CP violation in the radiative dileptonic B-meson decays

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Abstract

We investigate the CP violating asymmetry, the forward backward asymmetry and the CP violating asymmetry in the forward-backward asymmetry for the radiative dileptonic B-meson decays $B_d \to \gamma\,\ell^+\ell^-$for the $\ell = e,\,\mu,\,\tau$ channels. It is observed that these asymmetries are quite sizable and $B_d \to \gamma\,\ell^+\ell^-$decays seem promising for investigating CP violation.

1 Introduction

The rare B meson decays are one of the most important probes of the effective Hamiltonian governing the flavor-changing neutral current transitions $b \to s(d)\ell^+\ell^-$. Among them, the rare $B_{s,d} \to \gamma\,\ell^+\ell^-$ decays receive a special interest due to their relative cleanliness and sensitivity to new physics. They have been investigated in the framework of the SM for light and heavy lepton modes in refs.[1]-[5]. These decays have also been studied in models beyond the SM, such as MSSM [6] and different versions of the two Higgs doublet models (2HDM) [7]-[10], and it was reported that the new physics effects can give sizable contributions to the relevant observables.

For $b \to s\ell^+\ell^-$ transition, the matrix element contains the terms that receive contributions from $t\bar{t}$, $c\bar{c}$ and $u\bar{u}$ loops, which are proportional to the combination of $\xi_t = V_{tb}V_{ts}^*$, $\xi_c = V_{cb}V_{cs}^*$ and $\xi_u = V_{ub}V_{us}^*$, respectively. Smallness of ξ_u in comparison with ξ_c and ξ_t, together with the unitarity of the CKM matrix elements, bring about the consequence that matrix element for the $b \to s\ell^+\ell^-$ decay involves only one independent CKM factor ξ_t, so that the CP violation in this channel is suppressed in the SM [11, 12]. However, for $b \to d\ell^+\ell^-$ decay, all the CKM factors $\eta_t = V_{tb}V_{td}^*$, $\eta_c = V_{cb}V_{cd}^*$ and $\eta_u = V_{ub}V_{ud}^*$ are at the same order in the SM so that they can induce a CP violating asymmetry between the decay rates of the reactions $b \to d\ell^+\ell^-$ and $\bar{b} \to \bar{d}\ell^+\ell^-$ [13]. So, $b \to d\ell^+\ell^-$ decay seems to be suitable for establishing CP violation in B mesons. On the other hand, it should be noted that the detection of the $b \to d\ell^+\ell^-$ decay will probably be more difficult in the presence of a much stronger decay $b \to s\ell^+\ell^-$ and this would make the corresponding exclusive decay channels more preferable in search of CP violation. In this context, the exclusive $B_d \to \rho\,\ell^+\ell^-$ and $B_d \to \pi\,\ell^+\ell^-$ decays have been extensively studied

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in the SM [14] and beyond [15]-[19]. So, we think that it would be interesting and complementary to consider the remaining exclusive mode $B_d \to \gamma\,\ell^+\ell^-$.

In this paper, we would like to study the CP violation in the exclusive $B_d \to \gamma\,\ell^+\ell^-$decay in the context of the SM. $B_d \to \gamma\,\ell^+\ell^-$decay is induced by the pure-leptonic decay $B_d \to \ell^+\ell^-$, which is well known to have helicity suppression for light lepton modes, having branching ratios (BR) of the order of 10^{-15} for $\ell = e$ and 10^{-10} for $\ell = \mu$ channels [1]. However, when a photon line is attached to any of the charged lines in $B_d \to \ell^+\ell^-$ process, it changes into the corresponding radiative ones, $B_d \to \gamma\,\ell^+\ell^-$, so helicity suppression is overcome and larger branching ratios are expected. In [2]([3]), it was found that in the SM, $BR(B_d \to \ell^+\ell^-\gamma) = (1.5(1.5)\ ,\ 1.2(1.8)\ ,\ -(6.2)) \times 10^{-10}$ for $\ell = e, \mu, \tau$, respectively. Although these BR's are quite low, in models beyond the SM they can be enhanced by two (one) orders, as shown e.g. in [20]([21]) for $B_{s(d)} \to \gamma\,\ell^+\ell^-$ decay, so investigation of this process may also be interesting from the point of view of the new physics effects.

In $B_d \to \gamma\,\ell^+\ell^-$decays, depending on whether the photon is released from the initial quark or final lepton lines, there exist two different types of contributions, namely the so-called "structure dependent" (SD) and the "internal Bremsstrahlung" (IB) respectively, while contributions coming from the release of the free photon from any charged internal line will be suppressed by a factor of m_b^2/M_W^2. The SD contribution is governed by the vector and axial vector form factors and it is free from the helicity suppression. Therefore, it could enhance the decay rates of the radiative processes $B_d \to \ell^+\ell^-\gamma$ in comparison to the decay rates of the pure leptonic ones $B_d \to \ell^+\ell^-$. As for the IB part of the contribution, it is proportional to the ratio m_ℓ/m_B and therefore it is still helicity suppressed for the light charged lepton modes while it is expected to enhance the amplitude considerably for $\ell = \tau$ mode. However, we note that IB part of the amplitude does not contribute to CP violating asymmetry A_{CP} and the forward-backward asymmetry A_{FB} (see section 2).

We organized the paper as follows: In section 2, first the effective Hamiltonian is presented and the form factors are defined. Then, the basic formulas of the differential branching ratio dBR/dx, A_{CP}, A_{FB} and CP violating asymmetry in forward-backward asymmetry $A_{CP}(A_{FB})$ for $B_d \to \gamma\,\ell^+\ell^-$decay are introduced. Section 3 is devoted to the numerical analysis and discussion.

2 The theoretical framework of $B_d \to \gamma\,\ell^+\ell^-$decays

The leading order QCD corrected effective Hamiltonian which is induced by the corresponding quark level process $b \to d\,\ell^+\ell^-$, is given by [22]-[25]:

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}}\, V_{tb}V_{td}^* \Bigg\{ \sum_{i=1}^{10} \quad C_i(\mu)\, O_i(\mu) - \lambda_u \{C_1(\mu)[O_1^u(\mu) - O_1(\mu)] + C_2(\mu)[O_2^u(\mu) - O_2(\mu)]\} \Bigg\} \tag{1}$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}, \tag{2}$$

using the unitarity of the CKM matrix i.e. $V_{tb}V_{td}^* + V_{ub}V_{ud}^* = -V_{cb}V_{cd}^*$. The explicit forms of the operators O_i can be found in refs. [22, 23]. In Eq.(1), $C_i(\mu)$ are the Wilson coefficients calculated at a renormalization point μ and their evolution from the higher scale $\mu = m_W$ down to the low-energy scale $\mu = m_b$ is described by the renormalization group equation. For $C_7^{eff}(\mu)$

this calculation is performed in refs.[26, 27] in next to leading order. The value of $C_{10}(m_b)$ to the leading logarithmic approximation can be found e.g. in [22, 25]. We here present the expression for $C_9(\mu)$ which contains the terms responsible for the CP violation in $B_d \to \gamma\,\ell^+\ell^-$decay. It has a perturbative part and a part coming from long distance (LD) effects due to conversion of the real $\bar{c}c$ into lepton pair $\ell^+\ell^-$:

$$C_9^{eff}(\mu) = C_9^{pert}(\mu) + Y_{reson}(s)\ , \tag{3}$$

where

$$\begin{aligned} C_9^{pert}(\mu) &= C_9 + h(u,s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \\ &+ \lambda_u(3C_1 + C_2)] - \frac{1}{2}h(1,s)\left(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right) \\ &- \frac{1}{2}h(0,s)\left[C_3(\mu) + 3C_4(\mu) + \lambda_u(6C_1(\mu) + 2C_2(\mu))\right] \\ &+ \frac{2}{9}\left(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right)\ , \end{aligned} \tag{4}$$

and

$$\begin{aligned} Y_{reson}(s) &= -\frac{3}{\alpha_{em}^2}\kappa \sum_{V_i=\psi_i} \frac{\pi\Gamma(V_i \to \ell^+\ell^-)m_{V_i}}{m_B^2 s - m_{V_i} + i m_{V_i}\Gamma_{V_i}} \\ &\times \left[(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right) \\ &+ \lambda_u(3C_1(\mu) + C_2(\mu))]\,. \end{aligned} \tag{5}$$

In Eq.(4), $s = q^2/m_B^2$ where q is the momentum transfer, $u = \frac{m_c}{m_b}$ and the functions $h(u,s)$ arise from one loop contributions of the four-quark operators $O_1 - O_6$ and are given by

$$\begin{aligned} h(u,s) &= -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln u + \frac{8}{27} + \frac{4}{9}y \qquad (6)\\ &\quad -\frac{2}{9}(2+y)|1-y|^{1/2}\begin{cases} \left(\ln\left|\frac{\sqrt{1-y}+1}{\sqrt{1-y}-1}\right| - i\pi\right), & \text{for } y \equiv \frac{4u^2}{s} < 1 \\ 2\arctan\frac{1}{\sqrt{y-1}}, & \text{for } y \equiv \frac{4u^2}{s} > 1, \end{cases} \\ h(0,s) &= \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s + \frac{4}{9}i\pi\ . \qquad (7) \end{aligned}$$

The phenomenological parameter κ in Eq. (5) is taken as 2.3 (see e.g. [13]).

Neglecting the mass of the d quark, the effective short distance Hamiltonian for the $b \to d\ell^+\ell^-$ decay in Eq.(1) leads to the QCD corrected matrix element:

$$\begin{aligned} \mathcal{M} &= \frac{G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{td}^*\Big\{C_9^{eff}(m_b)\,\bar{d}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma^\mu\ell + C_{10}(m_b)\,\bar{d}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma^\mu\gamma_5\ell \\ &- 2C_7^{eff}(m_b)\,\frac{m_b}{q^2}\bar{d}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\ell}\gamma^\mu\ell\Big\}. \end{aligned} \tag{8}$$

The next step is to calculate the matrix element of the $B_d \to \gamma\,\ell^+\ell^-$decay. It can be written as the sum of the SD and IB parts

$$\mathcal{M}(B_d \to \ell^+\ell^-\gamma) = \mathcal{M}_{SD} + \mathcal{M}_{IB}\,. \tag{9}$$

It is evident from Eq.(8) that the following matrix elements are needed for the calculations of $\mathcal{M}_{SD}$ part :

$$\langle \gamma \left| \bar{d} \gamma_\mu (1 \mp \gamma_5) b \right| B \rangle = \frac{e}{m_B^2} \Big\{ \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} q^\lambda k^\sigma g(q^2) \pm i \Big[\varepsilon_\mu^* (kq) - (\varepsilon^* q) k_\mu \Big] f(q^2) \Big\} ,$$

$$\langle \gamma | \bar{d} i \sigma_{\mu\nu} q^\nu (1 \mp \gamma_5) b | B \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu\alpha\beta\sigma} \epsilon^{*\alpha} q^\beta k^\sigma \, g_1(q^2) \mp \, i \left[\epsilon_\mu^* (kq) - (\epsilon^* q) k_\mu \right] \, f_1(q^2) \right\}, \quad (10)$$

where ε_μ^* and k_μ are the four vector polarization and four momentum of the photon, respectively, and p_B is the momentum of the B meson. The form factors g, f, g_1, and f_1 were calculated in the framework of the light-cone QCD sum rules in [2, 4, 28].

The matrix element describing the SD part can be obtained from Eqs. (8)-(10),

$$\begin{aligned} \mathcal{M}_{SD} &= \frac{\alpha G_F}{4\sqrt{2}\,\pi} V_{tb} V_{td}^* \frac{e}{m_B^2} \bigg\{ \bar{\ell} \gamma^\mu (1 - \gamma_5) \ell \left[A_1 \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} q^\alpha k^\beta + i \, A_2 \Big(\varepsilon_\mu^* (kq) - (\varepsilon^* q) k_\mu \Big) \right] \\ &+ \bar{\ell} \gamma^\mu (1 + \gamma_5) \ell \left[B_1 \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} q^\alpha k^\beta + i \, B_2 \Big(\varepsilon_\mu^* (kq) - (\varepsilon^* q) k_\mu \Big) \right] \bigg\} , \end{aligned} \quad (11)$$

where

$$\begin{aligned} A_1 &= \frac{-2}{q^2} m_b C_7^{eff} g_1 + (C_9^{eff} - C_{10}) g \, , \\ A_2 &= \frac{-2}{q^2} m_b C_7^{eff} f_1 + (C_9^{eff} - C_{10}) f \, , \\ B_1 &= \frac{-2}{q^2} m_b C_7^{eff} g_1 + (C_9^{eff} + C_{10}) g \, , \\ B_2 &= \frac{-2}{q^2} m_b C_7^{eff} f_1 + (C_9^{eff} + C_{10}) f \, , \, . \end{aligned} \quad (12)$$

For the IB part, using

$$\langle 0 | \bar{d} \gamma_\mu \gamma_5 b | B \rangle = - \, i f_B p_{B\mu} \quad , \quad \langle 0 | \bar{d} \sigma_{\mu\nu} (1 + \gamma_5) b | B \rangle \, = \, 0 \;\; , \quad (13)$$

and conservation of the vector current, we get

$$\mathcal{M}_{IB} = \frac{\alpha G_F}{4\sqrt{2}\,\pi} V_{tb} V_{td}^* e f_B i \left\{ F \, \bar{\ell} \left(\frac{\not{\varepsilon}^* \not{p}_B}{2 p_1 k} - \frac{\not{p}_B \not{\varepsilon}^*}{2 p_2 k} \right) \gamma_5 \ell \right\} , \quad (14)$$

with

$$F = 4 m_\ell C_{10} \;\; . \quad (15)$$

Substituting Eqs.(11) and (14) into Eq. (9), squaring it and then averaging over the initial and summing over the final spins of the leptons and polarization of the photon, we find the photon energy distribution, after integration over the phase space, which is given by

$$\frac{d\Gamma}{dx} = \left| \frac{\alpha G_F}{4\sqrt{2}\,\pi} V_{tb} V_{td}^* \right|^2 \frac{\alpha}{(2\,\pi)^3} \, \pi \, m_B \, D(x) \quad (16)$$

where

$$
\begin{aligned}
D(x) = \quad & m_B^2 x^3 v \Bigg[\frac{1}{6}(|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2)(1-r-x) \\
& + r \, Re(A_1 \, B_1^* + A_2 \, B_2^*) \Bigg] + f_B m_\ell x^2 Re([A_1 + B_1] F^*) \, ln \frac{1+v}{1-v} \\
& - f_B^2 \left[2v \frac{1-x}{x} + \left(2 + \frac{4r}{x} - \frac{2}{x} - x \right) ln \frac{1+v}{1-v} \right] |F|^2
\end{aligned} \tag{17}
$$

where $x = 2E_\gamma / m_B$ is the dimensionless photon energy and $v = \sqrt{1 - \frac{4r}{1-x}}$ with $r = m_\ell^2 / m_B^2$.

We now consider the CP violating asymmetry, A_{CP}, between the $B_d \to \gamma \, \ell^+ \ell^-$ and $\bar{B}_d \to \gamma \, \ell^+ \ell^-$ decays, which is defined as follows:

$$
A_{CP}(x) = \frac{\Gamma(B_d \to \gamma \, \ell^+ \ell^-) - \Gamma(\bar{B}_d \to \gamma \, \ell^+ \ell^-)}{\Gamma(B_d \to \gamma \, \ell^+ \ell^-) + \Gamma(\bar{B}_d \to \gamma \, \ell^+ \ell^-)} \; . \tag{18}
$$

Using this definition we calculate the A_{CP} as:

$$
A_{CP} = \frac{\int H(x) \, dx}{\int (D(x) - H(x)) \, dx} \tag{19}
$$

where

$$
\begin{aligned}
H(x) = \quad & \frac{-2x^2}{3} \, Im \, \lambda_u \Bigg[2 \, Im \, \xi_2 \Bigg(C_7^{eff} (f \, f_1 + g \, g_1) m_b \, v \, x \left(\frac{x - 2r - 1}{1-x} \right) \\
& + 6 C_{10} g f_B m_\ell^2 \, ln \frac{1+v}{1-v} \Bigg) - (f^2 + g^2) m_B^2 v x (x - 2r - 1) \, Im \, \xi_1^* \xi_2 \Bigg] .
\end{aligned} \tag{20}
$$

In calculating this expression, we use the following parametrization:

$$
C_9^{eff} \equiv \xi_1 + \lambda_u \, \xi_2 \, . \tag{21}
$$

We note that in these integrals the Dalitz boundary for the dimensionless photon energy x is taken as

$$
\delta \leq x \leq 1 - \frac{4 m_\ell^2}{m_B^2} \, , \tag{22}
$$

since $|\mathcal{M}_{IB}|^2$ term has infrared singularity due to the emission of soft photon. In order to obtain a finite result, we follow the approach described in ref.[4] and impose a cut on the photon energy, i.e., we require $E_\gamma \geq 25$ MeV, which corresponds to detecting only hard photons experimentally. This cut requires that $E_\gamma \geq \delta \, m_B / 2$ with $\delta = 0.01$.

Next, we consider the forward-backward asymmetry, A_{FB}, in $B_d \to \gamma \, \ell^+ \ell^-$. Using the definition of differential A_{FB}

$$
A_{FB}(x) = \frac{\int_0^1 dz \frac{d\Gamma}{dz} - \int_{-1}^0 dz \frac{d\Gamma}{dz}}{\int_0^1 dz \frac{d\Gamma}{dz} + \int_{-1}^0 dz \frac{d\Gamma}{dz}} \, , \tag{23}
$$

where $z = \cos\theta$, θ is the angle between the momentum of the B-meson and that of ℓ^- in the c.m. frame of the dileptons $\ell^+ \ell^-$, we find

$$
A_{FB} = \frac{\int \, dx \, E(x)}{\int \, dx \, D(x)} \, , \tag{24}
$$

with

$$E(x) = -4\,v\,x^2\Bigg(m_B^2\,x\,\sqrt{(x-1)(x-1+4r)}\text{Re}(A_1A_2^*-B_1B_2^*)$$
$$+ \quad 4f_Bm_\ell\,v\left(\frac{x-1}{x-1+4r}\right)\ln\frac{4r}{x-1}\text{Re}((A_2-B_2)F^*)\Bigg)\,. \tag{25}$$

We have also a CP violating asymmetry in A_{FB}, $A_{CP}(A_{FB})$, which is an important measurable quantity in extracting precise information about free parameters of the models used. Since in the limit of CP conservation, one expects $A_{FB} = -\bar{A}_{FB}$ [29], where A_{FB} and $\bar{A}_{FB}$ are the forward-backward asymmetries in the particle and antiparticle channels, respectively, it is defined as

$$A_{CP}(A_{FB}) = A_{FB} + \bar{A}_{FB}\ , \tag{26}$$

Here, $\bar{A}_{FB}$ can be obtained by the replacement,

$$C_9^{eff}(\lambda_u) \to \bar{C}_9^{eff}(\lambda_u \to \lambda_u^*). \tag{27}$$

3 Numerical analysis and discussion

In Figs. (1-5), we present the dependence of the A_{CP}, A_{FB} and $A_{CP}(A_{FB})$ on the dimensionles photon energy x for the $B_d \to \gamma\,\ell^+\ell^-(\ell = e,\,\mu,\,\tau)$ decays for two different sets of parameters $(\rho,\,\eta) = (-0.07;\,0.34)$ and $(0.3;\,0.34)$ in the following Wolfenstein parametrization:

$$\lambda_u = \frac{\rho(1-\rho)-\eta^2-i\eta}{(1-\rho)^2+\eta^2} + O(\lambda^2). \tag{28}$$

We have also evaluated the average values of CP asymmetry $< A_{CP} >$, forward-backward asymmetry $< A_{FB} >$ and CP asymmetry in the forward-backward asymmetry $< A_{CP}(A_{FB}) >$ in $B_d \to \gamma\,\ell^+\ell^-$decay for the above sets of parameters $(\rho,\,\eta)$, and our results are displayed in Table 1 and 2 with and without including the long distance effects, respectively.

For the form factors $g,\ f,\ g_1$ and f_1, we have used the values calculated in the framework of light–cone QCD sum rules in refs. [2, 4, 28], which can be represented in the following dipole forms,

$$g(q^2) = \frac{g(0)}{\left(1-\frac{q^2}{m_g^2}\right)^2}, \qquad f(q^2) = \frac{f(0)}{\left(1-\frac{q^2}{m_f^2}\right)^2}\ ,$$
$$g_1(q^2) = \frac{g_1(0)}{\left(1-\frac{q^2}{m_{g_1}^2}\right)^2}\ , \qquad f_1(q^2) = \frac{f_1(0)}{\left(1-\frac{q^2}{m_{f_1}^2}\right)^2}\ , \tag{29}$$

where

$$g(0) = 1\,GeV\ ,\ f(0) = 0.8\,GeV\ ,\ g_1(0) = 3.74\,GeV^2\ ,\ f_1(0) = 0.68\,GeV^2\ ,$$
$$m_g = 5.6\,GeV\ ,\ m_f = 6.5\,GeV\ ,\ m_{g_1} = 6.4\,GeV\ ,\ m_{f_1} = 5.5\,GeV\ .$$

$(\rho;\,\eta)$	$< A_{CP} >$ $\ell = e$	$\ell = \mu$	$\ell = \tau$	$< A_{CP}(A_{FB}) >$ $\ell = e$	$\ell = \mu$	$\ell = \tau$	$< A_{FB} >$ $\ell = e$	$\ell = \mu$	$\ell = \tau$
$(0.3;\,0.34)$	0.118	0.114	0.103	-0.004	-0.004	0.038	-0.472	-0.460	-0.188
$(-0.07;\,0.34)$	0.061	0.059	0.050	0.004	0.004	0.021	-0.503	-0.490	-0.192

Table 1: The average values of A_{CP}, A_{FB} and $A_{CP}(A_{FB})$ in $B_d \to \gamma\,\ell^+\ell^-$for the three distinct lepton modes including the long distance effects.

$(\rho;\,\eta)$	$< A_{CP} >$ $\ell = e$	$\ell = \mu$	$\ell = \tau$	$< A_{CP}(A_{FB}) >$ $\ell = e$	$\ell = \mu$	$\ell = \tau$	$< A_{FB} >$ $\ell = e$	$\ell = \mu$	$\ell = \tau$
$(0.3;\,0.34)$	0.106	0.103	0.081	0.011	0.010	0.021	-0.520	-0.507	-0.200
$(-0.07;\,0.34)$	0.053	0.051	0.034	0.010	0.010	0.011	-0.529	-0.514	-0.200

Table 2: The same as Table (1), but without including the long distance effects.

In addition to these form factors, the input parameters and the initial values of the Wilson coefficients we used in our numerical analysis are as follows:

$$\begin{aligned}
& m_B = 5.28\,GeV\ ,\ m_b = 4.8\,GeV\ ,\ m_c = 1.4\,GeV\ ,\ f_B = 0.2\,GeV\ , \\
& m_\tau = 1.78\,GeV,\ m_\mu = 0.105\,GeV,\ |V_{tb}V_{td}^*| = 0.01, \\
& C_1 = -0.245,\ C_2 = 1.107,\ C_3 = 0.011,\ C_4 = -0.026,\ C_5 = 0.007, \\
& C_6 = -0.0314,\ C_7^{eff} = -0.315,\ C_9 = 4.220,\ C_{10} = -4.619.
\end{aligned} \tag{30}$$

In our numerical analysis, we take into account five possible resonances for the LD effects coming from the reaction $b \to d\,\psi_i \to d\,\ell^+\ell^-$, where $i = 1, ..., 5$ and divide the integration region into two parts for $\ell = \tau$: $\delta \le x \le 1 - ((m_{\psi_2} + 0.02)/m_B)^2$ and $1 - ((m_{\psi_2} - 0.02)/m_B)^2 \le x \le 1 - (2m_\ell/m_B)^2$, where $m_{\psi_2} = 3.686$ GeV is the mass of the second resonance and into three parts for $\ell = e$ and μ: $\delta \le x \le 1 - ((m_{\psi_2} + 0.02)/m_B)^2$, $1 - ((m_{\psi_2} - 0.02)/m_B)^2 \le x \le 1 - ((m_{\psi_1} + 0.02)/m_B)^2$ and $1 - ((m_{\psi_1} - 0.02)/m_B)^2 \le x \le 1 - (2m_\ell/m_B)^2$, where $m_{\psi_1} = 3.097$ GeV is the mass of the first resonance.

For reference, we present our SM predictions without long distance effects

$$BR(B_d \to \gamma\ell^+\ell^-) \ = \ (8.43, 8.52, 6.22) \times 10^{-10}\,, \tag{31}$$

for $\ell = e, \mu, \tau$. Here, we have used $\tau(B_d) = 1.5 \times 10^{-12}$ s, $|V_{tb}V_{td}^*| = 0.01$, and $(\rho;\eta) = (0.30; 0.34)$. Our result for $\ell = \tau$ is in a good agreement with ref.[3], but for $\ell = e$ and μ, they are larger than those of ref.[2] and [3].

In Fig.(1) and Fig.(2), we present the dependence of A_{CP} on the dimensionless photon energy x, for $B_d \to \gamma\,\ell^+\ell^-$decay for the Wolfenstein parameters $(\rho;\,\eta) = (-0.07;\,0.34)$ and $(\rho;\,\eta) = (0.3;\,0.34)$, respectively. The three distinct lepton modes $\ell =\ e,\ \mu,\ \tau$ are represented by the small dashed, dashed and solid curves, respectively. We observe that the A_{CP} for $\ell =\ e,\ \mu$ cases almost coincide, reaching up to 15 % for the intermediate values of x. The A_{CP} for $\ell =\ \tau$ mode exceeds the values of the other modes and reaches $40\ \%$ in the high-x region. We also observe from Tables 1 and 2 that including the LD effects in calculating $<\ A_{CP}\ >$ changes the results only $11 - 15\%$ for $\ell =\ e,\ \mu$ modes, while $\ell =\ \tau$ mode, it is quite sizable, $27 - 47\%$, depending on the sets of the parameters used for $(\rho;\,\eta)$.

The x dependence of A_{FB} for the $B_d \to \gamma \, \ell^+ \ell^- (\ell = e, \, \mu, \, \tau)$ decays are plotted in Fig.(3) for $(\rho; \, \eta) = (0.3; \, 0.34)$. Since we observe that this dependence is almost unchanged for the other set of $(\rho; \, \eta) = (-0.07; \, 0.34)$, we do not display it here. We see that A_{FB} is negative for all values of x, except in the resonance regions. $< A_{FB} >$ amounts to -50% for $\ell = e, \, \mu$ modes, but it stands smaller for $\ell = \tau$ mode, (-20%) as expected. The LD effects on $< A_{FB} >$ are about 10%, but in reverse manner, decreasing its magnitude in comparison to the values without LD contributions.

We present the dependence of the $A_{CP}(A_{FB})$ of $B_d \to \gamma \, \ell^+ \ell^-$decay on x in Fig.(4) and Fig.(5) again for two different sets of the Wolfenstein parameters. As for A_{CP}, $A_{CP}(A_{FB})$ for $\ell = e$, and $\ell = \mu$ modes almost coincide. They can have both signs and stand smaller for all values of x compared to the one of $\ell = \tau$ mode. For the latter case, $A_{CP}(A_{FB})$ is positive for all values of x except in the resonance region and it is at the order of magnitude 10^{-1}. LD effects seem to be quite significant for $< A_{CP}(A_{FB}) >$, decreasing its value by $60 - 80\%$ for $\ell = e, \, \mu$ modes, while $\ell = \tau$ mode, including LD effects increases $< A_{CP}(A_{FB}) >$ by $75 - 90\%$.

As a conclusion we can say that there is a significant A_{CP} and $A_{CP}(A_{FB})$ for the $B_d \to \gamma \, \ell^+ \ell^-$decay, although the branching ratios predicted for these channels are relatively small because of CKM suppression. Experimentally, to be able to measure the BR of $B_d \to \gamma \, \ell^+ \ell^-$decays, the required number of events, N, are $N \sim BR \times (number \; of \; B - mesons \; produced)$. Since approximately, 6×10^{11} B_d mesons are expected to be produced per year, we may hope that $B_d \to \gamma \, \ell^+ \ell^-$decays could be measured in LHC-B experiment in future.

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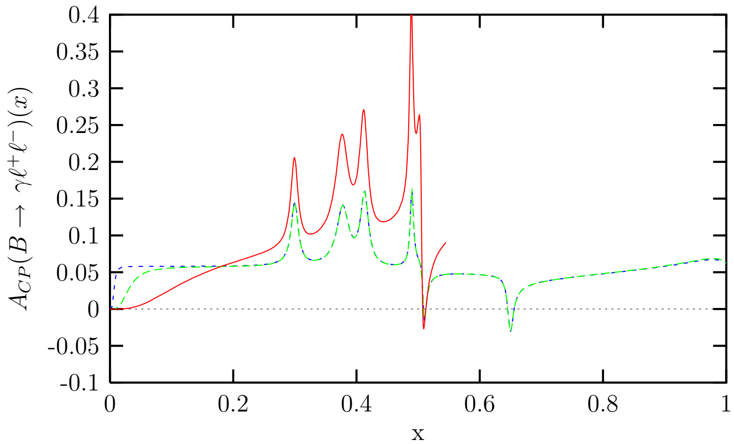

Figure 1: A_{CP} for $B_d \to \gamma \ell^+ \ell^-$decay for the Wolfenstein parameters $(\rho, \eta) = (-0.07; 0.34)$. The three distinct lepton modes $\ell = e, \mu, \tau$ are represented by the small dashed, dashed and solid curves, respectively.

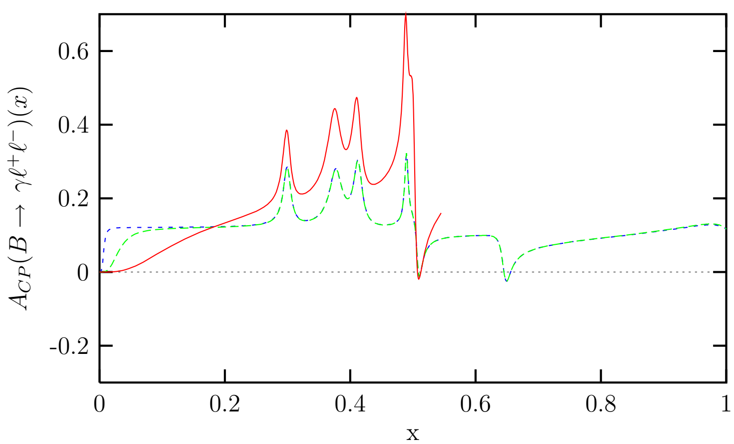

Figure 2: The same as Fig.(1) but for the Wolfenstein parameters $(\rho, \eta) = (0.3; 0.34)$

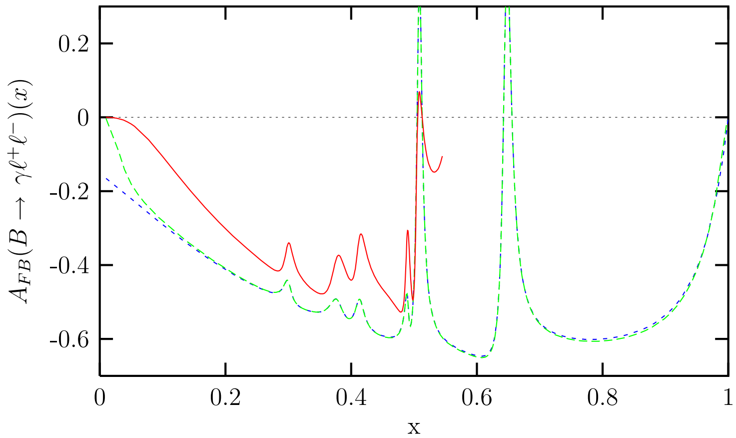

Figure 3: A_{FB} for $B_d \to \gamma\,\ell^+\ell^-$decay for the Wolfenstein parameters $(\rho,\,\eta) = (0.3;\,0.34)$. The three distinct lepton modes $\ell = \ e,\ \mu,\ \tau$ are represented by the small dashed, dashed and solid curves, respectively.

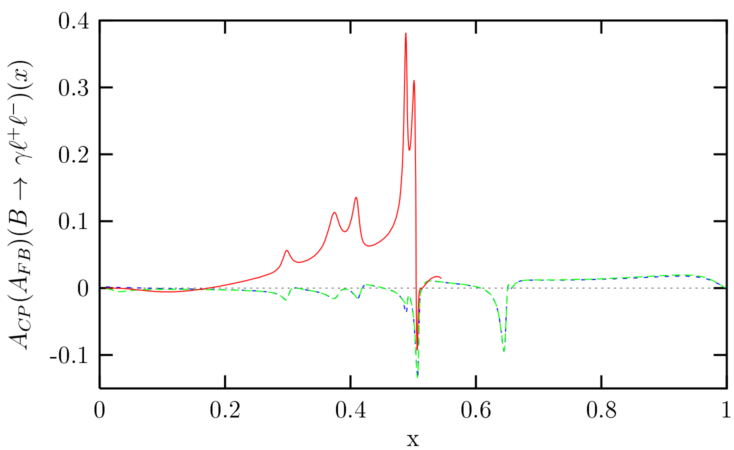

Figure 4: $A_{CP}(A_{FB})$ for $B_d \to \gamma\,\ell^+\ell^-$decay for the Wolfenstein parameters $(\rho,\,\eta) = (-0.07;\,0.34)$. The three distinct lepton modes $\ell = \ e,\ \mu,\ \tau$ are represented by the small dashed, dashed and solid curves, respectively.

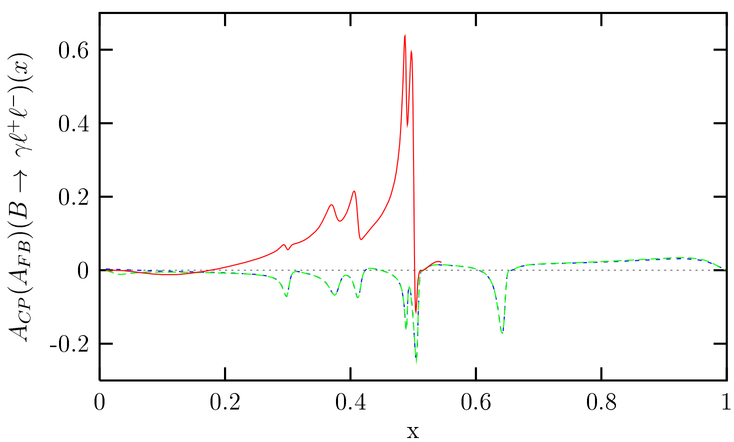

Figure 5: The same as Fig.(1) but for the Wolfenstein parameters $(\rho,\, \eta) = (0.3;\, 0.34)$