ORDER PICKING ORIENTED STORAGE ASSIGNMENT PROBLEM IN A VERTICAL LIFT MODULE SYSTEM

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ABSTRACT

ORDER PICKING ORIENTED STORAGE ASSIGNMENT PROBLEM IN A VERTICAL LIFT MODULE SYSTEM

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Since the advancements in technology paved the way for an increased consumer-manufacturer interaction, companies are forced to adapt a mass customization philosophy in their production operations. This philosophy requires a higher variation of raw materials to be stored by the manufacturer to fill the customer orders on time. However, this increased variation in the inventory will also mean increased requirements for the storage area. Using automated storage and retrieval systems (AS/RS) is one way of using the volume in warehouse buildings more efficiently. By storing trays into the shelves inside a shuttle, vertical lift modules (VLMs) are among the AS/RSs promising more dense storage. These units can also be combined and used as a system called "VLM Pod". Although using a VLM helps in using the volume more efficiently, retrieval of the trays in the VLM units may take some time and cause unwanted waits.

This study analyses a design-to-order manufacturing company's transaction history and discusses the factors that make the responsiveness of a warehouse important in such an environment. Then, the throughput related VLM decisions are discussed.

With the help of the observations from a time study and the transaction history of the company, various combinations of those throughput related decisions are simulated. After these considerations, an integer linear programming model is proposed for assigning parts to trays and trays to shelves in VLM units. The proposed model and its variants are used in a computational experiment on small data sets sampled from the company's transaction records. The optimal solutions for the small data sets are used in a picking simulation. According to the picking simulation with the small data sets, the proposed model yielded an average of 10% improvement in the completion time of all pick tasks, while the number of tray retrievals during picking is reduced by approximately 65%, compared to the current system at the analyzed company.

Keywords: vertical lift module, storage assignment, correlated item storage, mathematical programming, simulation

ÖZ

BİR DİKEY DEPOLAMA ÜNİTESİ SİSTEMİNDE SİPARİŞ TOPLAMA ODAKLI ÜRÜN YERLEŞTİRME PROBLEMİ

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Teknolojideki gelişmeler sayesinde artan müşteri-tedarikçi ilişkileri ile firmalar günümüzde sipariş usulü kitle pazarlamacılığı yaklaşımı ile üretim yapmaya zorlanmaktadır. Bu yaklaşım ile üreticiler, müşteri taleplerini zamanında karşılamak için yüksek çeşitlilikte hammadde depolamak zorunda kalmaktadır. Aynı zamanda, artan hammadde çeşitliliği, artan depo alanı ihtiyacı anlamına gelmektedir. Otomatik yerleştirme ve toplama sistemlerini kullanmak, ambar binalarındaki hacmi daha verimli kullanma yöntemlerinden biridir. Bir otomatik yerleştirme ve toplama sistemi olan dikey depolama üniteleri, tepsileri dikey bir kule içerisindeki raflara yerleştirerek yoğun depolama vadetmektedir. Ayrıca, bu üniteler bir arada, "Pod" adı verilen sistemler hâlinde de kullanılabilmektedir. Dikey depolama ünitelerini kullanmak mevcut hacmi verimli kullanmayı sağlarken, bir yandan da rafların getirilmesi sırasında istenmeyen bekleme sürelerine yol açabilir.

Bu çalışma, siparişe göre tasarım yöntemi ile imalat yapan bir firmanın geçmiş işlem kayıtlarını inceleyerek, benzer ortamlarda ambarların hızlı yanıt verebilirliğinin önemini arttıran etkenleri tartışmaktadır. Sonrasında, dikey depolama "Pod"larının çıktı performansını etkileyen kararlar tartışılmaktadır. Sistemin zaman etüdüyle elde edilen bilgiler ve şirketin işlem geçmişi yardımı ile farklı dikey depolama "Pod" kararları simüle edilmektedir. Elde edilen bulgulardan yararlanarak; parçaları tepsilere, tepsileri de raf ve dikey depolama ünitelerine atayacak bir tamsayılı doğrusal programlama modeli önerilmektedir. Önerilen model ve onun farklı biçimleri, bahsedilen firmanın kayıtlarından örneklenen küçük veri kümeleri ile çalıştırılmıştır. Buradan elde edilen en iyi çözümlerin performansı, bir sipariş toplama süreci simüle edilerek mevcut sistemle kıyaslanmış, önerilen çözümün denenen örneklerdeki işlerin bitiş zamanını %10 öne çektiği ve sipariş toplama sırasındaki tepsi çağırma sayısını %65 oranında azalttığı gözlemlenmiştir.

Anahtar Kelimeler: dikey depolama ünitesi, lokasyon atama, korelasyonlu ürün depolama, matematik programlama, simülasyon

To my family

for their support

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ABBREVIATIONS	
AE	Analyzed environment. Refers to the manufacturing firm that have provided its operational history data to be analyzed by the author. Real-life problem data sets are also obtained from this company and time studies had been conducted there.
AS/RS	Automated storage and retrieval systems
BOM	Bill of materials
CBS	Class based storage
COI	Cube-per-order index
ERP	Enterprise resource planning software
I/O	Input and output
LP	Linear programming
SKU	Stock keeping unit, refers to a distinct part type and used interchangeably with the word "part"
VLM	Vertical lift module

CHAPTER 1

INTRODUCTION

In recent years, increased technology usage and globalization have made the competition in markets more intense. First, there is an increase in the buyers' ability to negotiate on the prices and reach other alternatives. Moreover, there is a trend towards smaller lot sizes and higher customization in manufacturing operations. These factors force companies to reduce operational costs and response times to stay in the game. Therefore, improving productivity in warehousing operations is more important than before (Le-Duc and de Koster, 2007).

In such market conditions, automated storage and retrieval systems (AS/RSs), such as Vertical Carousels and Vertical Lift Modules (VLM) are becoming popular solutions to the problem of increasing the throughput and space efficiency in warehouses (MHI, 2012).

Vertical carousels and VLMs are mini-load parts-to-picker AS/RSs where parts are presented to the picker in containers. All the containers in vertical carousels move together, whereas VLMs have a lift system that retrieves or stores containers independently (see Figure 1.1). They both provide space efficiency compared to the traditional storage racks. However, as noted by Romaine (2004) and Jacobs et al. (2000), throughput advantages of these systems highly depend on the storage assignments of parts since the tray retrieval operations take a significant time and the number of tray retrievals in a picking task depends on the storage locations of the parts.

In the literature, there are many studies about vertical and horizontal carousel systems. However, only a few studies on VLMs exist (Dukic et al., 2015; Nicolas et al., 2018). This study aims to fill this gap by discussing the decision alternatives impacting the

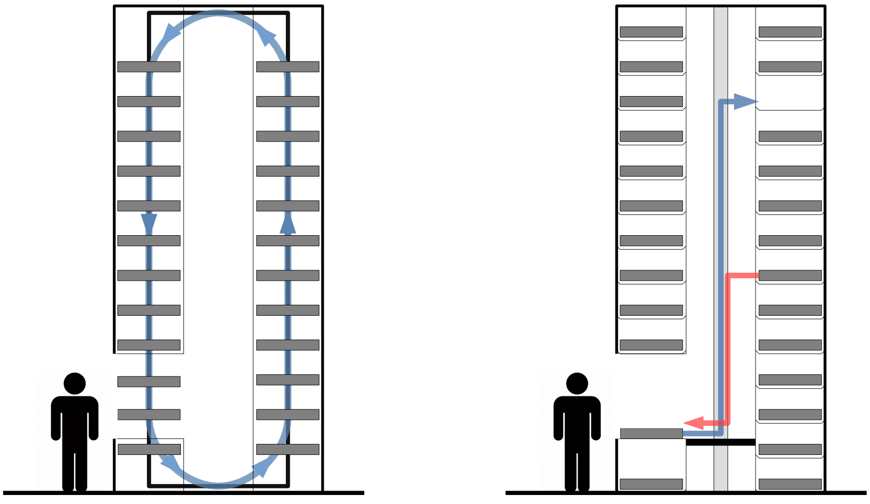

Figure 1.1: (a): Representation of a vertical carousel. (b): Representation of a vertical lift module.

system throughput in VLM pods and by proposing a storage assignment model to keep throughput of these systems high in a manufacturing company's warehouse. Besides, the proposed procedure will be tested on real-life data from a design-to-order manufacturing firm's warehouse.

Chapter 2 describes the analyses done on a manufacturing company's transaction history and highlights the business pains. Chapter 2 continues with the time study observations and discussion of the factors that make focusing on the throughput of a VLM system important. In Chapter 3, throughput related decisions in a VLM pod are discussed and analyzed in a VLM pod picking simulation and compared to make suggestions for the decision makers. After the simulation, the importance of the storage assignment problem in a VLM pod is emphasized, and the problem is described. Then, a review of the related literature is presented in Chapter 4, followed by the mathematical model for the problem in Chapter 5. Afterwards, Chapter 6 deals with the computational studies on the model using real-life data in an experiment setup, validates our model by simulating it using the time study observations and then discusses some options that may make the solution process faster. Finally, our findings and potential topics for future studies are discussed in Chapter 7.

CHAPTER 2

BACKGROUND INFORMATION

Warehousing operations can be categorized as receiving, storage, picking, and shipment in chronological order of occurrence (Çelik, 2009). First, materials to be stored are received from their sources, such as manufacturing or purchasing. Then, they are controlled according to the respective quality measures. After the quality control step, parts are stored with respect to the warehouse's location assignment method. Picking activities comprise collecting the demanded items from their locations. Following activities, such as ensuring the correctness of collected quantities, packing the items as ordered, and loading the vehicles are considered as shipment activities. These operations are summarized in Figure 2.1.

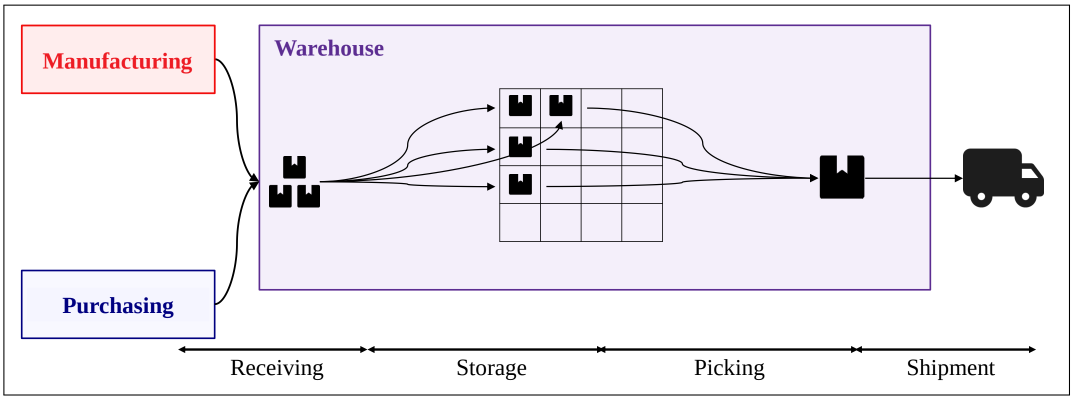

Figure 2.1: A basic representation of the operations during a part's life cycle at a typical production facility's warehouse.

In the literature, order picking is often defined as the most demanding activity among all warehouse operations, both in terms of cost and time (de Koster et al., 2007; Li et al., 2017; Sgarbossa et al., 2019; Tompkins et al., 2010). Therefore, as also stated by Frazelle (2001), order picking has great potential for increased productivity in a warehouse. Moreover, the current market conditions force suppliers to reduce their cycle times and increase the necessity of higher efficiency levels in order picking operations.

On the other hand, the physical area is still a constraint for warehouse managers. To use the area efficiently, warehouses generally consist of various sections with different physical attributes, each suitable for efficient storage of a different-sized set of parts. These sections often include traditional racks, narrow locations for small parts, automated storage and retrieval system (AS/RS) setups. A generalized representation of the operations in such warehouses can be seen in Figure 2.2, where a section is dedicated to a specific AS/RS setup, Vertical Lift Module (VLM) Pod.

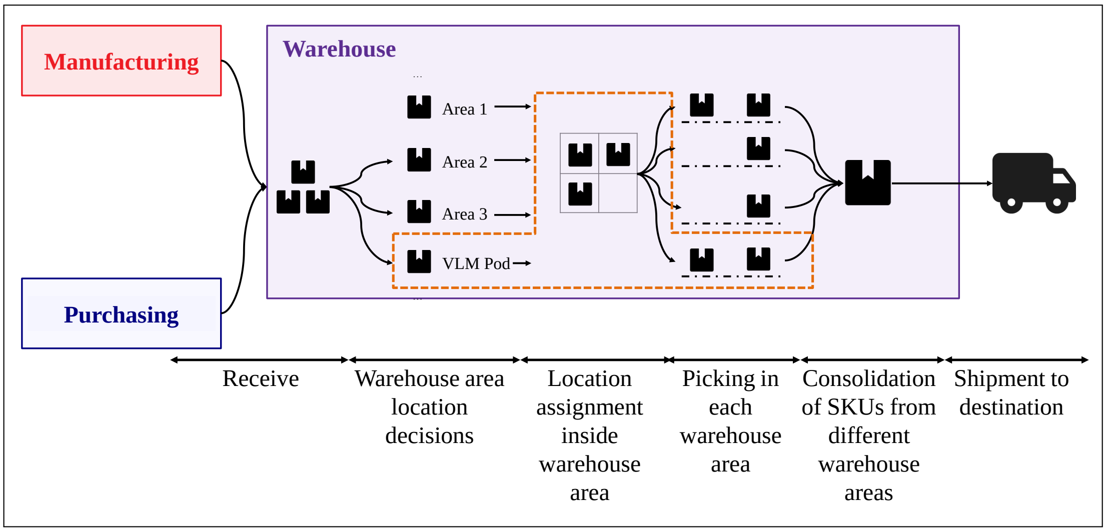

Figure 2.2: Summary of operations in a warehouse consisting of various areas. This study focuses on activities surrounded by the dashed line.

Aside from their benefits in floor space utilization, AS/RSs have advantages in reducing time spent walking in order picking times. Therefore, these systems are used by companies to adapt their warehousing operations for today's conditions (Dukic et al., 2015).

VLMs, which are examples of AS/RS types, have received attention in the literature in the recent years (Li et al., 2017). These systems can be used as a single unit, or can be combined to form a system called a "Pod". This modularity allows the warehouse designers to be more flexible in decisions related to storage capacity improvements.

In such setups, the operator is expected to be working on a unit while the other unit is busy retrieving the next tray. However, this can be achieved if only the load distribution among the units in the pod is balanced (Bozer and White, 1996). Therefore, balancing the workloads of individual units in a pod becomes a new challenge in such setups (Racca, 2015). Another challenge is determining the loading strategy since the system's picking performance depends on the storage assignment method (Jacobs et al., 2000).

This study's primary motivation is to focus on the highlighted operations in Figure 2.2 and provide an analysis on the operations history of a design-to-order armored land vehicles manufacturer in Ankara. This study will present detailed information about the environment, describe the mentioned analysis, and then make the problem statement.

2.1 Vertical Lift Modules (VLMs)

VLMs are parts-to-picker AS/RSs where several trays are stored in a high rectangular prism. In these systems, trays containing the requested parts are presented to the input/output (I/O) location. Trays in a VLM are generally divided by boxes to form individual storage locations. Once a tray is on the I/O location, the picker picks the part from its box with the help of indicators pointing the correct box. A representation of a VLM system can be seen in Figure 2.3. This setting is similar to end-of-aisle mini-load AS/RSs except for having only two columns in the y-axis. Although Battini et al. (2016) argue that VLMs are often used for storing less popular parts, warehouses that store parts of any activity class in VLMs also exist (Gullberg and Lundberg, 2017; Hulshof, 2019; Racca, 2015; Sinha, 2016; Tenhagen, 2018).

When the VLM units are integrated and assigned to one operator to form a "Pod", they can be considered as a multi aisle AS/RS with as many I/O locations as the number of VLM units forming that pod. A VLM pod's representation can be seen in Figure 2.4. This modular approach makes capacity improvement expansions easier after the system setup.

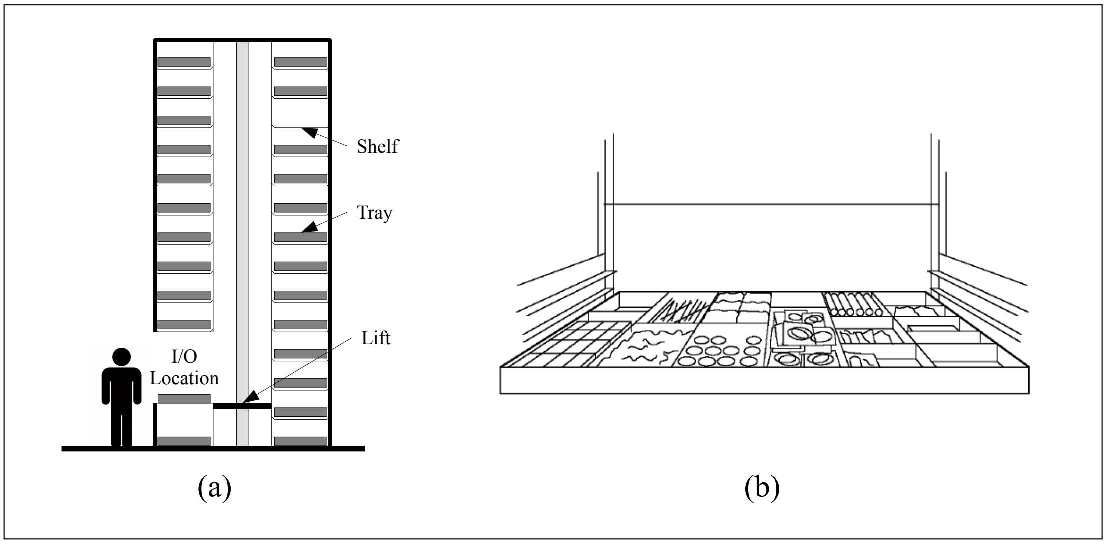

Figure 2.3: (a): Components of a VLM unit. (b): Representation of a VLM tray on I/O location from the operator's perspective (Retrieved from Romaine (2004)).

The advantages of using a VLM setup can be listed as space savings, lower labor costs with reduced walking distances, and increased control on the inventory on hand (Dukic et al., 2015). VLMs also provide improvements in ergonomics, which is an essential factor in operational success (Grosse et al., 2015). By having the I/O point in the "Green Zone" defined by Finnsgård et al. (2011), VLMs reduce the need for non-natural postures that may cause musculoskeletal issues during picking (Grosse et al., 2015).

Figure 2.4: A photograph of a VLM pod. Retrieved from Kardex AG (2018).

2.2 Analysis of a Job Shop Manufacturing Firm's History Data & Outcomes

As the first step of this study, an analysis of the operations record of an armored land vehicle manufacturing plant had been made. After the analysis, it has been found that the operational efficiency of a VLM pod in the company's warehouse could have been improved with the help of better decision support tools. This section will briefly describe the manufacturing environment and present the factors that make up the motivation of this study's focus, increasing the throughput in a VLM pod.

2.2.1 Company Overview

In a design-to-order manufacturing company, each order can be considered as a different project. An increased number of customers means high marginal complexity for such environments in many cases, because of distinct requirements on products in each order. In addition to the wide variety in the set of products, ordered quantities in the defense industry are low in general (Hartley, 2007; Rogerson, 1995). Being a manufacturer of armored defense vehicles, the analyzed environment (AE) is no exception to that. Therefore, it can be an excellent example of a company on the upper left corner of the product-process matrix defined by Hayes and Wheelwright (1979), which is a tool that can be used to get some insights on how a company operates based on its product and process characteristics. This matrix is shown in Figure 2.5.

Products of the AE have many different components, therefore many bill of materials (BOM) lines. The average number of BOM lines per product is more than 6,000 for this company. Manufacturers of similarly complex products can also be expected to have a similarly high number of BOM lines for their products. A sampling on the AE's BOM tables indicated that different products in the same order had common component ratios between 50%-90%, where the same metric was approximately 5% between two products from different orders.

With a wide variety of complex products, AE also has many distinct components. Among all the components in the database, AE has 73% of them defined as purchased from a supplier. This finding is in line with the generalizations of Hartley (2007) about the complexity of subcontractor networks in the defense industry.

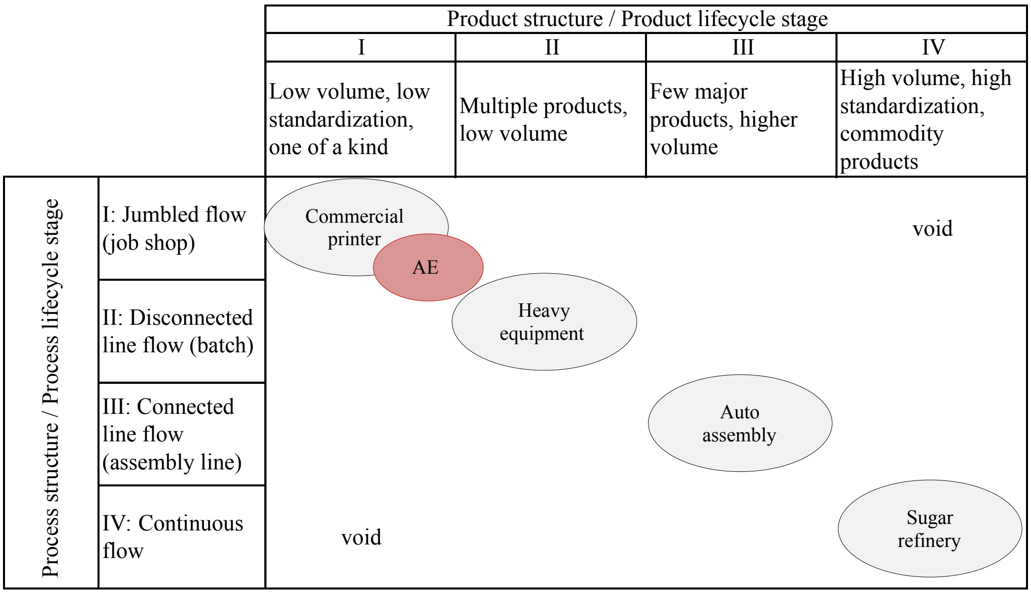

Figure 2.5: Product-process matrix diagram, example product and processes based on proposed model by Hayes and Wheelwright (1979) and analyzed environment's position (AE) on the matrix.

Being a job shop production environment, AE has different manufacturing disciplines. For each production discipline's operation, a new shop order is released. The number of released shop orders is kept evenly in each month to utilize the production capacity better (Figure A.1). However, since products that are being produced differ from time to time, the total number of components requested from the warehouse fluctuates each month (Figure A.2). This issue can be considered as the "bullwhip effect", which is a term describing that a small change in the customer demand results in higher impacts in the upstream nodes of the supply chain Gong and de Koster (2011).

After the manufacturing operations are completed, all products go through a series of inspections. Some of these items are rejected and returned to manufacturing as a component for rework shop orders. These rework shop orders create unexpected workloads at the warehouse. Count of the rework shop orders accounted for 26% of all released shop orders between January 2018 and March 2019. This information supports the survey outcomes of Rawabdeh (2005), a study that lists defects among the top issues causing unnecessary operations in job shop manufacturing firms. The monthly breakdown of the shop orders by their types in AE (repair or manufacture) can be seen in Figure A.3. This figure shows that the percentage of rework orders stays approximately the same among the months. That is, a high number of rework orders is a constant issue for the AE, rather than being an occasional problem. Although the number of rework shop orders can be estimated, the context of these orders cannot be known in advance. Therefore, activities in the warehouse of AE should be flexible enough to cope with these uncertainties.

2.2.2 Activity Profile of the Studied Real Life Warehouse

To understand the warehouse in AE better and be able to present its current situation, a warehouse activity profiling has been conducted following suggestions by Bartholdi (2017). In this section, insights on warehousing operations are presented in this manner.

AE's warehouse comprises various manual picking areas, each suitable for a set of parts with different physical dimensions. Distribution of the number of locations assigned and the number of order lines processed can be seen in Figures A.4 and A.5, respectively. With 2 VLM units in it, VLM pod is also one of these warehouse sections in this setting.

The warehouse had 40,000 different parts stored in various quantities as of March 2019. The average number of order lines per month was 49,143 for the period between January 2018 and May 2019. The breakdown of the number of processed order lines by months can be seen in Figure A.6.

Warehouse area decisions are made according to physical attributes and activity class of the parts to be stored in this company. A Pareto analysis o1 the distribution of picks over parts had shown that a high proportion of activities in the warehouse are because of a small portion of part types (Appendix A, Figure A.7). This outcome can be interpreted as a supporting fact for the activity-based class assignment decisions among warehouse areas.

Although there may be different pick order reasons, nearly 80% of the picks account for the shop order components. The breakdown of the sources of pick activities can be seen in Figure A.8.

Since MRP shows when a part will be required, each pick order of the warehouse has an expected delivery date at AE. Among all the pick lines released between January 2018 and March 2019, the late delivery ratio was 58%. Late delivery ratios for each month are plotted in Figure 2.6. This figure highlights the fluctuations in the workload of this warehouse. Please note that that this warehouse's operational capacity did not change drastically in the plotted months. Therefore, it may be inferred that the urgency levels of pick orders also fluctuate, causing higher late deliveries than other periods with similar workloads. Additional plots about the distribution of late deliveries by warehouse areas and order reasons are given in Figure A.9. These plots show that the "Narrow Aisle", "Main HL Trays" and "VLM System" are the areas supplying the highest amount of parts to the pick orders. Figure A.9 also highlights the fact that these three areas suffer from late pick order deliveries.

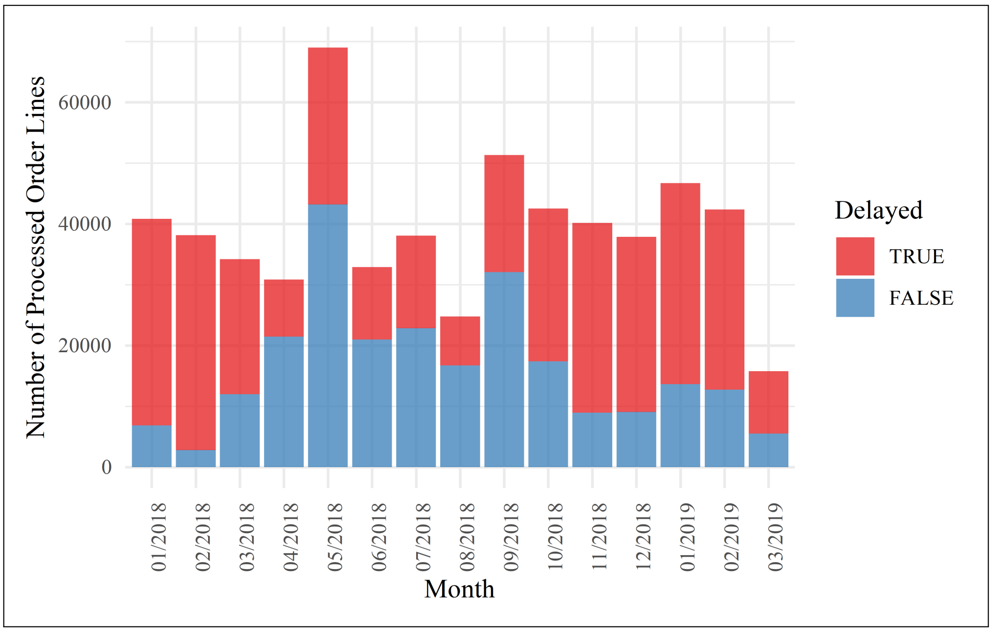

Figure 2.6: Lateness status of pick order lines by months.

2.2.3 Overview of the VLM Pod in AE's Warehouse

VLM Pod in AE's warehouse is used for storing parts with smaller physical dimensions of each activity class. The current setup has two units with 116 trays in each. Among 40,000 part types stored in the warehouse, the VLM system stores 6,000 of those parts in various quantities.

Location Assignments Inside VLM Pod

In the current setup, location assignments inside the VLM system, such as part-tray and part-box assignment decisions, are made temporarily to cope with the dynamic environment. In other words, a part-box assignment is removed from the system when the on-hand quantity becomes zero at the respective storage box.

After the company designs a new product, the arrived components of that product are manually assigned together on a set of trays by operators. However, if a part's stock diminishes, the location assignment of that part is also removed. When such parts arrive with no previously decided locations, assignments are made according to the "closest open location" principle defined by de Koster et al. (2007). With this method, the nearest available storage location will be assigned to the new part by the VLM unit's control software. Therefore, the first manual slotting configurations are not preserved in this dynamic environment with these methods. The impacts of the "closest open location" assignments can be seen as scattered distribution of the colors in Figure A.10.

In a VLM pod, tray-shelf assignments should also be considered as a component of location assignment decisions. In the current setup, these assignments are left to the VLM controller, which makes these decisions arbitrarily.

Order Picking Operations in VLM Pod

The VLM system picks account for 13% of the total picking activity in the analyzed warehouse (see Figure A.6). On average, this VLM system processes 12 pick orders, meaning 112 order lines (pick operations) each day. As in the whole warehouse's pick order reasons, most of the tasks are shop order component picks. Moreover, Figure A.12 shows a further detail, that the VLM system works for assembly shop orders for more than 95% of picks.

Jacobs et al. (2000) state that such AS/RSs are throughput constrained, rather than being storage space constrained. The findings at the AE support this statement, with nearly 40% of the locations as empty while the system struggles to satisfy orders before their due dates (see Figure A.9).

The VLM units at the AE complete 60 dual cycles each day for pick orders, where a dual cycle means storing a tray on a shelf and then retrieving another one. In picking operations of orders that require parts from both units, the operator does not alternate between units for each pick operation. Instead, the current system sequences all the picks from the same unit together. Here, the picker completes all picks from a unit, then moves to the next unit after all parts in the previous VLM unit are picked. A flow chart of the current pick process is given in Figure A.13. This VLM unit sequencing decision is discussed in more detail in the further parts of this study.

Finally, picking distribution over parts had also been analyzed. For this analysis, pick distributions of parts in the order history are plotted as a Pareto chart. As seen in Figure 2.7, picking distribution over parts follows the 80-20 rule since 80% of the activity is for only 20% of parts stored.

In such a VLM setup, cycle time mainly includes travel of lift for storing and retrieving trays. Therefore, the distribution of picks over trays may be a basic performance indicator of the current planning operations for the VLM system. A similar Pareto chart is also plotted for the distribution of picking over trays. This plot can be seen in Figure 2.8, which indicates that the 80-20 rule is not valid in the distribution of picks over trays. Combined with the outcomes in Figure 2.7, it means that location assignments do not follow the demand patterns in this setup.

The distribution of picks over parts is shown as a heat map of all trays in the VLM Pod in Appendix A, in Figure A.11.

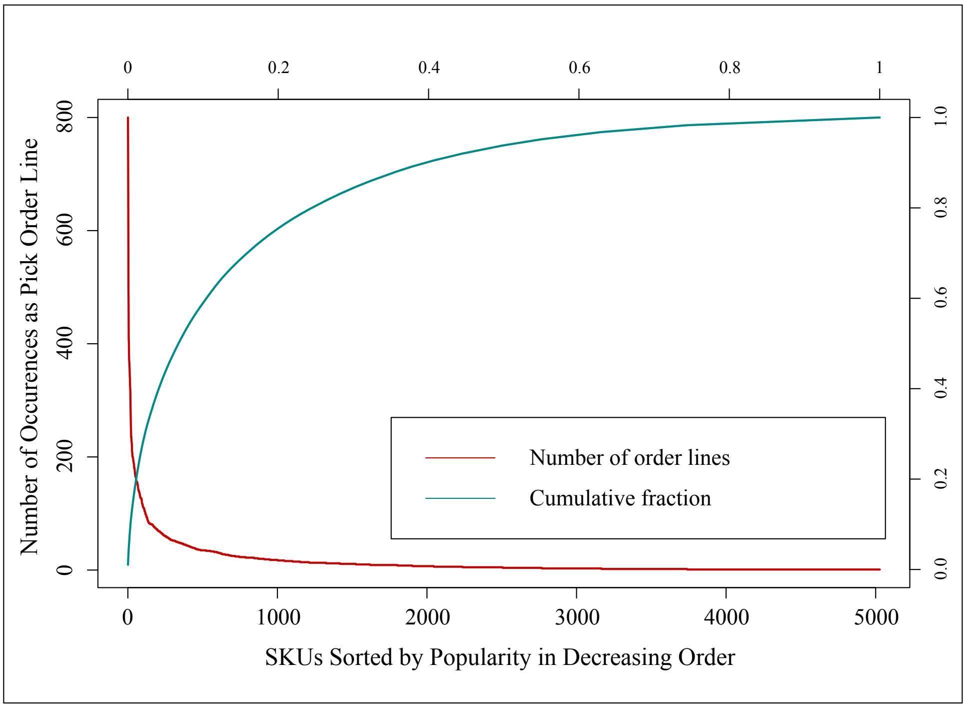

Figure 2.7: Distribution of picking operations over the parts in the VLM system.

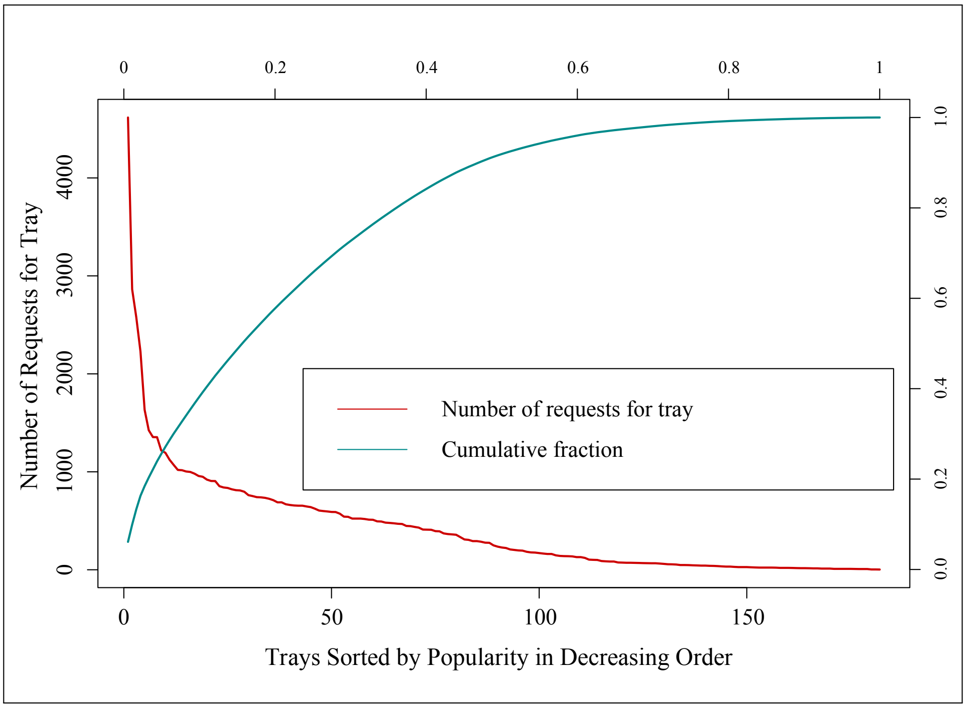

Figure 2.8: Distribution of picking operations over trays.

Balancing Workloads of the Individual Units

Balancing workloads of individual units in such AS/RS pods has a direct impact on the picking performances (Racca, 2015), so it is an important task. However, there is an imbalance between the workloads of two units in the analyzed system. For example, unit 1 had 3,882 different part types stored in it, while unit 2 had 2,220 different part types stored in it on a specific date. The number of the processed order lines per day also changes drastically between units. The difference in terms of workload between the two units can be seen in Figure 2.9 and Table A.1. According to Bozer and White (1996), such imbalances may cause lower throughput values in AS/RSs.

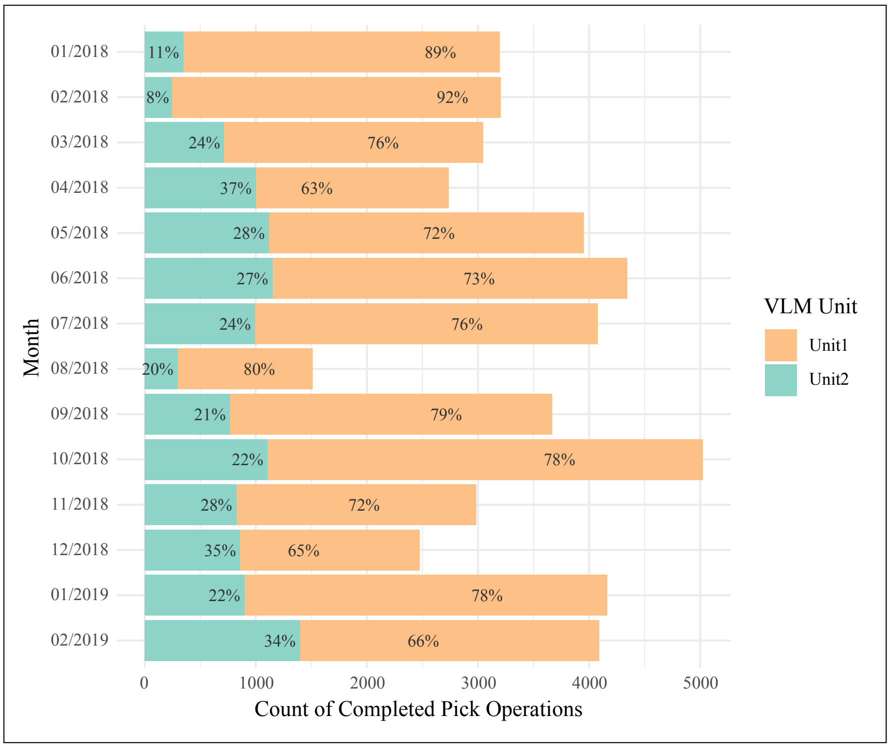

Figure 2.9: Number of processed order lines by each unit for the period between Jan. 2018 and Mar. 2019.

2.2.4 Factors that Make Responsiveness of Warehouses Important in Job Shop Environments

Responsiveness includes the ability to respond to external factors in reasonable times with a correct response (Barclay et al., 1996). Due to some features of the analyzed company's operations, the responsiveness of the warehouse is essential. These factors can be considered typical for manufacturing environments with low volume and a wide variety. Therefore, these observations are going to be discussed as more generic ones rather than specific issues of the AE.

A warehouse activity profiling study showed that there are some sources of complexities in the wider system of interest for decision-makers at the warehouses of job shop manufacturing environments with changing product offerings. These sources of uncertainties are highlighted in this section.

The sources of complexities in inventory management operations of the AE can be listed as follows:

- Design-to-order production firms in the defense industry are often obliged to store spare parts of their products for possible future requirements. This obligation causes higher storage requirements for warehouses and longer travel routes for picking or more time spent rearranging the locations.
- Warehouses are generally designed not to be homogeneous in terms of layout design. There may be different systems and layout designs in various areas because of the different part attributes (such as dimensions, weight, and electrostatic protection requirement) and usage reasons (stored as a spare part or stored for manufacturing).
- In job shop environments, the rework shop orders often account for a high percent of the total shop orders. Since rework operations are not planned at the beginning of a production process, these unexpected tasks are adding an extra workload on the warehouse.
- Even though the monthly number of released shop orders stay in an acceptable interval (Figure A.1), the number of component lines for all released shop orders in a month may deviate due to the different products that are being produced (Figure A.2). In other words, the manufacturing plan of the company leads to uneven workload distributions for the warehouse.
- Because of the high ratio of purchased parts in their inventory list, defense industry companies' operations highly depend on their suppliers' and other external parties' (such as customs, logistics suppliers) performances. Any delay in

the subcontractors' operations may render previous plans obsolete, eventually leading to unplanned operations.

2.3 Data Sources

In this study, all the analyses and computational experiments were performed using the data obtained from the AE. These data are described in this section.

2.3.1 Analyzed Company's Enterprise Resource Planning Software (ERP) Database

There are several types of data used both in the analysis step and in the computational experiments of the proposed procedures. A set of tables in the AE's database were used as inputs, and those tables are represented as an ER diagram in Figure 2.10.

The problem instances used in the computational experiments are sampled from tables *Part Component*, *Inventory Part*, *Warehouse Pick Orders*, and *Pick Order Lines*. The instance generation procedure is described in Section 6.1.

2.3.2 Time Study

Since information on the task times is crucial for any performance study, a time study was conducted at AE on the machine's operations and the picker's tasks.

2.3.2.1 Machine Time Study

VLM unit's operations can be divided into two main categories: single command and dual command operations. Single command operations start when there is no tray on the I/O location, end when the requested tray arrives at the I/O location. Dual command operations start with a tray on the I/O location, continue with storage of the previous tray, retrieval of the next tray, then end with the next tray on the I/O location.

The task breakdown of a dual command operation is represented in Figure 2.11. Descriptions and observations about the tasks are given in Table 2.1.

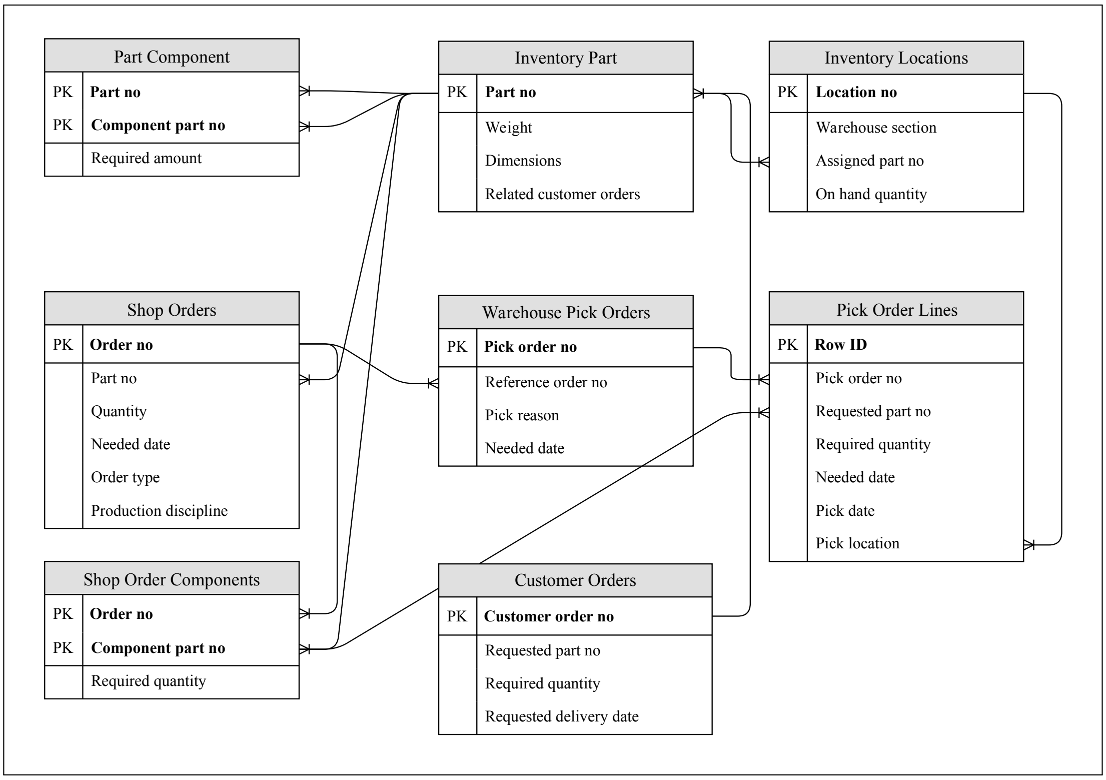

Figure 2.10: Relational schema of the database tables obtained from AE for this study.

The observations of the machine task time study have some variations. As also stated by Groover (2007), sources of these variations can include:

- Variations in the measurements,
- Observer error,
- Delays caused by the control unit of the VLM.

Because of these variations, the real value of the task times, T_r, can only be estimated within a confidence interval. Following the guideline presented by Groover (2007), we aimed to be 95% confident that the real value of the task time lies within $\pm 10\%$ of the mean of all observations, $\bar{x}$. Since the population variance is not known, student t distribution is used for constructing the general confidence interval statement. In this statement, α refers to the confidence level we want to achieve, s refers to the sample variance, and n refers to the sample size.

$$P(T_r \text{ lies within } \bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}) = 1 - \alpha \tag{2.1}$$

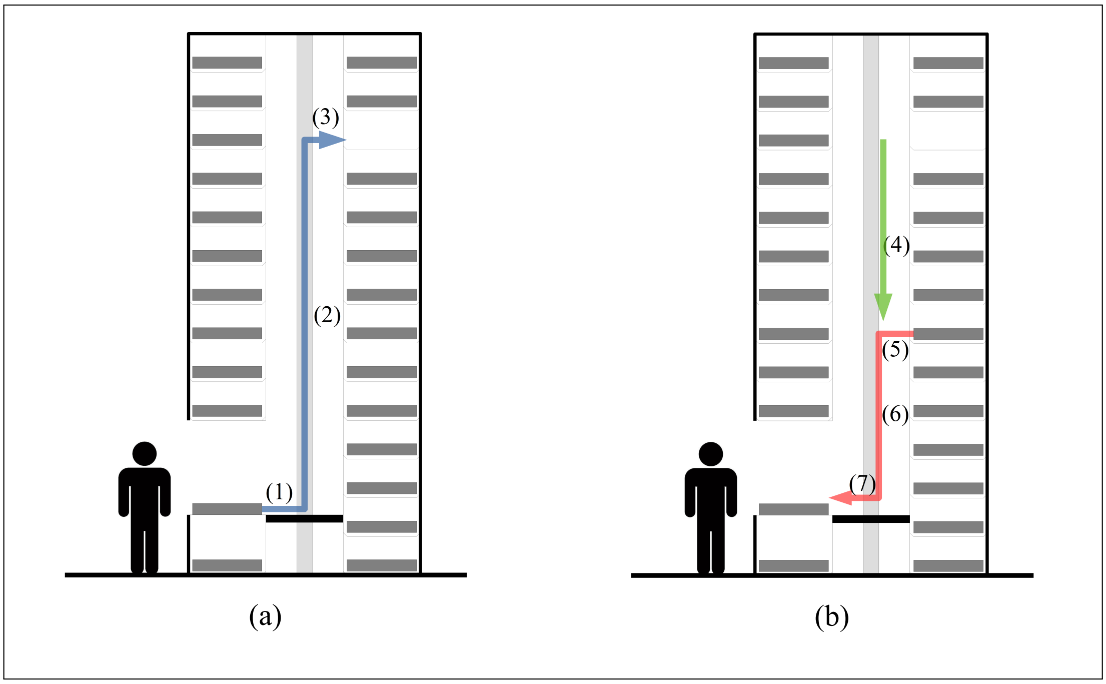

Figure 2.11: Representation of machine tasks listed in Table 2.1. Tasks (2), (4) and (6) are considered under the "vertical movement" categories. (a): Tray put tasks; (b): Tray retrieval tasks.

Before moving on, let us express the interval half-length in terms of the mean of the observations:

$$k\bar{x} = t_{\alpha/2}\frac{s}{\sqrt{n}} \tag{2.2}$$

According to equation 2.2, to construct a confidence interval that lies within $\pm 10\%$ of the mean of all observations, we would want to have $k = 0.1$. Rearranging equation 2.2 for n, the minimum number of the required observations can be found by calculating:

$$n_{min} = (\frac{st_{\alpha/2}}{k\bar{x}})^2 \tag{2.3}$$

Together with the task descriptions and the observation details, calculated n_{min} values for each task can also be found in Table 2.1. An example calculation for n_{min} value of operation 1 is below:

$$n_{minOp1} = (\frac{0.204(1.96)}{0.1(7.996)})^2 = 0.25005 \tag{2.4}$$

Table 2.1: List of the tasks of a VLM unit in a dual command operation and their observation results in the time study.

Task	Number of Observations	Mean Duration in Sec.	Std. Dev. of Duration in Sec.	n_{min}
1: Pull outgoing tray from opening	178	7.996	0.204	1
3: Horizontal movement of outgoing tray to its destination shelf	174	5.932	0.29	1
5: Horizontal movement of incoming tray from its shelf	169	5.901	0.285	1
7: Horizontal movement of incoming tray to the opening	171	8.571	0.276	1
Vertical movement for 5cms (dist <=500cms)	88	0.056	0.006	5
Vertical movement for 5cms (dist >500cms)	299	0.04	0.006	9

The result from Equation 2.4 rounds up to 1, and this shows that only one observation would be sufficient if we had known that sample deviation in the beginning. Since we had already completed 178 observations to get that sample variation and mean information, no additional observation for this task is necessary.

2.3.2.2 Operator Time Study

Being a key element in VLM pods, an operator has a potential impact on the performance of a setup. Therefore, the operator needs to be taken into account. For this purpose, operations have been observed in the current system. Although they might be valid only for the AE, the main outcomes of this time study can be listed as follows:

- The task times for picking an item from a tray can be considered equal for all the locations on a tray.
- Time spent picking the parts changes according to the quantity requested since the operator counts the items before completing the picking task.

In this direct time study, the operator's picking operation was divided into three main tasks: preparation for picking, actual picks of the parts, and walk between two VLM units. Besides the sources of variations defined in Section 2.3.2.1, introducing a manual worker means that the worker's pace may also be one source for the variation in the operator task time observations. Therefore, as in the previous section, n_{min} values have been calculated for the operator task time observations, too. Descriptions and time study results of these picking tasks can be seen in Table 2.2. Since all the n_{min} values turned out to be less than the actual number of observations in the study, only one session was conducted for obtaining the task times.

Table 2.2: List of the tasks of the operator in a picking operation and their observation results in the time study.

Task	**Number of Observations**	**Mean Duration in Sec.**	**Std. Dev. of Duration in Sec.**	n_{min}
1: Preparation & picking and packing (fixed time spent per pick order line)	150	10.509	1.514	8
K: Walk between units	63	7.092	0.741	5
Task duration for picking one item (order qty <=2)	20	6.762	0.526	3
Task duration for picking one item (order qty >=2 & order qty <=8)	60	2.243	0.394	13
Task duration for picking one item (order qty >8)	54	1.301	0.282	19

2.4 Motivation of the Study

As presented in Section 2.2.2, there are a set of factors that make responsiveness of the warehouse operations more important in the AE. However, many warehouse areas at the AE, including the VLM pod, struggle to satisfy pick orders at their requested dates (see Figure A.9).

In their studies, Romaine (2004) and Jacobs et al. (2000) argue that the storage assignments and order picking rules impact an AS/RS system's order picking performance. However, today, the commercial software used to manage VLM units, such as Power Pick Global and Kardex Direct Drive SAP, are not flexible enough to let the users make detailed modifications in location assignment and order picking methods.

Besides the lack of solutions as applications for the end-user, research on the related literature shows that among the studies dedicated to AS/RS, just a few focus on VLMs (Dukic et al., 2015; Meller and Klote, 2004; Nicolas et al., 2018; Rosi et al., 2016). Even though there are some studies proposing methods to make some decisions in Figure 2.12, to the best of our knowledge, there is no study that discusses all the factors affecting the throughput of a VLM system.

To conclude, the identified business pains at the AE direct us towards the lack of studies, methods, and applications for improving the operational efficiency of VLMs.

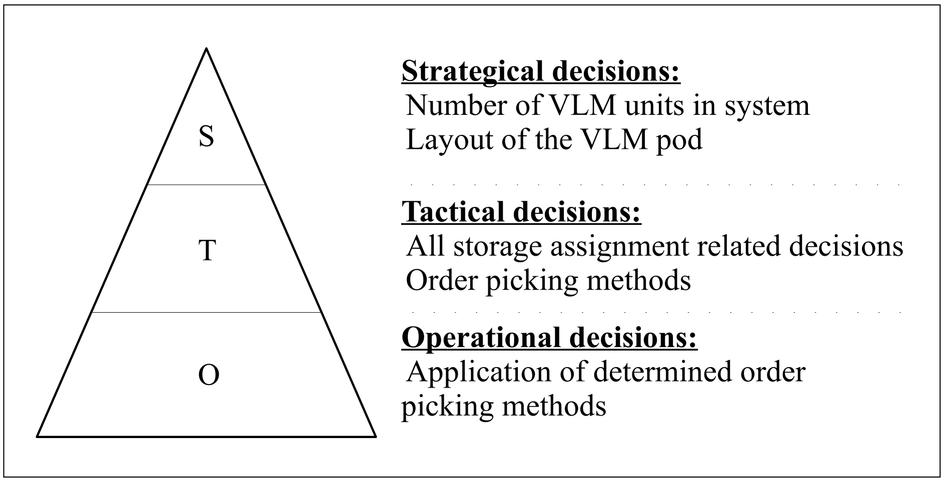

Figure 2.12: Summary of warehousing decisions in a VLM pod.

CHAPTER 3

PROBLEM STATEMENT

After our observations of the AE's warehouse and decision maker's responsibilities there, the system boundaries are determined as in Figure B.1 in Appendix B. Inputs of the system are;

- a set of parts to be stored,
- BOMs of the final products,
- a MPS indicating the manufacturing dates and quantities of each final product,
- picking task times and tray retrieval times.

This study considers responsiveness and throughput values as the main performance measures of a VLM pod in a production warehouse. Objective is to provide methods to design VLM pod configurations with high performance measures for production warehouses or to increase them in existing VLM pod configurations. After the analyses presented in Chapter 2 and the preliminary picking simulation of various VLM pods (see Appendix C), increased throughput is planned to be achieved by investigating the decision alternatives for a series of sub procedures.

- Storage location assignments,
- order picking process flow,
- design of the VLM pod (number of units in a pod, number of trays in a VLM unit, etc.)

These problems have some input/output relationships with each other (see Figure 3.1). In this study, storage assignment is considered as the main problem and alternative set of actions for the other sub problems are investigated as different scenarios for the storage assignment problem. In addition to defining the storage assignment problem, this chapter will also discuss the other throughput related decisions in a VLM pod and the reasoning behind putting storage assignments in the main focus of this study. Since the source of this study is a real life setup, its current decisions will be used as a benchmark in assessment of the proposed procedure. Current system in terms of the selected decision alternatives are described in Section 2.2.3 and summarized in Table 3.1. Moreover, current system's layout of the storage locations and their pick frequency can be seen in Appendix A, Figure A.11.

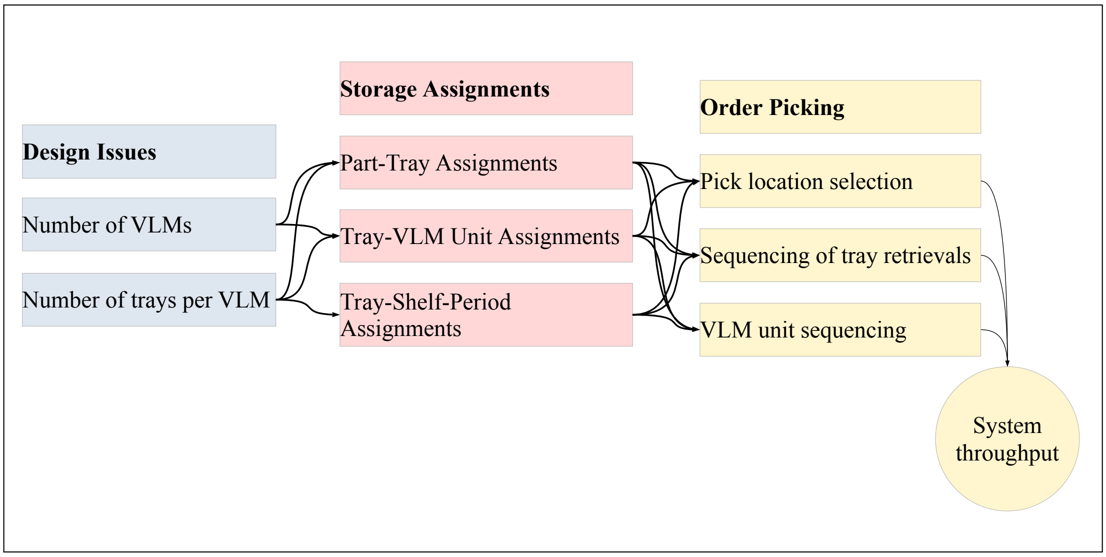

Figure 3.1: Concept map representing input/output relations between the defined sub problems and their environment. Note that rectangles represent decisions and circles represent outputs.

3.1 Order Based VLM Pod Storage Assignment Problem

As stated in Jacobs et al. (2000), storage assignment decisions have a direct impact on the picking performance of AS/RSs. Therefore, the location assignment problem is selected as the basis of this study. In this sub problem, we have

- a set of parts to be stored,
- BOMs of the final products,
- a MPS indicating the manufacturing dates and quantities of each final product in each time period (season),
- a list of expected manufacturing pick orders, which can be obtained by processing MPS and BOMs of final products,
- the number of trays per VLM unit,
- the number of locations per VLM tray,
- a VLM pod consisting a definite number of equal VLM units, and
- an order picking method that defines how the operator alternates between individual VLM units in the pod.

The objective is to minimize the expected number of tray retrievals and total distance traveled by the lift in VLM units. This is to be achieved by making these **decisions**:

- part-tray assignments,
- tray-shelf assignments for each season,
- tray-VLM unit assignments and
- pick order-part location assignments.

There is a yearly break at the AE each year. This break allows the operator of the VLM pod to reset all the part-tray assignments by changing these parts' trays. Therefore, our storage assignment method is assumed to be used each year, and the planning horizon is considered to be a year. According to our observations from the AE, a season is considered to be a six-month period. Unlike changing part-tray assignments, changing tray-shelf assignments in each VLM is not a labor-intensive task. Therefore, part-tray and tray-VLM unit assignments are the same for each season, where the only changing decisions among different seasons are tray-shelf assignments.

These storage assignment decisions are dependent on each one. Therefore, each problem's decision should be suitable with each one. For example, a turnover based Part-Tray assignment cannot be applied with random Tray-Shelf assignments.

Alternative decisions for the Part-Tray and the Tray-Shelf assignment problems for a specified season can be listed as follows:

- Random storage assignments for both the Part-Tray and the Tray-Shelf problems
- Class based storage (CBS) with activity classes for the Part-Tray problem and turnover based assignments for the Tray-Shelf problem
- CBS with correlation based classes for the Part-Tray problem and turnover based assignments for the Tray-Shelf problem
- CBS with activity classes for the Part-Tray problem and CBS with activity classes for the Tray-Shelf problem
- CBS with correlation based classes for the Part-Tray problem and CBS with activity classes for the Tray-Shelf problem

Alternative decisions for the Tray-VLM unit problem can be defined as all the Tray-VLM unit combinations that satisfy the maximum number of trays per VLM unit constraint.

Constraints in the storage assignment problem can be listed as follows:

- All parts must be assigned to a storage location in the VLM pod.
- Each location in the VLM pod can accommodate only one type of part.
- Each tray has a number of locations that can be used in location assignments.
- Each shelf can store only one tray at a time.
- Each VLM unit has the similar capacity in terms of the total number of trays.

Finally, the **assumptions** in the order based VLM pod storage assignment problem are as follows:

- Location capacities are defined in the VLM System database for each part.
- Parts can be stored in a fragmented location assignment concept, as presented by Ho and Sarma (2009).
- Tray sizes, the number of trays per unit and the number of VLM units are given.
- Each tray takes the same vertical space in the VLM unit.
- On hand quantities of each part are enough to satisfy planned production in the facility.
- Replenishment cycles are completed instantly when the VLM pod becomes idle.
- Operator picks are in line with the order picking method defined in this study.
- The bottleneck of the VLM pod is not the picking operator.

3.2 Order Picking Location Selection in Fragmented Storage

In the case of no stock splitting, the trays to retrieve for each order would be obvious. If stock splitting is allowed, a problem emerges with the objective of finding the smallest set of trays that can satisfy the given pick order. This decision takes storage location assignments as an input. Therefore, it must be addressed together with, or after the storage assignment decisions.

Constraints of the pick location selection problem are as follows:

- All parts in an order must be collected.
- Parts can only be picked from locations that has the requested part.

In the solution of this problem, following assumptions will be made:

- A storage location that is assigned to a part has enough number of materials to satisfy the ordered quantity. This is similar to the instant replenishment assumption in the order based VLM pod storage assignment problem.

- All the pick orders are assumed to consist of only the parts stored in the VLM pod.

3.3 Other Sub Problems Related with the Throughput of a VLM Pod

Aside from the previous problems, there are other decisions to be made in a VLM pod. Since the main aim of this study is to improve throughput performance of VLM pods, the other problems related with the throughput of a VLM pod will also be discussed. Moreover, readers can find the reasoning behind this study's focus on storage assignment problem in the discussions here.

Picking Sequence Among the VLM Units

After picking all the requested items from the tray on the input/out put location, the VLM unit starts a tray retrieval process. During this process, the operator cannot pick from that VLM unit. While aiming a higher operator utilization, there are two decision alternatives for the operator: walk to another VLM unit in the pod or wait for the VLM unit to retrieve the tray. Since this is a binary decision, the possible alternatives have been simulated using the transaction history of the AE and the time study observations presented in Section 2.3.2. According to the simulation's results, walking to the next VLM unit is the better decision in terms of the system's throughput. Therefore, we suggest the decision maker at the AE to implement "walk to the next unit" as the VLM unit sequencing rule in the VLM pod. Details of the simulation that supports our suggestion can be seen in Appendix C.

Tray Retrieval Sequencing

The sequence of tray retrievals may have an impact on the total distance traveled by the lifts in the VLM units. Determining the sequence of part retrievals in an order is another sub problem related to order picking. With the objective of finding the shortest path for the VLM lift during retrievals, all sequence combinations of the selected trays for picking can be considered as the set of alternative actions. However, this sequencing task's impact on the system's throughput is discussed in Appendix D and the potential impacts of a storage assignment model that minimizes the number of tray retrievals are found to be more significant. Therefore, this study will leave this sub problem as a topic for future studies.

The VLM Pod Design Problem

Since these AS/RS setups have high costs (Roodbergen and Vis, 2009), determining the best design configuration with a given workload is an important task. There are two main decisions to be assessed in this study: the number of VLMs in a pod and the number of trays per VLM unit. The issue of the number of VLM units have been investigated in the preliminary simulation in Appendix C, and with at least 92% confidence level, it is seen that having another VLM unit is not the best choice for the decision maker at the AE with the current set of parts and orders. However, the different design decisions have been used in the computational experiments in Chapter 6 to see the impacts of various design decisions on the throughput in different settings.

Table 3.1: List of current set of decisions for all the described sub problems.

Decision	**Current Decision in AE's System**
Number of VLMs	2
Number of trays per VLM	116
Part-Tray Assignments	Closest empty location, random
Tray-Shelf Assignments	Random
Tray-VLM Unit (Part-VLM Unit Assignments)	Random
Fragmented Storage	Not Implemented
Pick Location Selection	Not applicable since fragmented storage is not implemented.
Picking Sequence Among VLM Units	Wait for the next tray in the same unit, until all of them are picked
Tray Retrieval Sequencing	Random, following pick row numbers assigned by ERP

CHAPTER 4

LITERATURE REVIEW

This thesis provides different streams of studies about the VLMs in the related literature. The publications about each of the sub-problems in the literature will be reported. Besides, guideline given by Kitchenham (2004) is partially used with the help of "Publish or Perish" application (Harzing, 2007) and several on-line search engines that are listed in Table 4.1.

Table 4.1: List of search engines used in the literature review.

Search Engine
1. Google Scholar
2. Microsoft Academic
3. Web of Science

4.1 Vertical Lift Modules

In this section, studies that specifically focus on VLMs have been investigated. List of the research questions and search keywords used in this section can be seen in Tables 4.2 and 4.3, respectively.

According to the determined keywords and search engines, all publications related with VLMs have been examined. Instead of only searching for the term "vertical lift module", similar AS/RS categories, such as "mini-load AS/RS" and "split-platform AS/RS" are also used as search terms since they are referred in literature as setups that are similar to VLMs (Battini et al., 2016; de Koster et al., 2007; Yang et al., 2017).

Table 4.2: List of research questions used in the literature review on VLMs.

Research Question	
RQ1.	Are there any studies on performance assessment of VLMs?
RQ2.	Which methods have been used for storage assignments specifically in a VLM?
RQ3.	Which methods have been developed for order picking optimization specifically in a VLM?
RQ4.	Are there any studies on making strategical decisions for a VLM Pod?

Table 4.3: List of keywords used in the literature review. Results of the last four keywords have been investigated in terms of their proposed method's applicability to a VLM, since there are some common features between VLMs and the systems given in these keywords.

Search Keyword	
1.	Vertical lift module
2.	Vertical lift system
3.	Lean lift
4.	End of aisle AS/RS
5.	Split platform AS/RS
6.	Mini load AS/RS
7.	Vertical carousel

Table 4.4: List of exclusion criteria used in the literature review. An answer of "no" for at least one of these questions will lead to exclusion of a search result from the review process.

Exclusion Criteria	
1.	Is this study related with the throughput or performance of AS/RS setups?
2.	Does this study aim to provide methods for deciding the number of storage units and the number of operators?
3.	Are the results of this study applicable in a VLM setup?

Systems in the last four keywords have some differences to the VLMs. Therefore, results for these keywords have been investigated in terms of their proposed method's or presented analysis' applicability to a VLM. For example, despite being very similar to the VLMs in terms of physical appearance and operator interaction, proposed solutions for vertical carousels are generally take advantage of the rotating mechanism of the trays in carousel system. One example for that approach is the organ-pipe storage assignment, which has been proved to be optimal for vertical carousels in the case of no order batching and independent item demands (Bengü, 1995). This storage method is incompatible with VLMs since there is no rotating mechanism in VLMs. Studies that offer such methods are also excluded from the search results in this review.

After running queries for defined keywords, the results have been filtered according to the research questions and exclusion criteria listed in Table 4.4. Results of this review show that there are not many studies that specifically focus on VLMs in the literature dedicated to AS/RSs. Similar observations are also reported by various authors in their works (Dukic et al., 2015; Meller and Klote, 2004; Nicolas et al., 2018; Rosi et al., 2016).

First research question is about the presence of studies that focus on performance assessment of VLMs. Answer of this question is important, because current capacity in terms of throughput should be known in order to be able to make strategical decisions concerning an AS/RS setup. According to the search results, there are several studies on performance assessment of VLMs, each one having their own set of assumptions.

For example, Battini et al. (2016) do not consider the replenishment cycles, assuming them to be completed in another shift. On the other hand, Calzavara et al. (2019) assume random storage assignments and uniform demand distributions. Another example is the assumption of equal tray retrieval times for all trays, which is used by Mantel et al. (2007). Lastly, Nicolas et al. (2018) ignore the picker's operation times and considering these times to be negligible.

Based on the two server closed queuing model of Bozer and White (1996), Meller and Klote (2004) presented a throughput model for carousels and VLMs of either in "pod" configurations, or as single unit setups. In addition to that, Dukic et al. (2015) provided a throughput model for single unit "dual-tray VLM"s, a VLM type with two I/O points per unit. Both of these studies (Dukic et al., 2015; Meller and Klote, 2004) are providing analytical methods to determine throughput capacities of VLM setups by employing stochastic models based on the assumptions of random storage assignment and uniform demand distribution among stored parts. Moreover, both studies focus on picking and ignore replenishment operations while assuming these activities to be handled when system is idle.

Random storage assignment considerations of Dukic et al. (2015) and Meller and Klote (2004) are expanded by Sgarbossa et al. (2019) with the application of class based storage assignments on their throughput model. With this additional modification, benefits of class based location assignment in VLMs are shown.

On the other hand, Hur et al. (2004) argue that the assumptions of deterministic pick and put order distributions and deterministic order arrivals are not adequate for real life scenarios. To determine the throughput capacity of an AS/RS, they suggest a queuing model where the server is modeled as the storage/retrieval machine and arrivals are modeled as orders. However, since they are focused completely on an AS/RS, order picker is not mentioned, which is an important element of VLM systems.

Instead of analytical models, Rosi et al. (2016) present a simulation approach for one VLM's throughput, with the aim of finding the performances of VLMs in various dimensions. In their study, throughput of different unit dimension and lift speed combinations are compared in terms of trays retrieved per unit time.

To conclude, although there are studies in the literature on performance assessment of VLMs, the current set of studies in the literature still allows extensions in some areas. As stated by Sgarbossa et al. (2019), one of these potential future work topics is the inclusion of picking operator's activities into these throughput models. Moreover, the current literature can also be expanded with simulations of various order picking policies, or different location assignment policies, such as "order oriented" (Mantel et al., 2007) or "BOM based" (Hsieh and Tsai, 2001) class assignments. According to the search results in this review, fragmented storage of parts in the VLM pod in a dynamic environment is also one of the areas that provide opportunities for further research.

4.2 Storage Location Assignment

In their study, Roodbergen and Vis (2009) list the most popular types of storage assignment strategies for AS/RSs. We describe these methods before investigating the studies that are specific to VLMs.

Dedicated storage assignment method can be described as making fixed location assignments for each part. Although this method can have some benefits in traditional rack layouts in terms of operational practicality, these benefits generally do not apply to an AS/RS. On the other hand, dedicated storage assignment makes more locations busy, since locations are still staying assigned to a part, even though there is no on hand stock.

Random storage assignment is making random storage assignments for all parts in the warehouse. With this method, all available locations have equal probability to be assigned for a part.

Closest open storage assignment guides decision makers to assign parts to the nearest location at the time of arrivals. This method makes locations in traditional rack layouts to be utilized in a way that all parts are distributed starting around the I/O point. This is an advantage in traditional rack layouts since it reduces expected walking distances. However, as seen in Figure A.10, this method does not follow a similar pattern for VLMs since the distances of locations to the I/O point are not constant.

Full turnover based storage assignment is the method of making location decisions based on activity rates of parts. Such methods propose locating the most used parts closer to the I/O point. One example method is cube-per-order index (COI) (Heskett, 1963), which is a measure for each part, calculated by dividing required storage space by its expected number of requests. Then, the method proposes locating each part according to its COI metric. In addition to practical difficulties in the presence of changing demands, Schuur (2015) shows that this slotting strategy is not optimal when pick orders consist more than one item.

Class based storage assignment (CBS) is dividing the locations in a predefined number of groups, generally 2 or 3 (Graves et al., 1977), then making part-class allocations based on each item's demand frequency (Roodbergen and Vis, 2009). In addition to that, partitioning product components based on the BOMs is suggested by Hsieh and Tsai (2001) for manufacturing environments. Previously mentioned organ-pipe assignment for carousels (Bengü, 1995) can also be considered as a variation of class based storage assignments.

Among the studies that focus specifically on VLMs, Sgarbossa et al. (2019) and Battini et al. (2016) consider CBS with part-class allocations based on demand frequencies. Battini et al. (2016) conclude that CBS among trays (each tray assigned to a class) perform better than random storage and a CBS among the boxes on each tray. On the other hand, Mantel et al. (2007) criticize the use of COI in places where orders consist more than one item. Instead of COI, they suggest an order oriented location assignment strategy where the aim is to minimize expected picking times for orders. After a detailed analysis, authors show that their proposed method outperforms COI, but a comparison with CBS had not been made.

4.2.1 Correlated Storage Assignment for BOM Picking

Since the retrieval of a tray to the VLM's I/O point causes an idle time for the picker (Battini et al., 2016), focusing on minimizing the number of tray retrievals can also be a method of improving order picking times. Although the current literature on VLMs do not mention such objective functions in storage assignment decisions, Garfinkel (2005) proposes a similar method while considering warehouse areas as equal zones and trying to minimize the total number of travels between zones to pick the orders. Moreover, Garfinkel (2005) aggregates identical parts into the same storage area. However, even there are some operational difficulties in storing identical parts in many different locations, Ho and Sarma (2009) show that a "fragmented storage" approach is beneficial in terms of order picking performances. The main difficulties of fragmented storage listed by Ho and Sarma (2009) are about manual control operations of the on hand stock and managing the fragmented storage assignments of the parts. Such difficulties do not exist in a VLM since the on hand stock is tracked by a stock keeping software installed on the controller computer of VLM pods. Therefore, the stock splitting case in the minimum zone visits model of Garfinkel (2005) can be applied for the location assignments of parts in VLMs.

CHAPTER 5

MATHEMATICAL MODEL FOR THE STORAGE LOCATION ASSIGNMENT PROBLEM

5.1 Mathematical Model for Order Based VLM Pod Storage Assignment

In this section, we present our assumptions and mathematical formulation. Our assumptions, which are based on the observations in Chapter 2 can be listed as follows:

- All VLM units in a VLM pod are identical in terms of capacity and speed.
- Each location in the VLM system can accommodate only one part type.
- Storage locations (i.e., boxes in the trays) are large enough to accommodate any number of parts to be stored.
- Picked items are replenished in the VLM units as soon as the VLM unit becomes idle and the warehouse does not experience a stockout.
- All the picking operations are included in the input data, and no external pick orders arrive.
- Pick orders are not batched.
- Operator is not the bottleneck of the VLM pod.
- All storage locations have enough quantities of parts to satisfy all the pick orders assigned to be collected from that location.
- VLM lift's travel between two consecutive trays' locations in the unit takes negligible time compared to the whole duration of a dual cycle tray retrieval operation. See Appendix D for more detailed discussion on this assumption.

Our notation, including the sets, parameters and decision variables are given in Table 5.1. Note that, tray-shelf assignments are made for each demand season, in order to comply with the requirements of the environment with a dynamic demand.

Table 5.1: Notation used for mathematical formulation M_{QP}

Sets	
P	Set of parts
B	Set of pick orders
S	Set of VLM units
D	Set of pick order-part pairs, such that if part p is in pick order b, $(p, b) \in D$
T	Set of trays in a VLM unit
F	Set of shelves in a VLM unit. Smaller indices represent trays that are closer to the I/O location.
Z	Set of time periods (seasons)
Parameters	
n_{bz}	Demand frequency of pick order b, $b \in B$ in season z, $z \in Z$
C_{ts}	Number of locations (boxes) in tray t of VLM unit s, $t \in T, s \in S$
Decision Variables	
$x_{pts} =$	$\begin{cases} 1 & \text{if part } p \in P \text{ is assigned to tray } t \in T \text{ in VLM unit } s \in S \\ 0 & \text{otherwise} \end{cases}$
$y_{bts} =$	$\begin{cases} 1 & \text{if tray } t \in T \text{ in VLM unit } s \in S \text{ is used for pick order } b \in B \\ 0 & \text{otherwise} \end{cases}$
$w_{pbts} =$	$\begin{cases} 1 & \text{if part } p \in P \text{ is to be picked from tray } t \in T \text{ in VLM unit } s \in S \text{ for pick order } b \in B \\ 0 & \text{otherwise} \end{cases}$
$k_{tfsz} =$	$\begin{cases} 1 & \text{if tray } t \in T \text{ is assigned to shelf } f \in F \text{ in VLM unit } s \in S \text{ for period } z \in Z \\ 0 & \text{otherwise} \end{cases}$

$$(M_{QP}) \qquad \text{Minimize} \qquad \sum_{z\in Z}\sum_{f\in F}\sum_{b\in B}\sum_{t\in T}\sum_{s\in S} f\, n_{bz}\, y_{bts}\, k_{tfsz} \tag{5.1}$$

subject to:

$$\sum_{t\in T}\sum_{s\in S} x_{pts} \geq 1 \qquad \forall p \in P \tag{5.2}$$

$$\sum_{p\in P} x_{pts} \leq C_{ts} \qquad \forall t \in T, \forall s \in S \tag{5.3}$$

$$x_{pts} \geq w_{pbts}, \qquad \forall (b,p) \in D, \forall t \in T, \forall s \in S \tag{5.4}$$

$$\sum_{s\in S}\sum_{t\in T} w_{pbts} = 1 \qquad \forall (b,p) \in D \tag{5.5}$$

$$w_{pbts} \leq y_{bts} \qquad \forall (b,p) \in D, \forall t \in T, \forall s \in S \tag{5.6}$$

$$\sum_{t\in T} k_{tfsz} = 1 \qquad \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.7}$$

$$\sum_{f\in F} k_{tfsz} = 1 \qquad \forall t \in T, \forall s \in S, \forall z \in Z \tag{5.8}$$

$$x_{pts} \in \{0,1\} \qquad \forall p \in P, \forall t \in T, \forall s \in S \tag{5.9}$$

$$y_{bts} \in \{0,1\} \qquad \forall b \in B, \forall t \in T, \forall s \in S \tag{5.10}$$

$$w_{pbts} \in \{0,1\} \qquad \forall (b,p) \in D, \forall t \in T, \forall s \in S \tag{5.11}$$

$$k_{tfsz} \in \{0,1\} \qquad \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.12}$$

In this model, the objective function (5.1) minimizes the sum of weighted number of tray retrievals during order picking processes throughout the planning horizon, where trays assigned to higher shelves have higher weight values. As discussed in Chapter 3, the planning horizon is considered to be a year and a season is a six-month period, i.e., the planning horizon includes two seasons. Constraint (5.2) ensures that all the parts are assigned to at least one storage location in the VLM pod. Constraint (5.3) introduces the tray capacities C_{ts} to the model, for each tray $t \in T$ and each VLM unit $s \in S$, in terms of the number of different part types stored. Constraint (5.4) constructs the relation between decision variables x_{pts} and w_{pbts}, so that a part can only be picked from a location if it is assigned to that location. Since the main aim is to count the number of tray retrievals during order picking, decision variable y_{bts} is present in this model, to tell if a tray is required to be retrieved for picking a specific order. Constraint set (5.5) ensures that all ordered parts are collected. Constraint (5.6) links decision variables w_{pbts} and y_{bts}, so that a tray is marked as required only if a pick operation is planned for it. Constraint (5.7) forces the model to assign only one tray for each shelf in a VLM unit, where constraint 5.8 ensures that all trays are assigned to a shelf. Finally, constraint sets (5.9) - (5.12) define the set restrictions for the decision variables.

5.2 Linearized Version of Model M_{QP}

Since two decision variables are multiplied by each other in the objective function of model M_{QP}, it is a non-linear programming model with a non-linear objective function and linear constraints. However, it can be converted to a linear model by following the propositions of Li (1994). Model M_{ILP} utilizes this proposition and includes another decision variable, m_{btfsz}, which represents the multiplication of two other binary decision variables: y_{bts} and k_{tfsz}.

The linearization involves the addition of three sets of constraints ((5.14)-(5.16)) besides the new decision variables introduced in Table 5.2. Among those three, constraint (5.14) provides a lower bound of 1 for m_{btfsz} when k_{tfsz} and y_{bts} are both equal to 1. On the other hand, constraints (5.15) and (5.16) provide an upper bound of 0 for m_{btfsz} when at least one of k_{tfsz} or y_{bts} is equal to 0.

Table 5.2: Additional notation used for mathematical formulation M_{ILP}

Decision Variables
$m_{btfsz} = \begin{cases} 1 & \text{if } y_{bts} \text{ and } k_{tfsz} \text{ are both equal to } 1, b \in B, t \in T, f \in F, s \in S, z \in Z \\ 0 & \text{otherwise} \end{cases}$

$$(M_{ILP}) \qquad \text{Minimize} \qquad \sum_{z\in Z}\sum_{f\in F}\sum_{b\in B}\sum_{t\in T}\sum_{s\in S} f\, n_{bz}\, m_{btfsz} \tag{5.13}$$

subject to:

$$\sum_{t\in T}\sum_{s\in S} x_{pts} \geq 1 \qquad \forall p \in P \tag{5.2}$$

$$\sum_{p\in P} x_{pts} \leq C_{ts} \qquad \forall t \in T, \forall s \in S \tag{5.3}$$

$$x_{pts} \geq w_{pbts} \qquad \forall (b,p) \in D, \forall t \in T, \forall s \in S \tag{5.4}$$

$$\sum_{s\in S}\sum_{t\in T} w_{pbts} = 1 \qquad \forall (b,p) \in D \tag{5.5}$$

$$w_{pbts} \leq y_{bts} \qquad \forall (b,p) \in D, \forall t \in T, \forall s \in S \tag{5.6}$$

$$\sum_{t\in T} k_{tfsz} = 1 \qquad \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.7}$$

$$\sum_{f\in F} k_{tfsz} = 1 \qquad \forall t \in T, \forall s \in S, \forall z \in Z \tag{5.8}$$

$$x_{pts} \in \{0,1\} \qquad \forall p \in P, \forall t \in T, \forall s \in S \tag{5.9}$$

$$y_{bts} \in \{0,1\} \qquad \forall b \in B, \forall t \in T, \forall s \in S \tag{5.10}$$

$$w_{pbts} \in \{0,1\} \qquad \forall (b,p) \in D, \forall t \in T, \forall s \in S \tag{5.11}$$

$$k_{tfsz} \in \{0,1\} \qquad \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.12}$$

$$m_{btfsz} \geq y_{bts} + k_{tfsz} - 1 \qquad \forall b \in B, \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.14}$$

$$m_{btfsz} \leq y_{bts} \qquad \forall b \in B, \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.15}$$

$$m_{btfsz} \leq k_{tfsz} \qquad \forall b \in B, \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.16}$$

$$m_{btfsz} \in \{0,1\} \qquad \forall b \in B, \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \tag{5.17}$$

5.3 Relaxation of the Binary Set Constraints in M_{ILP}

We investigate the impact of relaxing binary set constraints in model M_{ILP} while considering the hierarchical relationships between each set of decision variables in the model. These relations are as follows: x_{pts} variables define the part location assignments, where w_{pbts}'s, part picking variables, are also based on. On the other hand, y_{bts} variables, which define an order's tray requirement, are also based on the w_{pbts} values. Lastly, k_{tfsz} values are based on the y_{bts} values. Considering this hierarchical relationship, we investigate the impact of relaxing integrality constraints on variables, starting with the linear relaxation of m_{btfsz} and including k_{tfsz}, w_{pbts}, and y_{bts} variables one by one.

5.3.1 Relaxation of the Binary Set Constraints on m_{btfsz}

We first relax the set constraints on m_{btfsz}, i.e., constraint (5.17). When y_{bts} and k_{tfsz} are binary variables, right hand side of the constraints (5.15) and (5.16) can only be 0 or 1, providing an upper bound of 0 or 1 for the decision variable m_{btfsz}. Likewise, constraint (5.14)'s right hand side value can be -1, 0, or 1, which can be considered as the lower bound for m_{btfsz} variables, since the model aims to minimize m_{btfsz} values. Therefore, constraint (5.17) can be replaced by the inequality (5.18), without violating the binary restrictions in the problem.

$$m_{btfsz} \geq 0 \qquad \forall b \in B, \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \quad (5.18)$$

The resulting model M_{ILP-m} is provided below.

$$(M_{ILP-m}) \qquad \text{Minimize} \qquad \sum_{z \in Z} \sum_{f \in F} \sum_{b \in B} \sum_{t \in T} \sum_{s \in S} f\, n_{bz}\, m_{btfsz} \qquad (5.13)$$

subject to: (5.2)-(5.12), (5.14)-(5.16), (5.18)

5.3.2 Relaxation of the Binary Set Constraints on m_{btfsz} and k_{tfsz}

Binary set constraints on k_{tfsz} variables can also be relaxed to take any value between 0 and 1 without hurting the validity of the model in terms of the binary structure of the solutions. In model M_{ILP-m}, the decision variables m_{btfsz} still continue to represent the multiplication of k_{tfsz} and y_{bts}'s if one of k_{tfsz} or y_{bts} variables is forced to be binary while the other one is allowed to take continuous values between 0 and 1. When one of k_{tfsz} and y_{bts} is fractional, right hand side of constraint (5.14) still provides a lower bound for m_{btfsz} that is equal to the multiplication of k_{tfsz} and y_{bts}, whereas the constraints (5.15) and (5.16) also continue to provide an upper bound for m_{btfsz}.

In this section, we argue that the same optimal solution will still be obtained when we relax the binary set constraints on both m_{btfsz} and k_{tfsz}. To prove our argument, we present a contradicting example.

Without loss of generality, let A, E, C and D be some index values for t, f, s and z, respectively. According to constraint (5.7), it is clear that at least one of the k_{Afsz} ($f \in F, s \in S, z \in Z$), variables must be given a non negative value. Let us assume that k_{AECD} is a fractional value. In this case, due to constraint (5.8), there would be at least one other $k_{AfCD} > 0$ ($f \neq E, f \in F$). Let the corresponding f index value be G for that decision variable, i.e., $f = G, G \neq E, k_{AECD} + k_{AGCD} = 1$. Note that m_{btfsz} variables still represent the multiplication of y_{bts} and k_{tfsz} values. Therefore, if tray A is ever used by any order, the objective function will include the corresponding m_{bAECD} and m_{bAGCD} values, which will be equal to k_{AECD} and k_{AGCD}, respectively. If the coefficients of m_{bAECD} and m_{bAGCD} are not equal, i.e., $G\, n_{bD} \neq E\, n_{bD}$, objective function will be minimized when binary assignments are made, instead of fractional values. If the resulting coefficients end up as equal, fractional assignments might occur, but there will also be at least one alternative optimal solution with binary k_{tfsz} values. To conclude, constraint (5.12) can be replaced by (5.19) without changing the binary structure of the obtained solution.

$$k_{tfsz} \geq 0 \qquad \forall t \in T, \forall f \in F, \forall s \in S, \forall z \in Z \quad (5.19)$$

The resulting model M_{ILP-mk} from this replacement is provided below.

$$(M_{ILP-mk}) \qquad \text{Minimize} \qquad \sum_{z\in Z}\sum_{f\in F}\sum_{b\in B}\sum_{t\in T}\sum_{s\in S} f\, n_{bz}\, m_{btfsz} \tag{5.13}$$

subject to: (5.2)-(5.11), (5.14)-(5.16), (5.18), (5.19)

5.3.3 Relaxation of the Binary Set Constraints on m_{btfsz}, k_{tfsz} and y_{bts}

In addition to the mentioned changes, models M_{ILP-m} and M_{ILP-mk} can be modified as to have continuous y_{bts} values. Since the w_{pbts} values are binary, constraint (5.6) provides a binary lower bound for the y_{bts} values. On the other hand, as the objective function actually tries to minimize y_{bts} values by keeping m_{btfsz} values minimum, y_{bts} values will be equal to the lower bound defined by constraint (5.6) in models M_{ILP-m} and M_{ILP-mk}. Therefore, equation (5.10) can be modified as:

$$y_{bts} \geq 0 \qquad \forall b \in B, \forall t \in T, \forall s \in S \tag{5.20}$$

Making this replacement on M_{ILP-mk} yields $M_{ILP-mky}$, which is provided below.

$$(M_{ILP-mky}) \qquad \text{Minimize} \qquad \sum_{z\in Z}\sum_{f\in F}\sum_{b\in B}\sum_{t\in T}\sum_{s\in S} f\, n_{bz}\, m_{btfsz} \tag{5.13}$$

subject to: (5.2)-(5.9), (5.11), (5.14)-(5.16), (5.18), (5.19), (5.20)

5.3.4 Relaxation of the Binary Set Constraints on w_{pbts} and m_{btfsz}

Binary set constraints on variable w_{pbts} in models M_{ILP} and M_{ILP-m} (i.e., where x_{pbts}, y_{bts} and k_{tfsz} variables are still binary) can be relaxed without violating the binary restrictions. Constraint (5.6) provides upper integer bounds for the w_{pbts} values. However, w_{pbts} values do not have a lower limit aside from constraint (5.5), which necessitates the sum of w_{pbts} over $s \in S, t \in T$. However, as in the case of relaxing k_{tfsz} variables, there's no advantage of assigning fractional values for w_{pbts} variables. If a w_{pbts} is assigned a fractional value, there must be at least one other $w_{pbts} > 0$ to satisfy constraint (5.5). If that is the case, constraint (5.4) will require more y_{bts} and x_{bts} variables to have a value of 1, which would increase the objective value. Therefore, constraint (5.11) in models M_{ILP} and M_{ILP-m} can be replaced by (5.21) while still preserving the binary structure of the solution.

$$w_{pbts} \geq 0 \qquad \forall p \in P, \forall b \in B, \forall t \in T, \forall s \in S \quad (5.21)$$

Resulting models from the mentioned replacements, M_{ILP-m} and M_{ILP-mw} can be seen below.

$$(M_{ILP-w}) \qquad \text{Minimize} \qquad \sum_{z \in Z} \sum_{f \in F} \sum_{b \in B} \sum_{t \in T} \sum_{s \in S} f \, n_{bz} \, m_{btfsz} \qquad (5.13)$$

subject to: (5.2)-(5.10), (5.12), (5.14)-(5.17), (5.21)

$$(M_{ILP-mw}) \qquad \text{Minimize} \qquad \sum_{z \in Z} \sum_{f \in F} \sum_{b \in B} \sum_{t \in T} \sum_{s \in S} f \, n_{bz} \, m_{btfsz} \qquad (5.13)$$

subject to: (5.2)-(5.10), (5.12), (5.14)-(5.16), (5.18), (5.21)

5.3.5 Relaxation of the Binary Set Constraints on m_{btfsz}, k_{tfsz}, y_{bts} and w_{pbts}

In addition to the previous models, we also consider relaxing the set constraints on decision variables $m_{btfsz}, k_{tfsz}, y_{bts}$ and w_{pbts} together. Unlike the previous relaxation, this one does not guarantee a binary solution, i.e., may yield a fractional solution. We analyze the structure of the solutions of this model and the time required to solve it to see if it could be useful in a potential rounding heuristic. The resulting model $M_{ILP-mkwy}$ is provided below.

$$(M_{ILP-mkwy}) \qquad \text{Minimize} \qquad \sum_{z\in Z}\sum_{f\in F}\sum_{b\in B}\sum_{t\in T}\sum_{s\in S} f\, n_{bz}\, m_{btfsz} \tag{5.13}$$

subject to: (5.2)-(5.9), (5.14)-(5.16), (5.18), (5.19), (5.20), (5.21)

CHAPTER 6

COMPUTATIONAL EXPERIMENTS

This chapter of the study introduces the experimental studies for solving the model presented in Chapter 5. After an introduction to the used problem instances, their sources and the experiment setup in Section 6.1, the discussion on the results and validation of the model are presented.

All computational experiments reported in this study were executed on a set of identical computers with Intel Core i7-4770S 3.1 GHz four-core CPUs and 2x8GB RAM. All methods used in the experiments were coded on C++, compiled on Microsoft Visual Studio 2017 15.5.5, and CPLEX software (version 12.9) was called by using the Concert library.

6.1 Problem Settings and Generation of the Problem Instances

Used problem instances in the computational experiments are small sets sampled from the analyzed environment's (AE) ERP database. The algorithm used in the generation of these small samples can be seen in Appendix E, Algorithm 3. With all the pick orders and their component lists on hand, this algorithm creates a small problem instance according to the given parameters. In this study, a problem setting is referred to a set of parameters for the target data sample, where a problem instance refers to the unique problem sets generated by using a previously determined setting. Table 6.1 shows the parameters of a setting and the values used in data generation phase of this study. Each combination of the decisions in Table 6.1 indicates a setting and 10 instances were generated for each of them.

Table 6.1: Levels for the parameters defining the problem setting.

Number of Orders (b)	Order Size Limits (p)	Number of VLM Units (s)	Tray Capacities (c)
5, 10, 15	[4, 8], [12, 16], [20, 24]	2, 3, 4	Low, Medium, High

A setting is defined by the following parameters of the problem:

- Number of pick orders,
- Upper and lower size limits for the pick orders, in terms of the number of requested unique part types,
- The number of VLM machines in the VLM pod,
- Capacities of the trays in terms of the number of different part types.

The size limits for the pick orders make Algorithm 3 only include the orders with a size within the limits. Therefore, total number of the parts in an instance depends on the random selection of pick orders. Moreover, Algorithm 3 determines the minimum number of trays according to the tray capacities and the number of parts in the instance. According to the number of parts in an instance, Algorithm 3 always assigns the minimum feasible number of trays. Therefore, there is no infeasible problem instance in our experimental study.

Although the current VLM pod in the AE had a fixed VLM pod configuration, the experiments also consider different VLM configuration and environment settings by changing the number of VLMs and tray capacities. Outcomes from the experiments with different settings will be beneficial in understanding the effect of VLM pod design on the solution procedures and the objective values of the obtained solutions.

Algorithm 3 starts the generation of a problem instance by taking these arguments: the pool of all pick orders, the number of orders, d, upper and lower limits for the number of components in the picked orders. Until the number of orders in the considered setting, d, is reached, the algorithm continues to pick orders with a size within the given limits. After there are d different orders in the picked list, parts in all the orders are combined in a unique list. The size of this list determines the number of parts defined in this problem instance, p. There is a one last check before considering this list of orders as a problem instance input. If there is not even one part that is used by more than one order, this order list is rejected since it is not likely for the AE to have all orders having disjoint sets of parts. Lastly, after obtaining the set of parts and the set of orders, these part and order sets are used to generate different VLM pod configuration settings. Therefore, one iteration of Algorithm 3 yields 9 different problem instances with the same parts and orders, but different number of VLMs and tray capacities. Note that the number of VLMs can be 2, 3 or 4 and tray capacities can be low, medium or high.

As shown in Table 6.1, each setting has a set of parameters: b, p, s and c. According to the number of decision variables and the number of constraints in the related models, settings with the lowest number of b, p, s and "High" tray capacity c values represent the models with smallest size, and hence they are expected to be solved faster. Therefore, all the experiments in this chapter start with the setting where $b = 5, p = [4, 8], s = 2, c =$"High". If a solution method fails to prove optimality in a time limit of 3 hours for a problem instance, we consider that problem instance as one that cannot be solved optimally with the selected method. It does not necessarily mean that our solver, CPLEX, was not able to find a feasible solution. If a problem setting has 3 or more instances that cannot be solved to optimality in the time limit, we do not consider solving the instances of any problem setting that is expected to be more difficult based on the parameters defining the size and the difficulty of the instance (i.e., parameters presented in Table 6.1).

Since Algorithm 3 only includes the demanded parts in a problem instance, models presented in Chapter 5 can be modified slightly. In the proposed models, constraint (5.2) forces the parts to be assigned to at least one storage location. However, if a part is requested in an order, constraints (5.4) and (5.5) already force the model to assign a storage location for that part. Therefore, constraint (5.2) can be removed if all the parts stored in the VLM unit are requested by at least one order in the planned period.

All the model alternatives were ran with and without the constraint (5.2), and a slight improvement in solution times were seen. Therefore, this section only reports the experiment results on the models without constraint (5.2).

In any case where unused parts need to be stored in a VLM, the decision maker can exclude the unused parts from the model's input. Then, those parts can be assigned to the remaining empty locations after obtaining the solution. Since unused parts will have no contribution to the objective function value, this approach (i.e., solving the model without constraint (5.2)) will still yield the same objective function value.

6.2 Computational Performance for the Proposed Models

Since version 12.5.1, CPLEX uses the proposition by Li (1994) to convert models with linear constraints, non-linear objective functions to linear models (Puget, 2013). Therefore, M_{QP} is not included in the experiments, since it will already be converted to model M_{ILP} by CPLEX in the preprocessing phase.

The experiments described in this section start by attempting to solve the instances in the easiest problem setting having the least number of decision variables. This setting is shown on the top-left corner of Table 6.2 (i.e., $b = 5, p = [4, 8], s = 2, c =$"High"). In the next steps, if 3 or more instances out of 10 cannot be solved to optimality within the allowed time limit for a setting, we did not consider the settings which are computationally more challenging. For this problem, increasing one of the b, p, or s values; or lowering the tray capacity c, while keeping the other 3 parameters unchanged, is considered to make a setting more complex, mainly due to the number of constraints and decision variables.

Experiments involving model M_{ILP} were able to reach optimality for 162 problem instances out of 810. The number of solved instances (n_S) for each problem setting can be seen in Table 6.2. In this table, highlighted cells indicate the settings with 3 or more unsolved instances. The cells with a "-" value indicate the settings that are more complex than any of the settings with 3 or more unsolved instances, and hence not considered. According to the reported values of solved instances, having a high tray capacity, i.e., closer to the average order size, makes the problem easier to solve. On the other hand, increased number of orders, increased number of parts and increased number of VLM units in a problem make the problem more difficult, so we observe degrading computational performance in those directions.

Table 6.2: Number of solved instances (n_S) out of 10 instances for each setting, using model M_{ILP}.

M_{ILP}, n_S		$s=2$			$s=3$			$s=4$		
		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	10	10	7	10	10	-	10	4	-
	$p \in [12,16]$	10	0	-	10	-	-	10	-	-
	$p \in [20,24]$	10	-	-	10	-	-	9	-	-
$b=10$	$p \in [4,8]$	10	0	-	10	-	-	3	-	-
	$p \in [12,16]$	1	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	10	-	-	8	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Since CPLEX cannot find the solutions using model M_{ILP} for most of the instances in our experiments, we tested the performance of modified versions of our model described in Section 5.3. This part of the study discusses those modifications (i.e., relaxation alternatives) and their performance as in terms of the number of solved instances for all settings and solution times. Please note that solving $M_{ILP-mkwy}$ may not yield an optimal solution due to the relaxation of w_{pbts} variables, as discussed in Section 5.3.5.

The number of problem instances solved to optimality can be seen in Table 6.3. According to those numbers, using model M_{ILP-mk} performs best. Detailed information about the number of solved instances for each setting and each model can be seen in Table 6.4. Please note that the solutions obtained using model $M_{ILP-mkwy}$ are different from the rest of the models presented since it allows fractional decision variables.

Table 6.3: Number of solved instances out of 810 for each model.

Model	Number of solved instances (n_S)
M_{ILP}	162
M_{ILP-m}	173
M_{ILP-mk}	193
$M_{ILP-mky}$	173
M_{ILP-w}	134
M_{ILP-mw}	156
$M_{ILP-mkwy}$	39

When looking at Tables 6.2 and 6.4, some changes in the model's limits can be noticed between each model. For example, relaxing the set constraints on the linearization variable, m_{btfsz}, causes a very small change in the number of solved instances. On the other hand, relaxing the set constraints on the k_{tfsz} variables increase the solvability particularly for the settings on the far right side on Table 6.4, where the total number of trays and VLM units become higher. This improvement is not surprising, considering the fact that k_{tfsz} variables represent the assignment decisions of trays to shelves in VLM units.

Solution time (in CPU seconds), number of unsolved instances (n_{US}) and the corresponding optimality gap values for each setting are reported in Tables 6.5-6.6. Each row in these tables represents a setting (defined by their p, b, c and s values), and the average and maximum values are reported across the instances in that setting. Aggregate measures, i.e., average and maximum, for solution time are calculated over the instances that can be solved in that problem setting. On the other hand, aggregate measures for the optimality gaps are calculated over the unsolved instances of the corresponding setting and presented as percentages in columns titled $\%Gap$. Table cells with N/A values represent the problem settings that are not considered since the corresponding model fails to solve more than 3 instances of easier problem instances.

The results show that some of the relaxation alternatives, such as M_{ILP-mk} provide improvements in solution times, thus the maximum problem size that can be solved in the time limits. According to the results, it is clear that having trays with higher capacities makes the problem easier to solve.

In these computational experiments consisting of using 7 different models, we identified some benefits in the solution times after relaxing the binary set constraints for some sets of decision variables. On the other hand, these models also showed that a real life scenario consisting of more parts than our experimental settings may require a high-end computing environment, or a different solution method that may make some sacrifices in terms of the solution quality, in order to get a solution in a reasonable amount of time. Model $M_{ILP-mkwy}$ was developed with that option in consideration: getting a non-integer solution and then obtaining a feasible solution by using rounding heuristics. In fact, models $M_{ILP-mkwy}$ and $M_{ILP-mkwy-red}$ yield solutions with fractional values for decision variables and can be used in rounding procedures. However, the solution times for these models were higher than the other model alternatives. Upon an investigation of the solution procedure, we detected that our solver, CPLEX, evaluates a much higher number of nodes during its search for the optimal solution, which makes using this model to obtain solutions to be rounded impractical.

Table 6.4: Number of solved instances out of 10 by using models M_{ILP-m}, M_{ILP-mk}, $M_{ILP-mky}$.

M_{ILP-m}		$s=2$			$s=3$			$s=4$		
		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	10	10	8	10	10	9	10	3	-
	$p \in [12,16]$	10	0	-	10	-	-	10	-	-
	$p \in [20,24]$	10	-	-	10	-	-	9	-	-
$b=10$	$p \in [4,8]$	10	0	-	10	-	-	5	-	-
	$p \in [12,16]$	1	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	10	-	-	8	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

M_{ILP-mk}		$s=2$			$s=3$			$s=4$		
		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	10	10	10	10	10	10	10	10	10
	$p \in [12,16]$	10	0	-	10	-	-	10	-	-
	$p \in [20,24]$	10	-	-	10	-	-	10	-	-
$b=10$	$p \in [4,8]$	10	0	-	10	-	-	5	-	-
	$p \in [12,16]$	2	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	9	-	-	7	-	-	-	-	-
	$p \in [12,16]$		-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

$M_{ILP-mky}$		$s=2$			$s=3$			$s=4$		
		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	10	10	10	10	10	8	10	9	8
	$p \in [12,16]$	10	0	-	10	-	-	8	-	-
	$p \in [20,24]$	10	-	-	10	-	-	9	-	-
$b=10$	$p \in [4,8]$	10	0	-	10	-	-	2	-	-
	$p \in [12,16]$	0	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	8	-	-	1	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

continued...

Table 6.4: Number of solved instances out of 10 by using models M_{ILP-m}, M_{ILP-mk}, $M_{ILP-mky}$.

M_{ILP-w}		$s = 2$			$s = 3$			$s = 4$		
		$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low
$b = 5$	$p \in [4, 8]$	10	10	4	10	10	-	10	1	-
	$p \in [12, 16]$	10	0	-	10	-	-	10	-	-
	$p \in [20, 24]$	10	-	-	10	-	-	9	-	-
$b = 10$	$p \in [4, 8]$	10	0	-	10	-	-	-	-	-
	$p \in [12, 16]$	0	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-
$b = 15$	$p \in [4, 8]$	-	-	-	-	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-

M_{ILP-mw}		$s = 2$			$s = 3$			$s = 4$		
		$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low
$b = 5$	$p \in [4, 8]$	10	10	4	10	10	-	10	1	-
	$p \in [12, 16]$	10	0	-	10	-	-	10	-	-
	$p \in [20, 24]$	10	-	-	10	-	-	9	-	-
$b = 10$	$p \in [4, 8]$	10	0	-	10	-	-	4	-	-
	$p \in [12, 16]$	0	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-
$b = 15$	$p \in [4, 8]$	10	-	-	8	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-

$M_{ILP-mkwy}$		$s = 2$			$s = 3$			$s = 4$		
		$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low
$b = 5$	$p \in [4, 8]$	10	5	-	6	-	-	-	-	-
	$p \in [12, 16]$	9	0	-	-	-	-	-	-	-
	$p \in [20, 24]$	4	-	-	-	-	-	-	-	-
$b = 10$	$p \in [4, 8]$	5	-	-	-	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-
$b = 15$	$p \in [4, 8]$	-	-	-	-	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-

Table 6.5: Solution time and optimality gap values for models M_{ILP}, M_{ILP-m}, M_{ILP-mk}, $M_{ILP-mky}$; in each problem setting.

p	b	c	s	M_{ILP}					M_{ILP-m}					M_{ILP-mk}					$M_{ILP-mky}$				
				n_{US}	CPU Time (sec.)		$\%Gap$		n_{US}	CPU Time (sec.)		$\%Gap$		n_{US}	CPU Time (sec.)		$\%Gap$		n_{US}	CPU Time (sec.)		$\%Gap$	
					avg.	max.	avg.	max.		avg.	max.	avg.	max.		avg.	max.	avg.	max.		avg.	max.	avg.	max.
4-8	5	High	2	-	0.26	0.58	-	-	-	0.18	0.23	-	-	-	0.26	0.38	-	-	-	0.50	1.27	-	-
4-8	5	High	3	-	1.83	8.00	-	-	-	1.70	6.57	-	-	-	2.00	6.18	-	-	-	4.35	13.01	-	-
4-8	5	High	4	-	231.82	1523.37	-	-	-	131.51	654.04	-	-	-	22.61	49.73	-	-	-	62.69	114.73	-	-
4-8	5	Mid	2	-	121.51	566.43	-	-	-	120.14	563.31	-	-	-	12.13	20.76	-	-	-	55.44	168.55	-	-
4-8	5	Mid	3	-	1230.77	4211.91	-	-	-	1190.75	4007.42	-	-	-	46.75	180.27	-	-	-	339.72	1041.00	-	-
4-8	5	Mid	4	6	6203.23	10731.50	11	14	7	4764.31	8240.03	9	15	-	733.74	2523.49	-	-	1	2492.35	7408.04	22	22
4-8	5	Low	2	3	498.96	861.51	6	12	2	515.28	836.99	3	3	-	110.80	281.31	-	-	-	1045.17	5401.23	-	-
4-8	5	Low	3	N/A	N/A	N/A	N/A	N/A	1	2139.21	9075.80	12	12	-	1079.49	9279.95	-	-	2	674.26	1869.22	16	19
4-8	5	Low	4	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	-	641.69	4867.10	-	-	2	1219.62	3090.00	11	13
4-8	10	High	2	-	2.89	8.10	-	-	-	2.92	8.63	-	-	-	15.85	32.33	-	-	-	25.97	116.58	-	-
4-8	10	High	3	-	192.03	484.11	-	-	-	187.27	482.01	-	-	-	266.01	676.30	-	-	-	848.95	2105.72	-	-
4-8	10	High	4	7	3603.61	5435.02	5	10	5	6446.22	10123.10	6	10	5	2941.92	4567.48	7	13	8	3287.52	5080.58	11	18
4-8	10	Mid	2	10	-	-	56	71	10	-	-	58	71	10	-	-	44	53	10	-	-	61	68
4-8	15	High	2	-	124.50	1071.06	-	-	-	141.35	1242.94	-	-	1	1132.65	3698.70	10	10	2	395.23	859.57	39	44
4-8	15	High	3	2	841.93	5720.34	4	4	2	841.20	5714.69	4	4	3	4788.64	7030.07	13	14	9	2690.66	2690.66	9	18
12-16	5	High	2	-	0.84	1.21	-	-	-	0.87	1.47	-	-	-	1.38	2.58	-	-	-	6.13	10.13	-	-
12-16	5	High	3	-	19.00	38.28	-	-	-	19.47	37.30	-	-	-	17.61	47.55	-	-	-	143.57	263.54	-	-
12-16	5	High	4	-	140.41	247.84	-	-	-	140.05	240.33	-	-	-	176.24	342.43	-	-	2	1783.51	6548.97	6	8
12-16	5	Mid	2	10	-	-	41	59	10	-	-	45	69	10	-	-	29	40	10	-	-	56	67
12-16	10	High	2	9	9890.13	9890.13	16	33	9	10149.70	10149.70	16	33	8	6635.10	7631.08	14	24	10	-	-	44	64
12-16	15	High	2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	10	-	-	62	82	N/A	N/A	N/A	N/A	N/A
20-24	5	High	2	-	156.97	402.63	-	-	-	159.62	403.89	-	-	-	17.33	32.75	-	-	-	169.33	843.26	-	-
20-24	5	High	3	-	29.33	41.59	-	-	-	29.86	43.12	-	-	-	23.90	32.10	-	-	-	330.84	1392.79	-	-
20-24	5	High	4	1	696.25	3954.05	5	5	1	1211.97	8600.02	4	4	-	444.74	1030.56	-	-	1	3435.05	7179.06	2	2

Table 6.6: Solution time and optimality gap values for models $M_{ILP} - w$, M_{ILP-mw}, $M_{ILP-mkwy}$; in each problem setting.

p	b	c	s	M_{ILP-w}					M_{ILP-mw}					$M_{ILP-mkwy}$				
				n_{US}	CPU Time (sec.)		$\%Gap$		n_{US}	CPU Time (sec.)		$\%Gap$		n_{US}	CPU Time (sec.)		$\%Gap$	
					avg.	max.	avg.	max.		avg.	max.	avg.	max.		avg.	max.	avg.	max.
4-8	5	High	2	-	0.18	0.31	-	-	-	0.18	0.26	-	-	-	24.12	83.34	-	-
4-8	5	High	3	-	1.16	5.86	-	-	-	0.72	1.48	-	-	4	1882.60	7133.36	39	44
4-8	5	High	4	-	112.34	397.27	-	-	-	111.41	392.38	-	-	N/A	N/A	N/A	N/A	N/A
4-8	5	Mid	2	-	98.03	358.52	-	-	-	102.29	358.80	-	-	5	382.51	1728.15	65	98
4-8	5	Mid	3	-	1065.28	3510.97	-	-	-	917.12	3344.48	-	-	N/A	N/A	N/A	N/A	N/A
4-8	5	Mid	4	9	4747.55	4747.55	10	18	9	5048.87	5048.87	10	18	N/A	N/A	N/A	N/A	N/A
4-8	5	Low	2	6	418.60	933.02	16	29	6	415.71	944.80	15	29	N/A	N/A	N/A	N/A	N/A
4-8	5	Low	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
4-8	5	Low	4	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
4-8	10	High	2	-	3.51	12.07	-	-	-	3.72	12.70	-	-	5	2583.09	8907.45	33	49
4-8	10	High	3	-	296.33	821.84	-	-	-	270.81	767.65	-	-	N/A	N/A	N/A	N/A	N/A
4-8	10	High	4	N/A	N/A	N/A	N/A	N/A	6	5242.71	8593.28	5	7	N/A	N/A	N/A	N/A	N/A
4-8	10	Mid	2	10	-	-	41	70	10	-	-	39	45	N/A	N/A	N/A	N/A	N/A
4-8	15	High	2	N/A	N/A	N/A	N/A	N/A	-	191.88	1630.49	-	-	N/A	N/A	N/A	N/A	N/A
4-8	15	High	3	N/A	N/A	N/A	N/A	N/A	2	1286.76	8652.17	3	3	N/A	N/A	N/A	N/A	N/A
12-16	5	High	2	-	1.18	2.02	-	-	-	1.12	1.92	-	-	1	1254.26	4385.32	5	5
12-16	5	High	3	-	29.93	52.35	-	-	-	28.61	52.36	-	-	N/A	N/A	N/A	N/A	N/A
12-16	5	High	4	-	374.47	1896.84	-	-	-	374.48	1900.70	-	-	N/A	N/A	N/A	N/A	N/A
12-16	5	Mid	2	10	-	-	55	79	10	-	-	55	79	10	-	-	100	100
12-16	10	High	2	10	-	-	16	32	10	-	-	16	32	N/A	N/A	N/A	N/A	N/A
12-16	15	High	2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
20-24	5	High	2	-	146.42	304.66	-	-	-	144.46	301.56	-	-	6	2496.87	9815.42	87	100
20-24	5	High	3	-	66.26	165.66	-	-	-	65.43	165.35	-	-	N/A	N/A	N/A	N/A	N/A
20-24	5	High	4	1	944.32	3522.30	9	9	1	936.22	3525.51	9	9	N/A	N/A	N/A	N/A	N/A

6.3 Quality of the Obtained Solutions

Based on our observations from the real-life system, AE's transaction history and the analyses regarding the throughput related VLM pod decisions; we propose storage assignments based on model M_{ILP} to increase throughput in VLM pods. M_{ILP} tries to increase throughput by reducing the time spent for tray retrievals through the use of an objective function of minimizing total weighted number of tray retrievals, where trays in higher shelves have higher weight values. This section discusses the performance of the obtained solutions in terms of the increase achieved in system throughput in VLM pods. Discussion in this section is based on the 196 instances that were solved to optimality in the experiments in Section 6.2. The distribution of the solved instances over problem settings can be seen in Table 6.7. Average of the optimal objective values for each problem setting can be seen in Table 6.8.

Values in Table 6.8 show that having smaller trays, thus, a higher number of trays in the VLM pod in such small instances hurts the objective function. This is not unexpected, because the objective function of the model is based on the number of tray retrievals. The optimal value also increases with an increase in the number of parts. Since the generated problem instances have bigger tray sizes (and hence lower number of trays) as the number of parts increase, the change in the objective function is less steep than the increase in the case with a higher number of trays.

Table 6.7: Number of obtained optimal values from the overall experimental runs for all used problem instances.

		$s = 2$			$s = 3$			$s = 4$		
		c =High	c =Mid	c =Low	c =High	c =Mid	c =Low	c =High	c =Mid	c =Low
$b = 5$	$p \in [4, 8]$	10	10	10	10	10	10	10	10	10
	$p \in [12, 16]$	10	0	0	10	0	0	10	0	0
	$p \in [20, 24]$	10	0	0	10	0	0	10	0	0
$b = 10$	$p \in [4, 8]$	10	0	0	10	0	0	6	0	0
	$p \in [12, 16]$	2	0	0	0	0	0	0	0	0
	$p \in [20, 24]$	0	0	0	0	0	0	0	0	0
$b = 15$	$p \in [4, 8]$	10	0	0	8	0	0	0	0	0
	$p \in [12, 16]$	0	0	0	0	0	0	0	0	0
	$p \in [20, 24]$	0	0	0	0	0	0	0	0	0

Table 6.8: Average optimal values obtained from all experimental runs.

avg. opt. val.		$s=2$			$s=3$			$s=4$		
		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	144.1	358.5	600.9	170.3	284.7	457.1	213.3	342.3	389.4
	$p \in [12,16]$	182.1	-	-	223.8	-	-	262.3	-	-
	$p \in [20,24]$	270.1	-	-	227.2	-	-	252.2	-	-
$b=10$	$p \in [4,8]$	254.1	-	-	274.2	-	-	298.3	-	-
	$p \in [12,16]$	424.5	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	389.2	-	-	377.6	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

In order to verify the proposed model's benefits on a VLM pod's throughput, the optimal solutions obtained in Section 6.2 were used in a picking simulation (simulation model is described in Appendix C). These picking simulations also consider the current storage assignment decisions used in the AE's VLM pod. This section presents comparison of the current system with the proposed one. Considered decisions and their alternative set of actions in this simulation are as follows:

Location Assignment Rules: In addition to the optimal solution for M_{ILP}, two other storage assignment rules are simulated. AE purchases all components for a customer order project in bulk and accepts them together at the warehouse. Therefore, each project's component parts are typically stored closer to each other at first location assignments. This grouping is similar to the pick order based grouping done by M_{ILP}, but with a low level of detail since there are less than 10 projects, but more than 12,000 different pick order types at the AE. However, as part locations are not fixed, closest open location assignments for the replenishments cause AE to reach random storage assignment in the long run (see Section 2.2.3 and Figure A.10 in Appendix A). Therefore, the current storage assignment scheme's performance is assumed to be between that of project based grouping and random assignment.

VLM Unit Sequencing: The orders consisting parts from more than one VLM unit in the pod are making the operator walk between VLM units. As discussed in Chapter

3, there are two alternative decisions for the operator after completing all the picks from a tray. The operator may wait for the next tray (will be referred as **"wait"**) or go to the other VLM unit on which the tray may be ready (will be referred as **"walk"**). Results from the preliminary VLM pod picking simulation described in Appendix C already show that walking to the other VLM unit during tray retrievals is the better choice for both of the random storage and turnover based location assignment cases.

Table 6.9: Simulated scenarios in the verification phase.

Scenario No.	Walk Duration Between Units (seconds)	Number of VLMs	Location Assignments	VLM Unit Sequencing Rule
1	7	2	M_{ILP}	Walk
2	7	2	M_{ILP}	Wait
3	7	2	Random	Walk
4*	7	2	Random	Wait
5	7	2	Project groups	Walk
6*	7	2	Project groups	Wait

This simulation considers 6 different scenarios for each of the problem instances which are summarized in Table 6.9. In the table, * indicates the scenarios reflecting the current real life situation at the AE. Performance measure values of the real-life system in AE is assumed to be between the performance measure values of scenarios 4 and 6. For each problem instance, 30 replications were run. The simulation steps can be seen in Algorithm 4 in Appendix E. Completion time of the last pick order (c_{max}), average operator utilization and number of tray retrievals are among the recorded performance measures.

Assumptions of the Simulation: This simulation assumes the following:

- All the VLM units in the VLM pod are identical.
- All trays have the same box layout and all trays have c boxes, where c is determined by the parameters of the simulated problem instance.
- Different part types cannot be stored in the same box.
- Boxes are large enough to accommodate any number of stored parts.

- After an order's picks are completed, the last tray stays on the I/O point of the VLM unit. In other words, dual cycle tray retrievals are used in between different orders.
- Each order's pick process starts from the closest tray to the I/O point.
- Task times follow normal distribution with the parameters found in time study presented in Section 2.3.2.
- There are enough on hand stock for all parts ordered during each replication. Therefore, there are no stock replenishment operations in the simulated period.
- Only pick tasks arrive into the system. Put and count operations are assumed to be carried on during periods that are not considered in this simulation.
- Operator works at a constant pace.

Observations: In this part, Scenario 1 is considered as our suggested system, whereas the current system is assumed to be between Scenario 4 and Scenario 6. Performance measures of these three scenarios are given in Tables 6.16-6.19, where a detailed list of the performance measures for all scenarios can be seen in Appendix E, Tables E.2 and E.3. All presented values in these tables are averages across 30 simulation replications for 10 problem instances of the indicated problem setting.

The average number of tray retrievals in Table 6.19 shows that M_{ILP} performs well in terms of minimizing the number of tray retrievals in the system. Since the objective function of M_{ILP} is based on the number of tray retrievals, this is expected. Average values of the number of tray retrievals in each scenario and setting are given in Table 6.19, where the average percentage improvement of the number of tray retrievals in Scenario 1, in comparison with Scenario 6 or Scenario 4, can be seen in Tables 6.10 and 6.11. In this table, improvement ratios of Scenario 6 and Scenario 4 are on the same cells, separated with a "-" in between. In overall, Scenario 1, Scenario 4 and Scenario 6 yielded the average number of tray retrievals as 125.8, 444.7 and 341.8, respectively. The improvements in the number of tray retrievals in Scenario 1 increase as the number of parts in the problem setting increase. Since scenarios 4 and 6 include random storage, they are expected to perform better in settings with less number of parts since the probability of having two parts on the same tray increases when the number of parts to locate decreases. On the other hand, as tray sizes get smaller (i.e., movement from the left hand side of Table 6.10 to the right), improvements obtained in Scenario 1 decreases. This shows that as tray sizes get smaller than the average order size, advantages of using M_{ILP} may decrease.

Table 6.10: % improvements in the number of tray retrievals in Scenario 1, compared with Scenario 4.

%ΔNum.Tray Ret.		$s=2$			$s=3$			$s=4$		
Scenario 1 vs. 4		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	72.4	54.5	42.4	67.2	57.8	43.7	65.3	60.2	44.9
	$p \in [12,16]$	76.5	-	-	81.8	-	-	78.1	-	-
	$p \in [20,24]$	75.3	-	-	83.5	-	-	82.5	-	-
$b=10$	$p \in [4,8]$	82.1	-	-	78.2	-	-	76.1	-	-
	$p \in [12,16]$	83.2	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	80.4	-	-	81.0	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Table 6.11: % improvements in the number of tray retrievals in Scenario 1, compared with Scenario 6.

%ΔNum.Tray Ret.		$s=2$			$s=3$			$s=4$		
Scenario 1 vs. 6		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	67.5	46.6	36.8	56.7	47.9	35.9	57.5	46.9	39.2
	$p \in [12,16]$	71.1	-	-	74.8	-	-	64.7	-	-
	$p \in [20,24]$	66.4	-	-	75.5	-	-	71.2	-	-
$b=10$	$p \in [4,8]$	77.9	-	-	73.5	-	-	70.2	-	-
	$p \in [12,16]$	78.5	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	74.4	-	-	75.5	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

When the results of this simulation are compared with the results from the picking simulation of the real-life VLM system (see Appendix C for the mentioned simulation), we notice that the operator utilization ratios are higher in the simulation of the small problem instances in each scenario. These high utilization ratios may also be an indicator of a bottleneck situation in the VLM pod. The operator's utilization includes the time spent for pick tasks and walks. On the other hand, a picking operation in a VLM pod consists of the operator's manual picks from the tray, tray retrievals in VLM units, and the occasional walks and waits. As stated in Section 2.3.2 and Appendix D, tray retrieval times depend on the height of the VLM unit. Since the small settings generated for the experimental study have small sets of parts, the total number of trays is also less than the real-life case in these reduced problem instances. The average number of trays per VLM unit among all of 196 solved problem instances is 2.6, which is extremely low compared to 114 trays per VLM in the real-life setup at the AE. Since a tray retrieval cycle's duration is dependent on the vertical distance traveled, problem settings with less number of trays will have faster tray retrievals. In such a case, the operator is more likely to act as the bottleneck of the system, and the picking cycle will be more likely to reflect the operator task times. As discussed in Chapter 3, if a VLM system has a highly utilized operator, making VLM tray retrieval times shorter should not be the priority of the decision-maker; therefore, M_{ILP} is less relevant in that case.

When the c_{max} values for Scenario 1 and Scenarios 6 and 4 are compared, the issue of high operator utilization levels should be considered, too. As seen in Table 6.17, there are some problem settings with operator utilization levels of more than 95%. Since M_{ILP} aims to reduce tray retrieval times, thus, also walks and waits caused by tray retrievals; the objective function of M_{ILP} may not reflect the situation well enough when the operator is the system's bottleneck. This does not mean that applying M_{ILP} will hurt the system's performance. It only means that the gains in the performance in terms of system throughput may not be high enough in such a case. The average c_{max} values for each problem setting and each of the considered scenarios in Table 6.16 also indicate this issue. Although there is no case where the system in Scenario 1 performs worse than Scenarios 4 and 6, using M_{ILP}, i.e., Scenario 1, in some of the problem settings, such as $b = 5, p = 12 - 16, s = 2, c =$High, this approach yields smaller improvements since the tray retrievals make up a smaller portion of the system throughput. Since the small tray counts in these considered settings are not likely to be encountered, we expect a real-life setup to have higher tray retrieval times. Percent improvements in the c_{max} values of Scenario 1 and Scenarios 4 and 6 can be seen in Tables 6.14 and 6.15. These results show that using M_{ILP} leads to improvements on the c_{max} values in all settings. As the number of VLM units and the number of trays increase, i.e., going from left to right in Table 6.14, improvements obtained from using M_{ILP} become higher. Since there are some cases where the operator is highly utilized, it is not easy to make a comment on the magnitude of average c_{max} improvements obtained by using M_{ILP}. However, these simulations show that M_{ILP} performs well in terms of reducing the picking times of a VLM system, compared to the applications at the AE. Since the improvements obtained from M_{ILP} in terms of the number of tray retrievals are between 65% and 70%, we can conclude that M_{ILP} performs well in terms of grouping the parts according to the pick orders.

Using M_{ILP} for the storage assignments yields improvements even in the cases where the operator was fully utilized. This is expected since the utilization figures also include the time spent walking. M_{ILP} reduces the total walk distances in picking tours, so a slight improvement may be expected even if the operator is fully utilized. Time spent walking in each scenario can be seen in Table 6.18, where the percent improvements of the same metric can be seen in Tables 6.12 and 6.13.

As a final note, although using a VLM unit sequencing rule as "walk" improves the system throughput, using M_{ILP} minimizes that decision's impact. If all the pick orders require at most one tray from each VLM unit, two alternative decisions for the VLM unit sequencing problem will become identical. The difference of c_{max} values between Scenario 1 and Scenario 2 is only 30 seconds out of 28,100, which shows that using M_{ILP} makes most of the pick orders require at most one tray from each VLM unit. This shows that M_{ILP} is successful both in grouping the parts of an order together and distributing trays among the VLM units evenly.

Table 6.12: % improvements in the time spent walking in Scenario 1, compared with Scenario 4.

%ΔWalk Time		$s=2$			$s=3$			$s=4$		
Scenario 1 vs. 4		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	69	54	42	61	46	34	48	44	34
	$p \in [12,16]$	74	-	-	68	-	-	64	-	-
	$p \in [20,24]$	78	-	-	66	-	-	63	-	-
$b=10$	$p \in [4,8]$	74	-	-	69	-	-	66	-	-
	$p \in [12,16]$	83	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	78	-	-	78	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Table 6.13: % improvements in the time spent walking in Scenario 1, compared with Scenario 6.

ΔWalk Time		$s=2$			$s=3$			$s=4$		
Scenario 1 vs. 6		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	61	42	25	49	30	25	38	35	28
	$p \in [12,16]$	64	-	-	60	-	-	56	-	-
	$p \in [20,24]$	61	-	-	58	-	-	50	-	-
$b=10$	$p \in [4,8]$	63	-	-	62	-	-	61	-	-
	$p \in [12,16]$	77	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	69	-	-	71	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Table 6.14: % improvements in the order picking completion times in Scenario 1, compared with 4.

$\%\Delta\, c_{max}$ Scenario 1 vs. 4		$s=2$ $c=$High	$s=2$ $c=$Mid	$s=2$ $c=$Low	$s=3$ $c=$High	$s=3$ $c=$Mid	$s=3$ $c=$Low	$s=4$ $c=$High	$s=4$ $c=$Mid	$s=4$ $c=$Low
$b=5$	$p \in [4,8]$	10.4	11.4	11.3	13.4	13.6	12.3	14.1	14.8	12.5
	$p \in [12,16]$	3.7	-	-	8.9	-	-	12.2	-	-
	$p \in [20,24]$	3.6	-	-	6.0	-	-	9.1	-	-
$b=10$	$p \in [4,8]$	12.6	-	-	15.1	-	-	15.9	-	-
	$p \in [12,16]$	8.3	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	12.8	-	-	15.1	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Table 6.15: % improvements in the order picking completion times in Scenario 1, compared with Scenario 6.

$\%\Delta\, c_{max}$ Scenario 1 vs. 6		$s=2$ $c=$High	$s=2$ $c=$Mid	$s=2$ $c=$Low	$s=3$ $c=$High	$s=3$ $c=$Mid	$s=3$ $c=$Low	$s=4$ $c=$High	$s=4$ $c=$Mid	$s=4$ $c=$Low
$b=5$	$p \in [4,8]$	10.0	10.3	13.0	11.7	10.9	11.1	11.2	10.0	11.3
	$p \in [12,16]$	4.0	-	-	7.0	-	-	8.2	-	-
	$p \in [20,24]$	4.3	-	-	4.0	-	-	5.7	-	-
$b=10$	$p \in [4,8]$	11.6	-	-	12.9	-	-	13.3	-	-
	$p \in [12,16]$	7.0	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	11.1	-	-	12.8	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Table 6.16: Average order picking completion times (c_{max}) in the considered scenarios.

Scenario 1		$s=2$			$s=3$			$s=4$		
	c_{max}	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	12755.52	13713.88	13778.98	12739.46	13358.76	13474.60	12933.01	16254.24	13579.05
	$p \in [12,16]$	33779.05	-	-	31463.11	-	-	31550.54	-	-
	$p \in [20,24]$	52267.52	-	-	52780.49	-	-	50978.23	-	-
$b=10$	$p \in [4,8]$	25099.71	-	-	25102.87	-	-	25721.57	-	-
	$p \in [12,16]$	60601.06	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	38840.55	-	-	39327.90	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Scenario 4		$s=2$			$s=3$			$s=4$		
	c_{max}	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	14231.02	15476.64	15539.17	14715.20	15463.32	15357.63	15058.57	19072.33	15513.70
	$p \in [12,16]$	35065.77	-	-	34544.59	-	-	35925.78	-	-
	$p \in [20,24]$	54206.91	-	-	56150.50	-	-	56082.23	-	-
$b=10$	$p \in [4,8]$	28712.84	-	-	29574.50	-	-	30569.20	-	-
	$p \in [12,16]$	66101.67	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	44557.90	-	-	46345.59	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Scenario 6		$s=2$			$s=3$			$s=4$		
	c_{max}	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	14170.62	15262.42	15600.31	14433.05	14922.21	15085.20	14566.10	18045.24	15183.18
	$p \in [12,16]$	35151.92	-	-	33840.12	-	-	34324.48	-	-
	$p \in [20,24]$	54640.20	-	-	54976.63	-	-	54067.38	-	-
$b=10$	$p \in [4,8]$	28407.79	-	-	28835.99	-	-	29670.80	-	-
	$p \in [12,16]$	65149.64	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p \in [4,8]$	43698.06	-	-	45111.67	-	-	-	-	-
	$p \in [12,16]$	-	-	-	-	-	-	-	-	-
	$p \in [20,24]$	-	-	-	-	-	-	-	-	-

Table 6.17: Average operator utilization levels (Op.Util.) in the considered scenarios.

Scenario 1		$s = 2$			$s = 3$			$s = 4$		
%Op.Util.		$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low
$b = 5$	$p \in [4, 8]$	95	92	91	97	96	94	99	97	96
	$p \in [12, 16]$	98	-	-	98	-	-	99	-	-
	$p \in [20, 24]$	98	-	-	99	-	-	99	-	-
$b = 10$	$p \in [4, 8]$	97	-	-	98	-	-	99	-	-
	$p \in [12, 16]$	98	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-
$b = 15$	$p \in [4, 8]$	96	-	-	98	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-

Scenario 4		$s = 2$			$s = 3$			$s = 4$		
%Op.Util.		$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low
$b = 5$	$p \in [4, 8]$	93	88	86	92	89	87	92	89	89
	$p \in [12, 16]$	100	-	-	95	-	-	93	-	-
	$p \in [20, 24]$	100	-	-	97	-	-	94	-	-
$b = 10$	$p \in [4, 8]$	93	-	-	92	-	-	92	-	-
	$p \in [12, 16]$	97	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-
$b = 15$	$p \in [4, 8]$	93	-	-	92	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-

Scenario 6		$s = 2$			$s = 3$			$s = 4$		
%Op.Util.		$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low	$c =$High	$c =$Mid	$c =$Low
$b = 5$	$p \in [4, 8]$	91	86	83	91	89	87	92	92	90
	$p \in [12, 16]$	97	-	-	96	-	-	95	-	-
	$p \in [20, 24]$	96	-	-	98	-	-	96	-	-
$b = 10$	$p \in [4, 8]$	91	-	-	92	-	-	93	-	-
	$p \in [12, 16]$	96	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-
$b = 15$	$p \in [4, 8]$	91	-	-	92	-	-	-	-	-
	$p \in [12, 16]$	-	-	-	-	-	-	-	-	-
	$p \in [20, 24]$	-	-	-	-	-	-	-	-	-

Table 6.18: Average time spent walking in the considered scenarios in seconds.

Scenario 1		$s=2$			$s=3$			$s=4$		
Walk time (s)		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p\in[4,8]$	495.0	886.6	1167.6	764.2	1110.6	1359.6	1123.5	1456.5	1487.3
	$p\in[12,16]$	658.2	-	-	993.6	-	-	1261.0	-	-
	$p\in[20,24]$	845.1	-	-	1085.1	-	-	1334.2	-	-
$b=10$	$p\in[4,8]$	833.2	-	-	1175.8	-	-	1438.0	-	-
	$p\in[12,16]$	1060.9	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p\in[4,8]$	1051.5	-	-	1293.1	-	-	-	-	-
	$p\in[12,16]$	-	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-

Scenario 4		$s=2$			$s=3$			$s=4$		
Walk time (s)		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p\in[4,8]$	1588.2	1939.4	2003.7	1935.6	2052.2	2056.8	2149.4	2594.7	2264.0
	$p\in[12,16]$	2525.7	-	-	3083.5	-	-	3497.9	-	-
	$p\in[20,24]$	3765.1	-	-	3227.4	-	-	3647.5	-	-
$b=10$	$p\in[4,8]$	3156.2	-	-	3765.5	-	-	4285.1	-	-
	$p\in[12,16]$	6357.9	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p\in[4,8]$	4861.4	-	-	5786.3	-	-	-	-	-
	$p\in[12,16]$	-	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-

Scenario 6		$s=2$			$s=3$			$s=4$		
Walk time (s)		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p\in[4,8]$	1270.9	1535.4	1565.1	1512.2	1590.3	1815.0	1803.9	2225.1	2075.9
	$p\in[12,16]$	1826.5	-	-	2483.7	-	-	2866.1	-	-
	$p\in[20,24]$	2176.2	-	-	2553.7	-	-	2657.1	-	-
$b=10$	$p\in[4,8]$	2278.0	-	-	3133.8	-	-	3691.9	-	-
	$p\in[12,16]$	4640.3	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p\in[4,8]$	3440.2	-	-	4505.2	-	-	-	-	-
	$p\in[12,16]$	-	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-

Table 6.19: Average of total number of tray retrievals in each replication and in each setting in the considered scenarios.

Scenario 1		$s=2$			$s=3$			$s=4$		
Num.Tray Ret.		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p\in[4,8]$	63.3	160.8	220.1	78.8	138.2	201.9	89.7	162.0	189.3
	$p\in[12,16]$	83.9	-	-	87.4	-	-	119.0	-	-
	$p\in[20,24]$	132.6	-	-	88.6	-	-	114.2	-	-
$b=10$	$p\in[4,8]$	81.2	-	-	101.8	-	-	117.2	-	-
	$p\in[12,16]$	164.0	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p\in[4,8]$	140.6	-	-	137.3	-	-	-	-	-
	$p\in[12,16]$	-	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-

Scenario 4		$s=2$			$s=3$			$s=4$		
Num.Tray Ret.		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p\in[4,8]$	229.4	353.1	382.4	239.9	327.5	358.9	258.2	406.9	343.3
	$p\in[12,16]$	356.5	-	-	481.4	-	-	543.5	-	-
	$p\in[20,24]$	536.4	-	-	537.1	-	-	653.4	-	-
$b=10$	$p\in[4,8]$	453.3	-	-	466.4	-	-	491.0	-	-
	$p\in[12,16]$	977.9	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p\in[4,8]$	719.1	-	-	723.5	-	-	-	-	-
	$p\in[12,16]$	-	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-

Scenario 6		$s=2$			$s=3$			$s=4$		
Num.Tray Ret.		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p\in[4,8]$	194.6	301.2	348.5	182.0	265.3	314.9	211.3	305.0	311.5
	$p\in[12,16]$	290.8	-	-	346.9	-	-	337.0	-	-
	$p\in[20,24]$	394.6	-	-	362.0	-	-	396.2	-	-
$b=10$	$p\in[4,8]$	367.5	-	-	384.7	-	-	393.2	-	-
	$p\in[12,16]$	762.5	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-
$b=15$	$p\in[4,8]$	549.9	-	-	559.4	-	-	-	-	-
	$p\in[12,16]$	-	-	-	-	-	-	-	-	-
	$p\in[20,24]$	-	-	-	-	-	-	-	-	-

CHAPTER 7

CONCLUSION

This study starts with an analysis of the business pains of a job shop manufacturing firm operating in the defense industry. With the help of our observations and discussions in the literature related to job shop manufacturing and defense industry, we list the factors making the responsiveness of a warehouse important in such environments. Although our findings show the importance of responsiveness of the warehouse, the analyzed company's warehouse fails to fulfill a high percent of the pick orders before their due dates. Further investigations show that one of the underlying problems of this issue is the lack of proper solutions for such environments' needs in an automated storage and retrieval system (AS/RS) setup, vertical lift module (VLM) system. VLMs are parts-to-picker AS/RSs that store parts on trays, store trays vertically, and retrieve trays to the input/output (I/O) location when needed. VLMs can also be integrated to form a "Pod", which is a combination of two or more VLM units.

In this study, the VLM Pod decisions related to the throughput of the system are discussed. A time study was conducted at the analyzed company, and a simulation model considering various order picking and storage assignment methods is developed to understand the problem better. The results of the simulation show that the cycle time of a picking task highly depends on the number of tray retrievals, which is determined by the used storage assignment method. In practice, VLM providers support random storage and turnover based storage assignments. On the other hand, most of the existing studies in the related literature also discuss turnover based storage assignments. However, turnover based storage assignment methods assume one-part pick lists or uncorrelated part demands, which are not suited well in a manufacturing environment where the pick orders are based on the bill of materials (BOM) lists.

Based on our observations, we propose an order picking oriented storage location assignment model formulation and its variations for VLM pods in manufacturing environments. The proposed model is tested on small problem instances sampled from the analyzed job shop manufacturing company. Afterward, the obtained optimal solutions for these small problems are used in a VLM pod picking simulation and tested against the current VLM pod configuration at the analyzed company. Results show that for the considered small problem settings, using the proposed location assignment model reduces the average number of tray retrievals for the order picking by 65%, and reduces the average time spent for picking all the orders by 10% in comparison with the current system at the analyzed company.

The proposed model in this study can be used in any warehouse's VLM pod if the pick order information is available beforehand, such as the bill of materials lists of the produced goods in a manufacturing company's warehouse. However, decision-makers must know the model's assumptions first. For example, since our model aims to reduce the walks and waits caused by tray retrievals, its solutions in a system with less walk and waits may have lower benefits in terms of throughput performance. Implementing a model that aims reducing the number of tray retrievals will increase operator utilization. Therefore, after our storage assignment model's implementation, decision makers should consider increasing the number of operators for further improvements in the VLM pod's throughput. Another finding is about the VLM pod design process. According to our computational experiments and the picking simulation, we suggest the practitioners to design VLM pods with tray capacities close to their average pick order sizes, if possible. As expected, tray capacities near the average order sizes increase the benefits obtained from the application of our model, and such tray sizes also reduce the solution times of the storage assignment model.

Besides the other warehouses' VLM pods, our model can also be useful in a problem related to computer science. With the increase in the collection and use of big data, clustered file systems, such as "Hadoop Distributed File System", are becoming more widespread. These file systems often use several servers in different racks and split large files among servers in this network. Each rack has a maximum bandwidth, and connecting to a new rack has a fixed time cost, which is latency. Therefore, file segments should be distributed to minimize the number of used racks while still using as many servers as possible to benefit from parallel processes.

This study contributes to the literature in different ways. First, simulation of various VLM order picking and storage assignment related decisions using real-life data helped us to quantify the impacts of some VLM pod decisions on the system's throughput performance. Moreover, unlike many other studies that assume tray retrieval times and operator task times to be constant, we conducted a time study on a very generic VLM pod and reported our observations regarding the task times of both the tray retrievals and the manual picking operations from the tray. We also emphasized the importance of implementing an order picking oriented storage assignment method in a manufacturing company's VLM using a combination of our findings and previous studies. Instead of using an artificial distance measure in the solution procedure, we modeled our storage assignment problem using a more direct measure based on the number of tray retrievals and total distance to the selected trays from I/O point. Finally, we validated our model by feeding its optimal solutions for the small problem instances obtained in the computational experiments.

Further consideration can be given to address the larger data sets. Although this problem is not one that needs to be solved frequently, getting a solution for the larger data sets using a consumer type computer might be a fruitful research area. If we consider the hierarchical relationships between decision variables, a decomposition method might be evaluated, i.e., $x_{pts}, y_{bts}, w_{pbts}$ in the first subproblem and k_{tfsz} in the second one. After that decomposition, the first subproblem may further be partitioned by separating the data according to the parts that are not related with each other. However, these approaches may not guarantee an optimal solution to the problem.

In this study, all the order picking and part related information are assumed to be readily available in the beginning of the planning horizon. A future study on removing this limitation might be useful. On the other side, discussions in this study on a VLM system's throughput can be extended to include dual-tray VLMs, which are VLMs with two I/O points. Finally, the simulation results in this study show that the operator is likely to be the bottleneck of the system once an order picking oriented storage assignment method is applied. Therefore, a VLM pod with more than one operator can also be a topic of further research.

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APPENDIX A

ANALYSIS OF THE MANUFACTURING COMPANY'S OPERATIONS

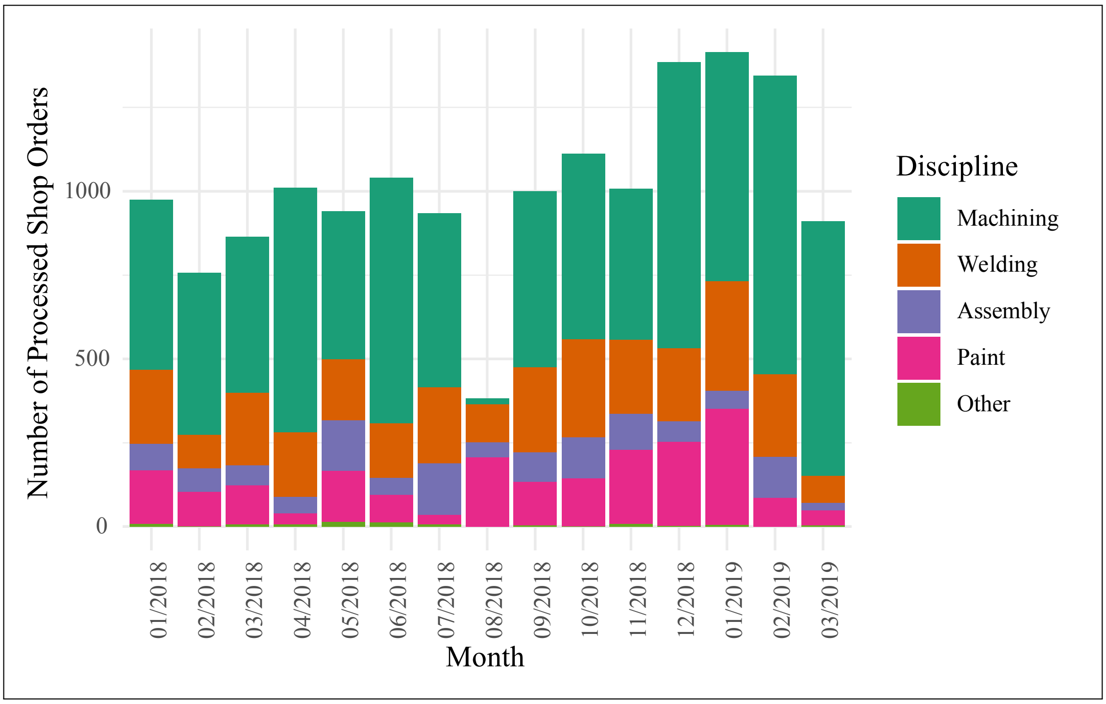

Figure A.1: Number of released manufacturing shop orders in each month between Jan. 2018 and Mar. 2019. In Aug. 2018, facility has been partially closed due to maintenance operations, so the unusual behavior is expected for Aug. 2018.

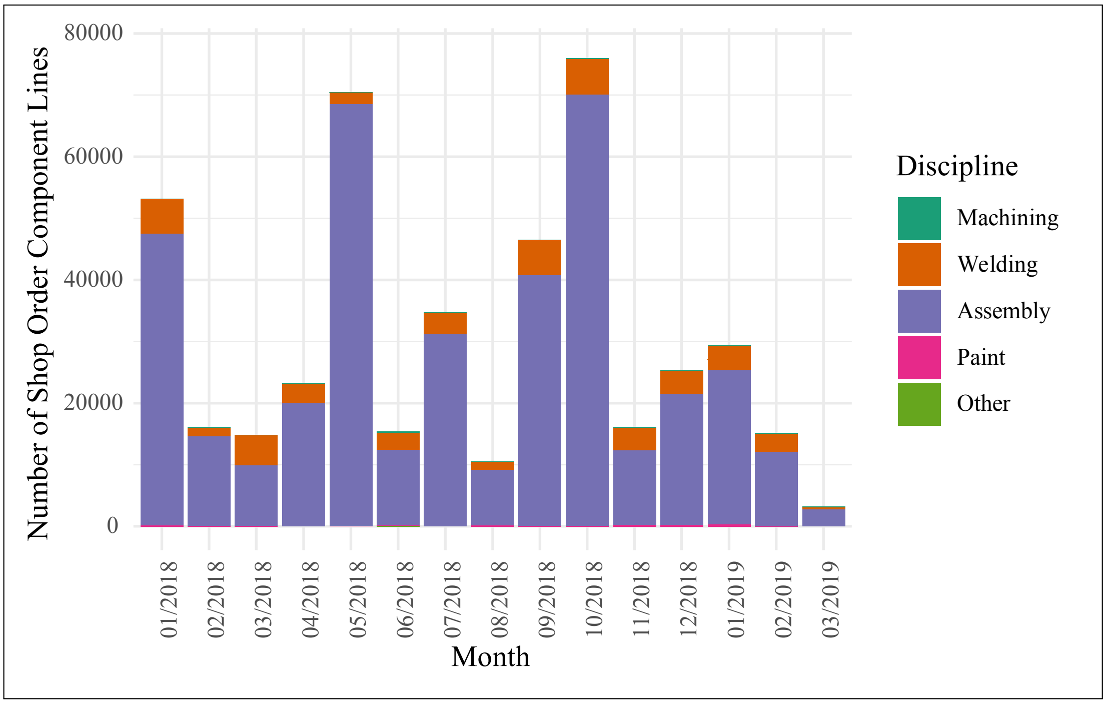

Figure A.2: Total number of component lines for all shop orders released in each month.

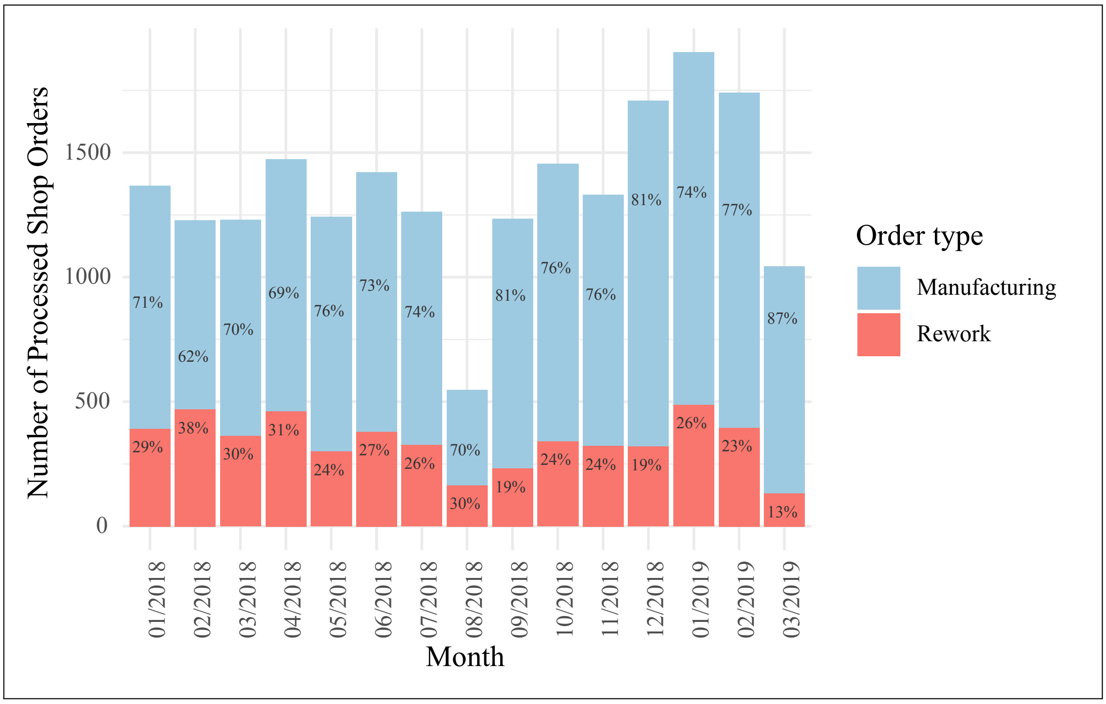

Figure A.3: Number of released manufacturing and rework shop orders in each month between Jan. 2018 and Mar. 2019.

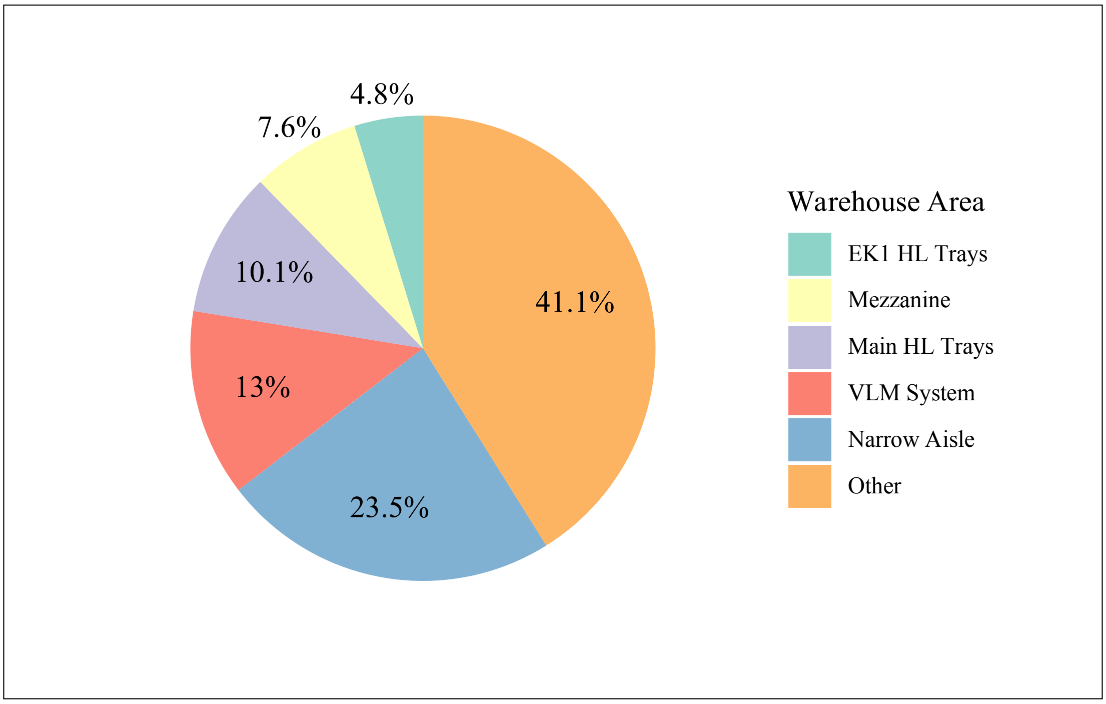

Figure A.4: Distribution of number of locations assigned in the most active warehouse areas.

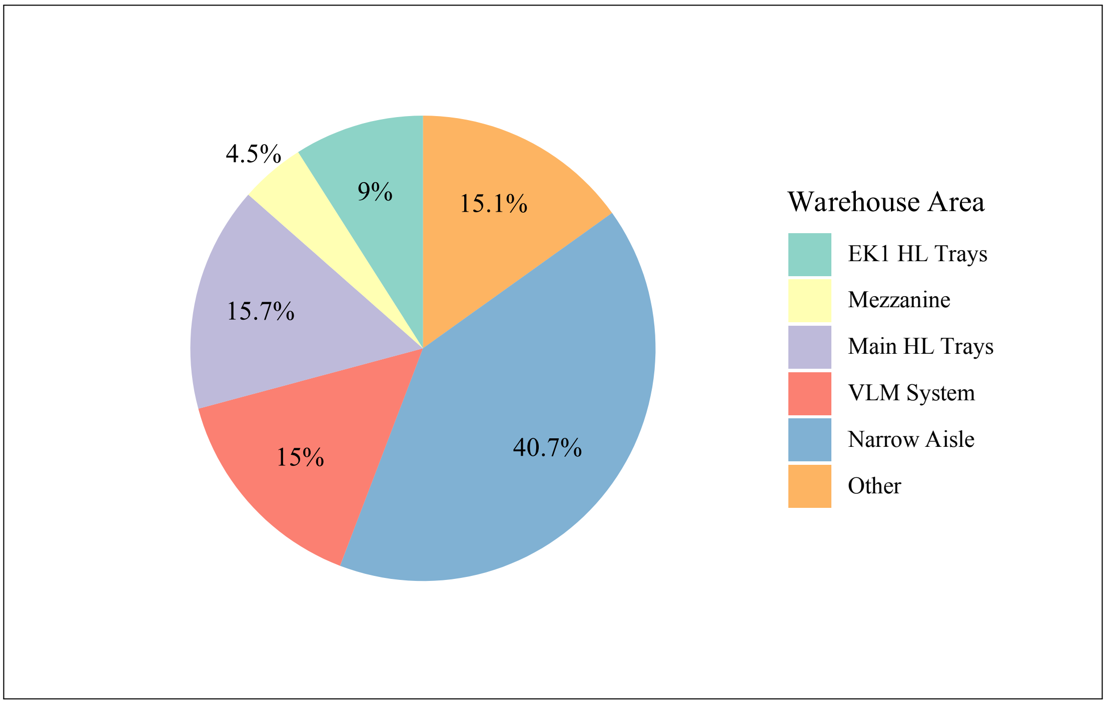

Figure A.5: Distribution of pick operations among the warehouse areas.

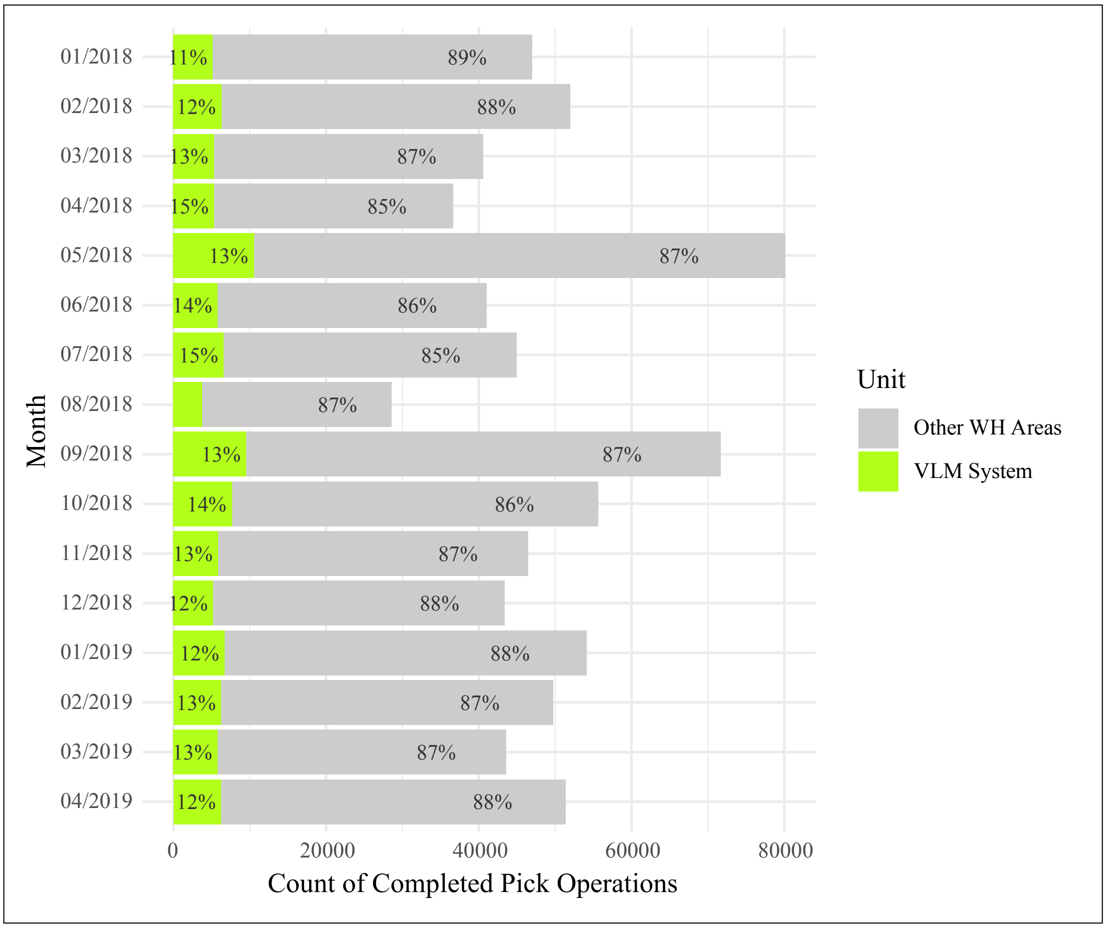

Figure A.6: Monthly breakdown of number of processed order lines.

Table A.1: Basic aggregate descriptors of the activities done in the analyzed VLM System.

	Value for Total Group	Value for Unit 1	Value for Unit 2
Number of orders processed per day	12.4	10.0*	5.3*
Number of order lines processed	112.4	88.9	23.6
Number of order lines per order	9.3	9.1*	4.5*
Number of tray retrievals per day	61	47.4	13.6

*: Some of the orders are being served by both units. Therefore, these values do not add up to the VLM System's totals since such orders are counted for both units.

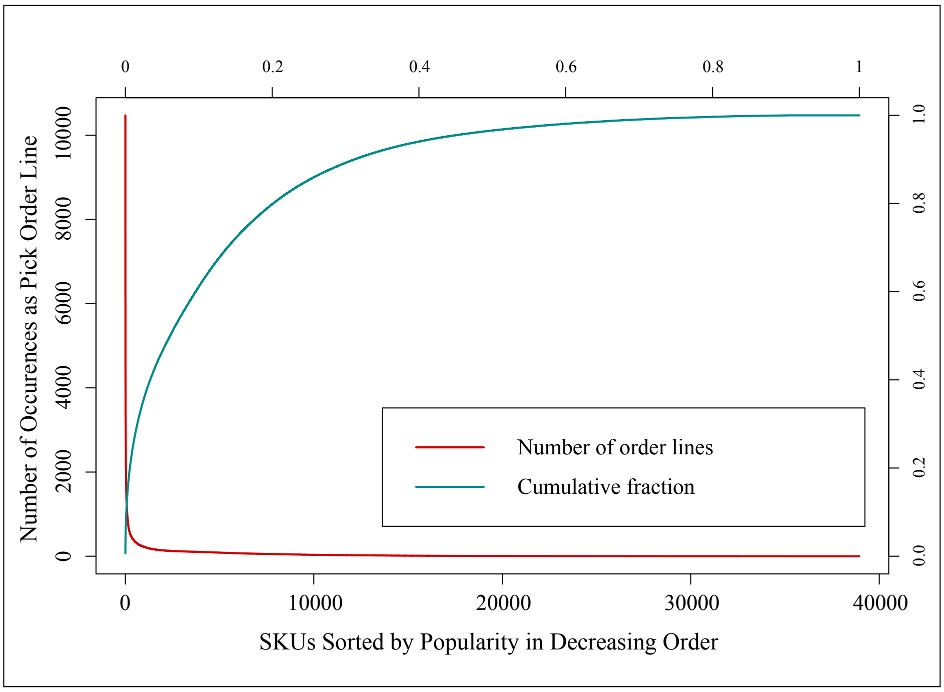

Figure A.7: Distribution of picks over SKUs across the whole warehouse. This Pareto chart justifies the current location area assignment process, which distributes parts among different warehouse areas according to their activity classes.

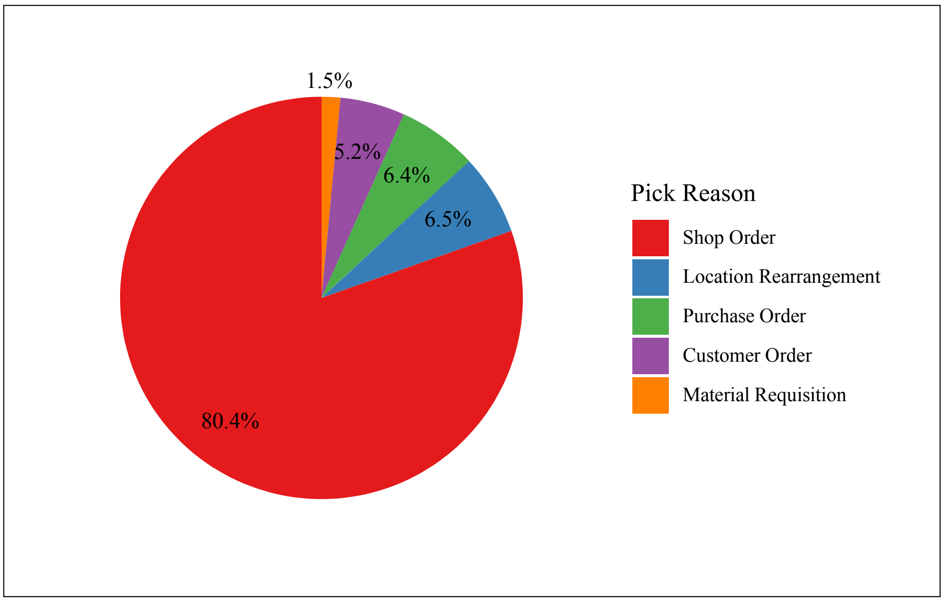

Figure A.8: Breakdown of warehouse pick order line reasons for orders between Jan. 2018 and Mar. 2019. High ratio of shop order picks support imply that the warehouse may benefit from a BOM based storage assignment method.

Figure A.9: Lateness status of previous pick orders for each warehouse area.

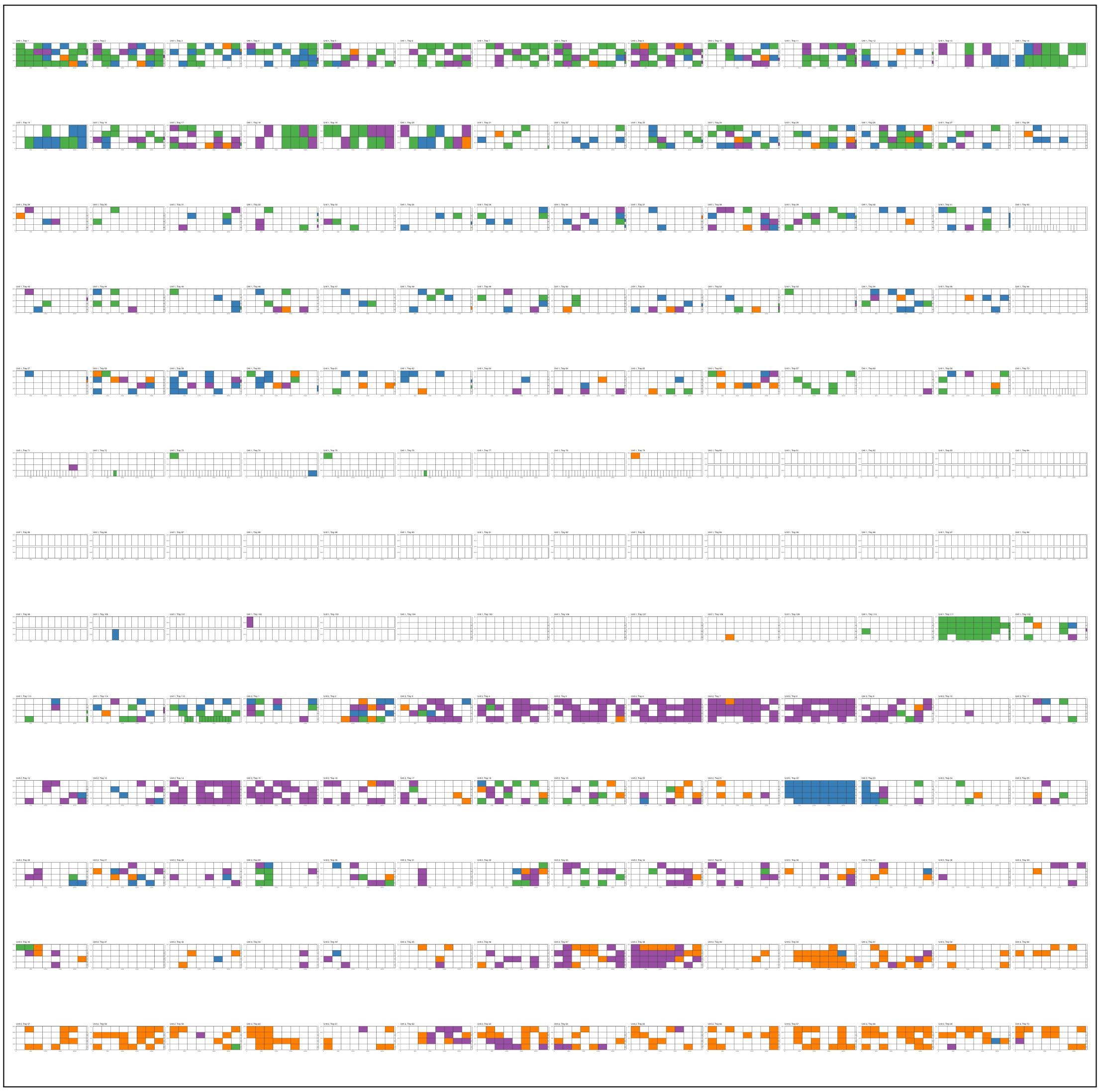

Figure A.10: Locations colored according to their related final product. Each color indicates a different final product and there is no location in this figure that is shared by more than one final product.

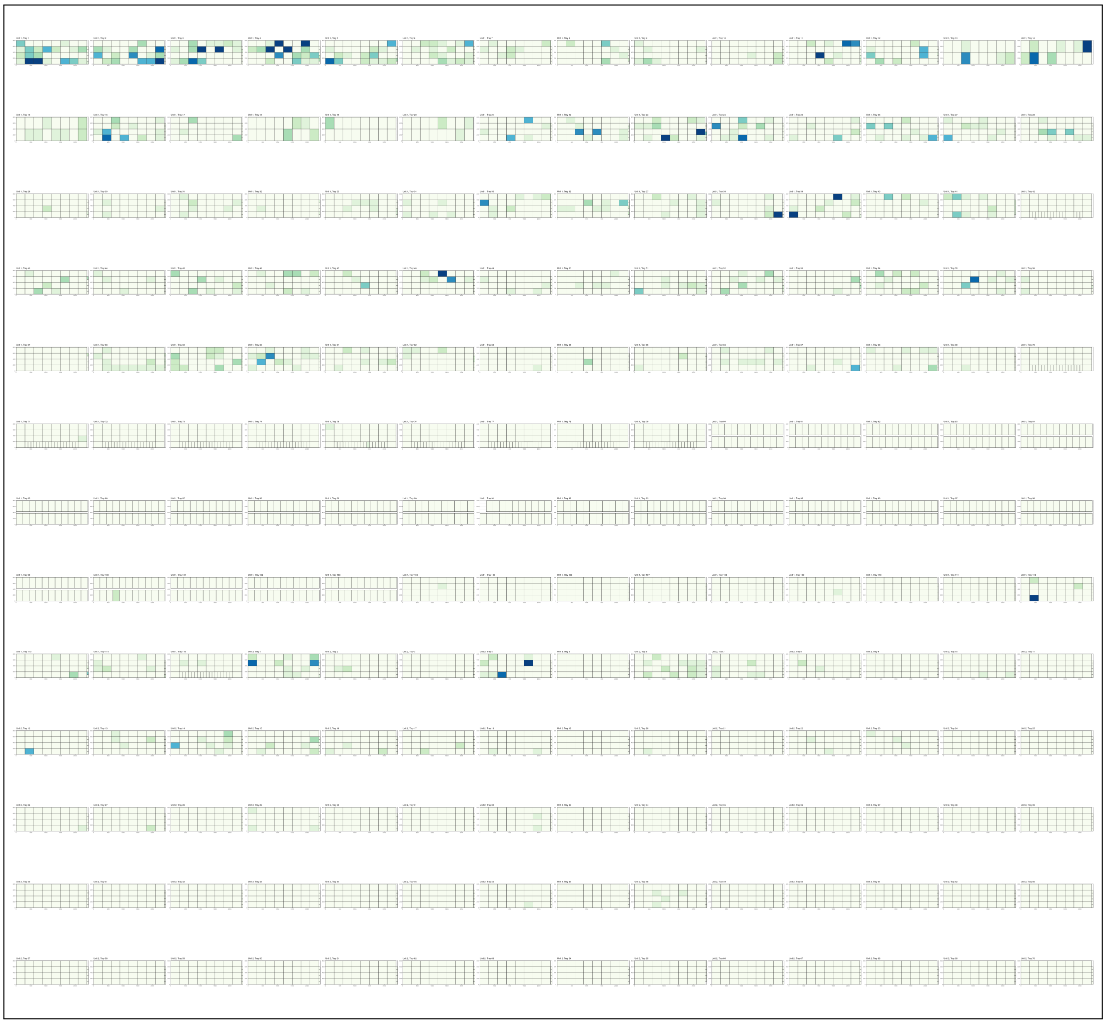

Figure A.11: Heatmap of all locations in the VLM System. Darker colors indicate more activity.

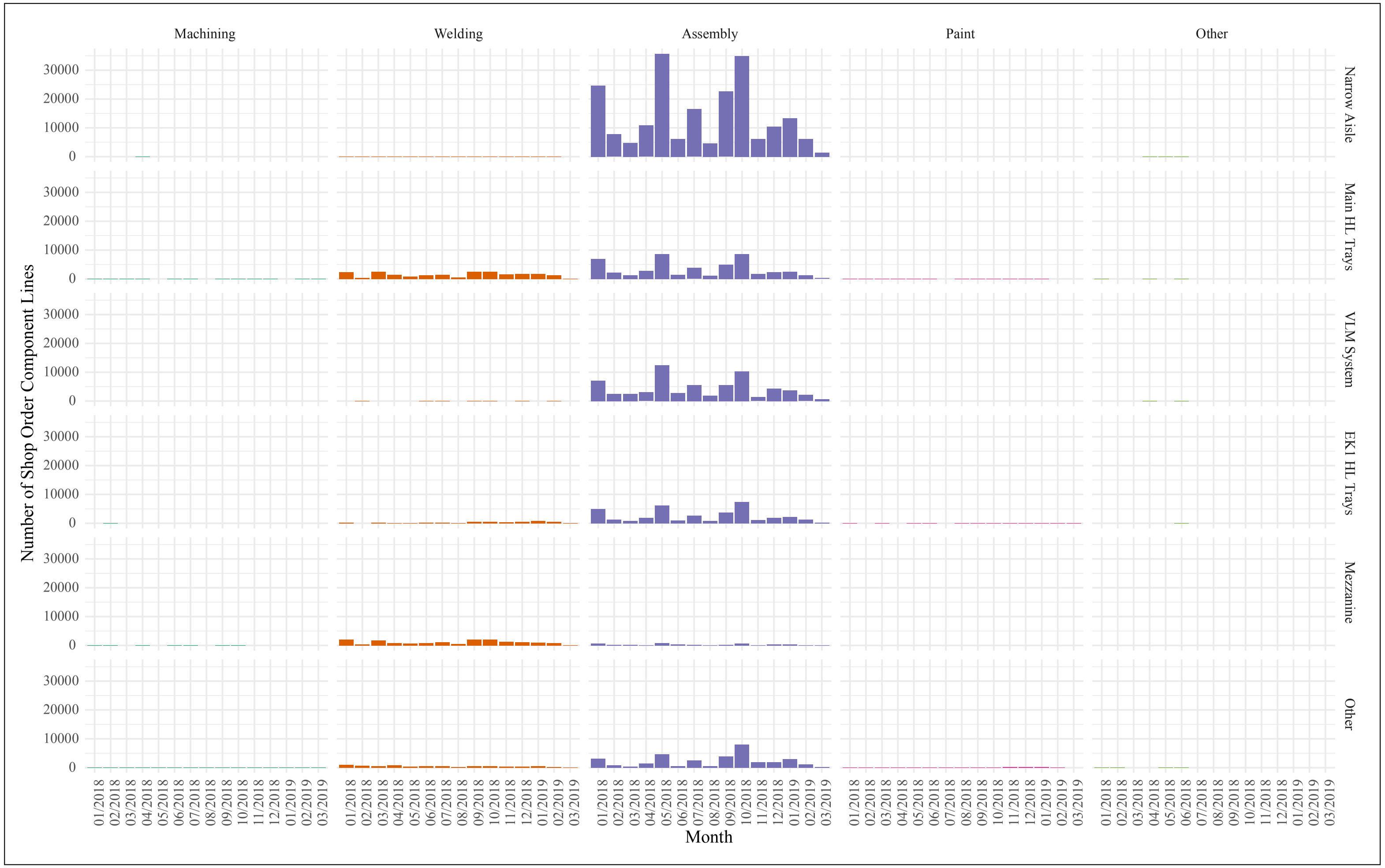

Figure A.12: Distribution of pick operations among the warehouse areas and shop order disciplines.

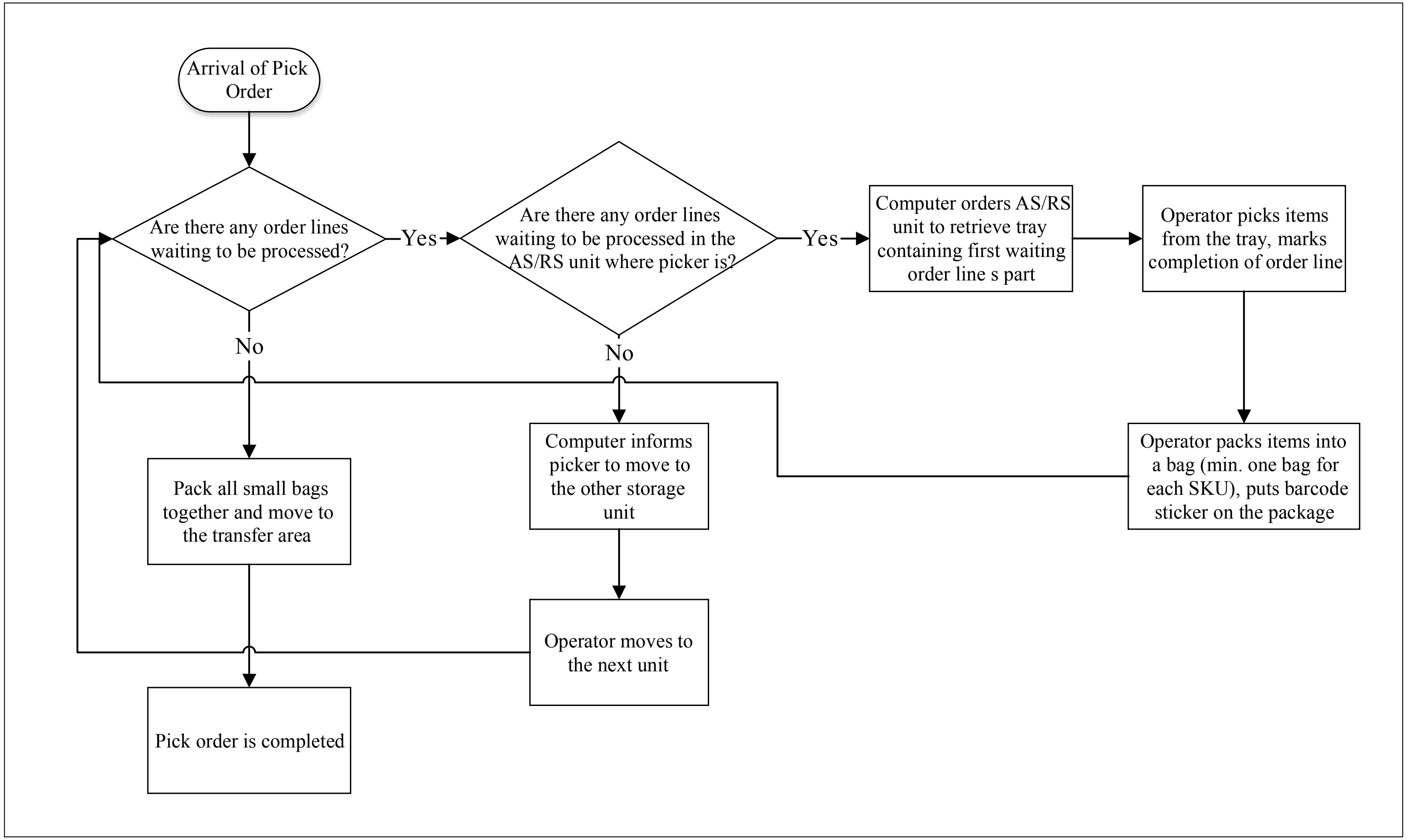

Figure A.13: Flow chart of the pick process in "wait for the next tray" VLM unit sequencing method.

APPENDIX B

PROBLEM DEFINITION

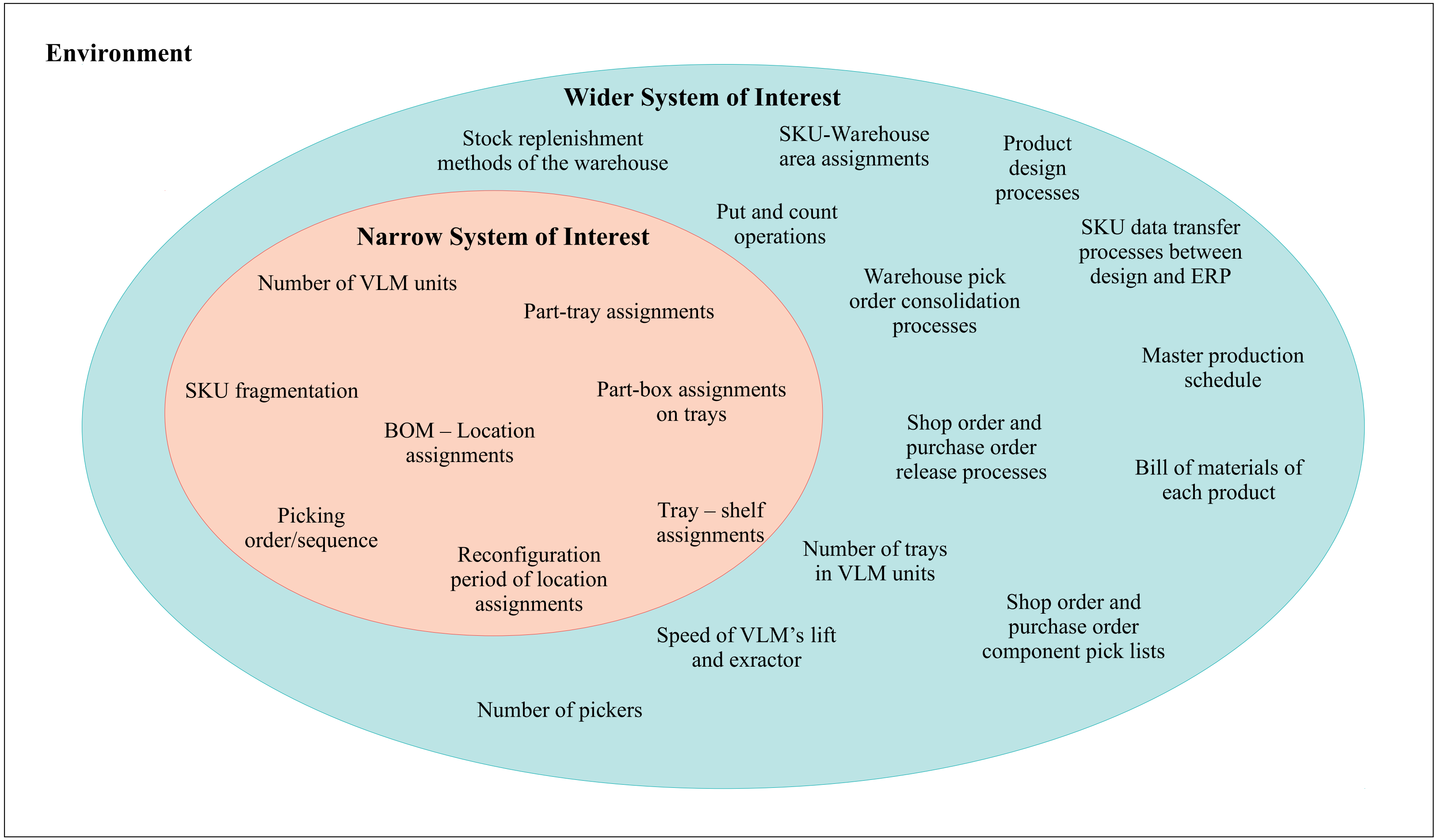

Figure B.1: Defined problem's boundaries.

APPENDIX C

SIMULATION OF VARIOUS VLM POD DECISIONS

Prior to the definition of the problem in this study, an order picking simulation has been run in order to have a better understanding of the impacts of some decisions on the throughput of a VLM system. This simulation's input data are sampled from the transaction history of the AE and the task time parameters have been gathered from the time study observations presented in Section 2.3.2. Main aim of this simulation is to compare basic alternative actions for some of the VLM pod related decisions in terms of picking throughput performance. Considered decisions and alternative actions can be listed as follows:

Location Assignment Rules: As discussed in Chapter 3, storage location assignment rules can be considered as a tactical decision impacting the throughput of VLM systems. Two basic storage assignment rules, **random storage assignments** and **full turnover based storage assignments** are going to be simulated.

Number of VLM Units: As a VLM pod design decision that may have an impact on the system's throughput, the same set of picking tasks will be simulated on a VLM pod consisting of **two VLM units** and on another one having **three VLM units**.

Distance Between VLM Units in the Pod: AE's VLM pod had two VLM units, and these units had a space of more than three meters long between units (see Figure C.1). Therefore, we wanted to see the impact of this space between individual VLM units in the VLM pod in AE, in terms of throughput performance. Therefore, we have two alternatives for the duration of the operator's walk between the VLM units: **3 seconds** and **7 seconds**. For the cases with 3 VLM units, we assumed distances between VLM unit pairs to be equal.

Figure C.1: VLM pod in the AE.

VLM Unit Sequencing: There may be situations that require the use of different trays on different VLM units to complete an order. In this case, the operator must decide whether to wait for the retrieval of the other tray on the same VLM unit (will be referred as "wait") or to go to the other VLM unit on which the tray may be ready (will be referred as "walk"). Best solution for this problem may change according to the task time parameters in each setup. This simulation aims to find an answer for the VLM pod in AE by using the parameters obtained from the time study and the transaction history of the company.

In this study, alternative actions for the mentioned decisions have been used as scenarios. These scenarios can be seen in Table C.1. A simulation of 100 replications have been conducted for each scenario. In each replication, 100 pick orders are randomly selected from a set of 1072 pick orders. Then, the operator's picking tasks and the tray movements in VLM units have been simulated according to the related scenario. Completion times for all tasks (will be referred as $cMax$) is used as the performance measure. Moreover, VLM lift utilization, total number of tray retrievals, total walk duration and the time spent waiting for the tray are also recorded as other performance measures.

Table C.1: Simulated scenarios and their parameters. * indicates the scenario reflecting the current real life situation at the AE.

Scenario No.	Walk Duration Between Units (seconds)	Number of VLMs	Location Assignments	VLM Unit Sequencing Rule
1	3	2	Random	Walk
2	3	2	Random	Wait
3	7	2	Random	Walk
4*	7	2	Random	Wait
5	3	2	Turnover based	Walk
6	3	2	Turnover based	Wait
7	7	2	Turnover based	Walk
8	7	2	Turnover based	Wait
9	3	3	Random	Walk
10	3	3	Random	Wait
11	7	3	Random	Walk
12	7	3	Random	Wait
13	3	3	Turnover based	Walk
14	3	3	Turnover based	Wait
15	7	3	Turnover based	Walk
16	7	3	Turnover based	Wait

C.1 Assumptions

This picking simulation assumes the following:

- All the VLM units in the VLM pod are identical.
- There is no excess storage capacity in the VLM pod.
- All trays have the same box layout and all trays have 38 boxes.
- Different part types cannot be stored in the same box.
- Boxes are large enough to accommodate any number of stored parts.
- After an order's picks are completed, the last tray stays on the I/O point of the VLM unit. In other words, dual cycle tray retrievals are used in between different orders.

- Each order's pick process starts from the closest tray to the I/O point.
- Task times follow normal distribution with the parameters found in time study presented in Section 2.3.2.
- There are enough on hand stock for all parts ordered during each replication. Therefore, there are no stock replenishment operations in the simulated period.
- Only pick tasks arrive into the system. Put and count operations are assumed to be carried on during periods that are not considered in this simulation.
- Operator works at a constant pace.

This simulation's algorithm is given in Algorithm 1. It has been coded and run on R. Three random number generator instances with different seed values have been used for order selection, operator task time generation and VLM unit tray retrieval time generation. Used random number generator instances follow the combined multiple recursive generator sequence proposed by L'Ecuyer (1996), which may be known as combMRG96a. It is a combination of two generators x_n and y_n:
$z_n = (x_n - y_n) \bmod m_1$
$x_n = (a_1 x_{n-1} + a_2 x_{n-2} + a_3 x_{n-3}) \bmod m_1$
$y_n = (b_1 y_{n-1} + b_2 y_{n-2} + b_3 y_{n-3}) \bmod m_2$
where $a_1 = 0$, $a_2 = 63308$, $a_3 = -183326$, $b_1 = 86098$, $b_2 = 0$, $b_3 = -539608$, $m_1 = 2^{31} - 1$ and $m_2 = 2145483479$.

Algorithm 1: Preliminary simulation

Input: pool of orders, time study observations for the machine and operator tasks

Output: for each scenario in Table C.1: completion time of all picks, operator utilization, VLM lift utilization, total number of tray changes, total walk duration, time spent waiting for the tray

for *each replication from **1** to **100*** **do**
- randomly select 100 orders from the list of all orders
- selectedOrderTasks = list of all pick tasks of the selected pick orders (requested part numbers, requested quantities)
- **for** *each line in selectedOrderTasks* **do**
  - generate operator's pick task times according to the time study results and append as new columns to the *selectedOrderTasks* table:
  - -fixed pick time
  - -variable pick time, depending on the number of items picked

for *each scenario in Table C.1* **do**
- assign storage locations for each part according to the current scenario, append location information (vlm.unit, tray.shelf.no) for the requested part as new columns to *selectedOrderTasks* table
- sort *selectedOrderTasks* according to the picking sequence defined in the current scenario
- generate walk durations according to the time study results, where needed
- generate tray retrieval durations according to the time study results, the scenario's picking sequence and the assigned storage locations
- **for** *each line **i** in selectedOrderTasks* **do**
  - get event times from function *eventTimes* (Algorithm 2)
- calculate and save: *completion time of all picks, operator utilization, VLM lift utilization, total number of tray retrievals, total walk duration, time spent waiting for the tray*

Algorithm 2: Function *eventTimes*, Time advancements and recording pick operation event times

Data: current pick task in *selectedOrderTasks* in Algorithm 1, complete *selectedOrderTasks* table in Algorithm 1, all related task durations for the pick tasks in *selectedOrderTasks*

Result: worker start and finish times, tray release and earliest usable times. All times are represented according to the simulation clock as seconds.

```
for each line i in selectedOrderTasks do
    if i is the first line of the VLM unit indicated in line i then
        tray.ready[i] = 40 // assume 40 seconds for the first tray's retrieval
        worker.start[i] = tray.ready[i]
        wait.for.tray[i] = 0
    else
        find j where j<i and selectedOrderTasks[j][vlm.unit] =
          selectedOrderTasks[i][vlm.unit]
        if selectedOrderTasks[i][tray.no] = selectedOrderTasks[j][tray.no]
          then
            tray.ready[i] = tray.ready[j]
        else
            tray.ready[i] = tray.release[j] + tray.time[i]
        worker.start[i] = max(tray.ready[i], worker.finish[i-1] +
          walk.if.applicable[i])
        wait.for.tray[i] = worker.start[i] - worker.finish[i-1] -
          walk.if.applicable[i]
    tray.release[i] = worker.start[i] +
      2/3(selectedOrderTasks[i][variable.pick.time]) +
      selectedOrderTasks[i][fixed.pick.time]
    worker.finish[i] = worker.start[i] +
      selectedOrderTasks[i][variable.pick.time] +
      selectedOrderTasks[i][fixed.pick.time]
```

C.2 Outcomes of the Preliminary VLM Pod Simulations

Since the same pick lists have been simulated for all the scenarios in each replication, they are easily comparable. To quantify the differences of each scenario with the current system (scenario 4), all of the alternatives with 7 seconds of walk duration have been compared with the current system first. The other walk duration alternative (3 seconds) is obviously better, so we excluded them to have a higher confidence level for the overall set of comparisons. With the walk duration alternatives removed, we have 8 scenarios (scenarios 3, 4, 7, 8, 11, 12, 15 and 16) including the current system. Therefore, $8 - 1 = 7$ confidence intervals will be constructed for $\mu_4 - \mu_3$, $\mu_4 - \mu_7$, $\mu_4 - \mu_8$ ... $\mu_4 - \mu_{16}$. To have a $1 - \alpha$ confidence level for the overall comparisons, each confidence level must be constructed at level $1 - \alpha/7$. The constructed confidence intervals on the mean differences can be seen in Table C.2. Each confidence interval is constructed at 99% confidence level. Therefore, according to the results, we can say (with at least 93% confidence level) that all the other alternatives are significantly better than the current system at the AE in terms of throughput performance.

Table C.2: Individual 99 percent confidence intervals for all comparisons with the current system (scenario 4) ($\mu_4 - \mu_j, j = 3, 7, 8, 11, 12, 15, 16$ and μ_i is the mean of all $cMax$ values for scenario i); * denotes significant difference

		Paired-t test	
j	$\overline{cMax_4} - \overline{cMax_j}$	Half-length	Interval
3	13,747.68	269.95	(13,477.74, 14,017.63)*
7	21,938.75	319.48	(21,619.27, 22,258.22)*
8	17,244.58	238.80	(17,005.78, 17,483.37)*
11	15,086.46	293.54	(14,792.93, 15,380)*
12	5,765.08	145.04	(5,620.032, 5,910.118)*
15	21,732.68	303.66	(21,429.02, 22,036.34)*
16	18,659.39	249.93	(18,409.45, 18,909.32)*

C.2.1 Impact of the Distance Between Units on the System Throughput

Even though it is already obvious that having a shorter distance between units is beneficial, we wanted to quantify the effect of the decision to put VLM units with a distance between them. To do this, each scenario have been compared with its alternative with a different walk duration. These alternatives can be seen in Table C.3. According to the results, with a VLM unit sequencing rule as "wait", the expected improvement on the completion time of all pick tasks is only 0.7%. However, the potential improvements would have been more if the VLM system had adopted a "pick and walk to the other unit" rule as a VLM unit sequencing decision for picking, since that rule would involve more walking for the operator. Since the time spent waiting for trays also increase with the shortened distance in "pick and walk" scenarios, it can be said that the VLM unit becomes to be the bottleneck in scenarios 1, 5, 9 and 11. Therefore, the decision maker should not expect improvements more than 3.1% from a change in the indicated walk distances.

Table C.3: Scenarios with 3 seconds of walk between units and their alternatives with a longer walk duration. ** denotes the current system.

Scenario (3 seconds of walk between units)				Alternative with a Walk Distance of 7 Seconds				
Scenario No	cMax (seconds) (average of 100 replications)	Total Walk Duration (seconds) (average of 100 replications)	Time Spent Waiting for Trays (seconds) (average of 100 replications)	Scenario No	cMax (seconds) (average of 100 replications)	Total Walk Duration (seconds) (average of 100 replications)	Time Spent Waiting for Trays (seconds) (average of 100 replications)	% Improvement on cMax if Distance was of 3 seconds
1	74,748	3,234	11,256	3	76,670	6,467	9,947	2.5
2	89,768	661	28,848	4**	90,417	1,323	28,840	0.7
5	66,473	2,141	4,075	7	68,479	4,281	3,943	2.9
6	72,527	652	11,617	8	73,173	1,304	11,614	0.9
9	73,032	2,917	9,856	11	75,331	5,833	9,243	3.1
10	83,678	979	22,440	12	84,652	1,959	22,439	1.2
13	66,765	1,982	4,525	15	68,685	3,965	4,466	2.8
14	70,821	941	9,622	16	71,758	1,882	9,621	1.3

C.2.2 Impact of the Storage Assignment Rule on the System Throughput

Although there can be different solutions to the storage assignment problem, only two simple rules have been simulated to keep the preliminary simulation as simple as possible. These rules were *turnover based storage assignment* where the most requested parts are located on trays that are closer to the I/O point of the VLM units, and *random storage assignment*. As in the previous chapters, pairwise comparisons between each scenario pairs where only different factor in each pair is the storage assignment rule. With this approach, we want to see if a storage assignment rule is dominating the other one in a VLM pod. As seen in Table C.4, with at least 92% confidence level, it can be said that a turnover based storage rule is dominating since it performs better in terms of system throughput performance in all of the scenarios. Please note that, as in Section C.2, the confidence level for all the intervals together is dependent on the number of comparisons made here, too.

Table C.4: Individual 99 percent confidence intervals for all comparisons between random storage assignment and turnover based storage assignment scenarios ($\mu_i - \mu_j, i \in$ set of scenarios with random storage, $j \in$ set of scenarios with the same set of decisions, except with a turnover based storage rule and μ_i is the mean of all $cMax$ values for scenario i); * denotes the cases where using turnover based storage assignments is significantly better.

		Paired-t test		
i	j	$\overline{cMax_i} - \overline{cMax_j}$	Half-length	Interval
1	5	8,274.71	200.39	(8074.32, 8475.10)*
2	6	17,240.27	238.84	(17001.43, 17479.11)*
3	7	8,191.07	185.69	(8005.38, 8376.75)*
4	8	17,244.58	238.80	(17005.78, 17483.37)*
9	13	6,266.44	235.81	(6030.64, 6502.25)*
10	14	12,856.16	196.84	(12659.32, 13053)*
11	15	6,646.21	210.23	(6435.99, 6856.44)*
12	16	12,894.31	196.49	(12697.83, 13090.8)*

C.2.3 Impact of the VLM Unit Sequencing Rules on the System Throughput

The similar approach of comparing the alternatives with respect to the factor in question is also applied for the VLM unit sequencing rules. The details of the comparison can be seen in Table C.5. Since all the confidence intervals on the mean differences do not include 0, it can be said with at least 92% confidence level that the VLM unit sequencing rule as "walk to the other VLM unit after picking" performs better in terms of the system throughput, therefore it is the dominating decision over the other alternative, which is waiting for the next tray in the same VLM unit.

Table C.5: Individual 99 percent confidence intervals for all comparisons between scenarios having different VLM unit sequencing rules: "wait for the next tray in same VLM unit" and "walk to the other VLM unit". ($\mu_i - \mu_j, i \in$ set of scenarios with VLM sequencing rule as "wait", $j \in$ set of the same scenarios, except with the VLM sequencing rule as "walk". μ_i is the mean of all $cMax$ values for scenario i); * denotes the cases where using a VLM sequencing rule as "walk" is significantly better.

		Paired-t test		
i	j	$\overline{cMax_i} - \overline{cMax_j}$	Half-length	Interval
2	1	15,019.64	277.51	(14742.13, 15297.14)*
4	3	13,747.68	269.95	(13477.74, 14017.63)*
6	5	6,054.08	151.56	(5902.51, 6205.64)*
8	7	4,694.17	134.23	(4559.94, 4828.40)*
10	9	10,646.00	258.95	(10387.05, 10904.95)*
12	11	9,321.39	238.13	(9083.26, 9559.52)*
14	13	4,056.28	110.79	(3945.49, 4167.08)*
16	15	3,073.29	96.17	(2977.12, 3169.46)*

C.2.4 Impact of the Number of VLM Units on the System Throughput

To make a comment about the impact of the number of VLM units on the system throughput, each scenario with 2 VLM units in Table C.1 has been compared with the one having all the same parameters with the exception of 3 VLM units. These pairwise comparisons and their outcomes can be seen in Table C.6. According to the results, at a confidence level of at least 92%, a conclusion about an alternative's dominance over the other one cannot be made. The comparisons between scenario 5 and scenario 13; and scenario 7 and scenario 15, show that an additional VLM unit may make the system worse when the storage assignments are made according to the turnover based assignment rule and VLM unit sequencing rule is "walk". Since all the scenarios have only one operator in this study, an additional VLM unit in the VLM pod can be expected to make the system perform worse in some cases. Therefore, the decision maker at the AE should consider the other decisions in the system before making a decision to invest in an additional VLM unit for the VLM pod.

Table C.6: Individual 99 percent confidence intervals for all comparisons between 2 VLM and 3 VLM scenarios ($\mu_i - \mu_j, i \in$ set of scenarios with 2 VLMs, $j \in$ set of the scenarios with the exactly same decisions, except with 3 VLMs and μ_i is the mean of all $cMax$ values for scenario i); * denotes the cases where using 3 VLMs is significantly better, where ** denotes the cases where using 2 VLMs is significantly better in terms of the system throughput.

		Paired-t test		
i	j	$\overline{cMax_4} - \overline{cMax_j}$	Half-length	Interval
1	9	1,716.44	275.65	(1440.78, 1992.09)*
2	10	6,090.07	145.63	(5944.45, 6235.7)*
3	11	1,338.78	251.12	(1087.67, 1589.9)*
4	12	5,765.08	145.04	(5620.03, 5910.12)*
5	13	-291.83	98.38	(-390.21, -193.45)**
6	14	1,705.96	33.98	(1671.99, 1739.94)*
7	15	-206.07	82.51	(-288.58, -123.56)**
8	16	1,414.81	31.85	(1382.97, 1446.66)*

C.3 Conclusion for the Preliminary Order Picking Simulation

According to the results and the mean improvements on the total task completion times, we suggest the decision maker to modify the current system's VLM unit sequencing rule as "walk to the next VLM unit" immediately. Moreover, when compared with the other factors, storage assignment rules' impact is high on the system's throughput performance (see Table C.4), therefore, storage assignment methods beyond the turnover based storage assignment may also improve the order picking performance and they should be investigated.

To conclude, the results of this simulation shows that the AE can benefit from approximately 24% improvements in the VLM pod throughput performance just by changing the VLM unit sequencing rule as "walk" and storage assignment method as "turnover based storage" (from scenario 4 to scenario 7). With this change, the average worker utilization (includes time spent walking) goes up from 68% to 94%, meaning there is little room left for further improvements since the picker is very likely to be the bottleneck of the system. Therefore, if further improvements are going to be searched for, the used methods must reduce the need for the operator's walk between units. In this manner, developing a storage assignment rule that tries to minimize the mean number of tray retrievals during a pick operation might result in even better improvements.

Table C.7: $cMax$ values for each scenario, replications from 1 to 25 in the simulation study.

	$cMax$ Values of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	67,214.42	82,738.71	69,052.47	83,398.02	58,460.29	64,977.12	60,577.83	65,628.00	66,295.72	76,311.08	68,396.66	77,251.89	59,154.11	63,189.63	61,121.58	64,124.05
2	70,129.03	84,188.19	71,939.15	84,823.77	61,934.89	67,731.94	63,824.97	68,349.96	67,804.08	79,220.86	70,159.54	80,171.07	61,964.45	66,117.84	63,797.54	67,011.87
3	65,505.37	81,123.16	67,498.09	81,801.16	58,670.25	64,185.38	60,564.39	64,840.17	65,421.97	75,442.22	67,678.63	76,421.82	59,061.63	62,522.96	60,930.26	63,469.71
4	72,448.07	87,321.64	74,546.29	87,974.74	63,958.64	69,610.96	66,022.13	70,299.31	69,743.99	80,579.80	72,454.52	81,608.88	64,676.19	68,191.19	66,591.94	69,169.21
5	65,374.40	81,461.85	67,361.90	82,105.89	57,632.51	63,723.56	59,668.22	64,340.40	64,255.81	75,240.75	66,589.80	76,196.15	57,570.85	61,956.82	59,572.39	62,885.56
6	63,581.38	76,974.37	65,340.25	77,597.01	56,282.01	61,543.16	58,196.99	62,160.66	62,615.84	71,335.96	64,588.39	72,263.62	56,724.79	59,940.97	58,502.27	60,845.50
7	86,552.59	102,112.54	88,304.43	102,741.42	78,150.83	85,068.10	80,084.54	85,673.63	84,432.14	96,530.58	86,591.04	97,489.50	78,558.85	83,011.96	80,429.98	83,934.09
8	75,351.76	88,285.38	77,239.22	88,895.64	66,229.71	71,783.61	68,156.64	72,383.37	72,478.89	82,593.44	74,743.55	83,510.77	66,203.50	70,238.64	68,085.87	71,103.26
9	82,552.86	97,218.81	84,629.16	97,883.50	73,676.09	79,097.50	75,673.53	79,778.04	80,950.28	90,658.92	83,462.43	91,681.04	74,000.39	77,337.85	75,864.32	78,315.91
10	87,716.28	100,658.70	89,587.72	101,276.20	79,769.23	84,397.97	81,597.93	85,019.33	85,117.18	94,016.96	87,357.56	94,953.17	79,441.89	82,990.23	81,264.97	83,884.52
11	60,781.87	77,297.62	62,635.48	77,996.34	51,759.95	58,067.48	53,738.45	58,760.59	59,650.52	70,730.68	61,832.78	71,763.49	51,707.36	56,273.15	53,722.13	57,267.91
12	79,185.16	92,357.00	80,981.99	93,000.26	70,104.57	75,727.72	72,058.85	76,380.20	75,864.14	86,416.45	78,339.67	87,394.26	70,355.38	74,133.08	72,235.97	75,074.27
13	61,849.42	77,553.08	63,630.95	78,204.23	53,992.19	59,887.58	55,879.38	60,541.87	60,076.22	71,683.49	62,337.33	72,649.13	54,056.78	58,075.24	55,926.30	59,018.87
14	76,076.76	91,984.34	78,007.01	92,636.43	68,081.25	75,263.40	70,246.77	75,891.08	74,065.69	85,462.04	76,544.83	86,437.01	69,169.55	73,449.92	71,103.66	74,374.12
15	70,966.49	85,761.63	72,951.39	86,416.88	63,736.46	68,589.70	65,661.67	69,247.33	70,892.91	80,717.66	73,156.68	81,690.74	63,368.07	66,972.14	65,272.27	67,904.92
16	71,692.29	87,579.66	73,638.88	88,235.23	63,941.49	69,705.21	66,032.17	70,364.14	70,082.90	80,776.47	72,484.04	81,765.75	64,129.22	67,924.15	66,110.06	68,884.01
17	66,549.55	80,691.13	68,536.01	81,339.34	57,733.86	63,809.55	59,732.71	64,464.46	63,059.48	75,013.89	65,486.10	76,032.73	57,466.43	61,811.68	59,469.27	62,784.28
18	73,462.56	86,628.40	75,298.23	87,260.74	66,563.99	71,485.21	68,371.46	72,115.72	72,868.37	81,924.54	75,000.93	82,884.25	66,251.19	69,992.32	68,127.73	70,914.40
19	60,188.17	75,408.24	62,048.12	76,066.53	52,165.67	58,328.32	54,154.38	58,969.09	59,140.18	69,198.05	61,364.08	70,161.87	52,199.59	56,598.70	54,164.17	57,536.57
20	86,647.98	101,845.85	88,574.98	102,514.78	78,935.67	85,084.88	80,985.08	85,745.55	83,826.73	95,893.78	86,211.65	96,868.10	78,842.79	83,254.35	80,825.55	84,218.50
21	64,921.21	79,804.57	66,672.41	80,479.94	56,115.28	62,423.88	58,148.63	63,106.19	63,356.75	74,258.02	65,614.51	75,286.32	55,899.37	60,613.81	57,970.76	61,593.31
22	64,584.49	80,052.47	66,457.72	80,714.53	56,704.74	62,573.68	58,657.72	63,226.31	63,059.05	73,190.79	65,163.28	74,161.89	56,580.24	60,983.78	58,517.64	61,908.32
23	75,234.12	90,048.66	77,264.29	90,682.70	67,618.64	73,100.02	69,633.67	73,721.42	75,204.44	84,261.44	77,284.62	85,198.56	67,929.96	71,368.12	69,790.23	72,285.84
24	81,967.91	97,166.11	83,866.95	97,783.38	74,633.99	80,664.15	76,646.32	81,290.42	80,303.82	91,006.17	82,551.18	91,974.41	75,280.43	78,978.99	77,101.21	79,887.41
25	74,032.02	91,108.21	76,270.69	91,766.30	66,572.52	73,043.29	68,762.48	73,699.98	74,927.79	84,588.09	77,139.43	85,583.00	66,748.11	71,243.23	68,862.90	72,213.28

Table C.8: $cMax$ values for each scenario, replications from 26 to 50 in the simulation study.

	$cMax$ Values of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
26	97,034.30	112,532.24	99,232.88	113,180.32	89,661.50	95,209.61	91,671.82	95,853.45	94,941.93	105,727.30	97,550.32	106,713.17	89,764.35	93,521.88	91,640.10	94,469.08
27	76,668.85	91,683.15	78,699.90	92,314.95	68,272.25	74,705.78	70,259.00	75,331.66	73,874.68	86,343.79	76,373.13	87,305.12	68,241.82	72,881.87	70,232.50	73,800.70
28	71,374.27	85,431.29	73,264.81	86,081.23	64,052.85	69,971.37	65,973.14	70,595.64	70,274.83	79,954.34	72,544.26	80,922.87	63,863.51	68,080.16	65,771.09	69,013.40
29	81,900.37	96,276.49	83,886.45	96,897.91	74,861.08	80,192.49	76,775.77	80,814.49	79,924.23	90,963.04	82,462.23	91,907.10	74,904.30	78,567.93	76,810.79	79,497.12
30	82,376.60	97,135.24	84,286.79	97,788.96	74,163.50	79,775.23	76,118.36	80,421.34	79,480.38	91,319.19	82,095.39	92,309.40	74,025.85	78,296.16	75,968.44	79,219.34
31	80,877.62	94,697.39	82,902.80	95,386.70	71,783.71	77,762.24	73,825.83	78,430.71	77,910.79	88,661.72	80,380.46	89,695.09	72,514.03	76,105.94	74,402.28	77,041.84
32	80,576.11	97,661.08	82,733.20	98,313.24	72,073.26	78,226.08	74,159.04	78,877.53	78,184.64	90,375.21	80,882.56	91,340.50	71,998.47	76,456.41	74,032.35	77,398.67
33	86,789.74	101,726.09	88,711.43	102,360.54	78,264.95	84,537.40	80,296.79	85,170.34	85,120.72	95,122.85	87,375.00	96,085.28	78,877.95	82,781.75	80,733.67	83,705.61
34	100,374.81	114,506.35	102,255.61	115,147.93	89,631.77	96,732.35	91,855.40	97,384.47	96,629.72	107,569.34	99,098.86	108,539.49	90,958.69	94,905.67	92,948.79	95,866.65
35	63,670.86	79,586.76	65,639.74	80,238.96	55,753.39	62,500.07	57,739.61	63,151.52	63,107.65	73,580.60	65,249.10	74,572.97	55,710.86	60,627.23	57,741.92	61,576.19
36	70,596.54	84,863.14	72,666.26	85,504.96	61,870.10	67,692.38	63,847.20	68,336.96	68,571.87	78,390.25	70,945.11	79,383.54	61,908.51	65,937.21	63,837.63	66,868.14
37	88,258.01	101,984.20	90,117.41	102,614.69	79,782.08	85,793.98	81,769.16	86,431.81	85,967.97	96,036.07	88,113.20	97,003.99	79,569.42	84,032.06	81,568.46	84,976.72
38	82,031.80	96,819.98	83,906.16	97,472.08	75,162.28	80,442.91	77,035.09	81,080.43	81,848.55	90,706.58	83,831.57	91,670.03	75,201.52	78,828.78	77,026.58	79,756.41
39	67,919.87	84,344.42	69,562.24	85,028.25	58,825.17	65,712.64	60,896.41	66,386.21	67,251.33	78,029.44	69,239.39	79,044.89	59,570.98	64,061.33	61,521.25	65,015.30
40	73,493.34	87,612.93	75,541.95	88,256.38	66,009.71	71,566.93	67,931.37	72,198.16	71,967.37	82,167.11	74,232.00	83,103.52	65,956.00	69,973.36	67,861.66	70,871.01
41	89,085.48	103,772.27	90,995.16	104,423.09	81,106.02	87,160.01	83,132.43	87,814.04	87,979.64	97,818.85	90,129.22	98,780.61	81,907.01	85,498.21	83,784.72	86,434.64
42	66,964.90	80,009.53	68,760.06	80,642.80	57,965.34	63,960.72	59,952.21	64,594.70	65,040.94	74,056.07	67,101.76	75,004.81	58,501.20	62,355.08	60,404.62	63,269.01
43	63,236.60	77,745.81	65,197.87	78,384.02	54,597.47	60,834.21	56,658.03	61,484.14	60,904.59	70,990.05	63,272.43	71,967.52	55,159.92	59,132.03	57,063.41	60,067.34
44	81,736.24	95,771.90	83,712.92	96,401.77	73,868.06	79,943.93	75,907.20	80,554.94	80,480.66	90,486.56	82,653.36	91,424.49	74,348.14	78,231.93	76,238.63	79,130.00
45	66,870.54	82,650.98	68,832.72	83,273.70	58,603.27	65,199.19	60,804.84	65,825.56	66,388.09	76,214.09	68,539.91	77,127.91	59,322.44	63,433.97	61,288.85	64,359.82
46	67,552.38	81,510.07	69,410.31	82,148.05	57,696.98	64,579.77	59,776.35	65,211.01	64,538.91	75,477.91	66,887.90	76,461.84	58,325.95	62,886.67	60,260.62	63,804.62
47	88,471.18	102,790.24	90,392.47	103,423.39	80,575.35	86,293.24	82,503.54	86,921.35	87,454.46	97,781.76	89,799.37	98,720.70	80,903.50	84,650.08	82,756.38	85,558.67
48	78,097.20	92,360.31	79,960.23	92,987.10	69,237.03	75,432.26	71,211.38	76,057.02	76,384.77	86,953.46	78,585.72	87,886.88	69,913.01	73,880.01	71,750.00	74,788.68
49	73,279.23	87,720.33	75,007.23	88,344.67	65,348.80	71,174.37	67,189.80	71,793.15	72,807.62	81,948.82	74,941.96	82,894.71	65,796.00	69,542.00	67,597.95	70,458.01
50	83,583.26	98,786.89	85,757.48	99,431.65	74,733.99	80,570.90	76,787.90	81,213.32	80,452.11	92,358.34	83,028.85	93,333.35	75,084.12	78,889.30	77,046.61	79,829.81

Table C.9: $cMax$ values for each scenario, replications from 51 to 75 in the simulation study.

	$cMax$ Values of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
51	81,211.59	96,409.95	83,094.40	97,088.64	71,838.81	77,797.47	73,888.00	78,477.55	78,146.17	89,839.46	80,542.47	90,867.71	71,957.57	76,153.18	73,905.21	77,115.93
52	96,375.31	111,933.73	98,500.20	112,581.08	88,321.61	94,684.16	90,333.44	95,323.31	95,451.84	106,966.72	97,679.41	107,918.80	88,645.34	93,002.58	90,616.04	93,939.18
53	63,145.75	80,019.48	65,312.24	80,687.72	55,426.98	61,875.49	57,548.12	62,542.63	61,127.59	73,184.11	63,753.57	74,189.09	55,821.84	60,211.66	57,840.22	61,160.22
54	70,842.42	85,128.27	72,737.81	85,780.18	62,844.80	68,713.84	64,755.65	69,355.72	67,332.96	78,853.17	69,770.50	79,826.48	63,330.12	67,295.72	65,175.86	68,205.14
55	67,943.76	81,619.37	69,707.90	82,258.07	60,356.28	65,383.86	62,077.30	66,000.97	67,623.96	75,988.05	69,544.08	76,941.18	60,214.21	63,769.59	61,937.15	64,666.67
56	67,846.91	81,657.19	69,701.81	82,326.57	59,382.19	64,938.30	61,291.92	65,600.06	66,516.39	75,951.98	68,643.11	76,946.16	60,153.66	63,249.42	61,914.59	64,189.52
57	68,740.92	85,649.48	70,798.90	86,289.42	62,102.90	68,142.92	64,073.40	68,761.77	68,011.28	79,280.49	70,419.45	80,231.41	62,612.15	66,331.84	64,428.42	67,249.12
58	69,386.06	84,369.72	71,117.17	85,034.01	60,403.50	66,487.14	62,390.79	67,130.97	66,049.97	77,857.24	68,407.03	78,831.71	60,548.86	64,745.42	62,488.46	65,689.08
59	71,401.64	85,506.98	73,267.04	86,139.19	63,325.57	69,067.67	65,237.59	69,695.78	69,751.67	79,874.75	71,975.69	80,832.32	63,851.77	67,533.54	65,644.38	68,446.01
60	103,675.81	117,789.36	105,527.58	118,437.29	95,163.75	100,898.44	97,157.91	101,551.59	100,268.65	111,839.95	102,644.06	112,814.16	95,969.89	99,384.53	97,732.83	100,314.13
61	68,030.57	82,507.67	69,772.63	83,149.02	60,231.69	65,744.21	62,144.87	66,383.47	65,170.38	76,437.80	67,517.01	77,388.65	60,434.50	64,119.35	62,258.05	65,040.08
62	66,606.60	83,271.60	68,426.29	83,920.10	57,707.22	64,480.05	59,671.82	65,112.56	65,341.65	77,366.72	67,515.47	78,342.18	57,778.42	62,576.17	59,743.13	63,517.41
63	85,253.53	99,384.10	87,156.03	100,033.59	77,330.82	82,559.75	79,185.46	83,188.55	83,615.56	93,717.13	85,987.74	94,673.45	77,736.95	81,041.34	79,472.29	81,947.36
64	64,697.43	79,705.37	66,492.53	80,342.73	55,992.81	62,347.50	57,966.52	62,987.95	60,714.37	73,553.54	63,086.97	74,534.28	56,375.32	60,499.10	58,259.03	61,416.69
65	66,175.91	81,081.79	68,027.51	81,736.60	58,114.27	64,710.62	60,280.57	65,380.55	64,342.52	75,431.53	66,736.84	76,422.10	58,749.55	62,910.44	60,805.56	63,883.93
66	73,399.71	88,465.91	75,531.09	89,104.09	65,983.46	71,331.15	67,961.07	71,968.79	72,012.19	81,800.33	74,430.23	82,767.63	66,116.24	69,830.59	67,983.87	70,740.61
67	71,210.94	86,234.90	72,912.87	86,889.46	62,811.65	68,922.59	64,694.73	69,578.39	70,257.09	79,818.27	72,330.39	80,808.82	63,118.54	67,115.21	65,000.13	68,063.90
68	72,416.95	88,564.86	74,288.55	89,232.15	64,420.13	70,609.99	66,487.27	71,266.93	71,077.91	81,823.81	73,421.58	82,820.64	64,741.29	68,685.56	66,691.07	69,656.35
69	68,281.45	82,884.89	70,115.71	83,529.31	59,725.17	66,660.65	61,831.10	67,319.22	66,255.07	76,927.55	68,611.44	77,905.09	59,872.35	64,778.91	61,901.27	65,729.10
70	86,951.64	102,083.80	89,020.30	102,735.15	78,414.22	84,072.25	80,401.06	84,711.29	86,192.72	95,694.79	88,295.52	96,663.64	78,630.75	82,600.17	80,535.61	83,532.68
71	69,769.87	84,278.14	71,740.53	84,960.20	61,154.34	67,226.08	63,200.49	67,881.19	66,934.99	78,000.18	69,357.44	78,998.05	61,218.49	65,600.27	63,237.31	66,542.04
72	78,682.18	93,459.29	80,635.61	94,124.52	72,220.34	77,144.18	74,086.41	77,796.65	78,263.67	87,793.01	80,451.09	88,782.01	71,963.32	75,563.71	73,763.93	76,501.10
73	71,431.92	87,591.43	73,412.08	88,234.90	62,365.99	68,894.86	64,373.68	69,538.98	69,842.92	81,034.03	72,166.19	81,986.95	62,596.89	67,150.15	64,528.92	68,083.05
74	62,620.75	78,733.00	64,645.45	79,387.70	54,006.56	60,742.37	56,153.79	61,379.87	60,299.54	71,880.60	62,590.14	72,854.12	54,692.94	58,899.73	56,710.53	59,865.83
75	75,576.95	90,397.87	77,342.63	91,007.67	67,177.90	73,808.38	69,235.59	74,424.63	72,769.29	84,314.09	75,105.95	85,262.55	67,464.29	72,050.00	69,406.28	72,965.66

Table C.10: $cMax$ values for each scenario, replications from 76 to 100 in the simulation study.

Replication	$cMax$ Values of Each Scenario															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
76	76,292.13	92,980.19	78,491.94	93,626.97	67,631.12	74,602.91	69,804.65	75,243.58	75,369.52	86,323.42	77,958.33	87,296.80	68,679.65	72,853.28	70,609.53	73,780.47
77	64,206.98	81,116.09	66,223.90	81,776.91	56,086.49	62,727.70	58,111.39	63,398.34	62,406.32	74,596.81	64,858.95	75,600.74	56,272.05	60,794.33	58,281.13	61,766.62
78	90,243.88	104,870.83	92,253.18	105,538.62	81,695.56	87,191.54	83,687.74	87,841.69	87,659.52	98,022.78	89,995.72	99,028.31	81,767.68	85,356.65	83,699.95	86,324.05
79	80,596.65	96,250.94	82,409.77	96,886.54	73,776.04	79,072.41	75,703.85	79,710.03	80,767.98	89,756.76	83,001.96	90,697.04	73,958.61	77,404.39	75,799.16	78,340.92
80	73,821.39	87,749.33	75,839.08	88,403.98	65,177.90	70,990.48	67,152.81	71,661.62	71,167.59	82,494.80	73,493.31	83,486.25	64,827.82	69,215.01	66,767.67	70,172.22
81	63,673.85	80,017.51	65,590.99	80,673.16	52,964.33	59,923.82	55,117.51	60,582.59	60,348.97	73,067.29	62,746.72	74,060.00	53,900.08	58,291.47	55,843.21	59,230.39
82	80,856.06	96,123.84	82,875.33	96,744.55	72,898.89	79,361.64	75,020.02	79,994.79	78,308.68	89,840.30	80,718.83	90,783.54	73,440.10	77,587.53	75,443.43	78,499.51
83	76,579.36	93,348.56	78,511.64	94,008.50	68,307.98	74,986.78	70,341.63	75,631.80	76,973.10	87,223.71	79,241.39	88,185.52	68,531.17	73,146.52	70,503.35	74,065.74
84	66,202.08	82,180.93	68,272.58	82,837.25	57,757.37	64,745.74	59,937.86	65,395.06	63,950.79	75,519.27	66,245.85	76,492.20	58,218.70	62,983.37	60,289.34	63,930.51
85	63,302.80	79,345.87	65,197.18	80,025.66	54,344.50	61,453.83	56,474.14	62,137.30	61,764.37	73,040.42	64,108.85	74,050.42	55,338.93	59,683.52	57,213.52	60,651.04
86	71,657.45	86,099.10	73,615.13	86,744.02	63,316.55	69,290.12	65,313.07	69,937.35	71,144.76	80,238.59	73,196.07	81,227.99	63,493.58	67,589.28	65,397.37	68,507.34
87	73,454.94	87,282.40	75,213.86	87,941.08	63,847.12	69,610.80	65,804.71	70,257.44	70,470.14	81,436.76	72,706.23	82,430.73	63,846.68	67,814.87	65,791.05	68,771.16
88	91,569.61	105,177.96	93,356.74	105,831.88	82,682.81	88,178.09	84,603.56	88,835.99	88,741.79	100,177.69	91,029.76	101,160.39	82,687.59	86,585.18	84,527.58	87,511.24
89	74,243.54	89,195.99	76,122.23	89,845.24	65,470.73	72,251.78	67,575.49	72,889.05	71,949.89	83,030.29	74,252.77	84,002.67	65,930.37	70,394.75	67,897.13	71,329.23
90	58,822.05	74,075.36	60,472.52	74,714.57	51,278.92	56,869.16	53,137.55	57,513.09	57,074.20	67,376.24	59,206.17	68,358.46	51,086.20	55,244.88	52,987.82	56,168.78
91	64,872.03	81,562.02	66,979.53	82,220.89	56,693.63	63,246.17	58,777.87	63,894.75	64,627.38	75,747.80	67,002.58	76,736.82	57,390.43	61,412.25	59,328.66	62,353.06
92	74,660.62	88,812.99	76,602.17	89,444.63	66,439.58	72,645.46	68,507.22	73,301.35	73,379.00	83,072.25	75,600.23	84,036.16	67,237.26	71,058.69	69,129.69	71,996.79
93	69,882.99	85,122.03	71,767.83	85,793.16	60,906.25	67,404.90	63,064.34	68,083.39	69,581.07	78,885.01	71,701.22	79,892.33	61,324.63	65,666.47	63,394.69	66,637.64
94	81,526.59	97,510.50	83,330.36	98,183.16	72,246.02	79,572.16	74,441.03	80,244.52	80,053.22	91,445.93	82,199.97	92,428.96	72,500.18	77,703.14	74,592.63	78,676.95
95	72,206.23	87,927.75	74,064.98	88,603.32	62,899.26	69,502.84	64,964.72	70,174.60	71,487.49	81,480.23	73,832.62	82,480.66	64,008.40	67,831.16	65,884.42	68,798.33
96	80,717.81	94,961.47	82,602.44	95,620.18	72,498.74	77,970.24	74,498.77	78,610.11	77,836.17	88,954.48	80,263.03	89,924.18	72,295.13	76,362.44	74,271.61	77,305.99
97	76,174.25	92,329.67	78,125.61	93,001.34	67,677.04	73,805.82	69,703.87	74,474.05	74,107.90	85,318.09	76,368.80	86,301.16	68,010.07	72,088.19	70,003.57	73,059.38
98	85,461.53	101,469.52	87,442.01	102,135.95	78,006.97	84,432.36	79,960.01	85,072.09	85,507.90	96,396.60	87,780.79	97,388.78	78,821.81	82,605.90	80,623.38	83,554.33
99	70,525.50	87,941.96	72,493.58	88,603.55	62,488.35	68,465.39	64,539.73	69,115.35	69,821.72	81,044.55	72,102.97	82,031.91	62,106.46	66,745.28	64,196.99	67,708.60
100	84,180.94	97,227.52	86,052.46	97,871.19	76,083.39	81,341.21	78,026.66	81,979.89	82,297.12	91,946.30	84,638.69	92,930.02	76,568.69	79,889.83	78,352.92	80,801.70

Table C.11: Operator's utilization levels for each scenario, replications from 1 to 25 in the simulation study.

	%Operator Utilization Levels of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	82	64	85	64	93	81	94	82	83	70	84	70	92	84	92	84
2	84	67	86	67	93	83	94	84	87	72	88	72	93	86	94	86
3	85	65	87	65	92	82	93	82	84	70	85	71	91	85	92	85
4	85	67	87	67	94	84	94	85	88	73	89	74	93	87	93	87
5	83	64	86	64	93	81	93	82	84	69	86	70	93	84	93	85
6	84	66	86	66	93	83	93	83	84	72	86	72	92	85	92	86
7	86	71	88	71	94	85	94	85	88	75	89	75	93	87	94	87
8	84	69	86	69	94	85	95	85	87	74	89	75	94	87	95	87
9	86	70	88	70	95	86	95	86	87	76	88	76	94	89	94	89
10	88	74	89	74	95	88	95	89	90	80	91	80	96	90	96	90
11	79	59	82	59	91	78	91	79	80	65	82	65	91	81	91	82
12	85	71	87	71	95	86	95	86	89	76	90	76	94	88	95	88
13	82	62	85	62	92	80	92	81	84	68	86	68	91	83	92	84
14	85	67	87	68	94	82	94	83	87	73	88	73	92	85	92	85
15	85	68	87	68	93	84	93	85	85	72	86	73	93	87	94	87
16	85	66	87	66	93	83	93	83	86	72	87	72	92	86	93	86
17	81	64	83	64	92	81	92	81	85	69	87	69	92	84	92	84
18	86	70	88	71	94	85	94	85	86	75	88	75	94	88	94	88
19	82	62	84	62	92	80	93	80	82	68	84	68	92	83	92	83
20	87	71	89	72	94	85	94	86	90	76	91	76	94	88	94	88
21	81	63	84	63	92	80	93	80	83	68	84	68	92	83	93	83
22	83	64	85	64	92	81	93	82	84	70	86	70	92	84	93	84
23	86	69	87	69	94	84	94	85	85	74	86	74	93	87	93	87
24	87	71	89	71	94	85	95	85	89	76	90	76	93	88	93	88
25	86	67	88	67	94	83	94	84	84	72	85	73	94	86	94	86

Table C.12: Operator's utilization levels for each scenario, replications from 26 to 50 in the simulation study.

	%Operator Utilization Levels of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
26	89	74	91	75	95	88	95	88	91	80	92	80	95	90	95	90
27	85	68	87	69	94	84	95	84	88	73	90	73	94	86	94	87
28	85	68	87	69	93	83	94	83	86	73	87	74	93	86	94	86
29	88	72	90	72	95	87	95	87	90	77	91	77	95	89	95	89
30	86	70	88	71	94	86	94	86	89	75	90	76	94	88	94	88
31	86	70	88	71	95	86	95	86	89	76	90	76	94	88	94	88
32	86	68	88	69	95	85	95	85	89	74	90	74	95	88	95	88
33	87	72	89	72	95	86	95	86	88	77	89	77	94	88	94	88
34	86	74	88	74	96	87	96	87	90	79	91	79	94	89	94	89
35	82	63	85	63	92	80	93	80	82	68	84	69	92	83	92	83
36	83	66	85	66	93	83	94	83	85	72	86	72	93	86	93	86
37	87	73	88	73	95	86	95	86	89	77	90	78	95	88	95	89
38	88	72	90	72	95	87	95	87	87	77	88	77	94	89	95	89
39	82	63	85	63	93	81	93	81	82	68	83	69	91	83	91	83
40	86	69	88	69	94	85	94	85	87	74	88	74	94	87	94	87
41	88	73	90	73	95	87	96	87	89	78	89	78	94	89	94	89
42	82	66	84	66	93	82	94	82	84	71	85	72	92	85	92	85
43	82	64	85	64	93	81	94	81	85	70	86	70	92	84	92	84
44	87	72	89	72	95	86	95	86	88	76	89	76	94	88	94	88
45	83	64	85	64	93	81	93	81	83	70	84	70	91	84	92	84
46	81	64	83	64	93	81	94	81	85	70	86	70	92	84	92	84
47	88	73	89	73	95	87	95	87	88	77	89	77	94	89	95	89
48	86	70	87	70	95	85	96	86	87	74	88	75	94	88	94	88
49	86	69	88	69	94	85	95	85	86	74	87	74	94	87	94	87
50	86	70	88	71	95	86	95	86	90	76	91	76	94	89	95	89

Table C.13: Operator's utilization levels for each scenario, replications from 51 to 75 in the simulation study.

	%Operator Utilization Levels of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
51	85	69	87	69	94	85	95	85	88	74	89	74	94	87	94	87
52	88	73	90	74	95	87	95	87	89	77	90	77	94	89	95	89
53	83	62	86	63	93	80	93	81	86	68	87	69	92	83	92	83
54	84	67	86	67	93	83	94	83	89	73	90	73	92	85	93	86
55	85	68	87	68	93	84	94	84	84	73	85	73	93	87	94	87
56	83	66	86	67	93	83	94	84	85	72	86	72	92	86	92	86
57	87	66	89	66	94	83	94	83	87	72	88	72	93	86	93	86
58	83	65	85	66	94	83	94	83	87	71	88	71	93	85	93	86
59	85	69	87	69	95	85	95	85	87	74	88	74	94	87	94	87
60	89	76	91	77	96	89	96	89	92	81	93	81	95	91	95	91
61	84	66	86	66	93	83	93	83	87	72	89	72	92	86	93	86
62	83	63	85	63	93	81	94	81	84	68	85	68	93	84	93	84
63	88	73	90	73	95	88	96	88	89	78	90	78	95	90	95	90
64	81	63	84	63	92	80	93	80	87	69	88	69	91	83	91	83
65	83	64	85	65	93	81	93	81	85	70	86	70	91	83	92	84
66	86	68	88	68	94	85	94	85	87	74	88	74	93	87	93	87
67	84	66	86	67	93	83	94	83	84	72	86	73	93	86	93	86
68	84	66	87	66	93	83	93	83	85	72	87	72	92	85	93	86
69	82	65	84	65	92	80	93	81	84	70	86	70	92	83	92	83
70	87	71	88	71	94	86	95	86	87	76	87	76	94	88	94	88
71	83	66	86	66	93	83	94	83	87	72	88	72	93	85	93	85
72	88	71	90	71	94	86	94	86	88	76	89	76	94	88	94	89
73	83	65	85	65	93	82	94	82	85	70	86	71	93	85	93	85
74	82	61	84	62	93	80	93	80	84	68	86	68	91	83	91	83
75	84	68	87	68	94	83	94	83	88	73	89	73	93	86	93	86

Table C.14: Operator's utilization levels for each scenario, replications from 76 to 100 in the simulation study.

	%Operator Utilization Levels of Each Scenario															
Replication	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
76	86	67	88	68	95	84	95	84	86	73	87	73	93	86	93	86
77	82	62	85	62	92	80	93	80	84	68	86	68	92	83	92	83
78	87	72	88	72	94	87	95	87	89	78	90	78	94	89	94	89
79	88	71	90	71	94	86	95	86	87	76	88	77	94	89	94	89
80	84	68	86	68	93	84	94	84	87	72	88	73	94	86	94	86
81	79	60	82	60	93	80	94	80	83	66	85	66	91	83	92	83
82	87	70	89	71	95	85	95	85	89	76	90	76	94	88	94	88
83	85	67	88	67	94	84	94	84	84	72	85	72	93	86	94	86
84	83	64	85	64	93	81	94	81	85	70	87	70	92	84	93	84
85	82	62	85	62	94	80	94	80	83	68	85	68	91	83	92	83
86	84	67	86	67	93	83	93	83	83	72	85	72	93	85	93	86
87	82	66	84	66	93	83	93	83	85	71	86	72	92	86	93	86
88	87	73	88	73	95	87	95	87	89	77	90	77	95	89	95	89
89	84	67	86	67	94	82	94	83	86	72	87	72	93	85	93	85
90	81	61	84	62	91	80	92	80	83	68	85	68	91	83	92	83
91	83	63	86	63	93	81	93	81	83	68	84	68	91	83	92	84
92	86	69	88	70	95	85	96	85	87	75	88	75	94	87	94	87
93	83	65	85	65	93	82	94	82	82	70	84	71	93	85	93	85
94	84	68	86	68	93	83	94	83	85	72	86	73	93	85	93	85
95	83	65	86	66	94	83	94	83	84	71	85	71	92	85	92	85
96	86	70	88	71	94	86	94	86	89	75	90	76	94	88	95	88
97	85	67	88	67	94	84	94	84	87	73	88	73	93	87	94	87
98	88	72	90	72	95	86	95	86	88	76	89	76	94	88	94	88
99	84	64	86	64	92	82	93	82	84	70	85	70	93	84	93	85
100	87	73	89	73	95	87	95	87	89	78	90	78	94	89	94	89

APPENDIX D

DISCUSSION OF THE IMPACT OF TRAY RETRIEVAL SEQUENCE ON THE SYSTEM THROUGHPUT

When the VLM pod's throughput is in consideration, the sequence of the tray retrievals may also impact the system's throughput. In order to justify this study's focus on the storage assignment rules, this chapter will discuss the impact of that sequencing decision on the throughput and compare it with the potential impacts of a storage assignment rule.

Tray retrievals in a VLM order picking task consist of dual cycles, where the cycle starts with the lift taking the previous tray from the I/O point and putting it into its shelf. Then, the cycle continues with the empty lift moving from the first tray's location to the next tray's location. Then the cycle ends when the lift brings the next tray into I/O point (see Figure D.1).

Meller and Klote (2004) present a single VLM throughput model, which also considers the dual cycle cases. This chapter will first explain the throughput model proposed in Meller and Klote (2004) and then discuss the impact of the tray retrieval sequence on the system's throughput using their model. The notation used in the mentioned study can be seen in Table D.1 and in Figure D.2.

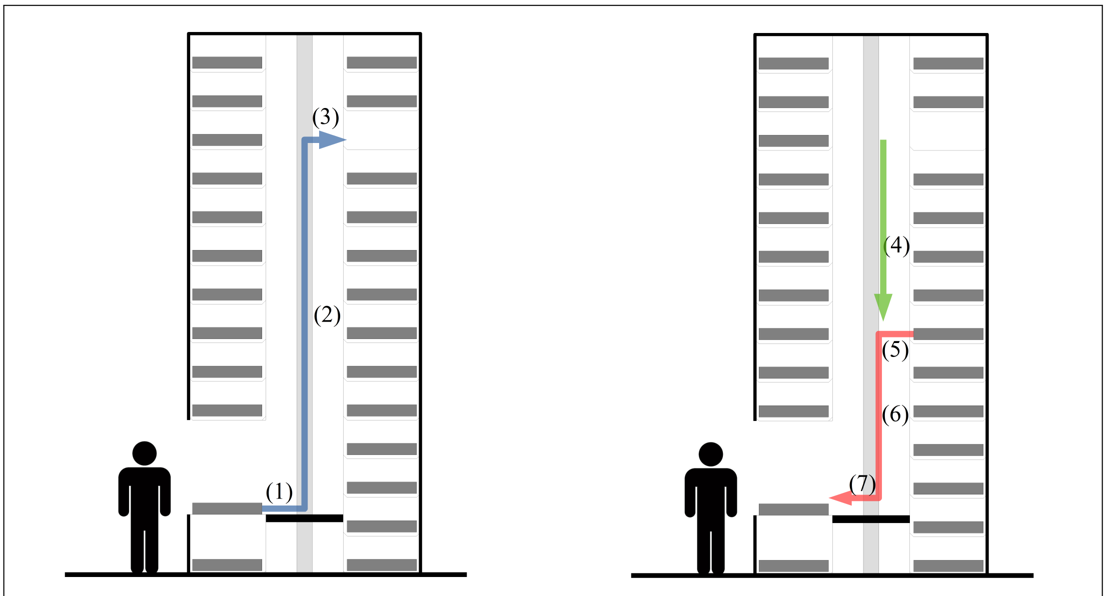

Figure D.1: Tasks in a dual cycle AS/RS operation. Tasks (1), (2) and (3) represent putting the previous tray into its place. Operation (4) represents the empty lift's movement from the previous tray's location to the next tray's location. Finally, tasks (5), (6) and (7) represent the retrieval of the next tray to the I/O location.

Table D.1: Notation used in the throughput model presented in Meller and Klote (2004).

H	height of the rack (VLM)
h_1, h_2, h_3	heights of the VLM sections
t_{0i}	expected travel time from/to the picking opening to/from section i
t_{ij}	expected travel time from/to section I to section j
p_1, p_2, p_3	probabilities of a storage/retrieval of a tray in corresponding sector
p_{ij}	probability that dual command cycle stores a tray in section i and retrieves a tray in section j
$E(DC)$	expected VLM crane travel time for dual command

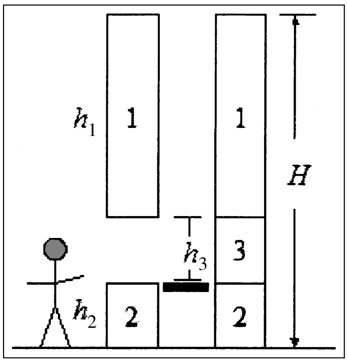

Figure D.2: A VLM unit with the mentioned sections (retrieved from Meller and Klote (2004)).

First of all, the authors assume random storage and random demand, so the expected travel times between the I/O point and Sections 1, 2 and 3 are:

$$
\begin{aligned}
t_{01} &= (h_3 + h_1/2)/v \\
t_{02} &= (h_2/2)/v \\
t_{03} &= (h_3/2)/v
\end{aligned}
\tag{D.1}
$$

Due to the random assignment assumption, expected travel times from section i of a VLM unit to section j of a VLM unit, $t_{ij}, i, j \in 1, 2, 3$ are:

$$
\begin{aligned}
t_{11} &= (h_1/3)/v \\
t_{12} = t_{21} &= (h_1/2 + h_3 + h_2/2)/v \\
t_{13} = t_{31} &= (h_1/2 + h_2/2)/v \\
t_{22} &= (h_2/3)/v \\
t_{23} = t_{32} &= (h_2/2 + h_3/2)/v \\
t_{33} &= (h_3/3)/v
\end{aligned}
\tag{D.2}
$$

With the random storage assignment assumption, probabilities of selecting a tray in section I of a VLM unit is the considered section's length divided by the sum of the lengths of all sections in a VLM unit:

$$
\begin{aligned}
p_1 &= 2h_1/(2H - h_3) \\
p_2 &= 2h_2/(2H - h_3) \\
p_3 &= h_3/(2H - h_3)
\end{aligned}
\tag{D.3}
$$

With these information, the authors define the expected travel times in a single cycle and a dual cycle tray retrieval task as follows:

$$E(SC) = \sum_{i=1}^{3} 2t_{0i}p_i \tag{D.4}$$

$$E(DC) = \sum_{i=1}^{3}\sum_{j=1}^{3}(t_{0i} + t_{ij} + t_{0j})p_i p_j \tag{D.5}$$

Therefore, the total travel time for a pick order task requesting n trays from the VLM unit is going to be:

$$E(OP) = 2E(SC) + (n-2)E(DC) \tag{D.6}$$

If we are only going to modify the tray retrieval sequence for improving the system's throughput, we would be making improvements only on the term $t_{ij}, i, j \in 1, 2, 3$ in the $E(OP)$. However, if we focus on a better storage assignment method that will;

- reduce n by putting the parts that are requested together on the same tray
- increase the probability of selecting a tray that is close to the I/O and reduce the same probability for the trays that are away from the I/O by sorting the stored parts in a VLM unit according to their popularity

the expected improvements on the $E(OP)$ in Equation D.6 can be higher since a storage assignment method will improve on modifying n and $p_i, i \in 1, 2, 3$ values, which are used as multipliers of the term $t_{ij} + t_{0i} + t_{0j}$, which is expected to be greater than t_{0i}. Therefore, having an improvement on the value of n will have higher impact on the system's throughput than having the same improvement on the values of $t_{ij}, i, j \in 1, 2, 3$. Moreover, a storage assignment method that places mostly used parts close to the I/O point will also reduce the expected value of $t_{ij}, i, j \in 1, 2, 3$, since $p_i, i \in 1, 2, 3$ values for the trays that are closer to the I/O will be higher in a such storage assignment method. Therefore, this study will first focus on developing a storage assignment method.

APPENDIX E

COMPUTATIONAL EXPERIMENTS

Table E.1: Matrix representing the difficulty advancement steps used in the experiments. As the indicated step numbers increase, the problems become computationally more challenging.

		$s=2$			$s=3$			$s=4$		
		$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low	$c=$High	$c=$Mid	$c=$Low
$b=5$	$p \in [4,8]$	1	2	3	2	3	4	3	4	5
	$p \in [12,16]$	2	3	4	3	4	5	4	5	6
	$p \in [20,24]$	3	4	5	4	5	6	5	6	7
$b=10$	$p \in [4,8]$	2	3	4	3	4	5	4	5	6
	$p \in [12,16]$	3	4	5	4	5	6	5	6	7
	$p \in [20,24]$	4	5	6	5	6	7	6	7	8
$b=15$	$p \in [4,8]$	3	4	5	4	5	6	5	6	7
	$p \in [12,16]$	4	5	6	5	6	7	6	7	8
	$p \in [20,24]$	5	6	7	6	7	8	7	8	9

Algorithm 3: Problem instance generation

Input: pool of orders, number of orders (b), upper limit for order component count (UL), lower limit for order component count (LL)

Result: A problem instance with b orders, each containing k distinct parts, where k is between UL and LL, and its subsets considering various VLM configuration settings.

repeat
    initialization;
    while $selected < b$ **do**
        pick a random order from the pool: $pickedOrd$;
        if $LL \leq$ *number of parts in* $pickedOrd \leq UL$ **then**
            select order to be used in the output;
            $selected = selected + 1$;
            $d = d+$ *number of parts in* $pickedOrd$;
            add $pickedOrd$ into $selectedOrders$;
            add all the parts in the selected order to the list $selectedParts$;
    remove duplicates from $selectedParts$;
    $p = size(selectedParts)$;
    Assign a random name for this problem instance;
until $p \neq d$;
for *all seasons in {season 1, season 2}* **do**
    randomly pick $roundUp(b/2)$ orders from $selectedOrders$ and assign random demand values between 10 and 15;
    assign random demand values between 3 and 8 for the remaining orders;
create three settings for a 2 VLM pod: small trays, medium trays, big trays;
create three settings for a 3 VLM pod: small trays, medium trays, big trays;
create three settings for a 4 VLM pod: small trays, medium trays, big trays;

Algorithm 4: Simulation to verify the proposed model

Input: list of parts, orders, location assignments ,time study observations for the machine and operator tasks

Output: for each scenario in Table 6.9: completion time of all picks, operator utilization, VLM lift utilization, total number of tray changes, total walk duration, time spent waiting for the tray

for *each replication from* ***1*** *to* ***100*** **do**

- selectedOrderTasks = list of all pick tasks of the selected pick orders (requested part numbers, requested quantities)
- **for** *each line in selectedOrderTasks* **do**
  - generate operator's pick task times according to the time study results and append as new columns to the *selectedOrderTasks* table:
  - -fixed pick time
  - -variable pick time, depending on the number of items picked

for *each scenario in Table C.1* **do**

- assign storage locations for each part according to the current scenario, append location information (vlm.unit, tray.shelf.no) for the requested part as new columns to *selectedOrderTasks* table
- sort *selectedOrderTasks* according to the picking sequence defined in the current scenario
- generate walk durations according to the time study results, where needed
- generate tray retrieval durations according to the time study results, the scenario's picking sequence and the assigned storage locations
- **for** *each line* ***i*** *in selectedOrderTasks* **do**
  - get event times from function *eventTimes* (Algorithm 2)
- calculate and save: *completion time of all picks, operator utilization, VLM lift utilization, total number of tray retrievals, total walk duration, time spent waiting for the tray*

Table E.2: Order picking completion time (c_{max}), $\%$ operator utilization, time spent walking (t_{walk}) and number of tray retrievals in the considered scenarios.

b	c	p	s	Scenario 1				Scenario 3				Scenario 5			
				c_{max}(sec.)	%Op.Util.	t_{walk}(sec.)	Num.Tray Ret.	c_{max}(sec.)	%Op.Util.	t_{walk}(sec.)	Num.Tray Ret.	c_{max}(sec.)	%Op.Util.	t_{walk}(sec.)	Num.Tray Ret.
5	High	4-8	2	12755.5	95	495.0	63.3	14996.5	86	1170.9	229.4	14658.9	87	989.1	194.6
5	High	4-8	3	12739.5	97	764.2	78.8	15259.7	87	1652.0	239.9	14601.4	89	1424.2	182.0
5	High	4-8	4	12933.0	99	1123.5	89.7	15427.3	88	1960.7	258.2	14842.1	90	1674.3	211.3
5	Mid	4-8	2	13713.9	92	886.6	160.8	16838.5	76	1165.5	353.1	16256.0	78	978.2	301.2
5	Mid	4-8	3	13358.8	96	1110.6	138.2	16247.8	82	1622.8	327.5	15459.1	84	1335.1	265.3
5	Mid	4-8	4	16254.2	97	1456.5	162.0	19882.8	83	2150.2	406.9	18519.4	88	1949.5	305.0
5	Low	4-8	2	13779.0	91	1167.6	220.1	16988.6	74	1176.6	382.4	16599.1	74	987.3	348.5
5	Low	4-8	3	13474.6	94	1359.6	201.9	16169.8	80	1602.8	358.9	15705.8	81	1465.1	314.9
5	Low	4-8	4	13579.0	96	1487.3	189.3	16085.4	84	1959.3	343.3	15799.9	84	1728.4	311.5
10	High	4-8	2	25099.7	97	833.2	81.2	30136.5	86	2412.6	453.3	29086.9	88	1906.9	367.5
10	High	4-8	3	25102.9	98	1175.8	101.8	30478.7	88	3324.1	466.4	29537.1	89	2811.0	384.7
10	High	4-8	4	25721.6	99	1438.0	117.2	31188.8	90	3989.4	491.0	30083.8	91	3472.7	393.2
15	High	4-8	2	38840.6	96	1051.5	140.6	46949.0	85	3575.7	719.1	44826.4	88	2847.8	549.9
15	High	4-8	3	39327.9	98	1293.1	137.3	47862.3	88	5004.0	723.5	45609.8	91	4238.1	559.4
5	High	12-16	2	33779.0	98	658.2	83.9	37124.2	91	1291.6	356.5	36475.6	92	1039.5	290.8
5	High	12-16	3	31463.1	98	993.6	87.4	36422.0	87	1961.2	481.4	34902.9	91	1849.9	346.9
5	High	12-16	4	31550.5	99	1261.0	119.0	37477.5	87	2571.0	543.5	35251.0	91	2311.7	337.0
10	High	12-16	2	60601.1	98	1060.9	164.0	72579.2	84	2569.8	977.9	68908.3	88	2426.9	762.5
5	High	20-24	4	52780.5	99	1334.2	114.2	58012.3	89	2492.5	653.4	55130.6	93	2021.4	396.2
5	High	20-24	2	50978.2	98	845.1	132.6	58457.7	88	1282.5	536.4	56382.2	91	1157.9	394.6
5	High	20-24	3	52267.5	99	1085.1	88.6	58279.8	91	1950.2	537.1	56329.2	94	1768.3	362.0
Average				28100.0	97	1089.5	127.2	32993.5	85	2232.6	468.5	31665.0	88	1923.0	360.9

Table E.3: Order picking completion time (c_{max}), $\%$ operator utilization, time spent walking (t_{walk}) and number of tray retrievals in the considered scenarios.

b	c	p	s	Scenario 2				Scenario 4				Scenario 6			
				c_{max}(sec.)	%Op.Util.	t_{walk}(sec.)	Num.Tray Ret.	c_{max}(sec.)	%Op.Util.	t_{walk}(sec.)	Num.Tray Ret.	c_{max}(sec.)	%Op.Util.	t_{walk}(sec.)	Num.Tray Ret.
5	High	4-8	2	12755.5	95	495.0	63.3	14231.0	93	1588.2	229.4	14170.6	91	1270.9	194.6
5	High	4-8	3	12739.5	97	764.2	78.8	14715.2	92	1935.6	239.9	14433.1	91	1512.2	182.0
5	High	4-8	4	12933.0	99	1123.5	89.7	15058.6	92	2149.4	258.2	14566.1	92	1803.9	211.3
5	Mid	4-8	2	13691.2	92	900.4	160.8	15476.6	88	1939.4	353.1	15262.4	86	1535.4	301.2
5	Mid	4-8	3	13291.6	96	1151.6	138.2	15463.3	89	2052.2	327.5	14922.2	89	1590.3	265.3
5	Mid	4-8	4	16241.6	97	1463.4	162.0	19072.3	89	2594.7	406.9	18045.2	92	2225.1	305.0
5	Low	4-8	2	13566.2	93	1277.9	220.1	15539.2	86	2003.7	382.4	15600.3	83	1565.1	348.5
5	Low	4-8	3	13406.1	95	1405.1	201.9	15357.6	87	2056.8	358.9	15085.2	87	1815.0	314.9
5	Low	4-8	4	13464.4	97	1532.6	189.3	15513.7	89	2264.0	343.3	15183.2	90	2075.9	311.5
10	High	4-8	2	25099.7	97	833.2	81.2	28712.8	93	3156.2	453.3	28407.8	91	2278.0	367.5
10	High	4-8	3	25102.9	98	1175.8	101.8	29574.5	92	3765.5	466.4	28836.0	92	3133.8	384.7
10	High	4-8	4	25721.6	99	1438.0	117.2	30569.2	92	4285.1	491.0	29670.8	93	3691.9	393.2
15	High	4-8	2	38840.6	96	1051.5	140.6	44557.9	93	4861.4	719.1	43698.1	91	3440.2	549.9
15	High	4-8	3	39327.9	98	1293.1	137.3	46345.6	92	5786.3	723.5	45111.7	92	4505.2	559.4
5	High	12-16	2	33745.2	98	662.3	83.9	35065.8	100	2525.7	356.5	35151.9	97	1826.5	290.8
5	High	12-16	3	31463.1	98	993.6	87.4	34544.6	95	3083.5	481.4	33840.1	96	2483.7	346.9
5	High	12-16	4	31511.5	99	1299.4	119.0	35925.8	93	3497.9	543.5	34324.5	95	2866.1	337.0
10	High	12-16	2	60601.1	98	1060.9	164.0	66101.7	97	6357.9	977.9	65149.6	96	4640.3	762.5
5	High	20-24	4	50978.2	99	1334.2	114.2	56082.2	94	3647.5	653.4	54067.4	96	2657.1	396.2
5	High	20-24	2	52267.3	98	848.0	132.6	54206.9	100	3765.1	536.4	54640.2	96	2176.2	394.6
5	High	20-24	3	52756.8	99	1112.7	88.6	56150.5	97	3227.4	537.1	54976.6	98	2553.7	362.0
Average				28071.7	97	1105.5	127.2	31346.0	93	3168.7	468.5	30721.1	92	2459.4	360.9