OUTP-01-02P

hep-ph/0101040

arXiv:hep-ph/0101040v2 17 Apr 2001

The chicken or the egg; or
Who ordered the chiral phase transition?

Ian I. Kogan [1], Alex Kovner[2] and Bayram Tekin [1]

[1]Theoretical Physics, Oxford University,
1 Keble Road, Oxford, OX1 3NP, UK

[2] Department of Mathematics and Statistics,
University of Plymouth, Plymouth PL4 8AA, UK

Abstract

We draw an analogy between the deconfining transition in the 2+1 dimensional Georgi-Glashow model and the chiral phase transition in 3+1 dimensional QCD. Based on the detailed analysis of the former [1] we suggest that the chiral symmetry restoration in QCD at high temperature is driven by the thermal ensemble of baryons and antibaryons. The chiral symmetry is restored when roughly half of the volume is occupied by the baryons. Surprisingly enough, even though baryons are rather heavy, a crude estimate for the critical temperature gives $T_c = 180$ Mev. In this scenario the binding of the instantons is not the cause but rather a consequence of the chiral symmetry restoration.

1 Introduction.

In this paper we suggest that the chiral symmetry restoration in QCD at high temperature is driven by the presence of baryons in the thermal ensemble. In this scenario the chiral symmetry is restored at the temperature at which the density of the baryons (and antibaryons) in the thermal ensemble is large enough so that they start to overlap in space.

There are two main properties of the baryon that render this proposal physically sensible. First, chiral properties of the baryon are the same as of a skyrmion in the effective chiral Lagrangian. That is, inside the baryon the chiral condensate has the opposite sign to that in the vacuum[2]. Thus if half of the space is filled with baryons, the average value of the chiral condensate vanishes and the chiral symmetry is restored. The second crucial property is that even though the baryons are heavy, they are spatially very large. Thus the temperature at which the baryons start overlapping in space is not of the order of their mass, but is significantly smaller. We will present some rough estimates of this temperature later on and will show that it is in the ball-park of 180 Mev.

This mechanism is in a way a competing mechanism to the instanton binding, which has been advocated and studied in [3]. According to the instanton binding scenario, it is the binding of instantons into "molecules" that drives the restoration of the chiral symmetry. In our scenario the symmetry is restored practically independently of the instanton dynamics. However once the symmetry restoration has taken place, the instantons are indeed bound in pairs by linear "potential". Thus the instanton binding is not the cause, but rather the consequence of the chiral symmetry restoration.

Before discussing QCD we would like to make our point on a simpler example, where one can show analytically that a similar mechanism is indeed responsible for a thermal phase transition. The case in point is the Georgi-Glashow model in 2+1 dimensions, and the transition is the deconfining phase transition. Many years ago Polyakov [4] showed that this theory is confining. Ever since this model has been used as a test ground for various ideas about the dynamics of confinement in 3+1 dimensional theories. It may perhaps seem surprising that we will be using it as a prototypical example for chiral rather than confining dynamics. But then again this remarkable model is full of surprises!

Let us first explain in what sense the dynamics of the 3D Georgi-Glashow model is similar to the chiral dynamics of QCD.

2 The Georgi-Glashow model - symmetries, anomalies, instantons and "baryons".

Consider the $SU(2)$ gauge theory with a scalar field in the adjoint representation in 2+1 dimensions.

$$S = -\frac{1}{2g^2}\int d^3x \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) + \int d^3x\left[\frac{1}{2}(D_\mu h^a)^2 + \frac{\lambda}{4}(h^a h^a - v^2)^2\right] \tag{1}$$

Here $A_\mu = \frac{i}{2}A^a_\mu\tau^a$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$, $h = \frac{i}{2}h^a\tau^a$, and $D_\mu h = \partial_\mu h + [A_\mu, h]$.

In the weakly coupled regime $v \gg g^2$, perturbatively the gauge group is broken to $U(1)$ by the large expectation value of the Higgs field. The photon associated with the unbroken subgroup is massless whereas the Higgs and the other two gauge bosons $W^\pm$ are heavy with the masses

$$M_H^2 = 2\lambda v^2, \qquad\qquad M_W^2 = g^2 v^2. \tag{2}$$

Thus perturbatively the theory behaves very much like electrodynamics with spin one charged matter.

This theory has a global symmetry which will play a very prominent role in the following discussion. This is the magnetic symmetry [5, 6]. Classically the following gauge invariant current is conserved

$$\tilde{F}^{\mu} = \epsilon^{\mu\nu\lambda}\hat{h}_a F^a_{\nu\lambda} - \frac{1}{e}\epsilon^{\mu\nu\lambda}\epsilon^{abc}\hat{h}_a(\mathcal{D}_\nu\hat{h})^b(\mathcal{D}_\lambda\hat{h})^c \tag{3}$$

where $\hat{h}^a = h^a/|h|$. This current defines a conserved charge through $\Phi = \int d^2x\tilde{F}_0(x)$. The continuous $U_M(1)$ magnetic symmetry generated by this charge is spontaneously broken in the vacuum, and the massless photon is the Goldstone boson which reflects this breaking in the spectrum.

However there are important quantum nonperturbative effects that change this picture in significant ways. Those are of course the effects of monopole-instantons. The theory supports stable Euclidean configurations with finite action

$$\begin{aligned} h^a(\vec{x}) &= \hat{x}^a h(r) \\ A^a_\mu(\vec{x}) &= \frac{1}{r}\left[\epsilon_{a\mu\nu}\hat{x}^\nu(1-\phi_1) + \delta^{a\mu}\phi_2 + (rA - \phi_2)\hat{x}^a\hat{x}_\mu\right] \end{aligned} \tag{4}$$

where $\hat{x}^a = x^a/r$. In the presence of such a monopole the magnetic current is not conserved, but rather has a non-vanishing divergence proportional to the monopole density.

$$\partial_\mu\tilde{F}_\mu = \frac{4\pi}{g}\rho \tag{5}$$

The $U_M(1)$ magnetic symmetry is thus *anomalous* in the quantum theory. It can be shown [6] that only the discrete Z_2 subgroup is unaffected by anomaly and thus remains a symmetry in the full quantum theory.

Due to this anomaly the photon becomes a pseudo-Goldstone boson and acquires a finite mass. This mass is proportional to the density of monopoles, and is exponentially small at weak coupling, $m^2_{ph} \propto \exp\{-4\pi M_W/g^2\}$.

Another effect of the monopoles is confinement of $W^\pm$ bosons. The physically transparent way to see this is to consider the effective low energy description of the model. As discussed in detail in [6, 1] the relevant degree of freedom at low energies is the scalar field V that creates a magnetic vortex of flux $2\pi/g$. Under the anomalous magnetic rotation by the angle α it transforms as

$$V \to e^{i\frac{2\pi}{g}\alpha} V \tag{6}$$

so that the conserved Z_2 subgroup ($\alpha = g/2$) acts on it by the sign change. The low energy effective Lagrangian in terms of the vortex field is

$$\mathcal{L} = \partial_\mu V^* \partial^\mu V - \lambda (V^* V - \mu^2)^2 - \frac{m^2}{4}(V^2 + V^{*2}) + \zeta(\epsilon_{\mu\nu\lambda}\partial_\nu V^* \partial_\lambda V)^2 \tag{7}$$

The coupling constants in eq.(7) are determined in the weakly coupled region from perturbation theory and dilute monopole gas approximation. In the weakly coupled region (assuming that the $W^\pm$ bosons are much lighter than the Higgs particle) we have

$$\begin{aligned} \mu^2 &= \frac{g^2}{8\pi^2} & \lambda &= \frac{2\pi^2 M_W^2}{g^2} \\ m &= m_{ph} & \zeta &\propto \frac{1}{g^4 M_W} \end{aligned} \tag{8}$$

Here m_{ph} is the exponentially small nonperturbative photon mass calculated by Polyakov [4].

As discussed extensively in [6] the W-bosons appear in this low energy description as solitons. They carry a unit winding number of the field V. Placing W at a point x forces the phase of V to wind along any curve that surrounds x. Due to the fact that the global symmetry of the effective Lagrangian is Z_2 and not $U(1)$, the lowest energy configuration that carries a unit winding is not rotationally symmetric hedgehog, but rather a quasi one dimensional string-like configuration see Fig.1.

The energy of this configuration is proportional to the length of the string with the string tension parametrically of order $g^2 m_{ph}$. A pair of heavy W^+ and W^- separated by a distance $R > 1/m_{ph}$ is connected by a string and is confined. In fact a more careful analysis [7] reveals that when the distance R is large this "adjoint" string splits in two "fundamental" ones. The fundamental string in the effective Lagrangian appears as a

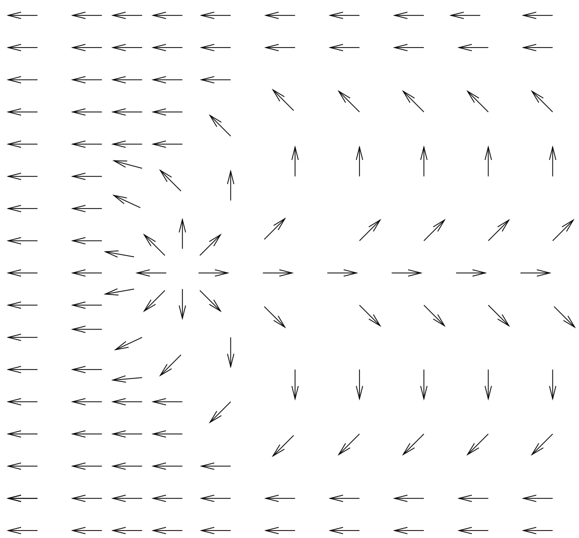

Figure 1: The string like configuration of the field V in the state of unit charge (W-boson).

domain wall separating two possible vacuum states of the field V, which are degenerate due to spontaneous breaking of the magnetic Z_2. As shown in [7] these fundamental strings repel each other, and thus it is energetically favorable for the adjoint string to split into two fundamental ones. Due to the linear confinement, the W bosons do not appear in the spectrum. The actual finite energy excitations are heavy W^+-W^- bound states. Such a state naturally looks like a domain of one vacuum inside the other one see Fig.2.[1] Thus inside the bound state the value of the order parameter V has the opposite sign that in the surrounding vacuum.

Many elements in the structure just discussed are very similar to QCD with massless fermions. The analogy we have in mind is the following.

- Classical axial $U_A(1)$ symmetry $\leftrightarrow$ Classical magnetic $U_M(1)$ symmetry .
- Axial anomaly due to instantons $\leftrightarrow$ Magnetic anomaly due to monopoles.
- Non-anomalous Z_{N_f} subgroup of $U_A(1)$ $\leftrightarrow$ Non-anomalous Z_2 subgroup of $U_M(1)$.

[1]The domain walls themselves of course have a finite thickness of order of the inverse photon mass.

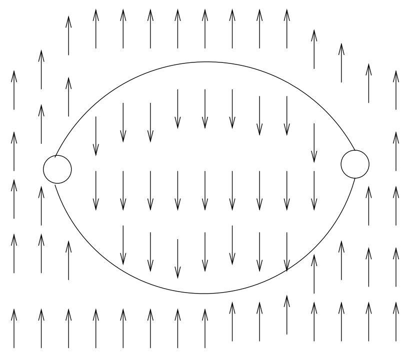

Figure 2: The W^+-W^- bound state as the domain of the other vacuum.

• Spontaneous breaking of Z_{N_f} by the chiral condensate $< \bar{\psi}\psi > \leftrightarrow$ Spontaneous breaking of Z_2 by the vortex condensate $< V >$.

• Heavy baryons-skyrmions: pockets of the other Z_{N_f} vacuum $\leftrightarrow$ Heavy $W^\pm$ bound states: pockets of the other Z_2 vacuum.

There is another important similarity between the baryons and the bound states in the Georgi - Glashow model. Both are heavy, but spatially large. In the Georgi - Glashow model, the mass of the bound state is roughly $M = 2M_W$, while the size D is of the order of the inverse photon mass. Thus there exists a parametric inequality $M >> D^{-1}$. In QCD of course there is no parametric inequality of this type, since the theory does not have a dimensionless coupling constant. Nevertheless the mass of the nucleon (940 Mev) is about ten times bigger than its inverse diameter (the radius is R=.88 fm)[8].

3 The deconfining phase transition.

While the zero temperature properties of the Georgi - Glashow model just described have been known for quite a while, the finite temperature deconfining phase transition has been studied only very recently[1]. The dynamics of this transition is quite interesting and turned out to be somewhat unexpected.

A natural, but as it turns out misleading way to think about the deconfining transition is in terms of the dynamics of the monopole “plasma”. At zero temperature the potential between monopoles is the 3D Coulomb potential $1/r$ and therefore the monopole gas is in the “plasma” phase. At finite temperature, when one of the dimensions is compactified the potential at distances $r > T$ turns into two dimensional Coulomb, that is logarithmic. The strength of the logarithmic interaction is proportional to the temperature, and at temperature $T_{BKT} = g^2/2\pi$ the monopoles bind in pairs via the Berezinsky-Kosterlitz-Thouless mechansim. Above this temperature the monopole gas is in the molecular phase. Since at zero temperature it is the monopole plasma effects that are responsible for confinement, one may be tempted to conclude that this BKT transition in the monopole gas is indeed the deconfining transition of the Georgi-Glashow model [9].

A more careful analysis however shows that the situation is much more interesting. The dynamics of the transition is completely different, and the critical temperature is half the value predicted by the monopole binding mechanism[1]. The real culprit are not the monopoles but rather the $W^{\pm}$ bosons, or equivalently their bound states. It may seem at first that W can not possibly affect the transition, since they are extremely heavy. However, even though their fugacity is very small at all temperatures of interest ($\exp\{-M_W/T\}$ with $T \propto g^2$), their effect is long range and therefore strongly affects the infrared properties of the system. As should be clear from the preceding discussion, the presence of W tends to disorder the vortex field V, since inside the confining strings which are attached to W the phase of V has maximal possible variations. Thus when the density of W's is large enough, the vacuum of V becomes disordered and the magnetic Z_2 symmetry restoration occurs. The magnetic symmetry restoration is indeed equivalent to deconfinement as discussed in detail in [10]. The analysis of [1] shows that the transition occurs at the temperature at which the fugacity of the W bosons becomes equal to the "fugacity" of monopoles and in the BPS limit one has

$$\exp\{-M_W/T_C\} = \exp\{-4\pi M_W/g^2\}, \qquad T_C = \frac{g^2}{4\pi} \tag{9}$$

At this temperature the mean distance between the W bosons in the thermal ensemble becomes equal (comparable) to the inverse mass of the photon. This point has a special significance in terms of the bound states of W^+ and W^-. As explained above these bounds states are essentially domains of the second vacuum ($< V >= -\mu$) inside the bulk vacuum $< V >= \mu$. The size of these domains is of the order of the inverse photon mass. Thus the transition occurs precisely at the temperature at which a finite fraction of the volume of the system is occupied by these domains of the second vacuum[2]. Indeed physically this is very reasonable. At the point when $< V >= \mu$ in half of the volume and $< V >= -\mu$ in the other half , the expectation value of V over the whole volume, and thus over the thermal ensemble vanishes. This is precisely where the symmetry restoring transition has to occur.

It was also shown in [1] that once the transition occurs, the potential between monopoles changes qualitatively. It becomes linear at large distances. Thus it is indeed true that the monopoles are bound in pairs above the transition. However this binding does not drive the phase transition but is rather the consequence of the transition which is driven by an entirely physically different mechanism - the overlap of the bound states in the thermal ensemble.

This picture of the transition is very simple and has a certain feel of universality about it. It seems very likely that a similar mechanism can operate in other cases. In particular in view of the similarities between the Georgi-Glashow model and chirally invariant QCD, we think that it is very interesting to explore whether the same mechanism is responsible

[2]The exact fraction of the volume was not calculated in [1]. It however follows from the results of [1] that this fraction is finite and not suppressed by an exponential factor of the type $\exp\{-AM_W/g^2\}$. Since the dependence of the W fugacity on the inverse temperature is exponential, this is enough to determine the critical temperature up to sub-leading corrections in powers of g^2/M_W.

for the chiral symmetry restoration. In the next section we will make some very rough estimates of the transition temperature assuming this is indeed the case.

4 Baryon driven chiral symmetry restoration.

Thus the picture of the chiral symmetry restoring phase transition we advocate is the following. At finite temperature the thermal ensemble contains some number of baryons and antibaryons. Inside the baryon the sign of the chiral condensate $\bar{\psi}\psi$ is opposite to that in the vacuum. As temperature increases the density of the baryons grows. At some point the density is large enough so that half of the volume is filled by the chiral condensate of the opposite sign. At this temperature the order parameter averaged over the thermal ensemble vanishes and the chiral symmetry is restored.

The factor that works against the symmetry restoration is the high mass of the baryon. On the other hand there are several factors that help. First, the size of the baryon is large. In the following estimates we will use for the radius of the baryon $R = .88$ fm[8]. Strictly speaking this is the charge radius, however the radius of the region of the wrong-sign-condensate is very similar [11]. Second, the entropy of the baryons is quite large. In the two flavor case we will take into account nucleon and delta, including their spin and isospin degrees of freedom. Third, the radius of the baryon itself depends on temperature and is believed to grow as the temperature rises. Although no reliable calculation of the swelling of the baryon size exists, it is reasonable to expect that the size increases by about 10-20% at the critical temperature due to the decrease of F_π. We will try to model this last effect in a very simplistic way.

To estimate the critical temperature we approximate the baryon ensemble by a non-relativistic ensemble of free noninteracting particles. The density of particles in such an ensemble is given by

$$n(T) = \sum_i N_i (\frac{M_i T}{2\pi})^{3/2} e^{-\frac{M_i}{T}} \tag{10}$$

where M_i is the mass of the particle of species i and N_i is the number of degrees of freedom with this mass.

We estimate the critical temperature by equating the fraction of the volume occupied by the particle to 1/2. In all the estimates we take the radius of all the relevant baryons to be equal. We will consider in the following the cases of 2 and 3 massless flavors as well as the realistic case of the massive strange quark.

Let us first consider the two flavor case. The only baryons important for the transition are the nucleon and the delta with $M_n = 938$ MeV and $M_\Delta = 1232$ MeV. We have checked numerically that including the Roper resonance does not affect the results. The fraction of the volume occupied by the nucleons and deltas at temperature T is

$$f(T) = 8\frac{4\pi R(T)^3}{3}\ (\frac{M_n T}{2\pi})^{3/2}\ e^{-\frac{M_n}{T}} \left\{1 + 4(\frac{M_\Delta}{M_n})^{3/2} e^{\frac{M_n - M_\Delta}{T}}\right\} \tag{11}$$

where the entropy factor is $2(2S+1)(2I+1)$ for particle-antiparticle, spin and isospin degrees of freedom. In this formula we allowed for the temperature dependence of the nuclear radius. Neglecting this effect first, we plot the fraction $f(T)$ in Fig.3. The striking feature of this plot is that all the action happens in the relatively narrow window between $T = 150$ Mev and $T = 215$ Mev. Note that this temperature range is indeed much lower than the baryon mass and is in the right ball-park for the chiral phase transition. The value of the critical temperature we extract from this graph is $T_c = 213$ Mev.

We next try to take into account the swelling of the baryon radius with temperature. Our simple ansatz for this dependence is

$$R(T) = R(0) + \frac{1}{M_\sigma(T)} - \frac{1}{M_\sigma(0)} \tag{12}$$

with M_σ - the mass of the σ-particle.

$$M_\sigma(T) = \frac{F_\pi(T)}{F_\pi(0)} M_\sigma(0) \tag{13}$$

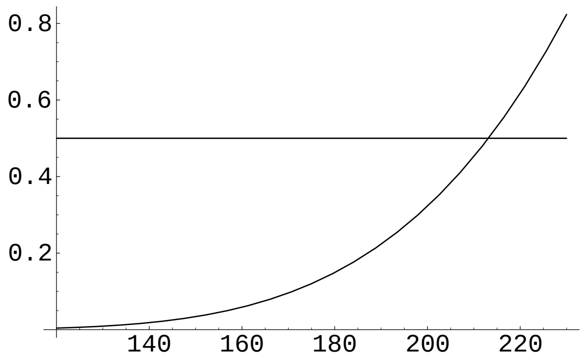

Figure 3: In two flavor QCD the fraction of the volume occupied by the nucleons and deltas as function of temperature. The phase transition temperature is $T = 213$ MeV. The radius of particles is assumed to be temperature independent.

The rationale for this is the following. The chiral order parameter $\bar{\psi}\psi$ couples directly to the σ - particle. Inside the nucleon the chiral order parameter has a negative sign. It has to relax to its vacuum value on the outside. This relaxation happens either through the "phase rotation" if σ-particle is very heavy, or through the change in the σ-field itself. In the latter case the distance over which it happens should be equal to the inverse σ-mass. Closer to the phase transition, σ becomes light and effective in the relaxation of the order parameter field. Eq.(12) is a simple interpolation between the low temperature situation, where σ is heavy and unimportant and the closer-to-criticality situation, where it does indeed contribute significantly to the size. The formula eq.(13) is just the simple linear σ - model type relation. We do not insist that eq.(12) has any precision, but we believe that it gives a rough estimate of the effect[3]. We take $M_\sigma(0) = 600$ MeV and $F_\pi(0) = 93$ MeV.

[3]We note that a similar effect of the change of F_π with temperature and the associate change in the size of the bound state is also present in the Georgi - Glashow model. Just like in QCD it is due to thermal fluctuations of the light particles, which in 3D are light photons. The reason we did not discuss it here, is that it is parametrically sub-leading. That is, it affects the correction to the value of the critical temperature at relative order g^2/M_W. Since QCD does not have a free parameter, the effect is likely to be more important in QCD and therefore should be taken into account. The effect of pions should disappear in $SU(N)$ theories for large N, since at large N both the inetraction of pions is weak and the ratio of the proton mass to its inverse size is parametrically large.

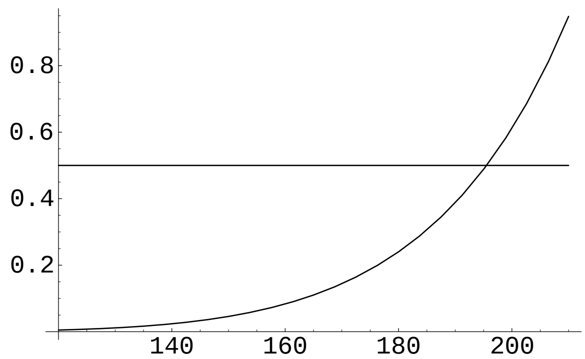

Figure 4: In two flavor QCD the fraction of the volume occupied by the nucleons and deltas. The dependence of the radius on temperature is given by eq.(14). The transition temperature is $T = 195.5$ MeV

To use this relation we still need to know the dependence of F_π on the temperature. In the lowest order in temperature it is given by [12]

$$F_\pi(T) = F_\pi(0)(1 - \frac{T^2}{12F_\pi(0)^2}) \tag{14}$$

The Padé resummed expression which should better represent the situation closer to criticality (T_c) has been proposed in [13].

$$\frac{F_\pi^2(T)}{F_\pi^2(0)} = \frac{1 - T^2/T_c^2}{1 - \frac{2}{3}(T^2/T_c^2)(1 - T^2/T_c^2)} \tag{15}$$

This formula assumes that the symmetry is $O(4) = SU(2) \times SU(2)$. Using eq.(14) the graph for the fraction $f(T)$ is given on Fig. 4. The critical temperature is $T = 195.5$ Mev.

Using $T_c = 195.5$ in eq.(15) we obtain Fig.5 with the critical temperature $T = 179$ Mev. Thus the swelling of the baryon radius has an effect of reducing the critical temperature by about 15%.

It is interesting to see how the value of the critical temperature depends on the number of flavors. For $N_f = 3$ case we should consider the baryon octet and decouplet.

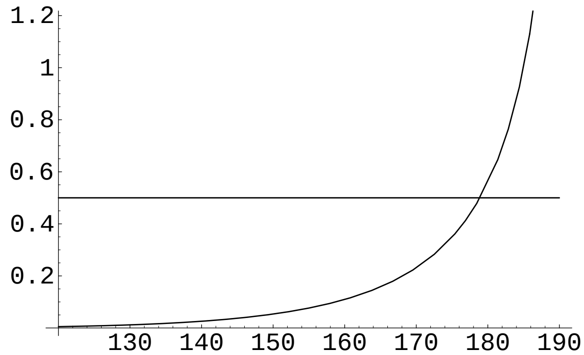

Figure 5: Same as in Fig.4 but with eq.(15) and $T_c = 195.5$. The transition temperature is $T = 179$ MeV

To get a rough idea here we will neglect the temperature dependence of the radius. In the idealized chirally symmetric three flavor case we take the octet mass as the mass of the nucleon and the decouplet mass as the mass of the delta. The resulting curve is plotted on Fig. 6.

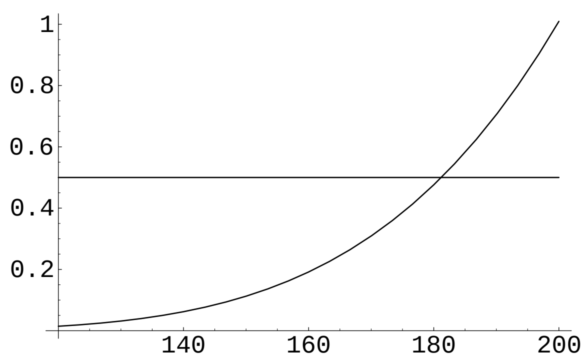

Figure 6: Fraction of the volume occupied by the baryons for three massless flavours. The transition temperature is $T = 181$ MeV

The critical temperature is $T = 181$ Mev. This is some 30 Mev lower than the corresponding value for the $N_f = 2$ case. The same trend exists in the lattice data [14]. In our approach this is easily understandable: it is the direct consequence of having roughly three times as many active baryons for $N_f = 3$ as for $N_f = 2$.

Taking instead the physical masses for the octet and decouplet members we get Fig. 7 with $T_c = 195.5$ Mev.

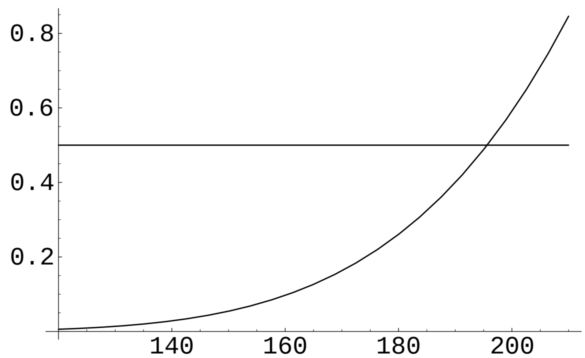

Figure 7: Same as in Fig.6 but with realistic baryon masses. The transition temperature is at $T = 195.5$ MeV

5 Discussion

The surprising result of our numerical estimates is that even though the baryon mass is around 1 Gev, the baryon overlap mechanism leads to critical temperature of order 180 Mev for $N_f = 2$, and about 30 Mev lower for $N_f = 3$. These numbers are perfectly reasonable and are in qualitative agreement with the lattice results which give $T_c = 173 \pm 8$ Mev for $N_f = 2$ and $T_c = 154 \pm 8$ Mev for $N_f = 3$ [14]. Of course our estimates are very rough and suffer from many uncertainties. For example, it is not clear that the fraction of the volume must be really 1/2. It may be enough to fill a smaller fraction, since the baryon has a pion tail which itself also contributes to disordering of the condensate. This would push the value of the critical temperature down. We also completely neglected the interaction between the baryons, which start to be important precisely in the region of densities we are interested in[4]. There is also an uncertainty of the dependence of the baryon radius on the temperature.

Our discussion of effects due to the thermal bath of mesons has been very rudimentary. Partly this effect has been taken into account by allowing for the temperature dependence of f_π(for more details see [12], [13] and references therein) which leads to the renormalization of the baryon parameters. This reduction in the value of f_π is due to direct disordering of the chiral vacuum by the thermal pions. The fact that the critical temperature we obtain is always lower than the input T_c in eq.(15) is in our view an indication that the disorder due to baryons takes precedence over the direct pion effects. The thermal production of vector and axial mesons we believe is less relevant since they are almost as heavy as baryons, but contrary to baryons have no direct disordering effect on the vacuum.

Since at this time we do not know how to take these effects into account in a well defined calculational framework, our discussion has been rather qualitative. It is however encouraging that the numbers fall in the right ball-park[5].

We note that our scenario relates to the instanton binding scenario of [3] in very much the same way as the actual transition in 3D Georgi-Glashow to the monopole binding

[4]We also neglected the fact that baryons are fermions. This effect is however rather small, and we have checked numerically that using Fermi-Dirac rather than Boltzmann distribution changes the value of the critical temperature by about 1 Mev.

[5]Due to the uncertainties in our estimates one has to be careful using them in some situations. For example a straightforward application of our argument would lead one to conclude that in the ordinary nuclear matter, chiral symmetry should be restored already at zero temperature, since the packing fraction of the baryons is close to one. In fact, however the critical density at which the chiral symmetry is restored is thought to be 2.5 to 3 times the nuclear matter density[15]. There is a significant difference however between the finite temperature and finite density situations. At finite temperature, due to the Boltzmann factor the dependence of the temperature on the packing fraction is essentialy logarithmic. Thus a change of order one in the packing fraction does not lead to significant change in the value of the critical temperature. On the other hand critical density is directly proportional to the packing fraction, and is thus very sensitive to any changes in it. One can certainly imagine dynamical effects which change the packing fraction from our naive estimates especially when a system is relatively dense. For example at finite chemical potential the size of the region inside the baryon where the order parameter is negative can shrink. This is consistent with the Skyrme model calculations of the sizes of baryons with higher baryon number[16]. A change of some 20 percent would be enough to push the effective packing fraction significantly below one, and thus push the system deep into chirally symmetric phase.

scenario [9]. In the chirally symmetric phase the potential between instantons should be linear whatever the mechanism that drives the transition is. This is simple to understand at high temperatures. Consider the correlation function of some local operator which is not invariant under the axial $U_A(1)$ but is invariant under the non-anomalous Z_{N_f} and also under the chiral $SU(N_f)\times SU(N_f)$. A good example of such an operator is 't Hooft's effective interaction vertex [17] T. At high temperature where the instanton gas is dilute and perturbation theory valid, the calculation of the correlation function $< T(x)T^*(y) >$ is dominated by the contribution of the instanton-antiinstanton pair at points x and y. One expects this correlation function to approach a constant value at large distance and the leading correction to be exponential $< T(x)T^*(y) >\propto [\exp\{-m|x-y|\}+\zeta]$. In terms of the instanton-antiinstanton potential this translates into linear potential which is screened at large distances. The screening is the consequence of the "breaking" of the string between instantons, whereby an extra instanton-antiinstanton pair appears when the distance $x-y$ is too large[18]. Thus just like in 3D we expect that the binding of instantons into pairs in the chirally symmetric phase is a consequence of the phase transition even if the transition itself is driven by a noninstanton mechanism.

An interesting property of the mechanism we suggest is a quite distinct large N_c behavior. The mass of the baryon is proportional to N_c. On the other hand the multiplicity of the lightest baryons scales as a power of N_c. For example in Skyrme model with 2 flavors one has $I=J=1/2, 3/2, 5/2, ..., N_c/2$ baryons with masses[6]

$$M = m_0 N_c + \frac{1}{N_c} m_1 I(I+1) \tag{16}$$

The degeneracy factor is $(2I+1)^2$ which (after summation over spins) leads to the overall extra factor N_c^3. Thus at large N_c the critical temperature predicted by the baryon overlap mechanism is $T_c \sim N_c/lnN_cT_0$ where T_0 is by order of magnitude of Λ_{QCD}. This temperature grows with N_C[7] On the other hand the deconfinement phase

[6]For discussion of more general case including the strange quark see for example [19].

[7]Interestingly although the temperature grows with N_C, at large N_c it is parametrically smaller than

transition temperature in the pure Yang-Mills theory is believed to be $O(1)$ in the large N_c limit and is parametrically smaller than T_c. Thus it is likely that at some critical number of colors the chiral symmetry restoration temperature becomes larger than the deconfinement temperature.

Some arguments have been advanced to the effect that if the chiral transition happens at lower temperature, it also drives deconfinement[20]. Thus at small N_c only one transition in QCD with fermions is observed. On the other hand if the deconfinement happens earlier, the chiral symmetry is not necessarily restored above this, first transition. In fact the common wisdom is that the confinement and the chiral symmetry breaking are due to different sectors of QCD dynamics. If chiral symmetry is still broken above the deconfining transition, the baryons should still exist there as bound states of quarks, even though the quarks themselves may be not confined. Thus the chiral symmetry restoring transition due to the baryon overlap mechanism can still run out its turn at $T_c = O(N_c)$. In this case for large enough number of colors the theory will have two distinct phase transitions: first the deconfining one and later the chirally restoring one. If the critical N_c is not too large, it may be possible to see the second transition in lattice simulations.

Another interesting issue is the fate of the hot chirally symmetric ground state when it is cooled. If the chiral transition is second or weakly first order there should be no appreciable hysteresis and thus during cooling the system should follow through the same states as during heating but in reverse order. This would imply production of baryon-antibaryon pairs in the initial stages of cooling and should lead to the production of baryon-rich final states in mid rapidity in collision processes which create quark-gluon plasma in the intermediate stage. If the transition is strongly first order there may be large hysteresis and cooling could proceed along a different root than heating.

Our suggestion in this paper is in large measure motivated by the analogy with the

the baryon mass with the suppression factor $1/lnN_c$.

3D Georgi - Glashow model. We should mention that the analogy is of course not perfect. The main new element in QCD is the existence of the continuous chiral symmetry in addition to the non-anomalous discrete axial one. Thus there are massless pions in the game, which was not the case in our 3D example. Thus for example the following question has to be answered. The direct consequence of the baryon mechanism is the vanishing of the chiral condensate. It however does not directly tell us that the pions become massive. In principle the situation when the order parameter vanishes, but there are still massless particles around is possible. It is in fact quite generic in 2 dimensional systems due to Coleman theorem. However it seems to us very unlikely that similar situation can be sustained in 4D. Thus we believe that once the condensate vanishes, pions will acquire a mass. It is interesting and important to identify a dynamical mechanism through which this is achieved[8].

Another aspect of QCD dynamics which is different compared to 3D Georgi-Glashow theory, is the role of instantons at zero temperature. In the Georgi-Glashow model, the monopole-instantons bring about the anomaly in the magnetic $U_M(1)$ symmetry, but they are not responsible for the spontaneous breaking of the residual Z_2 group. The spontaneous breaking is there already on the perturbative level. On the other hand in QCD it is believed that both the anomalous breaking of $U_A(1)$ and the spontaneous breaking of the residual chiral symmetry are due to instanton dynamics. Thus one may be more inclined to believe that the symmetry restoration transition in QCD is also linked to the instanton physics. However we stress that it is not at all necessary that the mechanism of the symmetry restoration is just elimination of the mechanism that brought about the symmetry breaking in the first place. Thus although it is logically possible that the instanton binding in QCD occurs at lower temperature than the baryon overlap, this question can only be settled by a reliable calculation. The numerical results

[8]We are grateful to Victor Petrov for raising this question and interesting and heated discussions on the subject.

of [3] indicate that the critical temperature for the instanton binding is by about 30 Mev lower than our estimate. However given the uncertainties of the calculation of [3] and even more so the qualitative level of our estimates here, we feel that much more work has to be done before a definite conclusion can be drawn on this point.

How does one distinguish between different possible mechanisms is not an easy question. On the qualitative level however, in the baryon overlap mechansim the symmetry restoration is due to large fluctuations of the phase of the order parameter rather than of its magnitude. Thus there should be a sharp distinction between this scenario and, say the transition in the linear σ - model. The quantity to measure in this case is the "square" of the order parameter, or in the case of two flavours, rather the 't Hooft vortex. If the transition is driven by large phase fluctuations, the average value of the 't Hooft vertex should change very little across the transition, since it is itself an invariant operator. If on the other hand, like in the linear σ - model the magnitude of the order parameter becomes small at criticality, so should the 't Hooft vertex. Such a measurement in the lattice gauge theory would be very interesting[9].

We think that the scenario we presented in this note is physically quite appealing and simple, and thus further work to check its validity is certainly warranted.

Acknowledgments

We are indebted to Victor Petrov for very interesting and stimulating discussions. We also acknowledge discussions with Witek Krasny and Misha Shifman. A.K. is supported by PPARC. The research of I.K. and B. T. are supported by PPARC Grant PPA/G/O/1998/00567.

References

[9] One has to be careful to appropriately smear the 't Hooft vertex to get rid of the ultraviolet contributions, which otherwise can easily blurr the picture

[1] G.Dunne, I.Kogan, A. Kovner and B. Tekin, “Deconfining phase transition in 2+1 D: The Georgi-Glashow model,” hep-th/0010201

[2] See for example, D. Diakonov and V. Petrov, “Nucleons as chiral solitons” hep-ph/0009006

[3] T. Schafer and E.V. Shuryak, “The instanton liquid in QCD at zero and finite temperature,” Phys. Rev. **D53**, 6522 (1996) hep-ph/9509337 and references therein.

[4] A.M. Polyakov, “Compact gauge fields and the infrared catastrophe,” Phys. Lett. **B 59** (1975) 82; “Quark confinement and topology of gauge groups,” Nucl. Phys. **B120**, 429 (1977);

[5] G. t’Hooft, “On The Phase Transition Towards Permanent Quark Confinement” Nucl. Phys. **B138**, 1 (1978);

[6] A. Kovner and B. Rosenstein, “Topological interpretation of electric charge, duality and confinement in (2+1)-dimensions,” Int. J. Mod. Phys. **A7**, 7419 (1992)

[7] A. Kovner and B. Rosenstein, “Strings and string breaking in 2+1 dimensional nonabelian theories,” J.High Energy Phys. **003** 9809 (1998), hep-th/9808031

[8] R. Rosenfelder, “Coulomb corrections to elastic electron-proton scattering and the proton charge radius“ nucl-th/9912031

[9] N.O. Agasyan and K. Zarembo, “Phase structure and nonperturbative states in three-dimensional adjoint Higgs model,” Phys. Rev. **D57**, 2475 (1998) hep-th/9708030.

[10] C.P. Korthals-Altes and A. Kovner, “Magnetic Z(N) symmetry in hot QCD and the spatial Wilson loop,” Phys. Rev. **D62**, 096008 (2000) hep-ph/0004052.

[11] V. Petrov, private communication.

[12] H. Leutwyler and A. V. Smilga, "Nucleons at finite temperature," Nucl. Phys. **B342**, 302 (1990).

[13] S. Jeon and J. Kapusta, "Pion decay constant at finite temperature in the nonlinear sigma model," Phys. Rev. **D54**, 6475 (1996) [hep-ph/9602400].

[14] F. Karsch, E. Laermann and A. Peikert, "Quark mass and flavor dependence of the QCD phase transition," hep-lat/0012023.

[15] A. Smilga, Nucl.Phys.**A654** 136c (1999) hep-ph/9901225 and references therein,

[16] M. Oka, K.F. Liu and H. Yu, Phys. rev. **D 34**, 1575 (1986)

[17] G. t'Hooft, "Computation of the quantum effects due to a four-dimensional pseudoparticle " Phys. Rev. **D 14**, 3432 (1976);

[18] I. Kogan, A. Kovner and B. Tekin, "Instanton molecules at high temperature: The Georgi-Glashow model and beyond,"hep-th/0101171.

[19] R. Dashen, E. Jenkins and A. Manohar, "The $1/N_c$ Expansion for Baryons" Phys.Rev.**D 49**, 4713 (1994), Erratum-ibid.bf D 51, 2489 (1995)., hep-ph/9310379

[20] S. Digal, E. Laermann, H. Satz, 'Deconfinement through chiral symmetry restoration in two-flavour QCD," hep-ph/0007175; "Deconfinement through chiral transition in 2 flavour QCD," hep-lat/0010046