

# Use of Computationally Efficient Method of Moments in the Optimization of Printed Antennas

Lale Alatan, *Student Member, IEEE*, M. I. Aksun, *Member, IEEE*,  
Kemal Leblebicioğlu, and M. Tuncay Birand, *Senior Member, IEEE*

**Abstract**— Derivation of the closed-form Green's functions and analytical evaluation of the method of moments (MOM) matrix entries have improved the computational efficiency of the significantly in the analysis of printed geometries. With this background in mind, an extension of this efficient numerical technique is to incorporate an optimization algorithm and to assess its potential as a computer-aided design (CAD) tool. Therefore, we have employed the Gradient search and Genetic algorithms, in conjunction with the electromagnetic (EM) simulation technique, to a number of representative examples of interest.

**Index Terms**— Green's functions, method of moments, printed antennas.

## I. INTRODUCTION

IN computer-aided design (CAD) of microstrip integrated circuits and antennas, it is of paramount importance to have access to an analysis program that is computationally efficient, though not at the expense of sacrificing the accuracy to any significance. This is because the analysis of microwave integrated circuits and printed antennas can be very time-consuming, and this can lead to an extremely inefficient design procedure, which typically calls for the circuit to be analyzed at each iteration of the design cycle. The method of moments (MoM), when applicable, is perhaps the most efficient and accurate technique for analyzing microstrip structures. However, it can still be computationally expensive if it employs a conventional approach based on the use of the Sommerfeld integrals. This has prompted us to develop certain efficient schemes that circumvent the evaluation of the Sommerfeld integrals and employ closed-form expressions, not only for the Green's functions but for the elements of the method of moments matrix as well. The derivation of the closed-form Green's functions has been detailed in [1], the expressions for the matrix elements have been presented in [2] and it has been demonstrated that the matrix fill time can be significantly reduced via the use of the tactics described therein. The procedure applied to obtain the port parameters of a printed

structure and the comparison of the numerical accuracy of the computationally efficient MoM with a commercially available software program are given in [3].

The focus of the present paper is on interactive design or optimization rather than on just analysis. Obviously, the more efficient the analysis, the greater is the speed with which we can carry out the design procedure. However, there are other strategies besides speeding up the analysis procedure that can help us in our goal to develop a useful CAD design tool. In this paper, we will discuss some of these strategies and will also present a number of optimization procedures for designing microwave integrated circuits and printed antennas.

One simple strategy for speeding up the computation is to recognize that in interactive design or optimization, typically the entire geometry of the circuit or antenna is not changed at each iteration, but that only a small part of it is either deleted or augmented. Thus, at each iteration step, only a few rows and columns are added to, or deleted from, the matrix system utilized in the previous iteration step. In this type of situation, the order recursive Gaussian elimination (ORGE) method [4], developed to improve the computational efficiency of solving the matrix equations for modified geometries, can be utilized very efficiently. This method constructs the solution of the modified matrix equation by utilizing the one generated in the previous iteration and, hence, eliminates the need to repeatedly solve the entire matrix equation. Since the computation time is significantly reduced by following this procedure, it becomes possible to make a real-time assessment of the effects caused by the modifications in the geometry that are introduced in the design process to improve the performance of the system.

With the incorporation of the schemes for improving the matrix fill and solution times, the MoM becomes a very efficient analysis technique for use in conjunction with an optimization procedure. It should be recognized, however, that different optimization algorithms may be employed for different applications to fit the characteristics of the problem [5], [6]. For instance, in one of the applications to be discussed in this paper, genetic algorithms (GA's) will be used to optimize the geometrical properties of a printed circuit configuration. In this method, the shape of the metallic etch that needs to be optimized is divided into subregions (cells) and GA's are applied to determine whether or not a particular cell is to be metallized. In a different type of application, the dimensions of the geometry are optimized

Manuscript received February 5, 1997; revised June 13, 1998. This work was sponsored in part by NATO's Scientific Affairs Division in the framework of the Science for Stability Programme, and in part by TUBITAK (The Scientific and Technical Research Council of Turkey) within the scope of COST 245 (European Cooperation in the Field of Scientific and Technical Research) project.

L. Alatan, K. Leblebicioğlu, and M. T. Birand are with the Department of Electrical and Electronics Engineering, Middle East Technical University, 06531 Ankara, Turkey.

M. I. Aksun is with the Department of Electrical and Electronics Engineering, Bilkent University, 06533 Ankara, Turkey.

Publisher Item Identifier S 0018-926X(99)04781-X.

by using the directional (gradient) search algorithms, such as the steepest descent or the conjugate gradient methods. The uniform meshing is not well suited for this type of application, and the dimensions of one or two subintervals are regarded as the variables to be optimized to meet the desired performance specifications.

The computationally efficient MoM is summarized in Section II, the descriptions of the problems to be optimized and a brief overview of the optimization algorithms are presented in Section III. The results are discussed in Section IV and some conclusions are drawn on the basis of these results in Section V.

## II. BRIEF OVERVIEW ON THE EFFICIENT MoM

Application of the spatial-domain MoM to the solution of mixed-potential integral equation results in a matrix equation whose entries contain two-dimensional (2-D) integrals over finite domains. These integrals involve spatial-domain vector and scalar type Green's functions which can be obtained from their spectral-domain counterparts via Hankel transformation which is computationally inefficient due to the highly oscillatory and slow convergent nature of the kernel of the transformation. Hence, the matrix-fill time in the application of the spatial-domain MoM is mainly determined by two processes: 1) calculation of the spatial-domain Green's functions and 2) evaluation of the 2-D integrals. The problems pertaining to the former process are eliminated by the use of the closed-form Green's functions method in which the spectral-domain Green's functions are approximated by complex exponentials and their spatial-domain counterparts are cast into a form of complex exponentials, via the Sommerfeld identity. The robustness and efficiency of the closed-form Green's functions method are improved by using a two-level approximation scheme as detailed in [1]. After approximating the spectral-domain Green's functions, the spatial domain Green's functions can be cast into a form of

$$\tilde{G} \cong \frac{1}{j2k_z} \sum_{n=1}^N a_n e^{-k_z \alpha_n} \Rightarrow G \cong \sum_{n=1}^N a_n \frac{e^{-jk_s r_n}}{r_n} \quad (1)$$

where  $r_n = \sqrt{x^2 + y^2 - \alpha_n^2}$  is a complex distance and  $k_s$  is the wave number in the source medium.

The 2-D integrals involved in the calculation of the matrix entries, which is the second process that contributes to the matrix-fill time, can be performed analytically by using the following procedure which is described here on a typical matrix entry as

$$Z_{xx}^{mn} = \langle T_{xm}, G_{xx}^A * B_{xn} \rangle + \frac{1}{\omega^2} \left\langle T_{xm}, \frac{\partial}{\partial x} \left( G_q * \frac{\partial}{\partial x} B_{xn} \right) \right\rangle \quad (2)$$

where  $\langle, \rangle$  denotes inner product, and  $T_{xm}$ ,  $B_{xn}$  are the testing and basis functions, respectively. The testing and basis functions are chosen to be rooftop functions which are triangular in the longitudinal direction and constant in the transverse direction. The first inner product of (2) is written

explicitly as

$$\begin{aligned} & \langle T_{xm}, G_{xx}^A * B_{xn} \rangle \\ &= \int_{D_T} \int dx dy T_{xm}(x, y) \\ & \cdot \int_{D_B} \int dx' dy' G_{xx}^A(x - x', y - y') B_{xn}(x', y'). \end{aligned} \quad (3)$$

By changing the order of integration and substituting the closed-form Green's functions given in (1), the inner product takes the form of

$$\begin{aligned} & \langle T_{xm}, G_{xx}^A * B_{xn} \rangle \\ &= \iint du dv G_{xx}^A(u, v) \\ & \cdot \iint dx dy T_{xm}(x, y) B_{xn}(x - u, y - v) \\ &= \sum_{n=1}^N a_n \iint du dv \frac{e^{-jk_s r_n}}{r_n} T_{xm} \otimes B_{xn} \end{aligned} \quad (4)$$

where  $\otimes$  denotes correlation function, which can be evaluated analytically. It has been shown that the 2-D integrals in (4) can be evaluated analytically by replacing the exponential term with its Taylor's series expansion whose center of expansion is chosen to be the mid point of each integration region [2]. The elimination of the numerical integrals reduces the computation time approximately by a factor of 40.

Although, the matrix fill time has been significantly reduced with this approach, there is still possibility for improvement. This is recognized from the fact that the Taylor series expansion is applied for every exponential function in the closed-form Green's function expression and the analytic integration is performed for every exponential. To avoid this repetition, the overall summation is approximated in the least square sense by a polynomial as

$$\begin{aligned} \sum_{n=1}^N a_n \frac{e^{-jk_s r_n}}{r_n} &\cong \frac{\beta_0}{R} + \beta_1 + \beta_2 R + \beta_3 R^2 \\ &+ \beta_4 R^3 + \beta_5 R^4 \end{aligned} \quad (5)$$

where  $R = \sqrt{x^2 + y^2}$ . As the resultant polynomial is a function only of  $R$ , the analytical integrations are performed only once. Since the polynomial approximation cannot be applied for the matrix entries which contain singularities, it is used in the calculation of all the matrix entries except for the self and adjacent terms. The improvement obtained in the matrix fill time is approximately 10–15 times as compared to the Taylor's series approximation method.

## III. OPTIMIZATION PROBLEMS AND ALGORITHMS

In the design of printed structures, the shape of the geometry plays an important role in changing the characteristics of the structure. For example, for a patch antenna, a circular polarization characteristic can be obtained either by placing stubs at the proper locations [see Fig. 1(a)] or by inserting a diagonal slot at the center of the antenna, as shown in

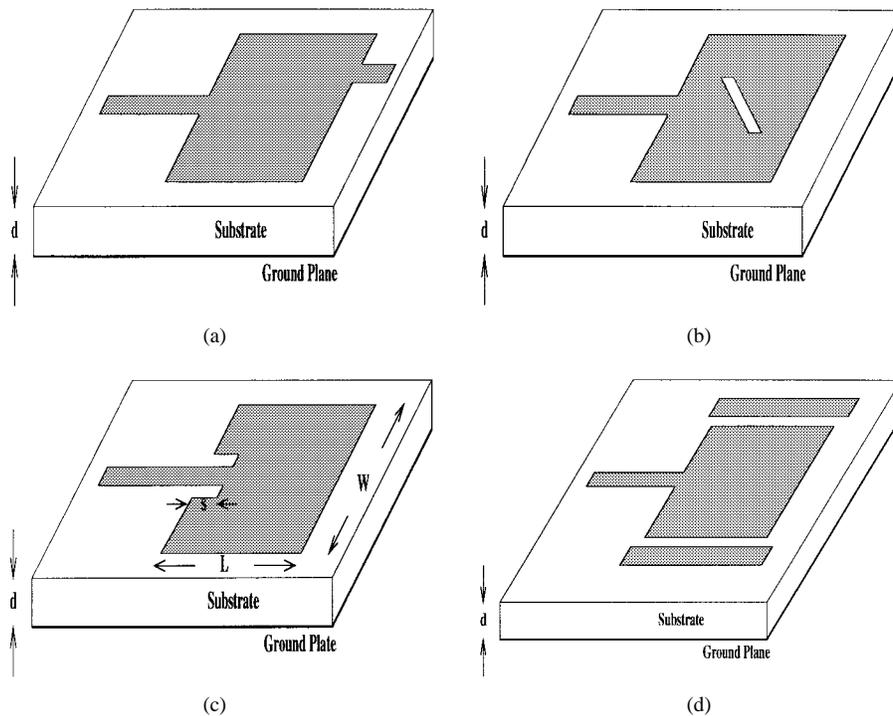


Fig. 1. Different microstrip patch antenna configurations.

[Fig. 1(b)]. In addition, the antenna can be tuned by using an inset-fed configuration as in [Fig. 1(c)], or the bandwidth can be increased by adding parasitic elements [Fig. 1(d)].

We observe that for all of these types of applications, the entire geometry is not modified but only a small portion of it is either deleted from, or added to, the geometry to obtain a desired specification. Consequently, it is unnecessary to solve the entire problem at each iteration during the optimization process. Instead, a better strategy is to select a region to be optimized at the beginning of the process and designate the remainder of the structure, which will remain unaltered, as the “main structure.” Next, the entire structure is divided into cells, the corresponding spatial-domain MoM matrix is filled by using the rooftop basis and testing functions, and the matrix thus computed is stored. The matrix entries corresponding to the main structure are selected from the stored matrix, and the lower upper (LU) decomposed form of the matrix is obtained by using the Gaussian elimination method. Next, the matrix entries corresponding to the “region to be modified” are selected and appended to the end of the LU decomposed matrix. Finally, the contribution of the modified region is calculated by using the ORGE method. The strategy outlined above enables us to observe the effects of any change in the modified region in a very efficient manner without recalculating the matrix entries at each iteration and without solving the matrix equation from the beginning.

For the type of design problems described above, it is quite convenient to use GA's for optimization. In GA's, the parameters that are going to be optimized are encoded in a binary form by using a string of “0's” and “1's”. The encoding process of this problem is very easy as the code of each cell in the modified region is set equal to one if it is metallized and zero otherwise. The length of the string resulting from

the above coding procedure is equal to the number of cells in the modified region, which is typically quite small. GA's are well suited for problems where one desires to find the optimal solution among a large number of admissible ones. For the present example, let us consider the case where the modified region is divided into  $5 \times 5 = 25$  cells. Then, since each cell can take on the value of either one or zero, there are  $2^{25} = 33\,554\,432$  possible solutions to the problem at hand. Therefore, in addition to the discreteness of the problem also due to the large search space, GA's truly excel when applied to this type of optimization problems.

Let us now turn to a different type of application where the shape of the printed structure is known, but it is desired to optimize the dimensions of the structure. For instance, to tune an inset-fed patch antenna, one may wish to optimize the depth of the notch and the length of the antenna. The use of a uniform meshing for this problem leads to a relatively large number of unknowns if one desires to observe the effects of small changes in the dimensions of the structure. Hence, to eliminate this problem, we use nonuniform meshing and optimize the width of the nonuniform sections. In the inset-fed antenna problem, the widths of the two sections (Fig. 2, Sections 1 and 2) are chosen to be the variables, and are optimized to obtain the desired input impedance level. In this problem, the shaded region shown in Fig. 2 is the main structure and the corresponding MoM matrix entries are calculated and the LU decomposed form of the matrix is stored before initiating the optimization algorithm. For these type of problems, the directional search type optimization algorithms are preferred, since the problem is simple enough to be solved with a traditional gradient search algorithm. In addition, the number of unknowns in this problem are small, and there is no need to encode the real unknowns.

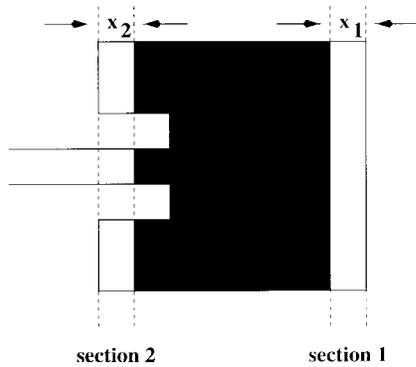


Fig. 2. Nonuniform meshing with variable widths  $x_1$  and  $x_2$ .

We will now proceed to present a brief overview of the aforementioned optimization algorithms that would help the reader in making the appropriate choice for the algorithm that is best suited for the problem at hand.

#### A. Directional (Gradient) Search Algorithms

The objective of an optimization problem is to minimize (or maximize) a cost function subject to certain constraints. In a directional search optimization, this can be achieved via the use of an iterative algorithm comprised of two steps, namely the “direction finding” and the “one-dimensional (1-D) search.” In the first step, an “improving” direction (descent direction for the minimization problem) is determined at the iteration point, and this is followed by a 1-D search where the next iteration point is calculated by finding a minimum in the “improving” direction, which has been determined earlier.

Among the different types of directional search algorithms, the most common are the steepest descent (SD), Newton’s, the conjugate gradient (CG), and quasi-Newton methods [7].

Generally, SD is the first choice in many of the applications because of its simplicity and globally convergent behavior. However, in many other applications, requiring a large number of variables, CG and quasi-Newton methods, which are also globally convergent, are preferred because of their better performance especially around optimum points. In this study, since the number of variables is only two (the width of the two variable sections), the SD method is preferred.

The most commonly used 1-D search algorithms, are the equal interval search, golden section search, dichotomous search, Fibonacci search methods, and the parabolic fit technique [8]. If it is desired to obtain a fixed interval of uncertainty with a minimum number of function evaluations, the Fibonacci search algorithm is the preferred choice.

#### B. Genetic Algorithms

GA’s [9] and simulated annealing [10] techniques, which are stochastic rather than deterministic in nature, provide two alternatives to traditional optimization schemes, and they employ directed random searches to locate optimal solutions in complex landscapes. Since the traditional optimization algorithms deal with the local properties of the iteration points, they may stagnate at a local extremum. In contrast,

the GA’s make use of a random search in addition to a systematic search and this prevents these methods from getting trapped into a local minimum or maximum. In fact, it has been proven that under certain conditions the GA’s converge to a global optimum, which, of course, is highly desirable [11].

GA’s are structured to solve real-world problems by imitating the processes occurring in natural evolution. The encoding mechanism maps each solution to a unique binary string. A set of strings constitutes a population and it evolves from generation to generation through the application of genetic operations, the most common of which are reproduction, crossover, and mutation.

Reproduction is based on the principle of survival of the fittest chromosome. Each string is associated with a fitness value that reflects its goodness relative to other members of the population. These fitness values are used to bias the selection of the chromosomes so that those with the best evaluations tend to reproduce more often than those whose evaluations are less superior. Several selection schemes are available, and among them, proportionate selection and the roulette wheel selection algorithms [12] are most frequently used.

The main operator to work on the parents is crossover, which is applied with a certain probability called crossover rate ( $p_c$ ). The recombination of the genetic material is simulated through the crossover mechanism by exchanging portions of chromosome strings between parents.

Mutation helps to regenerate the lost genetic material. It is performed by randomly changing one bit of the chromosome from zero to one or vice versa. The mutation rate  $p_m$  is the probability that a bit will be flipped. Mutation can be considered as the random search part of the algorithm and it enhances the possibility of finding a better solution.

In the application of GA’s most of the steps can be performed in a variety of ways (i.e., selection schemes) and there are many parameters ( $p_m$ ,  $p_c$ , population size) that require careful tuning. Therefore, one might need to try different combinations of possible choices of parameters and possible ways of applying genetic operators to get the best out of GA for every problem. In this study the main motivation for choosing the GA’s is not to use an optimization algorithm that converges to the global optimum but to use a procedure which fits the discrete nature of the problem.

## IV. RESULTS AND DISCUSSIONS

To illustrate applications of some of the optimization techniques mentioned above, as a first example the SD method with a Fibonacci search was used to optimize the dimensions of an inset-fed patch antenna. For typical patch widths ( $\approx \lambda/2$ ), the input impedance of the antenna is so high that it is necessary to feed the antenna with a very narrow transmission line to decrease the mismatch between the antenna and the feed network. To eliminate this problem, the antenna feed point should be shifted toward the center of the patch via the insertion of notches. An inset-fed patch antenna with dimensions  $7.62 \text{ mm} \times 7.62 \text{ mm}$  was analyzed at different frequencies for different notch depths and the

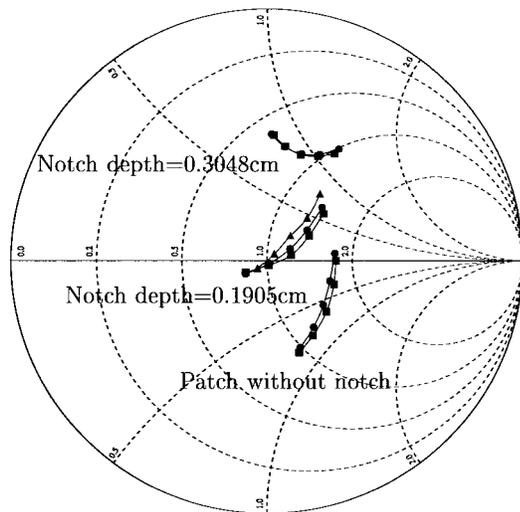


Fig. 3. Input impedance of an inset-fed patch antenna ( $7.62 \text{ mm} \times 7.62 \text{ mm}$ ) for different notch depths.  $\epsilon_r = 2.33$ ,  $d = 1.5748 \text{ mm}$ . Circle: present method; Square: numerical results in [14]; Triangle: experimental results in [14].  $f_{\text{start}} = 11.3 \text{ GHz}$ ,  $f_{\text{stop}} = 11.7 \text{ GHz}$ ,  $f_{\text{step}} = 0.1 \text{ GHz}$ .

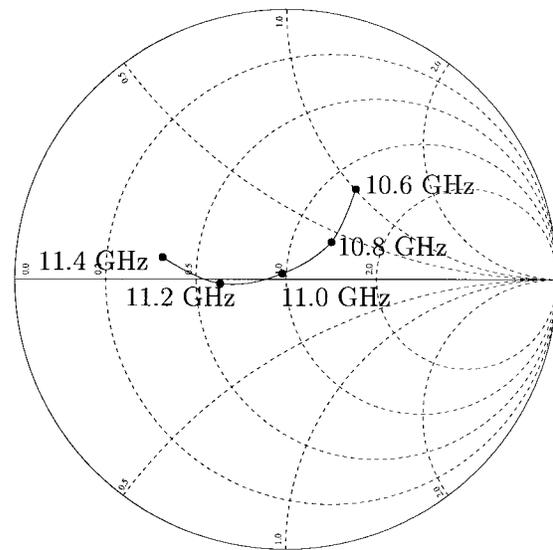


Fig. 4. Input impedance of the inset-fed patch antenna with  $L = 8.12 \text{ mm}$ ,  $W = 7.62 \text{ mm}$ , notch depth =  $1.88 \text{ mm}$ .

results were compared with the calculated and measured values presented in [14], as shown in Fig. 3. The permittivity and the thickness of the substrate are 2.33 and 1.5748 mm, respectively.

From Fig. 3, it can be observed that the antenna should be operated at 11.6 GHz, to have a good matching condition. However, to obtain a good operation at 11.0 GHz, the depth of the notch and the length of the antenna need to be optimized by making the widths of Section I and Section II (shown in Fig. 2) variable. For this example, the algorithm was found to converge within ten iterations and, at each iteration, the number of function evaluation is nine (five for Fibonacci search, two for the derivative with respect to  $x_1$ , and two for the derivative with respect to  $x_2$ ). As a result of optimization, a perfect match at 11 GHz was achieved when the length of the patch and the depth of the notch were chosen to be 8.12 mm and 1.88 mm, respectively. The input impedance variation around the resonance is given in Fig. 4.

In the next example, the GA was employed in the design of a circularly polarized antenna by removing some portions of the metallization from the opposite corners of a square patch antenna. In center-fed square patch antennas, circular polarization can be obtained by exciting two diagonal modes in such a way that the resonant frequency of one of them becomes slightly higher than that of the other. This can be achieved by disturbing the field variation of one of the diagonal modes [15]. This disturbance can be obtained either by placing a diagonal slot at the center or by perturbing the opposite corners of the patch. The frequency of operation should be adjusted such that the two diagonal modes are excited in phase quadrature.

With this background, we made use of the GA's to obtain the best possible axial ratio. A center-fed square patch antenna with parameters,  $\epsilon_r = 2.56$ ,  $d = 0.3175 \text{ cm}$ ,  $L = W = 6.12 \text{ cm}$  was divided into  $17 \times 17$  cells. The states of the  $25 \times 2 = 50$  cells, which are located at the opposite corners of the antenna (shown in Fig. 5) were chosen as

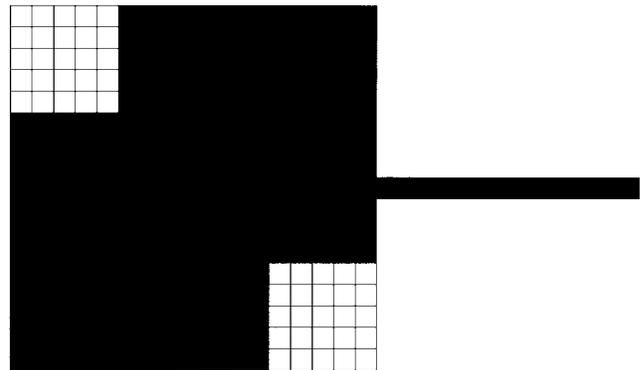


Fig. 5. Optimized region in the application of GA.

the parameters to be optimized. The state of a cell is one if it is metallized, zero if it is empty, and the length of a chromosome is 50. For these choices, the initial number of basis functions corresponding to the main structure is 467 and there are an additional 100 basis functions in the region to be modified. Central processing unit (CPU) of Sparc 20 workstation times for the evaluation of the matrix entries and the solution of matrix equations are given in Table I. From the table it can be observed that the cost function evaluation of a single chromosome requires at most 34.75 s (the CPU time is obtained for the case that all the modified region is metallized).

In the application of GA's, as the population gets larger there will be more chance to explore the different regions of the search space. However, in spite of all the acceleration techniques described herein, the evaluation time for the cost function is not very short, so there is a compromise in the choice of the population size. Consequently, before fixing the population size, several simulations were performed to obtain the table shown in Table II, which shows the average number of iterations required to find the optimum solution for different population sizes.

TABLE I  
CPU TIMES

Process	CPU time
Approximation of the Green's functions	7.94 sec
Matrix fill-time (467+100 = 567 basis functions)	18.1 sec
Matrix solution time for the main structure	34.46 sec
Matrix solution time for the modified region	$\leq 34.75$ sec

TABLE II  
AVERAGE NUMBER OF ITERATIONS FOR DIFFERENT POPULATION SIZES

Population size	40	70	100
Average number of iterations	15	20	11

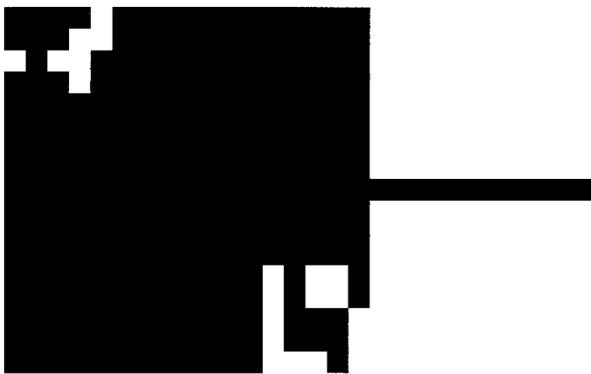


Fig. 6. Circularly polarized antenna geometry with  $17 \times 17$  divisions.

It is seen that although the convergence is obtained more rapidly for a large population, the population with size 40 gives the best performance as far as the total number of cost function evaluations are concerned. Due to these observations, the population size was chosen to be 40 and the first generation was initialized randomly.

The cost function, which is the axial ratio of the antenna, takes a value between one and infinity. A fitness function was used to assign a higher fitness value to the chromosome with a lower axial ratio. In addition to that, fitness scaling was employed to avoid the premature convergence which may arise due to the large range space  $([1, \infty])$  of the cost function.

In general, high crossover and mutation rates are preferred for small populations to increase the variety of the chromosomes within the population. Indeed, the crossover rate is somewhat equivalent to the ratio of the number of offsprings reproduced in one generation to the population size. As our population size was small, the number of offsprings was chosen to be equal to the population size. Parents were selected by using the roulette-wheel selection scheme and they

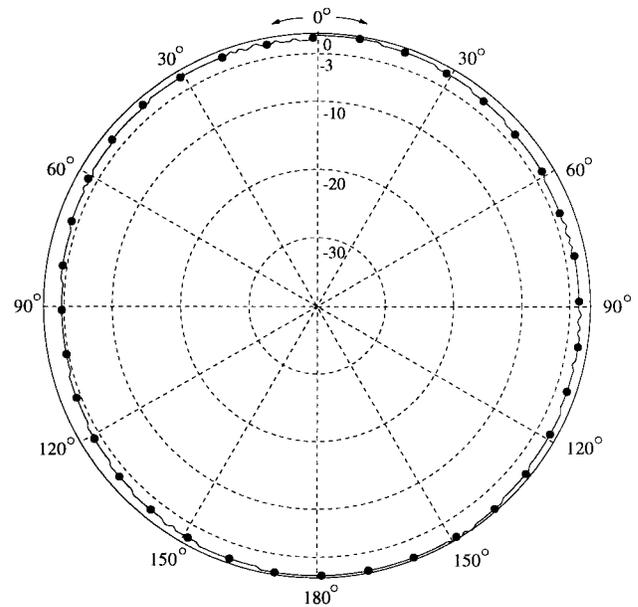


Fig. 7. Measured polarization pattern of the antenna shown in Fig. 6.



Fig. 8. Circularly polarized antenna geometry with  $21 \times 21$  divisions.

were used to reproduce 40 offsprings by applying a two-point crossover process.

In most of the GA applications, the mutation rate is not fixed throughout the simulation but it is adjusted according to the convergence behavior of the problem. In our application, using the crossover operation alone always provided good convergence rate and this could not be improved any further with the use of different mutation rates. Hence, the mutation rate was fixed to 0.003. Then, the elitist model was applied by retaining the chromosome with the best fitness value for the next generation, and by choosing the other 39 chromosomes randomly among the parents and the children. The initial generation was replaced by the new generation and the same steps were carried out until the required error criterion was obtained. At the end, the chromosome with the best fitness value was returned as the output. The best result at 1.48 GHz (axial ratio = 1.14) was obtained for the geometry shown in Fig. 6. This antenna was fabricated and we measured the polarization pattern of it by using the well-known procedure in which the test antenna is rotated opposite to a linearly polarized antenna and the signal level on the test antenna is

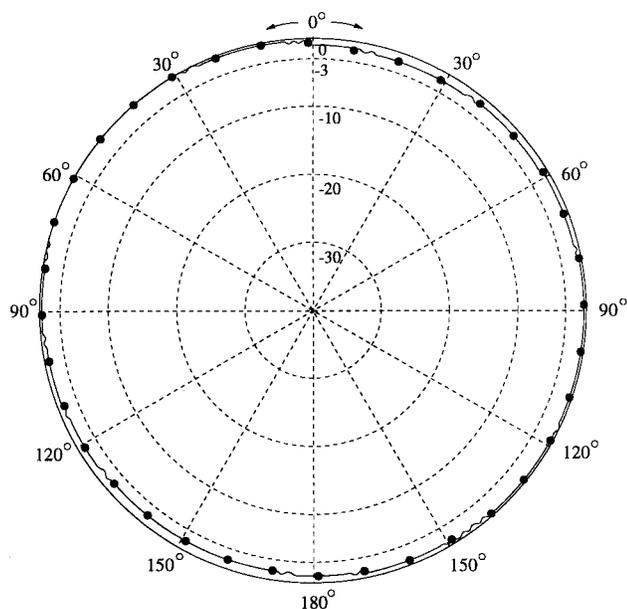


Fig. 9. Measured polarization pattern of the antenna shown in Fig. 8.

plotted with respect to the position of the linearly polarized one. The measured polarization pattern is plotted in Fig. 7, which shows that the measured axial ratio is 2.5 dB ( $=1.33$ ). The discrepancy between the measured and calculated axial ratio values is most probably due to the insufficient gridding of the structure. As the designed structure contains some discontinuities, a finer gridding is required to fit the abrupt variation of the current distribution. Due to these observations, the same design procedure was repeated by dividing the antenna into  $21 \times 21$  cells. The best result (axial ratio = 1.09) was obtained for the geometry shown in Fig. 8. The measured polarization pattern is given in Fig. 9, which shows that a good agreement between the calculated (0.748 dB) and measured (1.0 dB) axial ratio values was obtained. However, the best axial ratio in the measurement was obtained at 1.455 GHz, slightly different from the design frequency, which could be attributed to a small variation in the permittivity of the dielectric substrate.

## V. CONCLUSION

Recently, it was demonstrated that the use of closed-form Green's functions, and the analytical evaluation of the MoM matrix entries lead to an accurate EM simulation tool for the analysis of microwave circuits and antennas. A natural extension of the above effort is to incorporate this analysis technique into an optimization algorithm and to assess its potential as a CAD tool. With this background in mind, we have employed the gradient search and genetic algorithms, in conjunction with the EM simulation technique, to a number of representative examples of interest. The gradient search algorithm has been employed for the optimization of the input impedance of an inset-fed microstrip antenna, while the genetic algorithm has been used to design a circularly polarized microstrip antenna. We have pointed out that, for best results, one should choose the

optimization algorithm that is well adapted for the specific problem at hand and that not all the optimization algorithms are equally well suited for each and every design problem.

## REFERENCES

- [1] M. I. Aksun, "A robust approach for the derivation of the closed-form Green's functions," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 651–658, May 1996.
- [2] L. Alatan, M. I. Aksun, K. Mahadevan, and M. T. Birand, "Analytical evaluation of the MoM matrix elements," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 519–525, Apr. 1996.
- [3] N. Kinayman and M. I. Aksun, "Efficient and accurate EM simulation technique for analysis and design of MMIC's," *Int. J. MIMICAE*, vol. 7, pp. 344–358, Sept. 1997.
- [4] K. Naishadham and P. Misra, "Order recursive gaussian elimination and efficient CAD of microwave circuits," in *Proc. IEEE MTT-S Int. Microwave Symp.*, Orlando, FL, May 16–20, 1995, pp. 1435–1438.
- [5] K. Naishadham and P. Misra, "Order recursive method of moments: A powerful computational tool for microwave CAD and optimization," in *Proc. IEEE MTT-S Int. Microwave Symp.*, 1996, pp. 1463–1466.
- [6] A. John and R. H. Jansen, "Evolutionary generation of (M)MIC component shapes using 2.5D EM simulation and discrete genetic optimization," in *Proc. IEEE MTT-S Int. Microwave Symp.*, 1996, pp. 745–748.
- [7] D. G. Luenberger, *Introduction to Linear and Nonlinear Programming*, Reading, MA: Addison-Wesley, 1973.
- [8] J. S. Arora, *Introduction to Optimum Design*. New York: McGraw-Hill, 1988.
- [9] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*. Reading, MA: Addison-Wesley, 1989.
- [10] L. Davis, Ed., *Handbook of Genetic Algorithms*. London, U.K.: Pitman, 1987.
- [11] G. Rudolph, "Convergence analysis of canonical genetic algorithms," *IEEE Trans. Neural Networks*, vol. 5, pp. 96–104, Jan. 1994.
- [12] L. Davis, Ed., *Handbook of Genetic Algorithms*. New York: Van Nostrand Reinhold, 1991.
- [13] M. Srinivas and L. M. Patnaik, "Genetic algorithms: A survey," *IEEE Comput.*, vol. 27, pp. 17–26, June 1994.
- [14] S. C. Wu, M. G. Alexopoulos, and O. Fordham, "Feeding structure contribution to radiation by patch antennas with rectangular boundaries," *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 1245–1249, Oct. 1992.
- [15] P. C. Sharma and K. C. Gupta, "Analysis and optimized design of single feed circularly polarized microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-31, pp. 949–955, Nov. 1983.

**Lale Alatan** (S'91) received the B.Sc., M.Sc., and Ph.D. degrees from the Middle East Technical University (METU), Ankara, Turkey, in 1990, 1993, and 1997, respectively.

Her research interests include numerical analysis of printed structures, microstrip antennas, and optimization techniques.

**M. I. Aksun** (M'92) received the B.S. and M.S. degrees in electrical and electronics engineering from the Middle East Technical University, Ankara, Turkey, in 1981 and 1983, respectively, and the Ph.D. degree in electrical and computer engineering from the University of Illinois at Urbana-Champaign in 1990.

From 1990 to 1992, he was a Postdoctoral Fellow at the Electromagnetic Communication Laboratory, University of Illinois at Urbana-Champaign. Since 1992 he has been on the faculty of the department of electrical and electronics engineering at Bilkent University, Ankara, Turkey, where he is currently an Associate Professor. His research interests include the numerical methods for electromagnetics, microstrip antennas, and microwave and millimeter-wave integrated circuits.



**Kemal Leblebicioğlu** was born in Turkey on May 12, 1957. He received the B.S. degree in electrical and electronics engineering, in 1979, and the M.S. and Ph.D. degrees in mathematics, in 1982 and 1988, respectively, all from the Middle East Technical University (METU), Ankara, Turkey.

From 1980 to 1988 he was with the Department of Mathematics at the METU as a Graduate Assistant. Since 1988, he has been on the Faculty of the Department of Electrical and Electronics Engineering at METU, where he is currently an Associate

Professor. His research interests include the numerical methods for partial differential equations, inverse problems, optimization, optimal control theory, neural networks, neuro and fuzzy controllers, and image processing.

**M. Tuncay Birand** (S'67–M'72–SM'96) received the B.S., M.S., and Ph.D. degrees from Middle East Technical University (METU), Ankara, Turkey, in 1966, 1967, and 1971, respectively.

Dr. Birand served as the head of the EEE Department at the Ankara Gazi University (1982–1983; 1984–1988) and at METU (1991–1992). He was the dean of the Engineering Faculty at METU from 1992 to 1997. He also served as a member of the Senior Advisory Board of the Turkish Ministry of Transportation (1983–1984) and as the Electronics Division Head of the Computer Technologies and Electronics Research Institute of the Scientific and Technical Research Council, Turkey (1990–1991). From 1990 to 1997 he was the representative of Turkey in the Senior Officials Committee of the European Cooperation in the Field of Scientific and Technical Research (COST). He was also a member of the management committees of the European COST 223 and COST 245 actions. During the period he served as the Dean of the Faculty of Engineering at METU and has been active in promoting and steering the accreditation and internationalization activities in engineering education. He was a Visiting Scholar at the Queen Mary College, London, U.K. (1978–1979), and worked as a R&D scientist at ERA Technology Ltd., England (1979–1981). From 1988 to 1989 he was a Fulbright scholar at the University of Illinois, Urbana-Champaign. He also worked as the head of the "Antennas" sector at the International Research Center in Telecommunications, Transmission and Radar (IRCTR), Faculty of Information Technology and Systems, Delft University of Technology, The Netherlands, from 1997 to 1998, on a sabbatical leave of absence from METU. He has over 50 publications, the majority of which have appeared in refereed international journals and proceedings and is the author of an international patent related to millimeter wave antennas. He is a member of the editorial and reviewer boards of international and national journals. His interest areas are antennas, microwaves, applied electromagnetism, and engineering education.

Dr. Birand received the 1983 Science Promotion Award of the Scientific and Technical Research Council of Turkey. In 1997 he was awarded the Meritorious Achievement Award in Accreditation Activities by the Institute of Electrical and Electronics Engineers (IEEE). In the same year he received the Silver Badge of Honor of the UNESCO International Center for Engineering Education (UICEE).