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OPTIMAL OPERATION POLICIES FOR MULTI-ITEM VENDING MACHINES WITH PRODUCT DEDICATED VARIANT TOWERS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
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BY

HACI ŞAHİN

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OPTIMAL OPERATION POLICIES FOR MULTI-ITEM VENDING MACHINES WITH PRODUCT DEDICATED VARIANT TOWERS

submitted by **HACI ŞAHİN** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof. Dr. Yasemin Serin
Head of the Department, **Industrial Eng. Dept., METU** ________________

Prof. Dr. Serhan Duran
Supervisor, **Industrial Eng. Dept., METU** ________________

Assoc. Prof. Dr. Ertan Yakıcı
Co-Supervisor, **Industrial Eng. Dept., National Defence Uni.** ________________

Examining Committee Members:

Assoc. Prof. Dr. Seçil Savaşaneril
Industrial Eng. Dept., METU ________________

Prof. Dr. Serhan Duran
Industrial Eng. Dept., METU ________________

Assist. Prof. Dr. Mustafa Kemal Tural
Industrial Eng. Dept., METU ________________

Assoc. Prof. Dr. Ertan Yakıcı
Industrial Eng. Dept., National Defence University ________________

Assoc. Prof. Dr. Mustafa Alp Ertem
Industrial Eng. Dept., Çankaya University ________________

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ABSTRACT

OPTIMAL OPERATION POLICIES FOR MULTI-ITEM VENDING MACHINES WITH PRODUCT DEDICATED VARIANT TOWERS

Şahin, Hacı
Master of Science, Industrial Engineering
Supervisor: Prof. Dr. Serhan Duran
Co-Supervisor: Assoc. Prof. Dr. Ertan Yakıcı

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In this thesis we focus on modelling and solving the operation policy problem of a seller by managing its vending machines at different locations. Each vending machine has sale unit towers, which can hold different items; therefore, the decisions of which items should be kept at which location and with how many sale towers become important. Due to the uncertainties faced in demand, solution of the problem determines the allocation of towers to the items and machines throughout a time horizon. Stochastic optimization approach is employed to obtain less costly operations policy and experimental results support usage of the proposed model.

Keywords: Vending Machine, Inventory Management, Dynamic Pricing, Stochastic Programming

ÖZ

ÜRÜNE TAHSİSLİ, DEĞİŞKEN ÜNİTELİ, ÇOKLU ÜRÜN İÇEREN SATIŞ OTOMATLARI İÇİN OPTİMUM OPERASYON POLİTİKLARI

Şahin, Hacı
Yüksek Lisans, Endüstri Mühendisliği
Tez Yöneticisi: Prof. Dr. Serhan Duran
Ortak Tez Yöneticisi: Doç. Dr. Ertan Yakıcı

Aralık 2019, 78 sayfa

Bu tezde farklı lokasyonlarda satış otomatı bulunan bir satıcının operasyonel politikalarının modellenmesi ve çözülmesi amaçlanmaktadır. Her bir satış otomatı farklı sayıda ürün ünitesine sahip olduğundan satış otomatında hangi ürünlerin tutulacağı ve hangi miktarda tutulacağı kararları önem arz etmektedir. Talebin belirsiz olması nedeniyle uygulanan çözüm, ünitelerin ürünlere ve makinelere tahsisini belirli bir zaman diliminde ve tüm senaryolar için belirlemektedir. Daha düşük maliyetli operasyonel politikanın elde edilebilmesi için stokastik optimizasyon yaklaşımı sergilenmiş ve elde edilen sonuçlar önerilen matematiksel modelin kullanılmasını destekleyici nitelikte olmuştur.

Anahtar Kelimeler: Satış Otomatı, Envanter Yönetimi, Dinamik Fiyatlandırma, Stokastik Programlama

to my family

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CHAPTER 1

INTRODUCTION

1.1 History of Vending Machines

Vending machine is an automated machine, that is used as a sales channel, which provides beverages, cigarettes, food and other items to customers with payment options by cash, coin, credit card or mobile applications. The first vending machine, which serves dispensed holy water in Egyptian temples, is known to be invented by ancient Greek mathematician Heron of Alexandria. As it can be seen from the demonstration in Figure 1.1., when the coin was inserted, it fell on a pan attached to a lever, which opened the valve allowing the water flow.

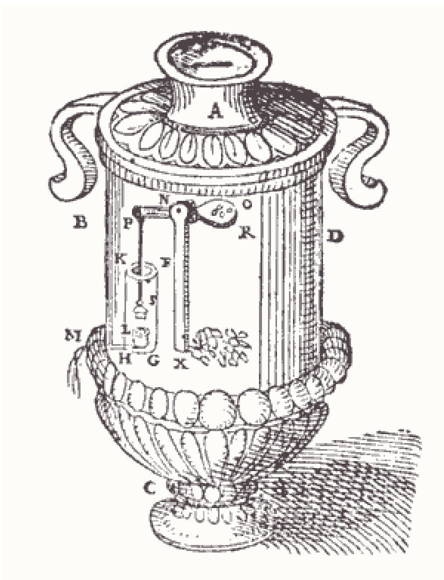


Figure 1.1. *Demonstration of the First Vending Machine, n.d., Retrieved from https://auroravms.com*

In early 17th century, portable vending machines dispensing tobacco are known to be used in the taverns of England. First fully automated vending machine, which dispensed stamps, appeared in 1867. After the automation, vending machines became popular to serve products in England and USA. Developments in vending machine industry allowed entrepreneurs to open the first coin operated restaurant, that served between 1902-1962, in Philadelphia.

Figure 1.2. *Coin Operated Restaurant in Philadelphia, 2017, Retrieved from http://georgerothert.com*

Globalization and individualization of people over the world increased the popularity of vending machines in 21st century and food, beverages and cigarette are the most common products sold in vending machines. Global vending machine market is expected to hit $ 30 billion by 2024 with a growth rate of 15 % yearly.

1.2 Marketing, Competition and Vending Machines

In recent years, the level of competition extremely increased for all markets. Due to globalization, many players offer similar products and services. In order to become stronger in this competitive environment, all players must act strategically and enhance their supply chain management activities. Some companies try to extend their market share from wholesale level to retail and lower levels. This strategy forces companies to reach customers by vending machines.

In recent years, vending machines are also being used for sales of valuable items like electronic devices including ipods, cell phones and digital cameras. Many companies use those machines as a sales channel. Especially beverage companies effectively use this channel. Vending machines are commonly located at offices/institutions, public transport hubs and retail sites. Developments in technology and IoT applications allow those machines to become smart/intelligent devices and make their management easier.

Figure 1.3. Vending Machines, *n.d., Retrieved from http://www.statesecrets.org*

1.3 Smart Vending Machines

Developments in technology allow companies to manage their vending machine operations in a better way utilizing inventory management, customer analytics, digital advertising and content management. It is possible to track inventory levels and current conditions of the items in a vending machine with the current technology.

Current market conditions make it possible for customers to be aware of the alternative prices for substitute items. Therefore, dynamic pricing is vital for the sustainability of companies. In order to be able to set optimum prices and adjust them over time, a company must have detailed information about its costs including operating costs, availability of supply, future demand and value perspective of the customers.

Vending machines provide following advantages to their owners. First, there is no need for cashier. Hence, that allows company to serve for 24 hours throughout the year with low labor costs. Second, those machines are convenient and they help customers to save time. Finally, overhead costs, operational costs and investment costs can be reduced by using vending machines.

1.4 Scope of the Thesis

The aim of this study is to develop a mathematical model to optimally manage vending machine operations including tower allocation, inventory management and replenishment policy. We have employed a stochastic programming approach to overcome variations in demand for the items.

In this thesis, we focus on the operation activities of the seller managing vending machines. Figure 1.4 shows demonstration of a vending machine with three towers. As it can be observed from the figure, the vending machines has unit towers for locating items for sale. Nature of the system requires locating only one item to each tower. Since each item has different size, the capacity of the tower differs for each product.

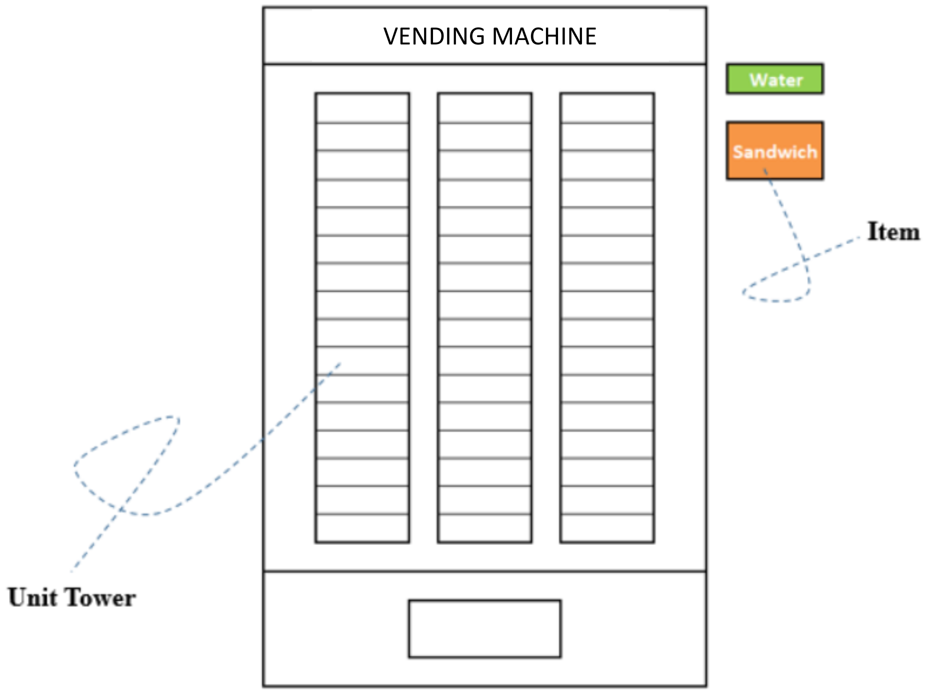


Figure 1.4. Demonstration of a vending machine

Operations of the vending machines include the following activities;

- Location of the vending machine: vending machines are assumed as an important sales channel in today's competitive marketing environment. Location of these machines plays a crucial role for sales amount and revenue. Although it is possible to change its location in time, sellers generally make this decision during the investment period. This fact makes the decision an investment policy rather than an operations policy.
- Capacity planning: capacity means number and size of the towers for vending machines. Vendors usually make this decision during the investment period. This fact makes the decision an investment policy rather than an operations policy.
- Allocation of towers: towers are scarce resource for locating items to the vending machines. Locating the correct item, that will generate most profit, to the tower is a crucial problem. Allocation performance directly affects the amount of profit generated.

- Replenishment of vending machines: items located at the vending machine will be sold and there will be need for replenishment. Performance of replenishment directly effects lost sales and profit of the vending machine. Replenishment policy also directly effects the holding period and holding cost of the items at the vending machines.
- Inventory level of the items: optimal setting of inventory level is crucial due to its direct effect on profitability of the item from lost sales and holding cost aspects.
- Routing of the vehicles: routing decision has an impact over replenishment and inventory level policies. Therefore, it has impact over holding cost and travelling cost.

The problem considered in this thesis is assumed to include location, capacity and routing decisions as input. We focus on tower allocation, replenishment policy and inventory level decisions in the context of operations policy. Demand is an important parameter that will drive before mentioned policies. In our case demand is uncertain and generated as a random variable in accordance with the given distribution.

We have assumed following issues listed below in order to well define the boundaries of the problem.

- Demand is uncertain and randomly generated in accordance with the related distribution. This uncertainty makes the problem stochastic.
- Demand is independent for each product.
- Each vending machine may include more than one tower and each tower can include only one type of item.
- Customer behavior, which covers immediate purchasing decision according to instantaneous valuation of the product, is myopic. Therefore, there is no consideration about the possible future price differentiations.
- Dynamic pricing is allowed.
- Backlogging is not allowed. If the product is not available, it is lost sales.

- Cost component only includes the procurement cost, holding cost and the delivery cost.
- Lead-time is assumed to be zero for supply centers. Hence, there will be no holding cost at the supply centers until the product is located in a vending machine.
- Replenishment activity can be also performed from locations other than supply centers. Therefore, increasing and decreasing the inventory level of any vending machine is possible.

In the light of these assumptions, we have proposed a mathematical model to seek an optimal solution that will answer following research questions:

1. Which items should be kept at each location and how many towers should be allocated for those items?
2. What should be the optimum inventory level of items at the beginning of each period?
3. Shall there be any item transfer between locations at each period of time?
4. What should be the price for each item at each location and each time period?

CHAPTER 2

REVIEW OF LITERATURE

In order to set the right price and adjust it over time, a company must have detailed information about operating costs, availability of supply, future demand and customer valuation of the product. Until last decade, tracking and keeping that information was very limited. According to Chen & Simchi-Levi (2012), growth of dynamic pricing is contributed by several factors. Advances in information technologies facilitate customer data collection and reduce the costs incurred due to changing prices. Therefore, many manufacturers and retailers have started applying dynamic pricing in order to improve efficiency of their operations. Recently, analytical models and decision support systems for customer data analysis and price optimization have been developed and implemented successfully in many industries.

According to Elmaghraby & Keskinocak (2003), characteristics of a market environment influence the type of dynamic pricing problem of the retailer. First of all, whether or not replenishment is possible during a planning horizon affects the inventory decisions. Second, demand dependence over time plays a crucial role in price optimization. Shelf life and customer knowledge about the product may affect the relation between demand over multiple periods. Another factor, seller's pricing ability is affected by the purchasing behavior of the customers. Myopic customer behavior, that makes purchasing decision according to value perception of the item immediately, allows the seller to ignore any effects of future pricing alternatives for current customer purchases. However, a strategic customer behavior, that takes the future path of prices into account, forces the seller to consider the effects of price changes on customers' purchasing decisions. In addition to those mentioned above, several factors may influence dynamic pricing decisions, such as demand seasonality, cross-elasticities and business rules.

The products considered in this study can be classified as replenishable and having time independent demand. Also the customer behavior is assumed to be myopic. We consider a set of products (with no demand correlation such as substitutability or complementarity), in contrast with the common assumption of single product in a monopoly market as almost all of the papers published before 2006 adopted [3].

We assume a set of selling locations where automated machines are located and inventory can be transferred among those locations. An automated machine has product tower (or towers) inside and a tower can include only one type of product. The cost component is assumed to be consisting of procurement cost, holding cost and delivery cost. It is also assumed that, there is no lead time in procurement and no holding cost until the product is located in an automated machine.

The seller's problem is to determine the pricing policy in order to balance demand and inventory by considering the combination of products for each location along with order, frequency and quantity of replenishment of those products in each location.

The introduced profit maximization problem has unique characteristics and to the best of our knowledge, there is no work in the literature defining this problem. Therefore, we are able to discuss here the most relevant problems introduced in the relevant literature.

Elmaghraby & Keskinocak (2003) reviews the literature, which assumes the case where the problem consists of pricing and procurement decisions, when inventory can be replenished. The literature in this category is divided into groups with respect to demand uncertainty, the attribute of cost functions and limit on the production/procurement capacity. The closest group of problems (to our problem) have uncertain (stochastic) demand, convex cost functions and uncapacitated procurement/production. In another relevant review paper (Chen & Simchi-Levi (2012)), the most relevant groups are defined as multi period models with convex ordering cost and multi period models with concave ordering cost. In the remaining part of this section, we will review the models with convex and concave cost function, respectively.

Federgruen & Heching (1999) compute optimal policies by the use of an efficient value iteration method. They show that dynamic pricing strategies are more beneficial when compared to static strategies. They also analyze the impact of uncertainties in demand uncertainties and price elasticities on optimal policies and their effect on profit. They finally observe that using dynamic pricing strategies dominate static strategies and provide extra benefit even in a stationary environment.

Zabel (1972), impressed by the simulation study in Nevins (1966) about pricing strategies and multi-period inventory management, study on a model which has stationary parameters with the assumption that it has a convex ordering cost and linear inventory holding cost. In addition, existing demand is lost if it not satisfied on time without any penalty cost. The author shows that it is difficult to extend the unique properties of optimal solutions from a single period model in Zabel (1970) to multi-period settings. Zabel (1972) extends the context of the study and focuses on an additive demand model. In this study demand has a concave structure and the random variable is distributed exponentially or uniformly. Zabel (1972) proves that extended base-stock list-price policy gives the best result and argues that the profit-to-go function has a concave structure.

When Thowsen (1975) studies additive demand further and develops a model similar to the model proposed by Zabel (1972) with linear ordering cost. The author assumes that if the demand is not satisfied and partially backlogged which compares full backlogging option and lost sales option as two extreme cases in a unified model, and a fraction of inventory may deteriorate from one period to the next.

A multi-period joint pricing and production model is studied by Chan et al. (2006). Structure of the problem includes discrete prices with non-stationary stochastic demand function. Their assumption is limited daily production capacity and backlogging is not allowed. They also propose policies and effective heuristics for the strategies for deterministic approximations. They show that, if capacity is not tight, delaying the production is better than delaying pricing.

Chen et al. (2008) makes a study that includes price adjustment costs within an integrated inventory and pricing model which has a stochastic demand. The model analyzed by Federgruen & Heching (1999), is similar to the one analyzed. Main difference is at each period, the price is compared with the previous periods price and if there is a difference it may involve a fixed price independent from the magnitude of the price change and a variable component proportional to the change in price. Since the general model is complex, Chen et al. (2008) focus on two special topics: the first one includes fixed ordering cost and fixed price adjustment cost whereas the second one only includes fixed price adjustment cost and no inventory carry over. For each case, they try to depict the optimal policy structure.

The models we stated up to now assume that the inventory is managed by a centralized point of view. Federgruen & Heching (2002) study the extension of Federgruen & Heching (1999) which include a distribution center and all retailer stores are served from this central point. Assumptions of this study does not only include that the distribution center does not hold any inventory, but also it will place an order, which will then be allocated to the retailer stores after related lead times. In parallel with this assumption, a single price will be determined across all retailer stores. Federgruen & Heching (2002) present an easy to tract approximation of the stochastic model. This model provides an optimal policy with a simple structure, and requires an extensive computational study to explain the benefits of dynamic pricing strategies.

To the best of our knowledge, there are a few papers studied multiple items in stochastic settings. Zhu & Thonemann (2009) study on the model Federgruen & Heching (1999) developed. They study with two substitutable products, which have dependency on the price of other product. They illustrate that for one-product problem the optimal inventory policy is similar to the base-stock policy problem. For one-product problem if the initial inventory level of products is low, the optimal decision is achieved with ordering both products. If the starting inventory levels of both products are high, not to order anything seem to be the optimal decision. When one of the product has a high initial inventory level and the other one has a low initial inventory level, the optimal decision is to order only the product with the low inventory level.

For models which include complementary or substitutable products, Chen et al. (2010b) illustrate the structure of the optimal production and pricing policy by developing a preservation result in two-dimensional space. If there are two substitutable products, they show refined structural property significantly simplifying some of the proofs in Zhu & Thonemann (2009), Ceryan et al. (2009) and Song & Xue (2007).

The papers we discussed up to here make an assumption, accepting ordering cost as linear or convex. In real world environment the ordering cost can be concave due to the result of economies of scale or incremental discounts provided by suppliers. This type of cost structures becomes an important indicator for multi-period models. When the value function is not concave, base-stock list-price policy is no more optimal. When we elaborate the literature, we mainly encounter studies, which try to define conditions where (s; S; p) policy is optimal. In a (s; S; p) type policy, inventory is replenished based on an (s; S) policy: which initiates re-ordering process (with an amount of S-s) if the inventory level is below dedicated s value at the beginning of period t. The selling price of the product may vary (not necessarily monotonically) according to initial inventory level at the beginning of the period.

The idea represented above was first analyzed by Thomas (1974) and he showed that (s; S; p) policy may not be the optimal solution if the prices are an element of a discrete set. The author defends that, (s; S; p) policy is optimal if the conditions are fairly general and the prices are continuous.

Chen & Simchi-Levi (2004a) shows that the (s; S; p) policy, that Thomas (1974) suggested, seem to be optimal when additive demand process exists. In addition, the authors develop an example which shows that, when the demand process is multiplicative, the value function may not be k-concave and (s; S; p) policy may not be the optimal policy. For the cases with multiplicative demand, Chen & Simchi-Levi (2004a) proposed the notion of symmetric k-concavity and introduced the concept of (s; S; A; p) policy. If the demand has general settings, (s; S; A; p) policy may not be optimal. Chen &Simchi-Levi (2004b) show that the concept of symmetric k-concavity that a stationary (s; S; p) policy is optimal for the infinite horizon model under either the discounted profit or the average profit criterion.

Feng & Chen (2004) try to find an efficient algorithm to understand the dynamics of the optimal policies. They prove that the nature of the optimal policy holds under slightly relaxed demand assumptions. On the other hand, Zhang & Fu (2005) try to find optimal (s; S; p) policy parameters with the help of simulation.

Polatoglu & Sahin (2000) deal with the same problem without backlogging option. They demonstrate that an (s; S; p) policy is not optimal under general conditions. They try to find the characteristics of the structure where (s; S; p) policy results the optimal solution.

Huh & Janakiraman (2008) develop an alternative approach to prove that for the problems with stationary cost and demand nature (s; S; p) policy results the optimal solution.

Multiple products, multiple locations and allowing transfer of multiple products between multiple selling locations complicates the problem considered in this thesis enough to make dynamic programming approach intractable. Since we assume linear cost functions, stochastic mixed integer linear programming seems more appropriate.

There are a few studies about the mathematical models for smart vending machine systems in the literature. Poon (2010) advanced an index to decide replenishment for smart vending machines and he tries to minimize shipping and stock-out costs by this index. Park and Yoon (2011) offered a two-phase solution model that depends on a nonlinear integer model for the smart vending machine supply chains with stock-out based.

Park and Yoo (2012) conducted an intuitive research work for intelligent automated vending machine with product substitution under the replenishment point, level-to-order planning and evaluation. They try to maximize the profit in case of successive replenishments. Also, in the objection function, the transportation cost, inventory carrying cost, operating cost, and substitution cost are component of the cost function and they subtract them from the total revenue. Moreover, they conducted experiments with five vending machine locations and these locations are uniform distribution on [0, 100]. Storehouse's location is (50, 50). Each of vending machines' average of daily demand are uniform distribution on [150, 210]. The values of five products are 0.525, 0.875, 1.225, 0.7, and 1.05, respectively. The rates of inventory carrying cost, operating cost, and substitution cost are 0.01, 0.1, and 0.05, respectively. Capacity of machine is 3200. The fixed cost is 100 and variable transportation cost is 0.4. The allowance ratio of lost sales is 0.3 in the experiments. Their heuristic proposition solves the problem as follows. First of all, proposition solves replenishment points of brands at vending machines. Second, it finds the number of product storage compartments. Third, heuristic decides the vehicle roads for daily replenishments. Their heuristic proposition generated satisfying solutions with 5.7% error rate on average when we compared with solving the integrated mathematical model.

CHAPTER 3

MATHEMATICAL MODEL

In this section, we present the required notation and the mathematical model that we used for solving the problem. We describe the mathematical model part by part to make it more comprehensible. After defining sets and the objective function of the model, we will explain all constraints one by one.

Objective function of the model focuses on maximizing profit, whereas constraints impose restrictions related to pricing, inventory balance, transportation, sales amount and capacity of towers.

3.1 Sets

The sets used in the model are given below:

$I = \{1,..., i,..., \|I\|\}$	: set of items,
$P_i = \{1,..., p_i,..., \|P_i\|\}$	: set of prices for item i,
$L = \{1,..., l,..., \|L\|\} \cup SC$	: set of locations (SC stands for the supply center),
$T = \{1,..., t,..., \|T\|\}$	: set of periods,
$M_{ll'} \subset T$	: set of time periods when items can be transferred from location l to l',
$S = \{1,..., s,..., \|S\|\}$	: set of scenarios.

3.2 Parameters:

$d(i,p_i,l,t,s)$	: forecasted demand for item i at location l, at period t when it is sold for price p_i at scenario s,
$c(i)$	: procurement cost of item i,
$h(i)$	: holding cost of item i for one period,
$dlv(l,l')$	: cost of travelling from location l to l'(l' is an alias for l),
$dst(l', l)$	: required time (periods) for transferring an item from location l' to l,
$tow(l)$	: number of available towers at location l,
$cap(i)$	: tower capacity for item i.
$pr(p_i)$	: price p_i for item i.

3.3 Decision Variables:

$x(i,p_i,l,t)$	: 1 if item i is available for sale at location l, at period t for price p_i; 0 o.w.,
$a(i,l,t) \in I^+$	: number of towers allocated to item i at location l at period t,
$n(i,l,t,s) \in I^+$	: inventory of item i at location l at the beginning of period t at scenario s,
$r(i,l,l',t,s) \in I^+$	: amount of item i transferred from location l to location l' at period t at scenario s,
$z(l,l',t,s)$	: 1 if any item is sent from location l to location l' at period t at scenario s; 0 o.w.,
$u(i,p_i,l,t,s) \in I^+$	: amount of item i sold at location l at period t for price p_i at scenario s,

3.4 Drivers of the Objective Function

Objective function of the model aims to maximize the average profit generated by all scenarios. In order to achieve this goal, our objective function considers generated revenue, procurement cost, holding cost and delivery cost components. Before optimization of the instance, the model generates demand value for all price levels. After calculating total profit for all scenarios, total profit is divided by |S|, number of scenarios, to find the average profit per scenario. A single tower allocation is made according to all scenarios. Therefore, it may not be the optimum solution for each individual scenario. Details of each component of the objective function is provided below.

3.4.1 Generated Revenue

$\sum_s \sum_i \sum_{p_i} \sum_{l:l\in(l\setminus \mathrm{SC})} \sum_t pr(p_i)u(i,p_i,l,t,s)$

Revenue is generated when an item is sold at a location. Price of item i is multiplied with the quantity of item i sold at location l (all locations excluding supply centers) for each scenario. This calculation is performed for all time periods and for all scenarios.

3.4.2 Procurement Cost

$\sum_s \sum_i c(i) \sum_{l:l\in SC}(n(i,SC,0,s) - n(i,SC,|\mathrm{T}|,s))$

Since all items are initially transferred from supply centers, all procurement cost can be calculated according to inventory change in supply centers. Lead time is assumed to be zero for all items, hence there is no need for holding inventory at supply centers between consecutive replenishments. To make calculations easier, initial inventory is assumed to be infinite (M=1.000.000) for each item at the supply centers. The difference between initial and final inventory gives the total amount of items subject to procurement cost.

3.4.3 Holding Cost

$\sum_s \sum_i \sum_{l:l\in(l\setminus \mathrm{SC})} \sum_t h(i)n(i,l,t,s)$

Holding cost is only accrued when the items are kept on the vending machines. Since lead time is assumed to be zero for supply centers, there is no holding cost at those locations. Holding cost is calculated by summation of inventory level of each item at the beginning of all time periods, for all scenarios and locations excluding supply centers.

3.4.4 Delivery Cost

$\sum_s \sum_t \sum_l \sum_{l'} dlv(l,l')z(l,l',t,s)$

Possible delivery routes are given as an input to the problem. However, usage of these routes is a decision of the mathematical model. If a route is used for transferring some items, transportation cost of that route is realized and considered as delivery cost. Total cost is calculated as sum of the costs for time periods where items transported from location l to location l'. Note that, we assume each location has its own vehicle (or an available transportation mean like taxi), and this vehicle is ready whenever it is needed.

3.5 Objective Function

Maximize

$$\frac{\begin{pmatrix}\sum_s \sum_i \sum_{p_i} \sum_{l:l\in(l\setminus \mathrm{SC})} \sum_t pr(p_i)u(i,p_i,l,t,s) - \sum_s \sum_i \sum_{l:l\in SC} c(i)\big(n(i,l,0,s) - n(i,l,|T|,s)\big) - \\ \sum_s \sum_i \sum_{l:l\in(l\setminus \mathrm{SC})} \sum_t h(i)n(i,l,t,s) - \sum_s \sum_t \sum_l \sum_{l'} dlv(l,l')z(l,l',t,s)\end{pmatrix}}{|S|}$$

subject to

3.6 Constraints

3.6.1 Price Constraint

Price of an item is decided for each time period and it can be differentiated for different locations and time periods. Dedication of the price of an item is obligatory if there is a sale of the item at that location. There cannot be more than one price for an item at a location for a time period.

(1) At most one price can be valid at a location for an item through a sale channel for each time period:

$$\sum_{p_i} \mathrm{x}(i, p_i, l, t) \leq 1 \qquad \forall\ i, l, t, c$$

3.6.2 Inventory Balance Constraint

Inventory level of an item at a location is derived by considering the previous inventory level, the items transferred to/from that location and the items sold at that location during the previous time period. This constraint provides balance of inventory at each location for all time periods.

(2) Inventory balance equations for items at sale locations

$$n(i, l, t, s) = n(i, l, t-1, s) - \sum_{l'} r(i, l, l', t - dst(l', l), s) + \sum_{l'} r(i, l', l, t, s) - \sum_{p_i} \mathrm{u}(i, p_i, l, t, s) \qquad \forall\ i, l, t > 0, s$$

$n(i, l \in (l \backslash \mathrm{SC}), t = 0, s)$ *is given and* $n(i, l = SC, t = 0, s) = M$ (= 1 million)

3.6.3 Transportation Between Locations

As stated earlier, transportation of the items between different locations is allowed. In order to be able to transport an item from one location to another, transportation mean must be present at the sending location at the appropriate time period. $M_{ll'}$ is defined as an input as the set of time periods when items can be transferred from location *l* to l'. Therefore, when *t* is not an element of the set $M_{ll'}$ items cannot be transported between locations *l* and l'.

(3) Items can be transferred from location *l* only if the transportation mean is present there:

$$z(l, l', t, s) = 0 \qquad \forall\ (l, l'), t \in T \setminus M_{ll'}, s$$

Constraint (4) can be seen as a complementary constraint of Constraint (3). Delivery cost component of the objective function is the multiplication of the delivery cost and the binary value showing transportation status between locations *l* and l'. If there are items transferred from location *l* to l', the model has to include cost of that transportation.

(4) Items need to be transferred if any item is sent from location *l* to l' at period *t*:

$$z(l, l', t, s)M \geq \sum_i r(i, l, l', t, s) \qquad \forall\ l, l', t, s$$

3.6.4 Sales Amount at a Location

Sales amount of an item at a location cannot exceed the current inventory level of the item at that location. Sales amount also cannot be more than demand at that time period.

(5) Number of items sold is an amount which is less than both inventory and demand:

$$u(i,p_i,l,t,s) \leq \; x(i,p_i,l,t)d(i,p_i,l,t,s) \qquad \forall\, i,p_i,l,t,s$$

$$\sum_{p_i} u(i,p_i,l,t,s) \leq \; n(i,l,t,s) \qquad \forall\, i,l,t,s$$

3.6.5 Minimum and Maximum Level of Inventory at Any Location

The proposed model aims to allocate towers to the items. In addition, for every item each tower has a maximum capacity for that specific item. Therefore, the amount of item *i* at a location cannot exceed the total capacity of allocated towers. At the same time, the empty allocated space for an item must be less than capacity of 1 tower.

(6) Inventory at a location should not exceed the capacity of allocated number of towers for that item

$$n(i,l,t,s) > cap(i)(a(i,l,t) - 1) \qquad \forall\, i,l \in (l \backslash \mathrm{SC}),t,s$$

$$n(i,l,t,s) \leq \; cap(i)a(i,l,t) \qquad \forall\, i,l \in (l \backslash \mathrm{SC}),t,s$$

3.6.6 Available Towers

Total number of towers at any location is limited. Allocated number of towers at any time period cannot exceed that total.

(7) Allocated towers at location l should not exceed number of available towers at that location

$$\sum_i a(i,l,t) \leq \; tow(l) \qquad\qquad \forall\, l,t$$

CHAPTER 4

NUMERICAL EXPERIMENTS

By conducting numerical experiments, we aim to understand whether our model provides efficient solutions to the problem with uncertainties. Our model considers the uncertainty in demand of the items by using scenarios. As stated earlier our model decides the inventory flow from the supply centers and other locations, the price of the item and the tower allocation for each item. In order to be able to evaluate success of the mathematical model, we will compare the results of the model with a heuristic approach.

4.1 Validation and Verification of the Model

In order to conclude that the proposed model works properly, we perform some functional tests and verify that the model gives the expected results. Details of the validation scenarios considered are provided in Appendix. Below we provide brief explanations of the scenarios used in validation and verification;

1. Check if sales occur from locations
   Scenario 1: 1 location, 1 item, 1 price, Demand exists, Initial inventory exists
2. Check if transfer occur from supply center to location in case of existing demand at location
   Scenario 2: 2 locations (1 of them is supply center), 1 item, 1 price, 1 route, Demand exists, No initial inventory at the location
3. Check if most profitable item is sold or not (high price or low cost)

Scenario 3: 2 locations (1 of them is supply center), 2 items, 1 price for each item, 1 route, Demand exists for both items, No initial inventory at locations, 1 tower

Scenario 4: 2 locations (1 of them is supply center), 2 items, 1 price for both items, 1 route, Demand exists for both items, No initial inventory at locations, 1 tower

4. Check if profit maximizing price is chosen, or not

   Scenario 5: 2 locations (1 of them is supply center), 1 item, 2 prices for the item, 1 route, High demand for low price and low demand for high price, No initial inventory at locations

5. Check if items with low demand are returned, or not

   Scenario 6: 2 locations (1 of them is supply center), 1 item, 1 price, 1 route, Demand exists, Initial inventory more than demand

6. Check if locations with low transportation cost are preferred or not

   Scenario 7: 3 locations (1 of them is supply center), 1 item, 1 price, 2 routes, Demand exists for all locations, No initial inventory at locations, Far location with transportation cost higher than revenue

7. Check if inventory transfer between locations is used, or not

   Scenario 8: 3 locations (1 of them is supply center), 1 item, 1 price, 2 routes, 1 location with initial inventory and without demand, other location with demand and without initial inventory, location with initial inventory is closer than supply center. No initial inventory at locations, Distant location has transportation cost higher than revenue

8. Check if items which has low holding cost, at locations are held or not

   Scenario 9: 1 location, 1 item, 2 prices for the item, Initial inventory, which is not sufficient for both time periods, at the location, holding cost is lower than marginal utility

9. Check if replenishment is done at lower cost periods or not

Scenario 10: 2 locations (1 of them is supply center), 1 item, 2 prices for the item, Demand exist for two different time periods, 2 routes (compatible with demand periods), replenishment for one period is more profitable

4.2 Settings

4.2.1 Details of the problem

After validation and verification of the model with simple problems, we conclude that the model functions properly. Next step is testing the model to gain insights with obtained solutions to our problem. A generated problem instance is considered which includes 1 day and 24 periods, 10 locations (including 2 supply centers), 8 different items, 3 prices for each product, 28 routes and 513 customer demands. 1 TL/km is accepted as the average travelling cost.

Table 4.1. includes information related to the items: name, their procurement cost, holding cost, tower capacities and alternative prices. As we stated earlier the costs of our problem consists of procurement cost, holding cost and transportation cost. Procurement cost and holding cost values in the table directly effects our objective function.

Table 4.1. Information About the Items for the Generated Instance

Item	*Procurement Cost*	*Holding Cost*	*Tower Capacity*	*Price_1*	*Price_2*	*Price_3*
Mobile Phone	1500	15	30	1750	2000	2300
Ipad	1750	17	20	2000	2400	2700
Kindle	500	5	20	700	850	1000
Wallet	200	2	40	250	300	400
Earphone	160	2	50	200	250	350
Battery	40	1	50	60	80	100
Watch	400	4	40	500	600	800
Gold (gr)	275	3	80	350	400	500
Mobile Phone	1500	15	30	1750	2000	2300
Ipad	1750	17	20	2000	2400	2700

Table 4.2. includes information about the locations: coordinates of the location and number of the towers at that location. Coordinates of the location and unit price of travel directly effects total transportation cost, which is one of the cost components in the objective function.

Table 4.2. Information About Locations for the Generated Instance

Location	*Latitude*	*Longitude*	*# of Towers*
Supply Center (Saray)	40,059329	32,618503	0
Supply Center (Ulus)	39,940261	32,859069	0
Next Level	39,910898	32,812458	5
Ankamall	39,951371	32,830734	8
Esenboğa Airport	40,115115	32,991288	6
Metromall	39,982687	32,60965	5
Kentpark	39,909469	32,775543	7
Göksu Park	39,994144	32,649378	5
Altınpark	39,966272	32,876612	4
Harikalar Diyari	39,976622	32,588181	4

We assume that all possible routes for item transfer between locations are given as an input to the model. Therefore, the model does not optimize the routes. Table 4.3. includes all information about the routes: origin and destination of the routes, distance between locations, required periods for transportation and arrival periods. Distance between locations directly effects transportation cost of the items. Arrival periods will affect holding cost due to the time difference between replenishment and sales.

Table 4.3. Information About Routes for Generated Instance

Origin	Destination	*Distance (km)*	*Required Periods*	*Arrival_1*	*Arrival_2*	*Arrival_3*
SC (Saray)	Göksu Park	20	0	1	7	14
SC (Saray)	Harikalar Diyarı	30	0	2	10	
Göksu Park	Metromall	10	0	4	16	
Metromall	Harikalar Diyari	10	0	6	20	
SC (Ulus)	Ankamall	10	0	2	13	18
SC (Ulus)	Next Level	20	0	5	17	
SC (Ulus)	Esenboğa Airport	80	1	3	11	
Next Level	Kentpark	10	0	7	18	
SC (Ulus)	Altınpark	10	0	8	15	
Göksu Park	SC (Saray)	20	0	11		
Harikalar Diyari	SC (Saray)	30	0	19		
Ankamall	SC (Ulus)	10	0	15		
Kentpark	SC (Ulus)	30	0	20		
Altınpark	Esenboğa Airport	70	1	9	21	
Esenboğa Airport	SC (Ulus)	80	1	5	16	

Figure 4.1. shows exact coordinates of the locations and routes between these locations on a map.

Figure 4.1. Route Map

Table 4.4. includes initial inventory level of locations for each item. Since there is no lead time for procurement of the items, there is no inventory hold at supply centers.

Table 4.4. Initial Inventory Level of Locations

Location	Item	*Inventory Level*
Ankamall	Mobile Phone	10
Ankamall	Battery	30
Ankamall	Gold (gr)	50
Esenboğa Airport	Mobile Phone	5
Esenboğa Airport	Battery	45
Esenboğa Airport	Watch	20
Esenboğa Airport	Earphone	30
Kentpark	Battery	20
Kentpark	Ipad	8
Göksu Park	Gold (gr)	10
Göksu Park	Earphone	14
Altınpark	Battery	15
Altınpark	Gold (gr)	10
Altınpark	Watch	7

Table 4.5. shows the demand summary for the items and the locations. Detailed demand table is provided in Appendix. Demands are assumed to be distributed normally.

Table 4.5. Information About Locations for Generated Instance

Item	*Number of Locations*	*Number of Prices*	*Number of Demand Period*
Mobile Phone	7	3	44
Ipad	6	3	14
Kindle	3	3	20
Wallet	7	3	16
Earphone	4	3	10
Battery	8	3	20
Watch	8	3	22
Gold (gr)	3	3	25

4.2.2 Size of the Problem

Table 4.6 includes the indicators showing size of the problem. As it can be observed from the table there are 8 items for sale at 8 locations, excluding supply centers. There are 3 price alternatives for each item and there are 28 routes defined as input for transporting items between locations.

Table 4.6. Information About Problem

Attribute	*Number*
Locations	10 (2 SCs)
Items	8
Periods	1 Day – 24 Periods (Hours)
Prices	3 for each item
Routes	28

Table 4.7 includes information about the parameters of the problem. As it can be observed from the table, we have uncertain demand at 171 demand points for 3 alternative prices. We have procurement cost, holding cost and tower capacity for each item. Each location, excluding supply centers, has a tower capacity.

Table 4.7. Information About Parameters of the Problem

Parameter	*Number*
$d(i,p_i,l,t,c,s)$	513*|S|
$c(i)$	8
$h(i)$	8
$dlv(l,l')$	15
$dst(l', l)$	15
$tow(l)$	8
$cap(i)$	8

Some of the decision variables are linked with scenarios, whereas the others are not. Price dedication and tower allocation are made according to scenarios but single allocation for all scenarios. Total number of pricing and allocation decision variables are 4.608 and 1.536 respectively. Amount of inventory, transfer amount, route usage and sales amount decision variables depend on the number of scenarios. As the number of scenarios increases number of these decision variables' also increases. For a 10-scenario case we have 188.544 decision variables.

Table 4.8. Information About Decision Variables

Decision Variable	*Number*
$x(i,p_i,l,t,c)$	4.608
$a(i,l,t)$	1.536
$n(i,l,t,s)$	1.536 x |S|
$r(i,l,l',t,s)$	10.752 x |S|
$z(l,l',t,s)$	1.344 x |S|
$u(i,p_i,l,t,c,s)$	4.608 x |S|

Table 4.9 includes information about number of constraints. We have only two constraints, whose number is independent of the number of scenarios. For 10-scenario case 152.448 constraints are considered for the problem.

Table 4.9. Information About Constraints

Constraint	*Number*
(1) Single Price	1.536
(2) Inventory Balance	1.536 x |S|
(3) Transportation	2.160 x |S| - 28
(4) Transportation	2.160 x |S|
(5) Sales Amount	6.144 x |S|
(6) Inventory	3.072 x |S|
(7) Tower	192

4.3 Results

4.3.1 Analysis on the Number of Scenarios

We run our model for the generated instance, described above, for 1, 10, 50, 100 scenarios for 10 times. Figure 4.2. shows the results of the model for 1 scenario. As observed from the figure objective function value deviates between TRY 134.894 and TRY 167.840 and run-time deviates between 0,4 and 0,6 seconds.

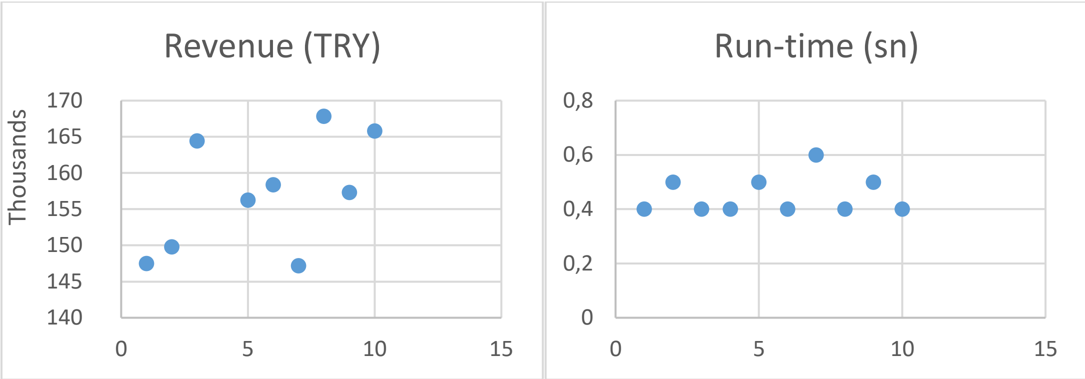


Figure 4.2 *Profit and Run-time for 1 Scenario Case*

For the same instance with 10 scenarios, the results can be found in Figure 4.3. As it can be observed from the figure, objective function value deviates between TRY 153.703 and TRY 162.658, whereas run-time deviates between 19 and 85 seconds.

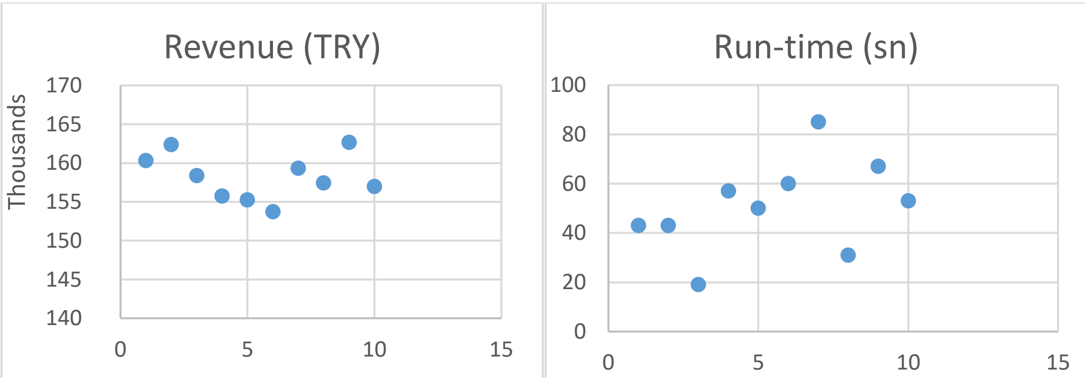


Figure 4.3. *Revenue and Run-time for 10 Scenario Case*

The model provides the results given in Figure 4.4. for the 50-scenario case. It can be observed from the figure, profit deviates between TRY 156.078 and TRY 158.388 where run-time deviates between 342 seconds and 422 seconds. It seems 50 scenarios provides robust solution for this problem. However, we have to keep in mind that the required scenarios for a robust solution might change for different problems.

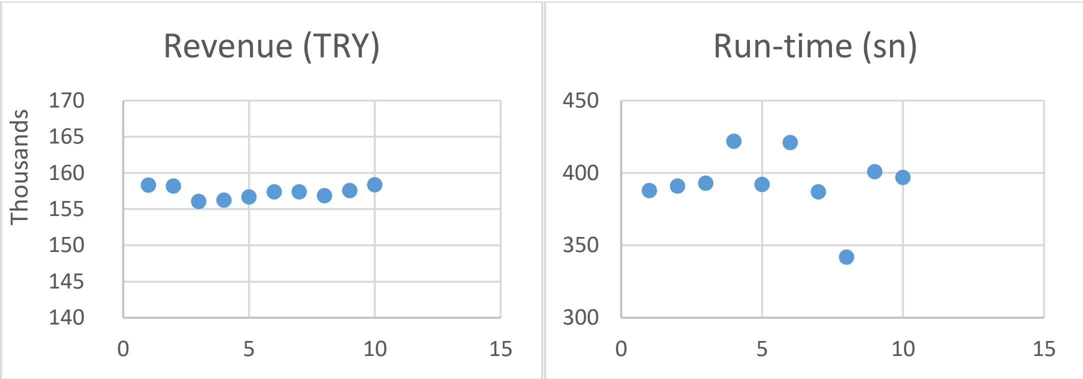


Figure 4.4. *Revenue and Run-time for 50 Scenario Case*

As it can be observed from Figure 4.5., for the case including 100 scenarios, profit deviates between TRY 156.034 and TRY 159.207 and run-time deviates between 1.275 and 2.942 seconds.

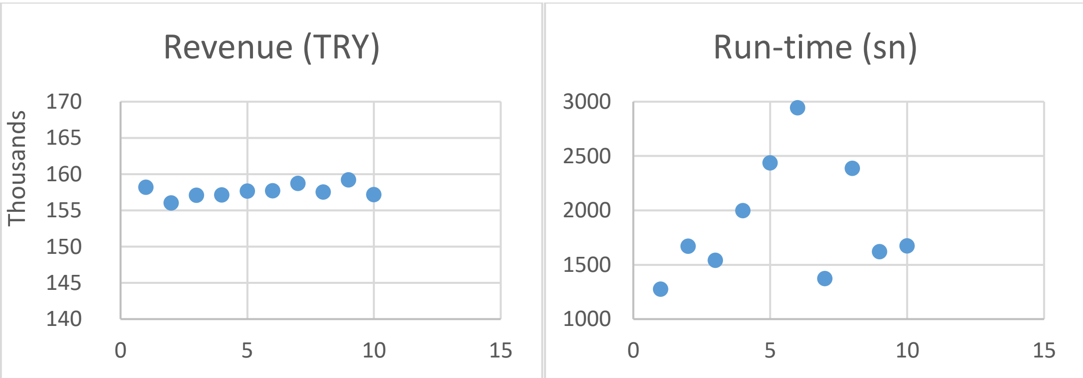


Figure 4.5. *Revenue and Run-time for 100 Scenario Case*

When we analyze the results of the model, the problem with 50 scenarios seem to be the best option. Although the results for 50-scenario instance are as robust as the results for 100-scenario instance, run-time is almost 1/5 of the run-time of 100-scenario instance.

4.3.2 Sales Amount for One Scenario

When we find the optimal solution for the problem, the sales summary of the mathematical model for a 50-scenario run is provided in the Table 4.6 below. Since the model might sell different quantities for different scenarios, here we provide most repetitive sale numbers for all scenarios. The objective function value of this instance is TRY 222.336,80 TRY. According to sales data a total amount of 767 items sold at 8 locations. As mentioned earlier the structure of the demand distributed normally and this fact brings stochastic nature of the instance.

Table 4.10. Sales Summary for Generated Instance

Location	Item	*Price*	*Quantity*
Altınpark	Battery	100	25
Altınpark	Gold (gr)	400	13
Altınpark	Gold (gr)	500	17
Altınpark	Mobile Phone	2300	3
Altınpark	Wallet	400	12
Altınpark	Watch	800	5
Ankamall	Battery	60	20
Ankamall	Battery	80	14
Ankamall	Battery	100	20
Ankamall	Earphone	350	3
Ankamall	Gold (gr)	350	5
Ankamall	Gold (gr)	500	7
Ankamall	Ipad	2400	9
Ankamall	Ipad	2700	6
Ankamall	Mobile Phone	1750	4
Ankamall	Mobile Phone	2000	13
Ankamall	Mobile Phone	2300	6
Ankamall	Watch	600	5
Ankamall	Watch	800	19
Esenboğa Airport	Battery	80	76
Esenboğa Airport	Battery	100	82
Esenboğa Airport	Earphone	200	9
Esenboğa Airport	Earphone	250	4
Esenboğa Airport	Earphone	350	16

Table 4.10 Continued

Esenboğa Airport	Ipad	2400	4
Esenboğa Airport	Ipad	2700	2
Esenboğa Airport	Kindle	850	6
Esenboğa Airport	Kindle	1000	2
Esenboğa Airport	Mobile Phone	1750	18
Esenboğa Airport	Mobile Phone	2000	14
Esenboğa Airport	Watch	600	8
Esenboğa Airport	Watch	800	5
Göksu Park	Battery	100	5
Göksu Park	Earphone	200	19
Göksu Park	Earphone	250	10
Göksu Park	Earphone	350	17
Göksu Park	Gold (gr)	400	14
Göksu Park	Gold (gr)	500	30
Göksu Park	Mobile Phone	2000	5
Göksu Park	Mobile Phone	2300	3
Göksu Park	Wallet	400	11
Göksu Park	Watch	800	13
Harikalar Diyari	Battery	100	21
Harikalar Diyari	Ipad	2400	2
Harikalar Diyari	Wallet	300	7
Harikalar Diyari	Wallet	400	20
Harikalar Diyari	Watch	600	4
Kentpark	Battery	80	23
Kentpark	Battery	100	25
Kentpark	Ipad	2400	8
Kentpark	Ipad	2700	2
Kentpark	Wallet	400	9
Kentpark	Watch	800	13
Metromall	Mobile Phone	2000	3
Metromall	Wallet	400	9

Table 4.10 Continued

Metromall	Watch	600	3
Metromall	Watch	800	8
Next Level	Battery	100	9
Next Level	Ipad	2700	3
Next Level	Kindle	850	13
Next Level	Mobile Phone	2000	4
Next Level	Mobile Phone	2300	2

4.3.3 Product Flow Between Locations

Table 4.11. shows flow of items between locations. According to this data 35 flow realized between locations. Total amount of 631 items flowed between locations. 3 of them is towards supply center. 13 of these flows are between locations other than supply center.

Table 4.11. Product Flow Between Locations for Generated Instance

Start Location	Destination	Item	*Quantity*
Altınpark	Esenboğa Airport	Mobile Phone	46
Altınpark	Esenboğa Airport	Watch	4
Altınpark	Esenboğa Airport	Necklace	3
Ankamall	Supply Center (Ulus)	Mobile Phone	1
Ankamall	Supply Center (Ulus)	Earphone	36
Ankamall	Supply Center (Ulus)	Tablet	82
Esenboğa Airport	Supply Center (Ulus)	Watch	15
Esenboğa Airport	Supply Center (Ulus)	Earphone	15
Göksu Park	Metromall	Wallet	3
Göksu Park	Metromall	Tablet	18
Göksu Park	Metromall	Watch	11
Metromall	Harikalar Diyari	Mobile Phone	7

Table 4.11 Continued

Next Level	Kentpark	Wallet	28
Next Level	Kentpark	Necklace	4
Next Level	Kentpark	Tablet	9
Next Level	Kentpark	Earphone	13
Supply Center (Saray)	Göksu Park	Wallet	5
Supply Center (Saray)	Göksu Park	Mobile Phone	34
Supply Center (Saray)	Göksu Park	Watch	35
Supply Center (Saray)	Göksu Park	Tablet	11
Supply Center (Saray)	Göksu Park	Necklace	25
Supply Center (Saray)	Göksu Park	Tablet	23
Supply Center (Saray)	Harikalar Diyari	Mobile Phone	23
Supply Center (Saray)	Harikalar Diyari	Wallet	2
Supply Center (Saray)	Harikalar Diyari	Necklace	19
Supply Center (Saray)	Harikalar Diyari	Wallet	5
Supply Center (Ulus)	Altınpark	Earphone	58
Supply Center (Ulus)	Altınpark	Necklace	21
Supply Center (Ulus)	Altınpark	Mobile Phone	9
Supply Center (Ulus)	Altınpark	Watch	12
Supply Center (Ulus)	Ankamall	Mobile Phone	24
Supply Center (Ulus)	Ankamall	Watch	3
Supply Center (Ulus)	Ankamall	Watch	17
Supply Center (Ulus)	Ankamall	Wallet	109
Supply Center (Ulus)	Ankamall	Necklace	23
Supply Center (Ulus)	Esenboğa Airport	Battery	71
Supply Center (Ulus)	Esenboğa Airport	Earphone	13
Supply Center (Ulus)	Esenboğa Airport	Ipad	7
Supply Center (Ulus)	Esenboğa Airport	Kindle	8
Supply Center (Ulus)	Esenboğa Airport	Mobile Phone	35
Supply Center (Ulus)	Esenboğa Airport	Watch	6
Supply Center (Ulus)	Next Level	Battery	37
Supply Center (Ulus)	Next Level	Ipad	4
Supply Center (Ulus)	Next Level	Kindle	15

Table 4.11 Continued

Supply Center (Ulus)	Next Level	Mobile Phone	7
Supply Center (Ulus)	Next Level	Wallet	9
Supply Center (Ulus)	Next Level	Watch	13

4.4 Further Analysis

We complete the whole picture by making some further analysis. Since our problem is a stochastic MILP problem, we use LP relaxation and sample average approximation (SAA) methods to understand the dynamics of the problem. LP relaxation is performed by relaxation of the integer and binary variables of the problem.

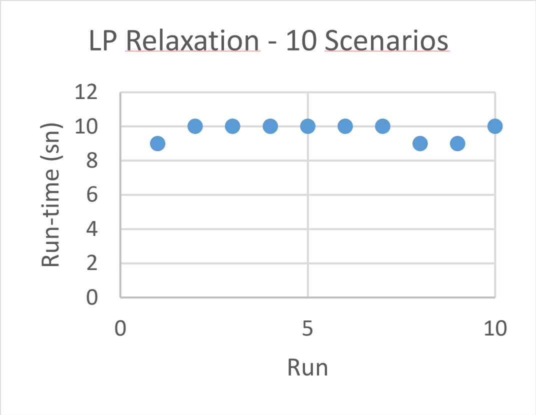
4.4.1 LP Relaxation

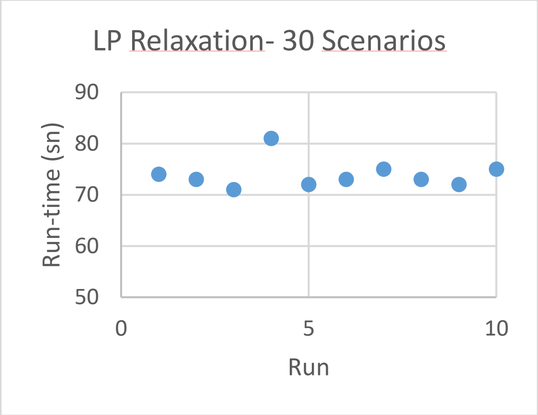
Our model includes 6 different decision variables. All those variables are integer or binary variables. Binary variables x(i,pi,l,t,c) and z(l,l´,t,s) are set between 0 and 1. Integer variables a(i,l,t), n(i,l,t,s), r(i,l,l´,t,s) and u(i,pi,l,t,c,s) are set to positive real numbers. Table 4.12 below shows the average value of the objective function for given number of scenarios for 10 runs. It can be observed from the table, when we relax all decision variables, our objective function increases by 4,71 % on the average.

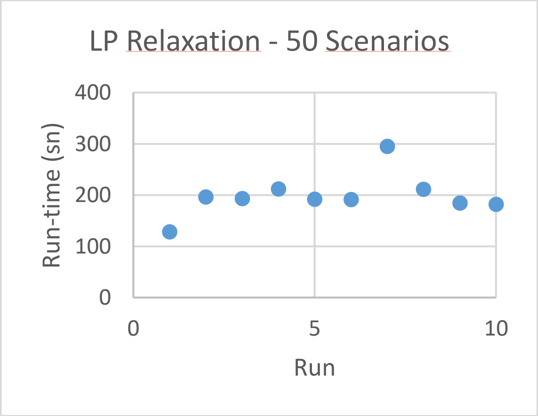
Table 4.12. Comparison of objective function value for MILP and LP Relaxation

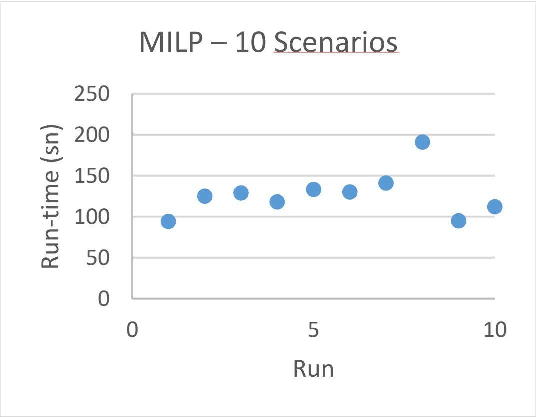
Number of Scenarios	MILP Result	LP Relaxation	Difference
10	223.000,35	231.195,36	3,67%
20	219.846,67	232.752,31	5,87%
30	221.194,50	230.085,02	4,02%
40	218.423,36	230.734,88	5,64%
50	221.086,27	230.779,66	4,38%
Average	220.710,23	231.109,45	4,71%

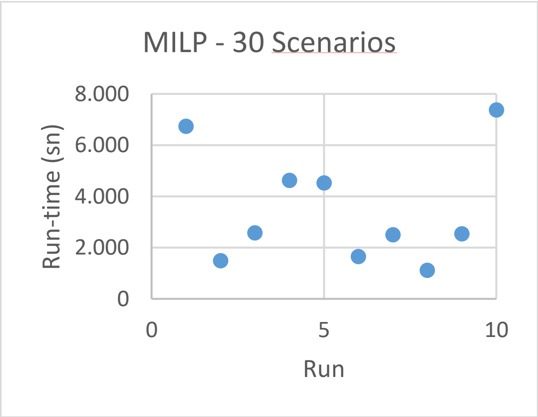
The graphs below show run-time for LP relaxation and MILP models for different number of scenarios. As it can be observed from the graphs, run-time for LP relaxation takes a time which is almost 1% of the run-time for MILP model for the same number of scenarios.











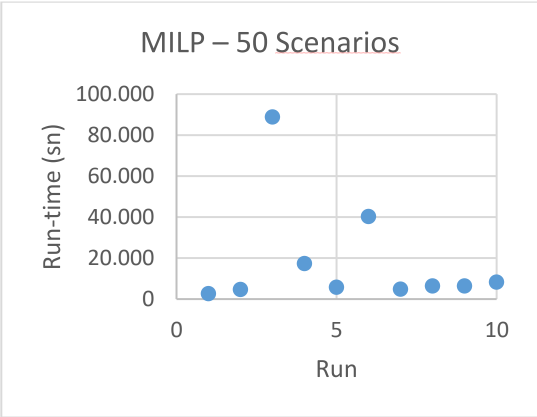


Figure 4.6. Run-time performance of MILP and LP Relaxed Mathematical Models

When we analyze LP relaxation results, there are two main reasons for the increase in the objective function value. The first one is related with the delivery cost. When the binary variable z(l,l´,t,s) is relaxed, it is set to a value which is very close to zero. Therefore, the relaxed mathematical model minimizes the delivery cost where it preserves conformance to the usage of transportation mean constraint. This result provides a negligible transportation cost between locations. The second reason is related with the revenue. We restrict the pricing activity by one price constraint, which forces to have only one price during a period for an item at a location. This constraint is managed by the binary variable x(i,pi,l,t) and it restricts the sale amount at a location during a time period. When we relax binary variable x(i,pi,l,t), we observe that more than one price could be given to an item at a location for the same period.

4.4.2 Sample Average Approximation

Sample average approximation is a method used for solving stochastic problems with deterministic models. In this technique the expected objective function of the stochastic problem is derived from a random sample. Normally our model generates numbers according to the given distribution of the demand as many as the scenarios require. In the sample average approximation model the mathematical model is no more stochastic. The random numbers are generated externally and generated numbers are used in the deterministic model to find the optimum solution for that specific generated numbers. In our model the solution is optimum for all scenarios, but it may not be optimum for each scenario. However, in sample approximation method we find optimal solution for each scenario. This fact results a better solution with sample average approximation when compared to our mathematical model. Table 4.13 below includes the comparison of the results of sample average approximation method and mathematical model for a 50-scenario case.

Table 4.13. SAA and Mathematical Model Results of a 50-Scenario Instance

Scenario Number	**SAA**	**Mathematical Model**
1	235.026	220.889
2	229.472	
3	230.303	
4	256.171	
5	232.042	
6	226.415	
7	204.961	
8	239.001	
9	228.638	
10	230.008	
11	217.650	
12	213.665	
13	233.421	
14	231.624	
15	242.864	
16	240.435	
17	226.077	
18	202.359	
19	224.991	
20	227.537	
21	214.504	
22	233.637	
23	234.599	
24	203.844	
25	244.160	

Table 4.13 Continued		
26	202.958	
27	234.436	
28	228.365	
29	227.022	
30	226.186	
31	223.212	
32	223.527	
33	245.177	
34	225.250	
35	208.674	
36	208.129	
37	214.856	
38	243.346	
39	189.880	
40	226.565	
41	247.781	
42	233.186	
43	226.982	
44	218.286	
45	238.976	
46	226.404	
47	232.643	
48	229.706	
49	227.649	
50	219.459	
Average	226.641,18	220.889

When we compare the results of mathematical model for 50-scenario case and sample average approximation method for each scenario, the latter creates 2,6 % higher profit from the same generated demands. As stated earlier this is due to the single price dedication and single allocation structure of the mathematical model for all scenarios.

Structure of the problem directly effects the objective function value improvement of the SAA method. As stated earlier, in our mathematical model single allocation is made for all scenarios in a run. However, SAA method results the optimal solution for each scenario and it may result different allocation and different price dedication for each scenario. If expected value for the sale amount is similar for each product, difference in tower allocation occur more. In the proposed problem there is no intention to create similar revenues for all items. When we create a problem with similar results at each route period, the sample average approximation method yields better result. Demand details of this instance is provided in Appendix. Table 4.14 below shows the comparison between results of sample average approximation method and mathematical model for the specially designed problem.

Table 4.14. SAA and Mathematical Model Results of a Specially Designed 50-Scenario Instance

#	**SAA**	**Mathematical Model**
1	153.866	
2	159.204	
3	154.254	
4	146.065	
5	136.870	
6	130.991	
7	154.371	
8	160.824	
9	160.100	
10	146.363	
11	140.962	
12	175.841	
13	150.800	143.618,91
14	144.086	
15	170.521	
16	146.677	
17	161.370	
18	147.537	
19	155.366	
20	156.355	
21	167.189	
22	159.834	
23	167.398	
24	160.397	
25	144.614	
26	137.571	

Table 4.14 Continued		
27	153.186	
28	151.676	
29	160.839	
30	149.720	
31	164.205	
32	158.415	
33	155.922	
34	158.108	
35	158.535	
36	156.851	
37	151.032	
38	157.132	
39	157.082	
40	138.081	
41	159.500	
42	140.084	
43	176.628	
44	153.468	
45	143.841	
46	159.821	
47	170.273	
48	153.099	
49	163.927	
50	175.161	
Average	155.120	142.818,33

The table shows us sample average approximation yields 8,6 % better result when compared to mathematical model with 50-scenario case. We know that it will absolutely yield a better result for 1+ scenarios of mathematical model results, but the performance of the SAA differs according to the allocation differentiations in deterministic solutions of each scenario.

4.4.3 Sample Average Approximation as a Heuristic Approach

We use sample average approximation as a second heuristic approach. We have price dedication and tower allocation for each individual scenario while using SAA method for one-scenario instances, while our mathematical model has only one price dedication and tower allocation for all scenarios. In order to have a good result which is close to our mathematical model solution, we use mostly recurrent SAA results for deriving single price dedication and single tower allocation for all scenarios. As in our mathematical model, we dedicate single price for an item at a location for each time period and single tower allocation for a location for each time periods according to most recurrent ones. According to those dedicated prices, we derive corresponding demand for that time period and run the model with given tower allocations, price decisions and corresponding demands. The Table 4.15 shows the results of those runs.

Table 4.15. SAA as a Heuristic Approach

#	SAA-Most Recurrent	Mathematical Model
1	213.459	220.889
2	216.585	
3	208.495	
4	243.362	
5	217.994	
6	212.304	
7	178.911	
8	221.145	
9	209.455	
10	212.041	
11	202.377	
12	201.280	
13	214.641	
14	209.455	
15	227.077	

Table 4.15 Continued		
16	221.598	
17	209.156	
18	185.095	
19	207.473	
20	216.160	
21	199.955	
22	219.366	
23	213.853	
24	185.526	
25	228.262	
26	192.810	
27	219.291	
28	209.766	
29	207.291	
30	208.660	
31	209.890	
32	207.186	
33	229.720	
34	207.126	
35	190.257	
36	194.067	
37	197.704	
38	228.248	
39	171.088	
40	215.237	
41	231.036	
42	218.705	
43	211.070	
44	203.135	
45	209.160	
46	209.415	
47	208.400	
48	213.932	
49	211.877	
50	204.427	
Average	209.690	220.889

Results of SAA based heuristic approach shows us that, we can obtain a solution with an objective function value which is 5 % less than that of the mathematical model.

Although it seems that we can create a good result with SAA based heuristic approach, the structure of the numerical example directly affects the performance of this approach.

4.5 Heuristic Approach

In order to evaluate the success of our mathematical model we have developed a heuristic approach for solving the same question. Proposed heuristic approach tries to maximize the demand met. At the same time, it tries to minimize total cost considered. Our model considers procurement cost, holding cost and transportation cost as the cost components of profit. Procurement cost is directly related with the total amount of sales. Steps of the heuristic approach is given below:

1. Sales locations are being supplied from supply centers. When there is no direct link between a supply center and a sales location, we have to hold the items at the locations between sales location and supply center. Sales locations without direct link with supply centers occupies towers of the in-between locations. While making allocation at these types of locations, we have to consider opportunity cost created at in-between locations. To track this opportunity cost relation, we define supply level, which shows number of locations between supply center and that location, for all locations. Supply centers are assumed to be level 0 (L0) locations. L (minimum level of supplying locations + 1) is set to be level of that location. Steps in the below is performed for all supply levels consecutively.
2. Period consolidation requirement is determined
    a. In order to understand consolidation requirement for demand time periods, demand periods and possible arrival periods for each location is sorted from closest to latest time period

b. If there is no transportation possibility between two consecutive demand periods those demand periods assumed to be equal to the earlier one
c. If there is no demand time between two consecutive transfer periods, earlier transfer will never be used.

3. Proposed heuristic approach aims maximizing profit from sales by dynamic pricing by using updated time periods
   a. For each time period, item and alternative prices, expected profit from each item is calculated by subtracting procurement cost of that item from expected revenue
   b. For alternative prices of an item at the same time period, price with highest profit generation is selected and the other price is eliminated
   c. After dedicating maximum profit generating price for each item, location and period, expected profit of one tower is calculated for each item by multiplying min (tower capacity, demand) and sales profit of that item
   d. After calculating maximum sales profit generation ability of one tower, those values are sorted from highest to lowest
   e. Starting from the highest one, allocation of first towers of that location is made for all periods
   f. After completion of first towers of all locations and periods, allocated capacity is subtracted from expected demand of those items and (c) and (d) are repeated and allocation of second tower is made for all locations and periods. This loop is repeated until all the towers are allocated or no more demand remains.
   g. After completion of previous loop, number of towers allocated to an item is dedicated. An iteration is made by comparing sales profit generation of current situation and potential situation by changing the price with the same tower allocation. If revenue increase can be supported by changing the price, new price is set and allocation

situation is updated. If any tower becomes empty by this change, steps (c), (d) and (e) are repeated to allocate emptied tower

   h. If there is no transportation mean to any location before first allocation, that allocation is cancelled and the plan is updated accordingly and this step is repeated till finding an available transportation mean

4. Our heuristic approach focuses on minimizing holding cost and transportation cost for all items.
   a. Starting from the latest allocation to minimize holding cost the closest time period to an allocation is found and it is set as the arrival period for those items to minimize holding cost
   b. Possibility of combining latest demand aggregation is checked. If it is possible, holding cost between two consecutive transfer and the second transportation cost are compared and if holding cost is cheaper, second transportation is cancelled. This step is repeated for aggregated demand if cancellation is made.
   c. If cancellation is not made step (b) is repeated for previous demand aggregation.

Proposed heuristic approach can only be applicable for 1 scenario cases.

4.5.1 Algorithm of Heuristic Approach

1. **for** each $l \in L$ **do**
2. calculate $L_l \in SC \Rightarrow L_l = 0$; $L_l \notin SC \Rightarrow$min ($L_l$ for locations with flow to destination location.
3. **end for**
4. **for** each p_i, i, t and l where $d(i,p_i,l,t,c) > 0$ **do**
5. calculate profit $d(i,p_i,l,t,c)(p_i - c(i))$ and set p_i with highest profit as selling price
6. set latest fulfillment period $f(i, l, t)$ to highest $t \in M_{ll'}$

7. **end for**
8. **for** each $n(i,l,t) > 0$
9. set $a(i,l,t) = x$ where $x \in I^+$ and $x \geq [n(i,l,t) / \text{cap}(i)]$
10. set remaining empty tower $\text{rtow}(l, t) = \text{tow}(l) - x$
11. **end for**
12. **for** each $L_l \in \{1, 2, 3, ...\}$, $l \in \{1,...,l,...,|L|\}$ and $f(i, l, t)$ **do**
13. **sort** $\min\{d(i,p_i,l,t,c,s), \text{tow}(i)\}(p_i - c(i))$ from highest to lowest
14. if $\text{rtow}(l, t) > 0$ at t, set $a(i, l, t) = a(i, l, t\text{-}1) + 1$ and
    set $\text{rtow}(l, t) = \text{rtow}(l, t\text{-}1) + 1$
15. if $d(i,p_i,l,t,c) > 0$ and $n(i,l,t) > 0$; $n(i,l,t+1) = n(i,l,t) - d(i,p_i,l,t,c)$
    $\text{rtow}(l, t+1) = x$ where $x \in I^+$ and $x \geq [n(i,l,t+1) / \text{cap}(i)]$
16. **end for**

4.5.2 Results of Heuristic Approach

Table 4.16. shows sales amounts of the proposed heuristic approach. When we analyze the sales results of the heuristic approach, at some time periods sales amount is higher than that of the mathematical model. The reason is the uncertain environment of the mathematical model. Since demand is uncertain and it is distributed by normal distribution in the mathematical model, at some time periods realized sales amount is lower than mean value of the distribution. In the proposed heuristic approach demand is assumed to be equal to the expected value of the distribution.

Table 4.16. Sales Amount of Heuristic Approach for Generated Instance

Item	Location	*Price*	*Quantity*
Battery	Altınpark	100	17
Gold (gr)	Altınpark	400	15
Gold (gr)	Altınpark	500	15
Mobile Phone	Altınpark	2300	1
Wallet	Altınpark	300	4

Table 4.16 Continued

Wallet	Altınpark	400	5
Watch	Altınpark	800	3
Battery	Ankamall	60	16
Battery	Ankamall	80	20
Battery	Ankamall	100	12
Earphone	Ankamall	350	4
Gold (gr)	Ankamall	500	9
Ipad	Ankamall	2400	9
Ipad	Ankamall	2700	7
Mobile Phone	Ankamall	1750	20
Mobile Phone	Ankamall	2000	6
Mobile Phone	Ankamall	2300	4
Watch	Ankamall	600	6
Watch	Ankamall	800	15
Battery	Esenboğa Airport	60	27
Battery	Esenboğa Airport	80	50
Battery	Esenboğa Airport	100	81
Earphone	Esenboğa Airport	200	9
Earphone	Esenboğa Airport	250	9
Earphone	Esenboğa Airport	350	12
Ipad	Esenboğa Airport	2400	1
Kindle	Esenboğa Airport	850	6
Kindle	Esenboğa Airport	1000	2
Mobile Phone	Esenboğa Airport	1750	15
Mobile Phone	Esenboğa Airport	2000	12
Wallet	Esenboğa Airport	300	2
Wallet	Esenboğa Airport	400	4
Watch	Esenboğa Airport	600	9
Watch	Esenboğa Airport	800	3
Battery	Göksu Park	100	2
Earphone	Göksu Park	200	43
Earphone	Göksu Park	250	4

Table 4.16 Continued

Earphone	Göksu Park	350	11
Gold (gr)	Göksu Park	400	8
Gold (gr)	Göksu Park	500	24
Mobile Phone	Göksu Park	2000	6
Mobile Phone	Göksu Park	2300	3
Wallet	Göksu Park	400	6
Watch	Göksu Park	800	12
Battery	Harikalar Diyari	80	3
Battery	Harikalar Diyari	100	14
Wallet	Harikalar Diyari	300	6
Wallet	Harikalar Diyari	400	17
Watch	Harikalar Diyari	600	4
Battery	Kentpark	80	21
Battery	Kentpark	100	20
Ipad	Kentpark	2400	7
Ipad	Kentpark	2700	1
Wallet	Kentpark	400	8
Watch	Kentpark	800	13
Mobile Phone	Metromall	2000	4
Wallet	Metromall	400	8
Watch	Metromall	600	4
Watch	Metromall	800	5
Battery	Next Level	100	12
Ipad	Next Level	2700	4
Kindle	Next Level	700	4
Kindle	Next Level	850	8
Mobile Phone	Next Level	2000	5
Mobile Phone	Next Level	2300	2

When compared with the results of mathematical model, heuristic approach does not have good efficiency for the locations without direct link to the supply centers.

Additionally, the heuristic approach cannot tackle with using insufficient towers. For example, consider that there is a tower requirement at Harikalar Diyarı and there is an idle tower at Metromall. Since unnecessary items cannot be send to Supply Center from Harikalar Diyarı until time period 15, it can be stored in the idle tower of Metromall until that time period. Therefore, tower requirement of the location Harikalar Diyarı could be satisfied as mathematical model fulfills. However, such a case cannot be handled by the proposed heuristic.

4.6 Comparison of Model and Heuristic Approach

With the heuristic approach, we try to find a reasonable solution to our problem. When we analyze the results of heuristic approach and the mathematical model, it is easily seen that our mathematical model increases the efficiency of operations policy of the vending machine management problem. To make a fair comparison between the heuristic approach and mathematical model, we solved 1-scenario case problem for 50 times with mathematical model. There instances are solved also by the heuristic approach.

Table y shows the results of heuristic approach and mathematical model for the same generated instances in terms of the total profit generated via sales of products.

Table 4.17. Comparison of Heuristic Approach and Mathematical Model

#	Heuristic Result	Mathematical Model	Comparison
1	209.297	227.564	8,03%
2	207.755	227.809	8,80%
3	192.332	203.077	5,29%
4	198.961	217.895	8,69%
5	216.305	235.226	8,04%
6	202.281	223.657	9,56%
7	199.110	220.781	9,82%

Table 4.17 Continued

8	222.224	240.661	7,66%
9	201.176	220.149	8,62%
10	205.983	225.071	8,48%
11	180.781	201.207	10,15%
12	213.571	235.988	9,50%
13	187.247	206.748	9,43%
14	214.299	232.033	7,64%
15	190.393	212.689	10,48%
16	218.558	239.842	8,87%
17	184.160	204.937	10,14%
18	206.109	223.656	7,85%
19	229.905	250.889	8,36%
20	201.500	219.148	8,05%
21	216.060	233.212	7,35%
22	229.828	251.184	8,50%
23	212.420	231.244	8,14%
24	205.941	228.661	9,94%
25	211.454	229.550	7,88%
26	187.783	205.234	8,50%
27	199.838	220.505	9,37%
28	192.329	210.674	8,71%
29	210.930	232.013	9,09%
30	201.360	220.706	8,77%
31	200.688	220.726	9,08%
32	232.850	254.418	8,48%
33	193.889	214.143	9,46%
34	226.755	247.732	8,47%
35	222.295	243.670	8,77%
36	200.362	221.408	9,51%
37	207.776	226.668	8,33%
38	211.166	230.363	8,33%
39	216.968	236.545	8,28%
40	223.278	244.771	8,78%
41	194.012	213.743	9,23%
42	196.321	217.019	9,54%
43	201.272	220.559	8,74%
44	232.364	251.899	7,76%
45	213.846	236.235	9,48%
46	222.737	243.858	8,66%
47	226.497	248.768	8,95%

Table 4.17 Continued

48	185.062	203.971	9,27%
49	218.346	239.243	8,73%
50	197.935	217.641	9,05%
Average	207.486	227.308	8,73%

According to the results, mathematical model provides 8,73 % better result on the average when compared to the heuristic approach.

CHAPTER 5

CONCLUSION

In this thesis, we develop a mathematical model to support operations policy of a multi-item vending machine. In our problem, the vendor has vending machines at many locations. The vendor has more than one item to sell at these locations. Some of the locations are supply centers and procurement of items does not have any lead time. Each vending machine has its own capacity managed by towers. Each tower of a vending machine can hold only one item. Therefore, assignment of the towers to the items plays a crucial role. Items at any location are not only replenished by supply centers, but also by other locations. Transfer period is another important decision. Since routing solely is a hard problem to deal with, in our model transfer periods and routes are given as input. Initial inventory level of the items at locations is also provided as input. Demand is assumed to have a stochastic nature rather than a deterministic value. Stochasticity of the demand is managed via use of scenarios.

In our model, objective function maximizes the profit. Since we manage stochastic demand by scenarios, we sum the profits for all scenarios and divide it by the number of scenarios. In the model, revenue is calculated by sum of sale volumes in TRY at all locations. The cost components include procurement cost, holding cost at all locations (excluding supply centers) and delivery cost occurred between pairs of locations. Holding cost at supply centers is assumed to be zero due to zero lead time for procurement.

The result of our experiments shows that, it is more efficient to use 50 scenarios for obtaining a robust result and for having a reasonable run-time. We have also observed that, the mathematical model provides better results than the heuristic approach.

To understand the dynamics of the problem, sample average approximation and LP relaxation methods are used. For the LP relaxation case, our mathematical model yields 8,3 % worse objective function value. We also compare the results of SAA and mathematical model. Since SAA finds the optimal solution for each scenario, it gives 2,6 % to 8,6 % better results when compared to the mathematical model. SAA solution is an upper bound for 50-scenario case.

The problem we address has similarities with shelf allocation and management problems of the retailers. Dynamics of shelf allocation problems may be analyzed in the future to understand the similarities and differences. In addition, we only utilize normal distribution for the demand in our mathematical model. In real life, demand may have different distributions. For future study, real world data can be used to analyze the past behavior of decision makers and a more realistic distribution can be utilized.

REFERENCES

[1] Chen, X., and D. Simchi-Levi (2012), Pricing and inventory management, R. Philipsand O. Ozer, eds., Oxford University Press, Oxford.

[2] W. Elmaghraby & P. Keskinocak (2003). `Dynamic Pricing in the Presence of Inventory Considerations: Research Overview, Current Practices, and Future Directions'. Management Science 49:1287-1309.

[3] M. Chen and Z-L. Chen (2015). Survey of Recent Dynamic Pricing Research, Production and Operations Management 24(5), pp. 704-731.

[4] A. Federgruen & A. Heching (1999). `Combined Pricing and Inventory Control under Uncertainty'. Operations Research 47:454-475.

[5] A. Federgruen & A. Heching (2002). `Multilocation Combined Pricing and Inventory Control'. Manufacturing & Service Operations Management 4(4):275-295.

[6] Y. Feng & Y. Chen (2004). `Optimality and optimization of a joint pricing and inventory-control policy for a periodic-review system'. Working paper .

[7] E. Zabel (1970). `Monopoly and Uncertainty'. The Review of Economic Studies 37(2):205-219.

[8] E. Zabel (1972). `Multiperiod monopoly under uncertainty'. Journal of Economic Theory 5(3):524-536.

[9] H. Zhang & M. Fu (2005). `Sample path derivatives for (s, S) inventory systems with price determination'. The Next Wave in Computing, Optimization, and Decision Technologies pp. 229-246.

[10] K. Zhu & U. Thonemann (2009). `Coordination of pricing and inventory control across products'. Naval Research Logistics 56(2):175-190.

[11] O. Ceryan, et al. (2009). `Managing Demand and Supply for Multiple Products Through Dynamic Pricing and Capacity Flexibility'. Working paper .

[12] L. Chan, et al. (2006). `Pricing, production, and inventory policies for manufacturing with stochastic demand and discretionary sales'. Manufacturing & Service Operations Management 8(2):149-168.

[13] X. Chen, et al. (2010b). `Preservation of Submodularity under Infimal Convolution in Two Dimensional Space and its Applications'. Working paper .

[14] X. Chen & D. Simchi-Levi (2004a). `Coordinating Inventory Control and Pricing Strategies with Random Demand and Fixed Ordering Cost: The Finite Horizon Case'. Operations Research 52(6):887-896.

[15] X. Chen & D. Simchi-Levi (2004b). `Coordinating Inventory Control and Pricing Strategies with Random Demand and Fixed Ordering Cost: The In_nite Horizon Case'. Mathematics of Operations Research 29(3):698-723.

[16] A. J. Nevins (1966). `Some Effects of Uncertainty: Simulation of a Model of Price'. The Quarterly Journal of Economics 80(1):73-87.

[17] J. Thomas (1974). `Price and Production Decisions with Random Demand'. Operations Research 22(3):513-518.

[18] G. T. Thowsen (1975). `A dynamic, nonstationary inventory problem for a price/quantity setting Firm'. Naval Research Logistics Quarterly 22(3):461-476.

[19] W. T. Huh & G. Janakiraman (2008). `(s, S) Optimality in Joint Inventory-Pricing Control: An Alternate Approach'. Operations Research 56:783-790.

[20] L. Polatoglu & I. Sahin (2000). `Optimal procurement policies under price-dependent demand'.I nt. J. Production Economics 65:141-171.

[21] J. Song & Z. Xue (2007). `Demand Management and Inventory Control for Substitutable Products'.Working paper .

[22] R. Yin & K. Rajaram (2007). `Joint pricing and inventory control with a Markovian demand model'. European Journal of Operational Research 182(1):113-126.

[23] C. Poon, T & L. Choy, K & K. Cheng, C & I. Lao, S. (2010). `A real-time replenishment system for vending machine industry'. IEEE International Conference on Industrial Informatics (INDIN). 10.1109/INDIN.2010.5549432.

[24] Park, Yang-Byung & Yoon, Sung-Joon. (2012). `A Decision Support System for the Operations of Vending Machine Supply Chains in a Green Logistics Environment'. IE interfaces. 25. 338-346. 10.7232/IEIF.2012.25.3.338.

[25] Park, Yang-Byung & Yoo, Jun-Soo. (2012). `A heuristic for the inventory management of smart vending machine systems'. Journal of Industrial Engineering and Management. 5. 10.3926/jiem.587.

APPENDIX

DETAILED INFORMATION ABOUT THE PROBLEM

Table A.1. Details of Demand for Generated Instance

Item	Location	Price	Period	Normal (μ, σ)
Mobile Phone	Esenboğa Airport	1750	4	Normal(8 ,2)
Mobile Phone	Esenboğa Airport	2000	4	Normal(4 ,2)
Mobile Phone	Esenboğa Airport	2300	4	Normal(2 ,1)
Mobile Phone	Esenboğa Airport	1750	7	Normal(14 ,4)
Mobile Phone	Esenboğa Airport	2000	7	Normal(11 ,3)
Mobile Phone	Esenboğa Airport	2300	7	Normal(6 ,2)
Mobile Phone	Esenboğa Airport	1750	9	Normal(7 ,2)
Mobile Phone	Esenboğa Airport	2000	9	Normal(5 ,1)
Mobile Phone	Esenboğa Airport	2300	9	Normal(2 ,1)
Mobile Phone	Esenboğa Airport	1750	12	Normal(10 ,3)
Mobile Phone	Esenboğa Airport	2000	12	Normal(4 ,1)
Mobile Phone	Esenboğa Airport	2300	12	Normal(1 ,1)
Mobile Phone	Kentpark	1750	1	Normal(21 ,4)
Mobile Phone	Kentpark	2000	1	Normal(6 ,2)
Mobile Phone	Kentpark	2300	1	Normal(3 ,1)
Mobile Phone	Kentpark	1750	4	Normal(10 ,3)
Mobile Phone	Kentpark	2000	4	Normal(8 ,2)
Mobile Phone	Kentpark	2300	4	Normal(5 ,1)
Mobile Phone	Ankamall	1750	3	Normal(9 ,2)
Mobile Phone	Ankamall	2000	3	Normal(6 ,2)
Mobile Phone	Ankamall	2300	3	Normal(3 ,1)
Mobile Phone	Ankamall	1750	8	Normal(4 ,1)
Mobile Phone	Ankamall	2000	8	Normal(3 ,1)
Mobile Phone	Ankamall	2300	8	Normal(1 ,2)
Mobile Phone	Ankamall	1750	11	Normal(5 ,2)
Mobile Phone	Ankamall	2000	11	Normal(4 ,1)
Mobile Phone	Ankamall	2300	11	Normal(2 ,1)
Mobile Phone	Ankamall	1750	14	Normal(6 ,2)
Mobile Phone	Ankamall	2000	14	Normal(4 ,2)
Mobile Phone	Ankamall	2300	14	Normal(3 ,1)

Table 7.1 Continued

Mobile Phone	Ankamall	1750	16	Normal(8 ,2)
Mobile Phone	Ankamall	2000	16	Normal(5 ,3)
Mobile Phone	Ankamall	2300	16	Normal(3 ,1)
Mobile Phone	Ankamall	1750	17	Normal(4 ,1)
Mobile Phone	Ankamall	2000	17	Normal(2 ,1)
Mobile Phone	Ankamall	2300	17	Normal(1 ,1)
Mobile Phone	Metromall	1750	8	Normal(5 ,2)
Mobile Phone	Metromall	2000	8	Normal(4 ,1)
Mobile Phone	Metromall	2300	8	Normal(2 ,1)
Mobile Phone	Next Level	1750	4	Normal(3 ,1)
Mobile Phone	Next Level	2000	4	Normal(2 ,1)
Mobile Phone	Next Level	2300	4	Normal(1 ,1)
Mobile Phone	Next Level	1750	9	Normal(6 ,1)
Mobile Phone	Next Level	2000	9	Normal(5 ,1)
Mobile Phone	Next Level	2300	9	Normal(3 ,1)
Mobile Phone	Next Level	1750	15	Normal(5 ,1)
Mobile Phone	Next Level	2000	15	Normal(3 ,1)
Mobile Phone	Next Level	2300	15	Normal(2 ,1)
Mobile Phone	Göksu Park	1750	4	Normal(7 ,1)
Mobile Phone	Göksu Park	2000	4	Normal(5 ,1)
Mobile Phone	Göksu Park	2300	4	Normal(4 ,1)
Mobile Phone	Göksu Park	1750	16	Normal(9 ,3)
Mobile Phone	Göksu Park	2000	16	Normal(6 ,1)
Mobile Phone	Göksu Park	2300	16	Normal(4 ,2)
Mobile Phone	Altınpark	1750	7	Normal(3 ,1)
Mobile Phone	Altınpark	2000	7	Normal(2 ,1)
Mobile Phone	Altınpark	2300	7	Normal(1 ,1)
Mobile Phone	Altınpark	1750	11	Normal(7 ,2)
Mobile Phone	Altınpark	2000	11	Normal(5 ,1)
Mobile Phone	Altınpark	2300	11	Normal(4 ,1)
Ipad	Esenboğa Airport	2000	2	Normal(6 ,2)
Ipad	Esenboğa Airport	2400	2	Normal(5 ,2)
Ipad	Esenboğa Airport	2700	2	Normal(3 ,1)
Ipad	Esenboğa Airport	2000	6	Normal(3 ,1)
Ipad	Esenboğa Airport	2400	6	Normal(2 ,1)
Ipad	Esenboğa Airport	2700	6	Normal(1 ,1)
Ipad	Esenboğa Airport	2000	10	Normal(5 ,1)
Ipad	Esenboğa Airport	2400	10	Normal(3 ,1)

Table 7.1 Continued

Ipad	Esenboğa Airport	2700	10	Normal(2 ,1)
Ipad	Esenboğa Airport	2000	14	Normal(4 ,1)
Ipad	Esenboğa Airport	2400	14	Normal(2 ,1)
Ipad	Esenboğa Airport	2700	14	Normal(1 ,1)
Ipad	Next Level	2000	3	Normal(5 ,1)
Ipad	Next Level	2400	3	Normal(3 ,1)
Ipad	Next Level	2700	3	Normal(2 ,1)
Ipad	Next Level	2000	8	Normal(6 ,1)
Ipad	Next Level	2400	8	Normal(4 ,1)
Ipad	Next Level	2700	8	Normal(3 ,1)
Ipad	Ankamall	2000	2	Normal(8 ,1)
Ipad	Ankamall	2400	2	Normal(6 ,1)
Ipad	Ankamall	2700	2	Normal(3 ,1)
Ipad	Ankamall	2000	12	Normal(4 ,1)
Ipad	Ankamall	2400	12	Normal(2 ,1)
Ipad	Ankamall	2700	12	Normal(1 ,1)
Ipad	Ankamall	2000	17	Normal(5 ,1)
Ipad	Ankamall	2400	17	Normal(4 ,1)
Ipad	Ankamall	2700	17	Normal(3 ,1)
Ipad	Ankamall	2000	19	Normal(4 ,1)
Ipad	Ankamall	2400	19	Normal(2 ,1)
Ipad	Ankamall	2700	19	Normal(1 ,1)
Ipad	Ankamall	2000	21	Normal(7 ,3)
Ipad	Ankamall	2400	21	Normal(6 ,2)
Ipad	Ankamall	2700	21	Normal(4 ,1)
Ipad	Harikalar Diyari	2000	6	Normal(4 ,1)
Ipad	Harikalar Diyari	2400	6	Normal(2 ,1)
Ipad	Harikalar Diyari	2700	6	Normal(1 ,1)
Ipad	Kentpark	2000	5	Normal(6 ,1)
Ipad	Kentpark	2400	5	Normal(4 ,1)
Ipad	Kentpark	2700	5	Normal(3 ,1)
Ipad	Kentpark	2000	7	Normal(7 ,1)
Ipad	Kentpark	2400	7	Normal(6 ,1)
Ipad	Kentpark	2700	7	Normal(4 ,1)
Ipad	Kentpark	2000	11	Normal(5 ,2)
Ipad	Kentpark	2400	11	Normal(3 ,1)
Ipad	Kentpark	2700	11	Normal(2 ,1)
Ipad	Metromall	2000	6	Normal(5 ,2)

Table 7.1 Continued

Ipad	Metromall	2400	6	Normal(4 ,1)
Ipad	Metromall	2700	6	Normal(1 ,1)
Kindle	Next Level	700	3	Normal(4 ,1)
Kindle	Next Level	850	3	Normal(2 ,1)
Kindle	Next Level	1000	3	Normal(1 ,1)
Kindle	Next Level	700	7	Normal(5 ,1)
Kindle	Next Level	850	7	Normal(4 ,1)
Kindle	Next Level	1000	7	Normal(2 ,1)
Kindle	Next Level	700	13	Normal(4 ,1)
Kindle	Next Level	850	13	Normal(2 ,1)
Kindle	Next Level	1000	13	Normal(1 ,1)
Kindle	Next Level	700	18	Normal(7 ,1)
Kindle	Next Level	850	18	Normal(6 ,1)
Kindle	Next Level	1000	18	Normal(4 ,1)
Kindle	Next Level	700	20	Normal(4 ,2)
Kindle	Next Level	850	20	Normal(2 ,1)
Kindle	Next Level	1000	20	Normal(1 ,1)
Kindle	Esenboğa Airport	700	2	Normal(6 ,3)
Kindle	Esenboğa Airport	850	2	Normal(4 ,1)
Kindle	Esenboğa Airport	1000	2	Normal(3 ,1)
Kindle	Esenboğa Airport	700	6	Normal(7 ,2)
Kindle	Esenboğa Airport	850	6	Normal(6 ,1)
Kindle	Esenboğa Airport	1000	6	Normal(4 ,1)
Kindle	Esenboğa Airport	700	8	Normal(5 ,2)
Kindle	Esenboğa Airport	850	8	Normal(3 ,2)
Kindle	Esenboğa Airport	1000	8	Normal(2 ,1)
Kindle	Kentpark	700	4	Normal(4 ,1)
Kindle	Kentpark	850	4	Normal(2 ,1)
Kindle	Kentpark	1000	4	Normal(1 ,1)
Kindle	Kentpark	700	6	Normal(6 ,1)
Kindle	Kentpark	850	6	Normal(4 ,1)
Kindle	Kentpark	1000	6	Normal(3 ,1)
Wallet	Harikalar Diyari	250	1	Normal(10 ,1)
Wallet	Harikalar Diyari	300	1	Normal(9 ,2)
Wallet	Harikalar Diyari	400	1	Normal(7 ,1)
Wallet	Harikalar Diyari	250	5	Normal(12 ,3)
Wallet	Harikalar Diyari	300	5	Normal(8 ,2)
Wallet	Harikalar Diyari	400	5	Normal(5 ,1)

Table 7.1 Continued

Wallet	Harikalar Diyari	250	8	Normal(9 ,2)
Wallet	Harikalar Diyari	300	8	Normal(6 ,1)
Wallet	Harikalar Diyari	400	8	Normal(4 ,1)
Wallet	Harikalar Diyari	250	14	Normal(6 ,2)
Wallet	Harikalar Diyari	300	14	Normal(4 ,1)
Wallet	Harikalar Diyari	400	14	Normal(2 ,1)
Wallet	Harikalar Diyari	250	18	Normal(7 ,2)
Wallet	Harikalar Diyari	300	18	Normal(6 ,1)
Wallet	Harikalar Diyari	400	18	Normal(3 ,1)
Wallet	Harikalar Diyari	250	21	Normal(9 ,3)
Wallet	Harikalar Diyari	300	21	Normal(8 ,1)
Wallet	Harikalar Diyari	400	21	Normal(4 ,2)
Wallet	Harikalar Diyari	250	6	Normal(11 ,2)
Wallet	Harikalar Diyari	300	6	Normal(9 ,2)
Wallet	Harikalar Diyari	400	6	Normal(7 ,1)
Wallet	Altınpark	250	9	Normal(9 ,1)
Wallet	Altınpark	300	9	Normal(6 ,1)
Wallet	Altınpark	400	9	Normal(5 ,1)
Wallet	Altınpark	250	12	Normal(8 ,1)
Wallet	Altınpark	300	12	Normal(6 ,1)
Wallet	Altınpark	400	12	Normal(3 ,1)
Wallet	Altınpark	250	15	Normal(9 ,2)
Wallet	Altınpark	300	15	Normal(6 ,2)
Wallet	Altınpark	400	15	Normal(5 ,1)
Wallet	Altınpark	250	19	Normal(10 ,3)
Wallet	Altınpark	300	19	Normal(8 ,1)
Wallet	Altınpark	400	19	Normal(6 ,2)
Wallet	Göksu Park	250	2	Normal(10 ,2)
Wallet	Göksu Park	300	2	Normal(9 ,1)
Wallet	Göksu Park	400	2	Normal(7 ,1)
Wallet	Göksu Park	250	6	Normal(8 ,1)
Wallet	Göksu Park	300	6	Normal(5 ,1)
Wallet	Göksu Park	400	6	Normal(4 ,1)
Wallet	Ankamall	250	1	Normal(11 ,1)
Wallet	Ankamall	300	1	Normal(9 ,1)
Wallet	Ankamall	400	1	Normal(8 ,1)
Wallet	Esenboğa Airport	250	4	Normal(8 ,1)
Wallet	Esenboğa Airport	300	4	Normal(7 ,1)

Table 7.1 Continued

Wallet	Esenboğa Airport	400	4	Normal(4 ,1)
Wallet	Esenboğa Airport	250	8	Normal(9 ,1)
Wallet	Esenboğa Airport	300	8	Normal(6 ,1)
Wallet	Esenboğa Airport	400	8	Normal(2 ,1)
Wallet	Metromall	250	9	Normal(11 ,2)
Wallet	Metromall	300	9	Normal(6 ,1)
Wallet	Metromall	400	9	Normal(4 ,1)
Wallet	Metromall	250	11	Normal(9 ,2)
Wallet	Metromall	300	11	Normal(5 ,2)
Wallet	Metromall	400	11	Normal(4 ,1)
Wallet	Metromall	250	14	Normal(6 ,1)
Wallet	Metromall	300	14	Normal(4 ,1)
Wallet	Metromall	400	14	Normal(3 ,1)
Wallet	Kentpark	250	3	Normal(10 ,1)
Wallet	Kentpark	300	3	Normal(8 ,1)
Wallet	Kentpark	400	3	Normal(6 ,1)
Wallet	Kentpark	250	6	Normal(8 ,1)
Wallet	Kentpark	300	6	Normal(7 ,1)
Wallet	Kentpark	400	6	Normal(5 ,1)
Wallet	Kentpark	250	12	Normal(10 ,2)
Wallet	Kentpark	300	12	Normal(6 ,1)
Wallet	Kentpark	400	12	Normal(9 ,1)
Earphone	Göksu Park	200	5	Normal(16 ,3)
Earphone	Göksu Park	250	5	Normal(13 ,2)
Earphone	Göksu Park	350	5	Normal(8 ,1)
Earphone	Göksu Park	200	8	Normal(20 ,3)
Earphone	Göksu Park	250	8	Normal(5 ,1)
Earphone	Göksu Park	350	8	Normal(4 ,1)
Earphone	Göksu Park	200	14	Normal(12 ,1)
Earphone	Göksu Park	250	14	Normal(6 ,1)
Earphone	Göksu Park	350	14	Normal(3 ,1)
Earphone	Göksu Park	200	17	Normal(16 ,2)
Earphone	Göksu Park	250	17	Normal(14 ,1)
Earphone	Göksu Park	350	17	Normal(7 ,1)
Earphone	Göksu Park	200	20	Normal(9 ,2)
Earphone	Göksu Park	250	20	Normal(5 ,2)
Earphone	Göksu Park	350	20	Normal(1 ,1)
Earphone	Göksu Park	200	22	Normal(14 ,2)

Table 7.1 Continued

Earphone	Göksu Park	250	22	Normal(5 ,1)
Earphone	Göksu Park	350	22	Normal(3 ,1)
Earphone	Esenboğa Airport	200	1	Normal(10 ,1)
Earphone	Esenboğa Airport	250	1	Normal(6 ,1)
Earphone	Esenboğa Airport	350	1	Normal(3 ,1)
Earphone	Esenboğa Airport	200	4	Normal(9 ,1)
Earphone	Esenboğa Airport	250	4	Normal(4 ,1)
Earphone	Esenboğa Airport	350	4	Normal(1 ,1)
Earphone	Esenboğa Airport	200	6	Normal(12 ,1)
Earphone	Esenboğa Airport	250	6	Normal(5 ,1)
Earphone	Esenboğa Airport	350	6	Normal(4 ,1)
Earphone	Esenboğa Airport	200	12	Normal(17 ,4)
Earphone	Esenboğa Airport	250	12	Normal(10 ,3)
Earphone	Esenboğa Airport	350	12	Normal(8 ,1)
Earphone	Esenboğa Airport	200	16	Normal(6 ,2)
Earphone	Esenboğa Airport	250	16	Normal(4 ,1)
Earphone	Esenboğa Airport	350	16	Normal(1 ,1)
Earphone	Kentpark	200	5	Normal(9 ,2)
Earphone	Kentpark	250	5	Normal(6 ,1)
Earphone	Kentpark	350	5	Normal(4 ,1)
Earphone	Kentpark	200	12	Normal(10 ,2)
Earphone	Kentpark	250	12	Normal(8 ,2)
Earphone	Kentpark	350	12	Normal(5 ,1)
Earphone	Ankamall	200	6	Normal(9 ,1)
Earphone	Ankamall	250	6	Normal(5 ,1)
Earphone	Ankamall	350	6	Normal(3 ,1)
Battery	Esenboğa Airport	60	1	Normal(16 ,1)
Battery	Esenboğa Airport	80	1	Normal(14 ,1)
Battery	Esenboğa Airport	100	1	Normal(8 ,1)
Battery	Esenboğa Airport	60	3	Normal(25 ,3)
Battery	Esenboğa Airport	80	3	Normal(21 ,2)
Battery	Esenboğa Airport	100	3	Normal(9 ,1)
Battery	Esenboğa Airport	60	5	Normal(20 ,2)
Battery	Esenboğa Airport	80	5	Normal(8 ,2)
Battery	Esenboğa Airport	100	5	Normal(5 ,1)
Battery	Esenboğa Airport	60	6	Normal(30 ,4)
Battery	Esenboğa Airport	80	6	Normal(24 ,2)
Battery	Esenboğa Airport	100	6	Normal(21 ,1)

Table 7.1 Continued

Battery	Esenboğa Airport	60	9	Normal(9 ,2)
Battery	Esenboğa Airport	80	9	Normal(6 ,1)
Battery	Esenboğa Airport	100	9	Normal(5 ,1)
Battery	Esenboğa Airport	60	10	Normal(17 ,3)
Battery	Esenboğa Airport	80	10	Normal(14 ,3)
Battery	Esenboğa Airport	100	10	Normal(8 ,1)
Battery	Esenboğa Airport	60	12	Normal(9 ,1)
Battery	Esenboğa Airport	80	12	Normal(5 ,1)
Battery	Esenboğa Airport	100	12	Normal(4 ,1)
Battery	Esenboğa Airport	60	14	Normal(22 ,2)
Battery	Esenboğa Airport	80	14	Normal(18 ,1)
Battery	Esenboğa Airport	100	14	Normal(9 ,1)
Battery	Esenboğa Airport	60	15	Normal(26 ,3)
Battery	Esenboğa Airport	80	15	Normal(15 ,2)
Battery	Esenboğa Airport	100	15	Normal(11 ,2)
Battery	Esenboğa Airport	60	17	Normal(10 ,2)
Battery	Esenboğa Airport	80	17	Normal(8 ,1)
Battery	Esenboğa Airport	100	17	Normal(4 ,1)
Battery	Esenboğa Airport	60	19	Normal(13 ,1)
Battery	Esenboğa Airport	80	19	Normal(9 ,1)
Battery	Esenboğa Airport	100	19	Normal(8 ,1)
Battery	Esenboğa Airport	60	20	Normal(6 ,1)
Battery	Esenboğa Airport	80	20	Normal(3 ,1)
Battery	Esenboğa Airport	100	20	Normal(1 ,1)
Battery	Esenboğa Airport	60	22	Normal(17 ,3)
Battery	Esenboğa Airport	80	22	Normal(15 ,3)
Battery	Esenboğa Airport	100	22	Normal(13 ,2)
Battery	Esenboğa Airport	60	23	Normal(24 ,4)
Battery	Esenboğa Airport	80	23	Normal(23 ,3)
Battery	Esenboğa Airport	100	23	Normal(18 ,3)
Battery	Next Level	60	3	Normal(9 ,2)
Battery	Next Level	80	3	Normal(6 ,1)
Battery	Next Level	100	3	Normal(4 ,1)
Battery	Next Level	60	7	Normal(11 ,2)
Battery	Next Level	80	7	Normal(6 ,1)
Battery	Next Level	100	7	Normal(3 ,1)
Battery	Next Level	60	11	Normal(10 ,3)
Battery	Next Level	80	11	Normal(8 ,1)

Table 7.1 Continued

Battery	Next Level	100	11	Normal(5 ,2)
Battery	Next Level	60	14	Normal(14 ,2)
Battery	Next Level	80	14	Normal(10 ,1)
Battery	Next Level	100	14	Normal(8 ,1)
Battery	Next Level	60	20	Normal(12 ,3)
Battery	Next Level	80	20	Normal(10 ,1)
Battery	Next Level	100	20	Normal(9 ,2)
Battery	Ankamall	60	2	Normal(9 ,2)
Battery	Ankamall	80	2	Normal(6 ,1)
Battery	Ankamall	100	2	Normal(5 ,1)
Battery	Ankamall	60	6	Normal(10 ,3)
Battery	Ankamall	80	6	Normal(8 ,2)
Battery	Ankamall	100	6	Normal(7 ,1)
Battery	Ankamall	60	10	Normal(14 ,2)
Battery	Ankamall	80	10	Normal(12 ,2)
Battery	Ankamall	100	10	Normal(9 ,1)
Battery	Ankamall	60	13	Normal(15 ,3)
Battery	Ankamall	80	13	Normal(13 ,2)
Battery	Ankamall	100	13	Normal(5 ,1)
Battery	Ankamall	60	18	Normal(18 ,2)
Battery	Ankamall	80	18	Normal(7 ,1)
Battery	Ankamall	100	18	Normal(2 ,1)
Battery	Metromall	60	4	Normal(15 ,3)
Battery	Metromall	80	4	Normal(14 ,2)
Battery	Metromall	100	4	Normal(12 ,1)
Battery	Metromall	60	5	Normal(9 ,2)
Battery	Metromall	80	5	Normal(7 ,1)
Battery	Metromall	100	5	Normal(4 ,1)
Battery	Metromall	60	9	Normal(11 ,1)
Battery	Metromall	80	9	Normal(7 ,1)
Battery	Metromall	100	9	Normal(5 ,1)
Battery	Metromall	60	12	Normal(14 ,1)
Battery	Metromall	80	12	Normal(12 ,1)
Battery	Metromall	100	12	Normal(9 ,1)
Battery	Kentpark	60	7	Normal(15 ,1)
Battery	Kentpark	80	7	Normal(8 ,1)
Battery	Kentpark	100	7	Normal(6 ,1)
Battery	Kentpark	60	9	Normal(9 ,1)

Table 7.1 Continued

Battery	Kentpark	80	9	Normal(6 ,1)
Battery	Kentpark	100	9	Normal(3 ,1)
Battery	Kentpark	60	15	Normal(19 ,1)
Battery	Kentpark	80	15	Normal(17 ,1)
Battery	Kentpark	100	15	Normal(5 ,1)
Battery	Kentpark	60	18	Normal(14 ,1)
Battery	Kentpark	80	18	Normal(10 ,1)
Battery	Kentpark	100	18	Normal(8 ,1)
Battery	Kentpark	60	20	Normal(11 ,2)
Battery	Kentpark	80	20	Normal(6 ,1)
Battery	Kentpark	100	20	Normal(5 ,2)
Battery	Kentpark	60	21	Normal(9 ,3)
Battery	Kentpark	80	21	Normal(7 ,2)
Battery	Kentpark	100	21	Normal(6 ,1)
Battery	Kentpark	60	23	Normal(12 ,2)
Battery	Kentpark	80	23	Normal(10 ,1)
Battery	Kentpark	100	23	Normal(5 ,2)
Battery	Göksu Park	60	6	Normal(8 ,2)
Battery	Göksu Park	80	6	Normal(7 ,2)
Battery	Göksu Park	100	6	Normal(5 ,2)
Battery	Göksu Park	60	9	Normal(11 ,1)
Battery	Göksu Park	80	9	Normal(8 ,1)
Battery	Göksu Park	100	9	Normal(6 ,1)
Battery	Altınpark	60	11	Normal(9 ,1)
Battery	Altınpark	80	11	Normal(7 ,1)
Battery	Altınpark	100	11	Normal(7 ,1)
Battery	Altınpark	60	14	Normal(14 ,1)
Battery	Altınpark	80	14	Normal(8 ,1)
Battery	Altınpark	100	14	Normal(7 ,1)
Battery	Altınpark	60	17	Normal(12 ,1)
Battery	Altınpark	80	17	Normal(10 ,1)
Battery	Altınpark	100	17	Normal(10 ,1)
Battery	Harikalar Diyari	60	10	Normal(8 ,2)
Battery	Harikalar Diyari	80	10	Normal(7 ,1)
Battery	Harikalar Diyari	100	10	Normal(5 ,2)
Battery	Harikalar Diyari	60	16	Normal(8 ,2)
Battery	Harikalar Diyari	80	16	Normal(6 ,2)
Battery	Harikalar Diyari	100	16	Normal(5 ,1)

Table 7.1 Continued

Battery	Harikalar Diyari	60	21	Normal(11 ,1)
Battery	Harikalar Diyari	80	21	Normal(9 ,2)
Battery	Harikalar Diyari	100	21	Normal(8 ,1)
Battery	Harikalar Diyari	60	23	Normal(10 ,1)
Battery	Harikalar Diyari	80	23	Normal(6 ,1)
Battery	Harikalar Diyari	100	23	Normal(5 ,1)
Watch	Next Level	500	2	Normal(10 ,1)
Watch	Next Level	600	2	Normal(9 ,1)
Watch	Next Level	800	2	Normal(7 ,1)
Watch	Ankamall	500	7	Normal(8 ,1)
Watch	Ankamall	600	7	Normal(7 ,1)
Watch	Ankamall	800	7	Normal(7 ,1)
Watch	Ankamall	500	10	Normal(8 ,1)
Watch	Ankamall	600	10	Normal(6 ,1)
Watch	Ankamall	800	10	Normal(5 ,1)
Watch	Ankamall	500	12	Normal(7 ,1)
Watch	Ankamall	600	12	Normal(7 ,1)
Watch	Ankamall	800	12	Normal(4 ,1)
Watch	Ankamall	500	17	Normal(5 ,1)
Watch	Ankamall	600	17	Normal(4 ,1)
Watch	Ankamall	800	17	Normal(2 ,1)
Watch	Ankamall	500	20	Normal(6 ,2)
Watch	Ankamall	600	20	Normal(6 ,1)
Watch	Ankamall	800	20	Normal(3 ,1)
Watch	Esenboğa Airport	500	4	Normal(4 ,2)
Watch	Esenboğa Airport	600	4	Normal(3 ,2)
Watch	Esenboğa Airport	800	4	Normal(1 ,1)
Watch	Esenboğa Airport	500	5	Normal(5 ,1)
Watch	Esenboğa Airport	600	5	Normal(4 ,1)
Watch	Esenboğa Airport	800	5	Normal(4 ,1)
Watch	Esenboğa Airport	500	10	Normal(4 ,1)
Watch	Esenboğa Airport	600	10	Normal(2 ,1)
Watch	Esenboğa Airport	800	10	Normal(1 ,1)
Watch	Esenboğa Airport	500	13	Normal(6 ,2)
Watch	Esenboğa Airport	600	13	Normal(6 ,1)
Watch	Esenboğa Airport	800	13	Normal(4 ,2)
Watch	Metromall	500	3	Normal(5 ,2)
Watch	Metromall	600	3	Normal(4 ,2)

Table 7.1 Continued

Watch	Metromall	800	3	Normal(3 ,1)
Watch	Metromall	500	15	Normal(5 ,1)
Watch	Metromall	600	15	Normal(5 ,1)
Watch	Metromall	800	15	Normal(4 ,1)
Watch	Metromall	500	18	Normal(6 ,1)
Watch	Metromall	600	18	Normal(4 ,1)
Watch	Metromall	800	18	Normal(1 ,1)
Watch	Metromall	500	20	Normal(7 ,1)
Watch	Metromall	600	20	Normal(7 ,1)
Watch	Metromall	800	20	Normal(4 ,1)
Watch	Kentpark	500	6	Normal(8 ,1)
Watch	Kentpark	600	6	Normal(8 ,1)
Watch	Kentpark	800	6	Normal(6 ,1)
Watch	Kentpark	500	9	Normal(7 ,1)
Watch	Kentpark	600	9	Normal(5 ,1)
Watch	Kentpark	800	9	Normal(3 ,1)
Watch	Kentpark	500	10	Normal(7 ,2)
Watch	Kentpark	600	10	Normal(6 ,1)
Watch	Kentpark	800	10	Normal(4 ,1)
Watch	Kentpark	500	14	Normal(6 ,1)
Watch	Kentpark	600	14	Normal(4 ,1)
Watch	Kentpark	800	14	Normal(4 ,1)
Watch	Kentpark	500	19	Normal(5 ,2)
Watch	Kentpark	600	19	Normal(3 ,1)
Watch	Kentpark	800	19	Normal(2 ,1)
Watch	Göksu Park	500	6	Normal(4 ,1)
Watch	Göksu Park	600	6	Normal(3 ,2)
Watch	Göksu Park	800	6	Normal(3 ,1)
Watch	Göksu Park	500	8	Normal(6 ,1)
Watch	Göksu Park	600	8	Normal(5 ,1)
Watch	Göksu Park	800	8	Normal(3 ,1)
Watch	Göksu Park	500	11	Normal(5 ,1)
Watch	Göksu Park	600	11	Normal(4 ,1)
Watch	Göksu Park	800	11	Normal(4 ,1)
Watch	Göksu Park	500	15	Normal(6 ,1)
Watch	Göksu Park	600	15	Normal(4 ,1)
Watch	Göksu Park	800	15	Normal(3 ,1)
Watch	Harikalar Diyari	500	2	Normal(7 ,1)

Table 7.1 Continued

Watch	Harikalar Diyari	600	2	Normal(5 ,1)
Watch	Harikalar Diyari	800	2	Normal(2 ,1)
Watch	Altınpark	500	7	Normal(6 ,2)
Watch	Altınpark	600	7	Normal(5 ,1)
Watch	Altınpark	800	7	Normal(5 ,2)
Gold (gr)	Ankamall	350	3	Normal(5 ,2)
Gold (gr)	Ankamall	400	3	Normal(3 ,1)
Gold (gr)	Ankamall	500	3	Normal(2 ,1)
Gold (gr)	Ankamall	350	4	Normal(5 ,1)
Gold (gr)	Ankamall	400	4	Normal(5 ,1)
Gold (gr)	Ankamall	500	4	Normal(4 ,1)
Gold (gr)	Ankamall	350	8	Normal(6 ,1)
Gold (gr)	Ankamall	400	8	Normal(4 ,1)
Gold (gr)	Ankamall	500	8	Normal(3 ,1)
Gold (gr)	Altınpark	350	4	Normal(5 ,1)
Gold (gr)	Altınpark	400	4	Normal(4 ,1)
Gold (gr)	Altınpark	500	4	Normal(4 ,1)
Gold (gr)	Altınpark	350	7	Normal(6 ,1)
Gold (gr)	Altınpark	400	7	Normal(4 ,1)
Gold (gr)	Altınpark	500	7	Normal(3 ,1)
Gold (gr)	Altınpark	350	9	Normal(6 ,1)
Gold (gr)	Altınpark	400	9	Normal(5 ,1)
Gold (gr)	Altınpark	500	9	Normal(5 ,1)
Gold (gr)	Altınpark	350	12	Normal(5 ,1)
Gold (gr)	Altınpark	400	12	Normal(4 ,1)
Gold (gr)	Altınpark	500	12	Normal(3 ,1)
Gold (gr)	Altınpark	350	15	Normal(6 ,2)
Gold (gr)	Altınpark	400	15	Normal(5 ,1)
Gold (gr)	Altınpark	500	15	Normal(4 ,2)
Gold (gr)	Altınpark	350	18	Normal(7 ,1)
Gold (gr)	Altınpark	400	18	Normal(5 ,1)
Gold (gr)	Altınpark	500	18	Normal(2 ,1)
Gold (gr)	Altınpark	350	20	Normal(6 ,2)
Gold (gr)	Altınpark	400	20	Normal(5 ,2)
Gold (gr)	Altınpark	500	20	Normal(3 ,1)
Gold (gr)	Altınpark	350	21	Normal(5 ,1)
Gold (gr)	Altınpark	400	21	Normal(4 ,1)
Gold (gr)	Altınpark	500	21	Normal(1 ,1)

Table 7.1 Continued

Gold (gr)	Göksu Park	350	6	Normal(7 ,1)
Gold (gr)	Göksu Park	400	6	Normal(6 ,1)
Gold (gr)	Göksu Park	500	6	Normal(6 ,1)
Gold (gr)	Göksu Park	350	7	Normal(5 ,2)
Gold (gr)	Göksu Park	400	7	Normal(5 ,1)
Gold (gr)	Göksu Park	500	7	Normal(4 ,1)
Gold (gr)	Göksu Park	350	10	Normal(7 ,1)
Gold (gr)	Göksu Park	400	10	Normal(6 ,2)
Gold (gr)	Göksu Park	500	10	Normal(4 ,1)
Gold (gr)	Göksu Park	350	12	Normal(6 ,1)
Gold (gr)	Göksu Park	400	12	Normal(4 ,1)
Gold (gr)	Göksu Park	500	12	Normal(3 ,1)
Gold (gr)	Göksu Park	350	15	Normal(7 ,1)
Gold (gr)	Göksu Park	400	15	Normal(5 ,1)
Gold (gr)	Göksu Park	500	15	Normal(4 ,1)
Gold (gr)	Göksu Park	350	17	Normal(9 ,1)
Gold (gr)	Göksu Park	400	17	Normal(7 ,1)
Gold (gr)	Göksu Park	500	17	Normal(7 ,1)
Gold (gr)	Göksu Park	350	18	Normal(5 ,1)
Gold (gr)	Göksu Park	400	18	Normal(5 ,1)
Gold (gr)	Göksu Park	500	18	Normal(2 ,1)
Gold (gr)	Göksu Park	350	20	Normal(6 ,1)
Gold (gr)	Göksu Park	400	20	Normal(5 ,1)
Gold (gr)	Göksu Park	500	20	Normal(5 ,1)
Gold (gr)	Göksu Park	350	22	Normal(8 ,1)
Gold (gr)	Göksu Park	400	22	Normal(8 ,2)
Gold (gr)	Göksu Park	500	22	Normal(4 ,1)