

# Strong coupling constant of negative parity nucleon with $\pi$ meson in light cone QCD sum rules

T. M. Aliev<sup>1 \*†</sup>, T. Barakat<sup>2 ‡</sup>, M. Savcı<sup>1 §</sup>

<sup>1</sup>Physics Department, Middle East Technical University, 06800 Ankara, Turkey

<sup>2</sup>Physics and Astronomy Department, King Saud University, Saudi Arabia

## Abstract

We estimate strong coupling constant between the negative parity nucleons with  $\pi$  meson within the light cone QCD sum rules. A method for eliminating the unwanted contributions coming from the nucleon–nucleon and nucleon–negative parity nucleon transition is presented. It is observed that the value strong coupling constant of the negative parity nucleon  $N^*N^*\pi$  transition is considerably different from the one predicted by the 3–point QCD sum rules, but is quite close to the coupling constant of the positive parity  $NN\pi$  transition.

PACS numbers: 14.20.Dh, 14.40.Be, 13.75.Gx, 11.55.Hx

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\*e-mail: taliev@metu.edu.tr

†permanent address:Institute of Physics,Baku,Azerbaijan

‡e-mail: tbarakat@KSU.EDU.SA

§e-mail: savci@metu.edu.tr

# 1 Introduction

The strong coupling constants of hadrons with mesons are the key quantity for understanding the dynamics of the existing hadron–hadron, hadron–meson, and photoproduction experiments. Among many couplings, only nucleon–pion coupling constant has been measured accurately from experiments. With increasing experimental information there appears the necessity for an accurate determination of the strong coupling constants of hadrons with pseudoscalar mesons. These coupling constants have so far been estimated within various approaches (relevant references can be found in [1]). The strong coupling constants of the octet baryons with pseudoscalar mesons are calculated in framework of the light cone QCD sum rules [1].

In the present note we calculate the  $N^*N^*\pi$  coupling constant in light cone QCD sum rules. Compared to the all other sum rules approaches that take only one positive parity baryon into consideration, the main novelty of the present calculation is that the contributions to the sum rules coming from the two positive parity  $N(938)$  and  $N(1440)$  states are taken into account. This fact makes the analysis of the sum rules more complicated in determination of the  $N^*N^*\pi$  coupling constant. In the present work a new method is explored for eliminating the unwanted contributions coming from the  $N(938) \rightarrow N(938)$ ,  $N(1440) \rightarrow N(1440)$ ,  $N^* \rightarrow N(938)$ ,  $N^* \rightarrow N(1440)$  transitions. In the following discussion we shall customarily denote  $N(1440)$  as  $N'$ . It should be noted here that the  $N^*N^*\pi$  coupling constant is determined in [2] in framework of the 3–point QCD sum rules.

The paper is organized as follows. In section 2 we derive the sum rules for the  $N^*N^*\pi$  coupling constant. In section 3 we present the numerical analysis of the sum rules for this parameter, and compare our result with the prediction of 3–point QCD sum rules.

## 2 Sum rules for the $N^*N^*\pi$ coupling

In this section sum rules for the  $N^*N^*\pi$  coupling constant within the light cone QCD sum rules method is derived. In determining this coupling constant we consider the following correlation function,

$$\Pi(p, q) = i \int d^4x e^{ipx} \left\langle \pi(q) \left| J_N(x) J_N^\dagger(0) \right| 0 \right\rangle, \quad (1)$$

where

$$J_N = \varepsilon^{abc} \left\{ (u^{a\dagger} C d^b) \gamma_5 u^c + \beta (u^{a\dagger} C \gamma_5 d^b) u^c \right\}, \quad (2)$$

is the interpolating current of the nucleon,  $a, b, c$  are the color indices,  $C$  is the charge conjugation operator, and  $\beta$  is an arbitrary parameter. The sum rules for the  $N^*N^*\pi$  coupling can be obtained by following the QCD sum rules procedure. On the one hand, the correlation function is calculated in terms of hadrons. On the other hand, it can be calculated in the deep Euclidian domain  $p^2 \ll 0$ ,  $(p+q)^2 \ll 0$  by using the operator product expansion (OPE) over twist. By matching then these two representations, the sum rules for the  $N^*N^*\pi$  coupling is obtained.

The hadronic part of the correlation function is obtained by inserting a complete set of baryons, and isolating the ground state contributions of the baryons as follows,

$$\Pi = \sum_{i,j} \frac{\langle 0 | J_N | N_i(p, s) \rangle \langle \pi(q) N_i | N_j(p+q) \rangle \langle N_j | \bar{J}_N | 0 \rangle}{(p_i^2 - m_i^2)[(p+q)^2 - m_j^2]}, \quad (3)$$

where  $i$  and  $j$  run over  $N$ ,  $N(1440)$  and  $N^*(1535)$ , and dots denote higher state contributions. The matrix elements in Eq. (3) are defined as,

$$\begin{aligned} \langle 0 | J_N | N(N') \rangle &= \lambda_{N(N')} u_{N(N')}(p), \\ \langle \pi(q) N_i | N_j(p+q) \rangle &= g_{N_i N_j \pi} \bar{u}_{N_i} \Gamma_j u_{N_j}, \\ \langle 0 | J_N | N^* \rangle &= \lambda_{N^*} \gamma_5 u_{N^*}(p), \end{aligned} \quad (4)$$

where

$$\Gamma_j = \begin{cases} i\gamma_5, & \text{for } N \rightarrow N, N' \rightarrow N', N' \rightarrow N, N^* \rightarrow N^* \text{ transitions,} \\ I, & \text{for } N^* \rightarrow N, N^* \rightarrow N' \text{ transitions.} \end{cases}$$

Using Eqs. (3) and (4), for the hadronic part of the correlation function we get,

$$\begin{aligned} \Pi &= A(\not{p} + m_N) i\gamma_5 (\not{p} + \not{q} + m_N) + B(\not{p} + m_{N'}) i\gamma_5 (\not{p} + \not{q} + m_{N'}) \\ &+ C(\not{p} + m_{N'}) i\gamma_5 (\not{p} + \not{q} + m_N) + D(\not{p} + m_N) i\gamma_5 (\not{p} + \not{q} + m_{N'}) \\ &+ E\gamma_5 (\not{p} + m_{N^*}) i\gamma_5 (\not{p} + \not{q} + m_{N^*}) (-\gamma_5) + F i\gamma_5 (\not{p} + m_{N^*}) (\not{p} + \not{q} + m_N) \\ &+ K i\gamma_5 (\not{p} + m_{N^*}) (\not{p} + \not{q} + m_{N'}) + L(\not{p} + m_N) (\not{p} + \not{q} + m_{N^*}) (-i\gamma_5) \\ &+ N(\not{p} + m_{N'}) (\not{p} + \not{q} + m_{N^*}) (-i\gamma_5), \end{aligned} \quad (5)$$

where

$$\begin{aligned} A &= \frac{|\lambda_N|^2 g_{NN\pi}}{(p^2 - m_N^2)[(p+q)^2 - m_N^2]}, \\ B &= \frac{|\lambda_{N'}|^2 g_{N'N'\pi}}{(p^2 - m_{N'}^2)[(p+q)^2 - m_{N'}^2]}, \\ C &= \frac{\lambda_N^* \lambda_{N'} g_{NN'\pi}}{(p^2 - m_{N'}^2)[(p+q)^2 - m_N^2]}, \\ D &= \frac{\lambda_{N'}^* \lambda_N g_{N'N\pi}}{(p^2 - m_N^2)[(p+q)^2 - m_{N'}^2]}, \\ E &= \frac{|\lambda_{N^*}|^2 g_{N^*N^*\pi}}{(p^2 - m_{N^*}^2)[(p+q)^2 - m_{N^*}^2]}, \\ F &= \frac{\lambda_{N^*}^* \lambda_N g_{N^*N\pi}}{(p^2 - m_{N^*}^2)[(p+q)^2 - m_N^2]}, \\ K &= \frac{\lambda_{N^*}^* \lambda_{N'} g_{N^*N'\pi}}{(p^2 - m_{N^*}^2)[(p+q)^2 - m_{N'}^2]}, \\ L &= \frac{\lambda_N^* \lambda_{N^*} g_{NN^*\pi}}{(p^2 - m_N^2)[(p+q)^2 - m_{N^*}^2]}, \\ N &= \frac{\lambda_{N'}^* \lambda_{N^*} g_{N'N^*\pi}}{(p^2 - m_{N'}^2)[(p+q)^2 - m_{N^*}^2]}. \end{aligned} \quad (6)$$

The correlation function can be calculated from the QCD side by using Wick's theorem. In calculation of the correlation function from theoretical side the expression of the light quark operators in the presence of an external field are needed, and it is calculated in [3]. It should be noted here that the quark propagator gets contributions from three-particle  $\bar{q}Gq$ , four-particle  $\bar{q}G^2q$ ,  $\bar{q}q\bar{q}q$  nonlocal operators. In further calculations we take into account only three-particle  $\bar{q}Gq$  operator and neglect contributions coming from four-particle operators. It is demonstrated in [3] that neglecting these contributions can be legitimated on the basis of an expansion over conformal spin. Under this approximation the light quark propagator in the background field is given by,

$$S_q(x) = \frac{i\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{ig_s}{16\pi^2x^2} \int_0^1 du \left[ \not{x}\bar{u}\sigma_{\alpha\beta}G^{\alpha\beta}(ux) + u\sigma_{\alpha\beta}G^{\alpha\beta}(ux) - \frac{1}{2}im_qx^2\sigma_{\alpha\beta}G^{\alpha\beta}(ux) \left( \ln \frac{-x^2\Lambda^2}{4} + 2\gamma_E \right) \right], \quad (7)$$

where  $\Lambda$  is the parameter separating the perturbative and nonperturbative domains, whose value is estimated to be  $\Lambda = (0.5 \div 1.0) GeV$  in [4].

Using the expression of the light quark propagator, the correlation function can be calculated from the QCD side straightforwardly in deep Euclidian region  $p^2 \rightarrow -\infty$ ,  $(p+q)^2 \rightarrow -\infty$  by using the operator product expansion over twist. In this calculation the matrix elements of the nonlocal operators  $\bar{q}(x)\Gamma q(0)$  and  $\bar{q}(x)\Gamma G_{\mu\nu}(ux)q(0)$  between the vacuum and pion states appear, where  $\Gamma$  corresponds to the matrices from full set of Dirac matrices. The matrix elements up to twist-4 are parametrized in terms of the pion distribution amplitudes in the following way [5-9]:

$$\begin{aligned} \langle \pi(p) | \bar{q}_1(x)\gamma_\mu\gamma_5q_1(0) | 0 \rangle &= -if_\pi q_\mu \int_0^1 du e^{i\bar{u}qx} \left( \varphi_\pi(u) + \frac{1}{16}m_\pi^2x^2\mathbb{A}(u) \right) \\ &\quad - \frac{i}{2}f_\pi m_\pi^2 \frac{x_\mu}{qx} \int_0^1 du e^{i\bar{u}qx} \mathbb{B}(u), \\ \langle \pi(p) | \bar{q}_1(x)i\gamma_5q_2(0) | 0 \rangle &= \mu_\pi \int_0^1 du e^{i\bar{u}qx} \phi_P(u), \\ \langle \pi(p) | \bar{q}_1(x)\sigma_{\alpha\beta}\gamma_5q_2(0) | 0 \rangle &= \frac{i}{6}\mu_\pi (1 - \tilde{\mu}_\pi^2) (q_\alpha x_\beta - q_\beta x_\alpha) \int_0^1 du e^{i\bar{u}qx} \phi_\sigma(u), \\ \langle \pi(p) | \bar{q}_1(x)\sigma_{\mu\nu}\gamma_5g_sG_{\alpha\beta}(vx)q_2(0) | 0 \rangle &= i\mu_\pi \left[ q_\alpha q_\mu \left( g_{\nu\beta} - \frac{1}{qx}(q_\nu x_\beta + q_\beta x_\nu) \right) \right. \\ &\quad - q_\alpha q_\nu \left( g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu) \right) \\ &\quad - q_\beta q_\mu \left( g_{\nu\alpha} - \frac{1}{qx}(q_\nu x_\alpha + q_\alpha x_\nu) \right) \\ &\quad \left. + q_\beta q_\nu \left( g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu) \right) \right] \\ &\quad \times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx} \mathcal{T}(\alpha_i), \end{aligned}$$

$$\begin{aligned}
\langle \pi(p) | \bar{q}_1(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) q_2(0) | 0 \rangle &= q_\mu (q_\alpha x_\beta - q_\beta x_\alpha) \frac{1}{qx} f_\pi m_\pi^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}_\parallel(\alpha_i) \\
&+ \left[ q_\beta \left( g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) \right. \\
&- \left. q_\alpha \left( g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) \right] f_\pi m_\pi^2 \\
&\times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}_\perp(\alpha_i), \\
\langle \pi(p) | \bar{q}_1(x) \gamma_\mu i g_s G_{\alpha\beta}(vx) q_2(0) | 0 \rangle &= q_\mu (q_\alpha x_\beta - q_\beta x_\alpha) \frac{1}{qx} f_\pi m_\pi^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}_\parallel(\alpha_i) \\
&+ \left[ q_\beta \left( g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) \right. \\
&- \left. q_\alpha \left( g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) \right] f_\pi m_\pi^2 \\
&\times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}_\perp(\alpha_i), \tag{8}
\end{aligned}$$

where

$$\mu_\pi = f_\pi \frac{m_\pi^2}{m_u + m_d}, \quad \tilde{\mu}_\pi = \frac{m_u + m_d}{m_\pi},$$

and  $\mathcal{D}\alpha = d\alpha_{\bar{q}} d\alpha_q d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g)$ . In these expressions  $\varphi_\pi(u)$  is the leading twist-two,  $\phi_\pi(u)$ ,  $\phi_\sigma(u)$ ,  $\mathcal{T}(\alpha_i)$  are the twist-three, and remaining ones are twist-four DAs, respectively, whose explicit expressions are given in the following section.

It follows from Eq. (5) that we have four independent Lorentz structures  $\not{p}\not{q}\gamma_5$ ,  $\not{q}\gamma_5$ ,  $\not{p}\gamma_5$  and  $\gamma_5$  for the problem under consideration. In principle, the relevant sum rules can be obtained by performing double Borel transformation over the variables  $-p^2$  and  $-(p+q)^2$  on the theoretical and hadronic parts, and matching the coefficients of the corresponding Lorentz structures. At this point we face the following problem. Among the nine coefficients given in Eq. (7) only coefficient  $E$  describes the  $N^*N^*\pi$  coupling constant. However, as has already been noted, we have only four independent Lorentz structures, and we need additional five more equations to determine the  $N^*N^*\pi$  coupling constant uniquely. Four of these additional five equations can be obtained by taking derivatives of the four equations with respect to the inverse Borel mass square. The fifth equation is obtained by taking second derivative of the coefficient of the structure  $\not{p}\not{q}\gamma_5$  with respect to again the inverse Borel mass square. From the numerical solution of these nine equations we get,

$$\begin{aligned}
g_{N^*N^*\pi} |\lambda_{N^*}|^2 e^{-m_{N^*}^2/M^2} &= 0.097\Pi_1^B + 0.029\Pi_2^B - 0.178\Pi_3^B - 0.143\Pi_4^B - 0.116\Pi_1^{B'} \\
&- 0.051\Pi_2^{B'} + 0.062\Pi_3^{B'} - 0.013\Pi_4^{B'} + 0.024\Pi_1^{B''} + \dots \tag{9}
\end{aligned}$$

where  $\Pi_1^B$ ,  $\Pi_2^B$ ,  $\Pi_3^B$ , and  $\Pi_4^B$  are the coefficients of the structures  $\not{p}\not{q}\gamma_5$ ,  $\not{q}\gamma_5$ ,  $\not{p}\gamma_5$  and  $\gamma_5$  after Borel transformations with respect to the variables  $-(p+q)^2$  and  $-p^2$  are performed, respectively;  $\Pi_i^{B'}$  stands for the first derivative of  $\Pi_i^B$  with respect to  $1/M_1^2$ , i.e.,  $d\Pi_i^B/d(1/M_1^2)$ , and  $\Pi_1^{B''}$  is the second derivative of  $\Pi_1^B$  with respect to  $1/M_1^2$ . Here we set  $m_\pi^2 = 0$ , and

dots correspond to contributions of continuum and higher states. These contributions can be calculated by using the hadron–quark duality, i.e., above some threshold in the  $(s_1, s_2)$  plane the hadronic spectral density is equal to the quark spectral density. Note that after taking derivatives of the invariant functions we set  $M_1^2 = M_2^2 = 2M^2$ . The expressions of the functions  $\Pi_1^B$ ,  $\Pi_2^B$ ,  $\Pi_3^B$  and  $\Pi_4^B$  are quite lengthy and for this reason we do not present them in the present work. Once Fourier and Borel transformations are carried out, continuum subtraction can be performed by using the following formula,

$$M^{2n} \rightarrow \frac{1}{\Gamma(n)} \int_0^{s_0} ds e^{-s/M^2} (s - n_{N^*}^2)^{n-1},$$

which leads to

$$M^2 \rightarrow M^2 \left(1 - e^{-s_0/M^2}\right).$$

For the higher twist terms that are proportional to the negative power of  $M^2$ , the subtraction procedure is not performed, since their contributions are negligibly small (for more details, see [5]).

Our final remark in this section is about the residue of the negative parity baryons, which is determined from the two–point correlation function,

$$\Pi(p^2) = i \int d^4x e^{ipx} \left\langle 0 \left| \text{T} \left( \bar{\eta}_N(x) \eta_N(0) \right) \right| 0 \right\rangle, \quad (10)$$

where  $\eta$  is given in Eq. (2). Saturating this correlation function with positive and negative parity baryons we get,

$$\Pi(p^2) = \frac{|\lambda_N|^2 (\not{p} + m_N)}{(p^2 - m_N^2)} + \frac{|\lambda_{N'}|^2 (\not{p} + m_{N'})}{(p^2 - m_{N'}^2)} + \frac{|\lambda_{N^*}|^2 (\not{p} + m_{N^*})}{(p^2 - m_{N^*}^2)}. \quad (11)$$

When we calculate this correlation function from theoretical side we get,

$$\Pi(p^2) = \not{p} \Pi_1(p^2) I \Pi_2(p^2), \quad (12)$$

where  $\Pi_i$  are the corresponding invariant functions. Performing Borel transformation over  $p^2$ , and equating the coefficients of the structures we get,

$$\begin{aligned} |\lambda_N|^2 e^{-m_N^2/M^2} + |\lambda_{N'}|^2 e^{-m_{N'}^2/M^2} + |\lambda_{N^*}|^2 e^{-m_{N^*}^2/M^2} &= \tilde{\Pi}_1^B, \\ m_N |\lambda_N|^2 e^{-m_N^2/M^2} + m_{N'} |\lambda_{N'}|^2 e^{-m_{N'}^2/M^2} - m_{N^*} |\lambda_{N^*}|^2 e^{-m_{N^*}^2/M^2} &= \tilde{\Pi}_2^B. \end{aligned} \quad (13)$$

Expressions of the invariant functions  $\tilde{\Pi}_1^B$  and  $\tilde{\Pi}_2^B$  are given in [10].

As can easily be seen from these equations there are six unknowns, namely,  $m_N$ ,  $\lambda_N$ ,  $m_{N'}$ ,  $\lambda_{N'}$ ,  $m_{N^*}$  and  $\lambda_{N^*}$ , and therefore we need six equations to be able to solve for these unknown parameters. Two of these equations are given in Eq. (13), and the remaining four equations can be obtained from Eq. (13) by taking first and second derivatives with respect to  $(-1/M^2)$ . Solving then these six equations we can determine  $\lambda_{N^*}$ . Our numerical analysis shows that  $|\lambda_{N^*}|^2$  is positive in the regions  $-1.0 \leq \cos \theta \leq -0.8$  (where  $\beta = \tan \theta$ ), and  $0.8 \leq \cos \theta \leq 1.0$ , and it is unphysical for all other values of  $\cos \theta$ . therefore in further numerical analysis we will use these domains in determination of the  $N^* N^* \pi$  coupling constant.

### 3 Numerical analysis

In this section numerical analysis for the sum rules of the strong coupling constant  $g_{N^*N^*\pi}$  obtained in the previous section is performed. In this analysis the values of the input parameters, as well as the expressions of the pion distribution amplitudes (DAs) are needed, which are the main ingredients of the light cone QCD sum rules. The expressions of the pion DAs are given as, [5–9]

$$\begin{aligned}
\varphi_{\mathcal{P}}(u) &= 6u\bar{u} \left[ 1 + a_1^{\mathcal{P}} C_1(2u-1) + a_2^{\mathcal{P}} C_2^{3/2}(2u-1) \right], \\
\mathcal{T}(\alpha_i) &= 360\eta_3\alpha_{\bar{q}}\alpha_q\alpha_g^2 \left[ 1 + w_3\frac{1}{2}(7\alpha_g-3) \right], \\
\phi_{\mathcal{P}}(u) &= 1 + \left[ 30\eta_3 - \frac{5}{2}\frac{1}{\mu_{\mathcal{P}}^2} \right] C_2^{1/2}(2u-1), \\
&\quad + \left( -3\eta_3w_3 - \frac{27}{20}\frac{1}{\mu_{\mathcal{P}}^2} - \frac{81}{10}\frac{1}{\mu_{\mathcal{P}}^2}a_2^{\mathcal{P}} \right) C_4^{1/2}(2u-1), \\
\phi_{\sigma}(u) &= 6u\bar{u} \left[ 1 + \left( 5\eta_3 - \frac{1}{2}\eta_3w_3 - \frac{7}{20}\mu_{\mathcal{P}}^2 - \frac{3}{5}\mu_{\mathcal{P}}^2a_2^{\mathcal{P}} \right) C_2^{3/2}(2u-1) \right], \\
\mathcal{V}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g(v_{00} + v_{10}(3\alpha_g-1)), \\
\mathcal{A}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g(0 + a_{10}(\alpha_q - \alpha_{\bar{q}})), \\
\mathcal{V}_{\perp}(\alpha_i) &= -30\alpha_g^2 \left[ h_{00}(1-\alpha_g) + h_{01}(\alpha_g(1-\alpha_g) - 6\alpha_q\alpha_{\bar{q}}) + h_{10}(\alpha_g(1-\alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2)) \right], \\
\mathcal{A}_{\perp}(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) \left[ h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g-3) \right], \\
B(u) &= g_{\mathcal{P}}(u) - \varphi_{\mathcal{P}}(u), \\
g_{\mathcal{P}}(u) &= g_0C_0^{1/2}(2u-1) + g_2C_2^{1/2}(2u-1) + g_4C_4^{1/2}(2u-1), \\
\mathbb{A}(u) &= 6u\bar{u} \left[ \frac{16}{15} + \frac{24}{35}a_2^{\mathcal{P}} + 20\eta_3 + \frac{20}{9}\eta_4 + \left( -\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3w_3 - \frac{10}{27}\eta_4 \right) C_2^{3/2}(2u-1) \right. \\
&\quad + \left( -\frac{11}{210}a_2^{\mathcal{P}} - \frac{4}{135}\eta_3w_3 \right) C_4^{3/2}(2u-1) \left. \right], \\
&\quad + \left( -\frac{18}{5}a_2^{\mathcal{P}} + 21\eta_4w_4 \right) [2u^3(10-15u+6u^2)\ln u \\
&\quad + 2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln \bar{u} + u\bar{u}(2+13u\bar{u})] , \tag{14}
\end{aligned}$$

where  $C_n^k(x)$  are the Gegenbauer polynomials, and

$$\begin{aligned}
h_{00} &= v_{00} = -\frac{1}{3}\eta_4, \\
a_{10} &= \frac{21}{8}\eta_4w_4 - \frac{9}{20}a_2^{\mathcal{P}}, \\
v_{10} &= \frac{21}{8}\eta_4w_4, \\
h_{01} &= \frac{7}{4}\eta_4w_4 - \frac{3}{20}a_2^{\mathcal{P}},
\end{aligned}$$

$$\begin{aligned}
h_{10} &= \frac{7}{4}\eta_4 w_4 + \frac{3}{20}a_2^{\mathcal{P}}, \\
g_0 &= 1, \\
g_2 &= 1 + \frac{18}{7}a_2^{\mathcal{P}} + 60\eta_3 + \frac{20}{3}\eta_4, \\
g_4 &= -\frac{9}{28}a_2^{\mathcal{P}} - 6\eta_3 w_3.
\end{aligned} \tag{15}$$

The values of the parameters  $a_1^{\mathcal{P}}$ ,  $a_2^{\mathcal{P}}$ ,  $\eta_3$ ,  $\eta_4$ ,  $w_3$ , and  $w_4$  entering Eq. (15) are listed in Table (1) for the pseudoscalar  $\pi$ ,  $K$  and  $\eta$  mesons.

	$\pi$	$K$
$a_1^{\mathcal{P}}$	0	0.050
$a_2^{\mathcal{P}}$ (set-1)	0.11	0.15
$a_2^{\mathcal{P}}$ (set-2)	0.25	0.27
$\eta_3$	0.015	0.015
$\eta_4$	10	0.6
$w_3$	-3	-3
$w_4$	0.2	0.2

Table 1: Parameters of the wave function calculated at the renormalization scale  $\mu = 1 \text{ GeV}$

The sum rules for the  $N^*N^*\pi$  coupling constant contains three additional auxiliary parameters, namely Borel mass  $M^2$ , continuum threshold  $s_0$  and the arbitrary number  $\beta$ . Obviously, the result for the  $N^*N^*\pi$  coupling constant should be independent of these parameters. This leads to the necessity to find such regions of these parameters where the strong coupling constant does not depend on them. This issue can be handled by the following procedure. The first attempt is to find a such a region of  $M^2$  at several predetermined fixed values of  $s_0$  and  $\beta$  so that  $N^*N^*\pi$  coupling constant is independent of its variation. The lower bound of  $M^2$  is determined from the condition that higher twist contributions are less than the leading twist contributions. The upper bound is obtained by requiring that higher states and continuum contributions constitute, say, 40% of the perturbative contribution. These conditions are both satisfied if the Borel mass parameter varies in the region  $1.5 \text{ GeV}^2 \leq M^2 \leq 2.5 \text{ GeV}^2$ . Note that this working region of  $M^2$  is also obtained from analysis of the magnetic moment of negative parity baryons [10].

In Figs. (1) and (2) we present the dependence of the strong coupling constant  $g_{N^*N^*\pi}$  on the Borel parameter  $M^2$  at the fixed values of the auxiliary parameter  $\beta = -0.5, -0.3, 0.0, 0.3, 0.5$  and at two fixed values of the continuum threshold  $s_0 = 4.0 \text{ GeV}^2$  and  $s_0 = 4.5 \text{ GeV}^2$ , respectively. It follows from these figures that  $g_{N^*N^*\pi}$  shows rather stable behavior to the variation of  $M^2$  in its working region.

The continuum threshold is the other arbitrary parameter of the sum rules. This parameter is related to the energy of the first excited state. Analysis of various



sum rules shows that  $\sqrt{s_0} = m_{ground} + \Delta$ , where  $m_{ground}$  is the ground state mass, and  $\Delta$  is the energy difference between ground and first excited states which varies in the domain  $0.3 GeV \leq \sqrt{s_0} \leq 0.8 GeV$ . In the present analysis we use the average value  $\sqrt{s_0} = (m_{ground} + 0.5) GeV$ .

We also studied the dependence of the  $N^*N^*\pi$  coupling constant on  $s_0$ , at four different values of the auxiliary parameter  $\beta = -0.5; -0.3; 0.0; 0.3, 0.5$ , and at two fixed values of the Borel mass parameter  $M^2 = 2.0 GeV^2$  and  $M^2 = 2.5 GeV^2$ . We observe that  $g_{N^*N^*\pi}$  is practically insensitive to the variations in  $s_0$ . The total result changes about 5–6%

The final stage of sum rules is to find such a region of  $\beta$  where  $g_{N^*N^*\pi}$  be independent of the variation in  $\beta$ . The arbitrary parameter varies in the domain  $-\infty \leq \beta \leq +\infty$ . This infinitely large region can be mapped into a more restricted domain by introducing the definition  $\beta = \tan \theta$ , by running  $\theta$  in the region  $0 \leq \cos \theta \leq \pi$ .

In Figs. (3) and (4) we present the dependence of the coupling constant  $g_{N^*N^*\pi}$  on  $\cos \theta$ , at two fixed values of the continuum threshold  $s_0 = 4.0 GeV^2$  and  $s_0 = 4.5 GeV^2$ , and at the fixed values of the Borel mass parameter  $M^2 = (1.5, 2.0, 2.5) GeV^2$ , respectively. We find that the coupling constant  $g_{N^*N^*\pi}$  is weakly dependent to the variation of  $\cos \theta$  in the region  $-1.0 \leq \cos \theta \leq -0.85$ . We also perform similar analysis at two more fixed values of the continuum threshold,  $s_0 = 4.2 GeV^2$  and  $s_0 = 4.8 GeV^2$ , which shows that the result for  $g_{N^*N^*\pi}$  changes at most 7–8%.

Taking into account the uncertainties coming from input parameters entering into the pion DAs, as well as from quark condensates, residues of  $N^*$  and from the parameters  $M^2$  and  $s_0$ , we finally get the following result,

$$g_{N^*N^*\pi} = (10 \pm 2).$$

Note that our prediction on  $N^*N^*\pi$  coupling is about 50% larger compared to that obtained in 3–point QCD sum rules [2]. This can be explained by the fact that in the limit  $q \rightarrow 0$  the result predicted by 3–point QCD sum rules is not reliable (for more details, see [16]).

Finally we compare our result on the  $N^*N^*\pi$  strong coupling constant with the predictions of the  $NN\pi$  coupling constant for the positive parity baryons. The  $g_{NN\pi}$  coupling constant is calculated in various works and the results obtained are summarized in the table below,

$$g_{NN\pi} = \begin{cases} 12 \pm 5 & [11, 12], \\ 9.76 \pm 2.04 & [13], \\ 13.3 \pm 1.2 & [14] \\ 14 \pm 4 & [1], \\ 13.5 \pm 0.5 & [15]. \end{cases}$$

When we compare our results on the strong coupling constants of negative parity baryons with pion with those similar coupling constants for the positive parity baryons, we observe that our predictions are quite close the results existing in literature for the positive parity nucleon pion coupling constant. Small difference in the results can be attributed to the different values of the input parameters, value of the residue, and continuum threshold  $s_0$ .

In summary, we calculate the strong coupling constant of negative parity baryons with pion in framework of the light cone QCD sum rules. The unwanted contributions coming

from positive-to-positive, and positive-to-negative parity transformations are eliminated by constructing combination of sum rules corresponding to different Lorentz structures. In the case of nucleons the situation becomes more challenging due to the second positive parity baryon  $N'(1440)$  in addition to the ground state  $N(938)$ . Our prediction on  $N^*N^*\pi$  coupling constant is in good agreement with those results for the positive parity baryons existing in literature, but considerably different from the value predicted by the 3-point QCD sum rules method.

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## Figure captions

**Fig. (1)** The dependence of the strong coupling constant  $g_{N^*N^*\pi}$  on the Borel parameter  $M^2$ , at the fixed value of the continuum threshold  $s_0 = 4.0 \text{ GeV}^2$ , and several fixed values of the auxiliary parameter  $\beta$ .

**Fig. (2)** The same as Fig. (1), but at the fixed value of the continuum threshold  $s_0 = 4.5 \text{ GeV}^2$ .

**Fig. (3)** The dependence of the strong coupling constant  $g_{N^*N^*\pi}$  on  $\cos\theta$ , at the fixed value of the continuum threshold  $s_0 = 4.0 \text{ GeV}^2$ , at several fixed values of  $M^2$ .

**Fig. (4)** The same as Fig. (3), but at the fixed value of the continuum threshold  $s_0 = 4.5 \text{ GeV}^2$ .

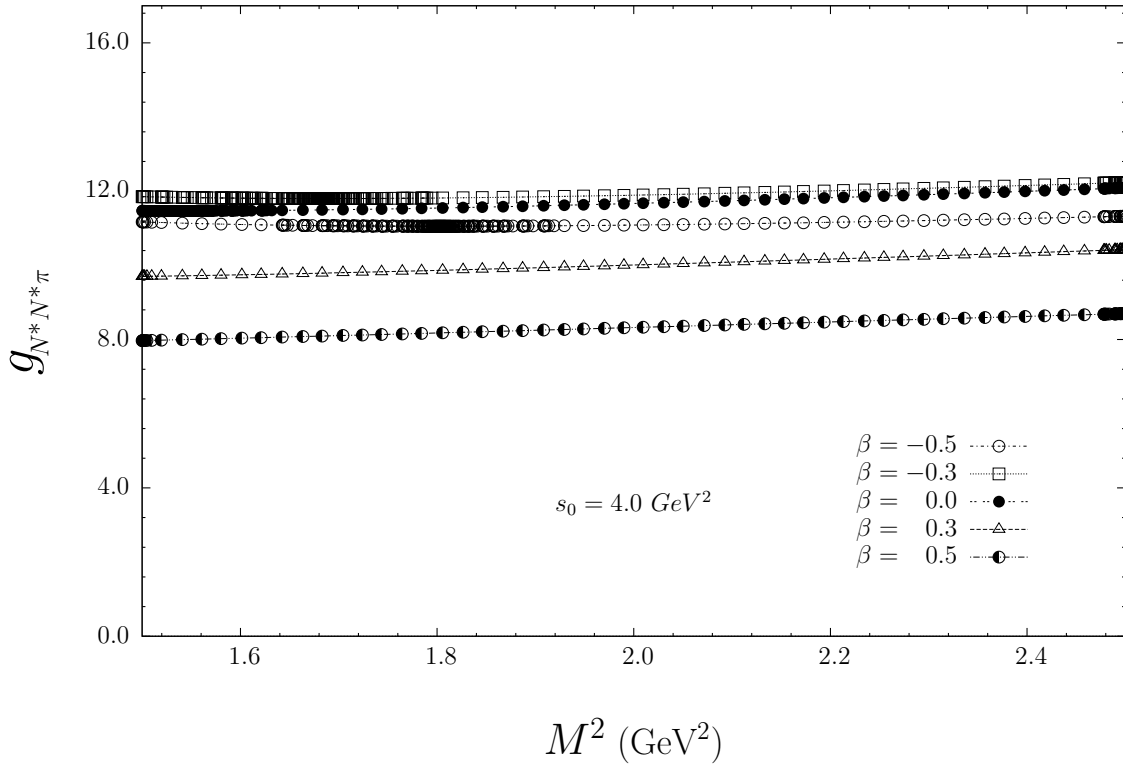


Figure 1:

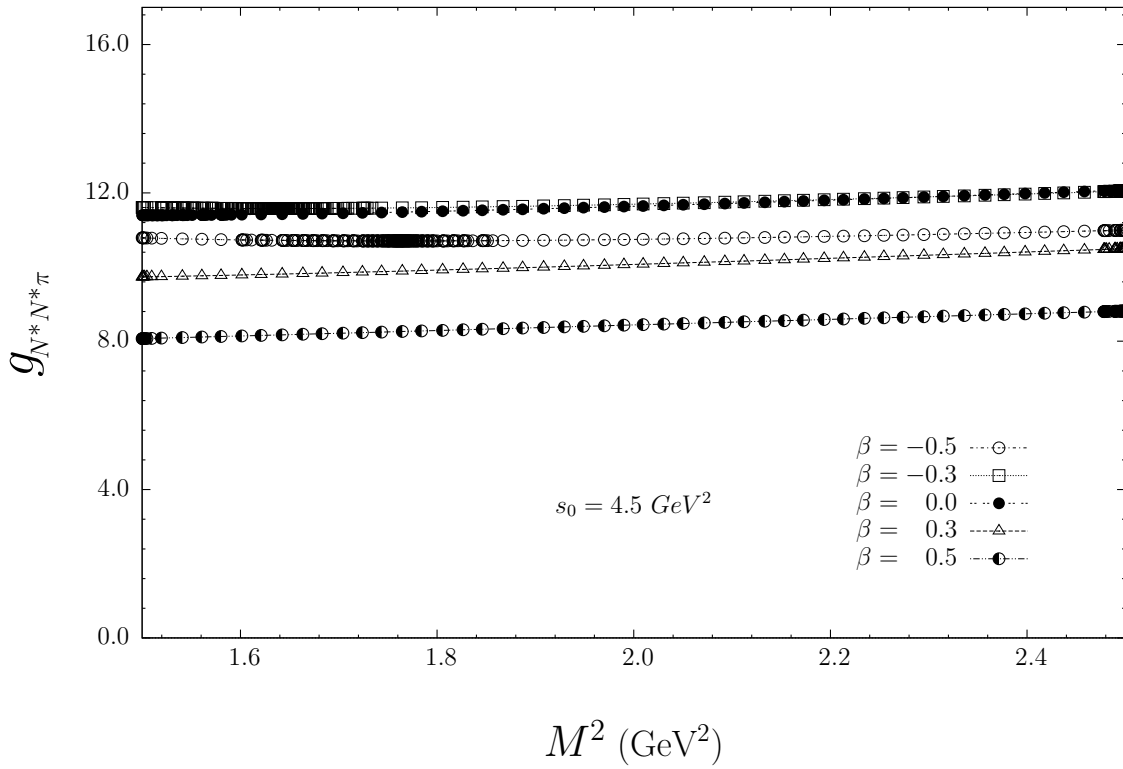


Figure 2:

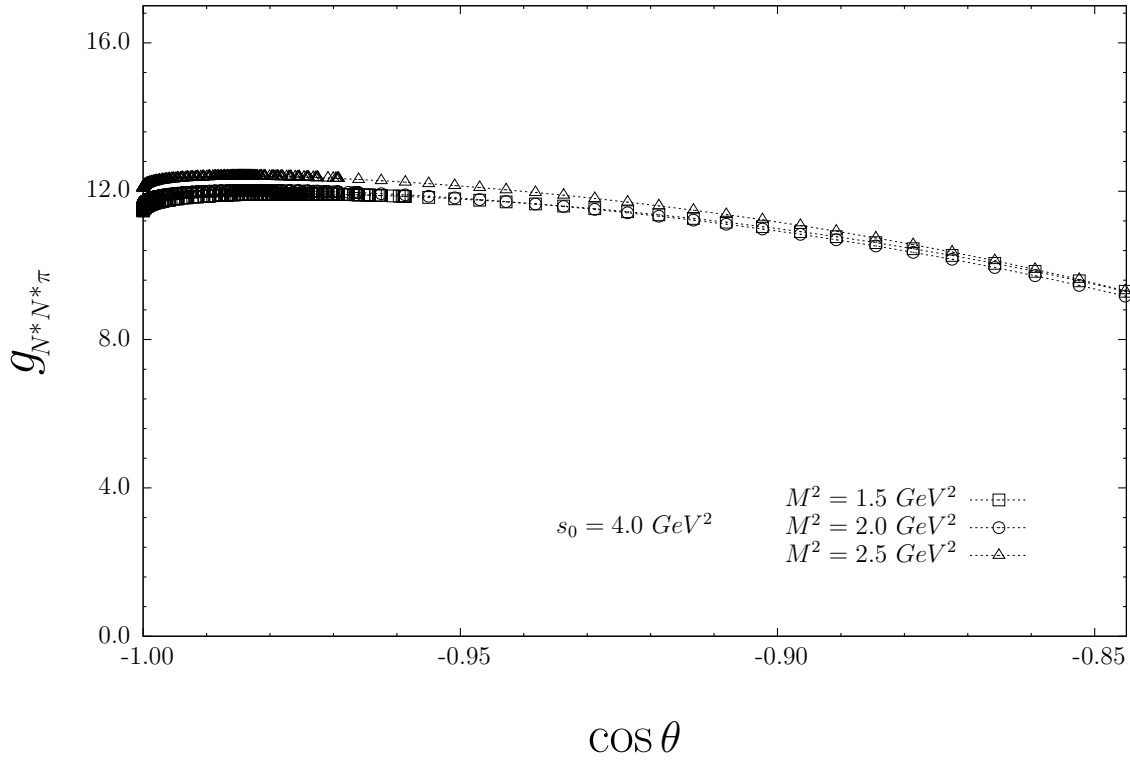


Figure 3:

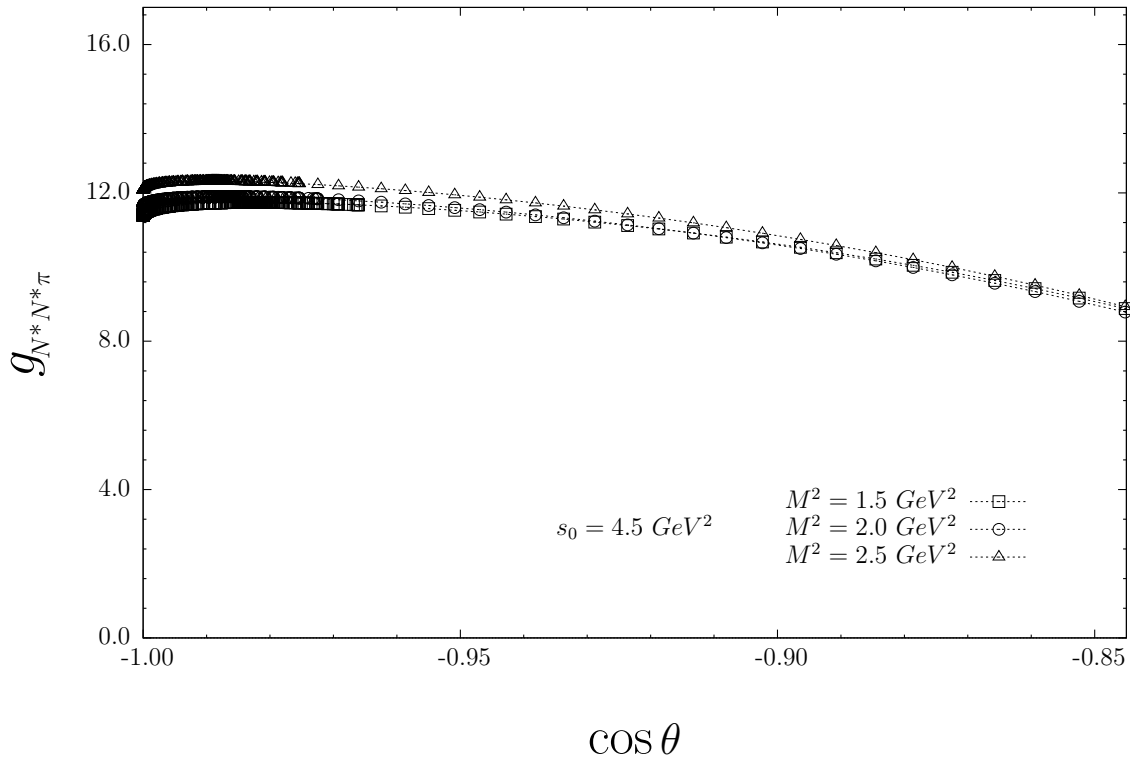


Figure 4: