

Width Effects on Near Threshold Decays of the Top Quark $t \rightarrow cWW, cZZ$ and of Neutral Higgs Bosons

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Abstract

The nonzero widths of heavy particles become significant when they appear in the final state of any decay occurring just around its kinematical threshold. To take into account such effects, a procedure, called the *convolution method*, was proposed by Altarelli, Conti and Lubicz. We expand their study which included only threshold effects for $t \rightarrow bWZ$ in the standard model. We discuss finite width effects in the three body decays $t \rightarrow cWW, cZZ$ and $A^0(h^0) \rightarrow tbW$ in the type III version of a two Higgs doublet model. In particular, we find a substantial enhancement in the decay $t \rightarrow cZZ$, which brings its branching ratio to $\text{BR}(t \rightarrow cZZ) \sim 10^{-3}$, and in the decay $A^0 \rightarrow tbW$, which, unlike the h^0 case, becomes competitive with the A^0 two-body decay modes.

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I. INTRODUCTION

The finite width of a particle is directly related to its instability. When its width is small with respect to its physical mass, finite width effects (FWE) are usually neglected except for decays in which a resonance can emerge when the particle appears as an intermediate state, or in decays that are kinematically allowed only very close to threshold and the particle is involved in either the initial or the final state. The former case is usually handled with the Breit-Wigner prescription, while the latter case, i.e., taking into account the FWE in processes occurring just around their kinematical threshold, needs special attention.

In this respect, there are two different methods proposed in the literature [1, 2, 3, 4]. They were referred to by Altarelli, Conti and Lubicz [1] as the *decay-chain method* (DCM) and the *convolution method* (CM).¹ In the first approach (i.e., the DCM), the dominant decay modes of the unstable final state particles are taken into account as subsequent decays to obtain the “total” decay rate and then the branching ratio for the “signal” (i.e., with the unstable particle in the finite state) is calculated by taking the ratio of the “total” decay rate to the multiplication of rates of the subsequent decay modes [1, 2]. This method requires kinematical cuts in order to maintain the direct connection between the “signal” and the total number of events. That is, since the observed final state (with its subsequent decay products) could be produced through other channels, kinematical cuts are required to minimize this undesired background. Therefore, this method leads to physical quantities which depend on kinematical cuts and so it inherits some degree of experimental difficulties.

Alternatively, in the CM the instability of a final state particle is described instead by a Breit-Wigner-like density function whose central value and half-width are governed by the width and the physical invariant mass of the particle. In this way, the unstable particle produced can be seen effectively as a real physical particle, having an invariant mass which is controlled by its density function. Although this method does not require any kinematical cut, it doubles the number of phase space integrals, making it computationally more challenging.

In this paper we employ the CM to study FWE in the three-body flavor changing rare top decays $t \rightarrow cWW$ and $t \rightarrow cZZ$, by including the widths of W and Z bosons. These

¹ An alternative approach has been recently discussed by Kuksa [5], based on the uncertainty relation for the mass of the unstable particle. This method has a close analogy to the *convolution method*.

decay modes and other two and three-body rare flavor changing top decays [6], can provide a unique testing ground for the standard model (SM) Glashow-Iliopoulos-Maiani (GIM) mechanism and may give hints about - beyond the SM - flavor changing physics such as may occur in some variations of Two-Higgs Doublet Models (2HDM's). FWE in these decay modes will be studied within the SM (in the case of $t \rightarrow cWW$) and in the context of the type III Two Higgs Doublet Model (in both $t \rightarrow cWW$ and $t \rightarrow cZZ$), which admits flavor changing neutral currents (FCNC) at the tree-level. The three-body top decays $t \rightarrow cWW, bWZ, cZZ$ have been considered before, without including FWE, in the SM [7, 8, 9], in 2HDM's [9, 10, 11, 12, 13], in a generic formalism including scalar, vector or fermion exchanges [14] and in topcolor-assisted Technicolor model [15]. In addition, the top decays $t \rightarrow bWh^0$ and $t \rightarrow bWA^0$ have been analyzed in the context of a general 2HDM [16]. Among the above decay modes, a simple threshold analysis shows that $t \rightarrow cZZ$ and $t \rightarrow bWZ$ are potentially the most sensitive to FWE. In particular, according to the recent CDF analysis based on the Tevatron RUN II data, the top mass is (1σ) [17]: $m_t = 173.5^{+2.7}_{-2.6} (stat) \pm 4.0 (syst)^2$. In fact, these later top mass measurements imply that for the stable Z-bosons case (i.e., without including FWE) the decay $t \rightarrow cZZ$ *cannot* occur if the top mass lies within its recent CDF and D0 1σ limits. We, therefore, expect FWE to be substantial in this decay. Indeed, we find that FWE (due to the rather large $\mathcal{O}(GeV)$ Z-width) can give $BR(t \rightarrow cZZ) \sim 10^{-5} - 10^{-3}$ (as opposed to null in the stable case), within some range of the allowed parameter space of the type III 2HDM. Moreover, even for the decay $t \rightarrow cWW$, for which the central value of the top-quark mass (i.e., $m_t = 173.5$) is about 10 GeV away from the kinematical threshold, we find that FWE from the unstable W-bosons can cause a several orders of magnitudes enhancement in the type III 2HDM with a light neutral Higgs of mass $m_{h^0} \lesssim 2m_W$, thus elevating the branching ratio from $BR(t \rightarrow cWW) \sim 10^{-9} - 10^{-8}$ to $BR(t \rightarrow cWW) \sim 10^{-4} - 10^{-3}$ in this case. Clearly, such large branching ratios would be accessible to the LHC and may even be detected at the Tevatron. A similar large enhancement due to FWE was found for the decay mode $t \rightarrow bWZ$ in both the CM [1] and the DCM [1, 2]. In particular, [1, 2] have found that, in the SM, the FWE increase this decay width by orders of magnitude (with respect to the

² Note that later D0 results from Tevatron RUN II, $m_t = 170.6 \pm 4.2 (stat) \pm 6.0 (syst)$ (see [17]), are based on less accumulated data and has larger statistical and systematic uncertainties.

stable final state gauge bosons), giving $\text{BR}(t \rightarrow bWZ) \simeq 2 \times 10^{-6}$ for $m_t \sim 176$ GeV.

To demonstrate the potential importance of FWE in neutral Higgs decays, we also examine the three-body neutral Higgs decays $h^0 \rightarrow tbW$ and $A^0 \rightarrow tbW$, within the type III 2HDM, assuming that either h^0 (the lighter CP-even neutral Higgs) or A^0 (the CP-odd neutral Higgs) have masses around $m_t + m_b + m_W$ (i.e., close to the threshold). It is well known that, for a SM-like Higgs, the two-body decay modes to the heaviest fermions and to the gauge bosons are dominant, since its couplings to these particles are proportional to their masses. Three-body sub-threshold decays (e.g., to W^*W or Z^*Z pairs) can also have sizable BR's despite the suppression factors involved [18]. In the context of the minimal supersymmetric extension of the SM (MSSM) sub-threshold three-body decays of especially heavy Higgs bosons might also have a large branching ratio [19]. In this paper we show that, including the top quark and the W boson width in the framework of the CM, the three-body Higgs decays $h^0 \rightarrow tbW$ and $A^0 \rightarrow tbW$ can be enhanced by about 3 orders of magnitudes in the type III 2HDM if they occur just around their kinematical threshold. For the case of $A^0 \rightarrow tbW$, such an enhancement can push its BR to the level of tens of percents and may, therefore, become critical for experimental searches of A^0 .

The paper is organized as follows: In Section II we describe the *convolution method*. In Section III we give a brief overview of the type III 2HDM. In section IV we examine the FWE in the top decays $t \rightarrow cWW, cZZ$ and in section V we study the FWE in the three-body Higgs decays $h^0 \rightarrow tbW$ and $A^0 \rightarrow tbW$. In Section VI we summarize our results.

II. THE CONVOLUTION METHOD

Particles with large width imply a large uncertainty in its mass from the mass uncertainty relation [20]. The CM can be used to include such large width effects in decays involving unstable particles in the final state. Consider for example the main top decay $t \rightarrow bW$. Since the W is unstable, we can define: $\Gamma(t \rightarrow bW) \equiv \Gamma = \sum_{i,j} \Gamma^0(t \rightarrow bf_i \bar{f}_j)$, where the sum runs over all the W decay modes. Furthermore, Γ can be decomposed into two parts corresponding to the transverse (Γ_T) and longitudinal (Γ_L) components of the intermediate W -boson (see e.g., [4]):

$$\Gamma = \Gamma_T + \Gamma_L , \tag{2.1}$$

where

$$\Gamma_T = \frac{1}{\pi} \sum_{ij} \int_{(m_i+m_j)^2}^{(m_t-m_b)^2} dp^2 \frac{\sqrt{p^2} \Gamma^0(t \rightarrow bW(p^2)) \Gamma^0(W(p^2) \rightarrow f_i \bar{f}_j)}{(p^2 - m_W^2)^2 + (\text{Im } \Pi_T(p^2))^2}, \quad (2.2)$$

and $\Gamma_L \propto f(m_i, m_j)$, with $f \rightarrow 0$ as $m_i, m_j \rightarrow 0$. Also, m_W is the mass of the W boson and $\text{Im } \Pi_T(p^2)$ and $\text{Im } \Pi_L(p^2)$ (appearing in Γ_L) are the absorptive parts of the transverse and longitudinal vacuum polarization tensor (see e.g., [4, 21]).

Using the Cutkotsky rule in the limit of massless fermion $m_i, m_j \rightarrow 0$ (f_i, f_j , are the fermions exchanged in the W self energy diagram), one obtains

$\text{Im } \Pi_L(p^2) \rightarrow 0$ and:

$$\text{Im } \Pi_T(p^2) = \sqrt{p^2} \sum_{i,j} \Gamma^0(W(p^2) \rightarrow f_i \bar{f}_j) = \frac{p^2}{m_W} \Gamma_W^0, \quad (2.3)$$

where Γ_W^0 is the usual on-shell decay width of W and $\sqrt{p^2} \geq m_i + m_j$. Thus, in this limit Γ reduces to:

$$\Gamma = \Gamma_T = \int_0^{(m_t-m_b)^2} dp^2 \rho(p^2, m_W, \Gamma_W^0) \Gamma^0(t \rightarrow bW(p^2)), \quad (2.4)$$

where $\rho(p^2, m_W, \Gamma_W^0)$ is the ‘‘invariant mass distribution function’’, given by:

$$\rho(p^2, m_W, \Gamma_W^0) = \frac{1}{\pi} \frac{\frac{p^2}{m_W} \Gamma_W^0}{(p^2 - m_W^2)^2 + \left(\frac{p^2}{m_W} \Gamma_W^0\right)^2}. \quad (2.5)$$

Eqs. (2.4) and (2.5) describe the factorization of the production and the decay modes of the W boson (in the limit of massless fermions). The case of a stable W boson (i.e., $\Gamma_W^0 \rightarrow 0$) makes $\rho \rightarrow \delta(p^2 - m_W^2)$ which sets $\Gamma = \Gamma^0(t \rightarrow bW)$, where Γ^0 is the width for an on-shell W without FWE.

The above prescription can be generalized to the case of a generic three-body decay of the form $a \rightarrow bV_1V_2$, where V_1 and V_2 are vector bosons:

$$\Gamma(a \rightarrow bV_1V_2) = \int_0^{(m_a-m_b)^2} dp_1^2 \int_0^{(m_a-m_b-\sqrt{p_1^2})^2} dp_2^2 \rho_1(p_1^2, m_{V_1}, \Gamma_{V_1}^0) \rho_2(p_2^2, m_{V_2}, \Gamma_{V_2}^0) \times \Gamma^0(a \rightarrow bV_1(p_1^2)V_2(p_2^2)). \quad (2.6)$$

Furthermore, for consistency of the CM one needs also the following modifications:

1. The sum over polarization vectors of a gauge-boson with an invariant mass p^2 should be taken as:

$$\sum_{\lambda} \epsilon_{\lambda}^{\mu}(p) \epsilon_{\lambda}^{\nu*}(p) = -g^{\mu\nu} + \frac{p^{\mu} p^{\nu}}{p^2} . \quad (2.7)$$

2. In calculating the “zeroth” width of the top-quark(Higgs) into the off-shell vector boson(s) [i.e., $\Gamma^0(a \rightarrow b V_1(p_1^2) V_2(p_2^2))$ in Eq. (2.6) or $\Gamma^0(\mathcal{H} \rightarrow a b(p_1) V(p_2^2))$ for $\mathcal{H} = h^0$ in Eq. (5.1)], the tree-level propagator of the massive vector bosons should be modified as (in the unitary gauge):

$$\frac{-i}{p^2 - m_V^2 + im_V \Gamma_V^0} \left[g^{\mu\nu} - \frac{p^{\mu} p^{\nu}}{m_V^2 - im_V \Gamma_V^0} \right] . \quad (2.8)$$

This substitution is required since in the CM the invariant mass p^2 is allowed to vanish, as can be seen from the integration limits in Eqs. (2.4) and (2.6).

3. In order to restore gauge invariance in the R_{ξ} -gauge, the Feynman rules in which masses of such resonant intermediate particles appear should be modified to be functions of the corresponding invariant masses. Such a modification is, however, not necessary in the unitary gauge that we have used in all our calculations [22].

III. THE TWO HIGGS DOUBLET MODEL OF TYPE III

One of the simplest extensions of the SM is obtained by enlarging the scalar sector with an additional $SU(2)_L$ doublet. In the most general case such a 2HDM gives rise to tree-level FCNC which are mediated by the physical Higgs bosons [23]. To avoid such potentially dangerous FCNC, one usually imposes an ad-hoc discrete symmetry [24] that leads to the type I or type II 2HDM (see for example [25] and [26]). An alternative way for suppressing FCNC in a general 2HDM (i.e., without imposing discrete symmetries) was suggested by Cheng and Sher in [27]. In the Cheng and Sher Ansatz the arbitrary flavor changing couplings of the scalars to fermions are assumed to be proportional to the square root of masses of the fermions participating in the Higgs Yukawa vertex (see below).³

³ The Cheng and Sher Ansatz ensures the suppression of FCNC within the first two generations of quarks, as required by the experimental constraints on FCNC in meson transitions, see [28].

Within the most general 2HDM one can always choose a basis where only one of the doublets acquires a vacuum expectation value (VEV): $\langle \Phi_1 \rangle = (0 \ v/\sqrt{2})^T$ and $\langle \Phi_2 \rangle = 0$. A general 2HDM in this basis is often referred to as the type III 2HDM (or Model III) [28, 29, 30]. With this choice of basis, Φ_1 corresponds to the usual SM doublet and all the new flavor changing couplings are attributed to Φ_2 . Note also that in this basis $\tan \beta = v_1/v_2$ has no physical meaning.⁴

As in any 2HDM, the physical Higgs sector of Model III consists of 3 neutral Higgs bosons (2 CP-even ones, h^0 and H^0 , and one CP-odd state A^0) and a charged scalar with its conjugate H^\pm . The neutral bosons are given, in terms of the original SU(2) doublets, as:

$$\begin{aligned} h^0 &= \sqrt{2} \left[-\left(\text{Re } \phi_1^0 - v \right) \sin \alpha + \text{Re } \phi_2^0 \cos \alpha \right], \\ H^0 &= \sqrt{2} \left[\left(\text{Re } \phi_1^0 - v \right) \cos \alpha + \text{Re } \phi_2^0 \sin \alpha \right], \\ A^0 &= -\sqrt{2} \text{Im } \phi_2^0. \end{aligned} \quad (3.1)$$

The flavor changing part of the Yukawa Lagrangian in Model III is given by [23, 28]:

$$\mathcal{L}_{Y,FC} = \xi_{ij}^U \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi_{ij}^D \bar{Q}_{iL} \phi_2 D_{jR} + \text{H.c.}, \quad (3.2)$$

where $\tilde{\phi}_2 = i\tau_2 \phi_2$, Q stands for the quark $SU(2)_L$ doublets, $U(D)$ for up-type (down-type) quark $SU(2)_L$ singlets and ξ^U , ξ^D are 3×3 non-diagonal matrices (in family space) that parametrize the strength of the FCNC vertices in the neutral Higgs sector. Adopting the Cheng and Sher Ansatz we set:⁵

$$\xi_{ij}^{U,D} = \lambda_{ij} \frac{\sqrt{m_i m_j}}{v}, \quad v = \left(\sqrt{2} G_F \right)^{-1/2}, \quad (3.3)$$

where for simplicity we assume the λ_{ij} 's to be real⁶ and symmetric (i.e., $\lambda_{ij}^* = \lambda_{ji}$) constants. For the Higgs-top-charm coupling we will take that $\lambda_{tc} = \lambda_{ct} \equiv \lambda \sim \mathcal{O}(1)$, which is compatible with all existing data, see [11, 28] for details.

Thus, for the top decays of our interest in this paper, the relevant terms in the Yukawa Lagrangian are [11]:

$$\mathcal{L}_{\mathcal{H}tc} = -\lambda \frac{\sqrt{m_c m_t}}{\sqrt{2}v} f_{\mathcal{H}} \mathcal{H} \bar{c} t,$$

⁴ “Switching on” $\tan \beta$ by allowing $\langle \Phi_2 \rangle \neq 0$ will not change any physical result.

⁵ Note that there is a factor of 1/2 difference between our definition for $\xi_{ij}^{U,D}$ in (Eq. 3.3) and the one used in [11]. This difference may be absorbed by redefining the arbitrary parameters λ_{ij} .

⁶ In this work we are not interested in CP-violating effects that may be driven by a possible phase contained in the λ_{ij} 's.

$$\mathcal{L}_{\mathcal{H}VV} = -gm_W G_V S_{\mathcal{H}} g_{\mu\nu} V^\mu V^\nu, \quad (3.4)$$

where $\mathcal{H} = h^0$ or H^0 , $V = W$ or Z and

$$\begin{aligned} f_{h^0;H^0} &= \cos \alpha; \sin \alpha, \\ S_{h^0;H^0} &= \sin \alpha; -\cos \alpha, \\ G_{W;Z} &= 1; \frac{m_Z^2}{m_W^2}. \end{aligned} \quad (3.5)$$

We will further need the $\mathcal{H}q_i q_i$ (with $\mathcal{H} = h^0, A^0$) and $H^\pm tb$ couplings [11, 28]:

$$\begin{aligned} \mathcal{L}_{\mathcal{H}q_i q_i} &= -\frac{m_{q_i}}{v} \bar{q}_i \left[h^0 \left(-\sin \alpha + \frac{\lambda_{ii}}{\sqrt{2}} \right) + iA^0 \frac{\lambda_{ii}}{\sqrt{2}} \gamma_5 \right] q_i, \\ \mathcal{L}_{H^- t \bar{b}} &= -\frac{1}{2v} V_{tb}^* H^- \bar{b} \left[\left(\lambda_{bb} m_b - \lambda_{tt} m_t \right) - \left(\lambda_{bb} m_b + \lambda_{tt} m_t \right) \gamma_5 \right] t. \end{aligned} \quad (3.6)$$

IV. FINITE WIDTH EFFECTS IN THE $t \rightarrow cWW$ AND $t \rightarrow cZZ$ DECAYS

In this section we will use the CM to evaluate the FWE in the top decays $t \rightarrow cWW$ and $t \rightarrow cZZ$. Kinematically, the naive threshold (i.e., not including FWE) for the decay $t \rightarrow cZZ$ is about 4 GeV away (i.e., larger) from the recent CDF 1σ limit (from Tevatron RUN II) on the top mass, $m_t(1\sigma) \leq 180.2$ GeV [17]. Also, as will be shown below, even for $t \rightarrow cWW$ the available phase space can be (depending on the top mass) small enough for the FWE to become significant.

We will consider the decay $t \rightarrow cWW$ at the tree-level in both the SM and Model III, while $t \rightarrow cZZ$ will be analysed only within Model III, since in the SM this decay is doubly suppressed by both one-loop factors and non-diagonal Cabibbo-Kobayashi-Maskawa (CKM) elements and is, therefore, unobservably small.

In the SM, the tree-level decay $t \rightarrow cWW$ proceeds via $t \rightarrow d^* W^+ \rightarrow cW^- W^+$ ($d = d, s$ or b quarks), with a BR of the order of $\mathcal{O}(10^{-14} - 10^{-13})$ (depending on the top-quark mass) if FWE are not taken into account [7]. The dominant SM diagram is $t \rightarrow b^* W^+ \rightarrow cW^- W^+$, since $V_{tb} \times V_{cb}$ is the largest out of the three possible products of CKM elements that enter this decay. In Model III there are two additional tree-level diagrams: $t \rightarrow ch^{0*} \rightarrow cW^+ W^-$ and $t \rightarrow cH^{0*} \rightarrow cW^+ W^-$ [10, 11]. In this case, we will use the Breit-Wigner prescription for the propagators of $\mathcal{H} = h^0$ or H^0 , i.e., $(q^2 - m_{\mathcal{H}}^2 + im_{\mathcal{H}} \Gamma_{\mathcal{H}})^{-1}$, where $\Gamma_{\mathcal{H}}$ is the total \mathcal{H}

width calculated from the dominant \mathcal{H} decay modes: $\mathcal{H} \rightarrow b\bar{b}, t\bar{t}, t\bar{c}, ZZ, WW, WW^*, ZZ^*$.^{7,8}

Using the CM, the partial decay width for $t \rightarrow cWW$ in any given model M can be written as [see Eq. (2.6)]:

$$\begin{aligned} \Gamma_{\text{conv}}^M(t \rightarrow cWW) = & \frac{1}{512\pi^3 m_t^3} \int_0^{(m_t - m_c)^2} dp_{W^+}^2 \left[\frac{p_{W^+}^2 \Gamma_W^0}{m_W \pi \left((p_{W^+}^2 - m_W^2)^2 + \left(\frac{p_{W^+}^2 \Gamma_W^0}{m_W} \right)^2 \right)} \right] \\ & \times \int_0^{(m_t - m_c - \sqrt{p_{W^+}^2})^2} dp_{W^-}^2 \left[\frac{p_{W^-}^2 \Gamma_W^0}{m_W \pi \left((p_{W^-}^2 - m_W^2)^2 + \left(\frac{p_{W^-}^2 \Gamma_W^0}{m_W} \right)^2 \right)} \right] \\ & \times \int_{(m_c + \sqrt{p_{W^-}^2})^2}^{(m_t - \sqrt{p_{W^+}^2})^2} dx_1 \int_{x_{2,\min}}^{x_{2,\max}} dx_2 \left| \mathcal{M}_{\text{conv}}^M(x_1, x_2, p_{W^+}^2, p_{W^-}^2) \right|^2, \end{aligned} \quad (4.1)$$

where the superscript M stands for the model used for the calculation of the convoluted amplitude $\mathcal{M}_{\text{conv}}^M$, and

$$\begin{aligned} x_{2,\min} &= (E_2 + E_3)^2 - \left(\sqrt{E_2^2 - p_{W^-}^2} + \sqrt{E_3^2 - p_{W^+}^2} \right)^2, \\ x_{2,\max} &= (E_2 + E_3)^2 - \left(\sqrt{E_2^2 - p_{W^-}^2} - \sqrt{E_3^2 - p_{W^+}^2} \right)^2, \\ E_2 &= \frac{x_1 - m_c^2 + p_{W^-}^2}{2\sqrt{x_1}}; \quad E_3 = \frac{-x_1 - p_{W^+}^2 + m_t^2}{2\sqrt{x_1}}. \end{aligned} \quad (4.2)$$

For the BR calculation, we approximate the total width of the top quark by its dominant decay $t \rightarrow bW$ which is computed at tree-level with the corresponding value of the top quark mass.

In Fig. 1 we plot the $\text{BR}(t \rightarrow cW^+W^-)$ as a function of the top quark mass in the SM, with and without FWE. The case of stable W 's in the final state (i.e. without FWE) is obtained by taking the limit $\rho(p_W^2, m_W^2, \Gamma_W^0) \rightarrow \delta(p_W^2 - m_W^2)$ [see Eq. (2.5)] which sets $p_{W^\pm}^2 = m_W^2$ in the integrand of Eq. (4.1). The decay $t \rightarrow cW^+W^-$ in the SM with stable W 's was calculated in [7] and our result for this case agrees with hers. From Fig. 1 we see that for the CDF central value of the top mass, $m_t = 173.5$ GeV, FWE can enhance the $\text{BR}(t \rightarrow cW^+W^-)$ by about an order of magnitude, reaching $\sim 2 \cdot 10^{-13}$. For the lower 1σ CDF limit $m_t \sim 167$ GeV, the enhancement due to FWE is of about two orders of

⁷ Note that in Model III the decay $\mathcal{H} \rightarrow t\bar{c}$ becomes important for $\lambda_{tc} \sim \mathcal{O}(1)$.

⁸ Depending on the \mathcal{H} mass, only the kinematically allowed decays will be included in $\Gamma_{\mathcal{H}}$.

magnitudes. Unfortunately, even with such large FWE in the decay $t \rightarrow cW^+W^-$, the BR in the SM is still too small to be measured - even at the LHC.

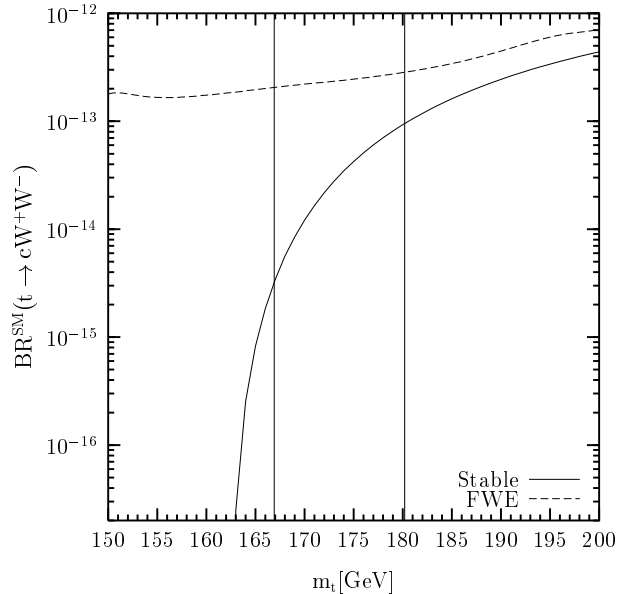


FIG. 1: The branching ratio for $t \rightarrow cWW$ in the SM, as a function of the top quark mass, without FWE (solid curve) and with FWE (dashed curve). The charm quark mass is set to $m_c = 1.87$ GeV and $V_{cb} = 0.046$. The vertical lines denote the recent (from Tevatron RUN II) lower and upper 1σ CDF limits on the top mass.

In Fig. 2 we show the $\text{BR}(t \rightarrow cW^+W^-)$ in Model III with $\lambda_{tc} = 1$, $m_{H^0} = 1$ TeV and $\alpha = \frac{\pi}{4}$ ⁹ (note that the SM tree-level contribution to $t \rightarrow cWW$, although included, is negligible in this case), as a function of m_t with and without FWE, for several values of the light Higgs mass $m_{h^0} = 130, 150, 170, \text{ and } 190$ GeV, and as a function of m_{h^0} with FWE, for the lower, upper and central CDF values of the top-quark mass $m_t = 166.9, 173.5, \text{ and } 180.2$ GeV. As was found in [10, 11], in Model III without FWE, the $\text{BR}(t \rightarrow cW^+W^-)$ can at most reach the level of $\text{few} \times 10^{-5}$ if m_t lies within its 1σ CDF limits and *only if* $m_{h^0} \sim m_t$. On the other hand, when FWE are “turned on”, a huge enhancement to the width arises within a large range of the Higgs mass. In particular, for $100 \text{ GeV} \lesssim m_{h^0} \lesssim 165 \text{ GeV}$, we

⁹ The dependence of $\text{BR}(t \rightarrow cW^+W^-)$ and $\text{BR}(t \rightarrow cZZ)$ on the Higgs mixing angle α in Model III can be found in [10, 11]. The maximum of these branching ratios with respect to α takes place at $\alpha = \pi/8$ (due to the dependency of the Higgs width on α) and not at $\alpha = \pi/4$ which is used through out our analysis.

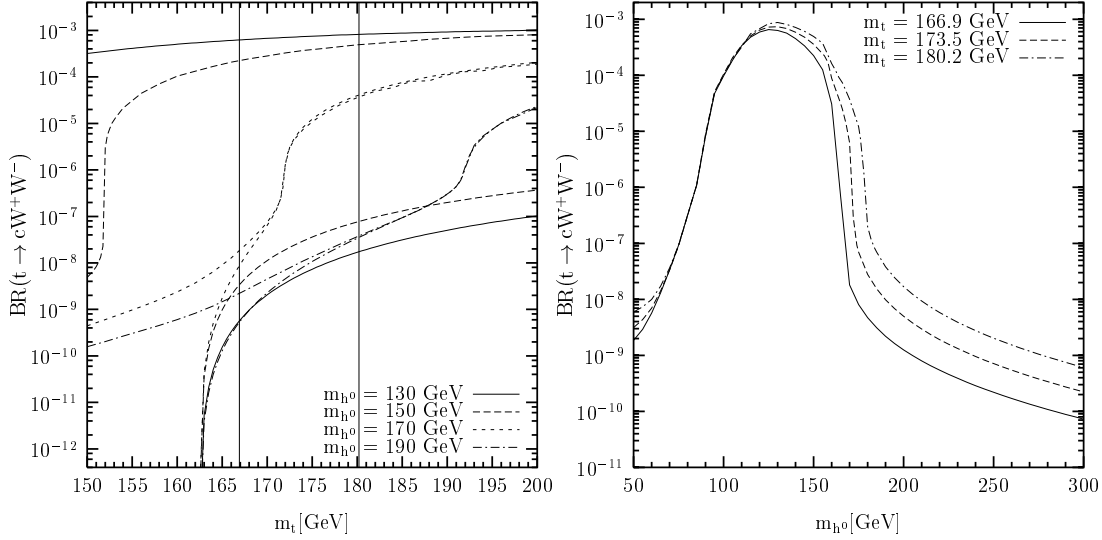


FIG. 2: The branching ratio for $t \rightarrow cWW$ in Model III, for $\lambda_{tc} = 1$, $m_{H^0} = 1$ TeV and $\alpha = \pi/4$. On the left: as a function of the top quark mass, without FWE (lower curves) and with FWE (upper curves), for $m_{h^0} = 130, 150, 170, 190$ GeV. On the right: as a function of the light Higgs mass m_{h^0} , for $m_t = 166.9, 173.5, 180.2$ GeV. See also caption to Fig. 1.

find $\text{BR}(t \rightarrow cW^+W^-) \gtrsim 10^{-4}$, if $167 \text{ GeV} \lesssim m_t \lesssim 180 \text{ GeV}$, in Model III when FWE are included. Note that, for the lower 1σ limit $m_t \sim 167$ GeV, i.e., close to the threshold for producing cWW , the FWE causes an up to six orders of magnitudes enhancement to the $\text{BR}(t \rightarrow cW^+W^-)$ if, e.g., $m_h \sim 130$ GeV.

For the decay $t \rightarrow cZZ$ in Model III we use the analytical results of $t \rightarrow cWW$ with the replacements $m_W \rightarrow m_Z / \cos \theta_W$ in the $\mathcal{H}VV$ vertex, $p_W^- \rightarrow p_{Z_1}$, $p_W^+ \rightarrow p_{Z_2}$ in Eq. (4.1) and with an additional overall factor of $1/2$ to take into account the symmetry factor for identical particles in the final state (i.e., Z bosons). Fig. 3 shows the scaled branching ratio $\text{BR}(t \rightarrow cZZ)/\lambda^2$ ($\lambda \equiv \lambda_{tc}$) in Model III with $m_{H^0} = 1$ TeV and $\alpha = \pi/4$ (see also footnote 9), as a function of m_t with and without FWE, for $m_{h^0} = 130, 150, 170,$ and 190 GeV, and as a function of m_{h^0} with FWE, for $m_t = 166.9, 173.5,$ and 180.2 GeV. Note that the decay $t \rightarrow cZZ$ is fundamentally different from $t \rightarrow cWW$, since, unlike $t \rightarrow cWW$, this decay channel *cannot* occur for stable Z -bosons if m_t lies within its 1σ limits. Thus, the inclusion of FWE in $t \rightarrow cZZ$ is crucial in this case. In particular, from Fig. 3 we see that a remarkably large $\text{BR}(t \rightarrow cZZ) \sim 10^{-5} - 10^{-3}$ is expected in Model III, if m_{h^0} lies within $90 \text{ GeV} \lesssim m_{h^0} \lesssim 170 \text{ GeV}$. Such a large BR will be accessible to the LHC and may even be

detected at the Tevatron.

Finally we note that, following [1] (who took $m_b = m_B$ for their calculation of $t \rightarrow bWZ$), we take $m_c = m_D = 1.87$ GeV.

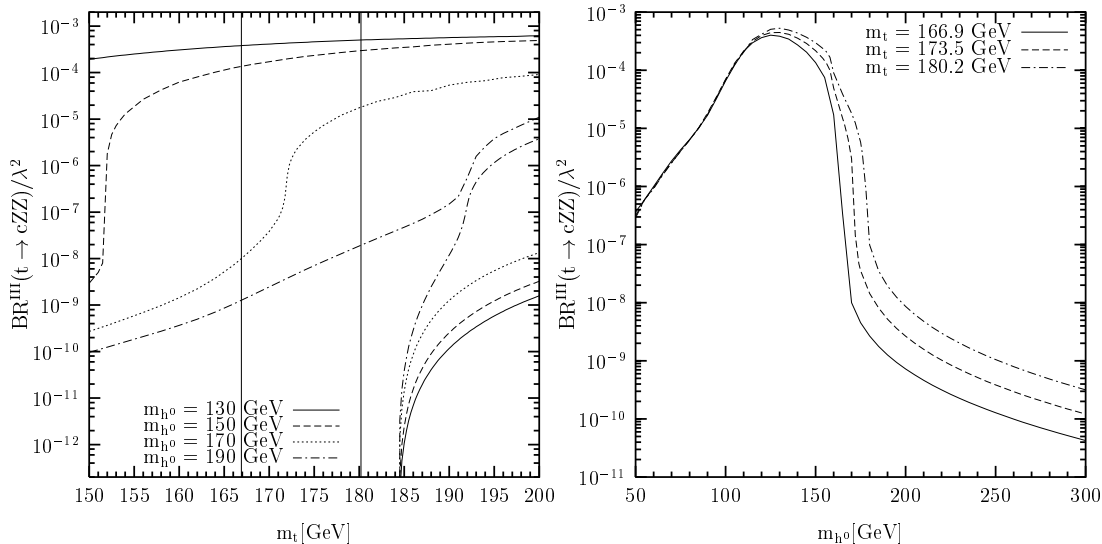


FIG. 3: The scaled branching ratio $BR(t \rightarrow cZZ)/\lambda^2$ in Model III, for $m_{H^0} = 1$ TeV and $\alpha = \pi/4$. On the left: as a function of the top quark mass, without FWE (lower curves) and with FWE (upper curves), for $m_{h^0} = 130, 150, 170, 190$ GeV. On the right: as a function of the light Higgs mass m_{h^0} , for $m_t = 166.9, 173.5, 180.2$ GeV. See also caption to Fig. 2.

V. FINITE WIDTH EFFECTS IN THE $A^0 \rightarrow (\bar{t}b + t\bar{b})W$ AND $h^0 \rightarrow (\bar{t}b + t\bar{b})W$ DECAYS

In this section we will examine FWE in three-body decays of neutral Higgs bosons in Model III. We will focus on the decay channels $A^0 \rightarrow \bar{t}bW^+$ and $h^0 \rightarrow \bar{t}bW^+$ which can have both theoretical and experimental advantages for Higgs searches and for investigating Higgs properties in the Higgs mass range $200 \text{ GeV} \lesssim m_{h^0}, m_{A^0} \lesssim 300 \text{ GeV}$.

The tree level diagrams contributing to these two decays in Model III are given in Fig. 5 (note that, for the A^0 decay, the diagram with an intermediate W -boson is missing, i.e., diagram (d), due to the absence of a tree-level A^0WW coupling). A formula analogous to

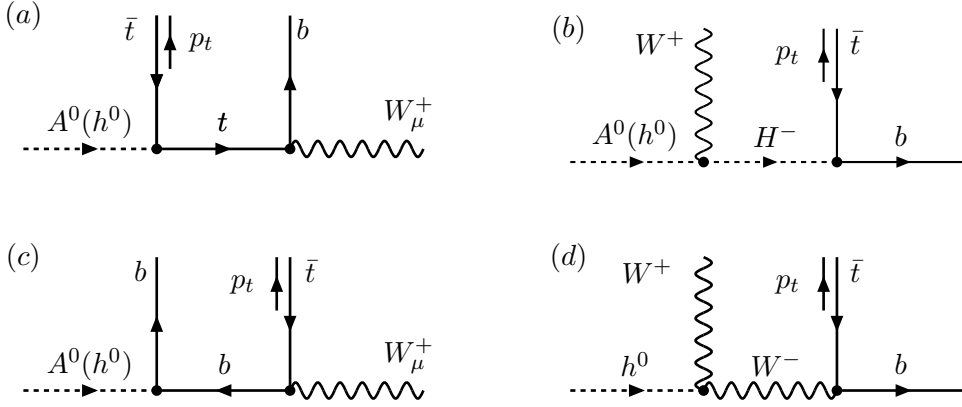


FIG. 4: Tree-level diagrams contributing to the decay $A^0(h^0) \rightarrow \bar{t}bW^+$ in Model III.

FIG. 5: Tree-level diagrams contributing to the decay $A^0(h^0) \rightarrow \bar{t}bW^+$ in Model III.

Eq. (2.6) can be given for Higgs decays as

$$\Gamma(\mathcal{H} \rightarrow b \bar{a} V) = \int_0^{(m_{\mathcal{H}} - m_b)^2} dp_1^2 \int_0^{(m_{\mathcal{H}} - m_b - \sqrt{p_1^2})^2} dp_2^2 \rho_1(p_1^2, m_t, \Gamma_a^0) \rho_2(p_2^2, m_V, \Gamma_V^0) \times \Gamma^0(\mathcal{H} \rightarrow b \bar{a}(p_1^2) V(p_2^2)), \quad (5.1)$$

where $\mathcal{H} = h^0$ or A^0 and $a(b)$ is the top(bottom) quark. Using the interaction terms in Section 3, we calculate the matrix element for each decay, where:

- The propagator of the intermediate W is taken from Eq. (2.8).
- In the calculation of Γ^0 in Eq. (5.1), the usual sum over the spins of the outgoing top-quark is modified to $\sum u(p_t)\bar{u}(p_t) = \not{p}_t + \sqrt{p_t^2}$ since, using the prescription of the CM, the final state top-quark is allowed to be off-shell.
- Throughout the following we assume that the Higgs mass spectrum respects $m_{h^0} < m_{A^0} \ll m_H^+, m_H^0$, setting $m_H^+ = m_H^0 = 1$ TeV. Thus, the contribution from the charged Higgs exchange, i.e., diagram (b) in Fig. 5, becomes negligible.
- The total width of A^0 is estimated from the decays $A^0 \rightarrow \tau\bar{\tau}, b\bar{b}, h^0 Z, h^0 Z^*, (t\bar{b} + \bar{t}b)W$, and the total width of h^0 is estimated from the decays $h^0 \rightarrow \tau\bar{\tau}, b\bar{b}, W^+W^-, ZZ$.
- We set all the relevant flavor diagonal λ 's of the Higgs Yukawa couplings in Eq. (3.6) to unity, i.e., $\lambda_{qq} = 1$.

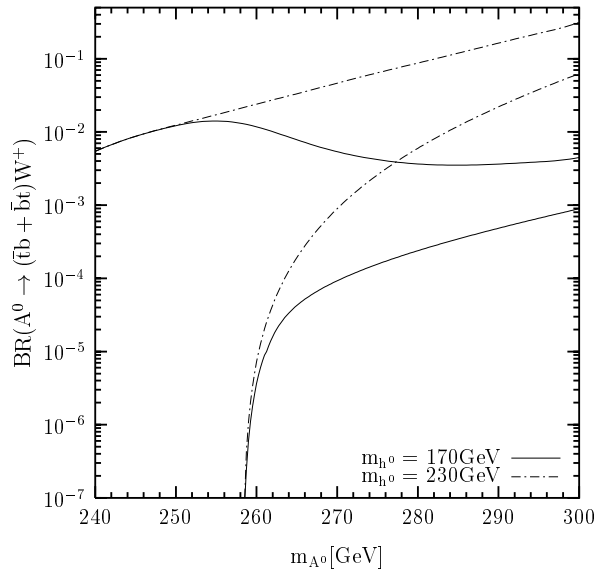


FIG. 6: The branching ratio $\text{BR}(A^0 \rightarrow \bar{t}bW^- + \bar{t}bW^+)$ in model III, as a function of the A^0 mass, with FWE (upper curves) and without FWE (lower curves), for $m_{h^0} = 170$ GeV (solid curves) and $m_{h^0} = 230$ GeV (dashed-dotted curves). Also, $m_t = 173.5$ GeV, $m_{H^+} = 350$ GeV and $\alpha = \pi/4$.

With the above assumptions, the remaining relevant input parameters (in Model III) for evaluating the branching ratios under consideration are m_{A^0} , m_{h^0} and the Higgs mixing angle α .

In Fig. 6 we depict the branching ratio of $A^0 \rightarrow (\bar{t}b + \bar{t}\bar{b})W$ as a function of m_{A^0} , for two values of the light Higgs mass $m_{h^0} = 170$ and 230 GeV and for $m_H^+ = 1$ TeV, $m_t = 173.5$ GeV and $\alpha = \pi/4$. We see that near threshold, i.e., $m_{A^0} \sim 260$ GeV, there is an enhancement of several orders of magnitude due to FWE, wherein the the branching ratio can reach $\text{BR}(A^0 \rightarrow (\bar{t}b + \bar{t}\bar{b})W) \sim 10^{-2}$. Away from threshold, the decay $A^0 \rightarrow (\bar{t}b + \bar{t}\bar{b})W$ is sensitive to the lightest neutral Higgs mass, m_{h^0} . In this case, the inclusion of FWE can increase the branching ratio by almost an order of magnitude, giving e.g. $\text{BR}(A^0 \rightarrow (\bar{t}b + \bar{t}\bar{b})W) \sim \text{few} \times 10^{-1}$ for $m_{A^0} \sim 300$ GeV and $m_{h^0} = 230$ GeV. Thus, FWE in the three-body decay $A^0 \rightarrow (\bar{t}b + \bar{t}\bar{b})W$ can become very significant – bringing its BR to the level of tens of percents and making it competitive with the A^0 two-body decays and, therefore, a viable experimental signature for studies of the properties of the Higgs sector.

Finally, let us consider the decay $h^0 \rightarrow (\bar{t}b + \bar{t}\bar{b})W$. In Fig. 7 we plot its branching ratio as a function of m_{h^0} for the same input parameters (of Model III) as in Fig. 6. In this case,

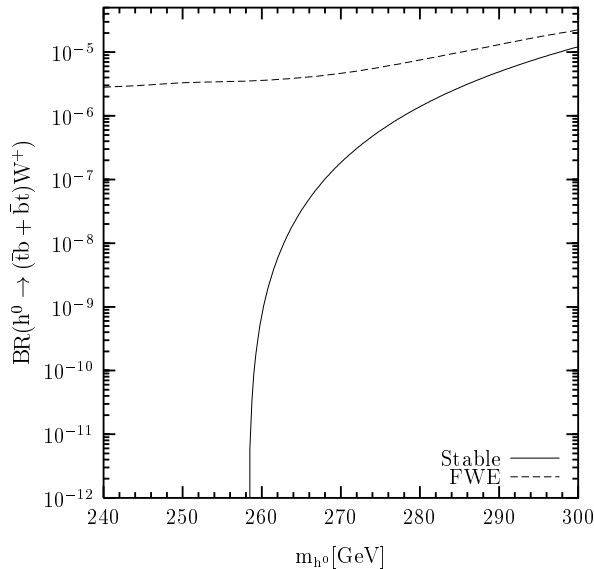


FIG. 7: The branching ratio $\text{BR}(h^0 \rightarrow \bar{t}bW^- + \bar{t}bW^+)$ in model III, as a function of the h^0 mass, with FWE (upper curve) and without FWE (lower curve), for $m_t = 173.5$ GeV, $m_{H^+} = 350$ GeV and $\alpha = \pi/4$.

in spite of the large enhancement near threshold due to FWE, the $\text{BR}(h^0 \rightarrow (\bar{t}b + \bar{t}b)W)$ remains rather small, i.e., at most of $\mathcal{O}(10^{-5})$, mainly due to the much larger h^0 total width caused by its tree-level decays to a pair of gauge-bosons $h^0 \rightarrow WW, ZZ$.

VI. SUMMARY

We have studied and emphasized the importance of FWE (finite width effects) in decays occurring just around their kinematical thresholds. For the inclusion of FWE we have adapted the so called CM (convolution method). In the CM, the unstable particle with 4-momentum p is treated as a real physical particle with an invariant mass $\sqrt{p^2}$ and effectively weighted by a Breit-Wigner-like density function, which, becomes a Dirac-delta function in the limit that the particle's total width approaches zero.

We first examined the FWE within the SM in the rare and flavor-changing tree-level top decay $t \rightarrow cW^+W^-$ and then extended our analysis to FWE in the tree-level top decays $t \rightarrow cW^+W^-$, $t \rightarrow cZZ$ and Higgs decays A^0 , $h^0 \rightarrow \bar{t}bW$ in a general two Higgs doublets model, the so called Model III, which gives rise to tree-level FCNC in the Higgs-fermion

sector. In all these case we find that FWE can become substantial – enhancing the branching ratios for the above decays by several orders of magnitudes near threshold.

Unfortunately, in the SM case, the top decay $t \rightarrow cW^+W^-$ remains too small to be of any value in the upcoming high energy colliders, i.e., $\text{BR}^{\text{SM}}(t \rightarrow cW^+W^-) \sim 10^{-13} - 10^{-12}$, in spite of the large enhancement due to FWE. On the other hand, in Model III, the large enhancement due to FWE in all these three-body top and Higgs decays can make a difference with respect to experimental studies in the upcoming hadron colliders. In particular, the branching ratios for the top-decays $t \rightarrow cW^+W^-$ and $t \rightarrow cZZ$ can reach the level of $10^{-4} - 10^{-3}$ near threshold – many orders of magnitudes larger than the corresponding branching ratio for the stable W and Z-bosons case (i.e., without FWE). For the $t \rightarrow cZZ$ decay, the inclusion of FWE is essential since such a large branching ratio arises even though the naive threshold for this decay is a few GeV away from the most recent 1σ upper limit on the top mass, $m_t(1\sigma) \sim 180$ GeV.

In the Higgs decays, FWE are more noticeable in the pseudo-scalar Higgs decay $A^0 \rightarrow (\bar{t}b + t\bar{b})W$, elevating its branching ratio to the level of tens of percents, thus making this three-body decay channel dominant and competitive with its two-body decays and, therefore, extremely important for experimental studies.

Thus, our study shows that FWE is essential for a proper treatment of otherwise neglected finite widths of particles which emerge at the final state of decays or scattering processes occurring just around the threshold.

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