Radiative Decays of Decuplet to Octet Baryons in Light Cone QCD

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Abstract

The radiative decays of decuplet to octet baryons are analyzed within the light cone QCD sum rules framework.The electromagnetic transition form factors for these decays are calculated at $q^2 = 0$ up to twist four accuracy for photon wave functions as well as including first order strange quark mass corrections. A comparison of our results with predictions of lattice theory and existing experimental data is presented.

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1 Introduction

The study of the electromagnetic transitions of the decuplet to octet baryons is an important issue for understanding of internal structure of baryons. Spin parity selection rules allow for magnetic dipole ($M1$), electric quadrapole ($E2$) and Coulomb quadrapole ($C2$) moments for these decays. These transitions can also give essential information about the wave function of the lowest lying baryons. For example in the $\Delta \rightarrow N\gamma^*$ transition if initial and final baryons' wave functions are spherically symmetric, then the $E2$ and $C2$ amplitude must vanish. The results of recent photo production experiments on nucleon at Bates [1] and Jefferson Lab [2] shows that these amplitudes are likely to be non zero. This result is an indication that these decays can contain many mysteries. For this reason, these decays must be carefully and completely studied both theoretically and experimentally.

The electromagnetic decays of baryons constitute an important class of decays for the determination of fundamental parameters of hadrons. For extracting these parameters, information about the non perturbative region of QCD is required. Therefore a reliable non-perturbative approach is needed. Among non-perturbative approaches, QCD sum rules [3] is more predictive in studying properties of hadrons.

QCD sum rules is based on the first principles of QCD. In this method, measurable quantities of hadrons in experiments are connected with QCD parameters, where hadrons are represented by corresponding interpolating quark currents taken at large virtualities and correlator of these quark currents is introduced. The main idea is to calculate this correlator with the help of operator product expansion (OPE) in the framework of QCD in the large Euclidean domain and represent this same correlator in terms of hadronic parameters in the other kinematical region. The sum rule for corresponding physical quantity is obtained by matching two representations of the correlator.

In the present work, we calculate the baryon decuplet to octet transition form factors in the framework of an alternative approach to the traditional QCD sum rules, i.e. light cone QCD sum rules method. This method is based on the OPE near the light cone, which is an expansion over the twist of the operators rather than dimensions as is the case in the traditional QCD sum rules. The non-perturbative dynamics encoded in the light cone wave functions determines the matrix elements of the non-local operators between vacuum and corresponding particle states (more about his method can be found in [4, 5]).

It should be mentions that the electromagnetic baryon decuplet to octet transitions has been investigated in chiral perturbation theory [6, 7, 8, 9], (partially) quenched chiral perturbation theory [10], in lattice simulations of quenched QCD in [11], in the large N_c limit of QCD [12]. In [13] the transition form factors for $\Delta \to N\gamma$ decay is calculated within the traditional QCD sum rules using external field method. This $\Delta \to N\gamma$ decay in QCD light cone QCD sum rules is investigates in [14].

The plan of this work is as follows. In section 2, we consider a generic correlator function,which yields light cone sum rules for decuplet to octet baryon transition form factors. Then this correlation function is calculated up to twist 4 accuracy for the photon wave functions including first order strange quark mass corrections. In this section we obtain sum rules for form factors of decuplet to octet baryons electromagnetic transition. In section 3, we present our numerical results and comparison with other approaches is presented.

2 Sum Rules for Electromagnetic Transition Moments for the Decuplet to Octet Baryon Transitions

In this section we calculate the baryon decuplet to octet electromagnetic transition form factors. We start our calculation by considering the following correlator function:

$$T_\mu(p,q) = i\int d^4x e^{ipx} \langle \gamma(q)| \mathcal{T}\left\{\eta_{\mathcal{O}}(x)\bar{\eta}_{\mathcal{D}\mu}(0)\right\}|0\rangle \tag{1}$$

where $\eta_{\mathcal{O}}$ and $\eta_{\mathcal{D}}$ are generic octet and decuplet interpolating quark currents respectively. We will consider the following baryon decuplet to octet electromagnetic transitions

$$\begin{aligned} \Sigma^{*+} &\to \Sigma^+\gamma \\ \Sigma^{*0} &\to \Sigma^0\gamma \\ \Sigma^{*0} &\to \Lambda\gamma \\ \Sigma^{*-} &\to \Sigma^-\gamma \\ \Xi^{*0} &\to \Xi^0\gamma \end{aligned}$$

$$\Xi^{*-} \;\; \rightarrow \;\; \Xi^{-}\gamma \tag{2}$$

First, the phenomenological part of the correlator function can be calculated by inserting a complete set of hadronic states to it. Saturating the correlator (Eq. (1)) by ground state baryons we get:

$$T_\mu(p,q) = \frac{\langle 0|\eta_{\mathcal{O}}|\frac{1}{2}(p)\rangle}{p^2 - m_{\mathcal{O}}^2}\langle\frac{1}{2}|\frac{3}{2}\rangle_\gamma \frac{\langle \frac{3}{2}(p+q)|\eta_{\mathcal{D}\mu}|0\rangle}{(p+q)^2 - M_{\mathcal{D}}^2} + \cdots \tag{3}$$

where $|\frac{1}{2}(p)\rangle$ and $|\frac{3}{2}(p)\rangle$ denote octet and decuplet baryons with momentum p, respectively, $m_{\mathcal{O}}$ and $M_{\mathcal{D}}$ denote the masses of the octet and decuplet baryons respectively, and $\cdots$ stand for the contributions of the higher states and the continuum. The matrix elements $\langle 0|\eta_{\mathcal{O}}|\frac{1}{2}(p)\rangle$ and $\langle \frac{3}{2}(p+q)|\eta_{\mathcal{D}\mu}|0\rangle$ are determined as:

$$\begin{aligned} \langle 0|\eta_{\mathcal{O}}|\frac{1}{2}(p)\rangle &= \lambda_{\mathcal{O}} u(p,s) \\ \langle \frac{3}{2}(p+q)|\eta_{\mathcal{D}\mu}|0\rangle &= \lambda_{\mathcal{D}} u_\mu(p+q,s') \end{aligned} \tag{4}$$

where $\lambda_{\mathcal{O}}$ and $\lambda_{\mathcal{D}}$ are the residues and u_μ is the Rarita-Schwinger spinor and s and s' are the four spin vectors of the octet and decuplet baryons respectively. The electromagnetic vertex of the decuplet to octet transition can be parameterized in terms of three form factors in the following way [19, 20]

$$\begin{aligned} \langle\frac{1}{2}|\frac{3}{2}\rangle_\gamma &= e u(p,s)\left\{G_1\left(q_\rho \not{\varepsilon} - \varepsilon_\rho \not{q}\right)\gamma_5\right. \\ &+ G_2\left((P\varepsilon)q_\rho - (Pq)\varepsilon_\rho\right)\gamma_5 \\ &+ \left. G_3\left((q\varepsilon)q_\rho - q^2\varepsilon_\rho\right)\gamma_5\right\} u^\rho(p+q) \end{aligned} \tag{5}$$

where $P = \frac{1}{2}\left(p + (p+q)\right)$ and ε is the photon polarization vector. Since in our case, photon is real, G_3 does not give any contribution to the considered decays and we need to know the values of the form factors G_1 and G_2 only at $q^2 = 0$.

For the experimental analysis, it is desirable to use such form factors which describe physical transition. This would correspond to a definite multipole or helicity transitions in a given reference frame. Linear combinations of the the form factors in Eq. (5) give magnetic dipole, G_M, electric quadrapole, G_E, and Coulomb quadrapole, G_C, form factors. The relations between two set of form factors (for completeness, we present the relation in general form when $q^2 \neq 0$. Our case corresponds to $q^2 = 0$):

$$\begin{aligned}
G_M &= \left[\left((3M_{\mathcal{D}} + m_{\mathcal{O}})(M_{\mathcal{D}} + m_{\mathcal{O}}) - q^2\right)\frac{G_1}{M_{\mathcal{D}}}\right. \\
&\quad \left. + \left(M_{\mathcal{D}}^2 - m_{\mathcal{O}}^2\right) G_2 + 2q^2 G_3\right] \frac{m_{\mathcal{O}}}{3(M_{\mathcal{D}} + m_{\mathcal{O}})} \\
G_E &= \left[\left(M_{\mathcal{D}}^2 - m_{\mathcal{O}}^2 + q^2\right)\frac{G_1}{M_{\mathcal{D}}} + \left(M_{\mathcal{D}}^2 - m_{\mathcal{O}}^2\right) G_2 + 2q^2 G_3\right] \frac{m_{\mathcal{O}}}{3(M_{\mathcal{D}} + m_{\mathcal{O}})} \\
G_C &= \left[2M_{\mathcal{D}} G_1 + \frac{1}{2}\left(3M_{\mathcal{D}}^2 + m_{\mathcal{O}}^2 - q^2\right) G_2\right. \\
&\quad \left. + \left(M_{\mathcal{D}}^2 - m_{\mathcal{O}}^2 + q^2\right) G_3\right] \frac{2m_{\mathcal{O}}}{3(M_{\mathcal{D}} + m_{\mathcal{O}})} \qquad (6)
\end{aligned}$$

Considering the expression for G_E, one sees that in the case we are interested in, i.e. $q^2 = 0$, G_E is proportional to $M_{\mathcal{D}} - m_{\mathcal{O}}$. As this quantity is very small ($(M_{\mathcal{D}} - m_{\mathcal{O}})/(M_{\mathcal{D}} + m_{\mathcal{O}})$ is $\sim 15\%$ for the $\Delta \to N$ transitions and is $\sim 5\%$ for the remaining transitions), small uncertainties in $M_{\mathcal{D}}$ or $m_{\mathcal{O}}$ leads to big uncertainties in the predictions for G_E. Although the sum rule predictions for the masses of the octet and decuplet baryons agree within errors with the experimental values, the error get amplified in the prediction for G_E. Thus it makes a big difference weather one uses the experimental value or the sum rule prediction for the values of $M_{\mathcal{D}}$ and $m_{\mathcal{O}}$. To give a quantitative idea about this uncertainty, if one considers the $\Delta \to N$ transitions, the experimental value of the mass difference of the Δ and the nucleon is $M_\Delta^{exp} - m_N^{exp} = 294\ MeV$, where as the mass prediction from the mass sum rules for the mass of the Δ can be as small as $M_\Delta = 1.06\ GeV$ [15] and the prediction for the mass of the nucleon can be as big as $m_N = 1.17\ GeV$ [16]. Thus one sees that, in the extreme cases, if one uses the predictions of the mass sum rules for the masses of the octet and decuplet baryons, even the sign of G_E might change. In our work, we use the experimental values for the masses of the octet and decuplet baryons. But our predictions on G_E should be considered as order of magnitude estimates.

In calculations, summation over spins of the Rarita-Schwinger spin vector is done using the relation:

$$\sum_s u_\alpha(p,s)\bar{u}_\beta(p,s) = -\left(\not{p} + M_{\mathcal{D}}\right)\left\{g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2p_\alpha p_\beta}{3M_{\mathcal{D}}^2} + \frac{p_\alpha\gamma_\beta - p_\beta\gamma_\alpha}{3M_{\mathcal{D}}}\right\} \qquad (7)$$

Using Eqs. (3-5) and (7), we see that correlator has numerous tensor structures not all of which are independent. The dependence can be removed by ordering gamma matrices in a specific order. For this purpose, the ordering $\not\varepsilon \not q \not p \gamma_\mu$ is chosen. With this ordering, the correlator function becomes:

$$\begin{aligned}
T_\mu &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m_{\mathcal{O}}^2}\frac{1}{(p+q)^2-M_{\mathcal{D}}^2}\,[\\
&\quad [\varepsilon_\mu (pq) - (\varepsilon p) q_\mu]\left\{-2G_1 M_{\mathcal{D}} - G_2 M_{\mathcal{D}} m_{\mathcal{O}} + G_2 (p+q)^2\right.\\
&\quad \left. + [2G_1 + G_2 (m_{\mathcal{O}} - M_{\mathcal{D}})] \not p + m_{\mathcal{O}} G_2 \not q - G_2 \not q \not p\right\}\gamma_5\\
&\quad + [q_\mu \not\varepsilon - \varepsilon_\mu \not q]\left\{G_1 (p^2 + M_{\mathcal{D}} m_{\mathcal{O}}) - G_1 (M_{\mathcal{D}} + m_{\mathcal{O}}) \not p\right\}\gamma_5\\
&\quad +2G_1 [\not\varepsilon (pq) - \not q (\varepsilon p)]\, q_\mu \gamma_5\\
&\quad -G_1 \not\varepsilon \not q \{m + \not p\}\, q_\mu \gamma_5\\
&\quad \text{other structures with } \gamma_\mu \text{ at the end or which are proportional to } (p+q)_\mu]
\end{aligned} \quad (8)$$

An essential reason for not considering the structures $\propto (p+q)_\mu$ and the structures which contain a γ_μ at the end is as follows: A spin-$\frac{3}{2}$ current can have a nonzero overlap with a spin-$\frac{1}{2}$ state:

$$\langle 0|\eta_{\frac{3}{2}\mu}|\frac{1}{2}(p)\rangle = (A' p_\mu + B' \gamma_\mu)\,\gamma_5 u(p) \quad (9)$$

Using $\gamma^\mu \eta_{\frac{3}{2}\mu} = 0$ one can easily obtain that $B' = -\frac{A'm}{4}$ Hence, in principle, spin-$\frac{1}{2}$ states can also give contribution to the correlation function. But given the ordering, they contribute only to the structures which contain a γ_μ at the end or which are proportional to $(p+q)_\mu$. In other words, spin-$\frac{1}{2}$ particles do not contribute to the structures that we have not omitted.

In order to obtain predictions for the values of G_1 and G_2, we need to choose two different structures to study. In deciding which structures to study, we have chosen structures which do not get contributions from the contact terms after the Borel transformations(for a discussion of the contact terms see [17]). In the exact $SU(3)_f$ limit, the contact terms vanish. But since we are interested also in the violations of $SU(3)$, the contact terms are in principle non-zero. In order to extract the physical interaction vertex, one has to subtract the contact terms from the correlation function. Among the structures in Eq. (8), the structures which receive contributions from the contact terms are $\varepsilon_\mu \gamma_5$, $\varepsilon_\mu \not q \gamma_5$, $\varepsilon_\mu \not p \gamma_5$, $q_\mu \gamma_5$, $q_\mu \not q \gamma_5$, and $q_\mu \not p \gamma_5$

We have studied all the possible pairs of structures. We obtained the best convergence for the pairs $\not\varepsilon \not p\gamma_5 q_\mu$ and $\not q \not p\gamma_5(\varepsilon p)q_\mu$, or $\gamma_5(\varepsilon p)q_\mu$ and $\not q \not p\gamma_5(\varepsilon p)q_\mu$. Although the structure $\not q \not p\gamma_5\varepsilon_\mu$ receives contributions from the contact terms, the contact terms vanish after the Borel transformation.

The structures $\not\varepsilon \not p\gamma_5 q_\mu$, $\gamma_5(\varepsilon p)q_\mu$, and $\not q \not p\gamma_5(\varepsilon p)q_\mu$ have the coefficients:

$$\begin{aligned} -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}(M_{\mathcal{D}}+m_{\mathcal{O}})\Sigma_6 &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}(M_{\mathcal{D}}+m_{\mathcal{O}})G_1 \\ e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M_{\mathcal{D}}\Sigma_9 &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M_{\mathcal{D}}(2G_1+G_2(m_{\mathcal{O}}-M_{\mathcal{D}})) \\ e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\Sigma_{12} &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}G_2 \end{aligned} \quad (10)$$

respectively, where

$$\begin{aligned} \Sigma_6 &= G_1 \\ \Sigma_9 &= 2G_1+G_2(m_{\mathcal{O}}-M_{\mathcal{D}}) \\ \Sigma_{12} &= G_2 \end{aligned} \quad (11)$$

Once the sum rules for Σ_i $(i=4,\ 6,\ 12)$ are obtained, the sum rules for G_X, $(X=1,\ 2,\ E,\ M)$ are obtained using the experimental masses of the octet and decuplet baryons through the relations obtained by inverting Eqs. (6) and (11):

In the deep Euclidean region where $p^2 << 0$ and $(p+q)^2 << 0$, the correlation function Eq. (1) can be calculated using OPE. For this purpose, one needs the explicit forms of the interpolating currents. Explicit forms of the interpolating quark currents for the corresponding decuplet members are as follows:

$$\begin{aligned} \eta_\mu^{\Sigma^{*0}} &= \sqrt{\frac{2}{3}}\epsilon^{abc}\left[\left(u^{aT}C\gamma_\mu d^b\right)s^c+\left(d^{aT}C\gamma_\mu s^b\right)u^c+\left(s^{aT}C\gamma_\mu u^b\right)d^c\right] \\ \eta_\mu^{\Sigma^{*+}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(d\to u) \\ \eta_\mu^{\Sigma^{*-}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(u\to d) \\ \eta_\mu^{\Delta^{*+}} &= \eta_\mu^{\Sigma^{*+}}(s\to d) \\ \eta_\mu^{\Delta^{*0}} &= \eta_\mu^{\Sigma^{*-}}(s\to u) \\ \eta_\mu^{\Xi^{*0}} &= \eta_\mu^{\Delta^0}(d\to s) \\ \eta_\mu^{\Xi^{*-}} &= \eta_\mu^{\Xi^{*0}}(u\to d) \end{aligned} \quad (12)$$

It is well known that the choice of the interpolating currents for the octet baryons is not unique. One can define two different currents with the quantum numbers of the octet baryons. The most general current is a linear combination of these two currents. In our studies we chose:

$$\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[\left(u^{aT}Cs^b\right)\gamma_5 d^c + t\left(u^{aT}C\gamma_5 s^b\right)d^c - \left(s^{aT}Cd^b\right)\gamma_5 u^c - t\left(s^{aT}C\gamma_5 d^b\right)u^c\right] \\
\eta^{\Sigma^+} &= \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d\to u) \\
\eta^{\Sigma^-} &= \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u\to d) \\
\eta^{p} &= \eta^{\Sigma^+}(s\to d) \\
\eta^{n} &= \eta^{\Sigma^-}(s\to u) \\
\eta^{\Xi^0} &= \eta^{n}(d\to s) \\
\eta^{\Xi^-} &= \eta^{\Xi^0}(u\to s) \\
\eta^{\Lambda} &= -\sqrt{\frac{1}{6}}\epsilon^{abc}\left[2\left(u^{aT}Cd^b\right)\gamma_5 s^c + 2t\left(u^{aT}C\gamma_5 d^b\right)s^c + \left(u^{aT}Cs^b\right)\gamma_5 d^c\right. \\
&+ \left. t\left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c + t\left(s^{aT}C\gamma_5 d^b\right)u^c\right] \qquad (13)
\end{aligned}$$

where t is an arbitrary parameter. The Ioffe current, generally used in the literature to study the properties of the octet baryons, corresponds to the choice $t=-1$ for this parameter. The current used in [11] corresponds to the limit $t\to\infty$. The physical quantities should be independent of the value of t. In our analysis, a region for the values of t is found requiring that the prediction should be independent of the value of t. The precise normalization of the octet and decuplet baryons are chosen such that in the $SU(3)_f$ limit, the mass sum rules are the same within a multiplet.

Note that, all the currents except the current of Λ can be obtained from the currents of Σ^0 and Σ^{*0} by simple substitutions. Hence, in the following we will give only the expressions for the $\Sigma^{*0}\to\Sigma^0$. The expressions for the other decays can be obtained using the following relationships:

$$\begin{aligned}
\Pi^{\Sigma^{*+}\to\Sigma^+} &= \Pi^{\Sigma^{*0}\to\Sigma^0}(d\to u) \\
\Pi^{\Sigma^{*-}\to\Sigma^-} &= \Pi^{\Sigma^{*0}\to\Sigma^0}(u\to d) \\
\Pi^{\Delta^+\to p} &= \Pi^{\Sigma^{*+}\to\Sigma^+}(s\to d) \\
\Pi^{\Delta^0\to n} &= \Pi^{\Sigma^{*-}\to\Sigma^-}(s\to u)
\end{aligned}$$

$$\begin{aligned}\Pi^{\Xi^{*0}\to\Xi^0} &= \Pi^{\Delta^0\to n}(d\to s)\\ \Pi^{\Xi^{*-}\to\Xi^-} &= \Pi^{\Xi^{*0}\to\Xi^0}(u\to d)\end{aligned}\tag{14}$$

Recently, it has been shown in [21, 22] that it is also possible to obtain the Λ current from that of the Σ^0 current by substitutions. For this purpose, note that

$$2\eta^{\Sigma^0}(d\leftrightarrow s) = -\sqrt{3}\eta^{\Lambda} - \eta^{\Sigma^0}\tag{15}$$

and hence

$$-\sqrt{3}\Pi^{\Sigma^{*0}\to\Lambda} = 2\Pi^{\Sigma^{*0}\to\Sigma^0}(d\leftrightarrow s) + \Pi^{\Sigma^{*0}\to\Sigma^0}\tag{16}$$

The correlator function receives three different types of contributions: perturbative contributions, non-perturbative contributions where photon is emitted from the freely propagating quark, i.e. at short distances, and non-perturbative contributions where the photon is emitted at long distances. In order to calculate the contribution of the terms which come from the long distance emission of the photon, the correlation function is expanded near to the light cone $x^2 = 0$.The expansion involves matrix elements of the nonlocal operators between vacuum and the one photon states, which are expressed in terms of photon wave functions with increasing twist. In other words all long distance effects are encoded in the matrix elements of the form $\langle\gamma(q)|\bar{q}(x_1)\Gamma q(x_2)|0\rangle$.

For the propagators of the quarks, we have used the following light quark propagator expanded up to linear order in the quark mass:

$$\begin{aligned}S_q(x) &= \frac{i\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right) - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\\ &\quad -ig_s\int_0^1 du\left[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(ux)\sigma_{\mu\nu} - ux^{\mu}G_{\mu\nu}(ux)\gamma^{\nu}\frac{i}{4\pi^2x^2}\right.\\ &\quad\left. -i\frac{m_q}{32\pi^2}G_{\mu\nu}\sigma^{\mu\nu}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right)\right]\end{aligned}\tag{17}$$

where Λ is the energy cut off separating perturbative and non perturbative regimes.

The matrix elements $\langle\gamma(q)|\bar{q}(x_1)\Gamma q(x_2)|0\rangle$ can be expanded on the light cone in terms of photon wave functions (in these expansions, we neglect the quark mass corrections) [23]:

$$\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 du e^{i\bar{u}qx}\left(\chi\varphi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right)$$
$$-\frac{i}{2(qx)}e_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du e^{i\bar{u}qx}h_\gamma(u)$$
$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = e_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\int_0^1 du e^{i\bar{u}qx}\psi^v(u)$$
$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -\frac{1}{4}e_q f_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta\int_0^1 du e^{i\bar{u}qx}\psi^a(u)$$
$$\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle\left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu\right)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{S}(\alpha_i)$$
$$\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}i\gamma_5(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle\left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu\right)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\tilde{\mathcal{S}}(\alpha_i)$$
$$\langle\gamma(q)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}(\alpha_i)$$
$$\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}(\alpha_i)$$
$$\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_s G_{\mu\nu}(vx)q(0)|0\rangle = e_q\langle\bar{q}q\rangle\left\{\left[\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\nu} - \frac{1}{qx}(q_\alpha x_\nu + q_\nu x_\alpha)\right)q_\beta\right.\right.$$
$$-\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\nu} - \frac{1}{qx}(q_\beta x_\nu + q_\nu x_\beta)\right)q_\alpha$$
$$-\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\mu} - \frac{1}{qx}(q_\alpha x_\mu + q_\mu x_\alpha)\right)q_\beta$$
$$\left.+\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{q.x}\right)\left(g_{\beta\mu} - \frac{1}{qx}(q_\beta x_\mu + q_\mu x_\beta)\right)q_\alpha\right]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1(\alpha_i)$$
$$+\left[\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu)\right)q_\nu\right.$$
$$-\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\nu\beta} - \frac{1}{qx}(q_\nu x_\beta + q_\beta x_\nu)\right)q_\mu$$
$$-\left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu)\right)q_\nu$$
$$\left.+\left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\nu\alpha} - \frac{1}{qx}(q_\nu x_\alpha + q_\alpha x_\nu)\right)q_\mu\right]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_2(\alpha_i)$$

$$
+\frac{1}{qx}(q_\mu x_\nu - q_\nu x_\mu)(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g)qx}\mathcal{T}_3(\alpha_i)
+\frac{1}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g)qx}\mathcal{T}_4(\alpha_i)\Big\} \qquad (18)
$$

where χ is the magnetic susceptibility of the quarks, $\varphi_\gamma(u)$ is the leading twist 2, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}$ and $\mathcal{V}$ are the twist 3 and $h_\gamma(u)$, $\mathbb{A}$, $\mathcal{T}_i$ $(i = 1,\ 2,\ 3,\ 4)$ are the twist 4 photon distribution amplitudes.

With all these input, after tedious calculations one obtains the expression of the correlation function in Eq. (1) in terms of QCD parameters. The two expressions of the correlation function, in two different kinematical regions are then matched using dispersion relation. The contributions of the higher states and the continuum are modeled using the quark-hadron duality. Finally, the sum rules are obtained after applying Borel transformations to the results in order both to suppress the contributions of the higher states and the continuum and to eliminate the polynomials on p^2 or $(p+q)^2$ which appear in dispersion relations.

Our final results for the coefficients defined in Eq. (10) are:

$$
\begin{aligned}
&-\sqrt{3}\lambda_{\Sigma^{*0}}\lambda_{\Sigma^0}(M_{\Sigma^{*0}}+m_{\Sigma^0})\Sigma_6 = \\
&\frac{1}{64\pi^4}(-1+t)\,(e_u+e_d-2e_s)\,M^6 \\
&-\frac{1}{24\pi^2}(-1+t)\,(e_u+e_d-2e_s)\,f_{3\gamma} i_1'(\mathcal{A},1-v)M^4 \\
&-\frac{1}{16\pi^2}\left[e_u\langle\bar u u\rangle\,(2m_d t + m_s(1+t)) + e_d\langle\bar d d\rangle\,(2m_u t + m_s(1+t))\right. \\
&\left.-e_s\langle\bar s s\rangle(m_u+m_d)(1+3t)\right] M^4\chi\varphi_\gamma(u_0) \\
&+\frac{1}{192\pi^2}(1-t)\,(e_u+e_d-2e_s)\,f_{3\gamma}\left[6\psi^a(u_0)-\bar u_0\psi^{a\prime}(u_0)+4\bar u_0\psi^v(u_0)\right]M^4 \\
&-\frac{m_0^2}{108M^2}\bar u_0\left[e_u\langle\bar u u\rangle\left[2\langle\bar s s\rangle(5+2t)-\langle\bar d d\rangle(5-t)\right]\right. \\
&\left.+e_d\langle\bar d d\rangle\left[2\langle\bar s s\rangle(5+2t)-\langle\bar u u\rangle(5-t)\right]-5e_s\langle\bar s s\rangle(\langle\bar u u\rangle+\langle\bar d d\rangle)\right]\tilde i_2(h_\gamma) \\
&-\frac{m_0^2}{432M^2}(1-t)\bar u_0\left(e_u(m_d\langle\bar d d\rangle+m_s\langle\bar s s\rangle)+e_d(m_u\langle\bar u u\rangle+m_s\langle\bar s s\rangle)\right. \\
&\left.-2e_s(m_u\langle\bar u u\rangle+m_d\langle\bar d d\rangle)\right)f_{3\gamma}\left(4\psi^v(u_0)-\psi^{a\prime}(u_0)\right) \\
&-\frac{m_0^2}{216M^2}(1+t)\left[e_u(\langle\bar d d\rangle m_s+\langle\bar s s\rangle m_d)+e_d(\langle\bar s s\rangle m_u+\langle\bar u u\rangle m_s)\right.
\end{aligned}
$$

$$
\begin{aligned}
&-2e_s(\langle\bar{d}d\rangle m_u+\langle\bar{u}u\rangle m_d)\big]\, f_{3\gamma}\psi^a(u_0)\\
&+\frac{1}{8\pi^2}\left[e_u m_u\left(\langle\bar{s}s\rangle(1+t)+2\langle\bar{d}d\rangle t\right)+e_d m_d\left(\langle\bar{s}s\rangle(1+t)+2\langle\bar{u}u\rangle t\right)\right.\\
&\left.-e_s m_s\left(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle\right)(1+3t)\right] M^2\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&-\frac{m_0^2}{144\pi^2}\left[e_u\left[\langle\bar{s}s\rangle(1+t)(m_d+5m_u)+\langle\bar{d}d\rangle(m_s(1+t)+10m_u t)\right]\right.\\
&+e_d\left[\langle\bar{s}s\rangle(1+t)(m_u+5m_d)+\langle\bar{u}u\rangle(m_s(1+t)+10m_d t)\right]\\
&\left.-e_s\left[2(1+t)(m_u\langle\bar{d}d\rangle+m_d\langle\bar{u}u\rangle)+5(1+3t)m_s(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)\right]\right]\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&-\frac{m_s}{24\pi^2}(e_u\langle\bar{u}u\rangle+e_d\langle\bar{d}d\rangle)i_1\Big((1+t)(\mathcal{T}_1+\mathcal{T}_3)-(3+t)(\mathcal{T}_2-\tilde{\mathcal{S}})-(1+3t)(\mathcal{T}_4+\mathcal{S}),1\Big)\times\\
&\times M^2\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&+e_s\frac{\langle\bar{s}s\rangle}{24\pi^2}(m_u+m_d)i_1\Big((3+t)\mathcal{T}_1-(1+t)(\mathcal{T}_2+\mathcal{T}_4+\mathcal{S}-\tilde{\mathcal{S}})+(1+3t)\mathcal{T}_3,1\Big)\times\\
&\times M^2\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&-\frac{1}{12\pi^2}(e_u\langle\bar{u}u\rangle m_d+e_d\langle\bar{d}d\rangle m_u)i_1\Big(\mathcal{T}_1+\mathcal{T}_2-\tilde{\mathcal{S}}+t(\mathcal{T}_3+\mathcal{T}_4+\mathcal{S}),1\Big)M^2\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&+e_u\frac{1}{16\pi^2}\left[\langle\bar{d}d\rangle\left(m_d(-1+t)+4(m_u-m_s)t\right)+\langle\bar{s}s\rangle\left(m_s(-1+t)+2m_u(1+t)-4m_d t\right)\right]M^2\\
&+e_d\frac{1}{16\pi^2}\left[\langle\bar{u}u\rangle\left(m_u(-1+t)+4(m_d-m_s)t\right)+\langle\bar{s}s\rangle\left(m_s(-1+t)+2m_d(1+t)-4m_u t\right)\right]M^2\\
&-e_s\frac{1}{8\pi^2}\left[\langle\bar{u}u\rangle\left(m_s(1+3t)-m_u(1-t)-4m_d t\right)+\langle\bar{d}d\rangle\left(m_s(1+3t)-m_d(1-t)-4m_u t\right)\right]M^2\\
&+\frac{1}{6}\left[e_u\langle\bar{u}u\rangle(\langle\bar{s}s\rangle(1+t)+2\langle\bar{d}d\rangle t)+e_d\langle\bar{d}d\rangle(\langle\bar{s}s\rangle(1+t)+2t\langle\bar{u}u\rangle)\right.\\
&\left.-e_s\langle\bar{s}s\rangle(1+3t)(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]M^2\chi\varphi_\gamma(u_0)\\
&-\frac{\bar{u}_0}{16\pi^2}\left[e_u\langle\bar{u}u\rangle(m_s(3+t)-2m_d)+e_d\langle\bar{d}d\rangle(m_s(3+t)-2m_u)\right.\\
&\left.-e_s\langle\bar{s}s\rangle(1+t)(m_u+m_d)\right]M^2\tilde{i}_2(h_\gamma)\\
&+\frac{1}{32\pi^2}\left[e_u\langle\bar{u}u\rangle(m_s(1+t)+2m_d t)+e_d\langle\bar{d}d\rangle(m_s(1+t)+2m_u t)\right.\\
&\left.-e_s\langle\bar{s}s\rangle(m_u+m_d)(1+3t)\right]M^2\mathbb{A}(u_0)
\end{aligned}
$$

$$
\begin{aligned}
&-e_s\frac{\langle\bar{s}s\rangle}{24\pi^2}(m_u+m_d)M^2\left[2(-1+t)i_1(\mathcal{T}_1,1)+2(1+t)i_1(\mathcal{T}_2,v)+(3+5t)i_1(\mathcal{T}_4,1)\right]\\
&-\frac{1}{24\pi^2}\left[(1+t)m_s(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)-e_s\langle\bar{s}s\rangle(m_u+m_d)(1+3t)\right]M^2\times\\
&\times\left[i_1\left(\mathcal{T}_2+2\mathcal{T}_3,1\right)-2i_1\left(\mathcal{T}_3-\mathcal{T}_4,v\right)\right]\\
&+\frac{1}{12\pi^2}\left(e_d\langle\bar{d}d\rangle m_u+e_u\langle\bar{u}u\rangle m_d\right)M^2\left[-3ti_1(\mathcal{S},1)-i_1(\tilde{\mathcal{S}},t-2v(1+t))+(-1+t)i_1(\mathcal{T}_1,1)\right.\\
&\left.-i_1(\mathcal{T}_2,t+2v)-2ti_1(\mathcal{T}_3,1-v)-ti_1(\mathcal{T}_4,1+2v)\right]\\
&+\frac{m_s}{24\pi^2}\left(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle\right)M^2\left[2(3+t)i_1(\mathcal{T}_2,v)+(3+7t)i_1(\mathcal{T}_4,1)\right]\\
&+\frac{m_s}{24\pi^2}\left(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle\right)M^2\left[(1+5t)i_1(\mathcal{S},1)-(1+t)i_1(\tilde{\mathcal{S}},1)-4i_1(\tilde{\mathcal{S}},v)\right]\\
&+\frac{t}{3}\left(\langle\bar{s}s\rangle(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)-2e_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right)\\
&+e_s\frac{\langle\bar{s}s\rangle}{24\pi^2}(m_u+m_d)M^2\left[(-1+t)i_1(\mathcal{S},1)+(1+3t)i_1(\tilde{\mathcal{S}},1)-4ti_1(\tilde{\mathcal{S}},v)\right]\\
&+e_u\frac{m_0^2}{144\pi^2}\left[\langle\bar{s}s\rangle(9m_dt-2m_s(-1+t))+\langle\bar{d}d\rangle(9m_st-2m_d(-1+t))\right]\\
&+e_d\frac{m_0^2}{144\pi^2}\left[\langle\bar{s}s\rangle(9m_ut-2m_s(-1+t))+\langle\bar{u}u\rangle(9m_st-2m_u(-1+t))\right]\\
&-e_s\frac{m_0^2}{72\pi^2}\left[\langle\bar{d}d\rangle(9m_ut-2m_d(-1+t))+\langle\bar{u}u\rangle(9m_dt-2m_u(-1+t))\right]\\
&-\frac{1}{24}\left[e_u\langle\bar{u}u\rangle(\langle\bar{s}s\rangle(1+t)+2\langle\bar{d}d\rangle t)+e_d\langle\bar{d}d\rangle(\langle\bar{s}s\rangle(1+t)+2\langle\bar{u}u\rangle t)\right.\\
&\left.-e_s\langle\bar{s}s\rangle(1+3t)(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]\left(\mathbb{A}(u_0)+\frac{10}{9}m_0^2\chi\varphi_\gamma(u_0)\right)\\
&-\frac{\bar{u}_0}{6}\left(e_u\langle\bar{u}u\rangle(2\langle\bar{d}d\rangle-(3+t)\langle\bar{s}s\rangle)+e_d\langle\bar{d}d\rangle(2\langle\bar{u}u\rangle-(3+t)\langle\bar{s}s\rangle)\right.\\
&\left.+e_s\langle\bar{s}s\rangle(1+t)(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right)\tilde{i}_2(h_\gamma)\\
&+\frac{\bar{u}_0}{48}(1-t)\left[e_u(\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s)+e_d(\langle\bar{u}u\rangle m_u+\langle\bar{s}s\rangle m_s)\right.\\
&\left.-2e_s(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d)\right]f_{3\gamma}\left(4\psi^v(\bar{u}_0)-\psi^{a\prime}(\bar{u}_0)\right)\\
&+\frac{e_u}{24}f_{3\gamma}\left[\langle\bar{d}d\rangle(m_d(1-t)+4m_st)+\langle\bar{s}s\rangle(m_s(1-t)+4m_dt)\right]\psi^a(u_0)\\
&+\frac{e_d}{24}f_{3\gamma}\left[\langle\bar{u}u\rangle(m_u(1-t)+4m_st)+\langle\bar{s}s\rangle(m_s(1-t)+4m_ut)\right]\psi^a(u_0)\\
&-\frac{e_s}{12}f_{3\gamma}\left[\langle\bar{d}d\rangle(m_d(1-t)+4m_ut)+\langle\bar{u}u\rangle(m_u(1-t)+4m_dt)\right]\psi^a(u_0)
\end{aligned}
$$

$$+e_s\frac{\langle\bar{s}s\rangle}{18}(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\left[2(1+t)i_1(\mathcal{T}_2,v)-2(1+2t)i_1(\mathcal{T}_2-\mathcal{T}_4,1)\right.$$
$$\left.+(1+3t)\left\{i_1(\mathcal{T}_1-\mathcal{T}_3,1)+2i_1(\mathcal{T}_3-\mathcal{T}_4,v)\right\}\right]$$
$$+\frac{\langle\bar{s}s\rangle}{18}(e_u\langle\bar{u}u\rangle+e_d\langle\bar{d}d\rangle)\left[-(1+t)i_1(\mathcal{T}_1-\mathcal{T}_3,1)\right.$$
$$-2(1+t)i_1(\mathcal{T}_3-\mathcal{T}_4,v)-2(1+2t)i_1(\mathcal{T}_4,1)$$
$$\left.-2(3+t)i_1(\mathcal{T}_2,v)+(2+t)i_1(\mathcal{T}_2,1)\right]$$
$$+\frac{e_u+e_d}{9}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\left[(-1+t)i_1(\mathcal{T}_2,1)+2i_1(\mathcal{T}_2,v)-ti_1(\mathcal{T}_1-\mathcal{T}_3,1)-2ti_1(\mathcal{T}_3-\mathcal{T}_4,v)\right]$$
$$+\frac{t}{9}\left[e_u\langle\bar{u}u\rangle(2\langle\bar{d}d\rangle-\langle\bar{s}s\rangle)+e_d\langle\bar{d}d\rangle(2\langle\bar{u}u\rangle-\langle\bar{s}s\rangle)-e_s\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]i_1(\mathcal{S},1)$$
$$+\frac{1}{9}\left[e_u\langle\bar{u}u\rangle(\langle\bar{d}d\rangle(1+t)-\langle\bar{s}s\rangle)+e_d\langle\bar{d}d\rangle(\langle\bar{u}u\rangle(1+t)-\langle\bar{s}s\rangle)\right.$$
$$\left.-te_s\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]i_1(\tilde{\mathcal{S}},1-2v)$$
$$-\frac{f_{3\gamma}}{36}(1-t)\left[e_u(\langle\bar{d}d\rangle m_d-\langle\bar{s}s\rangle m_s)+e_d(\langle\bar{u}u\rangle m_u-\langle\bar{s}s\rangle m_s)\right]i_1'(\mathcal{V},1-v)$$
$$+\frac{1}{36}(1-t)\left[e_u(\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s)+e_d(\langle\bar{u}u\rangle m_s+\langle\bar{s}s\rangle m_s)\right.$$
$$\left.-2e_s(\langle\bar{d}d\rangle m_d+\langle\bar{u}u\rangle m_u)\right]f_{3\gamma}i_1'(\mathcal{A},1-v) \qquad (19)$$

$$-\sqrt{3}\lambda_{\Sigma^{*0}}\lambda_{\Sigma^0}M_{\Sigma^{*0}}\Sigma_9=-\frac{\bar{u}_0}{32\pi^4}(1-t)(e_u+e_d-2e_s)M^6$$
$$-\frac{1}{8\pi^2}(1+t)\left[e_u\langle\bar{u}u\rangle(m_d+m_s)+e_d\langle\bar{d}d\rangle(m_u+m_s)\right.$$
$$\left.-2e_s\langle\bar{s}s\rangle(m_u+m_d)\right]M^2\left(M^2\chi\varphi_\gamma(u_0)-\frac{1}{2}\mathbb{A}(u_0)\right)$$
$$+\frac{1}{24\pi^2}(1-t)(e_u+e_d-2e_s)f_{3\gamma}\left[i_1'(\mathcal{A}-\mathcal{V},1-v)+\frac{3}{4}\psi^a(u_0)+3\tilde{i}_2(\psi_v)\right]M^4$$
$$+\frac{1}{4\pi^2}(1+t)\left[e_um_u(\langle\bar{d}d\rangle+\langle\bar{s}s\rangle)+e_dm_d(\langle\bar{u}u\rangle+\langle\bar{s}s\rangle)-2e_sm_s(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]\times$$
$$\times\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\left(M^2-\frac{5}{18}m_0^2\right)$$
$$-\frac{m_0^2}{72\pi^2}(1+t)\left[e_u(\langle\bar{d}d\rangle m_s+\langle\bar{s}s\rangle m_d)+e_d(\langle\bar{u}u\rangle m_s+\langle\bar{s}s\rangle m_u)\right.$$
$$\left.-2e_s(\langle\bar{d}d\rangle m_u+\langle\bar{u}u\rangle m_d)\right]\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)$$

$$
\begin{aligned}
&-\frac{1}{12\pi^2}(1+t)\left[e_s\langle\bar{s}s\rangle(m_u+m_d)i_1\left(\mathcal{S}-\tilde{\mathcal{S}}-2\mathcal{T}_1+\mathcal{T}_2-2\mathcal{T}_3+\mathcal{T}_4,1\right)\right.\\
&+(e_u\langle\bar{u}u\rangle m_d+e_d\langle\bar{d}d\rangle m_u)i_1\left(\mathcal{S}-\tilde{\mathcal{S}}+\mathcal{T}_1+\mathcal{T}_2+\mathcal{T}_3+\mathcal{T}_4,1\right)\\
&\left.-m_s(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)i_1\left(2\mathcal{S}-2\tilde{\mathcal{S}}-\mathcal{T}_1+2\mathcal{T}_2-\mathcal{T}_3+2\mathcal{T}_4,1\right)\right]M^2\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&-\frac{m_0^2}{3M^2}\bar{u}_0\left(e_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle+e_d\langle\bar{u}u\rangle\langle\bar{s}s\rangle-2e_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right)\\
&+\frac{m_0^2}{216M^2}(1+t)\left[e_u(\langle\bar{d}d\rangle m_s+\langle\bar{s}s\rangle m_d)+e_d(\langle\bar{u}u\rangle m_s+\langle\bar{s}s\rangle m_u)\right.\\
&\left.-2e_s(\langle\bar{d}d\rangle m_u+\langle\bar{u}u\rangle m_d)\right]f_{3\gamma}\left(4\tilde{i}_2(\psi^v)-\psi^a(u_0)\right)\\
&+\frac{1}{3}(1+t)\left[e_u\langle\bar{u}u\rangle(\langle\bar{d}d\rangle+\langle\bar{s}s\rangle)+e_d\langle\bar{d}d\rangle(\langle\bar{u}u\rangle+\langle\bar{s}s\rangle)-2e_s\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]M^2\chi\varphi_\gamma(u_0)\\
&-\frac{1}{4\pi^2}\left[e_u\langle\bar{u}u\rangle(2m_d+m_s(1+t))+e_d\langle\bar{d}d\rangle(2m_u+m_s(1+t))\right.\\
&\left.-e_s\langle\bar{s}s\rangle(m_u+m_d)(3+t)\right]M^2\tilde{\tilde{i}}_2(h_\gamma)\\
&+\frac{u_0}{4\pi^2}(1+t)\left[\langle\bar{d}d\rangle(2e_s+e_u)(m_u-m_s)+\langle\bar{u}u\rangle(2e_s+e_d)(m_d-m_s)\right.\\
&\left.+\langle\bar{s}s\rangle(e_u-e_d)(m_u-m_d)\right]M^2\\
&-\frac{\bar{u}_0}{4\pi^2}t\left[m_s(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)-2e_s(m_u\langle\bar{d}d\rangle+m_d\langle\bar{u}u\rangle)+e_sm_s(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right]M^2\\
&-\frac{\bar{u}_0}{8\pi^2}(1-t)\left[e_u(m_s\langle\bar{s}s\rangle+m_d\langle\bar{d}d\rangle)+e_d(m_s\langle\bar{s}s\rangle+m_u\langle\bar{u}u\rangle)-2e_s(m_u\langle\bar{u}u\rangle+m_d\langle\bar{d}d\rangle)\right]M^2\\
&+\frac{\bar{u}_0}{4\pi^2}\left[te_u\langle\bar{s}s\rangle(m_u-m_d)+te_d\langle\bar{s}s\rangle(m_d-m_u)\right.\\
&\left.+(3+t)(e_u\langle\bar{d}d\rangle m_u+e_d\langle\bar{u}u\rangle m_d-e_sm_s(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle))\right]M^2\\
&+\frac{m_s}{12\pi^2}(e_u\langle\bar{u}u\rangle+e_d\langle\bar{d}d\rangle)\left[i_1(\mathcal{S}-\tilde{\mathcal{S}}+\mathcal{T}_2+\mathcal{T}_4,1+3t+2v)-2(1+t)i_1(\mathcal{T}_3,1-v)\right]M^2\\
&+\frac{1}{12\pi^2}(e_u\langle\bar{u}u\rangle m_d+e_d\langle\bar{d}d\rangle m_u)\left[(1-3t)i_1(\mathcal{S}+\mathcal{T}_2,1)-4i_1(\mathcal{S}+\mathcal{T}_2,v)\right.\\
&\left.+(1+t)i_1(\tilde{\mathcal{S}}-\mathcal{T}_4,1+2v)-2(1+t)i_1(\mathcal{T}_3,1-v)\right]M^2\\
&-e_s\frac{\langle\bar{s}s\rangle}{6\pi^2}(m_u+m_d)i_1(\mathcal{S}-t\tilde{\mathcal{S}}+\mathcal{T}_2-2(1+t)\mathcal{T}_3+t\mathcal{T}_4,1-v)M^2\\
&+\frac{1}{3}(1+t)(e_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle+e_d\langle\bar{u}u\rangle\langle\bar{s}s\rangle-2e_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle)\\
&+\frac{m_0^2}{16\pi^2}u_0(1+t)\left[e_u(\langle\bar{d}d\rangle m_s+\langle\bar{s}s\rangle m_d)+e_d(\langle\bar{u}u\rangle m_s+\langle\bar{s}s\rangle m_u)-2e_s(\langle\bar{u}u\rangle m_d+\langle\bar{d}d\rangle m_u)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{m_0^2}{72\pi^2}\bar{u}_0(5-t)(e_u m_u + e_d m_d)\langle\bar{s}s\rangle + \frac{5}{36\pi^2}e_s m_0^2 m_s \bar{u}_0(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) \\
&-\frac{m_0^2}{72\pi^2}\bar{u}_0(15-t)(e_u\langle\bar{d}d\rangle m_u + e_d\langle\bar{u}u\rangle m_d) \\
&+\frac{7m_0^2}{144\pi^2}\bar{u}_0(1+t)\left[e_u(\langle\bar{d}d\rangle m_s + \langle\bar{s}s\rangle m_d) + e_d(\langle\bar{u}u\rangle m_s + \langle\bar{s}s\rangle m_u) - 2e_s(m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle)\right] \\
&+\frac{m_0^2}{72\pi^2}\bar{u}_0(1-t)\left[e_u(\langle\bar{d}d\rangle m_d + \langle\bar{s}s\rangle m_s) + e_d(\langle\bar{u}u\rangle m_u + \langle\bar{s}s\rangle m_s) - 2e_s(\langle\bar{u}u\rangle m_u + \langle\bar{d}d\rangle m_d)\right] \\
&-\frac{1}{12}(1+t)\left[e_u\langle\bar{u}u\rangle(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle) + e_d\langle\bar{d}d\rangle(\langle\bar{u}u\rangle + \langle\bar{s}s\rangle) - 2e_s\langle\bar{s}s\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)\right] \times \\
&\times\left(\mathbb{A}(u_0) + \frac{10}{9}m_0^2\chi\varphi_\gamma(u_0)\right) \\
&+\frac{1}{6}\left[e_u(\langle\bar{d}d\rangle m_s + \langle\bar{s}s\rangle m_d) + e_d(\langle\bar{u}u\rangle m_s + \langle\bar{s}s\rangle m_u) - 2e_s(\langle\bar{d}d\rangle m_u + \langle\bar{u}u\rangle m_d)\right] \times \\
&\times f_{3\gamma}\left(t\psi^a(u_0) - 4\tilde{i}_2(\psi^v)\right) \\
&+\frac{1-t}{24}\left[e_u(\langle\bar{d}d\rangle m_d + \langle\bar{s}s\rangle m_s) + e_d(\langle\bar{u}u\rangle m_u + \langle\bar{s}s\rangle m_s) - 2e_s(\langle\bar{d}d\rangle m_d + \langle\bar{u}u\rangle m_u)\right] \times \\
&\times f_{3\gamma}\left(\psi^a(u_0) + 4\tilde{i}_2(\psi^v)\right) \\
&+\frac{1}{3}\left[e_u\langle\bar{u}u\rangle(2\langle\bar{d}d\rangle + (1+t)\langle\bar{s}s\rangle) + e_d\langle\bar{d}d\rangle(2\langle\bar{u}u\rangle + (1+t)\langle\bar{s}s\rangle)\right. \\
&\left.-e_s\langle\bar{s}s\rangle(3+t)(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)\right]\tilde{\tilde{i}}_2(h_\gamma) \\
&+\frac{1}{18}(1-t)\left[e_u\langle\bar{d}d\rangle m_d + e_d\langle\bar{u}u\rangle m_u - e_s(\langle\bar{d}d\rangle m_d + \langle\bar{u}u\rangle m_u)\right] f_{3\gamma}i_1'(\mathcal{A}-\mathcal{V}, 1-v) \\
&-\frac{1}{9}\langle\bar{s}s\rangle(e_u\langle\bar{u}u\rangle + e_d\langle\bar{d}d\rangle)\left[i_1(\mathcal{S} - \tilde{\mathcal{S}} + \mathcal{T}_2 + \mathcal{T}_4, -1+t+2v)\right. \\
&\left.+(1+t)i_1(\mathcal{T}_1, 1) - (1+t)i_1(\mathcal{T}_3, 1-2v)\right] \\
&-e_s\frac{\langle\bar{s}s\rangle}{9}(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)\left[(-1+t)i_1(\mathcal{S} + \tilde{\mathcal{S}} + \mathcal{T}_2 - \mathcal{T}_4, 1) + 2i_1(\mathcal{S} - t\tilde{\mathcal{S}} + \mathcal{T}_2 + t\mathcal{T}_4, v)\right. \\
&\left.-2(1+t)i_1(\mathcal{T}_1, 1) + 2(1+t)i_1(\mathcal{T}_3, 1-2v)\right] \\
&+\frac{1}{9}\langle\bar{u}u\rangle\langle\bar{d}d\rangle(e_u + e_d)\left[2(-1+t)i_1(\mathcal{S} + \mathcal{T}_2, 1) + 4i_1(\mathcal{S} + \mathcal{T}_2, v) - 2(1+t)i_1(\tilde{\mathcal{S}} - \mathcal{T}_4, v)\right. \\
&\left.-(1+t)i_1(\mathcal{T}_1, 1) + (1+t)i_1(\mathcal{T}_3, 1-2v)\right] \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
&-\sqrt{3}\lambda_{\Sigma^{*0}}\lambda_{\Sigma^0}\Sigma_{12} = \\
&-\frac{u_0\bar{u}_0}{128\pi^4}(1-t)\left(e_u + e_d - 2e_s\right)M^4
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{6\pi^2}(1-t)\left[(e_d\langle\bar{d}d\rangle m_u+e_u\langle\bar{u}u\rangle m_d)\tilde{i}_1\left(\mathcal{T}_1+\mathcal{T}_2-\mathcal{T}_3-\mathcal{T}_4,1\right)\right.\\
&-m_s\left(e_u\langle\bar{u}u\rangle+e_d\langle\bar{d}d\rangle\right)\tilde{i}_1\left(\mathcal{T}_2-\mathcal{T}_4,1\right)\\
&\left.-e_s\langle\bar{s}s\rangle(m_u+m_d)\tilde{i}_1\left(\mathcal{T}_1-\mathcal{T}_3,1\right)\right]\left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\\
&+\frac{m_0^2}{72\pi^2M^2}(1-t)u_0\bar{u}_0\left(e_u(\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s)\right.\\
&\left.+e_d(\langle\bar{u}u\rangle m_u+\langle\bar{s}s\rangle m_s)-2e_s(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d)\right)\\
&+\frac{\bar{u}_0}{24M^2}(1-t)\left(e_u(\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s)+e_d(\langle\bar{u}u\rangle m_u+\langle\bar{s}s\rangle m_s)\right.\\
&\left.-2e_s(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d)\right)f_{3\gamma}\left(\psi^a(u_0)-4\tilde{i}_2(\psi^v)\right)\\
&+\frac{\bar{u}_0}{3M^2}\left[e_u\langle\bar{u}u\rangle(2\langle\bar{d}d\rangle-(3+t)\langle\bar{s}s\rangle)+e_d\langle\bar{d}d\rangle(2\langle\bar{u}u\rangle-(3+t)\langle\bar{s}s\rangle)\right.\\
&\left.+e_s\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)(1+t)\right]\tilde{\tilde{i}}_2(h_\gamma)\\
&+\frac{2}{9M^2}\left[e_s\langle\bar{s}s\rangle(1+t)(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)\tilde{i}_1\left(\mathcal{T}_2-\mathcal{T}_4,1-v\right)\right.\\
&+\langle\bar{u}u\rangle\langle\bar{d}d\rangle(e_u+e_d)\tilde{i}_1\left(\mathcal{T}_2-\mathcal{T}_4,1+t-2v\right)\\
&\left.-\langle\bar{s}s\rangle(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\tilde{i}_1\left(\mathcal{T}_2-\mathcal{T}_4,2+2t-(3+t)v\right)\right]\\
&-\frac{f_{3\gamma}}{18M^2}(1-t)\left[(e_u+e_d)m_s\langle\bar{s}s\rangle\left(i_1(\mathcal{A}-\mathcal{V},1)+2i_1(\mathcal{V},1-v)\right)\right.\\
&-\left(e_um_d\langle\bar{d}d\rangle+e_dm_u\langle\bar{u}u\rangle\right)i_1(\mathcal{A}-\mathcal{V},v)\\
&\left.-e_s\left(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d\right)i_1(\mathcal{A}+\mathcal{V},1-v)\right]\\
&+\frac{m_0^2}{216M^4}(1-t)\bar{u}_0\left(e_u(\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s)+e_d(\langle\bar{u}u\rangle m_u+\langle\bar{s}s\rangle m_s)\right.\\
&\left.-2e_s(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d)\right)f_{3\gamma}\left(4\tilde{i}_2(\psi^v)-\psi^a(u_0)\right)\\
&+\frac{m_0^2}{54M^4}\bar{u}_0\left[(-5+t)(e_u+e_d)\langle\bar{u}u\rangle\langle\bar{d}d\rangle-5e_s(1+t)\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\right.\\
&\left.+2\langle\bar{s}s\rangle(5+2t)(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\right]\tilde{\tilde{i}}_2(h_\gamma)\\
&-\frac{1}{96\pi^2}(1-t)\bar{u}_0\left(e_u+e_d-2e_s\right)M^2f_{3\gamma}\left(4\tilde{i}_2(\psi^v)-\psi^a(\bar{u}_0)\right)\\
&-\frac{1}{24\pi^2}(1-t)\left(e_u+e_d-2e_s\right)M^2f_{3\gamma}i_1(\mathcal{A}+\mathcal{V},1-v)\\
&-\frac{1}{6\pi^2}\left[(e_d\langle\bar{d}d\rangle m_u+e_u\langle\bar{u}u\rangle m_d)\left((-1+t)\tilde{i}_1(\mathcal{T}_1-\mathcal{T}_3,1)+2\tilde{i}_1(\mathcal{T}_2-\mathcal{T}_4,t-v)\right)\right.\\
&-e_s\langle\bar{s}s\rangle(m_u+d_d)\left((-1+t)\tilde{i}_1(\mathcal{T}_1-\mathcal{T}_3,1)-(1+t)\tilde{i}_1(\mathcal{T}_2-\mathcal{T}_4,1-v)\right)
\end{aligned}
$$

$$
\begin{aligned}
&-m_s(e_d\langle\bar{d}d\rangle + e_u\langle\bar{u}u\rangle)\left((1+3t)\tilde{i}_1(\mathcal{T}_2-\mathcal{T}_4,1)-(3+t)\tilde{i}_1(\mathcal{T}_2-\mathcal{T}_4,v)\right)\Big]\\
&-\frac{1}{16\pi^2}u_0\bar{u}_0(1-t)\left(e_u(\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s)+e_d(\langle\bar{u}u\rangle m_u+\langle\bar{s}s\rangle m_s)-2e_s(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d)\right)\\
&-\frac{\bar{u}_0}{8\pi^2}\left[e_u\langle\bar{u}u\rangle(2m_d-(3+t)m_s)+e_d\langle\bar{d}d\rangle(2m_u-(3+t)m_s)\right.\\
&\left.+e_s\langle\bar{s}s\rangle(m_u+m_d)(1+t)\right]\tilde{\tilde{i}}_2(h_\gamma)
\end{aligned}
\tag{21}
$$

where $\frac{1}{M^2}=\frac{1}{M_1^2}+\frac{1}{M_2^2}$, M_i^2 are the Borel parameters. The contributions of the continuum and the higher states are subtracted using the quark hadron duality by replacing M^{2n} by $M^{2n}E_n(x)$ and replacing $M^{2n}\ln\frac{M^2}{\Lambda^2}$ by $M^{2n}\left(\ln\frac{M^2}{\Lambda^2}-E_n(x,M^2)\right)$ for $n>0$, where $x=\frac{s_0}{M^2}$, s_0 is the continuum threshold,

$$
\begin{aligned}
E_n(x) &= 1-e^{-x}\sum_{i=0}^{n}\frac{x^i}{i!}\\
E_n(x,M^2) &= \frac{1}{\Gamma(n)}\int_x^\infty ds s^{n-1}\left(\ln\frac{sM^2}{\Lambda^2}-\psi(n)\right)
\end{aligned}
\tag{22}
$$

where $\psi(n)$ is the digamma function.

The functions $i_n,\ i'_n,\ \tilde{i}_n,\ \tilde{\tilde{i}}_n$ appearing in Eqs. (19-21) are defined as:

$$
\begin{aligned}
i_1(\varphi,f(v)) &= \int\mathcal{D}\alpha_i\int_0^1 dv\varphi(\alpha_{\bar{q}},\alpha_q,\alpha_g)f(v)\delta(k-\bar{u}_0)\\
i'_1(\varphi,f(v)) &= \int\mathcal{D}\alpha_i\int_0^1 dv\varphi(\alpha_{\bar{q}},\alpha_q,\alpha_g)f(v)\delta'(k-\bar{u}_0)\\
i''_1(\varphi,f(v)) &= \int\mathcal{D}\alpha_i\int_0^1 dv\varphi(\alpha_{\bar{q}},\alpha_q,\alpha_g)f(v)\delta''(k-\bar{u}_0)\\
\tilde{i}_1(\varphi,f(v)) &= \int\mathcal{D}\alpha_i\int_0^1 dv\int_0^{\alpha_{\bar{q}}+v\alpha_g} dk\varphi(\alpha_{\bar{q}},\alpha_q,\alpha_g)f(v)\delta(k-\bar{u}_0)\\
\tilde{i}_2(f) &= \int_{\bar{u}_0}^1 duf(u)\\
\tilde{\tilde{i}}_2(f) &= \int_{\bar{u}_0}^1 du(u-\bar{u}_0)f(u)
\end{aligned}
\tag{23}
$$

where $k=\alpha_{\bar{q}}+v\alpha_g$ when there is no integration over k, $u_0=\frac{M_1^2}{M_1^2+M_2^2}$ and $\int\mathcal{D}\alpha_i=\int_0^1 d\alpha_{\bar{q}}\int_0^1 d\alpha_q\int_0^1 d\alpha_g\delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g)$. Since, for the reactions under considerations, $m \simeq M$, we will set the Borel parameters to be equal, i.e $M_1^2 = M_2^2 = 2M^2$ and $u_0 = \frac{1}{2}$.

In order to obtain the values of the moments G_X, $(X = 1,\ 2,\ E,\ M)$, we also need the expression for the residues. The residues can be obtained from the mass sum rules. For completeness, we also include the mass sum rules that we used to obtain the values of the residues [24, 25]:

$$
\begin{aligned}
\lambda_{\Sigma^0}^2 e^{-\frac{m_\Sigma^2}{M^2}} \;=\; & \frac{M^6}{1024\pi^2}(5+2t+5t^2)E_2(x) - \frac{m_0^2}{96M^2}(-1+t)^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle \\
& -\frac{m_0^2}{16M^2}(-1+t^2)\langle\bar{s}s\rangle(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle) \\
& +\frac{3}{128m_0^2}m_0^2(-1+t^2)\left[m_s(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)+\langle\bar{s}s\rangle(m_u+m_d)\right] \\
& -\frac{1}{64\pi^2}(-1+t)^2\left(\langle\bar{d}d\rangle m_u+\langle\bar{u}u\rangle m_d\right)M^2E_0(x) \\
& -\frac{3}{64\pi^2}(-1+t^2)\left(m_s(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)+\langle\bar{s}s\rangle(m_u+m_d)\right)M^2E_0(x) \\
& +\frac{1}{128\pi^2}(5+2t+5t^2)\left(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d+\langle\bar{s}s\rangle m_s\right) \\
& +\frac{1}{24}\left[3\langle\bar{s}s\rangle(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)\left(-1+t^2\right)+(-1+t)^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right] \\
& +\frac{m_0^2}{256\pi^2}(-1+t)^2\left(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle\right) \\
& +\frac{m_0^2}{256\pi^2}(-1+t^2)\left[13m_s(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)+11\langle\bar{s}s\rangle(m_d+m_u)\right] \\
& -\frac{m_0^2}{192\pi^2}(1+t+t^2)\left(\langle\bar{u}u\rangle m_u+\langle\bar{d}d\rangle m_d-2m_s\langle\bar{s}s\rangle\right) \qquad (24)
\end{aligned}
$$

and

$$
\begin{aligned}
M_{\Sigma^{*0}}\lambda_{\Sigma^{*0}}^2 e^{-\frac{m_\Sigma^2}{M^2}} \;=\; & \left(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle+\langle\bar{s}s\rangle\right)\frac{M^4}{9\pi^2}E_1(x)-(m_u+m_d+m_s)\frac{M^6}{32\pi^4}E_2(x) \\
& -\left(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle+\langle\bar{s}s\rangle\right)m_0^2\frac{M^2}{18\pi^2}E_0(x) \\
& -\frac{2}{3}\left(1+\frac{5m_0^2}{72M^2}\right)\left(m_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle+m_d\langle\bar{s}s\rangle\langle\bar{u}u\rangle+m_s\langle\bar{d}d\rangle\langle\bar{u}u\rangle\right) \\
& +\left(m_s\langle\bar{d}d\rangle\langle\bar{s}s\rangle+m_u\langle\bar{d}d\rangle\langle\bar{u}u\rangle+m_d\langle\bar{s}s\rangle\langle\bar{u}u\rangle\right)\frac{m_0^2}{12M^2} \qquad (25)
\end{aligned}
$$

where $x = \frac{s_0}{M^2}$. (The mass sum rule for the Λ baryon can be obtained from Eq. (24) using the relation given in [22])

Note that, from the mass sum rules, one can only obtain the square of the residues. Hence, from the mass sum rules it is not possible to deduce the sign of the residues. Thus, in this work, we can not predict the absolute sign of the moments. For comparison with the experimental data, we will also consider the following ratio:

$$\mathcal{R}_{EM} = -\frac{G_E}{G_M} \tag{26}$$

Although we do not predict the sign of G_E and G_M separately, the sign of $\mathcal{R}_{EM}$ is predicted by light cone QCD sum rules. Our sign convention for the residues are such that $\lambda_{\mathcal{D}} > 0$ and $\lambda_{\mathcal{O}} > 0$ as $t \to \infty$.

An interesting limit to consider is the $SU(3)_f$ limit, i.e. the limit $m_u = m_d = m_s$ and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle$, which allows one to establish relationship between the transition amplitudes. In this limit, Eqs. (19-21) are proportional to $e_u + e_d - 2e_s$. Using Eq. (14), one obtains the following relations:

$$\begin{aligned} \Sigma^{\Xi^{*-} \to \Xi^-} &= \Sigma^{\Sigma^{*-} \to \Sigma^-} = 0 \\ 2\Sigma^{\Sigma^{*0} \to \Sigma^0} &= -\sqrt{3}\Sigma^{\Sigma^{*0} \to \Lambda} = \Sigma^{\Delta^+ \to p} = -\Sigma^{\Delta^0 \to n} = \Sigma^{\Sigma^{*+} \to \Sigma^+} = -\Sigma^{\Xi^{*0} \to \Xi^0} \end{aligned} \tag{27}$$

which are the well known $SU(3)_f$ relationships. Note that, Eq. (27) imply that $\mathcal{R}_{EM}$ is the same for all considered processes. Here, we should remind once more that the signs are not predictions of the LCQSR, only relative factors up to a sign are reproduced by the LCQSR. The original sign convention is chosen such that the $SU(3)$ relations are reproduced including the signs. Since we are neglecting the masses of the u and d quarks and the differences in their condensates, isospin subgroup of $SU(3)_f$ remains unbroken. Hence, the relations $\Sigma^{\Delta^+ \to p} = -\Sigma^{\Delta^0 \to n}$ still holds in the approximation that we are considering.

At the end of this section, we present the decay width for the decay $\mathcal{D} \to \mathcal{O}\gamma$ in terms of the multipole moments G_E and G_M:

$$\Gamma_\gamma = 3\frac{\alpha}{32}\frac{(M_{\mathcal{D}}^2 - m_{\mathcal{O}}^2)^3}{M_{\mathcal{D}}^3 m_{\mathcal{O}}^2}\left(G_M^2 + 3G_E^2\right) \tag{28}$$

or in terms of the helicity amplitudes used by PDG [26]:

$$\Gamma_\gamma = \frac{w^2}{\pi}\frac{m_{\mathcal{O}}}{2M_{\mathcal{D}}}\left(A_{1/2}^2 + A_{3/2}^2\right) \tag{29}$$

where w is the energy of the photon and the helicity amplitudes are defined as:

$$\begin{aligned} A_{1/2} &= -\eta\,(G_M - 3G_E) \\ A_{3/2} &= -\sqrt{3}\eta\,(G_M + G_E) \end{aligned} \tag{30}$$

where

$$\eta = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\frac{M_{\mathcal{D}}^2 - m_{\mathcal{O}}^2}{2m_{\mathcal{O}}}\right)^{1/2}\frac{e}{2m_{\mathcal{O}}}$$

3 Numerical Analysis

For the numerical values of the input parameters, the following values are used: $\langle\bar{u}u\rangle(1\ GeV) = \langle\bar{d}d\rangle(1\ GeV) = -(0.243)^3\ GeV^3$, $\langle\bar{s}s\rangle(1\ GeV) = 0.8\langle\bar{u}u\rangle(1\ GeV)$, $m_0^2(1\ GeV) = 0.8$ [27], $\chi(1\ GeV) = -4.4\ GeV^{-2}$ [28], $\Lambda = 300\ MeV$ and $f_{3\gamma} = -0.0039\ GeV^2$ [23]. The photon wave functions are: [23]

$$\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u}\left(1 + \varphi_2(\mu)C_2^{\frac{3}{2}}(u - \bar{u})\right) \\
\psi^v(u) &= 3\left(3(2u-1)^2 - 1\right) + \frac{3}{64}\left(15w_\gamma^V - 5w_\gamma^A\right)\left(3 - 30(2u-1)^2 + 35(2u-1)^4\right) \\
\psi^a(u) &= \left(1 - (2u-1)^2\right)\left(5(2u-1)^2 - 1\right)\frac{5}{2}\left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A\right) \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2\left(1 + w_\gamma^A\frac{1}{2}(7\alpha_g - 3)\right) \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2 \\
h_\gamma(u) &= -10\left(1 + 2\kappa^+\right)C_2^{\frac{1}{2}}(u - \bar{u}) \\
\mathbb{A}(u) &= 40u^2\bar{u}^2\left(3\kappa - \kappa^+ + 1\right) \\
&\quad +8(\zeta_2^+ - 3\zeta_2)\,[u\bar{u}(2 + 13u\bar{u}) \\
&\quad + 2u^3(10 - 15u + 6u^2)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})] \\
\mathcal{T}_1(\alpha_i) &= -120(2\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)\left((\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)\right) \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)\left((\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)\right)
\end{aligned} \tag{31}$$

The constants appearing in the wave functions are given as [23] $\varphi_2(1\ GeV) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$

Once the input parameter are determined, the next task is to find the continuum threshold, s_0, and a suitable region of the Borel mass, M^2, for each of the processes. To find an upper bound for the Borel mass parameter, M^2, we required the continuum contribution to be less then the contribution of continuum subtracted sum rules, and requiring that the contribution of the highest power of $\frac{1}{M^2}$ be less than 20% of the highest power of M^2, gave a lower bound on M^2. Using these constraints we found that for $\Delta \to N\gamma$ transitions, $s_0 = 3{-}4\ GeV^2$ and $0.9\ GeV^2 < M^2 < 1.2\ GeV^2$; for the $\Sigma^* \to \Sigma$ and $\Sigma^{*0} \to \Lambda$ transitions, $s_0 = 3 - 5\ GeV^2$ and $0.9\ GeV^2 < M^2 < 1.2\ GeV^2$; and for the $\Xi^* \to \Xi\gamma$ transitions, $s_0 = 3 - 5\ GeV^2$ and $1.1\ GeV^2 < M^2 < 1.4\ GeV^2$.

In Figs. (1) and (2), we depict the dependence of $|G_E|$ obtained from Σ_6 and Σ_{12}, and Σ_9 and Σ_{12}, respectively, on $\cos(\theta)$, where θ is defined through $t = \tan(\theta)$ for the decays $\Delta \to N$. Two common features of these graphs are that they become large near $\cos(\theta) = \pm 1$, and they go to zero at some finite value of $\cos(\theta)$ These behavior can be understood in the following way: The correlation function Eq. (1) is a linear function of t. From the correlation function, one can only obtain the product $\lambda_{\mathcal{O}}(t)\lambda_{\mathcal{D}}G_E$, and hence this product, obtained from the correlation function, is also a linear function of t. In particular, there is a point $t = t_0$ at which this product goes to zero. On the other hand, from the definition of $\lambda_{\mathcal{O}}(t)$, Eq. (4), it is also seen that the residue is also a linear function of t, which has to be determined using the mass sum rules Eq. (24), and consequently there is a point $t = t_0'$ at which the residue goes to zero. If one could make an exact calculation of the correlation function Eq. (1) and the mass sum rule Eq. (24), then one should obtain $t_0 = t_0'$, but due to the approximations used, these two points do not coincide. This is reflected in Figs. (1) and (2) as a point at which $|G_E|$ go to zero (at $t = t_0$) and a point at which $|G_E|$ becomes very large (near $t = t_0'$). These points and any region between them, as well as any enhancement/suppression near these points, is an artifact of the approximations used, and hence the suitable region for t should be away from these points.

Another point to observe comparing these two figures is that in Fig. (1), the points at which $|G_E|$ vanishes and that it gets very large are very close together whereas in Fig. (2), they are farther apart. In particular, in Fig.

(2), the Ioffe current, which corresponds to $\cos(\theta) = -\frac{1}{\sqrt{2}} \simeq -0.71$, is out of the suitable region for t. For the working region of $\cos(\theta)$, if one chooses $-0.5 \leq \cos(\theta) \leq 0.5$ for Fig. (1), and $-0.2 \leq \cos(\theta) \leq 0.3$ for Fig. (2), one obtains $|G_E| = 0.17 \pm 0.04$ and $|G_E| = 0.21 \pm 0.07$ respectively, i.e. within the respective working regions of t, using the sum rules for Σ_6 and Σ_{12} or Σ_9 and Σ_{12}, the predictions obtained for G_E are in agreement. But due to the better stability of the results obtained using the sum rules for Σ_6 and Σ_{12} with respect to variations of t and hence a larger working region of $\cos(\theta)$, from now on we will present only the results of the sum rules obtained from Σ_6 and Σ_{12}.

Figs. (3) and (4), are the same as Figs. (1) and (2), but for $|G_M|$. The general features of the dependence of $|G_M|$ on $\cos(\theta)$ is the same as those of $|G_E|$, i.e. Fig. (3) is more stable than Fig. (4) as a function of t and hence has a larger working region for $\cos(\theta)$.

In Figs. (5)-(30), the dependence of G_E and G_M on M^2 for various processes and different values of the continuum threshold is depicted for the Ioffe current and the limit $|t| \to \infty$ ($\cos(\theta) = 0$). The Ioffe current always gives smaller results due to its vicinity to the point where the correlation function vanishes as a function of t. From the figures it is seen that all of the sum rules are stable with respect to small variation of s_0 in the region of M^2 considered.

In Table (1), we show our results for the moments G_X ($X =\ E,\ M$) and also $\mathcal{R}_{EM}$ values for the central values of G_E and G_M. For the values in the table, the value of $\cos(\theta)$ is restricted to be $-0.5 \leq \cos(\theta) \leq 0.5$ The errors quoted are due to the variations of the Borel mass M^2, the continuum threshold s_0 and the t parameter. The largest uncertainty is due to the residual dependence on the value of t. The non zero values for the moments of the transitions $\Sigma^{*-} \to \Sigma^-$ and $\Xi^{*-} \to \Xi^-$ are purely due to $SU(3)_f$ violating effects. For the other decays, within theoretical errors, the values of G_M respect the $SU(3)_f$ flavor symmetry.

In Table (2), we present our results and the results obtained from lattice calculation [11]. Note that, the conventions that we use are different from the conventions used in [11]. This difference leads to a factor of $\sqrt{3/2}$ between our results and the results in [11]. Hence, to make a comparison, the values we quote in Table (2) are the results given in [11] multiplied by $\sqrt{2/3}$. We see that for the values of G_M, our results are in agreement with the results from lattice within error bars. The main reason for our large error bars are

Process	G_E	G_M	$\mathcal{R}_{EM}(\%)$
$\Delta^+ \to p$	0.17 ± 0.05	2.5 ± 1.3	-6.8
$\Delta^0 \to n$	-0.17 ± 0.05	-2.5 ± 1.3	-6.8
$\Sigma^{*+} \to \Sigma^+$	0.08 ± 0.02	2.1 ± 0.85	-3.8
$\Sigma^{*0} \to \Sigma^0$	0.034 ± 0.007	0.89 ± 0.38	-3.8
$\Sigma^{*0} \to \Lambda$	-0.13 ± 0.02	-2.3 ± 1.4	-5.7
$\Sigma^{*-} \to \Sigma^-$	-0.010 ± 0.004	-0.31 ± 0.10	-3.2
$\Xi^{*0} \to \Xi^0$	-0.09 ± 0.02	-2.2 ± 0.74	-4.1
$\Xi^{*-} \to \Xi^-$	0.011 ± 0.003	0.31 ± 0.11	-3.5

Table 1: The predictions on the moments for various decays. The magnetic moments are given in terms of natural magnetons

Process	G_E	$G_E^{[11]}$	G_M	$G_M^{[11]}$	$\mathcal{R}_{EM}(\%)$	$\mathcal{R}_{EM}^{[11]}(\%)$
$\Delta^+ \to p$	0.17 ± 0.05	-0.04(11)	2.5 ± 1.3	2.01(33)	-6.8	3(8)
$\Delta^0 \to n$	-0.17 ± 0.05	0.04(11)	-2.5 ± 1.3	-2.01(33)	-6.8	3(8)
$\Sigma^{*+} \to \Sigma^+$	-0.08 ± 0.02	-0.06(8)	2.1 ± 0.85	2.13(16)	-3.8	5(6)
$\Sigma^{*0} \to \Sigma^0$	-0.034 ± 0.007	-0.02(4)	0.89 ± 0.38	0.87(7)	-3.8	4(6)
$\Sigma^{*-} \to \Sigma^-$	-0.010 ± 0.004	0.020(10)	-0.31 ± 0.10	-0.38(4)	-3.2	8(4)
$\Xi^{*0} \to \Xi^0$	-0.09 ± 0.02	0.03(4)	-2.2 ± 0.74	-2.26(14)	-4.1	2.4(27)
$\Xi^{*-} \to \Xi^-$	0.011 ± 0.003	-0.018(7)	0.31 ± 0.11	0.38(3)	-3.5	7.4(30)

Table 2: Our results together with the results from lattice [11]

due to the residual t dependence of our results, where as in [11], only the limit $t \to \infty$ is considered.

For the values of G_E, out results are in agreement with the results of lattice calculations for the channels $\Sigma^{*+(0)} \to \Sigma^{+(0)}$. and the biggest discrepancy between out results and the lattice results are for the channels $\Sigma^{*-} \to \Sigma^-$ and $\Xi^{*-} \to \Xi^-$, where even within error bars, we do not agree even on the sign of G_E. Note that these two channels are the channels for which in the case of exact $SU(3)_f$ symmetry, $G_E = 0$. For the remaining channels $\Delta \to N$ and $\Xi^{*0} \to \Xi^0$, there is not agreements with the results of the lattice calculation but within error, we do agree on the sign of G_E for these channels.

In Table (3), we give our predictions for the helicity amplitudes and the decay widths for the corresponding decay. For the time being, the experimental data is available only for the decays [26] $\Delta \to N\gamma$ for which

Process	$A_{1/2}(GeV^{-1/2})$	$A_{3/2}(GeV^{-1/2})$	$\Gamma(MeV)$
$\Delta^+ \to p$	-0.12 ± 0.09	-0.27 ± 0.14	0.90 ± 0.73
$\Delta^0 \to n$	0.12 ± 0.09	0.27 ± 0.14	0.90 ± 0.73
$\Sigma^{*+} \to \Sigma^+$	-0.067 ± 0.033	-0.14 ± 0.05	0.11 ± 0.82
$\Sigma^{*0} \to \Sigma^0$	-0.029 ± 0.014	-0.057 ± 0.024	0.021 ± 0.015
$\Sigma^{*0} \to \Lambda$	0.088 ± 0.067	0.20 ± 0.11	0.47 ± 0.41
$\Sigma^{*-} \to \Sigma^-$	0.010 ± 0.004	0.020 ± 0.006	0.002 ± 0.001
$\Xi^{*0} \to \Xi^0$	0.067 ± 0.028	0.14 ± 0.05	0.14 ± 0.09
$\Xi^{*-} \to \Xi^-$	-0.010 ± 0.004	-0.019 ± 0.007	0.003 ± 0.002

Table 3: The predictions for the helicity amplitudes and the decay widths for various decays

$A_{1/2} = -0.135 \pm 0.005\ GeV^{-1/2}$, $A_{3/2} = -0.250 \pm 0.008\ GeV^{-1/2}$ and $\Gamma = 0.64 \pm 0.06\ MeV$. It is seen that our predictions for the amplitudes for these decays are in agreement with the experimental results. Recently, SELEX Collaboration has announced an upper bound for the radiative width of the decay $\Sigma^{*-} \to \Sigma^-\gamma$ as $\Gamma(\Sigma^{*-} \to \Sigma^-\gamma) < 9.5\ KeV$[30]. Our prediction for the width of this channel is $\Gamma = 2 \pm 1\ KeV$ which is below the experimental bound.

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Figure Captions

Fig. 1 The dependence of $G_E^{\Delta\to N}$, obtained from Σ_6 and Σ_{12}, on $\cos(\theta)$ for $s_0 = 3\ GeV^2$ and $s_0 = 4\ GeV^2$ for the value of the Borel parameter $M^2 = 1\ GeV^2$

Fig. 2 The same as Fig. 1 but for $G_E^{\Delta\to N}$ obtained from Σ_9 and Σ_{12}

Fig. 3 The same as Fig. 1 but for $G_M^{\Delta\to N}$

Fig. 4 The same as Fig. 2 but for $G_M^{\Delta\to N}$

Fig. 5 The dependence of $G_E^{\Delta^+\to p}$, obtained from Σ_6 and Σ_{12}, on the borel parameter M^2 for $s0 = 3\ GeV^2$ and $s_0 = 4\ GeV^2$. The lines with circles on them correspond to the Ioffe current, $t = -1$, whereas others correspond to $t = \infty$

Fig. 6 The same as Fig. 5 but for $G_M^{\Delta^+\to p}$

Fig. 7 The same as Fig. 1, but for the decay $\Sigma^{*+} \to \Sigma^+$ and for $s0 = 3,\ 4,\ 5\ GeV^2$

Fig. 8 The same as Fig. 5, but for the decay $\Sigma^{*+} \to \Sigma^+$ and for $s0 = 3,\ 4,\ 5\ GeV^2$

Fig. 9 The same as Fig. 7, but for $G_M^{\Sigma^{*+}\to\Sigma^+}$

Fig. 10 The same as Fig. 8, but for $G_M^{\Sigma^{*+}\to\Sigma^+}$

Fig. 11 The same as Fig. 7, but for the decay $\Sigma^{*0} \to \Sigma^0$

Fig. 12 The same as Fig. 8, but for the decay $\Sigma^{*0} \to \Sigma^0$

Fig. 13 The same as Fig. 9, but for the decay $\Sigma^{*0} \to \Sigma^0$

Fig. 14 The same as Fig. 10, but for the decay $\Sigma^{*0} \to \Sigma^0$

Fig. 15 The same as Fig. 7, but for the decay $\Sigma^{*0} \to \Lambda$

Fig. 16 The same as Fig. 8, but for the decay $\Sigma^{*0} \to \Lambda$

Fig. 17 The same as Fig. 9, but for the decay $\Sigma^{*0} \to \Lambda$

Fig. 18 The same as Fig. 10, but for the decay $\Sigma^{*0} \to \Lambda$

Fig. 19 The same as Fig. 7, but for the decay $\Sigma^{*-} \to \Sigma^-$

Fig. 20 The same as Fig. 8, but for the decay $\Sigma^{*-} \to \Sigma^-$

Fig. 21 The same as Fig. 9, but for the decay $\Sigma^{*-} \to \Sigma^-$

Fig. 22 The same as Fig. 10, but for the decay $\Sigma^{*-} \to \Sigma^-$

Fig. 23 The same as Fig. 7, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 24 The same as Fig. 8, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 25 The same as Fig. 9, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 26 The same as Fig. 10, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 27 The same as Fig. 7, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 28 The same as Fig. 8, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 29 The same as Fig. 9, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

Fig. 30 The same as Fig. 10, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

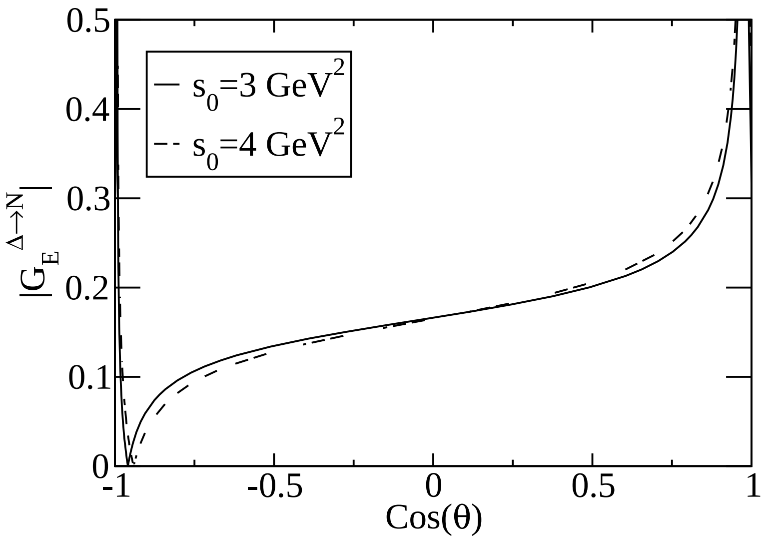


Figure 1: The dependence of $G_E^{\Delta\to N}$, obtained from Σ_6 and Σ_{12}, on $\cos(\theta)$ for $s_0 = 3\ GeV^2$ and $s_0 = 4\ GeV^2$ for the value of the Borel parameter $M^2 = 1\ GeV^2$

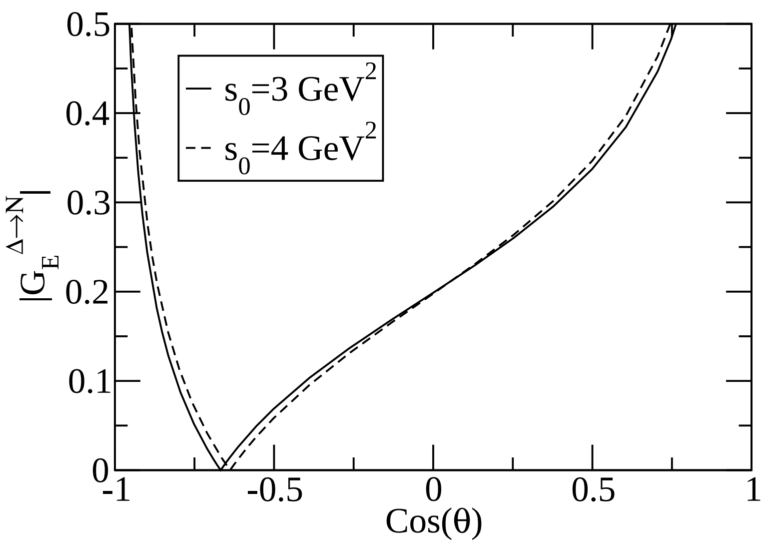


Figure 2: The same as Fig. 1 but for $G_E^{\Delta\to N}$ obtained from Σ_9 and Σ_{12}

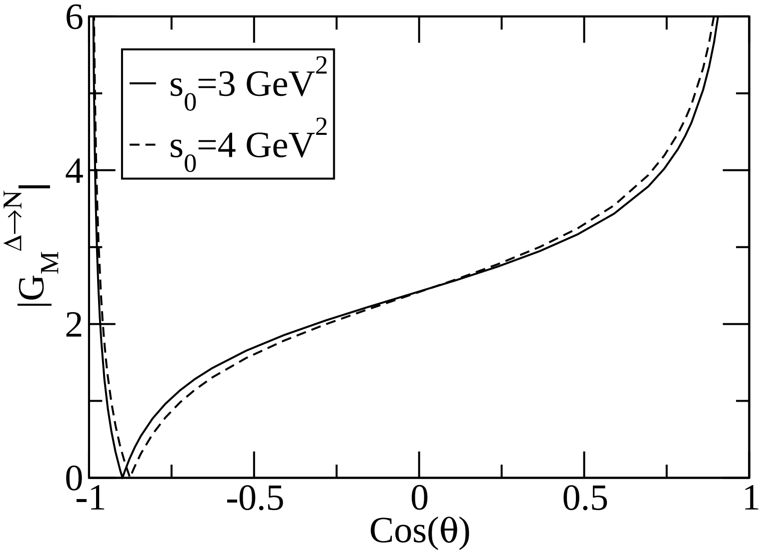


Figure 3: The same as Fig. 1 but for $G_M^{\Delta\to N}$

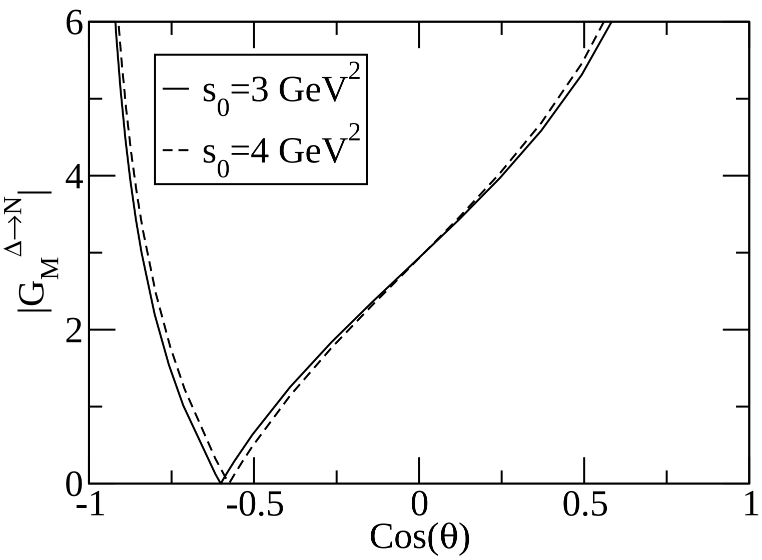


Figure 4: The same as Fig. 2 but for $G_M^{\Delta\to N}$

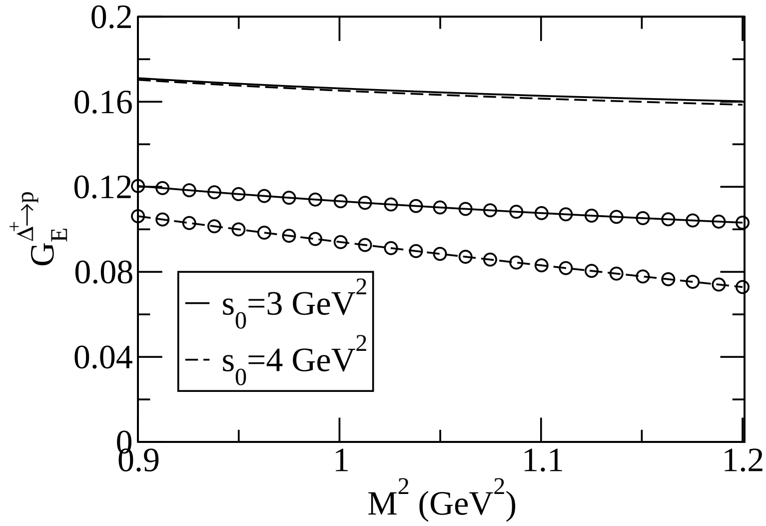


Figure 5: The dependence of $G_E^{\Delta^+\to p}$, obtained from Σ_6 and Σ_{12}, on the borel parameter M^2 for $s0 = 3\ GeV^2$ and $s_0 = 4\ GeV^2$. The lines with circles on them correspond to the Ioffe current, $t = -1$, whereas others correspond to $t = \infty$

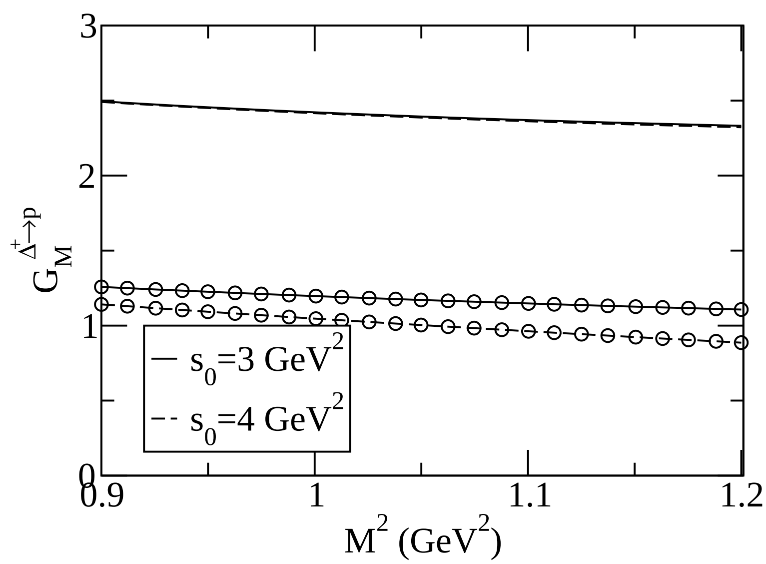


Figure 6: The same as Fig. 5 but for $G_M^{\Delta^+\to p}$

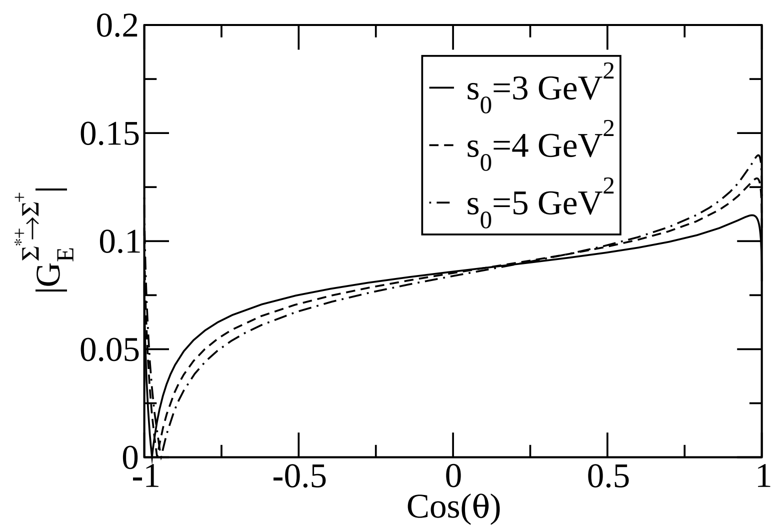


Figure 7: The same as Fig. 1, but for the decay $\Sigma^{*+} \to \Sigma^{+}$ and for $s0 = 3,\ 4,\ 5\ GeV^2$

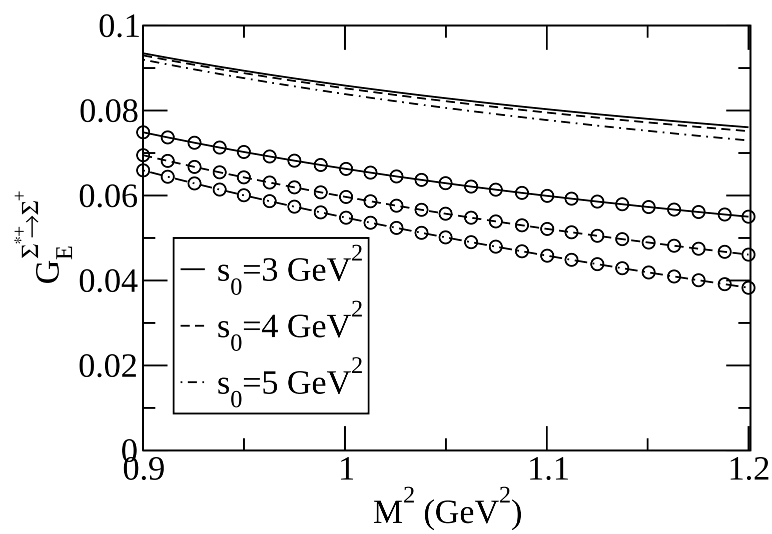


Figure 8: The same as Fig. 5, but for the decay $\Sigma^{*+} \to \Sigma^{+}$ and for $s0 = 3,\ 4,\ 5\ GeV^2$

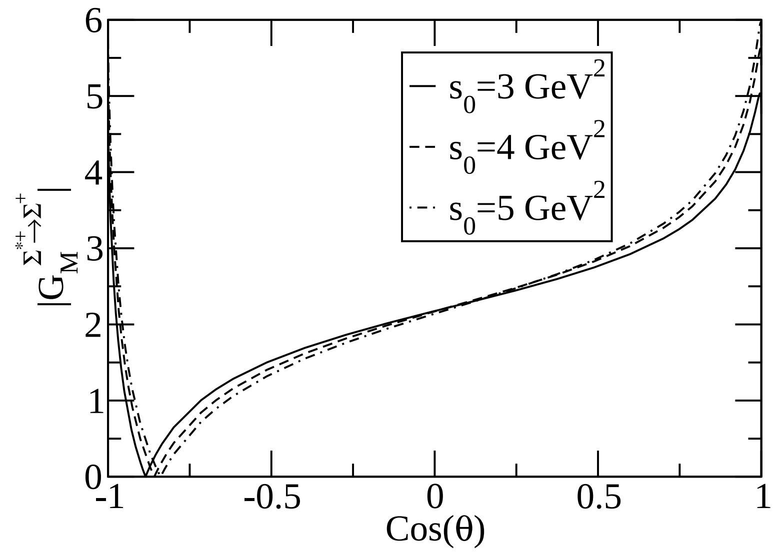


Figure 9: The same as Fig. 7, but for $G_M^{\Sigma^{*+}\to\Sigma^+}$

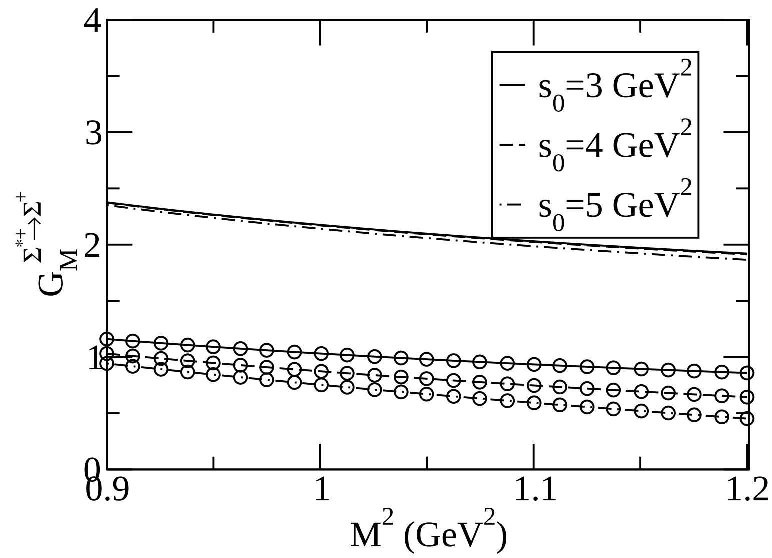
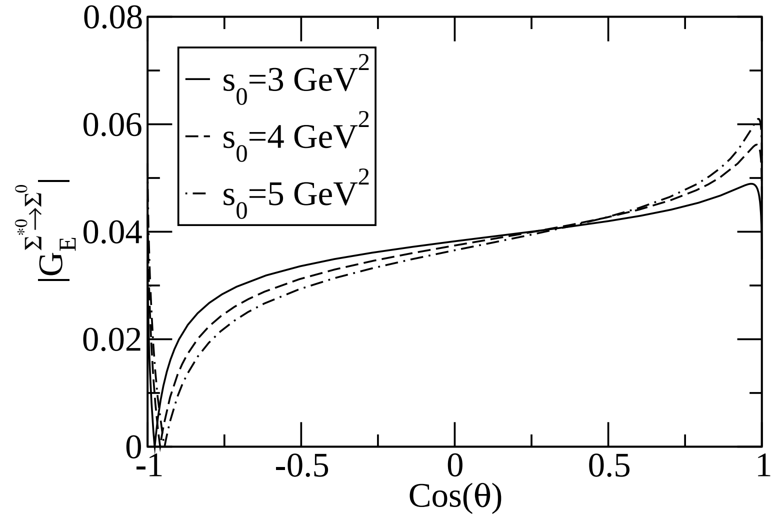


Figure 10: The same as Fig. 8, but for $G_M^{\Sigma^{*+}\to\Sigma^+}$



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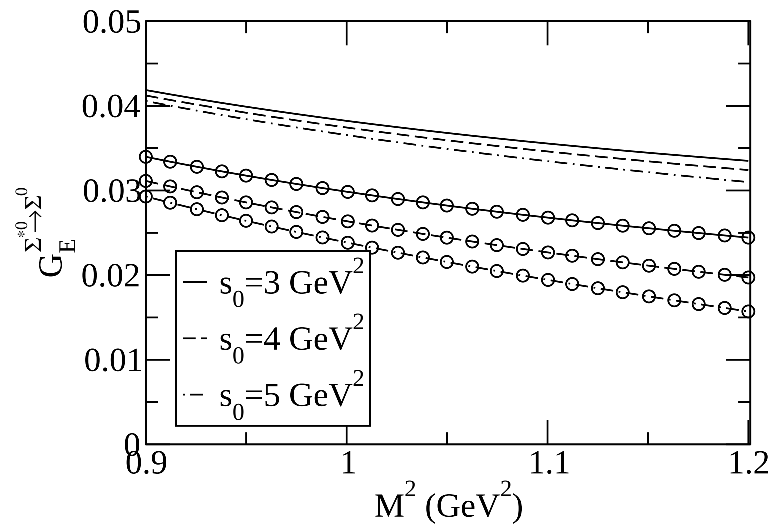

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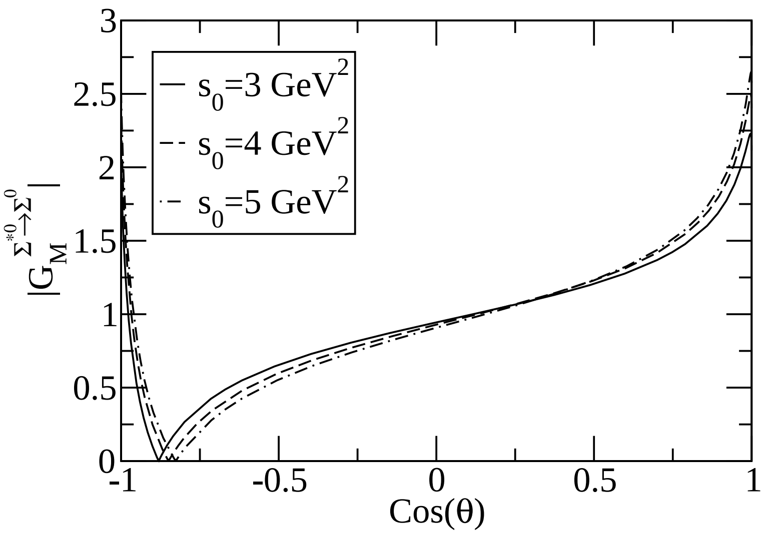

Figure 13: The same as Fig. 9, but for the decay $\Sigma^{*0} \to \Sigma^0$

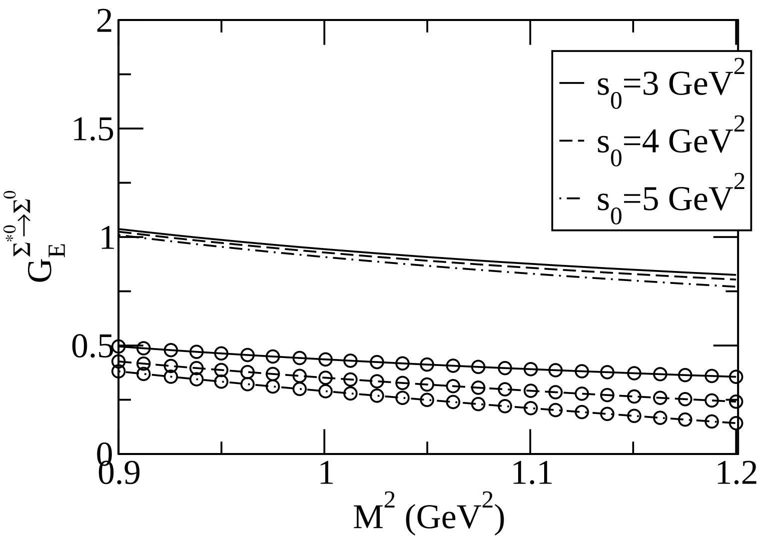

Figure 14: The same as Fig. 10, but for the decay $\Sigma^{*0} \to \Sigma^0$

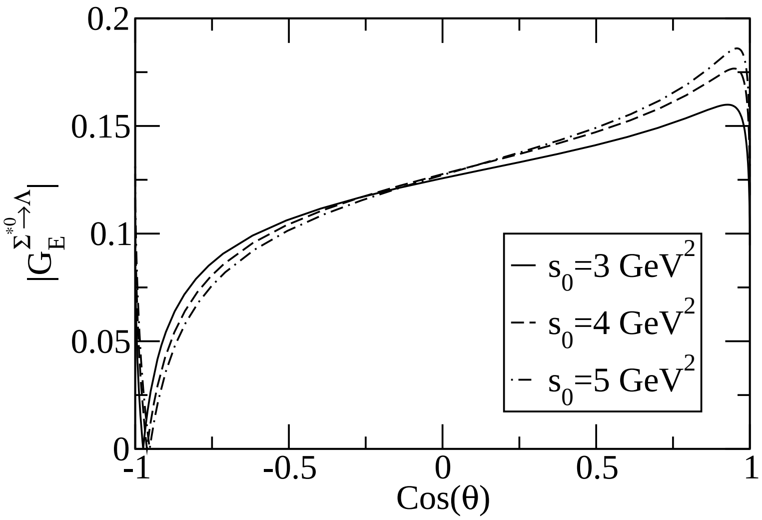

Figure 15: The same as Fig. 7, but for the decay $\Sigma^{*0} \to \Lambda$

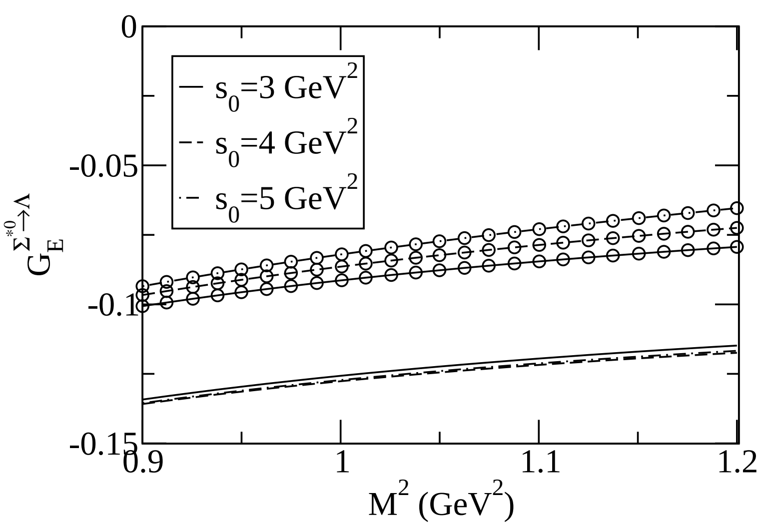

Figure 16: The same as Fig. 8, but for the decay $\Sigma^{*0} \to \Lambda$

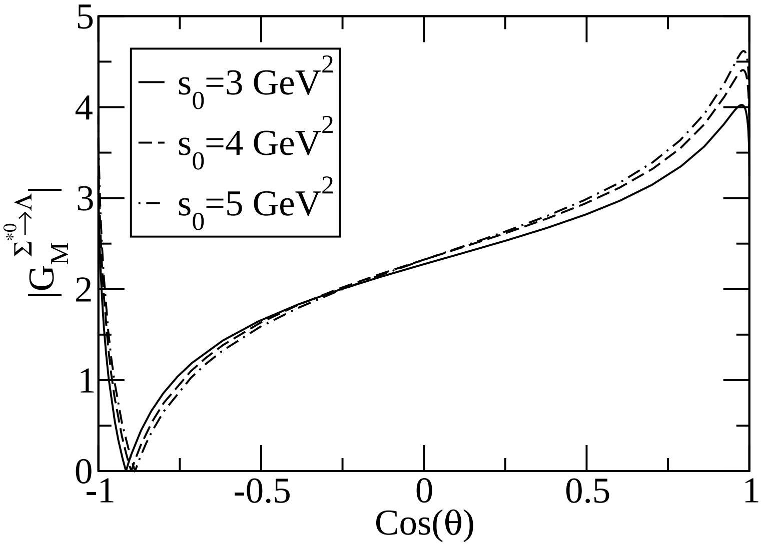

Figure 17: The same as Fig. 9, but for the decay $\Sigma^{*0} \to \Lambda$

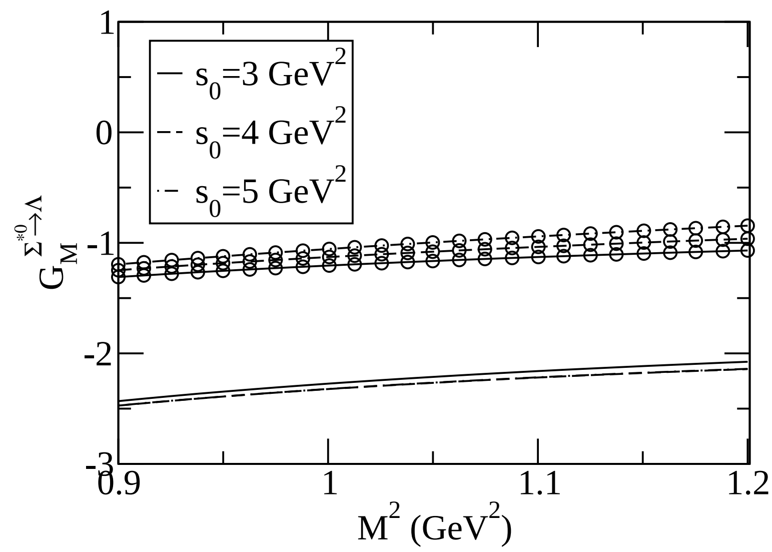

Figure 18: The same as Fig. 10, but for the decay $\Sigma^{*0} \to \Lambda$

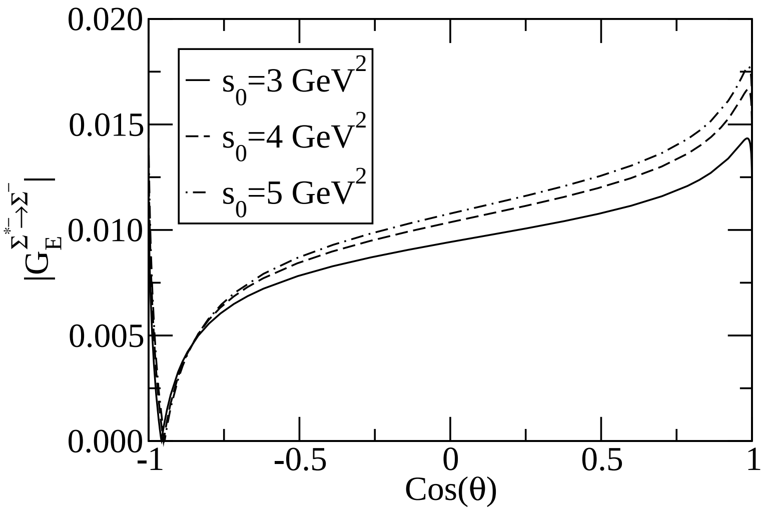

Figure 19: The same as Fig. 7, but for the decay $\Sigma^{*-} \to \Sigma^-$

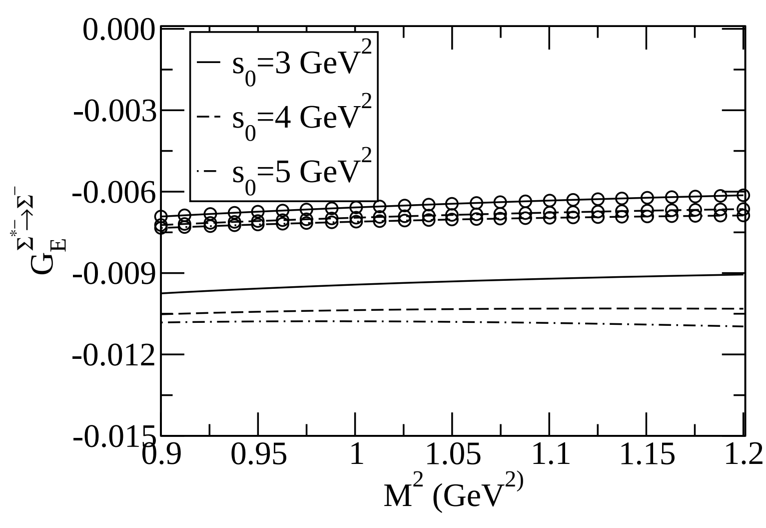

Figure 20: The same as Fig. 8, but for the decay $\Sigma^{*-} \to \Sigma^-$

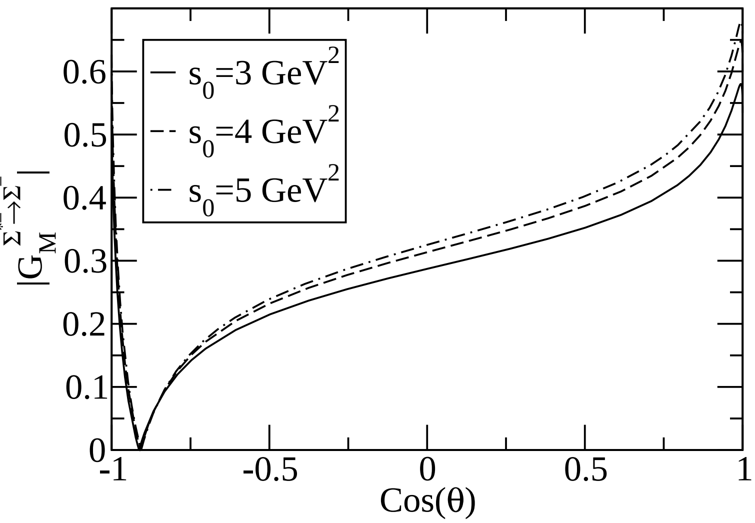

Figure 21: The same as Fig. 9, but for the decay $\Sigma^{*-} \to \Sigma^-$

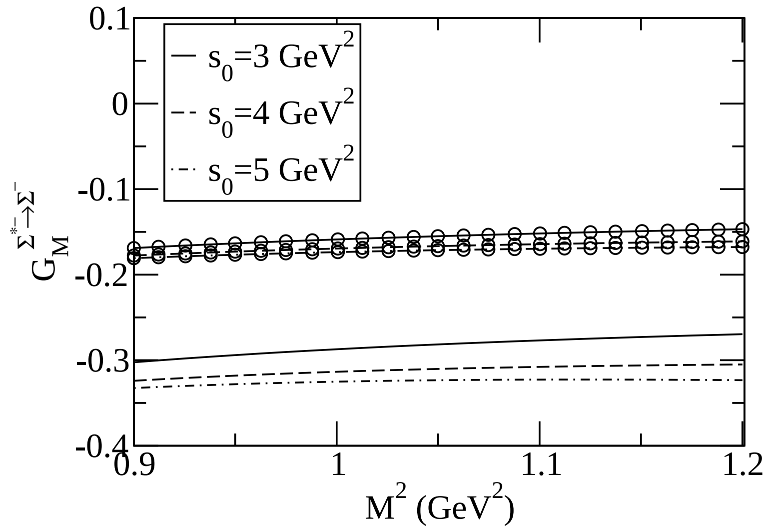

Figure 22: The same as Fig. 10, but for the decay $\Sigma^{*-} \to \Sigma^-$

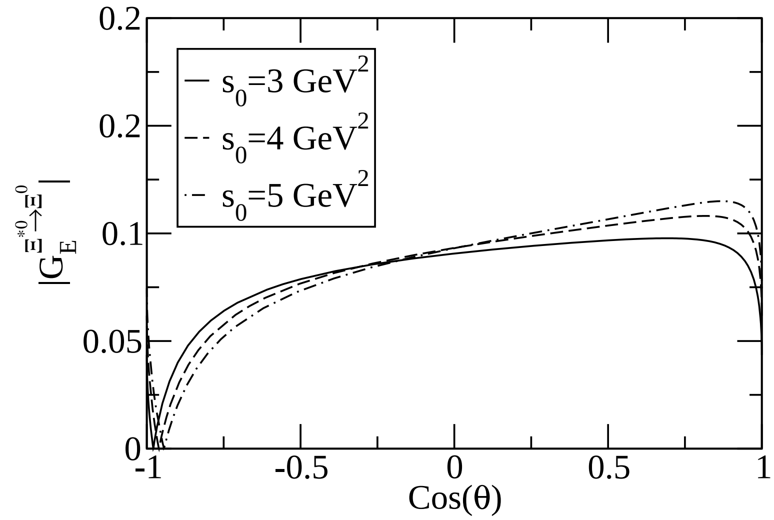

Figure 23: The same as Fig. 7, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

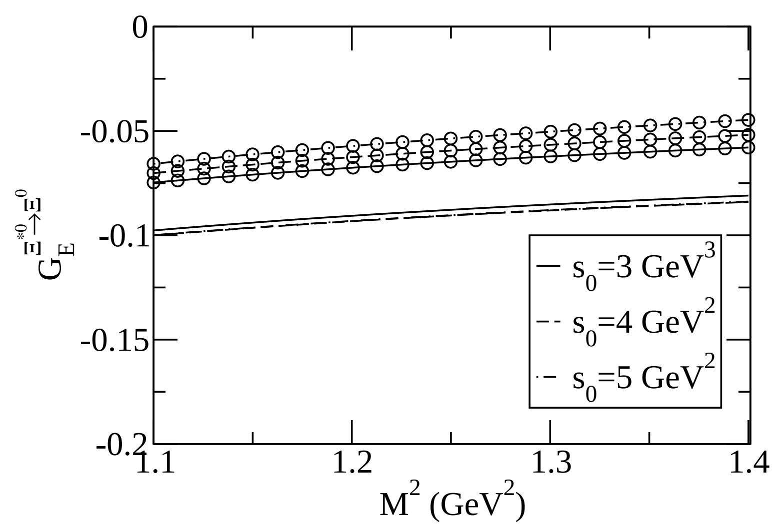

Figure 24: The same as Fig. 8, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

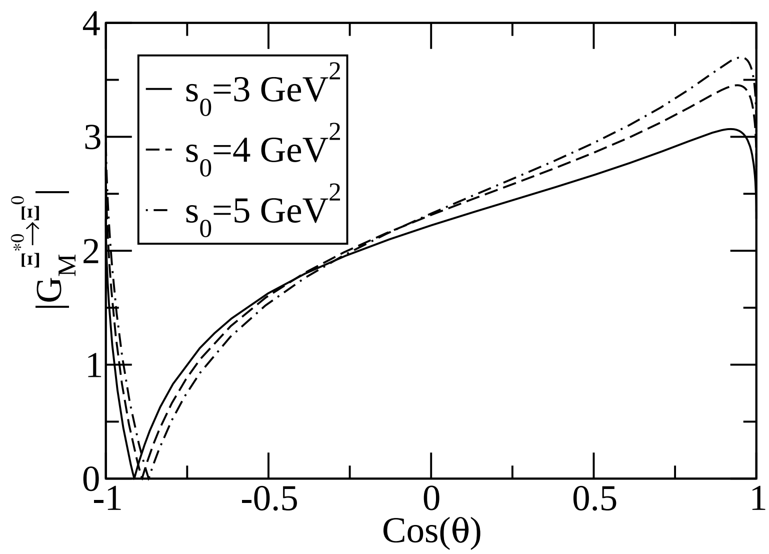

Figure 25: The same as Fig. 9, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

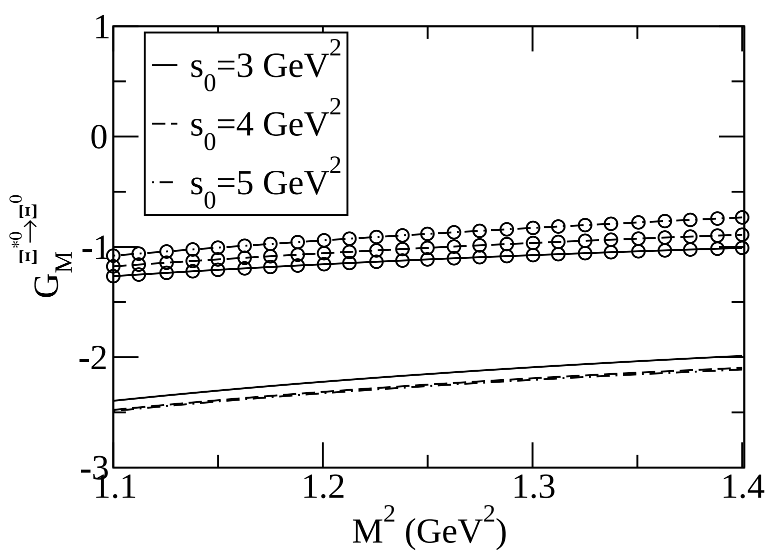

Figure 26: The same as Fig. 10, but for the decay $\Xi^{*0} \to \Xi^0$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

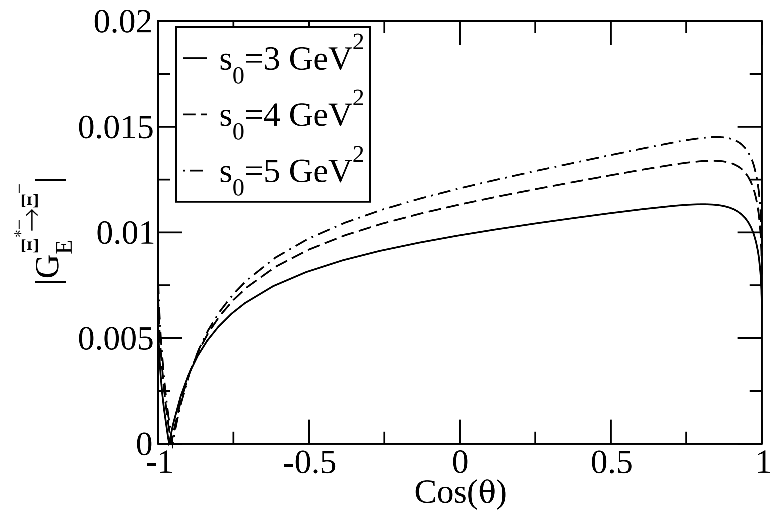

Figure 27: The same as Fig. 7, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

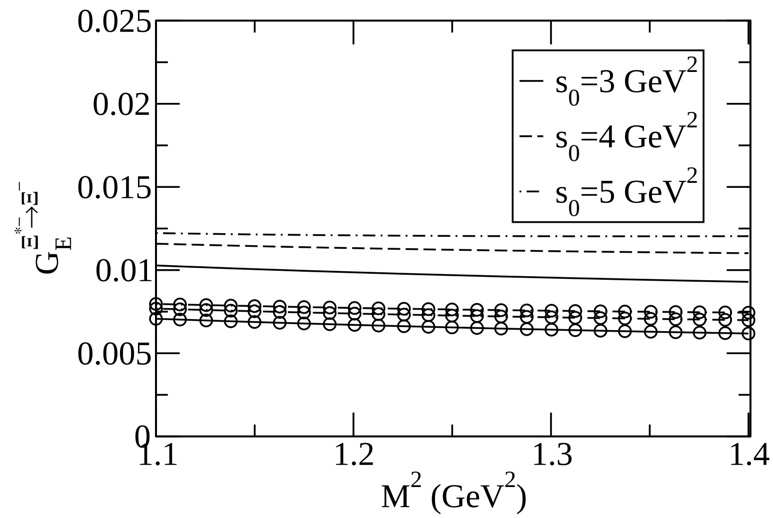

Figure 28: The same as Fig. 8, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

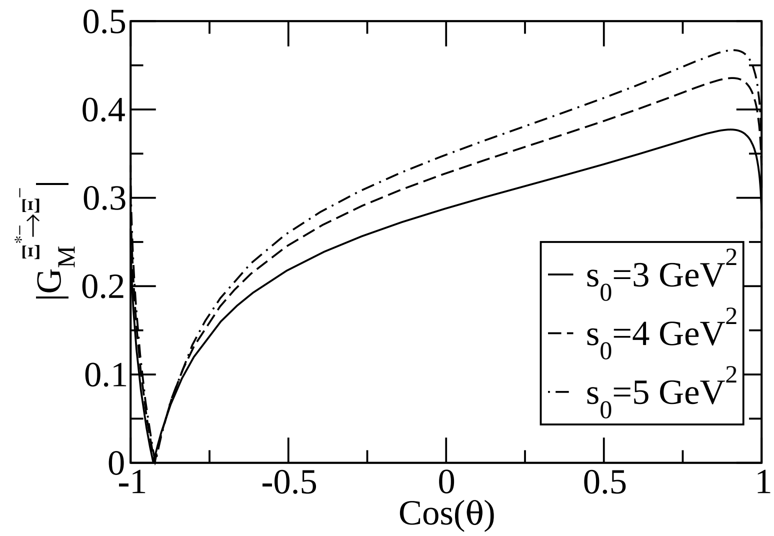

Figure 29: The same as Fig. 9, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$

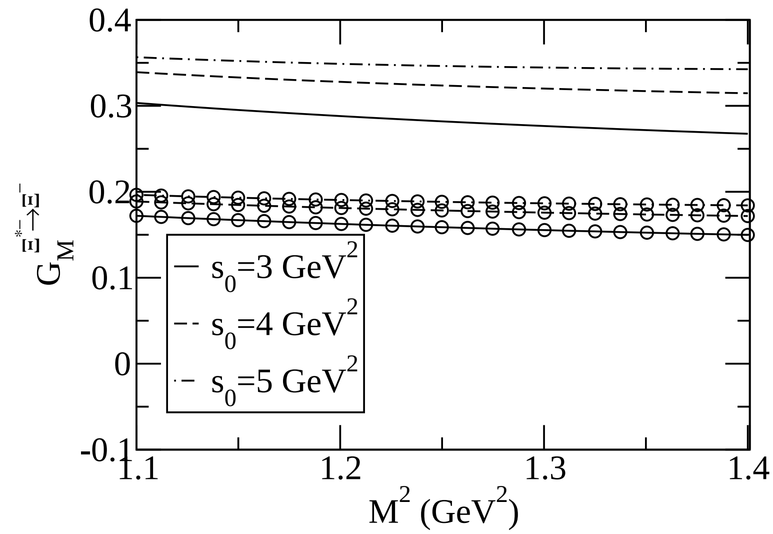

Figure 30: The same as Fig. 10, but for the decay $\Xi^{*-} \to \Xi^-$ and for the Borel parameter $M^2 = 1.2\ GeV^2$