

Vertices of the heavy spin-3/2 sextet baryons with light vector mesons in QCD

T. M. Aliev^{1 *†}, K. Azizi^{2 ‡}, M. Savcı^{1 §}

¹ Physics Department, Middle East Technical University, 06531 Ankara, Turkey

² Physics Division, Faculty of Arts and Sciences, Doğuş University,
Acıbadem-Kadıköy, 34722 Istanbul, Turkey

Abstract

All transitions among the heavy spin-3/2 sextet baryons with participation of the light vector mesons ($B_Q^* B_Q V$) are investigated in the framework of the light cone sum rules. These vertices are described by four coupling constants. The corresponding sum rules are derived for each coupling constant. It is shown that the correlation functions of different transitions can be described by only one invariant function.

*e-mail: taliev@metu.edu.tr

†permanent address: Institute of Physics, Baku, Azerbaijan

‡e-mail: kazizi@dogus.edu.tr

§e-mail: savci@metu.edu.tr

1 Introduction

During last few years, there have been significant progress in the experimental studies of heavy baryons. Especially, exciting discoveries have been made in the heavy baryon spectroscopy [1]. The $(\frac{1}{2})^+$ antitriplet states $(\Lambda_c^+, \Xi_c^+, \Xi_c^0)$ as well as the $(\frac{1}{2})^+$ and $(\frac{3}{2})^+$ sextet states $(\Omega_c^0, \Sigma_c, \Sigma_c')$ and $(\Omega_c^*, \Sigma_c^*, \Sigma_c^{*'})$ have been discovered, while for the s-wave bottom baryons only the $\Lambda_b, \Sigma_b, \Sigma_b^*$ and Ω_b have been established [2].

The experimental achievements simulate theoretical investigations in this area. These baryons are very useful in understanding the dynamics of the nonperturbative QCD. The heavy baryons provide rich laboratory for studying the predictions of the heavy quark effective theory (HQET). The heavy baryons are also very informative in investigating the polarization effects. This is due to the fact that part of the polarization of heavy quark is transferred to the heavy baryon.

Following the experimental discoveries of heavy hadrons, the interest of researchers is focused on the study of their weak, electromagnetic and strong decays. The semileptonic weak decays of heavy baryons can provide us invaluable knowledge about the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements and information about their internal structures. In studying the strong decays of heavy baryons, one needs to know the corresponding strong coupling constants. As is well known, formation of hadrons take place at low energy scale belongs to the nonperturbative region of QCD. Therefore, we can not calculate the strong coupling constants of baryons starting from the fundamental Lagrangian. For this reason, in calculating the strong coupling constants, some nonperturbative approaches are needed. The QCD sum rules method [3] is one of the most predictive one among a bunch of nonperturbative approaches. In the present work, we calculate the strong coupling constants of the transitions among sextet spin-3/2 baryons within the QCD light cone sum rules (LCSR) [4]. The main difference of the light cone and traditional versions of the sum rules is that, in the LCSR the operator product expansion (OPE) is performed over the twists rather than dimension of the operators, as is the case in the latter one. It should be noted that the coupling constants of heavy spin-1/2 baryons with light vector and pseudoscalar mesons are calculated within the LCSR method in [5, 6], the coupling constants of spin-3/2 to spin-1/2 heavy baryon–light mesons are calculated in [7, 8], and the coupling constants of spin-3/2 sextet heavy baryons with pseudoscalar mesons are calculated in [9].

The present study is organized in the following way. In section 2, first we obtain the structure independent relations among the correlation functions. Then, we derive sum rules for the coupling constants of the spin-3/2 sextet baryons with light vector mesons in terms of the invariant functions. In section 3, the numerical analysis of the sum rules obtained in section 2 is carried out.

2 Sum rules for the $B_Q^* B_Q^* V$ couplings in LCSR

This section is devoted to the calculation of the strong coupling constants among the heavy spin-3/2 sextet baryons with light vector mesons. For determination of these coupling

constants within LCSR, the following correlation function is introduced:

$$\Pi_{\mu\nu} = i \int d^4x e^{ipx} \left\langle V(q) \left| \mathcal{T} \left\{ \eta_\mu(x) \bar{\eta}_\nu(0) \right\} \right| 0 \right\rangle , \quad (1)$$

where $V(q)$ is the light vector meson with four momentum q , and η_μ and η_ν are the interpolating currents for the sextet heavy spin-3/2 baryons. The explicit form of interpolating current for the heavy spin-3/2 baryons is given as [10]

$$\eta_\mu = A \epsilon^{abc} \left\{ (q_1^{aT} C \gamma_\mu q_2^b) Q^c + (q_2^{aT} C \gamma_\mu Q^b) q_1^c + (Q^{aT} C \gamma_\mu q_1^b) q_2^c \right\} . \quad (2)$$

where the normalization constant A and the light quark content of all members are given in Table 1.

	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*- (0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*- (0)}$	$\Omega_{b(c)}^{*- (0)}$
q_1	u	u	d	u	d	s
q_2	u	d	d	s	s	s
A	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$

Table 1: The normalization constant and the light quark content of the heavy spin-3/2 sextet baryons.

According to the QCD sum rules philosophy, this correlation function is calculated in terms of hadrons from one side (phenomenological) and in terms of quark-gluon degrees of freedom from the other side (theoretical). In order to obtain the phenomenological side of the sum rules, the correlation function is saturated by a complete set of hadronic states that carries the same quantum numbers as the interpolating current η_μ . Isolating the ground state contribution one can easily obtain

$$\Pi_{\mu\nu} = \frac{\langle 0 | \eta_\mu(0) | B_Q^*(p_2) \rangle \langle B_Q^*(p_2) V(q) | B_Q^*(p_1) \rangle \langle B_Q^*(p_1) | \bar{\eta}_\nu(0) | 0 \rangle}{(p_2^2 - m_2^2)(p_1^2 - m_1^2)} + \dots , \quad (3)$$

In all following discussion we put $p_2 = p$ and $p_1 = p + q$. In order to obtain the expression of the correlation function from phenomenological side, we need to know the matrix elements entering into Eq. (3). These matrix elements are defined as

$$\langle 0 | \eta_\mu | B_Q^*(p + q) \rangle = \lambda_{B_Q^*} u_\mu(p) , \quad (4)$$

where $\lambda_{B_Q^*}$ is the residue of B_Q^* heavy baryon and $u_\mu(p)$ is the Rarita-Schwinger spinor. The matrix element $\langle B_Q^*(p) V(q) | B_Q^*(p + q) \rangle$ is determined with the help of four form factors g_1, g_2, g_3 and g_4 in the following way:

$$\begin{aligned} \langle B_Q^*(p) V(q) | B_Q^*(p + q) \rangle = & \bar{u}_\alpha(p) \left\{ g^{\alpha\beta} \left[\not{q} g_1 + 2p\epsilon \frac{g_2}{m_1 + m_2} \right] \right. \\ & \left. + \frac{q^\alpha q^\beta}{(m_1 + m_2)^2} \left[\not{q} g_3 + 2p\epsilon \frac{g_4}{m_1 + m_2} \right] \right\} u_\beta(p + q) , \quad (5) \end{aligned}$$

where ε_μ is the four polarization of the light vector meson.

Substituting Eqs. (4) and (5) into Eq. (3), and performing summation over spins of spin-3/2 baryons with the help of the formula

$$\sum_s u_\mu(p, s) \bar{u}_\nu(p, s) = (\not{p} + m) \left(g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{2p_\mu p_\nu}{3m^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3m} \right), \quad (6)$$

we can obtain the physical side of the correlation function.

At this point we face two principal drawbacks: a) the structures that appear are not all independent, b) the interpolating current for the heavy spin-3/2 baryon couples to the spin-1/2 states, i.e.,

$$\langle 0 | \eta_\mu | 1/2(p) \rangle = A \left(\gamma_\mu - \frac{4}{m} p_\mu \right) u(p). \quad (7)$$

It follows from this equation that the structures that proportional to γ_μ on the left and γ_ν on the right, as well as the terms that are proportional to p_μ and $(p+q)_\nu$ contain contributions coming from unwanted spin-1/2 states, and hence they should be removed. Both of the above-mentioned problems can be solved by ordering the Dirac matrices. In the present work, we choose the ordering of the Dirac matrices in the form $\gamma_\mu \not{q} \not{p} \gamma_\nu$.

Taking into account this procedure, we get the following expression for the physical side of the correlation function:

$$\begin{aligned} \Pi_{\mu\nu} = & \frac{\lambda_{B_{Q_1}^*} \lambda_{B_{Q_2}^*}}{[(p+q)^2 - m_1^2](p^2 - m_2^2)} \left\{ 2(\varepsilon p) g_{\mu\nu} \not{q} \left[g_1 + g_2 \frac{m_2}{m_1 + m_2} \right] \right. \\ & - 2(\varepsilon p) g_{\mu\nu} \not{q} \not{p} \frac{g_2}{m_1 + m_2} + q_\mu q_\nu \not{q} \not{p} \frac{g_3}{(m_1 + m_2)^2} - 2(\varepsilon p) q_\mu q_\nu \not{q} \not{p} \frac{g_4}{(m_1 + m_2)^3} \\ & + \text{other structures beginning with } \gamma_\mu \text{ and ending with } \gamma_\nu, \text{ or} \\ & \left. \text{terms that are proportional to } p_\mu \text{ or } (p+q)_\nu \right\}. \quad (8) \end{aligned}$$

In order to obtain sum rules for the combination $g_1 + g_2 \frac{m_2}{m_1 + m_2}$ and form factors g_2 , g_3 and g_4 , we choose the coefficients of the structures, $(\varepsilon p) g_{\mu\nu} \not{q}$, $(\varepsilon p) g_{\mu\nu} \not{q} \not{p}$, $q_\mu q_\nu \not{q} \not{p}$ and $(\varepsilon p) q_\mu q_\nu \not{q} \not{p}$, respectively.

As has already been noted, in order to obtain the sum rules for the coupling constants, the correlation function from the QCD side is needed. Before calculating the theoretical part of the correlation function we obtain the relations among invariant functions, which are quite efficient in calculation of the coupling constants g_i . For this purpose, we shall follow the works [5–9], where essential points of the relevant approach are presented. The main advantage of this approach is that it involves $SU(3)_f$ symmetry violation effects, as well as the fact that the obtained relations among invariant functions are all structure independent.

As an example, we consider the transition $\Sigma_b^{*0} \rightarrow \Sigma_b^0 \rho^0$. The invariant function corresponding to any structure can formally be written in the following form:

$$\Pi^{\Sigma_b^{*0} \rightarrow \Sigma_b^0 \rho^0} = g_{\rho^0 \bar{u}u} \Pi_1(u, d, b) + g_{\rho^0 \bar{d}d} \Pi_1'(u, d, b) + g_{\rho^0 \bar{b}b} \Pi_2(u, d, b). \quad (9)$$

It follows from Eq. (2) that the interpolating currents for the heavy spin-3/2 baryons are symmetric with respect to the light quark interchange, and therefore $\Pi_1'(u, d, b) = \Pi_1(d, u, b)$. Using the form of the interpolating current of ρ^0 meson, one can easily see that

$$g_{\rho^0 \bar{u}u} = -g_{\rho^0 \bar{d}d} = \frac{1}{\sqrt{2}}, g_{\rho^0 \bar{b}b} = 0, \quad (10)$$

as a result of which we get,

$$\Pi^{\Sigma_b^{*0} \rightarrow \Sigma_b^{*0} \rho^0} = \frac{1}{\sqrt{2}} \left[\Pi_1(u, d, b) - \Pi_1(d, u, b) \right]. \quad (11)$$

Obviously, $\Pi^{\Sigma_b^{*0} \rightarrow \Sigma_b^{*0} \rho^0} = 0$ in the $SU(2)_f$ limit.

Taking into account the quark content of ω and ϕ mesons we observe that

$$g_{\omega \bar{u}u} = -g_{\omega \bar{d}d} = \frac{1}{\sqrt{2}}, g_{\phi \bar{s}s} = 1. \quad (12)$$

The functions Π_1 , Π_1' and Π_2 in Eq. (9) physically correspond to radiation of ρ meson from u , d and b quarks, respectively.

Following the works [5–9], one can obtain relations among the invariant functions involving ρ , ω , K^* and ϕ mesons. These relations are given in the Appendix. It follows from the relations among the invariant functions presented in the main body of the text and in the appendix that all these transitions can be described in terms of a single universal function.

Now we turn back to our main problem, i.e., constructing the sum rules for the strong coupling constants describing the transitions among heavy spin-3/2 sextet baryons. Using the expressions of distribution amplitudes (DA's) for the light vector mesons, as well as the quark operators, the theoretical part of the sum rules can, in principle, be obtained in the standard way.

The theoretical part of the correlation function can be calculated in deep Euclidian region, $-p^2 \rightarrow \infty$, $-(p+q)^2 \rightarrow \infty$, using the OPE. The main nonperturbative inputs of the LCSR method is the DA's. In the problem under consideration, we need to know the DA's of the light vector mesons, which are given in [11–13]. In order to calculate the theoretical part of the correlation function, the expressions of the heavy and light quark propagators are needed, whose expressions are given in [14] and [15], respectively.

The final step for obtaining sum rules of the strong coupling constants, is equating the coefficients of the structures $(\varepsilon p)g_{\mu\nu}\not{q}$, $(\varepsilon p)g_{\mu\nu}\not{p}$, $q_\mu q_\nu \not{q}\not{p}$ and $(\varepsilon p)q_\mu q_\nu \not{q}\not{p}$ from both representations of the correlation function, and then applying double Borel transformation on the variables $-p^2$ and $-(p+q)^2$ on both sides, which suppresses the contributions of the higher states and continuum. As a result of these operations, we obtain the following sum rules for the strong coupling constants g_i :

$$g_1 + g_2 \frac{m_2}{m_1 + m_2} = \frac{1}{2\lambda_{B_{Q_1}^*} \lambda_{B_{Q_2}^*}} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2 + M_2^2}} \Pi_1^{(1)},$$

$$\begin{aligned}
g_2 &= -\frac{m_1 + m_2}{2\lambda_{B_{Q_1}^*} \lambda_{B_{Q_2}^*}} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2 + M_2^2}} \Pi_1^{(2)} , \\
g_3 &= -\frac{(m_1 + m_2)^2}{\lambda_{B_{Q_1}^*} \lambda_{B_{Q_2}^*}} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2 + M_2^2}} \Pi_1^{(3)} , \\
g_4 &= -\frac{(m_1 + m_2)^3}{2\lambda_{B_{Q_1}^*} \lambda_{B_{Q_2}^*}} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2 + M_2^2}} \Pi_1^{(4)} ,
\end{aligned} \tag{13}$$

where M_1^2 and M_2^2 are the Borel parameters in initial and final channels, respectively. It should be remembered that the relations among the invariant functions are structure independent, but their explicit expressions are structure dependent. For this reason, we provide the invariant functions with one extra upper index. The indices 1, 2, 3 and 4 correspond to the choice of the structures $(\varepsilon p)g_{\mu\nu}\not{q}$, $(\varepsilon p)g_{\mu\nu}\not{q}\not{p}$, $q_\mu q_\nu \not{q}\not{p}$ and $(\varepsilon p)q_\mu q_\nu \not{q}\not{p}$, respectively. In further numerical analysis, we set $M_1^2 = M_2^2 = M^2$ since the masses of the initial and final baryons are very close to each other. The residues of the heavy spin-3/2 baryons have been estimated within the QCD sum rules in [16].

3 Numerical analysis

This section is devoted to the numerical calculation of the sum rules for the coupling constants of the spin-3/2 to spin-3/2 heavy baryon transitions with the participation of the light vector mesons. The main input parameters entering the sum rules are the Borel mass parameter M^2 , the continuum threshold s_0 and DA's of the light vector mesons. The expressions of DA's are taken from [11–14]. The values of other input parameters are: $\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle = (0.012 \pm 0.01) \text{ GeV}^4$, $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$ [17], $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [18], $m_s(2 \text{ GeV}) = (111 \pm 6) \text{ MeV}$ at $\Lambda_{QCD} = 330 \text{ MeV}$ [19]. For the masses of the heavy hadrons we use the results of the work [20].

The continuum threshold s_0 and the Borel mass M^2 are the auxiliary parameters of the considered sum rules. For this reason, we should find the so-called "working region" of these parameters, where physical quantities are practically independent of them. The working region of M^2 is determined in the following way. The upper limit of M^2 is obtained by requiring that the contributions of higher states and continuum should be less than 40% of the total result of the correlation function. The lower limit of M^2 is determined by demanding that the contribution of the terms with higher powers of $1/M^2$ constitute (20–25)% of the contributions from the terms with highest power of M^2 . Taking both these conditions into account, we find that the "working region" of M^2 for the baryons with b -quark lies in the region $10 \leq M^2 \leq 20 \text{ GeV}^2$, while for the baryons containing c -quark it is $4 \leq M^2 \leq 8 \text{ GeV}^2$. The continuum threshold s_0 depends on the mass of the first excited state. In general, in the quark models, the energy difference between the first excited and ground states is about 0.5 GeV . Therefore, for the continuum threshold we use $s_0 \approx (m_{ground} + 0.5)^2 \text{ GeV}^2$. This leads us to choose the "working region" of the continuum threshold as $38 \text{ GeV}^2 \leq s_0 \leq 42 \text{ GeV}^2$ and $9 \text{ GeV}^2 \leq s_0 \leq 12 \text{ GeV}^2$ for the heavy baryons with b and c quarks, respectively.

In the present work, we study the dependence of the coupling constants g_1, g_2, g_3 and

g_4 on the Borel mass parameter M^2 in the range determined by its own working region, at several different values of s_0 . As an example, in Figs. (1)–(4) we present the results of our numerical analysis for the $\Xi_b^{*0} \rightarrow \Xi_b^{*0} \rho^0$ transition. We see from these figures that the coupling constants exhibit good stability on M^2 and s_0 . The predictions of the sum rules for the coupling constants g_1 , g_2 , g_3 and g_4 are presented in Tables (1) and (2). Note that in the Tables, we only present the modules of the strong coupling constants, since the sum rules method cannot predict the sign of the residues of the heavy baryons.

transition	g_1	g_2	g_3	g_4
$\Xi_b^{*0} \rightarrow \Xi_b^{*0} \rho^0$	9 ± 1	19 ± 2	50 ± 5	15 ± 2
$\Sigma_b^{*0} \rightarrow \Sigma_b^{*-} \rho^+$	18 ± 2	36 ± 4	100 ± 20	27 ± 3
$\Xi_b^{*0} \rightarrow \Sigma_b^{*+} K^{*-}$	30 ± 5	40 ± 4	110 ± 15	28 ± 3
$\Omega_b^{*-} \rightarrow \Xi_b^{*0} K^{*-}$	30 ± 5	41 ± 4	110 ± 20	28 ± 3
$\Sigma_b^{*+} \rightarrow \Xi_b^{*0} K^{*+}$	30 ± 5	40 ± 4	100 ± 20	28 ± 3
$\Xi_b^{*0} \rightarrow \Omega_b^{*-} K^{*+}$	35 ± 5	42 ± 5	100 ± 20	28 ± 3
$\Sigma_b^{*+} \rightarrow \Sigma_b^{*+} \omega$	16 ± 2	32 ± 3	90 ± 10	24 ± 4
$\Xi_b^{*0} \rightarrow \Xi_b^{*0} \omega$	10 ± 2	17 ± 2	45 ± 5	13 ± 2
$\Xi_b^{*0} \rightarrow \Xi_b^{*0} \phi$	17 ± 2	25 ± 3	85 ± 10	25 ± 5
$\Omega_b^{*-} \rightarrow \Omega_b^{*-} \phi$	40 ± 5	50 ± 6	140 ± 20	50 ± 10

Table 2: Coupling constants of the light vector mesons with heavy spin-3/2 baryons containing b quark.

transition	g_1	g_2	g_3	g_4
$\Xi_c^{*+} \rightarrow \Xi_c^{*+} \rho^0$	8 ± 1	13 ± 1	27 ± 3	16 ± 2
$\Sigma_c^{*+} \rightarrow \Sigma_c^{*0} \rho^+$	15 ± 1	18 ± 1	45 ± 5	10 ± 2
$\Xi_c^{*+} \rightarrow \Sigma_c^{*++} K^{*-}$	29 ± 2	19 ± 1	50 ± 10	13 ± 2
$\Omega_c^{*0} \rightarrow \Xi_c^{*+} K^{*-}$	18 ± 2	26 ± 4	50 ± 10	30 ± 4
$\Sigma_c^{*++} \rightarrow \Xi_c^{*+} K^{*+}$	27 ± 4	19 ± 1	50 ± 10	13 ± 2
$\Xi_c^{*+} \rightarrow \Omega_c^{*0} K^{*+}$	21 ± 6	27 ± 2	52 ± 10	31 ± 4
$\Sigma_c^{*++} \rightarrow \Sigma_c^{*++} \omega$	13 ± 2	16 ± 2	40 ± 5	9 ± 2
$\Xi_c^{*+} \rightarrow \Xi_c^{*+} \omega$	10 ± 2	11 ± 2	23 ± 3	17 ± 3
$\Xi_c^{*+} \rightarrow \Xi_c^{*+} \phi$	13 ± 2	12 ± 1	38 ± 5	9 ± 2
$\Omega_c^{*0} \rightarrow \Omega_c^{*0} \phi$	36 ± 6	35 ± 5	80 ± 10	45 ± 5

Table 3: The same as Table (1), but for the heavy baryons containing c quark.

The errors given in the Tables (1) and (2) can be attributed to the uncertainties of the input parameters and uncertainties inherit in M^2 and s_0 . At the end of this section, it

should be noted that part of the relevant coupling constants are calculated within the same framework in [21], which are considerably different compared to the results presented in this work. In our opinion, these discrepancies between our results and those that are given in [21] could be due to the following reasons: the residues of Ω_Q^* , Ξ_Q^* , Σ_Q^* given in [21] are several times of magnitude larger compared to ours. Moreover, the results presented in [21] do not satisfy the relations between invariant functions.

In conclusion, in the present work, we have calculated the coupling constants of heavy sextet spin-3/2 to spin-3/2 transitions with the participation of the light vector mesons in LCSR. It is shown that the relations among the correlation functions responsible for different transitions are described in terms of only one universal function. These relations are all structure independent, while the explicit expression of this universal function is structure dependent. The values of the four coupling constants that appear in the parametrization of the $B_Q^*B_Q^*V$ vertex are obtained.

Appendix

In this appendix we present the expressions of the correlation functions in terms of invariant function Π_1 .

$$\begin{aligned}
\Pi^{\Sigma_b^{*+} \rightarrow \Sigma_b^{*+} \rho^0} &= \sqrt{2} \Pi_1(u, u, b) , \\
\Pi^{\Sigma_b^{*-} \rightarrow \Sigma_b^{*-} \rho^0} &= -\sqrt{2} \Pi_1(d, d, b) , \\
\Pi^{\Xi_b^{*0} \rightarrow \Xi_b^{*0} \rho^0} &= \frac{1}{\sqrt{2}} \Pi_1(u, s, b) , \\
\Pi^{\Xi_b^{*-} \rightarrow \Xi_b^{*-} \rho^0} &= -\frac{1}{\sqrt{2}} \Pi_1(d, s, b) , \\
\Pi^{\Sigma_b^{*+} \rightarrow \Sigma_b^{*0} \rho^+} &= \sqrt{2} \Pi_1(d, u, b) , \\
\Pi^{\Sigma_b^{*0} \rightarrow \Sigma_b^{*-} \rho^+} &= \sqrt{2} \Pi_1(u, d, b) , \\
\Pi^{\Xi_b^{*0} \rightarrow \Xi_b^{*-} \rho^+} &= \Pi_1(d, s, b) , \\
\Pi^{\Sigma_b^{*0} \rightarrow \Sigma_b^{*+} \rho^-} &= \sqrt{2} \Pi_1(d, u, b) , \\
\Pi^{\Sigma_b^{*-} \rightarrow \Sigma_b^{*0} \rho^-} &= \sqrt{2} \Pi_1(u, d, b) , \\
\Pi^{\Xi_b^{*-} \rightarrow \Xi_b^{*0} \rho^-} &= \Pi_1(u, s, b) , \\
\Pi^{\Xi_b^{*0} \rightarrow \Sigma_b^{*+} K^{*-}} &= \sqrt{2} \Pi_1(u, u, b) , \\
\Pi^{\Xi_b^{*-} \rightarrow \Sigma_b^{*0} K^{*-}} &= \Pi_1(u, d, b) , \\
\Pi^{\Omega_b^{*-} \rightarrow \Xi_b^{*0} K^{*-}} &= \sqrt{2} \Pi_1(s, s, b) , \\
\Pi^{\Sigma_b^{*+} \rightarrow \Xi_b^{*0} K^{*+}} &= \sqrt{2} \Pi_1(u, u, b) , \\
\Pi^{\Sigma_b^{*0} \rightarrow \Xi_b^{*-} K^{*+}} &= \Pi_1(u, d, b) , \\
\Pi^{\Xi_b^{*0} \rightarrow \Omega_b^{*-} K^{*+}} &= \sqrt{2} \Pi_1(s, s, b) , \\
\Pi^{\Xi_b^{*0} \rightarrow \Sigma_b^{*0} \bar{K}^{*0}} &= \Pi_1(d, u, b) , \\
\Pi^{\Xi_b^{*-} \rightarrow \Sigma_b^{*-} \bar{K}^{*0}} &= \sqrt{2} \Pi_1(d, d, b) , \\
\Pi^{\Omega_b^{*-} \rightarrow \Xi_b^{*-} \bar{K}^{*0}} &= \sqrt{2} \Pi_1(s, s, b) , \\
\Pi^{\Sigma_b^{*0} \rightarrow \Xi_b^{*0} K^{*0}} &= \Pi_1(d, u, b) , \\
\Pi^{\Sigma_b^{*-} \rightarrow \Xi_b^{*-} K^{*0}} &= \sqrt{2} \Pi_1(d, d, b) , \\
\Pi^{\Xi_b^{*-} \rightarrow \Omega_b^{*-} K^{*0}} &= \sqrt{2} \Pi_1(s, s, b) , \\
\Pi^{\Sigma_b^{*0} \rightarrow \Sigma_b^{*0} \omega} &= \frac{1}{\sqrt{2}} \left[\Pi_1(u, d, b) + \Pi_1(d, u, b) \right] , \\
\Pi^{\Sigma_b^{*+} \rightarrow \Sigma_b^{*+} \omega} &= \sqrt{2} \Pi_1(u, u, b) , \\
\Pi^{\Sigma_b^{*-} \rightarrow \Sigma_b^{*-} \omega} &= \sqrt{2} \Pi_1(d, d, b) , \\
\Pi^{\Xi_b^{*0} \rightarrow \Xi_b^{*0} \omega} &= \frac{1}{\sqrt{2}} \Pi_1(u, s, b) , \\
\Pi^{\Xi_b^{*-} \rightarrow \Xi_b^{*-} \omega} &= \frac{1}{\sqrt{2}} \Pi_1(d, s, b) ,
\end{aligned}$$

$$\begin{aligned}\Pi^{\Xi_b^{*0} \rightarrow \Xi_b^{*0} \phi} &= \Pi_1(s, u, b) , \\ \Pi^{\Xi_b^{*-} \rightarrow \Xi_b^{*-} \phi} &= \Pi_1(s, d, b) , \\ \Pi^{\Omega_b^{*-} \rightarrow \Omega_b^{*-} \phi} &= 2\Pi_1(s, s, b) .\end{aligned}$$

Similar relations can be obtained for the heavy baryons containing c quark by making the replacement $b \rightarrow c$, and adding a positive unit charge to the charge of each heavy baryon.

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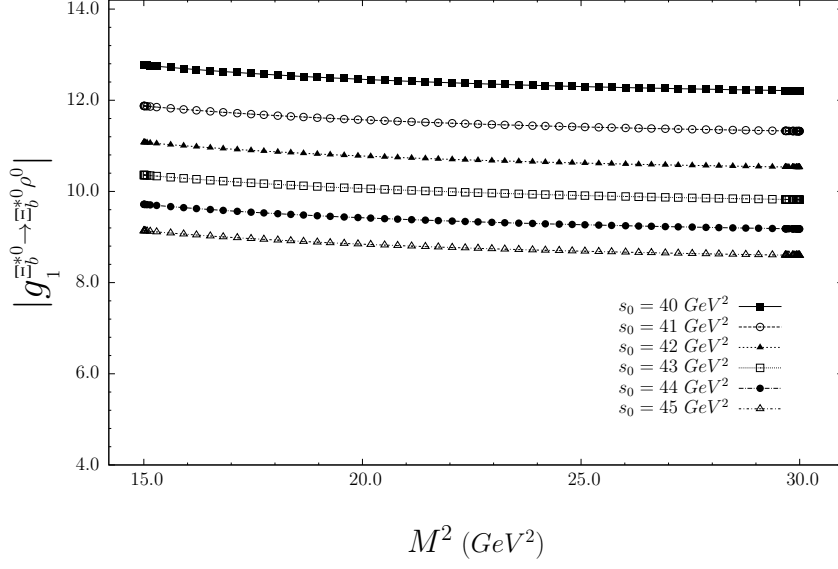


Figure 1: The dependence of the strong coupling constant g_1 for the $\Xi_b^{*0} \rightarrow \Xi_b^0 \rho^0$ transition on the Borel mass parameter M^2 at several different fixed values of the continuum threshold s_0 .

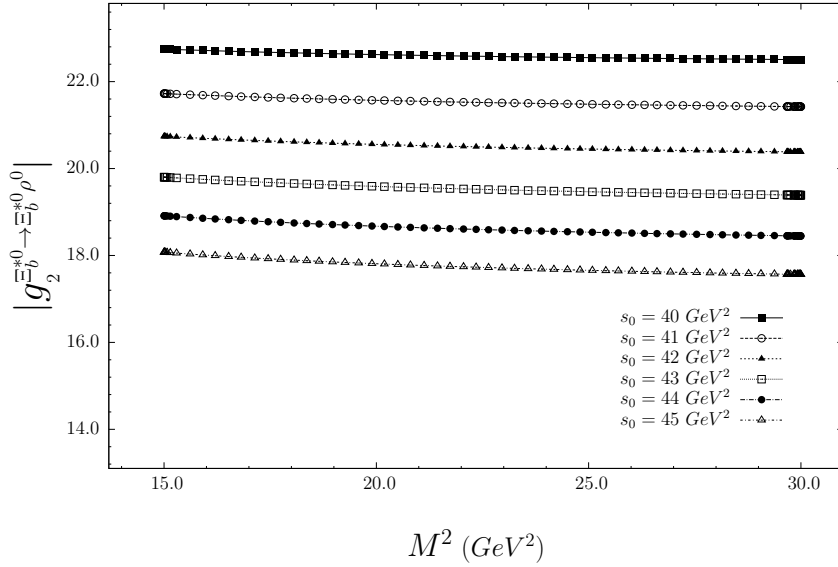


Figure 2: The same as Fig. (1), but for the strong coupling constant g_2 .

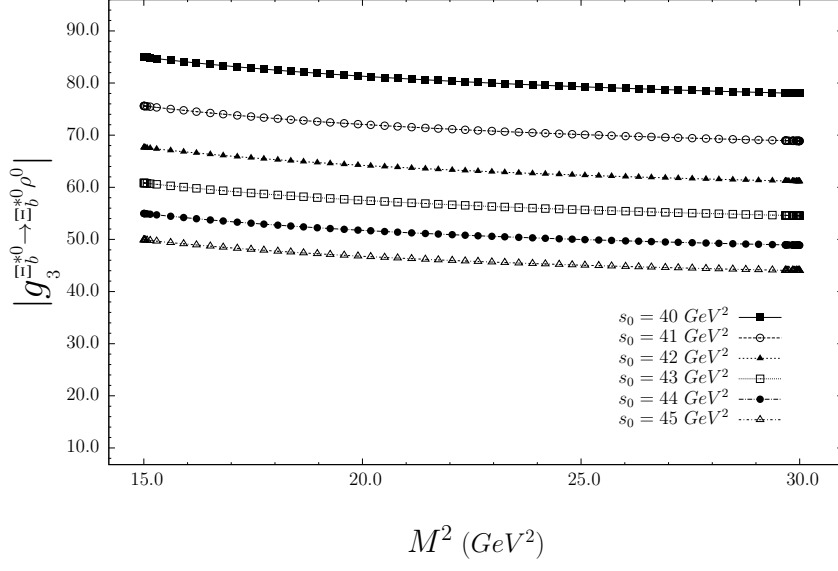


Figure 3: The same as Fig. (1), but for the strong coupling constant g_3 .

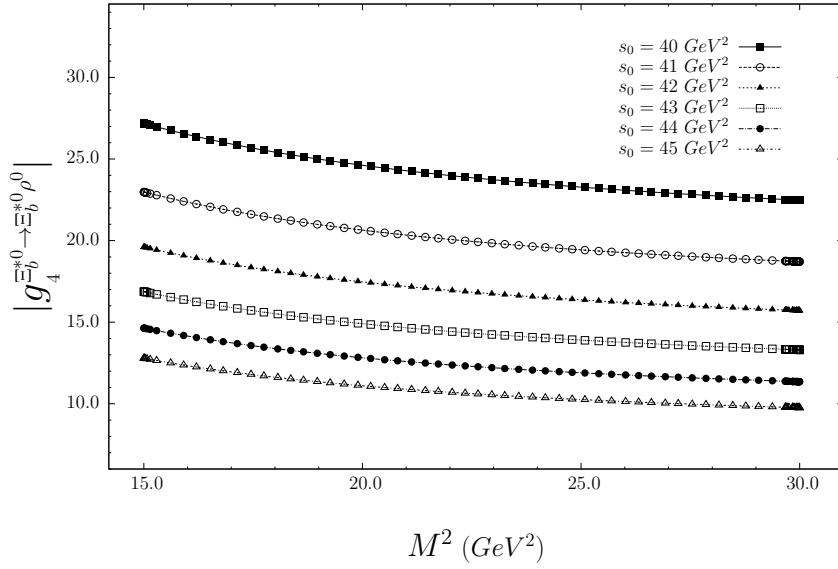


Figure 4: The same as Fig. (1), but for the strong coupling constant g_4 .