

# Magnetic moment of $X_Q$ state with $J^{PC} = 1^{+\pm}$ in light cone QCD sum rules

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## Abstract

The magnetic moments of the recently observed resonance  $X_b(5568)$  by DO Collaboration and its partner with charm quark are calculated in the framework of the light cone QCD sum rules, by assuming that these resonances are represented as tetra-quark states with quantum numbers  $J^{PC} = 1^{+\pm}$ . The magnetic moment can play critical role in determination of the quantum numbers, as well as giving useful information about the inner structure of these mesons.

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# 1 Introduction

Observation of the charmonium like  $X(3872)$  resonance by the Belle Collaboration [1] opens new directions in particle physics. At present, more than 23 new exotic particles are discovered experimentally (for a review, see [2]). The usual thing about these discoveries is that, these states denoted as XYZ family, cannot be described by the usual quark–antiquark picture, and these are indications that they have more complicated structures. Experiments are usually focused on the measurement of their masses, identifications of their spins, parities, as well as decay widths.

These experimental observations have stimulated theoretical studies in this direction. In theoretical analysis for studying the properties of exotic states, usually two pictures, tetra–quark or molecular picture (bound states of two mesons), are used (for a review see [3, 4] and [5]). Recently the DO collaboration has reported the observation of the  $X_b(5568)$  state [6], whose analysis assigned the quantum numbers  $J^{PC} = 0^{++}$ , while did not exclude the possibility  $J^{PC} = 1^{++}$ . Later however, LHCb [6] and CMS [7] Collaborations did not confirm the existence of this state. Therefore, the observed  $X_b(5568)$  resonance by DO Collaboration still needs solid experimental conformation. The analysis of the mass, decay width  $X_b \rightarrow B_s^0 \pi$  has already been performed with the quantum number  $J^{PC} = 0^{++}$  in many works (see [9]–[18]).

In the present work, we assume that  $X_b(5568)$  state and its charm partner (hereafter we will denote these states as  $X_Q$ ) has quantum numbers  $J^{PC} = 1^{+\pm}$ , and calculate the magnetic moment of these exotic states. Obviously, the  $X_Q$  state with the quantum numbers  $J^{PC} = 0^{++}$  has zero magnetic moment.

The paper is organized as follows. In section 2, we consider  $X_b(5568)$  state, and its partner with  $c$ –quark as a tetra–quark state with the quantum numbers  $J^{PC} = 1^{+\pm}$ , and calculate the magnetic moment of these states in the framework of the light cone QCD sum rules method. Section 3 is devoted to the numerical analysis of the sum rules obtained for this exotic state.

## 2 The $X_Q$ meson magnetic moment in light cone QCD sum rules

In calculating the magnetic moment of the  $X_Q$  meson in the framework of the light cone QCD sum rules, we start by considering the following correlation function,

$$\Pi_{\mu\nu\rho}(p, q) = - \int d^4y e^{ipx} \langle 0 | T \{ J_\mu^{X_Q}(x) J_\nu(y) J_\rho^{X_Q^\dagger}(0) \} | 0 \rangle . \quad (1)$$

Here, the current  $J^{X_Q}$  with the quantum number  $J^{PC} = 1^{+\pm}$  describes the corresponding  $X_Q$  meson, and has the form,

$$J_\mu^{X_Q} = \frac{1}{\sqrt{2}} \varepsilon^{abc} \varepsilon^{ade} \left\{ (s^{bT}(x) C \gamma_5 u^c) (\bar{Q}^d(x) C \gamma_\mu \bar{d}^{eT}) \pm (s^{bT}(x) C \gamma_\mu u^c) (\bar{Q}^d(x) C \gamma_5 \bar{d}^{eT}) \right\} \quad (2)$$

where  $a, b, c, d, e$  are the color indices,  $C$  is the charge conjugation operator. The electromagnetic current  $J_\nu$  is given as,

$$J_\nu = e_s \bar{s} \gamma_\nu s + e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d + e_Q \bar{b} \gamma_\nu b ,$$

where  $e_q$  is the corresponding electric charge.

From technical point of view, it is quite useful to rewrite the correlation function by introducing the background electromagnetic plane wave field

$$F_{\mu\nu} = i (\varepsilon_\nu^\lambda q_\mu - \varepsilon_\mu^\lambda q_\nu) ,$$

where  $\varepsilon_\nu$  and  $q_\mu$  are the polarization and four-momentum of the background electromagnetic field. The correlator can then be written as,

$$\Pi_{\mu\nu\rho}(p, q)\varepsilon^{(\lambda)\nu} = i \int d^4y e^{ipx} \langle 0 | T \{ J_\mu^{X_Q}(x) \bar{J}_\rho^{X_Q}(0) \} | 0 \rangle_F . \quad (3)$$

The subscript in this expression means that the expectation value is evaluated in the background electromagnetic  $F_{\mu\nu}$  field. The correlation function (1) can be obtained by expanding Eq. (3) in powers of the background field, and keeping only terms linear in  $F_{\mu\nu}$ , which corresponds to the single photon emission (see [19] and [20] for more details about the background field).

We first calculate the correlation function in Eq. (1) in terms of the hadronic degrees of freedom. To do that, we saturate it with complete set of intermediate states having the same quantum numbers as the interpolating current of  $X_Q$ , and then isolate the contributions of the ground state, from which we get,

$$\Pi_{\mu\nu\rho}(p, q)\varepsilon^{(\lambda)\nu} = \frac{\langle 0 | J_\mu^{X_Q} | X_Q(p) \rangle}{p^2 - m_{X_Q}^2} \langle X_Q(p) | X_Q(p+q) \rangle_F \frac{\langle X_Q(p+q) | \bar{J}_\rho^{X_Q} | 0 \rangle}{(p+q)^2 - m_{X_Q}^2} + \dots \quad (4)$$

where the dots represent the contributions coming from the higher states and continuum, and  $q$  is the momentum of the photon. The matrix element  $\langle 0 | X_\mu^{X_Q} | X_Q \rangle$  is determined as,

$$\langle 0 | X_\mu^{X_Q} | X_Q \rangle = \lambda_{X_Q} \varepsilon_\mu^\lambda . \quad (5)$$

Using the time reversal and parity invariance, the electromagnetic vertex of the two vector mesons in presence of the electromagnetic background field can be defined in terms of the three form factors as [21]:

$$\begin{aligned} \langle X_Q(p, \varepsilon^\lambda) | X_Q(p+q, \varepsilon^{\lambda'}) \rangle_F &= -\varepsilon^\tau (\varepsilon^\lambda)^\alpha (\varepsilon^{\lambda'})^\beta \left\{ G_1(Q^2) g_{\alpha\beta} (2p+q)_\tau + G_2(Q^2) (q_\alpha g_{\tau\beta} - q_\beta g_{\tau\alpha}) \right. \\ &\quad \left. - \frac{1}{2m_{X_Q}^2} G_3(Q^2) q_\alpha q_\beta (2p+q)_\tau \right\} , \end{aligned} \quad (6)$$

where  $\varepsilon^\tau$  is the photon, and  $(\varepsilon^\lambda)^\alpha$ ,  $(\varepsilon^{\lambda'})^\beta$  are the polarizations of  $X_Q$  mesons. The form factors  $G_i(Q^2)$  can be written in terms of charge  $F_C(Q^2)$ , magnetic  $F_M(Q^2)$  and quadrupole  $F_D(Q^2)$  form factors in the following way.

$$\begin{aligned} F_C(Q^2) &= G_1(Q^2) + \frac{2}{3}\eta F_D(Q^2) , \\ F_M(Q^2) &= G_2(Q^2) , \\ F_D(Q^2) &= G_1(Q^2) - G_2(Q^2) + (1 + \eta)G_3(Q^2) , \end{aligned} \quad (7)$$

where  $\eta = Q^2/4m_{X_Q}^2$ . At zero momentum-square transfer, the form factors  $F_C(Q^2 = 0)$ ,  $F_M(Q^2 = 0)$ , and  $F_{\mathcal{D}}(Q^2 = 0)$  are related to the electric charge, magnetic moment  $\mu$  and the quadrapole moment  $\mathcal{D}$  in the following way:

$$\begin{aligned} eF_C(0) &= e, \\ eF_M(0) &= 2m_{X_Q}\mu, \\ eF_{\mathcal{D}} &= m_{X_Q}^2\mathcal{D}. \end{aligned} \quad (8)$$

Using Eqs. (4), (5) and (6), and performing summation over polarization of the  $X_Q$  meson, and imposing the condition  $q \cdot \varepsilon = 0$ , at  $Q^2 = 0$  the correlation function takes the form,

$$\begin{aligned} \varepsilon^\nu \Pi_{\mu\nu\rho} &= \lambda_{X_Q}^2 \varepsilon^\tau \frac{1}{m_{X_Q}^2 - (p+q)^2} \frac{1}{m_{X_Q}^2 - p^2} \left\{ 2p_\tau F_C(0) \left[ g_{\mu\rho} - \frac{p_\mu q_\rho - p_\rho q_\mu}{m_{X_Q}^2} \right] \right. \\ &\quad \left. + F_M(0) \left[ q_\mu g_{\rho\tau} - q_\rho g_{\mu\tau} + \frac{1}{m_{X_Q}^2} p_\tau (p_\mu q_\rho - p_\rho q_\mu) \right] - [F_C(0) + F_{\mathcal{D}}(0)] \frac{p_\tau}{m_{X_Q}^2} q_\mu q_\rho \right\} \quad (9) \end{aligned}$$

In order to determine the magnetic moment of  $X_Q$  meson from Eq. (9) we choose the structure  $(p \cdot \varepsilon)(p_\mu q_\nu - p_\nu q_\mu)$ . The reason why we prefer this structure is that it contains more powers of momentum, which exhibits good convergence of the operator product expansion (OPE), and hence leads to more reliable determination of the magnetic moment  $\mu$ . In result, for the coefficient of the structure  $(p \cdot \varepsilon)(p_\mu q_\nu - p_\nu q_\mu)$  of the correlation function from hadronic side we obtain,

$$\Pi = \frac{\lambda_{X_Q}^2}{m_{X_Q}^2} \frac{1}{(m_{X_Q}^2 - p^2)[m_{X_Q}^2 - (p+q)^2]} \mu, \quad (10)$$

where  $\mu$  is the magnetic moment of  $X_Q$  meson in natural units, i.e., in units of  $e/2m_{X_Q}$ .

In constructing the sum rule for the magnetic moment of  $X_Q$  meson, we further need to calculate the correlation function in terms of quark and gluon degrees of freedom.

Using the explicit form of the interpolating current given in Eq. (2) we get the following expression for the correlation function in terms of the relevant propagators as,

$$\begin{aligned} \Pi_{\mu\rho} \varepsilon_\nu^\lambda &= \frac{i}{2} \epsilon^{abc} \epsilon^{ade} \epsilon^{a'b'c'} \epsilon^{a'd'e'} \int d^4x e^{ipx} \left\langle 0 \left| \text{Tr} \gamma_5 \tilde{S}_s^{bb'}(x) \Gamma_5 S_u^{cc'}(x) \text{Tr} \gamma_\mu \tilde{S}_Q^{dd'}(-x) \Gamma_\rho S_d^{ee'}(-x) \right. \right. \\ &\quad \pm \text{Tr} \gamma_\mu \tilde{S}_s^{bb'}(x) \Gamma_5 S_u^{cc'}(x) \text{Tr} \gamma_5 \tilde{S}_Q^{dd'}(-x) \Gamma_\rho S_d^{ee'}(-x) \\ &\quad \pm \text{Tr} \gamma_5 \tilde{S}_s^{bb'}(x) \Gamma_\mu S_u^{cc'}(x) \text{Tr} \gamma_\rho \tilde{S}_Q^{dd'}(-x) \Gamma_5 S_d^{ee'}(-x) \\ &\quad \left. + \text{Tr} \gamma_\rho \tilde{S}_s^{bb'}(x) \Gamma_\mu S_u^{cc'}(x) \text{Tr} \gamma_5 \tilde{S}_Q^{dd'}(-x) \Gamma_5 S_d^{ee'}(-x) \right| 0 \rangle_F, \end{aligned} \quad (11)$$

where  $\tilde{S} = CS^T C$ . It follows from this expression that in order to calculate the correlation function from the QCD side, the expressions of the light and heavy quark propagators are needed in presence of the external field. The light cone expansion of the light quark propagator is calculated in [22] whose expression in  $x$ -space is,

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left( 1 - i \frac{m_q}{4} \not{x} \right) - \frac{\langle \bar{q}q \rangle}{192} m_0^2 x^2 \left( 1 - i \frac{m_q}{6} \not{x} \right)$$

$$\begin{aligned}
& - \frac{ig_s}{16\pi^2 x^2} \int_0^1 du \left[ \not{x} \bar{u} \sigma_{\alpha\beta} G^{\alpha\beta}(ux) + u \sigma_{\alpha\beta} \not{x} G^{\alpha\beta}(ux) \right. \\
& \left. - \frac{1}{2} i m_q x^2 \sigma_{\alpha\beta} G^{\alpha\beta}(ux) \left( \ln \frac{-x^2 \Lambda^2}{4} + 2\gamma_E \right) \right], \tag{12}
\end{aligned}$$

where  $\Lambda$  is the cut-of energy separating the perturbative and nonperturbative domains,  $\gamma_E$  is the Euler constant,  $m_q$  is the light quark mass. The light cone expansion of the heavy quark operator in the  $x$ -space is given as,

$$\begin{aligned}
S_Q(x) = & \frac{m_Q^2}{4\pi^2} \left\{ \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{\not{x}}{-x^2} K_2(m_Q \sqrt{-x^2}) \right\} \\
& - \frac{g_s}{16\pi^2} \int_0^1 du G_{\mu\nu}(ux) \left[ (\sigma^{\mu\nu} \not{x} + \not{x} \sigma^{\mu\nu}) \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma^{\mu\nu} K_0(m_Q \sqrt{-x^2}) \right] + \dots \tag{13}
\end{aligned}$$

where  $K_i(m_Q \sqrt{-x^2})$  are the modified Bessel functions. Substituting Eqs. (12) and (13) in Eq. (11), one can calculate the correlation function from the QCD side. This correlation function receives both perturbative, i.e., when photon interacts perturbatively with quark propagators, and nonperturbative, i.e., photon interacts with light quarks at large distance, contributions.

It should be noted here that, in calculating the perturbative contributions one of the free quark operators, i.e., the first two terms in Eqs. (12) and (13) are replaced by,

$$S^{free} \rightarrow \int d^4 y S^{free}(x-y) \not{A}(y) S^{free}(y),$$

and the remaining three propagators are taken as the free ones. In calculation of the nonperturbative contributions it is necessary to replace one of the light quark propagators by,

$$S_{\alpha\beta}^{ab} \rightarrow -\frac{1}{4} (\bar{q}^a \Gamma_i q^b) ((\Gamma_i)_{\alpha\beta}), \tag{14}$$

where  $\Gamma_i$  are the full set of Dirac matrices, and the remaining quark propagators are taken as in Eqs. (12) and (13). Once Eq. (14) is plugged in Eq. (11), there appear matrix elements such as  $\langle \gamma(q) | \bar{q}(x) \Gamma_i q(0) | 0 \rangle$  and  $\langle \gamma(q) | \bar{q}(x) \Gamma_i G_{\alpha\beta} q(0) | 0 \rangle$ , which are needed in calculating the nonperturbative contributions. In addition to these matrix elements, in principle, non-local operators such as  $\bar{q} G^2 q$  and  $\bar{q} q \bar{q} q$  are expected to appear. But it is known that the contributions of such operators are small, which is justified by the conformal spin expansion [22, 23], and therefore we shall neglect them. The matrix elements  $\langle \gamma(q) | \bar{q}(x) \Gamma_i q(0) | 0 \rangle$  and  $\langle \gamma(q) | \bar{q}(x) \Gamma_i G_{\alpha\beta} q(0) | 0 \rangle$  are expressed in terms of the photon distribution amplitudes whose expressions are given below.

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle = & -i e_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left( \chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\
& - \frac{i}{2(qx)} e_q \langle \bar{q} q \rangle \left[ x_\nu \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left( \varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u)
\end{aligned}$$

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q}q \rangle \left\{ \left[ \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left( g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&\quad - \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left( g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&\quad - \left( \varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left( g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&\quad \left. + \left( \varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left( g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&\quad + \left[ \left( \varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left( g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&\quad - \left( \varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left( g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&\quad - \left( \varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left( g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&\quad \left. + \left( \varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left( g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&\quad + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
&\quad \left. + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \right\},
\end{aligned}$$

where  $\varphi_\gamma(u)$  is the leading twist-2,  $\psi^v(u)$ ,  $\psi^a(u)$ ,  $\mathcal{A}$  and  $\mathcal{V}$  are the twist-3, and  $h_\gamma(u)$ ,  $\mathbb{A}$ ,  $\mathcal{T}_i$  ( $i = 1, 2, 3, 4$ ) are the twist-4 photon distribution amplitudes (DAs), and  $\chi$  is the magnetic susceptibility. The values of the input parameters in photon DAs are calculated in [20]. The measure  $\mathcal{D}\alpha_i$  is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g).$$

In order to obtain the sum rules for the magnetic moment of the  $X_Q$ , and its partner with  $c$ -quark it is necessary to match the hadronic and QCD side representations of the

correlation function. Performing double Borel transformation over the variables  $-p^2$  and  $-(p+q)^2$ , which suppress the higher states and continuum contributions, we finally get the sum rules for the magnetic moment of  $X_Q$  state. Note that Borel transformations are carried out with the help of the formulas,

$$\mathcal{B}\left\{\frac{1}{(p^2 - m_1^2)[(p+q)^2 - m_2^2]}\right\} \rightarrow e^{-m_1^2/M_1^2 - m_2^2/M_2^2}, \quad (15)$$

in the hadronic part, and

$$\mathcal{B}\left\{\frac{1}{[m^2 - \bar{u}p^2 - u(p+q)^2]^\alpha}\right\} \rightarrow (M^2)^{(2-\alpha)}\delta(u - u_0)e^{-m^2/M^2}, \quad (16)$$

where we replace

$$M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}, \text{ and } u_0 = \frac{M_1^2}{M_1^2 + M_2^2}.$$

in the theoretical part.

Since the mass of the final and initial  $X_Q$  mesons are equal to each other, we set  $M_1^2 = M_2^2 = 2M^2$ , which leads to  $u_0 = 1/2$ . Substituting these relations into the sum rules obtained for the magnetic moment, we get

$$\mu = \frac{e^{m_{X_Q}^2/M^2} m_{X_Q}^2}{\lambda_{X_Q}^2} \left\{ \Pi^B \right\}. \quad (17)$$

Explicit expression of  $\Pi^B$  for the  $1^{++}$  state is given in the Appendix.

We can easily see from this expression that, in order to calculate magnetic moment residue  $\lambda_{X_Q}$  is also needed. This residue can be determined by considering the two-point correlation function,

$$\Pi_{\mu\nu} = i \int d^4x e^{ipx} \langle 0 | \text{T} \{ J_\mu^{X_Q}(x) \bar{J}_\nu^{X_Q}(0) \} | 0 \rangle. \quad (18)$$

Saturating this correlation function with  $X_Q$  mesons, the phenomenological part of (18) can be written as,

$$\Pi_{\mu\nu}^{phys} = \frac{\lambda_{X_Q}^2}{m_{X_Q}^2 - p^2} \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots \quad (19)$$

Choosing the coefficient of the structure  $g_{\mu\nu}$  which describes the contribution of the pure  $1^+$  state, and performing Borel transformation over  $-p^2$ , the physical part of the correlation function can be written as

$$\Pi_2^{phys B} = \lambda_{X_Q}^2 e^{-m_{X_Q}^2/M^2}. \quad (20)$$

To be able to construct sum rules for  $\lambda_{X_Q}$ , the correlation function (18) must be calculated from the QCD side. The expansion of the correlation function in terms of the heavy and light quark propagators is given by Eq. (11) in the absence of background field  $F$ .

Substituting Eq. (12) and (13) into Eq. (18), and choosing the coefficient of the structure  $g_{\mu\nu}$ , and performing Fourier and Borel transformations, for the residue of  $X_Q$  we get

$$\lambda_{X_Q}^2 = e^{m_{X_Q}^2/M^2} \Pi_2^{(theor B)}, \quad (21)$$

where subscript 2 stands for the two-point correlation function. The expression for  $\Pi_2^{(theor B)}$  is quite lengthy and not illuminating, and therefore we do not present it here explicitly. It should be noted here that, from the numerical analysis of the sum rules for the masses of the  $1^{++}$  and  $1^{+-}$   $X_c$  states, we get

$$m_{X_c} \simeq (2.55 \pm 0.15) GeV.$$

### 3 Numerical analysis

The key ingredient of the light cone QCD sum rules for the magnetic moment is the photon distribution amplitudes (DAs). The values of DAs are determined in [20], which we will use in our numerical calculations.

Sum rules for the magnetic moment contain, in addition to DAs, also many input parameters, such as the masses of the strange, charm and bottom quarks, values of the quark condensates. In our numerical analysis we use,  $\overline{m}_b(m_b) = (4.18 \pm 0.03) GeV$ ,  $\overline{m}_c(m_c) = (1.275 \pm 0.025) GeV$  (in  $\overline{MS}$  scheme),  $m_s(2 GeV) = (95 \pm 5) MeV$ ,  $\langle \overline{q}q \rangle|_{1 GeV} = (-0.24 \pm 0.01)^3 GeV^3$  [24, 25], and we use  $\chi(1 GeV) = -2.85 GeV^{-2}$  for the value of the magnetic susceptibility which is obtained in [26].

The sum rules for the magnetic moment of  $X_Q$  contain also two auxiliary parameters, i.e., Borel mass parameter  $M^2$  and the continuum threshold  $s_0$ . So in determination of the magnetic moment of  $X_Q$  meson we should find such regions of these auxiliary parameters so that the dependence of magnetic moment on them would be minimal.

The working region of the Borel parameter is determined from the convergence of the operator product expansion series and suppression of the contributions of the higher states and the continuum. The upper bound of the Borel parameter is fixed by requiring that the contribution of higher states and the continuum constitutes less than 40% of the contribution coming from the perturbative part. The lower bound is fixed from the condition that higher twist contributions are less than the leading twist contributions. Having these restrictions imposed we obtain  $5 GeV^2 \leq M^2 \leq 8 GeV^2$  for the  $X_b(5568)$ ; and  $2 GeV^2 \leq M^2 \leq 4 GeV^2$  for the  $X_c$  mesons, respectively, for the working regions of Borel mass-square parameter.

The working region of the continuum threshold  $s_0$  is determined in the following way. The difference  $\sqrt{s_0} - m_{ground}$ , where  $m_{ground}$  is the ground state mass, is the energy necessary to excite this particle to its first excited state. The analysis in various sum rules predicts that this difference varies in the domain from  $0.3 GeV$  to  $0.8 GeV$ . In further analysis we shall use  $\sqrt{s_0} - m_{ground} = 0.5 GeV$ .

As an example in Figs. 1 and 2 we present the dependence of the magnetic moment  $\mu$  of the  $1^{++}$  and  $1^{+-}$   $X_b(5568)$  states on  $M^2$ , at several fixed values of the continuum threshold  $s_0$ . We observe from the figures that the magnetic moment exhibits indeed good stability in the working region of  $M^2$ .



The dependence of the magnetic moment  $\mu$  on  $s_0$ , at various fixed values of  $M^2$  is also studied, and it is observed that the magnetic moment  $\mu$  is almost stable under variation of  $s_0$  in its working region. Our final results for the magnetic moments of these states are,

$$\mu = \begin{cases} (0.22 \pm 0.07)\mu_N, & \text{for } X_b \ 1^{++} \\ (0.24 \pm 0.08)\mu_N, & \text{for } X_b \ 1^{+-} \\ (0.075 \pm 0.015)\mu_N, & \text{for } X_c \ 1^{++} \\ (0.10 \pm 0.02)\mu_N, & \text{for } X_c \ 1^{+-} \end{cases}$$

where the error in the result can be attributed to the variations in  $M^2$  and  $s_0$ , as well as to the uncertainties in the values of input parameters. It follows from these results that the magnetic moment of  $X_b(5568)$  meson is large enough to be measured in future experiments.

In summary, in the present work we calculate the magnetic moment of exotic  $X_b(5568)$  state and its partner with  $c$ -quark state with the quantum numbers  $J^{PC} = 1^{+\pm}$  in framework of the QCD sum rules method. Measurement of the magnetic of  $X_b(5568)$  meson in future experiments can be very useful in determining the quantum numbers, as well as understanding the internal structure of these exotic states.

## Appendix: Expression of the invariant function $\Pi^B$ for the $1^{++} X_Q$ state

Coefficient of  $\frac{1}{2}(p_\mu q_\nu - p_\nu q_\mu)\varepsilon \cdot p$  structure.

$$\begin{aligned}
\Pi^B = & \frac{e^{-m_Q^2/M^2}}{331776\pi^2 M^4} f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_Q \left[ 47 \langle \bar{d}d \rangle (e_s + e_u) - 2e_u \langle \bar{s}s \rangle + 2e_s \langle \bar{u}u \rangle \right] \psi^a (1 - u_0) \\
& - \frac{e^{-m_Q^2/M^2}}{165888\pi^2 m_Q M^2} f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 \left[ 47 \langle \bar{d}d \rangle (e_s + e_u) - 2e_u \langle \bar{s}s \rangle + 2e_s \langle \bar{u}u \rangle \right] \psi^a (1 - u_0) \\
& + \frac{1}{6144\pi^4} m_0^2 m_Q M^2 \left[ 2 \langle \bar{d}d \rangle e_b + (e_b - 2e_d) (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \right] (\mathcal{I}_1 - 2m_Q^2 \mathcal{I}_2 + m_Q^4 \mathcal{I}_3) \\
& + \frac{1}{36864\pi^4} f_{3\gamma} \langle g_s^2 G^2 \rangle m_Q^2 M^2 (\mathcal{I}_2 - m_Q^2 \mathcal{I}_3) \left[ 2e_d i_2(\mathcal{V}, 1) + (e_s + e_u) \tilde{i}_3(\mathcal{V}, 1) \right] \\
& - \frac{1}{1152\pi^4} \langle \bar{d}d \rangle \langle g_s^2 G^2 \rangle m_Q M^2 (3\mathcal{I}_2 - 4m_Q^2 \mathcal{I}_3) \left[ e_s + e_u - 2e_d \tilde{j}(h_\gamma) \right] \\
& + \frac{1}{18432\pi^4} m_Q M^2 \left\{ 6m_0^2 \left[ (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) (\mathcal{I}_1 - 4m_Q^2 \mathcal{I}_2 + 3m_Q^4 \mathcal{I}_3) \right. \right. \\
& - \langle \bar{d}d \rangle (e_s + e_u) (24\mathcal{I}_1 - 95m_Q^2 \mathcal{I}_2 + 71m_Q^4 \mathcal{I}_3) \left. \left. \right] - 2 \langle \bar{d}d \rangle e_d \langle g_s^2 G^2 \rangle (3\mathcal{I}_2 - 4m_Q^2 \mathcal{I}_3) i_1(\mathcal{S}, 1) \right. \\
& - \langle g_s^2 G^2 \rangle (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) (3\mathcal{I}_2 - 4m_Q^2 \mathcal{I}_3) \tilde{i}_2(\tilde{\mathcal{S}}, 1) \\
& + 8f_{3\gamma} m_Q (\mathcal{I}_2 - m_Q^2 \mathcal{I}_3) (e_s + e_u) (\langle g_s^2 G^2 \rangle + 96 \langle \bar{d}d \rangle m_Q \pi^2) \psi^a (1 - u_0) - e_d \langle g_s^2 G^2 \rangle \psi^a (u_0) \left. \right\} \\
& - \frac{1}{3072\pi^6} m_Q M^4 \left\{ 12 \langle \bar{d}d \rangle e_d m_Q^2 \pi^2 (\mathcal{I}_2 - 2m_Q^2 \mathcal{I}_3 + m_Q^4 \mathcal{I}_4) i_1(\mathcal{S}, 1) \right. \\
& - 2\pi^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) (\mathcal{I}_1 - 6m_Q^2 \mathcal{I}_2 + 9m_Q^4 \mathcal{I}_3 - 4m_Q^6 \mathcal{I}_4) \tilde{i}_2(\tilde{\mathcal{S}}, 1) \\
& - m_Q (\mathcal{I}_2 - 2m_Q^2 \mathcal{I}_3 + m_Q^4 \mathcal{I}_4) \left[ e_d \langle g_s^2 G^2 \rangle - (e_s + e_u) (\langle g_s^2 G^2 \rangle + 96 \langle \bar{d}d \rangle m_Q \pi^2) \right. \\
& \left. \left. + 192 \langle \bar{d}d \rangle e_d m_Q \pi^2 \tilde{j}(h_\gamma) \right] \right\} \\
& - \frac{1}{3072\pi^4} f_{3\gamma} m_Q^2 M^6 \left\{ 6e_d m_Q^2 (\mathcal{I}_3 - 2m_Q^2 \mathcal{I}_4 + m_Q^4 \mathcal{I}_5) i_2(\mathcal{V}, 1) \right. \\
& + (e_s + e_u) (\mathcal{I}_2 - 3m_Q^4 \mathcal{I}_4 + 2m_Q^6 \mathcal{I}_5) \tilde{i}_3(\mathcal{V}, 1) \\
& + 48m_Q^2 (\mathcal{I}_3 - 2m_Q^2 \mathcal{I}_4 + m_Q^4 \mathcal{I}_5) \left[ (e_s + e_u) \psi^a (1 - u_0) - e_d \psi^a (u_0) \right] \left. \right\} \\
& - \frac{1}{512\pi^6} m_Q^2 M^8 \left[ 4(e_d - e_s - e_u) m_Q^2 (\mathcal{I}_3 - 3m_Q^2 \mathcal{I}_4 + 3m_Q^4 \mathcal{I}_5 - m_Q^6 \mathcal{I}_6) \right. \\
& + e_b (\mathcal{I}_2 - 4m_Q^2 \mathcal{I}_3 + 6m_Q^4 \mathcal{I}_4 - 4m_Q^6 \mathcal{I}_5 + m_Q^8 \mathcal{I}_6) \left. \right] \\
& - \frac{e^{-m_Q^2/M^2}}{221184m_Q \pi^4} \left\{ \langle g_s^2 G^2 \rangle m_0^2 \left[ 47 \langle \bar{d}d \rangle (e_s + e_u) - 2e_u \langle \bar{s}s \rangle + 2e_s \langle \bar{u}u \rangle \right] (1 - 3m_Q^2 e^{m_Q^2/M^2} \mathcal{I}_2) \right. \\
& + 16f_{3\gamma} \pi^2 \left[ 6m_0^2 m_Q^2 e^{m_Q^2/M^2} (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) (\mathcal{I}_1 - 2m_Q^2 \mathcal{I}_2) \right. \\
& \left. \left. - 3 \langle \bar{d}d \rangle (e_s + e_u) m_0^2 m_Q^2 e^{m_Q^2/M^2} (48\mathcal{I}_1 - 95m_Q^2 \mathcal{I}_2) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + 8\langle \bar{d}d \rangle (e_s + e_u) \langle g_s^2 G^2 \rangle (1 - 3m_Q^2 e^{m_Q^2/M^2} \mathcal{I}_2) \Big] \psi^a(1 - u_0) \Big\} \\
& + \frac{1}{2304\pi^2} e_d f_{3\gamma} m_0^2 m_Q (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) (\mathcal{I}_1 - m_Q^2 \mathcal{I}_2) \psi^a(u_0).
\end{aligned}$$

The functions  $i_n$  ( $n = 1, 2$ ),  $\tilde{i}_n$  ( $n = 1, 2$ ), and  $\tilde{j}(f(u))$  are defined as:

$$\begin{aligned}
i_1(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta(k - u_0), \\
\tilde{i}_1(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta(\tilde{k} - u_0), \\
i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(k - u_0), \\
\tilde{i}_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(\tilde{k} - u_0), \\
\tilde{j}(f(u)) &= \int_{u_0}^1 du (u - u_0) f(u), \\
\mathcal{I}_n &= \int_{m_Q^2}^{s_0} ds \frac{e^{-s/M^2}}{s^n},
\end{aligned}$$

(22)

where

$$k = \alpha_q + \alpha_g(1 - v), \quad \tilde{k} = \alpha_{\bar{q}} + \alpha_g v.$$

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## Figure captions

**Fig. (1)** The dependence of the magnetic moment of  $X_b(5568) 1^{++}$  state on the Borel parameter  $M^2$ , at several fixed values of the continuum threshold  $s_0$ .

**Fig. (2)** The same as Fig. (1), but for the  $X_b(5568) 1^{+-}$  state.

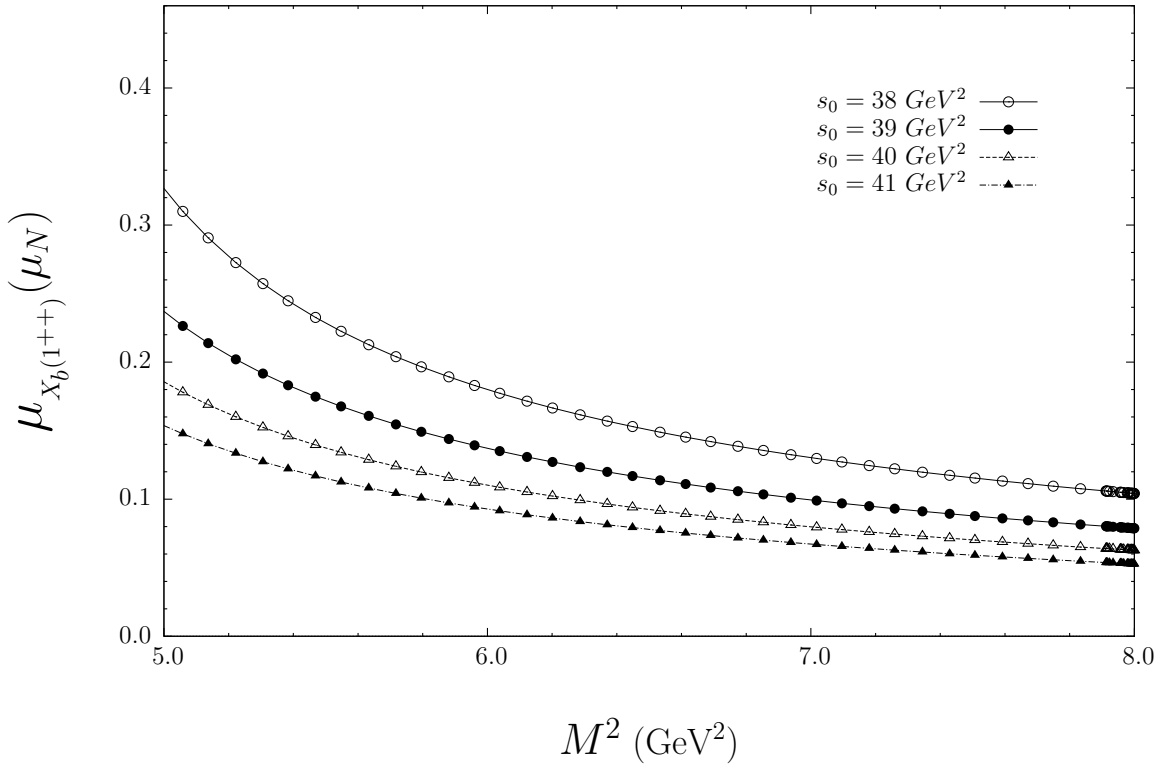


Figure 1:

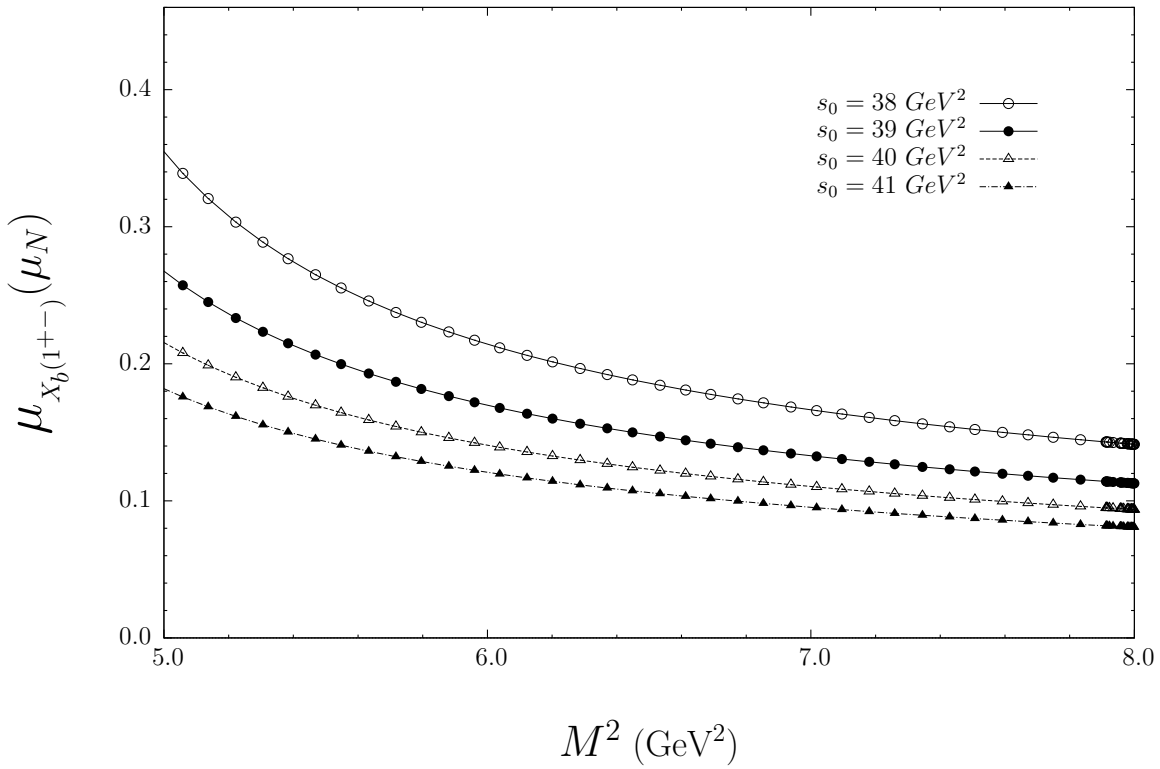


Figure 2: