb or, the # base units in 1 group. The answer will be in base units, or pints.
2/3 pint in 3/4 lb
*How many in one group

Groups → Base Units
1 bowl → 3/4 pint
1/2 bowl → ? pint.
$\frac{3}{4} \times \frac{2}{3} = ?$
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
Neither, this problem is asking for
$\frac{3}{4} \times \frac{2}{3} = ?$
3/4 pint in 1 bowl; 2/3 bowl; ? pint in 2/3 bowl

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
? lb → 2/3 gallon
$\frac{3}{4} \times ? = 2/3$
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? $\frac{2}{3} \div \frac{3}{4} =$
This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # of groups, or pounds. The answer will be in groups, or pounds.
*How many groups.

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
$\frac{2}{3}$ lb → ? gallon
$3/4 \times \frac{2}{3} = ?$
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Neither, this question is asking for how many gallons in 2/3 lb (group), or $\frac{3}{4} \times \frac{2}{3} = ?$
3/4 gallon in 1 lb; 2/3 lb; ? gallons in 2/3 lb</td>
<td>2.d.
• Multiplication/Division
  ○ PT 2 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
• Group and Base Units
  ○ PT 2 included information about the number of groups and units explicitly and correctly.
• Multiplicative situation interpretation
  ○ PT 2 mentioned this situation as “how many groups”
  ○ Thus, PT 2 interpreted this situation as QDS.</td>
</tr>
</table>

B (continued).

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<td>PT 2
MB</td>
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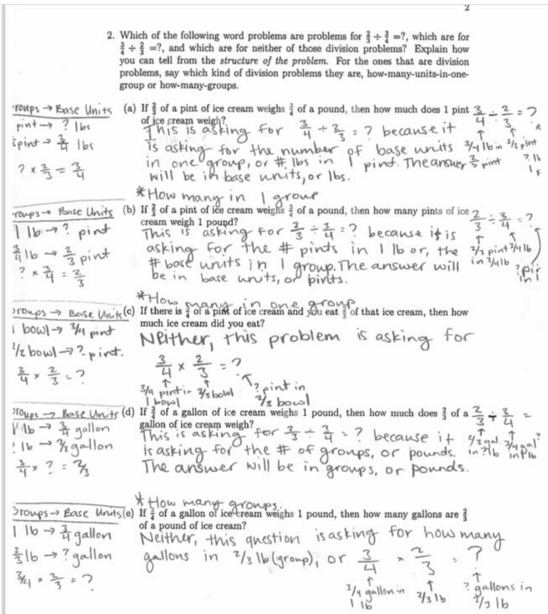

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

Groups → Base Units
1 pint → ? lbs
$\frac{2}{3}$ pint → $\frac{3}{4}$ lbs
$? \times \frac{2}{3} = \frac{3}{4}$

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

This is asking for $\frac{3}{4} \div \frac{2}{3} = ?$ because it is asking for the number of base units in one group, or # lbs in 1 pint. The answer will be in base units, or lbs.

$\frac{3}{4} \div \frac{2}{3} = ?$
3/4 lb in 2/3 pint
2/3 pint

*How many in 1 group

Groups → Base Units
1 lb → ? pint
$\frac{3}{4}$ lb → $\frac{2}{3}$ pint
$? \times \frac{3}{4} = \frac{2}{3}$

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # pints in 1 lb or, the # base units in 1 group. The answer will be in base units, or pints.

$\frac{2}{3} \div \frac{3}{4} = ?$
2/3 pint in 3/4 lb
3/4 lb

*How many in one group

Groups → Base Units
1 bowl → 3/4 pint
1/2 bowl → ? pint.
$\frac{3}{4} \times \frac{2}{3} = ?$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

Neither, this problem is asking for $\frac{3}{4} \times \frac{2}{3} = ?$
3/4 pint in 1 bowl
2/3 bowl
? pint in 2/3 bowl

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
? lb → 2/3 gallon
$\frac{3}{4} \times ? = 2/3$

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

This is asking for $\frac{2}{3} \div \frac{3}{4} = ?$ because it is asking for the # of groups, or pounds. The answer will be in groups, or pounds.

$\frac{2}{3} \div \frac{3}{4} =$
2/3 gal. in ? lb
3/4 gal in 1 lb

*How many groups.

Groups → Base Units
1 lb → $\frac{3}{4}$ gallon
$\frac{2}{3}$ lb → ? gallon
$3/4 \times \frac{2}{3} = ?$

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

Neither, this question is asking for how many gallons in 2/3 lb (group), or $\frac{3}{4} \times \frac{2}{3} = ?$
3/4 gallon in 1 lb
2/3 lb
? gallons in 2/3 lb

</td>
<td>

2.e.

- Multiplication/Division
  - PT 2 used multiplication with multiplication operation sign ×
- Group and Base Units
  - PT 2 included information about the number of groups and units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 2 interpreted this situation as multiplication situation.
  - Thus, PT 2 interpreted this situation as MS.

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C. PT 3's solutions for the five sub questions

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<td>PT 3
AC</td>
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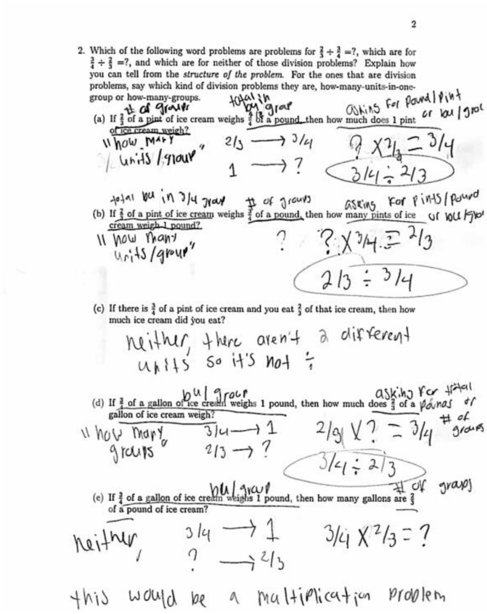

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
# of groups   total in by group   asking for pound/pint or bu/group
"how many units/group"   2/3 → 3/4   1 → ?   ? X 2/3 = 3/4   3/4 ÷ 2/3

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
total bu in 3/4 group   # of groups   asking for pints/pound or bu/group
"how many units/group"   ?   ? X 3/4 = 2/3   2/3 ÷ 3/4

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
neither, there aren't a different units so it's not ÷

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
bu/group   asking for total pounds or # of groups
"how many groups"   3/4 → 1   2/3 → ?   2/3 X ? = 3/4   3/4 ÷ 2/3

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
bu/group   # of groups
neither,   3/4 → 1   ? → 2/3   3/4 X 2/3 = ?
this would be a multiplication problem
</td>
<td>
2.a.

- Multiplication/Division
  - PT 3 used the division operation sign ÷
    $$\frac{3}{4} \div \frac{2}{3}$$
- Groups and base units
  - PT 3 included information about the groups and units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 3 mentioned this situation as "how many units/group"
  - Thus, PT 3 interpreted this situation as PDS
</td>
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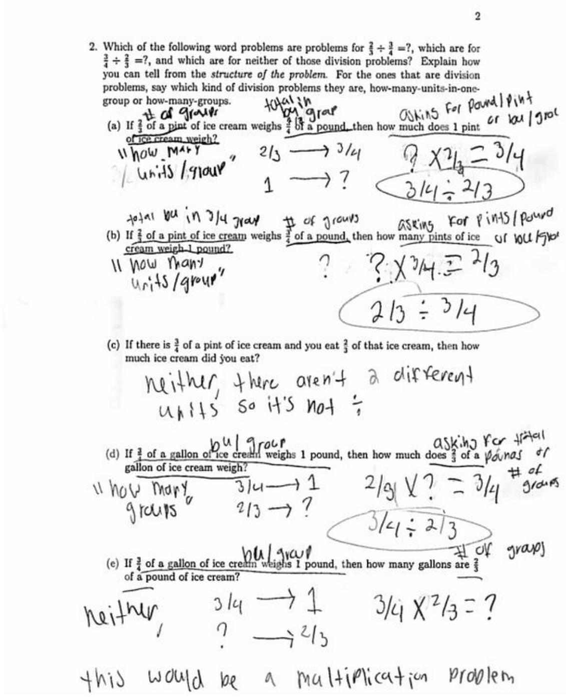
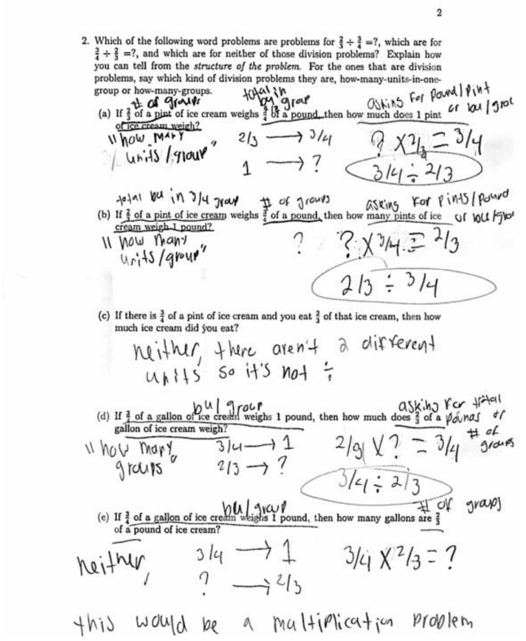
</table>

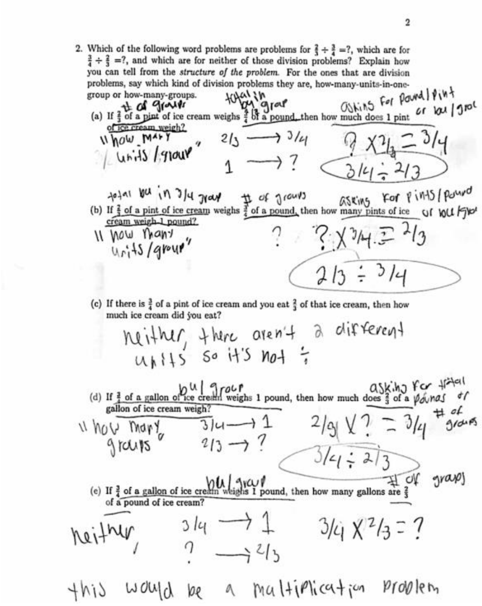
C (continued).

<table>
<tr>
<td>PT 3
AC</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
# of groups   total in by group   asking for pound / pint or bu / group
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
"how many units / group" $2/3 \longrightarrow 3/4$ $1 \longrightarrow ?$ $? \times 2/3 = 3/4$ $3/4 \div 2/3$
total bu in 3/4 group   # of groups   asking for pints / pound or bu / group
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
"how many units / group"   ? $? \times 3/4 = 2/3$ $2/3 \div 3/4$
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
neither, there aren't a different units so it's not ÷
bu / group   asking for total pounds or # of groups
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
"how many groups" $3/4 \longrightarrow 1$ $2/3 \longrightarrow ?$ $2/3 \times ? = 3/4$ $3/4 \div 2/3$
bu / group   # of groups
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
neither, $3/4 \longrightarrow 1$ $? \longrightarrow 2/3$ $3/4 \times 2/3 = ?$
this would be a multiplication problem</td>
<td>2.b.
• Multiplication/Division
  o PT 3 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
• Groups and base units
  o PT 3 included information about units and groups explicitly and correctly.
• Multiplicative situation interpretation
  o PT 3 mentioned this situation as “how many units/group”
  o Thus, PT 3 interpreted this situation as PDS.</td>
</tr>
</table>

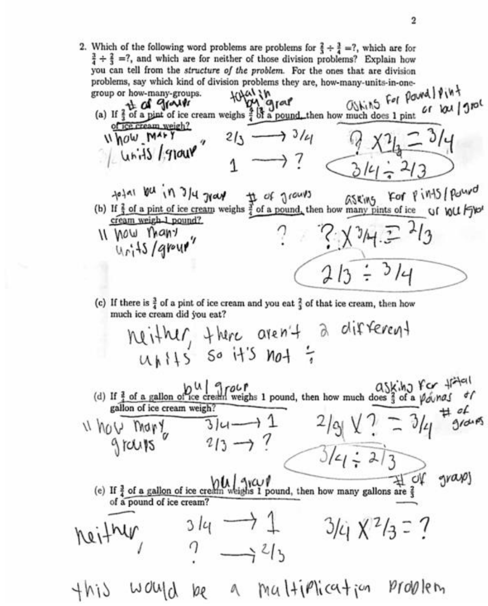
C (continued).

PT 3
AC
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	# of groups total in by group asking for pound/pint or bu/group
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	"how many units/group" 2/3 → 3/4 1 → ? ? X 2/3 = 3/4 3/4 ÷ 2/3
	total bu in 3/4 group # of groups asking for pints/pound or bu/group
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	"how many units/group" ? ? X 3/4 = 2/3 2/3 ÷ 3/4
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	neither, there aren't a different units so it's not ÷
	bu/group asking for total pounds or # of groups
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	"how many groups" 3/4 → 1 2/3 → ? 2/3 X ? = 3/4 3/4 ÷ 2/3
	bu/group # of groups
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	neither, 3/4 → 1 ? → 2/3 3/4 X 2/3 = ?
	this would be a multiplication problem
		2.c.
		• Multiplication/Division
		  ○ PT 3 did not show the multiplication
		• Groups and base units
		  ○ PT 3 did not include information about groups and base units
		• Multiplicative situation interpretation
		  ○ PT 3 interpreted this situation as MS.

C (continued).

PT 3
AC
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	"how many units/group" 2/3 → 3/4 1 → ? ? X 2/3 = 3/4 3/4 ÷ 2/3
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	"how many units/group" ? ? X 3/4 = 2/3 2/3 ÷ 3/4
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	neither, there aren't a different units so it's not ÷
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	"how many groups" 3/4 → 1 2/3 → ? 2/3 X ? = 3/4 3/4 ÷ 2/3
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	neither, 3/4 → 1 ? → 2/3 3/4 X 2/3 = ?
	this would be a multiplication problem
		2.d.
		• Multiplication/Division
		  ○ PT 3 used the division operation sign ÷ and showed the equation wrongly
		$$\frac{3}{4} \div \frac{2}{3}$$
		• Groups and base units
		  ○ PT 3 included information about units and groups explicitly and correctly.
		• Multiplicative situation interpretation
		○ PT 3 mentioned this situation as "how many groups" problem
		PT 3 interpreted it as QDS.

C (continued).

<table>
<tr>
<td>PT 3
AC</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

# of groups   total in by group   asking for pound/pint or bu/group

"how many units/group"   2/3 → 3/4   1 → ?   ? × 2/3 = 3/4   3/4 ÷ 2/3

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

total bu in 3/4 group   # of groups   asking for pints/pound or bu/group

"how many units/group"   ?   ? × 3/4 = 2/3   2/3 ÷ 3/4

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

neither, there aren't a different units so it's not ÷

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

bu/group   asking for total pounds or # of groups

"how many groups"   3/4 → 1   2/3 → ?   2/3 × ? = 3/4   3/4 ÷ 2/3

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

bu/group   # of groups

neither,   3/4 → 1   ? → 2/3   3/4 × 2/3 = ?

this would be a multiplication problem
</td>
<td>2.e.

Multiplication/Division
PT 3 used multiplication with multiplication operation sign ×
$$\frac{3}{4} \times \frac{2}{3}$$
Group and Base Units
PT 3 included information about groups and base units explicitly and correctly.
Multiplicative situation interpretation
PT 3 interpreted this situation as MS

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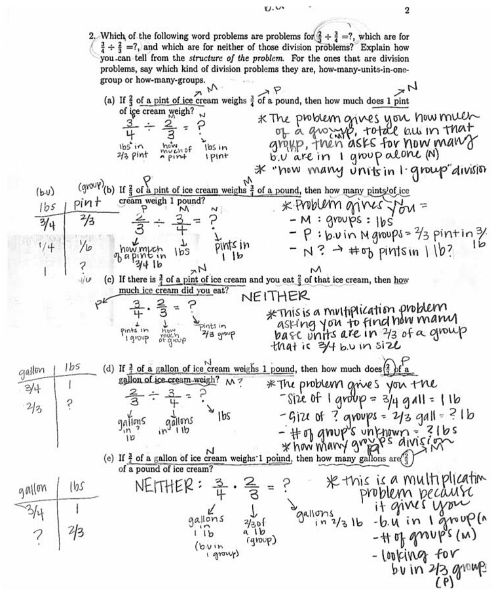
</table>

D. PT 4's solutions for the five sub questions

<table>
<tr>
<td>PT 4
EE</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{3}{4} \div \frac{2}{3} = ?$ lbs in 2/3 pint; how much of a pint; lbs in 1 pint

*The problem gives you how much of a group, total b.u in that group, then asks for how many b.u are in 1 group alone (N)
* "how many units in 1 group" division

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\frac{2}{3} \div \frac{3}{4} = ?$ how much of a pint in 3/4 lb; lbs; pints in 1 lb

*Problem gives you =
- M : groups : lbs
- P : b.u in M groups = 2/3 pint in 3/4 lb
- N ? → # of pints in 1 lb?

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

NEITHER

$\frac{3}{4} \cdot \frac{2}{3} = ?$ pints in 1 group; how much of group; pints in 2/3 group

*This is a multiplication problem asking you to find how many base units are in 2/3 of a group that is 3/4 b.u in size

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{2}{3} \div \frac{3}{4} = ?$ gallons in ? lb; gallons in 1 lb; lbs

*The problem gives you the
- Size of 1 group = 3/4 gall = 1 lb
- Size of ? groups = 2/3 gall = ? lb
- # of group's unknown = ? lbs
*how many groups division

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

NEITHER : $\frac{3}{4} \cdot \frac{2}{3} = ?$ gallons in 1 lb (b.u in 1 group); 2/3 of a lb (group); gallons in 2/3 lb

*this is a multiplication problem because it gives you
- b.u in 1 group
- # of groups (M)
- looking for b.u in 2/3 group (P)
</td>
<td>
2.a.

- Multiplication/Division
  - PT 4 used the division operation sign ÷
    $$\frac{3}{4} \div \frac{2}{3}$$
- Groups and base units
  - PT 4 included information about group and base unit explicitly and correctly.
- Multiplicative situation interpretation
  - PT 4 interpreted this situation as mentioned "how much of a group, total base unit in that group, then asks for 'how many bu are in 1 group alone (N)".
  - Thus, PT 4 interpreted this situation as PDS.
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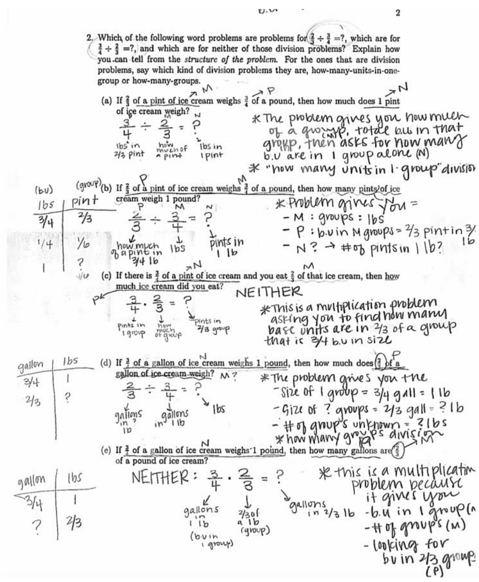
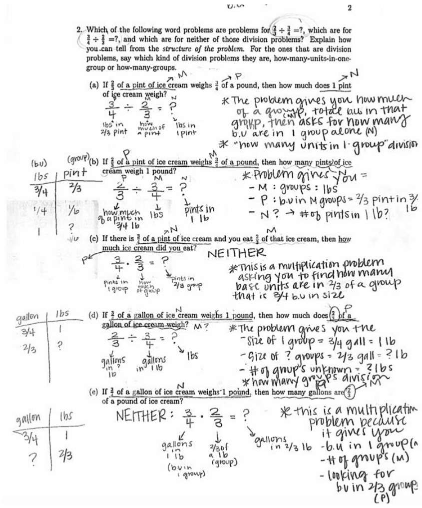
D (continued).

<table>
<tr>
<td>PT 4
EE</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3} = ?$
* The problem gives you how much of a group, total b.u in that group, then asks for how many b.u are in 1 group alone (N)
* "how many units in 1 group" division
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3} \div \frac{3}{4} = ?$
* Problem gives you =
- M : groups : lbs
- P : b.u in M groups = 2/3 pint in 3/4 lb
- N ? → # of pints in 1 lb?
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
NEITHER
$\frac{3}{4} \cdot \frac{2}{3} = ?$
* This is a multiplication problem asking you to find how many base units are in 2/3 of a group that is 3/4 b.u in size
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{2}{3} \div \frac{3}{4} = ?$
* The problem gives you the
- Size of 1 group = 3/4 gall = 1 lb
- Size of ? groups = 2/3 gall = ? lb
- # of group's unknown = ? lbs
* how many groups division
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
NEITHER : $\frac{3}{4} \cdot \frac{2}{3} = ?$
* this is a multiplication problem because it gives you
- b.u in 1 group
- # of groups (M)
- looking for b.u in 2/3 group (P)</td>
<td>2.b.
• Multiplication/Division
  ○ PT 4 used the division operation sign ÷
$\frac{2}{3} \div \frac{3}{4}$
• Groups and base units
  ○ PT 4 included information about group and base unit explicitly and correctly.
• Multiplicative situation interpretation
  ○ PT 4 mentioned that this question is asking # of base units in 1 group.
  ○ Although PT 4 explicitly did not state if it is PDS, PT 4 included definition for PDS in the solution.</td>
</tr>
</table>

D (continued).

<table>
<tr>
<td>PT 4
EE</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{3}{4} \div \frac{2}{3} = ?$ lbs in 2/3 pint; how much of a pint; lbs in 1 pint

*The problem gives you how much of a group (M), total b.u in that group (P), then asks for how many b.u are in 1 group alone (N)

* "how many units in 1 group" division

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

lbs (b.u)	pint (group)
3/4	2/3
1/4	1/6
1	?

$\frac{2}{3} \div \frac{3}{4} = ?$ how much of a pint in 3/4 lb; lbs; pints in 1 lb

*Problem gives you =
- M : groups : lbs
- P : b.u in M groups = 2/3 pint in 3/4 lb
- N ? → # of pints in 1 lb?

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

NEITHER

$\frac{3}{4} \cdot \frac{2}{3} = ?$ pints in 1 group; how much of group; pints in 2/3 group

*This is a multiplication problem asking you to find how many base units are in 2/3 of a group that is 3/4 b.u in size

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

gallon	lbs
3/4	1
2/3	?

$\frac{2}{3} \div \frac{3}{4} = ?$ gallons in ? lb; gallons in 1 lb; lbs

*The problem gives you the
- size of 1 group = 3/4 gall = 1 lb
- size of ? groups = 2/3 gall = ? lb
- # of group's unknown = ? lbs

*how many groups division

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

gallon	lbs
3/4	1
?	2/3

NEITHER: $\frac{3}{4} \cdot \frac{2}{3} = ?$ gallons in 1 lb (b.u in 1 group); 2/3 of a lb (group); gallons in 2/3 lb

*this is a multiplication problem because it gives you
- b.u in 1 group (N)
- # of groups (M)
- looking for bu in 2/3 group (P)
</td>
<td>
2.c.

- Multiplication/Division
  - PT 4 used the multiplication with the multiplication $$\frac{3}{4} \times \frac{2}{3}$$
- Groups and base units
  - PT 4 included information about group and base unit explicitly and correctly.
- Multiplicative situation interpretation
  - PT 4 interpreted this situation as MS.
</td>
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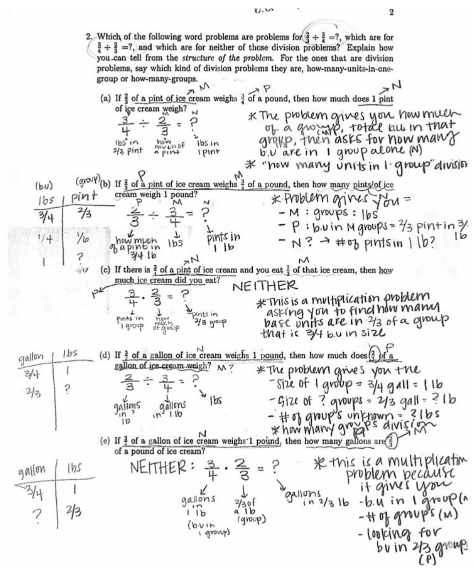
</table>

D (continued).

<table>
<tr>
<td>PT 4
EE</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4}$ =?, which are for $\frac{3}{4} \div \frac{2}{3}$ =?, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3} = ?$
lbs in 2/3 pint; how much of a pint; lbs in 1 pint
* The problem gives you how much of a group (M), total b.u in that group, then asks for how many b.u are in 1 group alone (N)
* "how many units in 1 group" division
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
(b.u) (group)
lbs | pint
3/4 | 2/3
1/4 | 1/6
1 | ?
$\frac{2}{3} \div \frac{3}{4} = ?$
how much of a pint in 3/4 lb; lbs; pints in 1 lb
* Problem gives you =
- M : groups : lbs
- P : b.u in M groups = 2/3 pint in 3/4 lb
- N ? → # of pints in 1 lb?
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
NEITHER
$\frac{3}{4} \cdot \frac{2}{3} = ?$
pints in 1 group; how much of group; pints in 2/3 group
* This is a multiplication problem asking you to find how many base units are in 2/3 of a group that is 3/4 b.u in size
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
gallon | lbs
3/4 | 1
2/3 | ?
$\frac{2}{3} \div \frac{3}{4} = ?$
gallons in ? lb; gallons in 1 lb; lbs
* The problem gives you the
- Size of 1 group = 3/4 gall = 1 lb
- Size of ? groups = 2/3 gall = ? lb
- # of group's unknown = ? lbs
* how many groups division
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
gallon | lbs
3/4 | 1
? | 2/3
NEITHER : $\frac{3}{4} \cdot \frac{2}{3} = ?$
gallons in 1 lb (b.u in 1 group); 2/3 of a lb (group); gallons in 2/3 lb
* this is a multiplication problem because it gives you
- b.u in 1 group (N)
- # of groups (M)
- looking for b.u in 2/3 group (P)</td>
<td>2.d.
• Multiplication/Division
  o PT 4 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
• Groups and base units
  o PT 4 included information about group and base unit explicitly and correctly.
• Multiplicative situation interpretation
  o PT 4 mentioned it as “how many groups division” problem
Thus, PT 4 interpreted it as QDS.</td>
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D (continued).

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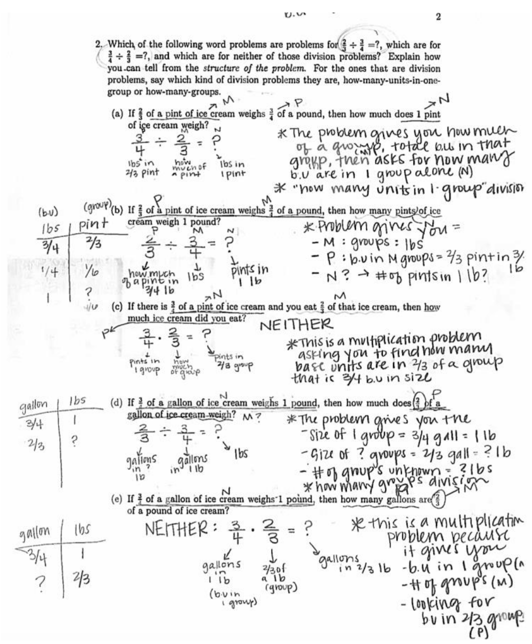
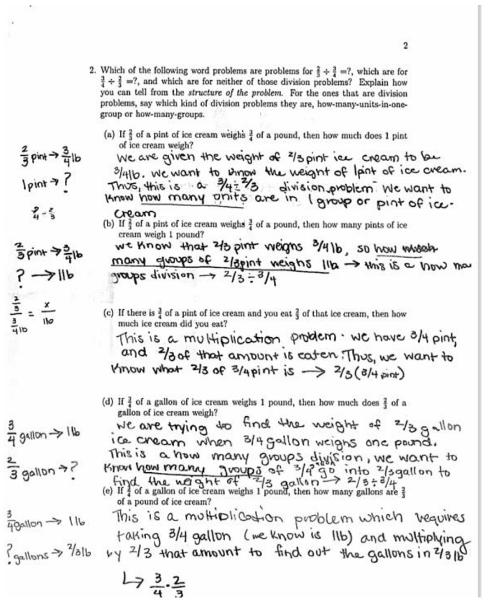
EE</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3} = ?$
lbs in 2/3 pint; how much of a pint; lbs in 1 pint
* The problem gives you how much of a group, total b.u in that group, then asks for how many b.u are in 1 group alone (N)
* "how many units in 1 group" division
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3} \div \frac{3}{4} = ?$
how much of a pint in 3/4 lb; lbs; pints in 1 lb
* Problem gives you =
- M : groups : lbs
- P : b.u in M groups = 2/3 pint in 3/4 lb
- N ? → # of pints in 1 lb?
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
NEITHER
$\frac{3}{4} \cdot \frac{2}{3} = ?$
pints in 1 group; how much of group; pints in 2/3 group
* This is a multiplication problem asking you to find how many base units are in 2/3 of a group that is 3/4 b.u in size
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{2}{3} \div \frac{3}{4} = ?$
gallons in ? lb; gallons in 1 lb; lbs
* The problem gives you the
- Size of 1 group = 3/4 gall = 1 lb
- Size of ? groups = 2/3 gall = ? lb
- # of group's unknown = ? lbs
* how many groups division
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
NEITHER : $\frac{3}{4} \cdot \frac{2}{3} = ?$
gallons in 1 lb (b.u in 1 group); 2/3 of a lb (group); gallons in 2/3 lb
* this is a multiplication problem because it gives you
- b.u in 1 group
- # of groups (M)
- looking for b.u in 2/3 group (P)</td>
<td>2.e.
Multiplication/Division
PT 4 used the multiplication operation sign ×
$\frac{3}{4} \times \frac{2}{3}$
Group and Base Units
PT 4 included information about group and base unit explicitly and correctly
Multiplicative situation interpretation
PT 4 interpreted this situation as MS.</td>
</tr>
</table>

E. PT 5's solutions for the five sub questions

<table>
<tr>
<td>PT 5
JG</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
1 pint → ?
$\frac{3}{4} \div \frac{2}{3}$

We are given the weight of 2/3 pint ice cream to be 3/4 lb. We want to know the weight of 1 pint of ice cream. Thus, this is a 3/4 ÷ 2/3 division problem. We want to know how many units are in 1 group or pint of ice-cream

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
? → 1 lb
$\frac{\frac{2}{3}}{\frac{3}{4}\text{lb}} = \frac{x}{1\text{lb}}$

we know that 2/3 pint weighs 3/4 lb, so how ~~much~~ many groups of 2/3 pint weighs 1 lb → this is a how ma groups division → 2/3 ÷ 3/4

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

This is a multiplication problem. we have 3/4 pint, and 2/3 of that amount is eaten. Thus, we want to know what 2/3 of 3/4 pint is → 2/3 (3/4 pint)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{3}{4}$ gallon → 1 lb
$\frac{2}{3}$ gallon → ?

We are trying to find the weight of 2/3 gallon ice cream when 3/4 gallon weighs one pound. This is a how many groups division, we want to know how many groups of 3/4 go into 2/3 gallon to find the weight of 2/3 gallon → 2/3 ÷ 3/4

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

$\frac{3}{4}$ gallon → 1 lb
? gallons → 2/3 lb

This is a multiplication problem which requires taking 3/4 gallon (we know is 1 lb) and multiplying by 2/3 that amount to find out the gallons in 2/3 lb

↳ $\frac{3}{4} \cdot \frac{2}{3}$
</td>
<td>
2.a.

- Multiplication/Division
  - PT 5 used the division operation sign ÷

    $$\frac{3}{4} \div \frac{2}{3}$$

- Groups and base units
  - PT 5 did not include information about groups and base units
- Multiplicative situation interpretation
  - PT 5 mentioned this situation as "how many units are in 1 group or pint of ice cream"
  - Thus, PT 5 interpreted this situation as PDS
</td>
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E (continued).

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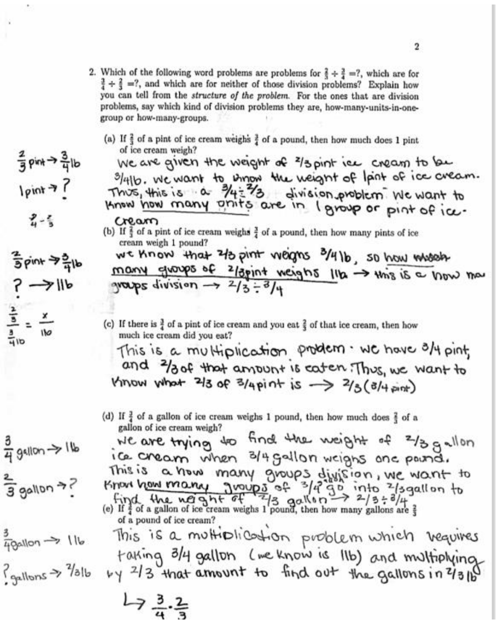
<tr>
<td>PT 5
JG</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
1 pint → ?
$\frac{3}{4} \div \frac{2}{3}$
We are given the weight of 2/3 pint ice cream to be 3/4 lb. We want to know the weight of 1 pint of ice cream. Thus, this is a 3/4 ÷ 2/3 division problem. We want to know how many units are in 1 group or pint of ice-cream
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
? → 1 lb
$\frac{\frac{2}{3}}{\frac{3}{4}\text{lb}} = \frac{x}{1\text{lb}}$
we know that 2/3 pint weighs 3/4 lb, so how ~~much~~ many groups of 2/3 pint weighs 1 lb → this is a how ma groups division → 2/3 ÷ 3/4
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
This is a multiplication problem. We have 3/4 pint, and 2/3 of that amount is eaten. Thus, we want to know what 2/3 of 3/4 pint is → 2/3 (3/4 pint)
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{3}{4}$ gallon → 1 lb
$\frac{2}{3}$ gallon → ?
We are trying to find the weight of 2/3 gallon ice cream when 3/4 gallon weighs one pound. This is a how many groups division, we want to know how many groups of 3/4 go into 2/3 gallon to find the weight of 2/3 gallon → 2/3 ÷ 3/4
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
$\frac{3}{4}$ gallon → 1 lb
? gallons → 2/3 lb
This is a multiplication problem which requires taking 3/4 gallon (we know is 1 lb) and multiplying by 2/3 that amount to find out the gallons in 2/3 lb
↳ $\frac{3}{4} \cdot \frac{2}{3}$</td>
<td>2.b.
• Multiplication/Division
○ PT 5 used the division operation sign ÷
$\frac{2}{3} \div \frac{3}{4}$
• Groups and base units
○ PT 5 did not include information about groups and base units
• Multiplicative situation interpretation
○ PT 5 mentioned this situation as “how many groups division”
○ Thus, PT 5 interpreted this situation as QDS not PDS. That is wrong.</td>
</tr>
</table>

E (continued).

<table>
<tr>
<td>PT 5
JG</td>
<td>2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
1 pint → ?
$\frac{3}{4} \div \frac{2}{3}$
We are given the weight of 2/3 pint ice cream to be 3/4 lb. We want to know the weight of 1 pint of ice cream. Thus, this is a 3/4 ÷ 2/3 division problem. We want to know how many units are in 1 group or pint of ice cream

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
? → 1 lb
$\frac{\frac{2}{3}}{\frac{3}{4}\text{lb}} = \frac{x}{1\text{lb}}$
we know that 2/3 pint weighs 3/4 lb, so how ~~much~~ many groups of 2/3 pint weighs 1 lb → this is a how ma groups division → 2/3 ÷ 3/4

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
This is a multiplication problem. we have 3/4 pint, and 2/3 of that amount is eaten. Thus, we want to know what 2/3 of 3/4 pint is → 2/3 (3/4 pint)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{3}{4}$ gallon → 1 lb
$\frac{2}{3}$ gallon → ?
We are trying to find the weight of 2/3 gallon ice cream when 3/4 gallon weighs one pound. This is a how many groups division, we want to know how many groups of 3/4 go into 2/3 gallon to find the weight of 2/3 gallon → 2/3 ÷ 3/4

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
$\frac{3}{4}$ gallon → 1 lb
? gallons → 2/3 lb
This is a multiplication problem which requires taking 3/4 gallon (we know is 1 lb) and multiplying by 2/3 that amount to find out the gallons in 2/3 lb
→ $\frac{3}{4} \cdot \frac{2}{3}$</td>
<td>2.c.
• Multiplication/Division
o PT 5 used the multiplication with the multiplication operation sign but PT 5 misplaced numbers
$\frac{2}{3} \times \frac{3}{4}$
• Groups and base units
o PT 5 did not include information about groups and base units
• Multiplicative situation interpretation
o PT 5 interpreted this situation as MS.</td>
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E (continued).

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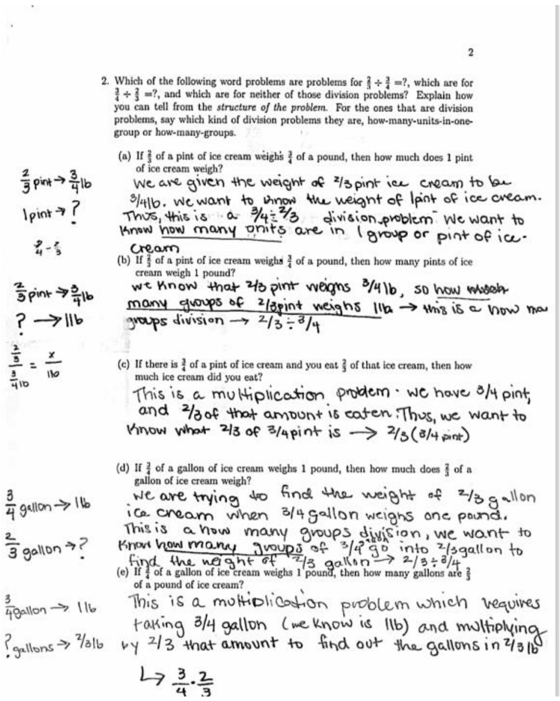
JG</td>
<td>
2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
1 pint → ?
$\frac{3}{4} \div \frac{2}{3}$
We are given the weight of 2/3 pint ice cream to be 3/4 lb. We want to know the weight of 1 pint of ice cream. Thus, this is a 3/4 ÷ 2/3 division problem. We want to know how many units are in 1 group or pint of ice-cream

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
? → 1 lb
$\frac{\frac{2}{3}}{\frac{3}{4}\text{lb}} = \frac{x}{1\text{lb}}$
we know that 2/3 pint weighs 3/4 lb, so how ~~much~~ many groups of 2/3 pint weighs 1 lb → this is a how ma[ny] groups division → 2/3 ÷ 3/4

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
This is a multiplication problem. we have 3/4 pint, and 2/3 of that amount is eaten. Thus, we want to know what 2/3 of 3/4 pint is → 2/3(3/4 pint)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{3}{4}$ gallon → 1 lb
$\frac{2}{3}$ gallon → ?
We are trying to find the weight of 2/3 gallon ice cream when 3/4 gallon weighs one pound. This is a how many groups division, we want to know how many groups of 3/4 go into 2/3 gallon to find the weight of 2/3 gallon → 2/3 ÷ 3/4

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
$\frac{3}{4}$ gallon → 1 lb
? gallons → 2/3 lb
This is a multiplication problem which requires taking 3/4 gallon (we know is 1 lb) and multiplying by 2/3 that amount to find out the gallons in 2/3 lb
↳ $\frac{3}{4} \cdot \frac{2}{3}$</td>
<td>2.d.
• Multiplication/Division
  ○ PT 5 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
• Groups and base units
  ○ PT 5 did not include groups and base units
• Multiplicative situation interpretation
  ○ PT 5 mentioned this situation as “how many groups of $\frac{3}{4}$ gallon go into $\frac{2}{3}$ gallon” problem Thus, PT 5 interpreted this situation as QDS.</td>
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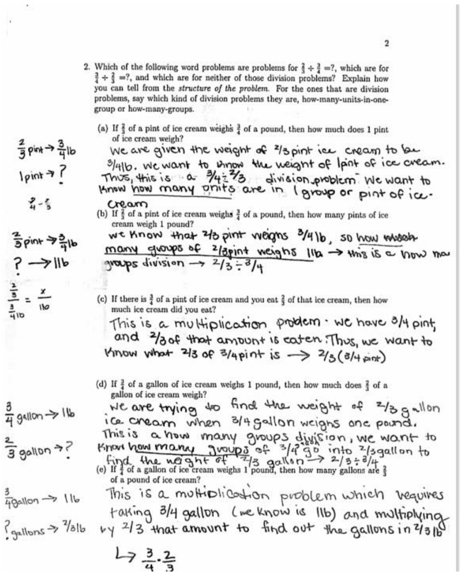
E (continued).

<table>
<tr>
<td>PT 5
JG</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
1 pint → ?
$\frac{3}{4} \div \frac{2}{3}$

We are given the weight of 2/3 pint ice cream to be 3/4 lb. We want to know the weight of 1 pint of ice cream. Thus, this is a 3/4 ÷ 2/3 division problem. We want to know how many units are in 1 group or pint of ice-cream

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\frac{2}{3}$ pint → $\frac{3}{4}$ lb
? → 1 lb
$\frac{\frac{2}{3}}{\frac{3}{4}\text{lb}} = \frac{x}{1\text{lb}}$

we know that 2/3 pint weighs 3/4 lb, so how ~~much~~ many groups of 2/3 pint weighs 1 lb → this is a how many groups division → 2/3 ÷ 3/4

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

This is a multiplication problem. we have 3/4 pint, and 2/3 of that amount is eaten. Thus, we want to know what 2/3 of 3/4 pint is → 2/3(3/4 pint)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{3}{4}$ gallon → 1 lb
$\frac{2}{3}$ gallon → ?

we are trying to find the weight of 2/3 gallon ice cream when 3/4 gallon weighs one pound. This is a how many groups division, we want to know how many groups of 3/4 go into 2/3 gallon to find the weight of 2/3 gallon → 2/3 ÷ 3/4

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

$\frac{3}{4}$ gallon → 1 lb
? gallons → 2/3 lb

This is a multiplication problem which requires taking 3/4 gallon (we know is 1 lb) and multiplying by 2/3 that amount to find out the gallons in 2/3 lb

→ $\frac{3}{4} \cdot \frac{2}{3}$
</td>
<td>
2.e.

- Multiplication/Division
  - PT 5 used the multiplication operation sign ×

$$\frac{3}{4} \times \frac{2}{3}$$

- Group and Base Units
  - PT 5 did not include information about groups and base units
- Multiplicative situation interpretation
  - PT 5 interpreted this situation interpreted it as MS
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F. PT 6's solutions for the five sub questions

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JH</td>
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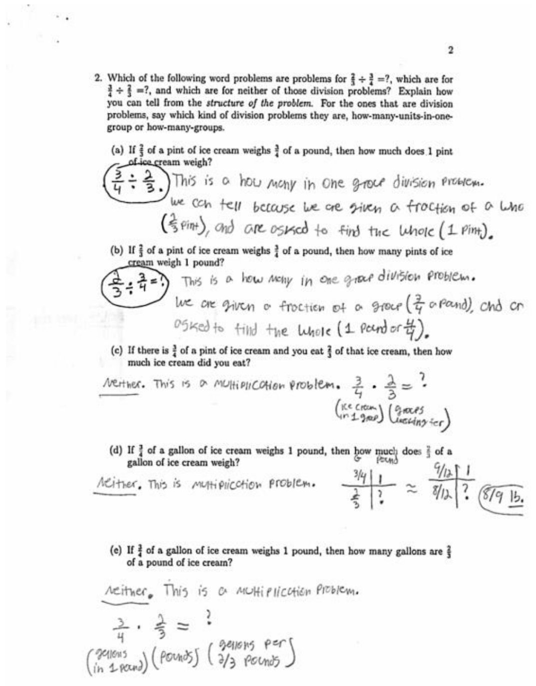

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\frac{3}{4} \div \frac{2}{3}$. This is a how many in one group division problem. we can tell because we are given a fraction of a who ($\frac{2}{3}$ pint), and are asked to find the whole (1 pint).

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\frac{2}{3} \div \frac{3}{4} = ?$ This is a how many in one group division problem. we are given a fraction of a group ($\frac{3}{4}$ a pound), and are asked to find the whole (1 pound or $\frac{4}{4}$).

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

Neither. This is a multiplication problem. $\frac{3}{4} \cdot \frac{2}{3} = ?$ (ice cream in 1 group) (groups looking for)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

G Pound

Neither. This is multiplication problem. 3/4 | 1, $\frac{2}{3}$ | ? $\approx$ 9/12 | 1, 8/12 | ? 8/9 lb.

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

Neither. This is a multiplication problem.

$\frac{3}{4} \cdot \frac{2}{3} = ?$

(gallons in 1 pound) (pounds) (gallons per 2/3 pounds)
</td>
<td>
2.a.

- Multiplication/Division
  - PT 6 used the division operation sign ÷

    $$\frac{3}{4} \div \frac{2}{3}$$
- Groups and base units
  - PT 6 did not include information about groups and base units
- Multiplicative situation interpretation
  - PT 6 mentioned this situation as a "how many in one group division problem"
  - Thus, PT 6 interpreted this situation as PDS
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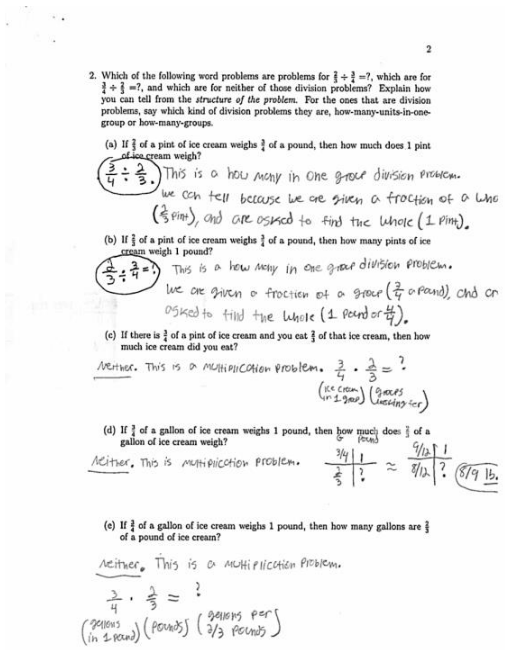
F (continued).

<table>
<tr>
<td>PT 6
JH</td>
<td>
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2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3}$. This is a how many in one group division problem. we can tell because we are given a fraction of a who ($\frac{2}{3}$ pint), and are asked to find the whole (1 pint).
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3} \div \frac{3}{4} = ?$ This is a how many in one group division problem. we are given a fraction of a group ($\frac{3}{4}$ a pound), and are asked to find the whole (1 pound or $\frac{4}{4}$).
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
Neither. This is a multiplication problem. $\frac{3}{4} \cdot \frac{2}{3} = ?$
(ice cream in 1 group) (groups eating ter)
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
G pound
Neither. This is multiplication problem.
3/4 | 1
$\frac{2}{3}$ | ?
≈
9/12 | 1
8/12 | ?
8/9 lb.
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Neither. This is a multiplication problem.
$\frac{3}{4} \cdot \frac{2}{3} = ?$
(gallons in 1 pound) (pounds) (gallons per 2/3 pounds)</td>
<td>2.b.
Multiplication/Division
PT 6 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
Groups and base units
PT 6 included information about groups and base units.
Multiplicative situation interpretation
PT 6 mentioned this situation as “how many in one group division” problem
Thus, PT 6 interpreted this situation as PDS.</td>
</tr>
</table>

F (continued).

<table>
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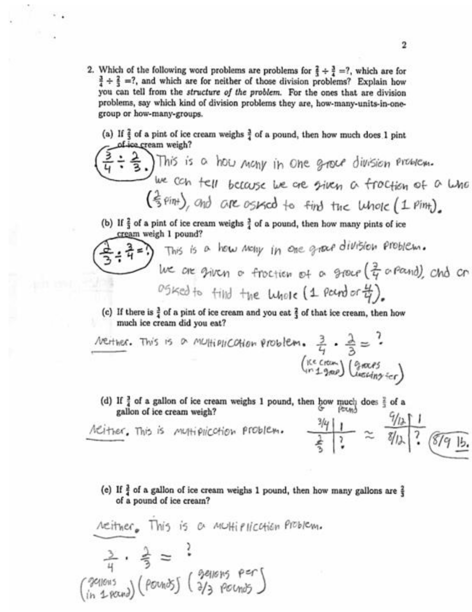
<td>PT 6
JH</td>
<td>
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Neither. This is a multiplication problem. $\frac{3}{4} \cdot \frac{2}{3} = ?$
(ice cream in 1 group) (groups [illegible])
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
G pound
Neither. This is multiplication problem. 3/4 | 1 ; $\frac{2}{3}$ | ? $\approx$ 9/12 | 1 ; 8/12 | ? 8/9 lb.
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Neither. This is a multiplication Problem.
$\frac{3}{4} \cdot \frac{2}{3} = ?$
(gallons in 1 pound) (pounds) (gallons per 2/3 pounds)</td>
<td>2.c.
• Multiplication/Division
  o PT 6 used the multiplication $\frac{3}{4} \times \frac{2}{3}$
• Groups and base units
  o PT 6 included information about groups and base units.
• Multiplicative situation interpretation
  o PT 6 interpreted this situation as MS</td>
</tr>
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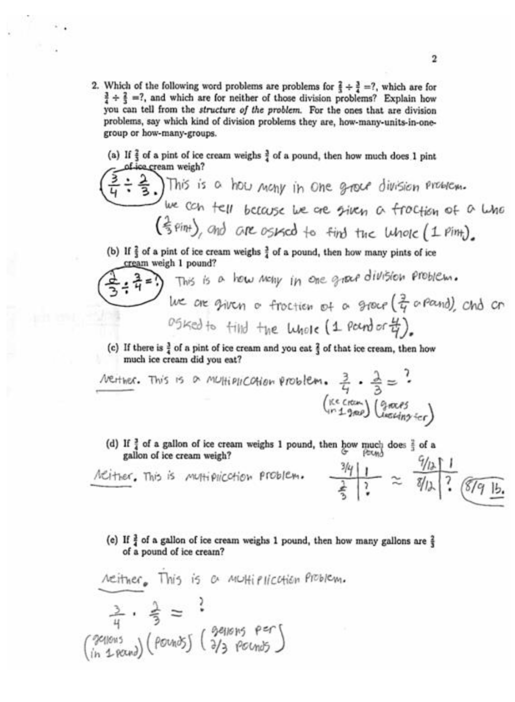
F (continued).

<table>
<tr>
<td>PT 6
JH</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

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(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

Neither. This is a multiplication problem. $\frac{3}{4} \cdot \frac{2}{3} = ?$ (ice cream in 1 group) (groups looking for)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

G / pound

Neither. This is multiplication problem. $\frac{3/4 \mid 1}{\frac{2}{3} \mid ?} \approx \frac{9/12 \mid 1}{8/12 \mid ?}$ 8/9 lb.

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

Neither. This is a multiplication problem.

$\frac{3}{4} \cdot \frac{2}{3} = ?$

(gallons in 1 pound) (pounds) (gallons per 2/3 pounds)
</td>
<td>
2.d.

- Multiplication/Division
  - PT 6 used the multiplication, but PT 6 should use division.
- Groups and base units
  - PT 6 did not include information about groups and base units.
- Multiplicative situation interpretation
  - PT 6 interpreted this situation as MS not QDS. That is wrong.
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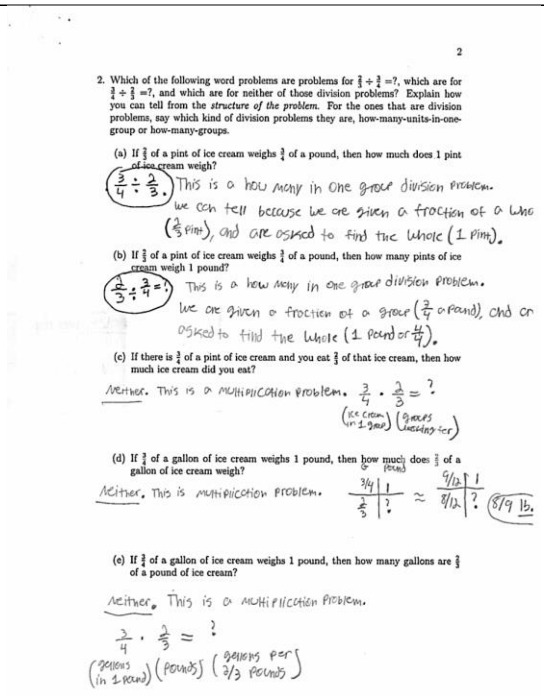
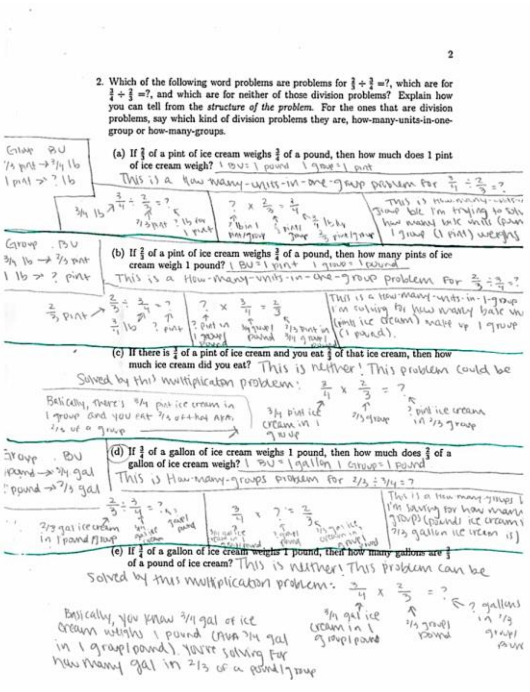
F (continued).

PT 6
JH
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	$\frac{3}{4} \div \frac{2}{3}$. This is a how many in one group division problem. we can tell because we are given a fraction of a who ($\frac{2}{3}$ pint), and are asked to find the whole (1 pint).
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	$\frac{2}{3} \div \frac{3}{4} = ?$ This is a how many in one group division problem. we are given a fraction of a group ($\frac{3}{4}$ a pound), and ar asked to find the whole (1 pound or $\frac{4}{4}$).
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	Neither. This is a multiplication problem. $\frac{3}{4} \cdot \frac{2}{3} = ?$ (ice cream in 1 group) (groups eating ter)
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	Neither. This is multiplication problem. G pound 3/4 | 1 ; $\frac{2}{3}$ | ? $\approx$ 9/12 | 1 ; 8/12 | ? 8/9 lb.
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	Neither. This is a multiplication problem.
	$\frac{3}{4} \cdot \frac{2}{3} = ?$
	(gallons in 1 pound) (pounds) (gallons per 2/3 pounds)
		2.e.
		• Multiplication/Division
		  ○ PT 6 used the multiplication operation sign $\times$
		$$\frac{3}{4} \times \frac{2}{3}$$
		• Group and Base Units
		  ○ PT 6 did not include information about groups and base units.
		• Multiplicative situation interpretation
		  ○ PT 6 interpreted this situation as MS

G. PT 7's solutions for the five sub questions

<table>
<tr>
<td>PT 7
LH</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? 1 BU = 1 pound 1 group = 1 pint
This is a how-many-units-in-one-group problem for $\frac{3}{4} \div \frac{2}{3} = ?$
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? 1 BU = 1 pint 1 group = 1 pound
This is a How-many-units-in-one-group problem for $\frac{2}{3} \div \frac{3}{4} = ?$
This is a how-many-units-in-1-group I'm solving for how many base units (pints ice cream) make up 1 group (1 pound).
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is neither! This problem could be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? 1 BU = 1 gallon 1 Group = 1 pound
This is How-many-groups problem for $2/3 \div 3/4 = ?$
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is neither! This problem can be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$
Basically, you know 3/4 gal of ice cream weighs 1 pound (AKA 3/4 gal in 1 group/pound). You're solving for how many gal in 2/3 of a pound/group</td>
<td>2.a.
• Multiplication/Division
  ○ PT 7 used the division operation sign ÷ $\frac{3}{4} \div \frac{2}{3}$
• Groups and base units
  ○ PT 7 included information about groups and base units.
• Multiplicative situation interpretation
  ○ PT 7 mentioned this situation as " how-many-units-in-one-group" problem.
  ○ Thus, PT 7 interpreted this situation as PDS</td>
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G (continued).

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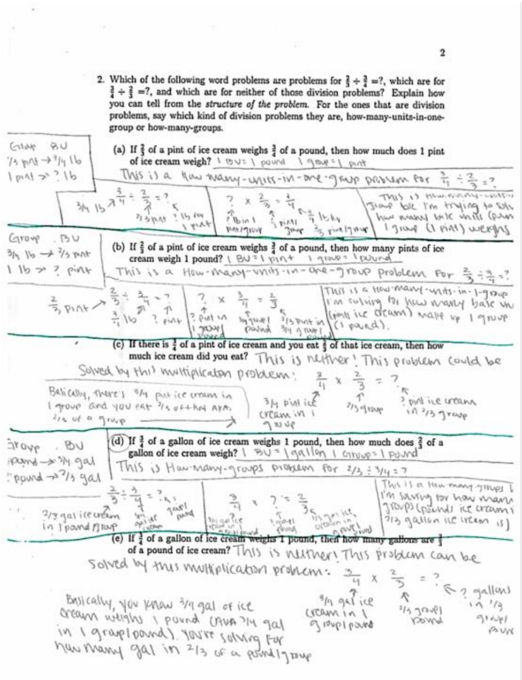
<td>PT 7
LH</td>
<td>
</td>
<td>2.b.

Multiplication/Division
PT 7 used the division operation sign ÷ $$\frac{2}{3} \div \frac{3}{4}$$
Groups and base units
PT 7 included information about groups and units explicitly and correctly.
Multiplicative situation interpretation
PT 7 mentioned this situation as “ how-many-units-in-one-group” problem.
Thus, PT 7 interpreted this situation as PDS.
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</table>

G (continued).

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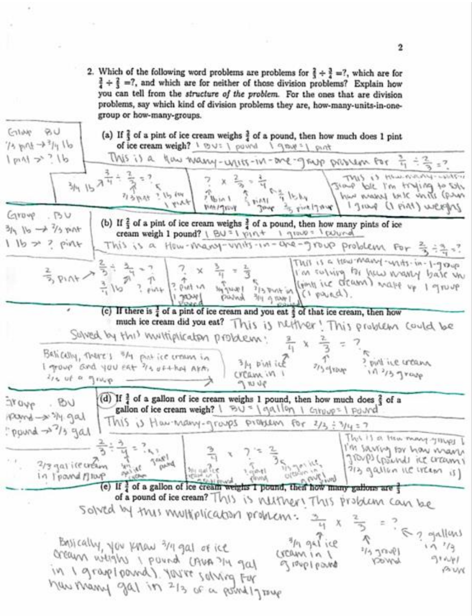


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? 1 BU = 1 pound 1 group = 1 pint

This is a How-many-units-in-one-group problem for $\frac{3}{4} \div \frac{2}{3} = ?$

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? 1 BU = 1 pint 1 group = 1 pound

This is a How-many-units-in-one-group problem for $\frac{2}{3} \div \frac{3}{4} = ?$

This is a How-many-units-in-1-group I'm solving for how many base units (pints ice cream) make up 1 group (1 pound).

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is neither! This problem could be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$

Basically, there's 3/4 pint ice cream in 1 group and you eat 2/3 of that AKA 2/3 of a group

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? 1 BU = 1 gallon 1 Group = 1 pound

This is How-many-groups problem for $2/3 \div 3/4 = ?$

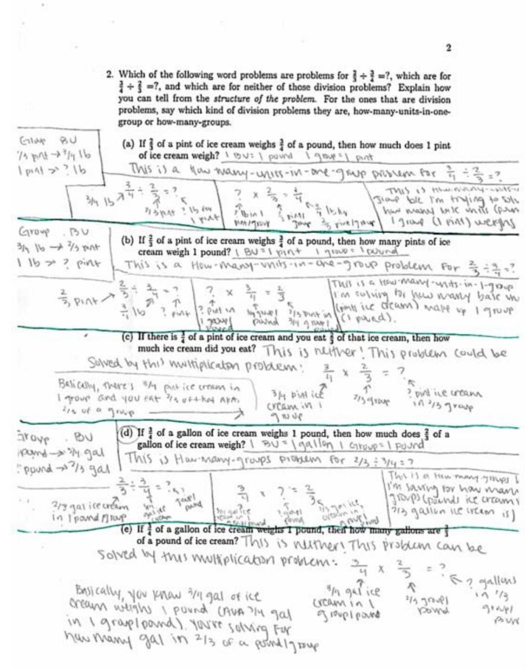
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is neither! This problem can be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$

Basically, you know 3/4 gal of ice cream weighs 1 pound (AKA 3/4 gal in 1 group/pound). You're solving for how many gal in 2/3 of a pound/group
</td>
<td>2.c.

Multiplication/Division
PT 7 used the multiplication $\frac{3}{4} \times \frac{2}{3}$
Groups and base units
PT 7 included information about groups and base units explicitly and correctly.
Multiplicative situation interpretation
PT 7 interpreted this situation as MS

</td>
</tr>
</table>

G (continued).

PT 7
LH
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? 1 BU = 1 pound 1 group = 1 pint
	This is a how-many-units-in-one-group problem for $\frac{3}{4} \div \frac{2}{3} = ?$
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? 1 BU = 1 pint 1 group = 1 pound
	This is a How-many-units-in-one-group problem for $\frac{2}{3} \div \frac{3}{4} = ?$
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is neither! This problem could be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? 1 BU = 1 gallon 1 Group = 1 pound
	This is How-many-groups problem for $2/3 \div 3/4 = ?$
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is neither! This problem can be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$
	Basically, you know 3/4 gal of ice cream weighs 1 pound (AKA 3/4 gal in 1 group/pound). You're solving for how many gal in 2/3 of a pound/group
		2.d.
		• Multiplication/Division
		  ○ PT 7 used the division operation sign ÷
		$$\frac{2}{3} \div \frac{3}{4}$$
		• Groups and base units
		  ○ PT 7 included groups and base units
		• Multiplicative situation interpretation
		  ○ PT 7 mentioned this situation as "how many groups" problem
		  ○ Thus, PT 7 interpreted this situation as QDS.

G (continued).

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LH</td>
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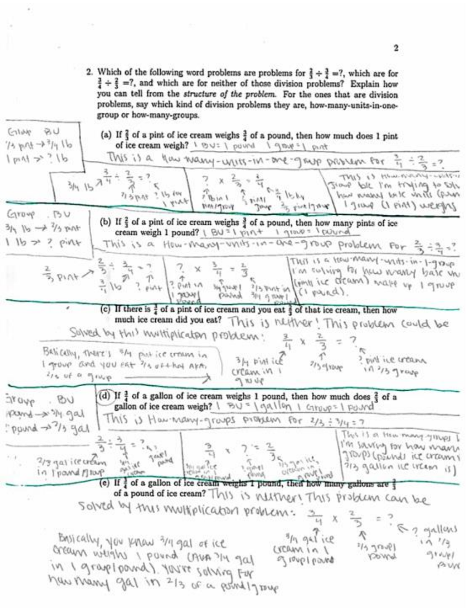

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

Group BU
2/3 pint → 3/4 lb
1 pint → ? lb

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? 1 BU = 1 pound 1 group = 1 pint
This is a how-many-units-in-one-group problem for $\frac{3}{4} \div \frac{2}{3} = ?$

Group BU
3/4 lb → 2/3 pint
1 lb → ? pint

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? 1 BU = 1 pint 1 group = 1 pound
This is a How-many-units-in-one-group problem for $\frac{2}{3} \div \frac{3}{4} = ?$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is neither! This problem could be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$

Group BU
1 pound → 3/4 gal
? pound → 2/3 gal

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? 1 BU = 1 gallon 1 Group = 1 Pound
This is How-many-groups problem for $2/3 \div 3/4 = ?$

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is neither! This problem can be solved by this multiplication problem: $\frac{3}{4} \times \frac{2}{3} = ?$

Basically, you know 3/4 gal of ice cream weighs 1 pound (AKA 3/4 gal in 1 group/pound). You're solving for how many gal in 2/3 of a pound/group
</td>
<td>
2.e.

- Multiplication/Division
  - PT 7 used the multiplication operation sign × $\frac{3}{4} \times \frac{2}{3}$
- Group and Base Units
  - PT 7 included information about groups and base units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 7 interpreted this situation as MS.
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H. 8's solutions for the five sub questions

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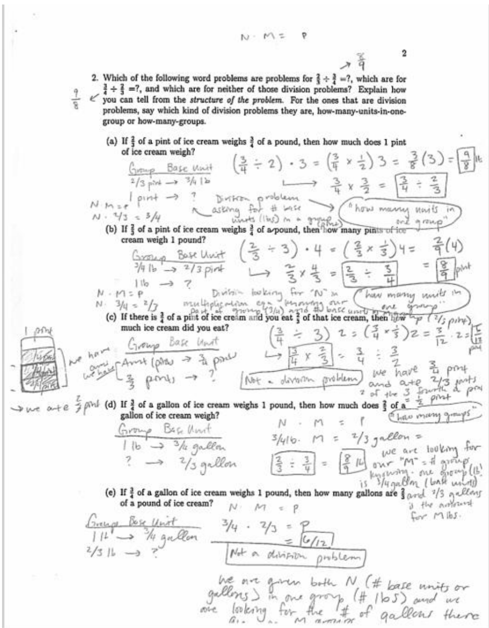
<tr>
<td>PT 8
CH</td>
<td>
N · M = P
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
Group Base Unit
2/3 pint → 3/4 lb
1 pint → ?
$(\frac{3}{4} \div 2) \cdot 3 = (\frac{3}{4} \times \frac{1}{2}) 3 = \frac{3}{8}(3) = \boxed{\frac{9}{8}}$ lb
$\frac{3}{4} \times \frac{3}{2} = \boxed{\frac{3}{4} \div \frac{2}{3}}$
"how many units in one group"
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
Group Base Unit
3/4 lb → 2/3 pint
1 lb → ?
$(\frac{2}{3} \div 3) \cdot 4 = (\frac{2}{3} \times \frac{1}{3}) 4 = \frac{2}{9}(4) = \boxed{\frac{8}{9}}$ pint
$\frac{2}{3} \times \frac{4}{3} = \boxed{\frac{2}{3} \div \frac{3}{4}}$
"how many units in one group"
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
Not a division problem
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
Group Base Unit
1 lb → 3/4 gallon
? → 2/3 gallon
N · M = P
$\boxed{\frac{2}{3} \div \frac{3}{4}} = \boxed{\frac{8}{9}\text{ lb}}$
"how many groups"
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Group Base Unit
1 lb → 3/4 gallon
2/3 lb → ?
3/4 · 2/3 = P = $\boxed{6/12}$
Not a division problem</td>
<td>2.a.
• Multiplication/Division
  o PT 8 used the division operation sign ÷
• Groups and base units
  o PT 8 included information about groups and base units.
• Multiplicative situation interpretation
  o PT 8 mentioned this situation as "how many units in one group" problem
  o Thus, PT 8 interpreted this situation as PDS.</td>
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H (continued).

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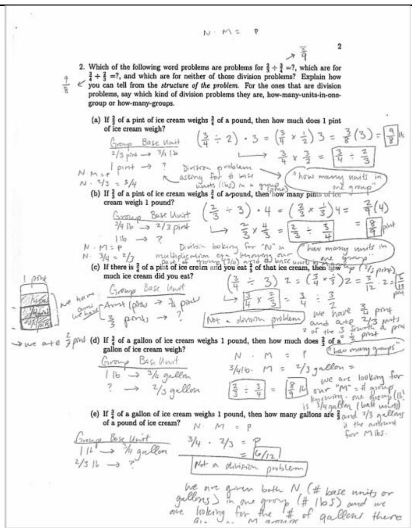
<td>PT 8
CH</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?</td>
<td>2.b.
• Multiplication/Division
  o PT 8 used the division operation sign ÷
• Groups and base units
  o PT 8 included information about groups and units.
• Multiplicative situation interpretation
  o PT 8 interpreted this situation as PDS.</td>
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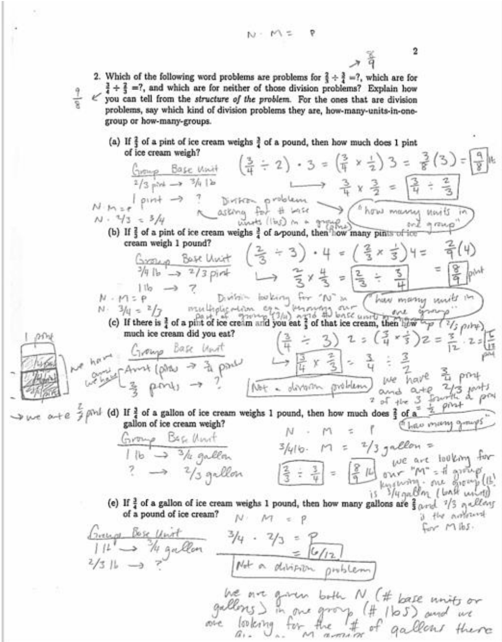
<td>
N · M = P
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4}$ =?, which are for $\frac{3}{4} \div \frac{2}{3}$ =?, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
Group Base Unit
2/3 pint → 3/4 lb
1 pint → ?
N · M = P
N · 2/3 = 3/4
$\left(\frac{3}{4} \div 2\right) \cdot 3 = \left(\frac{3}{4} \times \frac{1}{2}\right) 3 = \frac{3}{8}(3) = \boxed{\frac{9}{8}}$ lb
$\frac{3}{4} \times \frac{3}{2} = \boxed{\frac{3}{4} \div \frac{2}{3}}$
Division problem asking for # base units (lbs) in a group (pint) → "how many units in one group"
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
Group Base Unit
3/4 lb → 2/3 pint
1 lb → ?
N · M = P
N · 3/4 = 2/3
$\left(\frac{2}{3} \div 3\right) \cdot 4 = \left(\frac{2}{3} \times \frac{1}{3}\right) 4 = \frac{2}{9}(4) = \boxed{\frac{8}{9}}$ pint
$\frac{2}{3} \times \frac{4}{3} = \boxed{\frac{2}{3} \div \frac{3}{4}}$
Division looking for "N" in multiplication eqn knowing our part of group (3/4) and # base units in ... "how many units in one group"
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
1 pint
Group Base Unit
Amt (pint) → 3/4 pint
2/3 pints → ?
$\left(\frac{3}{4} \div 3\right) 2 = \left(\frac{3}{4} \times \frac{1}{3}\right) 2 = \frac{3}{12} \cdot 2 = \boxed{\frac{6}{12}}$ pint
$\boxed{\frac{3}{4} \times \frac{2}{3}} \div \frac{3}{4} \div \frac{3}{2}$
Not a division problem
we have 3/4 pint and ate 2/3 parts 2 of the 3 fourth a pint = 1/2 pint
→ we ate 2/3 pint
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
"how many groups"
Group Base Unit
1 lb → 3/4 gallon
? → 2/3 gallon
N · M = P
3/4 lb · M = 2/3 gallon =
$\boxed{\frac{2}{3} \div \frac{3}{4}} = \boxed{\frac{8}{9}}$ lb
we are looking for our "M" = # group knowing one group (lb) is 3/4 gallon (base units) and 2/3 gallons is the amount for M lbs.
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
Group Base Unit
1 lb → 3/4 gallon
2/3 lb → ?
N · M = P
3/4 · 2/3 = P
= $\boxed{6/12}$
Not a division problem
we are given both N (# base units or gallons) in one group (# lbs) and we are looking for the # of gallons there</td>
<td>2.c.
• Multiplication/Division
o PT 8 used the multiplication $\frac{3}{4} \times \frac{2}{3}$
• Groups and base units
o PT 8 included information about groups and base units explicitly and correctly.
• Multiplicative situation interpretation
o PT 8 interpreted this situation as MS.</td>
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H (continued).

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CH</td>
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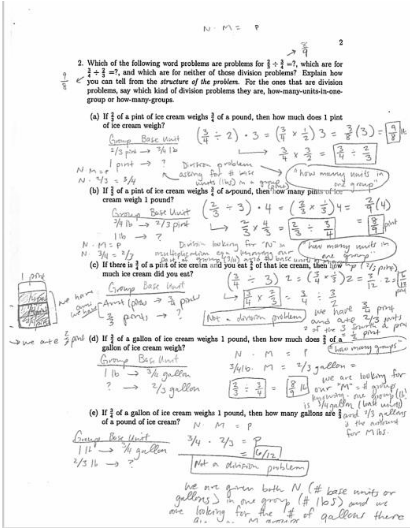


N · M = P

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =$?, which are for $\frac{3}{4} \div \frac{2}{3} =$?, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

Group Base Unit
2/3 pint → 3/4 lb
1 pint → ?

$(\frac{3}{4} \div 2) \cdot 3 = (\frac{3}{4} \times \frac{1}{2}) 3 = \frac{3}{8}(3) = \boxed{\frac{9}{8}}$ lb

$\frac{3}{4} \times \frac{3}{2} = \boxed{\frac{3}{4} \div \frac{2}{3}}$

N · M = P
N · 2/3 = 3/4

Division problem asking for # base units (lbs) in a group (pint)

"how many units in one group"

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

Group Base Unit
3/4 lb → 2/3 pint
1 lb → ?

$(\frac{2}{3} \div 3) \cdot 4 = (\frac{2}{3} \times \frac{1}{3}) 4 = \frac{2}{9}(4)$

$\frac{2}{3} \times \frac{4}{3} = \boxed{\frac{2}{3} \div \frac{3}{4}} = \boxed{\frac{8}{9}}$ pint

N · M = P
N · 3/4 = 2/3

Division looking for "N" in multiplication eqn knowing our part of group (3/4) and # base units

"how many units in one group"

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

Group Base Unit
Amnt (pints) → 3/4 pint
2/3 pints → ?

$(\frac{3}{4} \div 3) 2 = (\frac{3}{4} \times \frac{1}{3}) 2 = \frac{3}{12} \cdot 2 = \boxed{\frac{6}{12}}$ pint

$\boxed{\frac{3}{4} \times \frac{2}{3}}$

Not a division problem

we have 3/4 pint and ate 2/3 parts of the 3 fourths a pint = 1/2 pint

1 pint

we ate 2/3 pint

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

"how many groups"

Group Base Unit
1 lb → 3/4 gallon
? → 2/3 gallon

N · M = P
3/4 lb · M = 2/3 gallon

$\boxed{\frac{2}{3} \div \frac{3}{4}} = \boxed{\frac{8}{9}\text{ lb}}$

we are looking for our "M" = # groups knowing one group (lb) is 3/4 gallon (base units) and 2/3 gallons is the amount for M lbs.

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

Group Base Unit
1 lb → 3/4 gallon
2/3 lb → ?

N · M = P
3/4 · 2/3 = P
= $\boxed{6/12}$

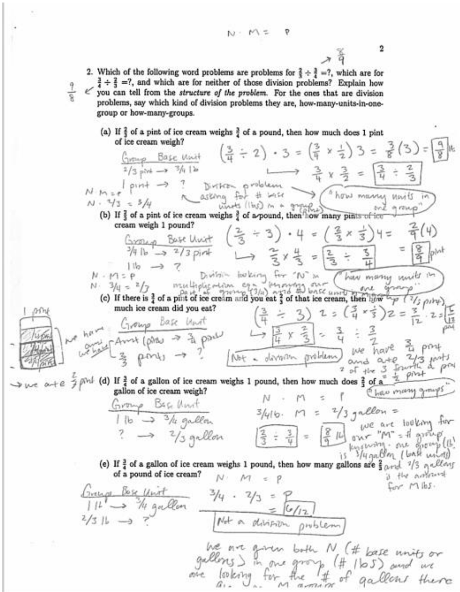
Not a division problem

we are given both N (# base units or gallons) in one group (# lbs) and we are looking for the # of gallons there
</td>
<td>
2.d.

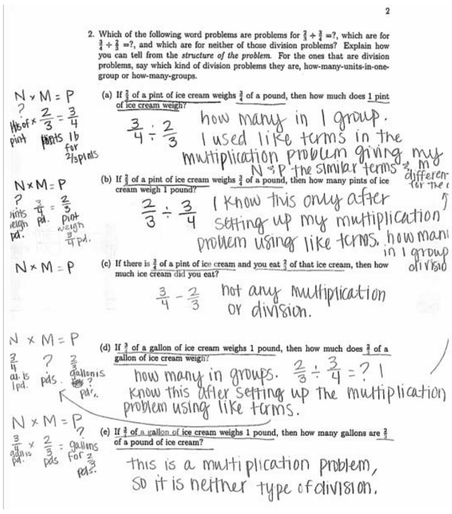
- Multiplication/Division
  - PT 8 used the division operation sign ÷

    $$\frac{2}{3} \div \frac{3}{4}$$
- Groups and base units
  - PT 8 included groups and base units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 8 mentioned this situation as "how many groups" problem
  - PT 8 interpreted this situation as QDS.
</td>
</tr>
</table>

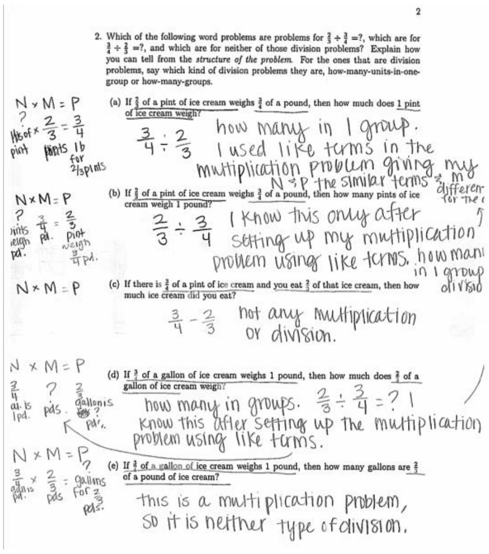
H (continued).

PT 8
CH
		2.e.
		• Multiplication/Division
		  ○ PT 8 used the multiplication operation sign ×
		$$\frac{3}{4} \times \frac{2}{3}$$
		• Group and Base Units
		  ○ PT 8 included information about groups and base units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 8 interpreted this situation as MS

I. PT 9's solutions for the five sub questions

PT 9
CJ
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	$N \times M = P$
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	$\frac{3}{4} \div \frac{2}{3}$ how many in 1 group. I used like terms in the multiplication problem giving my N ÷ P the similar terms & M differen- for the
	$N \times M = P$
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	$\frac{2}{3} \div \frac{3}{4}$ I know this only after setting up my multiplication problem using like terms. how many in 1 group division
	$N \times M = P$
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	$\frac{3}{4} - \frac{2}{3}$ not any multiplication or division.
	$N \times M = P$
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	how many in groups. $\frac{2}{3} \div \frac{3}{4} = ?$ I know this after setting up the multiplication problem using like terms.
	$N \times M = P$
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	$\frac{3}{4} \times \frac{2}{3} =$ gallons for $\frac{2}{3}$ pds.
	this is a multiplication problem, so it is neither type of division.
		2.a.
		• Multiplication/Division
		  ○ PT 9 used the division operation sign ÷ $\frac{3}{4} \div \frac{2}{3}$
		• Groups and base units
		  ○ PT 9 included information about groups and base units.
		• Multiplicative situation interpretation
		  ○ PT 9 mentioned "how many in 1 group"
		  ○ PT 9 interpreted it as PDS

I (continued).

PT 9
CJ
	
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	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
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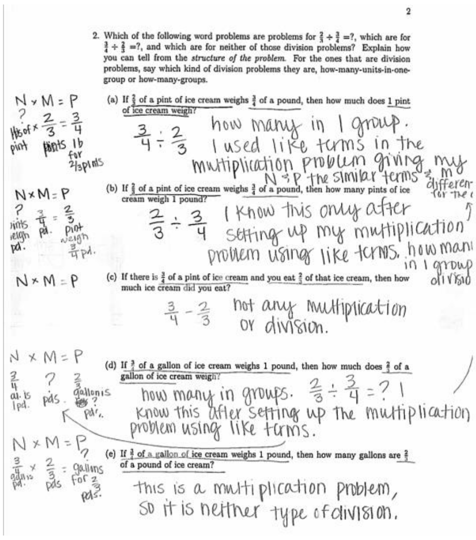
	$N \times M = P$
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	this is a multiplication problem, so it is neither type of division.
		2.b.
		• Multiplication/Division
		  ○ PT 9 used the division operation sign ÷
		$\frac{2}{3} \div \frac{3}{4}$
		• Groups and base units
		  ○ PT 9 included information about groups and units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 9 mentioned "how many in 1 group division"
		  ○ Thus, PT 9 interpreted this situation as PDS.

I (continued).

<table>
<tr>
<td>PT 9
CJ</td>
<td>
2
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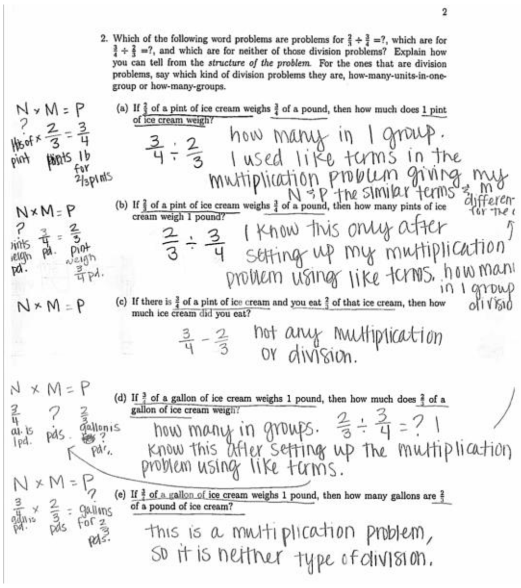
this is a multiplication problem, so it is neither type of division.</td>
<td>2.c.
Multiplication/Division
PT 9 did not use multiplication or division and PT 9’s solution is incorrect.
Groups and base units
PT 9 did not include information about groups and base units explicitly and correctly.
Multiplicative situation interpretation
PT 9 did not interpret this situation as MS</td>
</tr>
</table>

I (continued).

<table>
<tr>
<td>PT 9
CJ</td>
<td>



2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

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$\frac{3}{4} \div \frac{2}{3}$ how many in 1 group. I used like terms in the multiplication problem giving my N ÷ P the similar terms & M different for the ...

$N \times M = P$

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$\frac{2}{3} \div \frac{3}{4}$ I know this only after setting up my multiplication problem using like terms. how many in 1 group division

$N \times M = P$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

$\frac{3}{4} - \frac{2}{3}$ not any multiplication or division.

$N \times M = P$

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

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this is a multiplication problem, so it is neither type of division.

</td>
<td>

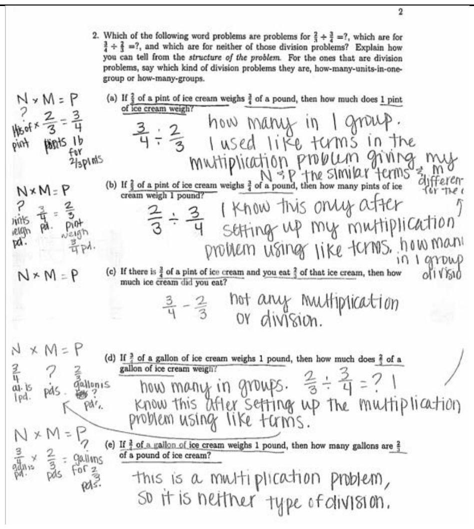
2.d.

- Multiplication/Division
  - PT 9 used the division operation sign ÷

    $$\frac{2}{3} \div \frac{3}{4}$$
- Groups and base units
  - PT 9 included groups and base units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 9 mentioned this situation as "how many in groups" problem
  - PT 9 interpreted this situation as PDS not QDS, so it is incorrect.

</td>
</tr>
</table>

I (continued).

PT 9
CJ
	
	2
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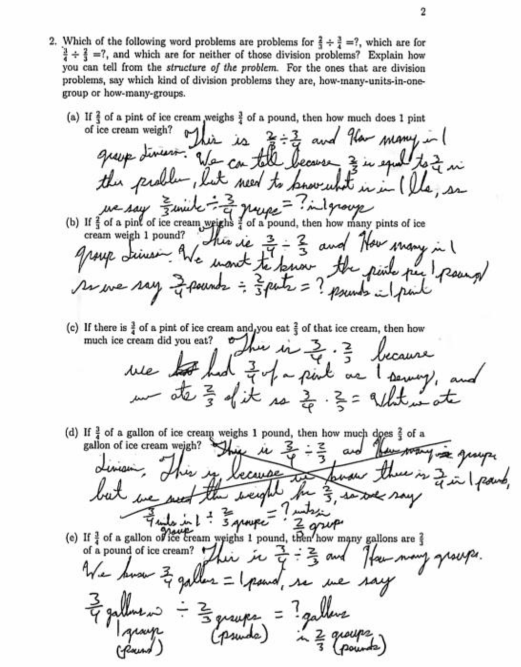
	this is a multiplication problem, so it is neither type of division.
		2.e.
		• Multiplication/Division
		  ○ PT 9 used the multiplication operation sign ×
		$\frac{3}{4} \times \frac{2}{3}$
		• Group and Base Units
		  ○ PT 9 included information about groups and base units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 9 interpreted this situation as MS

J. PT 10's solutions for the five sub questions

<table>
<tr>
<td>PT 10
CM</td>
<td>


2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

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(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? This is $\frac{3}{4} \div \frac{2}{3}$ and How many in 1 group division. We want to know the pints per 1 pound so we say $\frac{3}{4}$ pounds $\div \frac{2}{3}$ pints $=$? pounds in 1 pint

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is $\frac{3}{4} \cdot \frac{2}{3}$ because we ~~had~~ had $\frac{3}{4}$ of a pint as 1 serving, and we ate $\frac{2}{3}$ of it so $\frac{3}{4} \cdot \frac{2}{3} =$ What we ate

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? ~~This is $\frac{3}{4} \div \frac{2}{3}$ and How many in groups division. This is because we know there is $\frac{3}{4}$ in 1 pound, but we need the weight for $\frac{2}{3}$, so we say $\frac{3}{4}$ units in 1 group $\div \frac{2}{3}$ groups $=$? units in $\frac{2}{3}$ group~~

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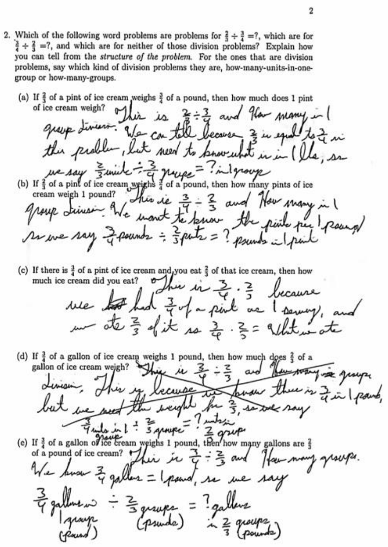
</td>
<td>2.a.

- Multiplication/Division
  - PT 10 used the division operation sign ÷, but PT 10 misplaced the numbers.
    $$\frac{2}{3} \div \frac{3}{4}$$
- Groups and base units
  - PT 10 did not include information about groups and base units.
- Multiplicative situation interpretation
  - PT 10 mentioned "how many in 1 group"
  - PT 10 interpreted it as PDS
</td>
</tr>
</table>

J (continued).

<table>
<tr>
<td>PT 10
CM</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
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(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? This is $\frac{3}{4} \div \frac{2}{3}$ and How many in 1 group division. We want to know the pints per 1 pound so we say $\frac{3}{4}$ pounds $\div \frac{2}{3}$ pints $=$? pounds in 1 pint
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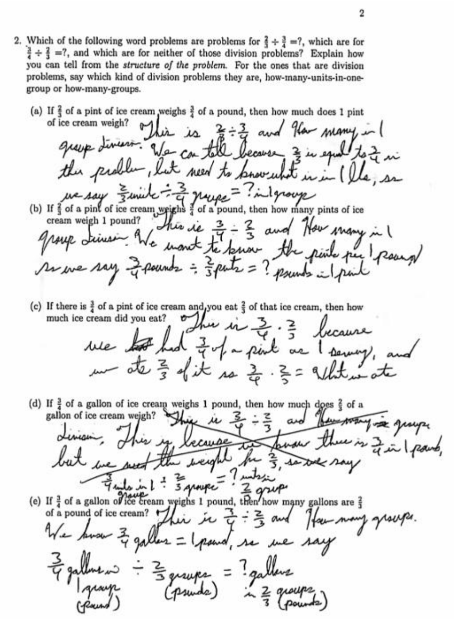
<td>2.b.
• Multiplication/Division
  ○ PT 10 used the division operation sign ÷ but PT 10 misplaced the numbers.
$$\frac{3}{4} \div \frac{2}{3}$$
• Groups and base units
  ○ PT 10 did not include information about groups and units explicitly and correctly.
• Multiplicative situation interpretation
  ○ PT 10 mentioned “how many in 1 group division”
  ○ Thus, PT 10 interpreted this situation as PDS.</td>
</tr>
</table>

J (continued).

PT 10 CM	
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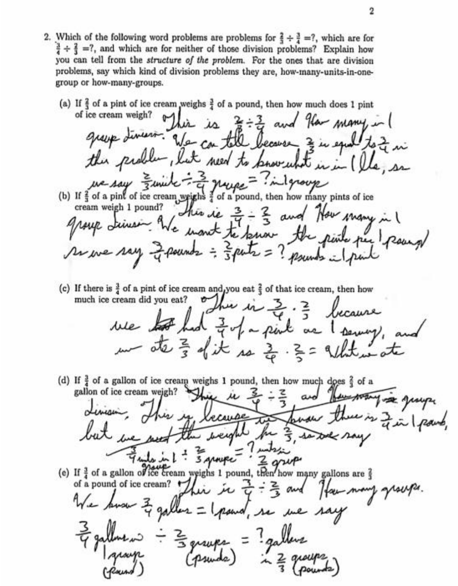
		2.c.
		• Multiplication/Division
		  ○ PT 10 used the multiplication $\frac{3}{4} \times \frac{2}{3}$
		• Groups and base units
		  ○ PT 10 did not include information about groups and base units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 10 did not include any information about the interpretation about the multiplicative situation explicitly.

J (continued).

<table>
<tr>
<td>PT 10
CM</td>
<td>
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2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? This is $\frac{2}{3} \div \frac{3}{4}$ and How many in 1 group division. We can tell because $\frac{2}{3}$ is equal to $\frac{3}{4}$ in the problem, but need to know what is in 1 lb, so we say $\frac{2}{3}$ units $\div \frac{3}{4}$ groups $=$? in 1 group
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? This is $\frac{3}{4} \div \frac{2}{3}$ and How many in 1 group division. We want to know the pints per 1 pound so we say $\frac{3}{4}$ pounds $\div \frac{2}{3}$ pints $=$? pounds in 1 pint
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is $\frac{3}{4} \cdot \frac{2}{3}$ because we ~~had~~ had $\frac{3}{4}$ of a pint as 1 serving, and we ate $\frac{2}{3}$ of it so $\frac{3}{4} \cdot \frac{2}{3} =$ What we ate
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? ~~This is $\frac{3}{4} \div \frac{2}{3}$ and How many in groups division. This is because we know there is $\frac{3}{4}$ in 1 pound, but we need the weight for $\frac{2}{3}$, so we say $\frac{3}{4}$ units in 1 group $\div \frac{2}{3}$ groups $=$? units in $\frac{2}{3}$ group~~
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is $\frac{3}{4} \div \frac{2}{3}$ and How many groups. We know $\frac{3}{4}$ gallon $=$ 1 pound, so we say $\frac{3}{4}$ gallons in 1 group (pound) $\div \frac{2}{3}$ groups (pounds) $=$? gallons in $\frac{2}{3}$ groups (pounds)</td>
<td>2.d.
Multiplication/Division
PT 10 used the division operation sign ÷ but PT 10 misplaced the numbers.
$$\frac{3}{4} \div \frac{2}{3}$$
Groups and base units
PT 10 did not include groups and base units correctly.
Multiplicative situation interpretation
PT 10 mentioned this situation as “how many groups” problem
PT 10 interpreted this situation as QDS.</td>
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J (continued).

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<td>PT 10
CM</td>
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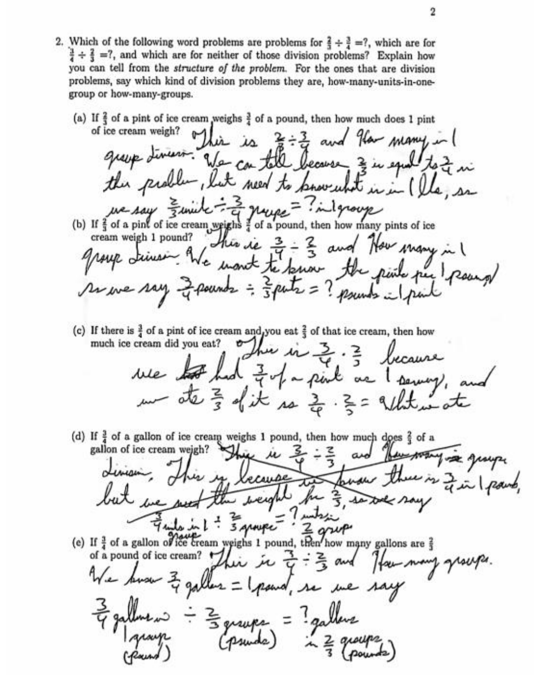

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? This is $\frac{2}{3} \div \frac{3}{4}$ and How many in 1 group division. We can tell because $\frac{2}{3}$ is equal to $\frac{3}{4}$ in the problem, but need to know what is in 1 lb, so we say $\frac{2}{3}$ units $\div \frac{3}{4}$ groups $= ?$ in 1 group

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? This is $\frac{3}{4} \div \frac{2}{3}$ and How many in 1 group division. We want to know the pints per 1 pound so we say $\frac{3}{4}$ pounds $\div \frac{2}{3}$ pints $= ?$ pounds in 1 pint

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This is $\frac{3}{4} \cdot \frac{2}{3}$ because we ~~had~~ had $\frac{3}{4}$ of a pint as 1 serving, and we ate $\frac{2}{3}$ of it so $\frac{3}{4} \cdot \frac{2}{3} =$ What we ate

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? This is $\frac{3}{4} \div \frac{2}{3}$ and How many in groups division. This is because we know there is $\frac{3}{4}$ in 1 pound, but we need the weight for $\frac{2}{3}$, so we say $\frac{3}{4}$ units in 1 group $\div \frac{2}{3}$ groups $= ?$ units in $\frac{2}{3}$ group

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is $\frac{3}{4} \div \frac{2}{3}$ and How many groups. We know $\frac{3}{4}$ gallons = 1 pound, so we say $\frac{3}{4}$ gallons in 1 group (pound) $\div \frac{2}{3}$ groups (pounds) $= ?$ gallons in $\frac{2}{3}$ groups (pounds)
</td>
<td>
2.e.

- Multiplication/Division
  - PT 10 used the division operation sign ÷ but it should be multiplication.

$$\frac{3}{4} \div \frac{2}{3}$$

- Group and Base Units
  - PT 10 did not include information about groups and base units correctly.
- Multiplicative situation interpretation
  - PT 10 did not interpret this situation correctly.
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K. PT 11's solutions for the five sub questions

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SP</td>
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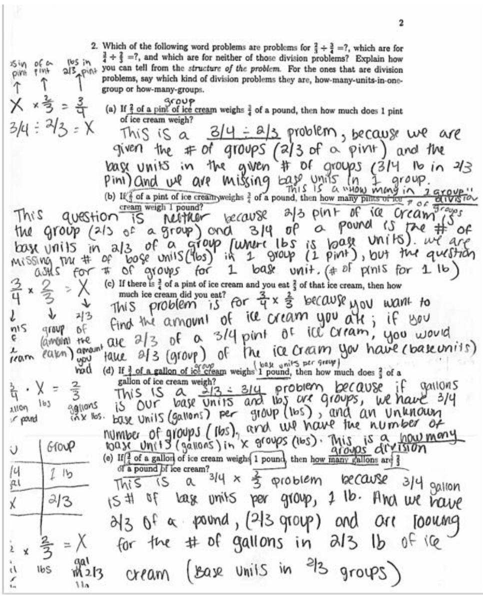

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

$X \times \frac{2}{3} = \frac{3}{4}$

$3/4 \div 2/3 = X$

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group. This is a "how many in 1 group" division

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). we are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)

$\frac{3}{4} \times \frac{2}{3} = X$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would take 2/3 (group) of the ice cream you have (base units)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{3}{4} \cdot X = \frac{2}{3}$

This is a 2/3 ÷ 3/4 problem because if gallons is our base units and lbs are groups, we have 3/4 base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

This is a $3/4 \times \frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups)

</td>
<td>

2.a.

- Multiplication/Division
  - PT 11 used the division operation sign ÷
    $$\frac{3}{4} \div \frac{2}{3}$$
- Groups and base units
  - PT 11 included information about groups and base units.
- Multiplicative situation interpretation
  - PT 11 mentioned "how many in 1 group"
  - PT 11 interpreted it as PDS

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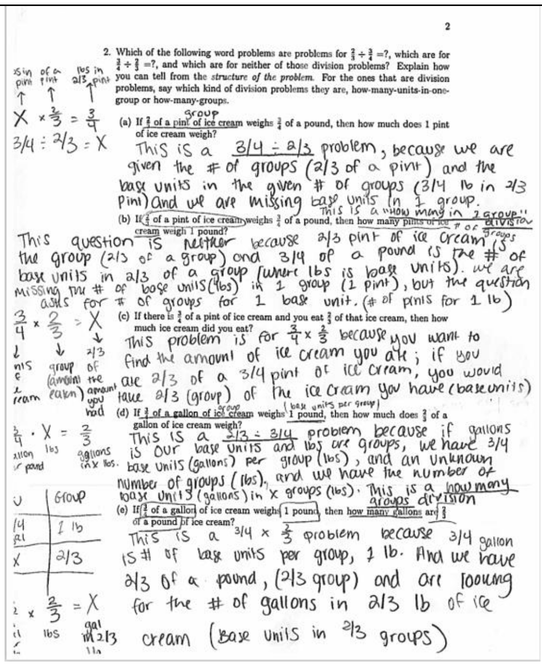
K (continued).

PT 11
SP
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	$X \times \frac{2}{3} = \frac{3}{4}$
	$3/4 \div 2/3 = X$
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group. This is a "how many in 1 group" division
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). we are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)
	$\frac{3}{4} \times \frac{2}{3} = X$
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would take 2/3 (group) of the ice cream you have (base units)
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	$\frac{3}{4} \cdot X = \frac{2}{3}$
	This is a 2/3 ÷ 3/4 problem because if gallons is our base units and lbs are groups, we have 3/4 base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	This is a 3/4 × $\frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups)
		2.a.
		• Multiplication/Division
		  ○ PT 11 used the division operation sign ÷
		$$\frac{3}{4} \div \frac{2}{3}$$
		• Groups and base units
		  ○ PT 11 included information about groups and base units.
		• Multiplicative situation interpretation
		  ○ PT 11 mentioned "how many in 1 group"
		  ○ PT 11 interpreted it as PDS

K (continued).

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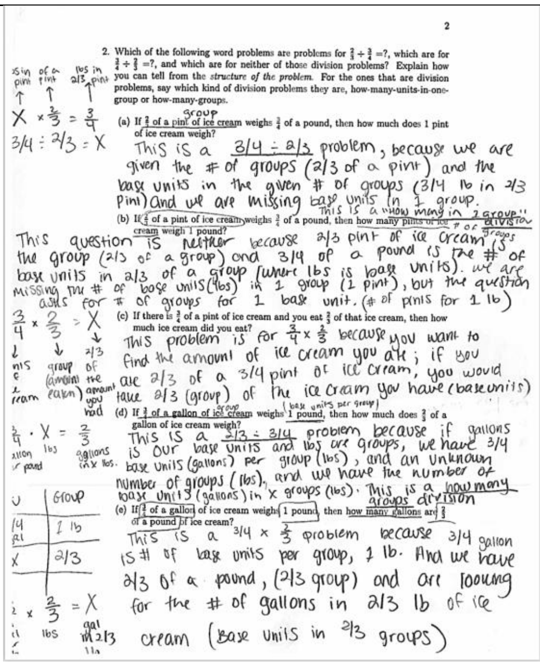
SP</td>
<td>
2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

$X \times \frac{2}{3} = \frac{3}{4}$

$3/4 \div 2/3 = X$

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group. This is a "how many in 1 group" division

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). We are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)

$\frac{3}{4} \times \frac{2}{3} = X$

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would take 2/3 (group) of the ice cream you have (base units)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\frac{3}{4} \cdot X = \frac{2}{3}$

This is a 2/3 ÷ 3/4 problem because if gallons and lbs are groups, we have 3/4 is our base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

This is a 3/4 × $\frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups)

$\frac{3}{4} \times \frac{2}{3} = X$</td>
<td>2.b.

Multiplication/Division
PT 11 used the multiplication operation sign and this is incorrect.
$$\frac{3}{4} \times \frac{2}{3}$$
Groups and base units
PT 11 included information about groups and units explicitly and correctly.
Multiplicative situation interpretation
PT 11 interpreted this situation as MS and this is incorrect.
</td>
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K (continued).

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<tr>
<td>PT 11
SP</td>
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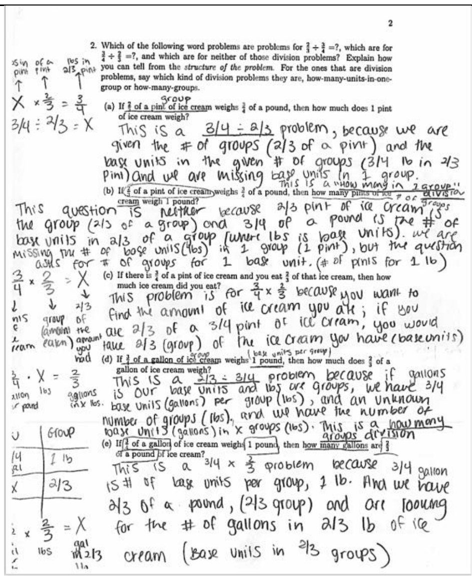

2

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

lbs in pint, of a pint, lbs in 2/3 pint

$X \times \frac{2}{3} = \frac{3}{4}$

$3/4 \div 2/3 = X$

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

group

This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

This is a "How many in 1 group" division

# of groups

This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). We are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)

$\frac{3}{4} \times \frac{2}{3} = X$

units, group (amount eaten), 2/3 of the amount you had

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would have 2/3 (group) of the ice cream you have (base units)

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

group, (base units per group)

$\frac{3}{4} \cdot X = \frac{2}{3}$

gallon per pound, lbs, gallons in x lbs.

This is a 2/3 ÷ 3/4 problem because if gallons is our base units and lbs are groups, we have 3/4 base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division

	Group
3/4 gal	1 lb
X	2/3

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

This is a $3/4 \times \frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups)

$\times \frac{2}{3} = X$

lbs, gal in 2/3 lb
</td>
<td>
2.b.

- Multiplication/Division
  - PT 11 used the multiplication operation sign and this is incorrect.

    $$\frac{3}{4} \times \frac{2}{3}$$
- Groups and base units
  - PT 11 included information about groups and units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 11 interpreted this situation as MS and this is incorrect.
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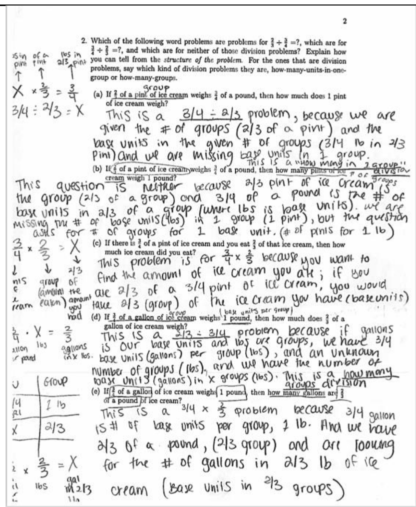
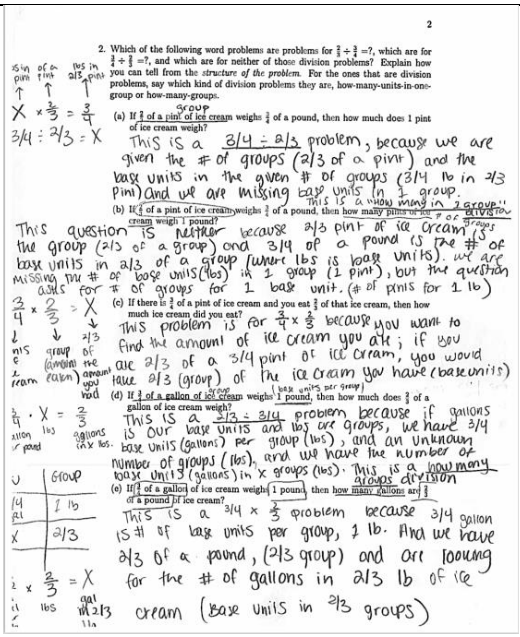
</table>

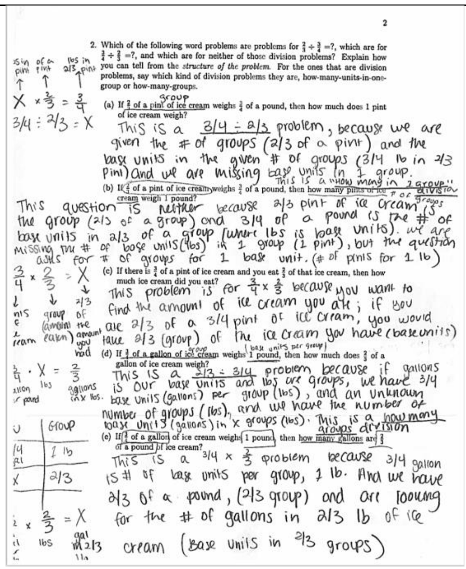
K (continued).

PT 11 SP	 2 2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups. $X \times \frac{2}{3} = \frac{3}{4}$ $3/4 \div 2/3 = X$ (a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh? This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group. (b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? This is a "How many in 1 group" division This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). We are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb) $\frac{3}{4} \times \frac{2}{3} = X$ (c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat? This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would take 2/3 (group) of the ice cream you have (base units) (d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh? $\frac{3}{4} \cdot X = \frac{2}{3}$ This is a 2/3 ÷ 3/4 problem because if gallons is our base units and lbs are groups, we have 3/4 base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division (e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream? This is a 3/4 × $\frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups) $\frac{3}{4} \times \frac{2}{3} = X$	2.c. • Multiplication/Division ○ PT 11 used the multiplication $\frac{3}{4} \times \frac{2}{3}$ • Groups and base units ○ PT 11 did not include information about groups and base units explicitly and correctly. • Multiplicative situation interpretation ○ PT 11 did not include any information about the interpretation about the multiplicative situation explicitly.

K (continued).

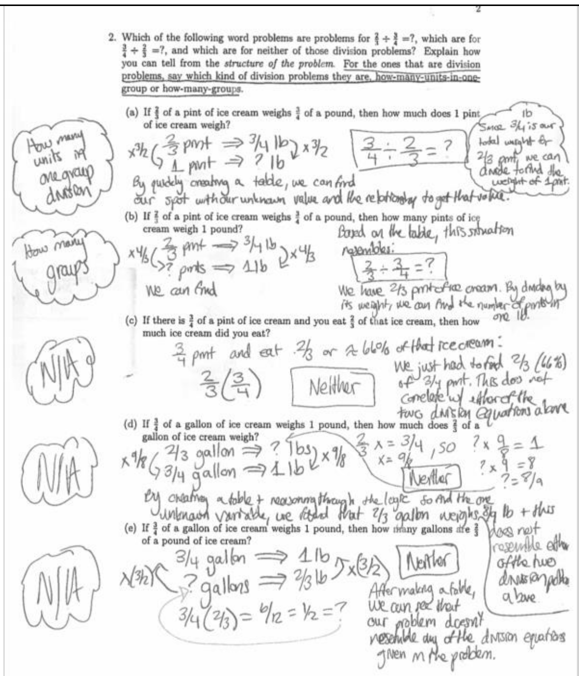
PT 11
SP
	
	2
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	lbs in pint, of a pint, lbs in 2/3 pint
	$X \times \frac{2}{3} = \frac{3}{4}$
	$3/4 \div 2/3 = X$
	(a) If $\frac{2}{3}$ of a pint of ice cream (group) weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group. This is a "how many in 1 group" division
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound? (# of groups)
	This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). We are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)
	$\frac{3}{4} \times \frac{2}{3} = X$
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would take 2/3 (group) of the ice cream you have (base units)
	group (amount eaten); 2/3 of the amount you had
	(d) If $\frac{3}{4}$ of a gallon of ice cream (group) weighs 1 pound (base units per group), then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	$\frac{3}{4} \cdot X = \frac{2}{3}$
	This is a 2/3 ÷ 3/4 problem because if gallons is our base units and lbs are groups, we have 3/4 base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division
	gallons in x lbs.
	Group
	1 lb
	2/3
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	This is a $3/4 \times \frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups)
	$\times \frac{2}{3} = X$
	lbs gal in 2/3
		2.d.
		• Multiplication/Division
		  ○ PT 11 used the division operation sign ÷
		$\frac{2}{3} \div \frac{3}{4}$
		• Groups and base units
		  ○ PT 11 included groups and base units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 11 mentioned this situation as "how many groups" problem
		  ○ PT 11 interpreted this situation as QDS.

K (continued).

<table>
<tr>
<td>PT
11
SP</td>
<td>
2
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
$X \times \frac{2}{3} = \frac{3}{4}$
$3/4 \div 2/3 = X$
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
This is a 3/4 ÷ 2/3 problem, because we are given the # of groups (2/3 of a pint) and the base units in the given # of groups (3/4 lb in 2/3 pint) and we are missing base units in 1 group. This is a "how many in 1 group" division
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
This question is neither because 2/3 pint of ice cream is the group (2/3 of a group) and 3/4 of a pound is the # of base units in 2/3 of a group (where lbs is base units). We are missing the # of base units (lbs) in 1 group (1 pint), but the question asks for # of groups for 1 base unit. (# of pints for 1 lb)
$\frac{3}{4} \times \frac{2}{3} = X$
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
This problem is for $\frac{3}{4} \times \frac{2}{3}$ because you want to find the amount of ice cream you ate; if you ate 2/3 of a 3/4 pint of ice cream, you would take 2/3 (group) of the ice cream you have (base units)
$\frac{3}{4} \cdot X = \frac{2}{3}$
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
This is a 2/3 ÷ 3/4 problem because if gallons is our base units and lbs are groups, we have 3/4 base units (gallons) per group (lbs), and an unknown number of groups (lbs), and we have the number of base units (gallons) in x groups (lbs). This is a how many groups division
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
This is a $3/4 \times \frac{2}{3}$ problem because 3/4 gallon is # of base units per group, 1 lb. And we have 2/3 of a pound, (2/3 group) and are looking for the # of gallons in 2/3 lb of ice cream (Base units in 2/3 groups)</td>
<td>2.e.

Multiplication/Division
PT 9 used the multiplication operation sign ×
$\frac{3}{4} \times \frac{2}{3}$
Group and Base Units
PT 11 included information about groups and base units explicitly and correctly.
Multiplicative situation interpretation
PT 11 did not mention anything explicitly for multiplicative situation.
</td>
</tr>
</table>

L. PT 12's solutions for the five sub questions

<table>
<tr>
<td>PT 12
AS</td>
<td>


2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

How many units in one group division

$\frac{2}{3}$ pint ⇒ 3/4 lb, 1 pint ⇒ ? lb (×3/2)

$\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$

By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.

Since 3/4 lb is our total weight for 2/3 pint, we can divide to find the weight of 1 pint.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

How many groups

$\frac{2}{3}$ pint ⇒ 3/4 lb, ? pints ⇒ 1 lb (×4/3)

We can find

Based on the table, this situation resembles: $\boxed{\frac{2}{3} \div \frac{3}{4} = ?}$

We have 2/3 pint of ice cream. By dividing by its weight, we can find the number of pints in one lb.

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

N/A

$\frac{3}{4}$ pint and eat 2/3 or ≈ 66% of that ice cream:

$\frac{2}{3}\left(\frac{3}{4}\right)$ $\boxed{\text{Neither}}$

We just had to find 2/3 (66%) of 3/4 pint. This does not correlate w/ either of the two division equations above

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

N/A

2/3 gallon ⇒ ? lbs, 3/4 gallon ⇒ 1 lb (×9/8)

$\frac{2}{3}x = 3/4$, so $x = 9/8$; $? \times \frac{9}{8} = 1$, $? \times 9 = 8$, $? = 8/9$

$\boxed{\text{Neither}}$

By creating a table + reasoning through the logic to find the one unknown variable, we found that 2/3 gallon weighs 8/9 lb + this does not resemble either of the two division problems above

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

N/A

3/4 gallon ⇒ 1 lb, ? gallons ⇒ 2/3 lb (×(3/2))

3/4(2/3) = 6/12 = 1/2 = ?

$\boxed{\text{Neither}}$

After making a table, we can see that our problem doesn't resemble any of the division equations given in the problem.
</td>
<td>
2.a.

- Multiplication/Division
  - PT 12 used the division operation sign ÷
    $$\frac{3}{4} \div \frac{2}{3}$$
- Groups and base units
  - PT 12 did not include information about groups and base units.
- Multiplicative situation interpretation
  - PT 12 mentioned this situation as "how many units in one group" problem
  - Thus, PT 12 interpreted this situation as PDS
</td>
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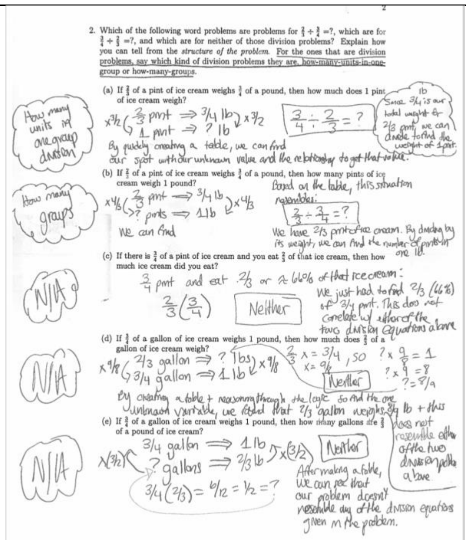
</table>

L (continued).

<table>
<tr>
<td>PT 12
AS</td>
<td>

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

How many units in one group division

$\times 3/2$ (2/3 pint ⇒ 3/4 lb ; 1 pint ⇒ ? lb) $\times 3/2$ $\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$

By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.

Since 3/4 is our total weight for 2/3 pint, we can divide to find the weight of 1 pint.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

How many groups

$\times 4/3$ (2/3 pint ⇒ 3/4 lb ; ? pints ⇒ 1 lb) $\times 4/3$

we can find

Based on the table, this situation resembles: $\boxed{\frac{2}{3} \div \frac{3}{4} = ?}$

We have 2/3 pint of ice cream. By dividing by its weight, we can find the number of pints in one lb.

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

N/A

3/4 pint and eat 2/3 or ≈ 66% of that ice cream:

$\frac{2}{3}\left(\frac{3}{4}\right)$ Neither

We just had to find 2/3 (66%) of 3/4 pint. This does not correlate w/ either of the two division equations above

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

N/A

$\times 9/8$ (2/3 gallon ⇒ ? lbs ; 3/4 gallon ⇒ 1 lb) $\times 9/8$

$\frac{2}{3}x = 3/4$, so $x = 9/8$; ? × 9/8 = 1 ; ? × 9 = 8 ; ? = 8/9

Neither

By creating a table + reasoning through the logic to find the one unknown variable, we found that 2/3 gallon weighs 8/9 lb + this does not resemble either of the two division problems above

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

N/A

$\times (3/2)$ (3/4 gallon ⇒ 1 lb ; ? gallons ⇒ 2/3 lb) $\times (3/2)$ Neither

3/4(2/3) = 6/12 = 1/2 = ?

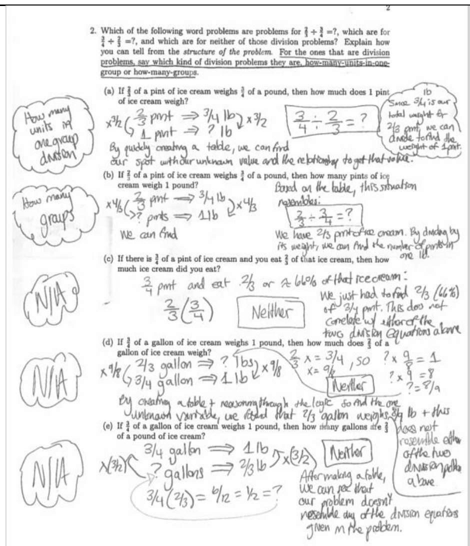
After making a table, we can see that our problem doesn't resemble any of the division equations given in the problem.</td>
<td>2.b.

- Multiplication/Division
  - PT 8 used the division operation sign ÷

$$\frac{2}{3} \div \frac{3}{4}$$

- Groups and base units
  - PT 12 included information about groups and units.
- Multiplicative situation interpretation
  - PT 12 interpreted this situation as QDS, and this interpretation is incorrect.</td>
</tr>
</table>

L (continued).

<table>
<tr>
<td>PT 12
AS</td>
<td>

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

How many units in one group division

$\times 3/2$ (2/3 pint ⇒ 3/4 lb ; 1 pint ⇒ ? lb) $\times 3/2$ $\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$

By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.

Since 3/4 is our total weight for 2/3 pint, we can divide to find the weight of 1 pint. lb

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

How many groups

$\times 4/3$ (2/3 pint ⇒ 3/4 lb ; ? pints ⇒ 1 lb) $\times 4/3$

We can find

Based on the table, this situation resembles: $\boxed{\frac{2}{3} \div \frac{3}{4} = ?}$

We have 2/3 pint of ice cream. By dividing by its weight, we can find the number of pints in one lb.

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

N/A

3/4 pint and eat 2/3 or ≈ 66% of that ice cream:

$\frac{2}{3}\left(\frac{3}{4}\right)$ $\boxed{\text{Neither}}$

We just had to find 2/3 (66%) of 3/4 pint. This does not correlate w/ either of the two division equations above

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

N/A

$\times 9/8$ (2/3 gallon ⇒ ? lbs ; 3/4 gallon ⇒ 1 lb) $\times 9/8$

$\frac{2}{3}x = 3/4$, so $x = 9/8$ $\boxed{\text{Neither}}$ $? \times \frac{9}{8} = 1$ $? \times 9 = 8$ $? = 8/9$

By creating a table + reasoning through the logic to find the one unknown variable, we found that 2/3 gallon weighs 8/9 lb + this does not resemble either of the two division problems above

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

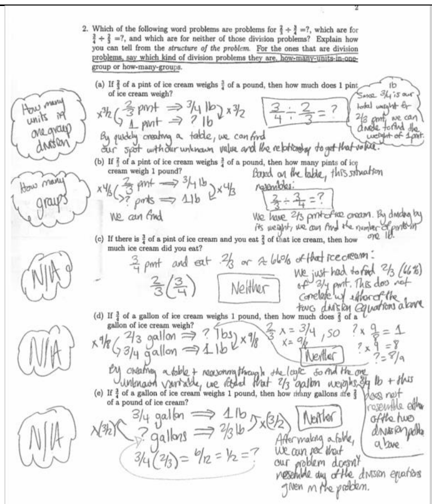
N/A

$\times(3/2)$ (3/4 gallon ⇒ 1 lb ; ? gallons ⇒ 2/3 lb) $\times(3/2)$ $\boxed{\text{Neither}}$

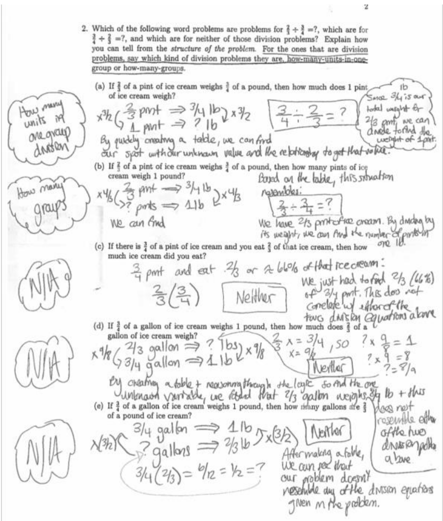
$3/4(2/3) = 6/12 = 1/2 = ?$

After making a table, we can see that our problem doesn't resemble any of the division equations given in the problem.</td>
<td>2.c.
• Multiplication/Division
○ PT 12 used the multiplication but misplaced N and M $\frac{2}{3} \times \frac{3}{4}$
• Groups and base units
○ PT 12 did not include any information about groups and base units
• Multiplicative situation interpretation
○ PT 12 interpreted this situation as MS</td>
</tr>
</table>

L (continued).

<table>
<tr>
<td>PT 12
AS</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
How many units in one group division
$\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$
By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.
Since 3/4 is our total weight for 2/3 pint, we can divide to find the weight of 1 pint.
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
How many groups
Based on the table, this situation resembles:
$\boxed{\frac{2}{3} \div \frac{3}{4} = ?}$
We have 2/3 pint of ice cream. By dividing by its weight, we can find the number of pints in one lb.
(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
N/A
$\frac{3}{4}$ pint and eat 2/3 or ≈ 66% of that ice cream:
$\frac{2}{3}\left(\frac{3}{4}\right)$ $\boxed{\text{Neither}}$
We just had to find 2/3 (66%) of 3/4 pint. This does not correlate w/ either of the two division equations above
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
N/A
2/3 gallon ⇒ ? lbs
3/4 gallon ⇒ 1 lb
$\boxed{\text{Neither}}$
By creating a table + reasoning through the logic to find the one unknown variable, we found that 2/3 gallon weighs 8/9 lb + this does not resemble either of the two division problems above
(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
N/A
3/4 gallon ⇒ 1 lb
? gallons ⇒ 2/3 lb
3/4(2/3) = 6/12 = 1/2 = ?
$\boxed{\text{Neither}}$
After making a table, we can see that our problem doesn't resemble any of the division equations given in the problem.</td>
<td>2.d.
Multiplication/Division
PT 12 used the division but did not show explicitly.
Groups and base units
PT did not include an information about groups and base units explicitly.
Multiplicative situation interpretation
PT 12 interpreted this situation as MS, but this interpretation is incorrect.</td>
</tr>
</table>

L (continued).

<table>
<tr>
<td>PT 12
AS</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
How many units in one group division
$\times 3/2$ (2/3 pint ⇒ 3/4 lb ; 1 pint ⇒ ? lb) $\times 3/2$ $\boxed{\frac{3}{4} \div \frac{2}{3} = ?}$
By quickly creating a table, we can find our spot with our unknown value and the relationship to get that value.
Since 3/4 is our total weight for 2/3 pint, we can divide to find the weight of 1 pint.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
How many groups
$\times 4/3$ (2/3 pint ⇒ 3/4 lb ; ? pints ⇒ 1 lb) $\times 4/3$
we can find
Based on the table, this situation resembles: $\boxed{\frac{2}{3} \div \frac{3}{4} = ?}$
We have 2/3 pint of ice cream. By dividing by its weight, we can find the number of pints in one lb.

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
N/A
3/4 pint and eat 2/3 or ≈ 66% of that ice cream:
$\frac{2}{3}\left(\frac{3}{4}\right)$ $\boxed{\text{Neither}}$
We just had to find 2/3 (66%) of 3/4 pint. This does not correlate w/ either of the two division equations above

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
N/A
$\times 9/8$ (2/3 gallon ⇒ ? lbs ; 3/4 gallon ⇒ 1 lb) $\times 9/8$
$\frac{2}{3}x = 3/4$, so $x = 9/8$ $? \times \frac{9}{8} = 1$ $? \times 9 = 8$ $? = 8/9$
$\boxed{\text{Neither}}$
By creating a table + reasoning through the logic to find the one unknown variable, we found that 2/3 gallon weighs 8/9 lb + this does not resemble either of the two division problems above

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
N/A
$\times(3/2)$ (3/4 gallon ⇒ 1 lb ; ? gallons ⇒ 2/3 lb) $\times(3/2)$ $\boxed{\text{Neither}}$
3/4(2/3) = 6/12 = 1/2 = ?
After making a table, we can see that our problem doesn't resemble any of the division equations given in the problem.</td>
<td>2.e.
Multiplication/Division
PT 12 used the multiplication operation sign ×
$\frac{3}{4} \times \frac{2}{3}$
Group and Base Units
PT 12 did not include any information about groups and base units explicitly.
Multiplicative situation interpretation
PT 12 interpreted this situation as MS.</td>
</tr>
</table>

M. PT 13's solutions for the five sub questions

<table>
<tr>
<td>PT 13
WY</td>
<td>

2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?

$\underbrace{\frac{3}{4}}_{\text{total base units (pound)}} \div \underbrace{\frac{2}{3}}_{\text{\# of groups (pint)}} = \underbrace{?}_{\text{\# of base units in 1 group}}$

$\frac{3}{4}$ of a pound is $\frac{2}{3}$ of a pint of ice cream. so $\frac{3}{4}$ is the total base units and $\frac{2}{3}$ is the group This is a "how many in one group" question.

(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?

$\underbrace{\frac{2}{3}}_{\text{total base units}} \div \underbrace{\frac{3}{4}}_{\text{\# of groups}} = \underbrace{?}_{\text{how many base units in 1 group}}$

This is a "how many in one group" problem

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?

$\underbrace{\frac{3}{4}}_{\text{base units in 1 group}} \times \underbrace{\frac{2}{3}}_{\text{\# of group}} = \underbrace{?}_{\text{total base units in } \frac{2}{3} \text{ groups}}$

This is a multiplication problem since this problem was asking $\frac{2}{3}$ of $\frac{3}{4}$.

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?

$\underbrace{\frac{2}{3}}_{\text{total base units (gallon)}} \div \underbrace{\frac{3}{4}}_{\text{\# of base unit make 1 group (pound)}} = \underbrace{?}_{\text{\# of groups}}$

This question is a "how many groups" question because we know the total base units and how many base unit make 1 group

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?

$\underbrace{\frac{3}{4}}_{\text{base units (gallon) in 1 group (pound)}} \times \underbrace{\frac{2}{3}}_{\text{\# of groups}} = \underbrace{?}_{\text{total base units in } \frac{2}{3} \text{ groups}}$

This is a multiplication problem since it ask us to take $\frac{2}{3}$ of $\frac{3}{4}$

</td>
<td>

2.a.

- Multiplication/Division
  - PT 13 used the division operation sign ÷
    $$\frac{3}{4} \div \frac{2}{3}$$
- Groups and base units
  - PT 13 included information about groups and base units explicitly and correctly.
- Multiplicative situation interpretation
  - PT 13 mentioned this situation as "how-many in one group" problem.
  - Thus, PT 13 interpreted this situation as PDS

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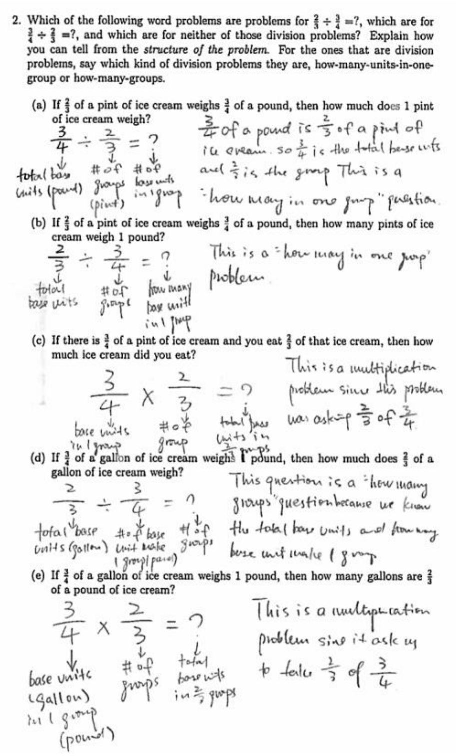
</table>

M (continued).

<table>
<tr>
<td>PT 13
WY</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3} = ?$
total base units (pound) / # of groups (pint) / # of base units in 1 group
$\frac{3}{4}$ of a pound is $\frac{2}{3}$ of a pint of ice cream. so $\frac{3}{4}$ is the total base units and $\frac{2}{3}$ is the group This is a "how many in one group" question.

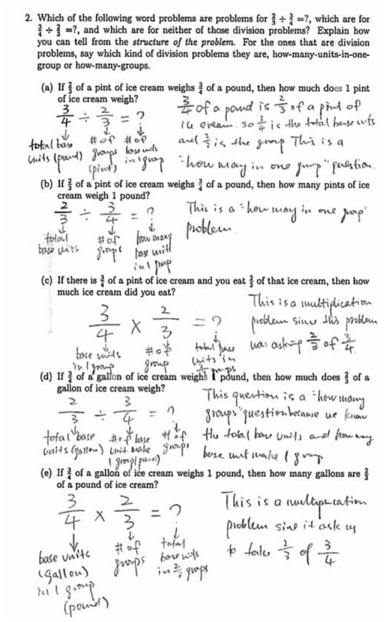
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3} \div \frac{3}{4} = ?$
total base units / # of groups / how many base units in 1 group
This is a "how many in one group" problem

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
$\frac{3}{4} \times \frac{2}{3} = ?$
base units in 1 group / # of group / total base units in $\frac{2}{3}$ groups
This is a multiplication problem since this problem was asking $\frac{2}{3}$ of $\frac{3}{4}$.

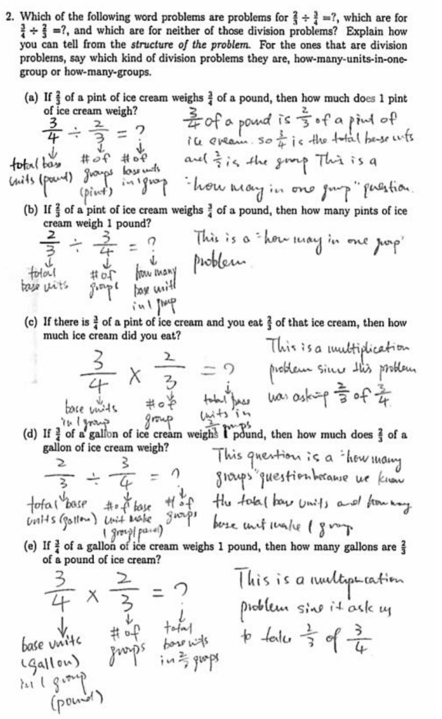
(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{2}{3} \div \frac{3}{4} = ?$
total base units (gallon) / # of base unit make 1 group (pound) / # of groups
This question is a "how many groups" question because we know the total base units and how many base unit make 1 group

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
$\frac{3}{4} \times \frac{2}{3} = ?$
base units (gallon) in 1 group (pound) / # of groups / total base units in $\frac{2}{3}$ groups
This is a multiplication problem since it ask us to take $\frac{2}{3}$ of $\frac{3}{4}$</td>
<td>2.b.
• Multiplication/Division
  ○ PT 13 used the division operation sign ÷
$\frac{2}{3} \div \frac{3}{4}$
• Groups and base units
  ○ PT 13 included information about groups and units explicitly and correctly.
• Multiplicative situation interpretation
  ○ PT 13 mentioned this situation as “how-many units” problem.
  ○ Thus, PT 13 interpreted this situation as PDS.</td>
</tr>
</table>

M (continued).

PT 13
WY
	
	2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.
	(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
	$\frac{3}{4} \div \frac{2}{3} = ?$
	total base units (pound) / # of groups (pint) / # of base units in 1 group
	$\frac{3}{4}$ of a pound is $\frac{2}{3}$ of a pint of ice cream. so $\frac{3}{4}$ is the total base units and $\frac{2}{3}$ is the group This is a "how many in one group" question.
	(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
	$\frac{2}{3} \div \frac{3}{4} = ?$
	total base units / # of groups / how many base unit in 1 group
	This is a "how many in one group" problem.
	(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
	$\frac{3}{4} \times \frac{2}{3} = ?$
	base units in 1 group / # of group / total base units in $\frac{2}{3}$ groups
	This is a multiplication problem since this problem was asking $\frac{2}{3}$ of $\frac{3}{4}$
	(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
	$\frac{2}{3} \div \frac{3}{4} = ?$
	total base units (gallon) / # of base unit make 1 group (pound) / # of groups
	This question is a "how many groups" question because we know the total base units and how many base unit make 1 group.
	(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
	$\frac{3}{4} \times \frac{2}{3} = ?$
	base units (gallon) in 1 group (pound) / # of groups / total base units in $\frac{2}{3}$ groups
	This is a multiplication problem since it ask us to take $\frac{2}{3}$ of $\frac{3}{4}$
		2.c.
		• Multiplication/Division
		  ○ PT 13 used the multiplication $\frac{3}{4} \times \frac{2}{3}$
		• Groups and base units
		  ○ PT 13 included information about groups and base units explicitly and correctly.
		• Multiplicative situation interpretation
		  ○ PT 13 interpreted this situation as MS

M (continued).

<table>
<tr>
<td>PT 13
WY</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} =?$, which are for $\frac{3}{4} \div \frac{2}{3} =?$, and which are for neither of those division problems? Explain how you can tell from the *structure of the problem*. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
$\frac{3}{4} \div \frac{2}{3} = ?$
total base units (pound) / # of groups (pint) / # of base units in 1 group
$\frac{3}{4}$ of a pound is $\frac{2}{3}$ of a pint of ice cream. so $\frac{3}{4}$ is the total base units and $\frac{2}{3}$ is the group This is a "how many in one group" question.

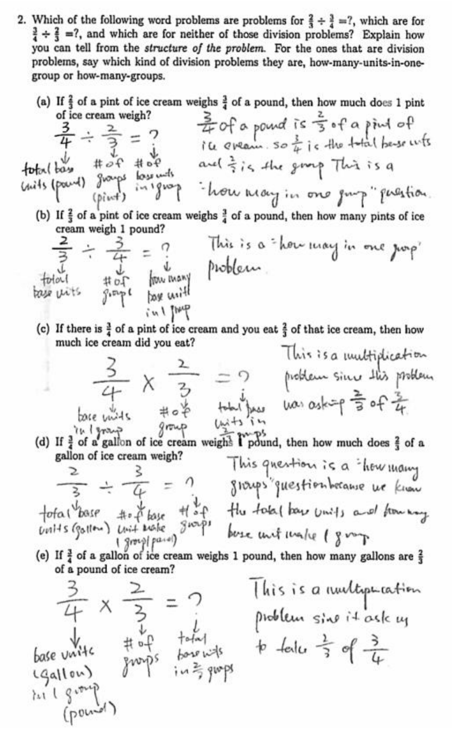
(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
$\frac{2}{3} \div \frac{3}{4} = ?$
total base units / # of groups / how many base units in 1 group
This is a "how many in one group" problem

(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
$\frac{3}{4} \times \frac{2}{3} = ?$
base units in 1 group / # of group / total base units in $\frac{2}{3}$ groups
This is a multiplication problem since this problem was asking $\frac{2}{3}$ of $\frac{3}{4}$

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
$\frac{2}{3} \div \frac{3}{4} = ?$
total base units (gallon) / # of base unit make 1 group (pound) / # of groups
This question is a "how many groups" question because we know the total base units and how many base unit make 1 group

(e) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how many gallons are $\frac{2}{3}$ of a pound of ice cream?
$\frac{3}{4} \times \frac{2}{3} = ?$
base units (gallon) in 1 group (pound) / # of groups / total base units in $\frac{2}{3}$ groups
This is a multiplication problem since it ask us to take $\frac{2}{3}$ of $\frac{3}{4}$</td>
<td>2.d.
• Multiplication/Division
  ○ PT 13 used the division operation sign ÷
$$\frac{2}{3} \div \frac{3}{4}$$
• Groups and base units
  ○ PT 13 included groups and base units
• Multiplicative situation interpretation
  ○ PT 13 mentioned this situation as “how many groups” problem
  ○ Thus, PT 13 interpreted this situation as QDS</td>
</tr>
</table>

M (continued).

<table>
<tr>
<td>PT 13
WY</td>
<td>
2. Which of the following word problems are problems for $\frac{2}{3} \div \frac{3}{4} = ?$, which are for $\frac{3}{4} \div \frac{2}{3} = ?$, and which are for neither of those division problems? Explain how you can tell from the structure of the problem. For the ones that are division problems, say which kind of division problems they are, how-many-units-in-one-group or how-many-groups.

(a) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how much does 1 pint of ice cream weigh?
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total base units (pound) / # of groups (pint) / # of base units in 1 group
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(b) If $\frac{2}{3}$ of a pint of ice cream weighs $\frac{3}{4}$ of a pound, then how many pints of ice cream weigh 1 pound?
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total base units / # of groups / how many base units in 1 group
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(c) If there is $\frac{3}{4}$ of a pint of ice cream and you eat $\frac{2}{3}$ of that ice cream, then how much ice cream did you eat?
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base units in 1 group / # of group / total base units in $\frac{2}{3}$ groups
This is a multiplication problem since this problem was asking $\frac{2}{3}$ of $\frac{3}{4}$.

(d) If $\frac{3}{4}$ of a gallon of ice cream weighs 1 pound, then how much does $\frac{2}{3}$ of a gallon of ice cream weigh?
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base units (gallon) in 1 group (pound) / # of groups / total base units in $\frac{2}{3}$ groups
This is a multiplication problem since it ask us to take $\frac{2}{3}$ of $\frac{3}{4}$</td>
<td>2.e.
Multiplication/Division
PT 13 used the multiplication operation sign × $\frac{3}{4} \times \frac{2}{3}$
Group and Base Units
PT 13 included information about groups and base units explicitly and correctly.
Multiplicative situation interpretation
PT 13 interpreted this situation as MS.</td>
</tr>
</table>