Radiative decays of the heavy tensor mesons in light cone QCD sum rules

T. M. Aliev* and M. Savcı†

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

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The transition form factors of the radiative decays of the heavy tensor mesons to heavy pseudoscalar and heavy vector mesons are calculated in the framework of the light-cone QCD sum rules method at the point $Q^2 = 0$. Using the obtained values of the transition form factors at the point $Q^2 = 0$, the corresponding decay widths are estimated. The results show that the radiative decays of the heavy-light tensor mesons could potentially be measured in the future planned experiments at LHCb.

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I. INTRODUCTION

With the recent developments in experimental techniques, many new particles have been discovered [1–5]. Some of the newly discovered particles were already predicted by the quark model, but many were not, and understanding their properties requires a new perspective beyond the conventional quark model. The heavy tensor mesons $\mathcal{D}_2(2460)$, $\mathcal{D}_{s_2}(2573)$, $B_2(5747)$, and $B_{s_2}(5840)$ predicted by the conventional quark model have all been discovered in the experiments, and their masses and decay widths have been measured [6]. A more refined analysis to study the properties of these particles will be conducted at LHCb and Belle II.

Following the discovery of the heavy tensor mesons, their strong, electromagnetic, and weak decays need to be investigated. Note that the strong decays $\mathcal{D}_2^0(2460) \to \mathcal{D}^+(\mathcal{D}^{*+})\pi^-$, $\mathcal{D}_2^+(2460) \to \mathcal{D}^0\pi^+$ [7–10], $\mathcal{D}_{s_2}^+(2573) \to \mathcal{D}^0 K^+$ [7], $B_2^0(5747) \to B^{*+}\pi^-$, [11,12], and $B_{s_2}^0(5840) \to B^+K^-$ [11,12] have already been observed in the experiments.

These observations have stimulated a series of many studies. For example, the strong coupling constants of the aforementioned decays have been calculated using the three-point sum rules [13–15] and light-cone QCD sum rules (LCSR) methods [16].

In the present work, we study the radiative decays of the heavy tensor mesons in the framework of the LCSR. Radiative decays constitute one of the most promising classes of decays for gathering information about the electromagnetic properties of hadrons, which are important for revealing their internal structure. It should be emphasized here that so far the radiative decays of the heavy tensor mesons have not been observed in the experiments, and our results might indicate that these decays can potentially be measured at LHCb.

The paper is organized as follows. In Sec. II, we formulate the LCSR for the transition form factors at the point $Q^2 = 0$. In Sec. III, we perform a numerical analysis of these form factors at the point $Q^2 = 0$ and calculate the corresponding decay widths. The last section contains our conclusion.

II. LIGHT-CONE QCD SUM RULES FOR THE HEAVY TENSOR → HEAVY PSEUDOSCALAR (VECTOR) MESON + PHOTON

Before presenting the details of the calculation, a few words about our notation are in order. In the present work the states of the heavy tensor, heavy vector, and heavy pseudoscalar mesons are denoted by the generic notations T_Q, V_Q, and P_Q, respectively.

The $T_Q \to P_Q(V_Q)\gamma$ decay is described by the following correlator:

$$\Pi_{\mu\nu\alpha(\rho)}(p,q) = -\int d^4x \int d^4y e^{i(px+qy)} \times \langle 0|J_{T_Q\mu\nu}(x) J_\alpha^{e\ell}(y) J_{P_Q}(0)(J_{V_Q\rho}(0))|0\rangle, \quad (1)$$

where

$$J_{T_Q\mu\nu}(x) = \frac{1}{2}\left[\bar{q}(x)\gamma_\mu \overset{\leftrightarrow}{\mathcal{D}}_\nu(x) Q(x) + \bar{q}(x)\gamma_\nu \overset{\leftrightarrow}{\mathcal{D}}_\mu(x) Q(x)\right],$$

$$J_{P_Q}(J_{V_Q\rho}) = \bar{q}i\gamma_5 Q(\bar{q}\gamma_\rho Q)$$

are the interpolating currents of the heavy tensor and heavy pseudoscalar (heavy vector) mesons, respectively, and

$$J_\alpha^{e\ell}(y) = e_q\bar{q}\gamma_\alpha q + e_Q\bar{Q}\gamma_\alpha Q$$

*taliev@metu.edu.tr
†savci@metu.edu.tr

is the electromagnetic current, where e_q and e_Q are the electric charges of the light and heavy quarks, respectively. The covariant derivative $\overleftrightarrow{\mathcal{D}}_\mu(x)$ is defined as

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2}[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x)],$$

where

$$\overrightarrow{\mathcal{D}}_\mu(x) = \overrightarrow{\partial}_\mu(x) - ig\frac{\lambda^a}{2}A^a_\mu(x),$$
$$\overleftarrow{\mathcal{D}}_\mu(x) = \overleftarrow{\partial}_\mu(x) + ig\frac{\lambda^a}{2}A^a_\mu(x).$$

In this expression, λ^a are the Gell-Mann matrices and $A^a_\mu(x)$ is the external field.

The correlator given in Eq. (1) can be rewritten in the presence of the electromagnetic background field of a plane wave

$$F_{\mu\nu} = i(\varepsilon^{(\gamma)}_\mu q_\nu - \varepsilon^{(\gamma)}_\nu q_\mu)$$

in the following form:

$$\Pi_{\mu\nu\alpha(\rho)}\varepsilon^{(\gamma)\alpha} = i\int d^4x e^{ipx}\langle 0|J_{T_Q\mu\nu}(x)J_{P_Q}(J_{V_Q\rho})(0)|0\rangle_F, \quad (2)$$

where the subscript F stands for the vacuum expectation value evaluated in the presence of the background electromagnetic field $F_{\mu\nu}$. The expression for the correlation function given in Eq. (1) can be obtained by expanding Eq. (2) in powers of the background field by taking into account only the terms linear in $F_{\mu\nu}$, which corresponds to single-photon emission (for more details about the background field method and its applications, see Refs. [17,18]).

In order to calculate any physical quantity using the QCD sum rules method, the correlation function needs to be computed in two different kinematical domains. On the one hand, the main contribution to the correlation function (2) originates in the region where $p^2 \simeq m^2_{T_Q}$ and $(p+q)^2 \simeq m^2_{P_Q}(m^2_{V_Q})$. On the other hand, the same correlation function can be investigated in the deep Euclidean domain where $p^2 \ll 0$, $(p+q)^2 \ll 0$, using the operator product expansion (OPE). As is well known, in the LCSR method the OPE is performed over the twists of the operators rather than their canonical dimensions, which is the case in the standard sum rules approach. The physical part of the correlation function (1) is obtained by inserting a complete set of the corresponding mesonic states, and then isolating the ground-state tensor and pseudoscalar (vector) mesons:

$$\Pi_{\mu\nu\alpha(\rho)}(p,q) = \frac{\langle 0|J_{T_Q\mu\nu}|T_Q(p)\rangle}{(m^2_{T_Q}-p^2)}\frac{\langle T_Q(p)|J^{e\ell}_\alpha|P_Q(V_Q)(p+q)\rangle}{[m^2_{P_Q(V_Q)}-(p+q)^2]}\langle P_Q(V_Q)|J_{P_Q}(J_{V_Q\rho})|0\rangle + \cdots, \quad (3)$$

where dots denote the higher state contributions, and $p' = p+q$. The matrix elements in Eq. (3) are defined as follows:

$$\begin{aligned}
\langle 0|J_{T_Q\mu\nu}|T_Q(p)\rangle &= f_{T_Q}m^3_{T_Q}\epsilon_{\mu\nu}(p),\\
\langle P_Q|J_{P_Q}|0\rangle &= \frac{f_{P_Q}m^2_{P_Q}}{m_Q+m_q},\\
\langle T_Q(p)|J^{e\ell}_\alpha|P_Q(p+q)\rangle &= g\varepsilon_{\alpha\rho\lambda\tau}\epsilon^{\rho\xi}p'_\xi p^\lambda q^\tau,\\
\langle V_Q(\varepsilon,p')|J_{V_Q\rho}|0\rangle &= f_{V_Q}m_{V_Q}\varepsilon^*_\rho,\\
\langle T_Q(p)|J^{e\ell}_\alpha|V_Q(p')\rangle &= h_1\epsilon_{\alpha\beta}\varepsilon^\beta + h_2\epsilon_{\alpha\beta}p'^\beta(\varepsilon\cdot p) + h_3\varepsilon_\alpha\epsilon_{\beta\tau}p'^\beta p'^\tau + h'_4 p_\alpha\epsilon_{\beta\tau}\varepsilon^\beta p'^\tau\\
&\quad + h'_5 p'_\alpha\epsilon_{\beta\tau}\varepsilon^\beta p'^\tau + h'_6 p_\alpha\epsilon_{\beta\tau}p'^\beta p'^\tau(\varepsilon\cdot p) + h'_7 p'_\alpha\epsilon_{\beta\tau}p'^\beta p'^\tau(\varepsilon\cdot p).
\end{aligned} \quad (4)$$

In these expressions, $\epsilon_{\alpha\beta}$ and ε_α are the tensor and vector meson polarizations, f_{P_Q} and f_{V_Q} are the decay constants of the heavy pseudoscalar and vector mesons, m_{P_Q} and m_{V_Q} are their masses, m_Q and m_q are the heavy and light quark masses, and g and h_i are the form factors responsible for the $T_Q \to P_Q$ and $T_Q \to V_Q$ transitions, respectively.

Substituting these matrix elements into the physical part of the corresponding correlation functions given in Eq. (1), we get

$$\Pi_{\mu\nu\alpha}\varepsilon^{(\gamma)\alpha}(q) = \frac{1}{m^2_{T_Q}-p^2}\frac{1}{m^2_{P_Q}-p'^2}f_{T_Q}m^3_{T_Q}\epsilon_{\mu\nu}\frac{f_{P_Q}m^2_{P_Q}}{m_Q+m_q}g\varepsilon_{\alpha\rho\lambda\tau}p^\lambda q^\tau p'_\xi\epsilon^{\rho\xi}\varepsilon^{(\gamma)\alpha}(q), \quad (5)$$

$$\Pi_{\mu\nu\alpha\rho}\varepsilon^{(\gamma)\alpha}(q) = \frac{1}{m_{T_Q}^2 - p^2}\frac{1}{m_{V_Q}^2 - p'^2} f_{T_Q} m_{T_Q}^3 f_{V_Q} m_{V_Q} \epsilon_{\mu\nu}(p)\varepsilon^{(\gamma)\alpha}\varepsilon_\rho^*\{h_1\epsilon_{\alpha\beta}\varepsilon^\beta + h_2\epsilon_{\alpha\beta}p'^\beta(\varepsilon\cdot p) + h_3\varepsilon^\alpha\epsilon_{\beta\tau}p'^\beta p'^\tau + h_4' p_\alpha\epsilon_{\beta\tau}\varepsilon^\beta p'^\tau + h_5' p_\alpha'\epsilon_{\beta\tau}\varepsilon^\beta p'^\tau + h_6' p_\alpha\epsilon_{\beta\tau}p'^\beta p'^\tau(\varepsilon\cdot p) + h_7' p_\alpha'\epsilon_{\beta\tau}p'^\beta p'^\tau(\varepsilon\cdot p)\}. \quad (6)$$

By summing over the spins of the tensor and vector mesons with the help of the identities

$$\epsilon_{\mu\nu}(p)\epsilon_{\alpha\beta}^*(p) = \frac{1}{2}\mathcal{P}_{\mu\alpha}\mathcal{P}_{\nu\beta} + \frac{1}{2}\mathcal{P}_{\mu\beta}\mathcal{P}_{\nu\alpha} - \frac{1}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta}, \quad \text{where } \mathcal{P}_{\alpha\beta} = -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_{T_Q}^2},$$
$$\varepsilon_\alpha(p')\varepsilon_\beta^*(p') = \mathcal{P}'_{\alpha\beta}, \quad \text{where } \mathcal{P}'_{\alpha\beta} = -g_{\alpha\beta} + \frac{p'_\alpha p'_\beta}{m_{V_Q}^2}, \quad (7)$$

the physical parts of the correlation functions are

$$\Pi_{\mu\nu\alpha}\varepsilon^{(\gamma)\alpha}(q) = \frac{f_{T_Q}m_{T_Q}^3}{m_{T_Q}^2 - p^2}\frac{f_{P_Q}m_{P_Q}^2}{m_{P_Q}^2 - p'^2}\frac{\varepsilon^{(\gamma)\alpha}(q)}{m_Q + m_q} g\left\{\frac{1}{2}\varepsilon_{\alpha\mu\lambda\tau}p^\lambda q^\tau\left(p'_\nu - \frac{p_\nu(p\cdot p')}{m_{T_Q}^2}\right) + (\mu\leftrightarrow\nu)\right\} + \cdots, \quad (8)$$

$$\Pi_{\mu\nu\alpha\rho}\varepsilon^{(\gamma)\alpha}(q) = \frac{f_{T_Q}m_{T_Q}^3}{m_{T_Q}^2 - p^2}\frac{f_{V_Q}m_{V_Q}}{m_{P_Q}^2 - p'^2}\varepsilon^{(\gamma)\alpha}(q)\left\{\frac{1}{2}\left(\mathcal{P}_{\mu\alpha}\mathcal{P}_{\nu\beta} + \mathcal{P}_{\mu\beta}\mathcal{P}_{\nu\alpha} - \frac{2}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta}\right)[h_1\mathcal{P}'^\beta_\rho + h_2 q^\beta p^\xi\mathcal{P}'_{\rho\xi}] + \frac{1}{2}\left(\mathcal{P}_{\mu\lambda}\mathcal{P}_{\nu\tau} + \mathcal{P}_{\mu\tau}\mathcal{P}_{\nu\lambda} - \frac{2}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\lambda\tau}\right)[h_3 q^\lambda q^\tau\mathcal{P}'_{\alpha\rho} + h_4 p_\alpha q^\tau\mathcal{P}'^\lambda_\rho + h_5 p_\alpha q^\lambda q^\tau p^\xi\mathcal{P}'_{\rho\xi}] + (\mu\leftrightarrow\nu)\right\} + \cdots, \quad (9)$$

where $h_4 = h_4' + h_5'$, $h_5 = h_6' + h_7'$, and dots denote the contributions coming from the excited states and continuum.

In order to determine the form factor g for the $T_Q \to P_Q\gamma$ transition, we choose the coefficient of the structure $\varepsilon_{\alpha\mu\lambda\tau}p^\lambda q^\tau q^\nu$. The situation is much more complicated for the vector $T_Q \to V_Q\gamma$ transition, as there are numerous possible structures. In this case not all transition form factors are independent. Indeed, by using gauge invariance one can easily obtain

$$h_1 + h_4(p\cdot q) = 0,$$
$$-h_2 + h_3 - h_5(p\cdot q) = 0.$$

It follows from these relations that we have only three independent form factors. Using these relations, the matrix element $\Pi_{\mu\nu\alpha\rho}\varepsilon^{(\gamma)\alpha}$ can be written as

$$\Pi_{\mu\nu\alpha\rho}\varepsilon^{(\gamma)\alpha}(q) = \frac{f_{T_Q}m_{T_Q}^3}{m_{T_Q}^2 - p^2}\frac{f_{V_Q}m_{V_Q}}{m_{P_Q}^2 - p'^2}\left\{\frac{h_1}{2}\left[\varepsilon^{(\gamma)\alpha} - \frac{1}{p\cdot q}(\varepsilon^{(\gamma)}\cdot p)q^\alpha\right]\mathcal{P}'^\beta_\rho\left(\mathcal{P}_{\alpha\mu}\mathcal{P}_{\nu\beta} + \mathcal{P}_{\mu\beta}\mathcal{P}_{\alpha\nu} - \frac{2}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta}\right) + \frac{h_2}{2}\left[\varepsilon^{(\gamma)\alpha} - \frac{1}{p\cdot q}(\varepsilon^{(\gamma)}\cdot p)q^\alpha\right]\mathcal{P}'_{\rho\xi}q^\beta q^\xi\left(\mathcal{P}_{\alpha\mu}\mathcal{P}_{\nu\beta} + \mathcal{P}_{\mu\beta}\mathcal{P}_{\alpha\nu} - \frac{2}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta}\right) + \frac{h_3}{2}\left[\varepsilon^{(\gamma)\alpha} - \frac{1}{p\cdot q}(\varepsilon^{(\gamma)}\cdot p)q^\alpha\right]\mathcal{P}'_{\alpha\rho}q^\beta q^\tau\left(\mathcal{P}_{\mu\beta}\mathcal{P}_{\nu\tau} + \mathcal{P}_{\mu\tau}\mathcal{P}_{\nu\beta} - \frac{2}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\beta\tau}\right). \quad (10)$$

As a result, in this transition we have three independent form factors h_i, $(i = 1,..,3)$, and hence we need three independent equations to determine them. In other words, three different structures are needed. In principle, any three structures can be chosen to determine the three transition form factors. It is known that structures with the highest number of momenta make the vacuum expectation values of the higher-dimensional operators numerically less important, and the operator product expansion exhibits good convergence (see, for example, Ref. [19]). For this reason, we choose the structures $(\varepsilon^{(\gamma)}\cdot p)q_\mu g_{\nu\rho}$, $(\varepsilon^{(\gamma)}\cdot p)p_\mu q_\nu q_\rho$, and $\varepsilon_\rho^{(\gamma)}q_\mu q_\nu$ to determine the form factors.

Having obtained the representation of the correlator function from the physical side, our next job is to calculate it in the deep Euclidean domain using the OPE. For this purpose, the explicit expressions of the interpolating currents for the heavy tensor and pseudoscalar (vector) mesons should be inserted into Eq. (2), and as a result we get

$$\Pi_{\mu\nu\alpha(\rho)}\varepsilon^{(\gamma)\alpha}(q) = i\int d^4x e^{ipx}\left\langle 0\left|\left[\frac{1}{2}\bar{q}(x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu Q(x) + \mu\leftrightarrow\nu\right]\bar{Q}(0)i\gamma_5 q(0)(\bar{Q}(0)\gamma_\rho q(0))\right|0\right\rangle_F. \tag{11}$$

In order to perform the OPE, we need the expressions for the light and heavy quark propagators in the presence of the gluonic and electromagnetic background fields. In the Fock-Schwinger gauge, where the path-ordering exponents can be omitted, these propagators can be written as

$$\begin{aligned} S_q(x) = {} & \frac{i\not{x}}{2\pi^2x^4} - \frac{im_q}{4\pi^2x^2} - \frac{i}{16\pi^2x^2}\int_0^1 du\{g[\bar{u}\not{x}\sigma_{\alpha\beta} + u\sigma_{\alpha\beta}\not{x}]G^{\alpha\beta} + e_q[\bar{u}\not{x}\sigma_{\alpha\beta} + u\sigma_{\alpha\beta}\not{x}]F^{\alpha\beta}(ux)\} \\ & - \frac{im_q}{32\pi^2}\int_0^1[g_sG_{\alpha\beta}\sigma^{\alpha\beta} + e_qF_{\alpha\beta}G^{\alpha\beta}]\left(\ln\frac{-x^2\Lambda^2}{4} + 2\gamma_E\right), \end{aligned} \tag{12}$$

$$\begin{aligned} S_Q(x) = {} & \frac{m_Q^2}{4\pi^2}\left\{\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} + \frac{i\not{x}}{(\sqrt{-x^2})^2}K_2(m_Q\sqrt{-x^2})\right\} \\ & - \frac{g_s}{16\pi^2}\int_0^1 duG_{\mu\nu}(ux)\left[(\sigma^{\mu\nu}\not{x} + \not{x}\sigma^{\mu\nu})\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma^{\mu\nu}K_0(m_Q\sqrt{-x^2})\right], \end{aligned} \tag{13}$$

where $K_i(m_Q\sqrt{-x^2})$ are the modified Bessel functions, and Λ is the parameter separating the nonperturbative and perturbative domains, whose value was calculated in Ref. [20] to be $\Lambda = (0.5 \pm 0.1)$ GeV. Note that the contributions of the nonlocal operators $\bar{q}G^2q$ and $\bar{q}q\bar{q}q$ are small (see Ref. [21]), and these contributions are all neglected in Eqs. (12) and (13).

Using the explicit expressions for the heavy and light quark propagators, the correlator function(s) given in Eq. (11) can be calculated. The correlator functions contain perturbative and nonperturbative parts. The perturbative part corresponds to the case when a photon interacts with the quark propagator perturbatively. The perturbative contribution is obtained by taking into account the first two terms in the quark propagator and a photon field that interacts with the quark field perturbatively.

The nonperturbative contribution is obtained by replacing the light quark propagator by

$$S_{\alpha\beta}(x-y) \to -\frac{1}{4}(\Gamma_k)_{\alpha\beta}\bar{q}^a\Gamma_kq^b,$$

where $\Gamma_k = \{I, \gamma_\mu, \gamma_5, i\gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2}\}$ is the full set of Dirac matrices. In this case, the matrix elements of two- and three-particle nonlocal operators appear between the vacuum and the photon states. The matrix elements are parametrized in terms of the photon distribution amplitudes (DAs) as follows [17]:

$$\begin{aligned} \langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = {} & -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int_0^1 due^{i\bar{u}(q\cdot x)}\left(\chi\varphi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right) \\ & - \frac{i}{2(q\cdot x)}e_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu^{(\gamma)} - q_\mu\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right) - x_\mu - \left(\varepsilon_\nu^{(\gamma)} - q_\nu\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\right]\int_0^1 due^{i\bar{u}(q\cdot x)}h_\gamma(u), \\ \langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = {} & e_qf_{3\gamma}\left(\varepsilon_\mu^{(\gamma)} - q_\mu\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\int_0^1 due^{i\bar{u}(q\cdot x)}\psi^v(u), \\ \langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = {} & -\frac{1}{4}e_qf_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{(\gamma)\nu}q^\alpha x^\beta\int_0^1 due^{i\bar{u}(q\cdot x)}\psi^a(u), \\ \langle\gamma(q)|\bar{q}(x)g_sG_{\mu\nu}(vx)q(0)|0\rangle = {} & -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{S}(\alpha_i), \\ \langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}i\gamma_5(vx)q(0)|0\rangle = {} & -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\tilde{\mathcal{S}}(\alpha_i), \\ \langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5q(0)|0\rangle = {} & e_qf_{3\gamma}q_\alpha(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{A}(\alpha_i), \\ \langle\gamma(q)|\bar{q}(x)g_sG_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle = {} & e_qf_{3\gamma}q_\alpha(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_ie^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{V}(\alpha_i), \end{aligned}$$

$$\begin{aligned}
\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_s G_{\mu\nu}(vx)q(0)|0\rangle &= e_q\langle\bar{q}q\rangle\Bigg[\Bigg[\left(\varepsilon_\mu^{(\gamma)} - q_\mu\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\left(g_{\alpha\nu} - \frac{1}{q\cdot x}(q_\alpha x_\nu + q_\nu x_\alpha)\right)q_\beta - \left(\varepsilon_\mu^{(\gamma)} - q_\mu\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\\
&\times\left(g_{\beta\nu} - \frac{1}{q\cdot x}(q_\beta x_\nu + q_\nu x_\beta)\right)q_\alpha - \left(\varepsilon_\nu^{(\gamma)} - q_\nu\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\left(g_{\alpha\mu} - \frac{1}{q\cdot x}(q_\alpha x_\mu + q_\mu x_\alpha)\right)q_\beta\\
&+\left(\varepsilon_\nu^{(\gamma)} - q_\nu\frac{\varepsilon^{(\gamma)}\cdot x}{q.x}\right)\left(g_{\beta\mu} - \frac{1}{q\cdot x}(q_\beta x_\mu + q_\mu x_\beta)\right)q_\alpha\Bigg]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{T}_1(\alpha_i)\\
&+\Bigg[\left(\varepsilon_\alpha^{(\gamma)} - q_\alpha\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\left(g_{\mu\beta} - \frac{1}{q\cdot x}(q_\mu x_\beta + q_\beta x_\mu)\right)q_\nu\\
&-\left(\varepsilon_\alpha^{(\gamma)} - q_\alpha\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\left(g_{\nu\beta} - \frac{1}{q\cdot x}(q_\nu x_\beta + q_\beta x_\nu)\right)q_\mu\\
&-\left(\varepsilon_\beta^{(\gamma)} - q_\beta\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\left(g_{\mu\alpha} - \frac{1}{q\cdot x}(q_\mu x_\alpha + q_\alpha x_\mu)\right)q_\nu\\
&+\left(\varepsilon_\beta^{(\gamma)} - q_\beta\frac{\varepsilon^{(\gamma)}\cdot x}{q\cdot x}\right)\left(g_{\nu\alpha} - \frac{1}{q\cdot x}(q_\nu x_\alpha + q_\alpha x_\nu)\right)q_\mu\Bigg]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{T}_2(\alpha_i)\\
&+\frac{1}{q\cdot x}(q_\mu x_\nu - q_\nu x_\mu)(\varepsilon_\alpha^{(\gamma)}q_\beta - \varepsilon_\beta^{(\gamma)}q_\alpha)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{T}_3(\alpha_i)\\
&+\frac{1}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{T}_4(\alpha_i)\Bigg\},\\
\langle\gamma(q)|\bar{q}(x)e_q F_{\mu\nu}(vx)q(0)|0\rangle &= -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu^{(\gamma)}q_\nu - -\varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{S}^\gamma(\alpha_i),\\
\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}F_{\mu\nu}(vx)q(0)|0\rangle &= e_q\langle\bar{q}q\rangle\frac{1}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon_\mu^{(\gamma)}q_\nu - \varepsilon_\nu^{(\gamma)}q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)(q\cdot x)}\mathcal{T}_4^\gamma(\alpha_i),
\end{aligned}$$

where $\varphi_\gamma(u)$ is the leading twist-2 photon DA, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}$, and $\mathcal{V}$ are the twist-3 photon DAs, $h_\gamma(u)$, $\mathbb{A}$, $\mathcal{S}$, $\tilde{\mathcal{S}}$, $\mathcal{S}^\gamma$, $\mathcal{T}_i$ ($i = 1, 2, 3, 4$), and $\mathcal{T}_4^\gamma$ are the twist-4 photon DAs, χ is the magnetic susceptibility, and the measure $\mathcal{D}\alpha_i$ is given by

$$\int\mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}}\int_0^1 d\alpha_q\int_0^1 d\alpha_g\delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g).$$

After calculating the correlation functions in the deep Euclidean domain, separating the coefficients of the structures $\epsilon_{\alpha\mu\lambda\tau}p^\lambda q^\tau q^\nu$ for the $T_Q \to P_Q\gamma$ transition and the coefficients of the structures $(\varepsilon^{(\gamma)}\cdot p)q_\mu g_{\nu\rho}$, $(\varepsilon^{(\gamma)}\cdot p)p_\mu q_\nu q_\rho$, and $\varepsilon_\rho^{(\gamma)}q_\mu q_\nu$ for the $T_Q \to V_Q\gamma$ transition, and matching them with the corresponding coefficients of these structures on the phenomenological side, we get the desired sum rules for the transition form factors at $Q^2 = 0$. We then perform Borel transformations over the variables $(-p^2)$ and $-(p+q)^2$ in order to suppress the contributions of the high states and the continuum, and equate the coefficients of the aforementioned structures, from which we obtain the following sum rules for the corresponding form factors:

$$-\frac{f_{T_Q}m_{T_Q}^3 f_{P_Q}m_{P_Q}^2}{2(m_Q + m_q)}e^{-(m_{P_Q}^2/M_1^2 + m_{T_Q}^2/M_2^2)}g = \Pi^{(P)}, \tag{14}$$

$$\begin{aligned}
-\frac{f_{T_Q}f_{V_Q}m_{T_Q}^3 m_{V_Q}}{(m_{T_Q}^2 - m_{V_Q}^2)}e^{-(m_{V_Q}^2/M_1^2 + m_{T_Q}^2/M_2^2)}h_1 &= \Pi_1^{(V)},\\
\frac{f_{T_Q}f_{V_Q}m_{T_Q}}{4m_{V_Q}}e^{-(m_{V_Q}^2/M_1^2 + m_{T_Q}^2/M_2^2)}[2h_1 + (m_{T_Q}^2 + m_{V_Q}^2)h_2 - 4m_{V_Q}^2 h_3] &= \Pi_2^{(V)},\\
-f_{T_Q}f_{V_Q}m_{T_Q}^3 m_{V_Q}e^{-(m_{V_Q}^2/M_1^2 + m_{T_Q}^2/M_2^2)}h_3 &= \Pi_3^{(V)}.
\end{aligned}\tag{15}$$

The expressions for the invariant functions $\Pi^{(P)}$ and $\Pi_i^{(V)}$ are presented in the Appendix.

The continuum subtraction procedure for the LCSR is given in detail in Ref. [22]. In our calculations we set $M_1^2 = M_2^2 = 2M^2$ (in this case $u_0 \to 1/2$), and the subtraction is performed by using the formula

$$(M^2)^n e^{-(m_Q^2/M^2)} \to \frac{1}{\Gamma(n)} \int_{m_Q^2}^{s_0} ds e^{-s/M^2} (s - m_Q^2)^{(n-1)} (n \geq 1).$$

A few words about the choice of $M_1^2 = M_2^2 = 2M^2$ are in order. The sum rules for the transition between the states with different masses require two Borel mass parameters. The difference between the masses of the initial and final states is small, and hence we can take $M_1^2 = M_2^2 = 2M^2$. Obviously, taking equal Borel mass parameters would cause additional uncertainty, and we estimate that this uncertainty is about (10–15)%.

We see from Eq. (14) that the form factor g responsible for the $T_Q \to V_Q \gamma$ decay can be directly calculated. In order to determine the form factors h_i for the $T_Q \to P_Q \gamma$ decay, the coupled set of equations given in Eq. (15) should be solved where h_i are expressed in term of the combinations of the $\Pi_i^{(B)}$.

III. NUMERICAL ANALYSIS

In this section we perform a numerical analysis of the sum rules obtained in the previous section for the relevant transition form factors at the point $Q^2 = 0$.

The magnetic susceptibility χ was estimated within the LCSR in Ref. [23] to have the value $\chi(1\ \text{GeV}) = -(2.85 \pm 0.50)\ \text{GeV}^{-2}$, which we shall use in further numerical calculations. For the heavy quark masses we use their $\overline{\text{MS}}$ values, i.e., $\bar{m}_c(\bar{m}_c) = (1.28 \pm 0.003)$ GeV and $\bar{m}_b(\bar{m}_b) = (4.16 \pm 0.03)$ GeV [24,25]. The values of the other input parameters are presented in Table I. In addition to these input parameters, the LCSR also contain the main nonperturbative parameters, namely, DAs. The expressions for the photon DAs [17] that we need in our calculations are

$$\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u}[1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u})], \\
\psi^v(u) &= 3[3(2u-1)^2 - 1] + \frac{3}{64}(15w_\gamma^V - 5w_\gamma^A)[3 - 30(2u-1)^2 + 35(2u-1)^4], \\
\psi^a(u) &= [1 - (2u-1)^2][5(2u-1)^2 - 1]\frac{5}{2}\left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A\right), \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2\left[1 + w_\gamma^A\frac{1}{2}(7\alpha_g - 3)\right], \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2, \\
h_\gamma(u) &= -10(1 + 2\kappa^+) C_2^{\frac{1}{2}}(u - \bar{u}), \\
\mathbb{A}(u) &= 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})], \\
\mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)], \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)], \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}.
\end{aligned} \tag{16}$$

The constants entering the above DAs are borrowed from Refs. [17,31] and their values are given in Table II.

Besides the input parameters that are presented in Tables I and II, sum rules contain two additional parameters, namely, the continuum threshold s_0 and the Borel mass parameter M^2. The sum rules calculations demand that the physical calculations should not depend on these auxiliary parameters. Hence, the working region of M^2 is determined by requiring that the following conditions are satisfied: i) the continuum and higher-states contributions are suppressed compared to the pole contribution, ii) the perturbative contributions are dominant over the nonperturbative ones, and iii) the OPE series converges. The upper bound of M^2 is determined by the condition that the higher-states contribution should be less than 40% with respect to the contributions coming from the perturbative ones, i.e.,

TABLE I. The values of the input parameters.

$\langle\bar{q}q\rangle(1\ \mathrm{GeV})$	$(-0.246^{+0.028}_{-0.019})^3\ \mathrm{GeV}^3$ [26]
$\langle\bar{s}s\rangle(1\ \mathrm{GeV})$	$0.8\times(-0.246^{+0.028}_{-0.019})^3\ \mathrm{GeV}^3$ [27]
m_0^2	$(0.8\pm0.1)\ \mathrm{GeV}^2$ [27]
$m_s(2\ \mathrm{GeV})$	96^{+8}_{-4} MeV [6]
$f_{\mathcal{D}_2}$	0.0228 ± 0.0068 [28]
$f_{\mathcal{D}_{s_2}}$	0.023 ± 0.011 [29]
f_{B_2}	0.0050 ± 0.0005 [30]
$f_{B_{s_2}}$	0.0060 ± 0.0005 [30]
$f_{\mathcal{D}}$	(0.210 ± 0.011) GeV [30]
$f_{\mathcal{D}_s}$	(0.259 ± 0.010) GeV [30]
$f_{\mathcal{D}^*}$	(0.263 ± 0.021) GeV [30]
$f_{\mathcal{D}_s^*}$	(0.308 ± 0.021) GeV [30]
f_B	(0.192 ± 0.013) GeV [30]
f_{B_s}	(0.231 ± 0.016) GeV [30]
f_{B^*}	$(0.196^{+0.028}_{-0.027})$ GeV [30]
$f_{B_s^*}$	(0.255 ± 0.019) GeV [30]

TABLE II. The values of the constant parameters entering into the distribution amplitudes at the renormalization scale $\mu = 1$ GeV [17,31].

φ_2	κ	κ^+	ξ_1	ξ_1^+	ξ_2	ξ_2^+	$f_{3\gamma}(\mathrm{GeV}^2)$	ω_γ^V	ω_γ^A
0.0	0.2	0.0	0.4	0.0	0.3	0.0	$(-4.0\pm2.0)\times10^{-3}$	3.8 ± 1.8	-2.1 ± 1.0

$$\frac{\int_{m_Q^2}^{s_0}\rho(s)e^{-s/M^2}ds}{\int_{m_Q^2}^{\infty}\rho(s)e^{-s/M^2}ds} < 0.4.$$

The lower limit of M^2 is obtained from the condition that the OPE series—that is, the higher-twist contributions—should be smaller than the leading-twist contributions. These conditions lead to the following working region of M^2:

$$4.0\ \mathrm{GeV}^2 \le M^2 \le 10.0\ \mathrm{GeV}^2 \quad \text{for the } \mathcal{D}_2\to\mathcal{D}(\mathcal{D}^*)\gamma,$$
$$12.0\ \mathrm{GeV}^2 \le M^2 \le 20.0\ \mathrm{GeV}^2 \quad \text{for the } B_2\to B(B^*)\gamma.$$

The values of the continuum threshold s_0 are obtained from the analysis of the mass sum rules, and are given as $(s_0)_{\mathcal{D}_2} = (8.5\pm0.5)\ \mathrm{GeV}^2$, $(s_0)_{\mathcal{D}_{s_2}} = (9.5\pm0.5)\ \mathrm{GeV}^2$ [16,29], $(s_0)_{B_2} = (39\pm1)\ \mathrm{GeV}^2$, and $(s_0)_{B_{s_2}} = (41\pm1)\ \mathrm{GeV}^2$ [30].

Having determined the working regions of M^2 and s_0, we now study the dependence of g and h_i on M^2 at several fixed values of s_0. We observe that, indeed, g and h_i demonstrate good stability with respect to the variation in M^2 in its working region.

The dependences of g and h_i on s_0 at several fixed values of M^2 are also analyzed. We find that these couplings exhibit very weak dependences on the variation of s_0. Our final results for the transition form factors g and h_i are presented in Table III.

Having obtained the form factors, the decay widths of the corresponding transitions can be estimated. The width(s) for the generic $A\to B\gamma$ is given by the following expression:

$$\Gamma(A\to B\gamma) = \frac{\alpha}{4m_A^3}\frac{1}{2s_A+1}(m_A^2-m_B^2)|M|^2,$$

where s_A is the spin of the initial particle A. Using this expression for the decay width and substituting the values of g and h_i from Table III, below we list the numerical values for the decay widths of the radiative decays of the heavy-light tensor mesons under consideration:

TABLE III. The values of the form factors g and h_i at the point $Q^2=0$.

	$g(\mathrm{GeV}^{-2})$	h_1	$h_2(\mathrm{GeV}^{-2})$	$h_3(\mathrm{GeV}^{-2})$
$\mathcal{D}_2^0\to\mathcal{D}^0\gamma$	-0.34 ± 0.03			
$\mathcal{D}_2^+\to\mathcal{D}^+\gamma$	0.26 ± 0.03			
$\mathcal{D}_{s_2}\to\mathcal{D}_s\gamma$	0.13 ± 0.03			
$\mathcal{D}_2^0\to\mathcal{D}^{*0}\gamma$		-0.025 ± 0.001	1.21 ± 0.19	0.87 ± 0.13
$\mathcal{D}_2^+\to\mathcal{D}^{*+}\gamma$		-0.15 ± 0.02	-1.40 ± 0.30	-0.90 ± 0.20
$\mathcal{D}_{s_2}\to\mathcal{D}_s^*\gamma$		-0.13 ± 0.02	-1.10 ± 0.20	-0.70 ± 0.08
$B_2^-\to B^-\gamma$	-0.35 ± 0.03			
$B_2^0\to B^0\gamma$	0.14 ± 0.02			
$B_{s_2}\to B_s\gamma$	0.080 ± 0.004			
$B_2^-\to B^{*-}\gamma$		0.95 ± 0.10	4.60 ± 0.60	2.80 ± 0.40
$B_2^0\to B^{*0}\gamma$		0.17 ± 0.03	-1.30 ± 0.15	-1.10 ± 0.11
$B_{s_2}\to B_s^*\gamma$		0.12 ± 0.02	-1.00 ± 0.10	-0.80 ± 0.04

$$\Gamma(\mathcal{D}_2^0 \to \mathcal{D}^0\gamma) = (3.19 \pm 0.54)\ \text{keV},$$
$$\Gamma(\mathcal{D}_2^+ \to \mathcal{D}^+\gamma) = (1.86 \pm 0.46)\ \text{keV},$$
$$\Gamma(\mathcal{D}_{s_2} \to \mathcal{D}_s\gamma) = (2.32 \pm 0.46)\ \text{keV},$$
$$\Gamma(\mathcal{D}_2^0 \to \mathcal{D}^{*0}\gamma) = (5.54 \pm 1.69)\ \text{keV},$$
$$\Gamma(\mathcal{D}_2^+ \to \mathcal{D}^{*+}\gamma) = (15.40 \pm 5.10)\ \text{keV},$$
$$\Gamma(\mathcal{D}_{s_2} \to \mathcal{D}_s^*\gamma)) = (10.20 \pm 3.50)\ \text{keV},$$
$$\Gamma(B_2^- \to B^-\gamma) = (1.61 \pm 0.29)\ \text{keV},$$
$$\Gamma(B_2^0 \to B^0\gamma) = (0.26 \pm 0.08)\ \text{keV},$$
$$\Gamma(B_{s_2} \to B_s\gamma) = (0.088 \pm 0.008)\ \text{keV},$$
$$\Gamma(B_2^- \to B^{*-}\gamma) = (50.70 \pm 11.22)\ \text{keV},$$
$$\Gamma(B_2^0 \to B^{*0}\gamma) = (0.60 \pm 0.17)\ \text{keV},$$
$$\Gamma(B_{s_2} \to B_s^*\gamma)) = (0.36 \pm 0.08)\ \text{keV}.$$

Using the experimental values of the decay widths of the tensor mesons under consideration—which are $\Gamma(\mathcal{D}_2) = (46.7 \pm 1.2)$ MeV, $\Gamma(\mathcal{D}_{s_2}) = (16.9 \pm 0.8)$ MeV, $\Gamma(B_2) = (20 \pm 5)$ MeV, and $\Gamma(B_{s_2}) = (1.47 \pm 0.33)$ MeV—we observe that the branching ratios of the considered radiative decays are of the order of 10^{-3}–10^{-5}. Referring to these results, we can comment that the radiative decays of the heavy-light tensor mesons are quite accessible at LHCb.

IV. CONCLUSION

In the present work the radiative decays of the tensor mesons to heavy-light pseudoscalar and vector mesons were studied within the LCSR. For this purpose, we first calculated the transition form factors entering into the matrix element of the relevant decays. Using the values of the relevant form factors at the point $Q^2 = 0$, we then estimated the corresponding decay widths. We observed that the branching ratios of the considered decays are larger than 10^{-5}, and therefore they could potentially be measured at LHCb in the near future.

APPENDIX: EXPRESSIONS FOR THE INVARIANT FUNCTIONS $\Pi^{(P)}$ AND $\Pi_i^{(V)}$

In this appendix we present the expressions for the invariant functions $\Pi^{(P)}$ and $\Pi_i^{(V)}$ in Eqs. (14) and (15).

1. $T_Q \to P_Q\gamma$ transition

$\epsilon_{\alpha\mu\lambda\tau} p^\lambda q^\tau q^\nu$ structure:

$$\begin{aligned}
\Pi^{(P)} &= \frac{e^{-m_Q^2/M^2}}{6912M^8} e_q \langle g_s^2 G^2\rangle m_Q^4 \langle\bar{q}q\rangle(1+2u_0)\mathbb{A}(u_0) \\
&+ \frac{e^{-m_Q^2/M^2}}{6912M^6\pi^2} m_Q^2 \{48 e_Q m_0^2 m_Q m_q \pi^2 \langle\bar{q}q\rangle + e_q \langle g_s^2 G^2\rangle [3m_q e^{m_Q^2/M^2}(\mathcal{J}_0 + m_Q^2 \mathcal{J}_1) \\
&+ 2\pi^2 \langle\bar{q}q\rangle(1-2u_0)\mathbb{A}(u_0) + 2\pi^2 f_{3\gamma} m_Q (1+2u_0)\psi^a(u_0)]\} \\
&+ \frac{e^{-m_Q^2/M^2}}{6912M^4\pi^2} m_Q \{144 e_Q m_0^2 m_Q \pi^2 \langle\bar{q}q\rangle - 3e_q \langle g_s^2 G^2\rangle m_Q m_q [2\gamma_E - e^{m_Q^2/M^2}(2\mathcal{I}_1 + \mathcal{J}_1 + 2m_Q^2(\mathcal{I}_2 + \mathcal{J}_2))] \\
&- 2e_q \langle g_s^2 G^2\rangle \pi^2 [2m_Q \langle\bar{q}q\rangle(1+2u_0)\chi\varphi_\gamma(u_0) - f_{3\gamma}(1-6u_0)\psi^a(u_0)]\} \\
&- \frac{e^{-m_Q^2/M^2}}{2304M^2\pi^2} \{e_q \langle g_s^2 G^2\rangle [2m_Q + m_q(1+2\gamma_E)] \\
&- 48 e_Q (m_0^2 - 2m_Q m_q)\pi^2 \langle\bar{q}q\rangle + e_q m_Q^2 [24\pi^2 \langle\bar{q}q\rangle(1+2u_0)\mathbb{A}(u_0) \\
&- \langle g_s^2 G^2\rangle m_q e^{m_Q^2/M^2}(2\mathcal{I}_2 + \mathcal{J}_2 + m_Q^2(3\mathcal{I}_3 + 2\mathcal{J}_3))]\} \\
&- \frac{e^{-m_Q^2/M^2}}{1728 m_Q M^2} e_q \langle g_s^2 G^2\rangle [2m_Q \langle\bar{q}q\rangle \chi\varphi_\gamma(u_0) + f_{3\gamma}\psi^a(u_0)] \\
&- \frac{M^2}{32\pi^2} m_Q^3 \{2e_Q[\mathcal{I}_2 - m_Q(m_Q + m_q)\mathcal{I}_3] - e_q m_Q [2m_Q \mathcal{I}_3 - 2m_q \mathcal{J}_3 - m_Q^2 m_q (\mathcal{I}_4 + 3\mathcal{J}_4)]\} \\
&+ \frac{e^{-m_Q^2/M^2}}{24} M^2 e_q \langle\bar{q}q\rangle(1+2u_0)\chi\varphi_\gamma(u_0).
\end{aligned}$$

2. $T_Q \to V_Q\gamma$ transition

$(\varepsilon^{(\gamma)}\cdot p)q_\mu g_{\nu\rho}$ *structure*:

$$\begin{aligned}\Pi_1^{(V)} =& -\frac{M^4}{16\pi^2}(e_Q-e_q)m_Q^6(3\mathcal{I}_4-4m_Q^2\mathcal{I}_5)+\frac{M^2}{16\pi^2}m_Q^4\{2e_q m_Q m_q\mathcal{I}_3+e_Q[\mathcal{I}_2-m_Q(m_Q+2m_q)\mathcal{I}_3]\}\\ &-\frac{M^2e^{-m_Q^2/M^2}}{24}\{e_q f_{3\gamma}[4\tilde{j}_1(\psi^v)+\psi^a(u_0)]\}-\frac{e^{-m_Q^2/M^2}}{432M^6}m_Q^3\langle\bar{q}q\rangle[6e_Q m_0^2 m_Q m_q-e_q\langle g_s^2G^2\rangle\tilde{j}_2(h_\gamma)]\\ &-\frac{e^{-m_Q^2/M^2}}{1728M^4}m_Q\{24e_Q m_0^2 m_Q(3m_Q-m_q)\langle\bar{q}q\rangle+e_q\langle g_s^2G^2\rangle[4\langle\bar{q}q\rangle\tilde{j}_2(h_\gamma)-f_{3\gamma}m_Q(4\tilde{j}_1(\psi^v)+\psi^a(u_0))]\}\\ &-\frac{e^{-m_Q^2/M^2}}{1728\pi^2 m_Q M^2}\{3e_q\langle g_s^2G^2\rangle m_Q^2 m_q-72e_Q m_Q[2m_Q^2 m_q-m_0^2(m_Q-m_q)]\pi^2\langle\bar{q}q\rangle\\ &+e_q\langle g_s^2G^2\rangle\pi^2[4\langle\bar{q}q\rangle\tilde{j}_2(h_\gamma)-f_{3\gamma}m_Q(4\tilde{j}_1(\psi^v)+\psi^a(u_0))]\}\\ &+\frac{e^{-m_Q^2/M^2}}{1152m_Q\pi^2}[96e_Q m_Q(2m_Q-m_q)\pi^2\langle\bar{q}q\rangle+e_q\langle g_s^2G^2\rangle(m_Q+2m_q-4m_Q^5 e^{m_Q^2/M^2}\mathcal{I}_3)]-\frac{e^{-m_Q^2/M^2}}{6}[e_q m_Q\langle\bar{q}q\rangle\tilde{j}_2(h_\gamma)].\end{aligned}$$

$(\varepsilon^{(\gamma)}\cdot p)p_\mu q_\nu q_\rho$ *structure*:

$$\begin{aligned}\Pi_2^{(V)} =& \frac{M^2}{16\pi^2}m_Q^4[(2e_Q-e_q)\mathcal{I}_3)-m_Q^2(6e_Q-4e_q)\mathcal{I}_4+4(e_Q-e_q)m_Q^4\mathcal{I}_5]\\ &-\frac{e^{-m_Q^2/M^2}}{864M^8}e_q\langle g_s^2G^2\rangle m_Q^3\langle\bar{q}q\rangle[(1+2u_0)\tilde{j}_2(h_\gamma)+2\tilde{j}_3(h_\gamma)]\\ &+\frac{e^{-m_Q^2/M^2}}{3456M^6}m_Q\{48e_Q m_0^2 m_Q m_q\langle\bar{q}q\rangle+e_q\langle g_s^2G^2\rangle[4f_{3\gamma}m_Q(3+2u_0)\tilde{j}_1(\psi^v)-4(1-6u_0)\langle\bar{q}q\rangle\tilde{j}_2(h_\gamma)+24\langle\bar{q}q\rangle\tilde{j}_3(h_\gamma)\\ &+f_{3\gamma}m_Q(8\tilde{j}_2(\psi^v)-(1+2u_0)\psi^a(u_0))]\}\\ &+\frac{e^{-m_Q^2/M^2}}{1728m_Q M^4}\{72e_Q m_0^2 m_Q m_q\langle\bar{q}q\rangle+e_q\langle g_s^2G^2\rangle[12f_{3\gamma}m_Q\tilde{j}_1(\psi^v)+4\langle\bar{q}q\rangle\tilde{j}_2(h_\gamma)-f_{3\gamma}m_Q\psi^a(u_0)]\}\\ &-\frac{e^{-m_Q^2/M^2}}{1152M^2\pi^2}\{96e_Q m_q\pi^2\langle\bar{q}q\rangle-e_q\langle g_s^2G^2\rangle[1+m_Q^2 e^{m_Q^2/M^2}(\mathcal{I}_2-4m_Q^2\mathcal{I}_3)]-96e_q m_Q\pi^2\langle\bar{q}q\rangle[(1+2u_0)\tilde{j}_2(h_\gamma)+2\tilde{j}_3(h_\gamma)]\}\\ &-\frac{e^{-m_Q^2/M^2}}{48}e_q f_{3\gamma}\{4(3+2u_0)\tilde{j}_1(\psi^v)+8\tilde{j}_2(\psi^v)-(1+2u_0)\psi^a(u_0)\}.\end{aligned}$$

$\varepsilon_\rho^{(\gamma)}q_\mu q_\nu$ *structure*:

$$\begin{aligned}\Pi_3^{(V)} =& -\frac{M^4e^{-m_Q^2/M^2}}{16\pi^2}\{2e_q[1-m_Q^6 e^{m_Q^2/M^2}(\mathcal{I}_4-m_Q^2\mathcal{I}_5)]+e_Q m_Q^2 e^{m_Q^2/M^2}[\mathcal{I}_2-2m_Q^4(\mathcal{I}_4+m_Q^2\mathcal{I}_5)]\}\\ &-\frac{e^{-m_Q^2/M^2}}{3456M^8}[e_q\langle g_s^2G^2\rangle m_Q^5(1+2u_0)\langle\bar{q}q\rangle\mathbb{A}(u_0)]-\frac{1}{1152M^6\pi^2}e_q gGgGm_Q^3 m_q[\mathcal{J}_0+m_Q^2\mathcal{J}_1]\\ &-\frac{e^{-m_Q^2/M^2}}{1728M^6}m_Q^3\{24e_Q m_0^2 m_Q m_q\langle\bar{q}q\rangle-e_q\langle g_s^2G^2\rangle\{6\langle\bar{q}q\rangle u_0\mathbb{A}(u_0)-(1+2u_0)\langle\bar{q}q\rangle\tilde{j}_1(h_\gamma)\\ &+4\langle\bar{q}q\rangle[u_0\tilde{j}_2(h_\gamma)+\tilde{j}_3(h_\gamma)]-f_{3\gamma}m_Q(1+2u_0)\psi^a(u_0)]\}\end{aligned}$$

$$+\frac{1}{1152M^4\pi^2}e_q\langle g_s^2G^2\rangle m_Q m_q[3\mathcal{J}_0-m_Q^2(3\mathcal{I}_1-\mathcal{J}_1)-3m_Q^4(\mathcal{I}_2+\mathcal{J}_2)]$$
$$+\frac{e^{-m_Q^2/M^2}}{6912M^4\pi^2}m_Q\{12m_Q[e_q\gamma_E\langle g_s^2G^2\rangle m_Q m_q-8e_Q m_0^2(3m_Q-2m_q)\pi^2\langle\bar{q}q\rangle]$$
$$+12e_q\langle g_s^2G^2\rangle\pi^2\langle\bar{q}q\rangle(1-2u_0)\mathbb{A}(u_0)-4e_q\langle g_s^2G^2\rangle\pi^2\langle\bar{q}q\rangle(1-6u_0)\tilde{j}_1(h_\gamma)$$
$$+e_q\langle g_s^2G^2\rangle\pi^2[8f_{3\gamma}m_Q(1+2u_0)\tilde{j}_1(\psi^v)+16\langle\bar{q}q\rangle(2-3u_0)\tilde{j}_2(h_\gamma)-48\langle\bar{q}q\rangle\tilde{j}_3(h_\gamma)]$$
$$+4e_q\langle g_s^2G^2\rangle m_Q\pi^2[2m_Q\langle\bar{q}q\rangle(1+2u_0)\chi\varphi_\gamma(u_0)f_{3\gamma}(4\tilde{j}_2(\psi^v)-(1-5u_0)\psi^a(u_0)$$
$$+\psi^v(u_0)+2u_0\psi^v(u_0))]-f_{3\gamma}e_q\langle g_s^2G^2\rangle m_Q\pi^2(1+2u_0)\psi^{a\prime}(u_0)\}$$
$$+\frac{1}{1152M^2\pi^2}e_q\langle g_s^2G^2\rangle m_Q m_q\{6\mathcal{I}_1+3\mathcal{J}_1-m_Q^2[\mathcal{I}_2-2\mathcal{J}_2+m_Q^2(9\mathcal{I}_3+6\mathcal{J}_3)]\}$$
$$+\frac{e^{-m_Q^2/M^2}}{3456m_Q M^2\pi^2}\{3m_Q[e_q\langle g_s^2G^2\rangle m_Q(2m_Q+m_q-2\gamma_E m_q)+8e_Q(5m_0^2+12m_Q^2)m_q\pi^2\langle\bar{q}q\rangle]$$
$$+e_q\pi^2[72m_Q^4\langle\bar{q}q\rangle(1+2u_0)\mathbb{A}(u_0)+\langle g_s^2G^2\rangle(4\langle\bar{q}q\rangle\tilde{j}_1(h_\gamma)-8\langle\bar{q}q\rangle\tilde{j}_2(h_\gamma)$$
$$+m_Q\{4m_Q\langle\bar{q}q\rangle(1-6u_0)\chi\varphi_\gamma(u_0)+f_{3\gamma}[8\tilde{j}_1(\psi^v)+2\psi^a(u_0)+4\psi^v(u_0)-\psi^{a\prime}(u_0)]\})]\}$$
$$-\frac{e^{-m_Q^2/M^2}}{576m_Q\pi^2}e_q\gamma_E m_q(\langle g_s^2G^2\rangle+72m_Q^2M^2)$$
$$-\frac{1}{16\pi^2}m_Q^3M^2\{e_Q(m_Q+2m_q)\mathcal{I}_2-e_Q m_Q^3\mathcal{I}_3-e_q m_q[\mathcal{J}_2+m_Q^2(\mathcal{I}_3+2\mathcal{J}_3)]\}$$
$$-\frac{e^{-m_Q^2/M^2}}{96}e_q M^2\{8(f_{3\gamma}+2f_{3\gamma}u_0)\tilde{j}_1(\psi^v)+8m_Q\langle\bar{q}q\rangle(1+2u_0)\chi\varphi_\gamma(u_0)$$
$$+4f_{3\gamma}[4\tilde{j}_2(\psi^v)-(2+u_0)\psi^a(u_0)+\psi^v(u_0)+2u_0\psi^v(u_0)]-f_{3\gamma}(1+2u_0)\psi^{a\prime}(u_0)\}$$
$$+\frac{1}{1152\pi^2}e_q m_Q\{72m_Q^2m_q(\mathcal{J}_1+m_Q^2\mathcal{J}_2)-\langle g_s^2G^2\rangle[2m_Q^3\mathcal{I}_3-m_q(6\mathcal{I}_2+3\mathcal{J}_2+m_Q^2[\mathcal{I}_3+2\mathcal{J}_3-m_Q^2(11\mathcal{I}_4+6\mathcal{J}_4)])]\}$$
$$+\frac{e^{-m_Q^2/M^2}}{1728m_Q\pi^2}\{3e_q\langle g_s^2G^2\rangle(m_Q-m_q)+144e_Q m_Q(2m_Q-m_q)\pi^2\langle\bar{q}q\rangle$$
$$+4e_q\pi^2[18m_Q^2\langle\bar{q}q\rangle(\mathbb{A}(u_0)+(1+2u_0)\tilde{j}_1(h_\gamma)-4[u_0\tilde{j}_2(h_\gamma)+\tilde{j}_3(h_\gamma)])$$
$$-\langle g_s^2G^2\rangle\langle\bar{q}q\rangle\chi\varphi_\gamma(u_0)+18f_{3\gamma}m_Q^3(1+2u_0)\psi^a(u_0)]\}, \tag{A1}$$

where s_0 is the continuum threshold, $M_1^2=M_2^2=2M^2$, and

$$u_0=\frac{M_1^2}{M_1^2+M_2^2}=\frac{1}{2},\qquad M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2}.$$

The integral functions $\mathcal{I}_n(n=1,\ldots,5)$, $\mathcal{J}_n(n=0,\ldots,4)$, and $\tilde{j}_n(f(u_0))(n=1,2,3)$ are defined as

$$\mathcal{I}_n=\int_{m_b^2}^{s_0}ds\frac{e^{-s/M^2}}{s^n},$$
$$\mathcal{J}_n=\int_{m_b^2}^{s_0}ds\frac{\ell n[M^2(s-m_Q^2)/(\Lambda^2 s)]}{s^n}e^{-s/M^2},$$
$$\tilde{j}_n(f)=\int_{u_0}^{1}du(u-u_0)^{(n-1)}f(u),\quad\text{and},\qquad \psi'^a(u_0)=\frac{d\psi^a(u)}{du}\bigg|_{u=u_0}.$$

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