# Radiative decay of the $X(3872)$ as a mixed molecule-charmonium state in effective field theory

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### A B S T R A C T

Assuming that $X(3872)$ is a mixture between $2P$ charmonium and $\bar{D}D^*$ molecular states with $J^{PC} = 1^{++}$, an analysis of $X(3872)$ radiative decays into $J/\psi\gamma$ and $\psi(2S)\gamma$ is presented. The modification of the radiative branching ratio due to possible constructive or destructive interferences between the meson-loop and the short-distance contact term, which is modeled by a charm quark loop, is shown. The model predictions are shown to be compatible with the experimentally determined ratio of the mentioned branching fractions for a wide range of the $X(3872)$ charmonium content. In the case of the destructive interference, a strong restriction on the charmonium admixture is found.

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## 1. Introduction

The $X(3872)$ state was first observed by Belle [1] through the channel $B^\pm \to J/\psi\pi^+\pi^- K^\pm$ and its quantum numbers were determined as $1^{++}$ [2]. The averaged mass of the $X(3872)$ is $3871.69 \pm 0.17$ MeV, and the full width is small, $\Gamma < 1.2$ MeV, which is not easily accommodated in the potential quark models. Moreover, its mass does not fit into the traditional quark model as non-relativistic bound state of charm quarks. The properties of $X(3872)$ turned out to be difficult to reconcile with a pure $c\bar{c}$ state in a quark model picture [3], but see also Ref. [4]. Despite the other possibilities including a molecular state consisting of a $D$ and $\bar{D}^*$ [5–12], tetraquark [13–15], $c\bar{c} - D\bar{D}^*$ mixing [3,16–19] or radial excitation of the $P$-wave charmonium [4,20,21], the structure of the $X(3872)$ is not yet fully understood. Since the mass of the $X(3872)$ is extremely close to the $D^0\bar{D}^{*0}$ threshold, many authors have suggested that it is a loosely bound state of $D\bar{D}^*$. In addition, predominantly molecular description of $X(3872)$ is also favored by the experimental ratio [22] of decay fractions of $X(3872)$ into $J/\psi\pi^+\pi^-$ and $J/\psi\pi^+\pi^-\pi^0$ final states [12,23].

Another puzzling observation about $X(3872)$ is its radiative decays. The ratio of the branching fractions into final states with a photon and a $J/\psi$ or $\psi(2S)$ has been measured [24,25] as

$$R_{\psi\gamma} = \frac{B_r(X \to \psi(2S)\gamma)}{B_r(X \to J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29. \quad (1)$$

Various quark model calculations describing the $X(3872)$ as a radially excited $\chi_{c1}(2P)$ charmonium state predict a wide range of values for this ratio. However, the results are very sensitive to quark model details since the radiative decay matrix element is proportional to the overlap integral of the initial state and the final state wave functions. An alternative discussion is presented in the work of Swanson et al. [26], where using vector meson dominance, it is argued that if $X(3872)$ is a predominantly molecular state, the ratio is predicted as $4 \times 10^{-3}$ which is three orders smaller than the observed ratio. Contrary to the claim in this study, in Ref. [27] it was demonstrated that the observed ratio allows the $X(3872)$ to be a hadronic molecule with the dominant component $D\bar{D}^*$. The charmonium admixture in a molecular picture of $X(3872)$ has been studied within an effective field theory framework in Ref. [19]. There it was concluded that the observed ratio can be explained, if one assumes that the compact component of the $X(3872)$ is 5–12%. In addition, the production rate of $X(3872)$ in the $p\bar{p}$ collisions which is about $1/20$ of the rate of $\psi(2S)$ can be accommodated, by a rough estimate, with an admixture of approximately 5% of a $c\bar{c}$ component in its wave function [28]. This size is also consistent with the findings of Ref. [29] where the $c\bar{c}$ core component of the $X(3872)$ is about 6%.

Within the molecular description of the $X(3872)$, triangular $DD^{(*)}\bar{D}^{(*)}$ and simple $D\bar{D}^*$ loop contributions to the radiative amplitude, without explicitly considering the short-range contributions, were computed in Ref. [27]. In an exploratory study of Ref. [30], the size of the counter-term was estimated in an effective field theory framework allowing for both a molecular as well as a compact component of the $X(3872)$. However, the meson
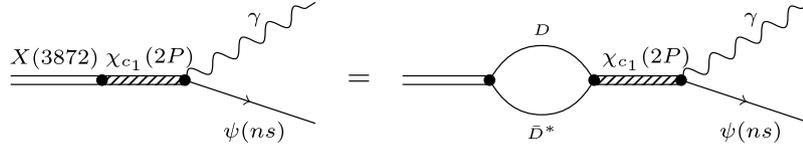
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**Fig. 1.** Decay mechanism for the transition $X(3872) \to \psi(nS)$ through an intermediate charmonium $\chi_{c1}(2P)$ state [30].

loop contribution in the $X(3872) \to J/\psi\gamma$ mode and possible interferences effects between the meson-loop and the counter-term contributions in both of the decays are neglected. Moreover, it was claimed in Ref. [19] that the relative phases of the coupling constants are uncertain and they can be fixed by an analysis of the branching ratio data.

In this study, we investigate the effects of short-range contributions to the radiative decays of the $X(3872)$ into $\psi(2S)\gamma$ and $J/\psi\gamma$ in an effective field theory allowing a $\chi_{c1}(2P)$ charmonium admixture in the molecular state. We demonstrate that the relative phase of the couplings are important to determine whether the charmonium content of the $X(3872)$ is nontrivial.

## 2. Formalism

As mentioned in the introduction, the triangular $DD^{(*)}\bar{D}^{(*)}$ and simple $D\bar{D}^*$ loop contributions to the radiative decays were calculated from diagrams Fig. 1(a-e) in the work by Guo et al. [27] within an effective theory framework. In [27], the contributions to the loop amplitude from these diagrams are written as

$$\mathcal{M}^{loop}_{\mu\sigma\lambda} = \frac{1}{\sqrt{2}} e g_{XD D^*} g_{\psi D D^*} m \sqrt{m_X m_\psi}$$
$$\times \int \frac{d^4 k}{4\pi} S^\nu_\sigma(k) S(k-p) J_{\mu\nu\lambda}(k), \quad (2)$$

where $J_{\mu\nu\lambda}$ tensor includes the electric and magnetic contributions and $S^\nu_\sigma(k)$ and $S(k-p)$ are the $D^*$, $D$ propagators, respectively. The couplings $g_2$ and $g'_2$ are used for the spin symmetric couplings of $\psi(nS)D^{(*)}\bar{D}^{(*)}$, $g_{\psi D D^*}$, for $n = 1$ and $n = 2$ respectively. Finally, the coupling constant of the $X(3872)$ to $\bar{D}D^*$, $g_{XD D^*}$, can be expressed as follows in terms of the probability, $\tilde{Z}_{X(3872)}$, to find the charmonium component $\chi_{c_1}(2P)$ in the physical wave function of the $X(3872)$ [30]

$$g_{XD D^*} = \left( \frac{-f_\Lambda^2}{G'_{QM}} \tilde{Z}_{X(3872)} \right)^{1/2}, \quad (3)$$

where $f_\Lambda$ is Gaussian regulator for on-shell mesons which depends on the masses of the involved mesons and $G'_{QM}$ is derivative of the meson-loop function with respect to energy. For the numerical analysis, the coupling constant, $g_{XD D^*}$ values are taken from Table I of Ref. [30], where the pole position of the $X(3872)$ was fixed at in the first Riemann sheet as a $1^{++}$ bound state[1] and the dressed $\chi_{c1}(2P)$ pole was located in the second Riemann sheet.

Since the loop integral in the amplitude (Eq. (2)) is divergent, one needs to include a counter-term to renormalize the ultraviolet divergences of the loop diagrams. After the renormalization procedure, the counter-term modeled by a charm quark loop in Fig. 1 provides a finite contribution to the total decay width. To estimate the strength of the short range interaction we use the effective

field theory approach of Ref. [30] which incorporates possible mixing between the molecular $D\bar{D}^*$ and $\chi_{c1}(2P)$ charmonium state. The contribution from short range interaction depicted in diagram Fig. 1 can be obtained as

$$\mathcal{A}_{\mu\sigma\lambda} = -i\sqrt{2}(\tilde{Z}_{X(3872)} \times f(\tilde{Z}_{X(3872)})^{1/2}$$
$$\times m_X m_\psi \delta^{nS2P} v^\eta \epsilon_{\sigma\mu\rho\eta} (v.q g^\rho_\lambda - q^\rho v_\lambda), \quad (4)$$

where $f(\tilde{Z}_{X(3872)})$ is the dressed and bare charmonium propagator ratio squared [30]. As is seen in Eq. (4), $\mathcal{A}_{\mu\sigma\lambda}$ depends on the $\chi_{c1}(2P)\psi(nS)\gamma$ coupling $\delta^{nS2P}$. This coupling is one of the greatest uncertainties of the present calculation. It can be written as

$$\delta^{nS2P} = \left( \frac{4\pi\alpha e_c^2}{3} \right)^{1/2} \langle \psi(nS)|r|\chi_{c1}(2P) \rangle, \quad (5)$$

where $e_c$ is the charm quark electric charge, $\alpha$ is the fine-structure constant and the overlap integral of the initial state and the final state wave functions $\langle \psi(nS)|r|\chi_{c1}(2P) \rangle$ can be calculated by using quark model wave functions.

Finally, the total amplitude can be expressed as follows

$$\mathcal{M}^{full} = (\mathcal{A}_{\mu\sigma\lambda} + \mathcal{M}^{loop}_{\mu\sigma\lambda}) \epsilon^\sigma_{(X)}(p) \epsilon^\mu_{(\psi)}(p-q) \epsilon^\lambda_{(\gamma)}(q). \quad (6)$$

### 2.1. General remarks

As pointed out in Ref. [27], quark model calculations predict a wide range for the radiative branching fractions $R_{\psi\gamma}$ assuming a $\chi_{c1}(2P)$ $c\bar{c}$ nature for the $X(3872)$, where the decay width results are very sensitive to quark model details in particular in the $J/\psi$ mode. The $\delta^{nS2P}$ couplings used in Ref. [30] are based on non-relativistic quark model of Ref. [20]. Here, two different quark model estimates for the overlap integrals, Set 1 and Set 2, of the initial state and the final state wave functions are considered to see dependence of the predictions presented in this work on the coupling constants, $\delta^{nS2P}$ (see Table 1).

In the analysis of Ref. [27], the dependence of the radiative branching ratios on the coupling of $X(3872)$ to the charmed mesons, $g_{XD D^*}$, cancels in the ratio since only the loop contributions are considered. However, in the present work, since the charm quark loop contribution does not contain this coupling, the predicted ratio depends on it. For the numerical analysis, the values of the $g_{XD D^*}$ are taken from Table I of Ref. [30]. Moreover, the ratio $R_{\psi\gamma}$ obtained in Ref. [27] depends on the ratio of the couplings $r_{g_2,g'_2}$, while it is separately dependent on $g_2$ and $g'_2$ in the study presented here. In Ref. [19], it was found that $r_{g_2,g'_2} \simeq 2$ from the analysis of Ref. [32]. In addition, using vector dominance arguments, the coupling constant, $g_2$, of $J/\psi$ to the charm meson-antimeson pair was estimated as about 2 GeV$^{-3/2}$ in Ref. [33]. Model independent estimates for that coupling were given in a range of 2.1–2.9 GeV$^{-3/2}$ in Ref. [34–36]. In line with these considerations, in the present work, $g_2$ is taken as 2.5 GeV$^{-3/2}$, and the results are analyzed for various values of $r_{g_2,g'_2}$. Note that results for smaller values of $g_2$ are qualitatively similar, nevertheless results for the value $g_2 = 2$ GeV$^{-3/2}$ can be found in the Appendix A.

On the other hand, the loop integrals in Eq. (2) are scale dependent. In Eq. (6), the cut-off dependence of the $\mathcal{M}^{loop}_{\mu\sigma\lambda}$ should

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[1] The analysis of Ref. [31], based on the different pole structures on the line shapes of the $X(3872)$ in $B$ decays to $KJ/\psi\pi^+\pi^-$ and $KJ/\psi D^0\bar{D}^{0*}$, finds solutions with either a bound state or a virtual state pole.

**Table 1**

Radiative decays of $X(3872)$ as a radially excited $\chi_{c1}(2P)$ charmonium state are based on quark model estimates. [†]: Overlap integrals are estimated from the widths given in Table III of Ref. [20] for the $\chi_{c1}(2p) \to \psi(nS)\gamma$ $E_1$ radiative transitions calculated by the non-relativistic quark model. $m_{J\psi} = 3096.92$ MeV, $m_{\psi(2S)} = 3686.11$ MeV and the mass predicted in Ref. [20] for the $\chi_{c_1}(2P)$ state have been used (see [30] for details).

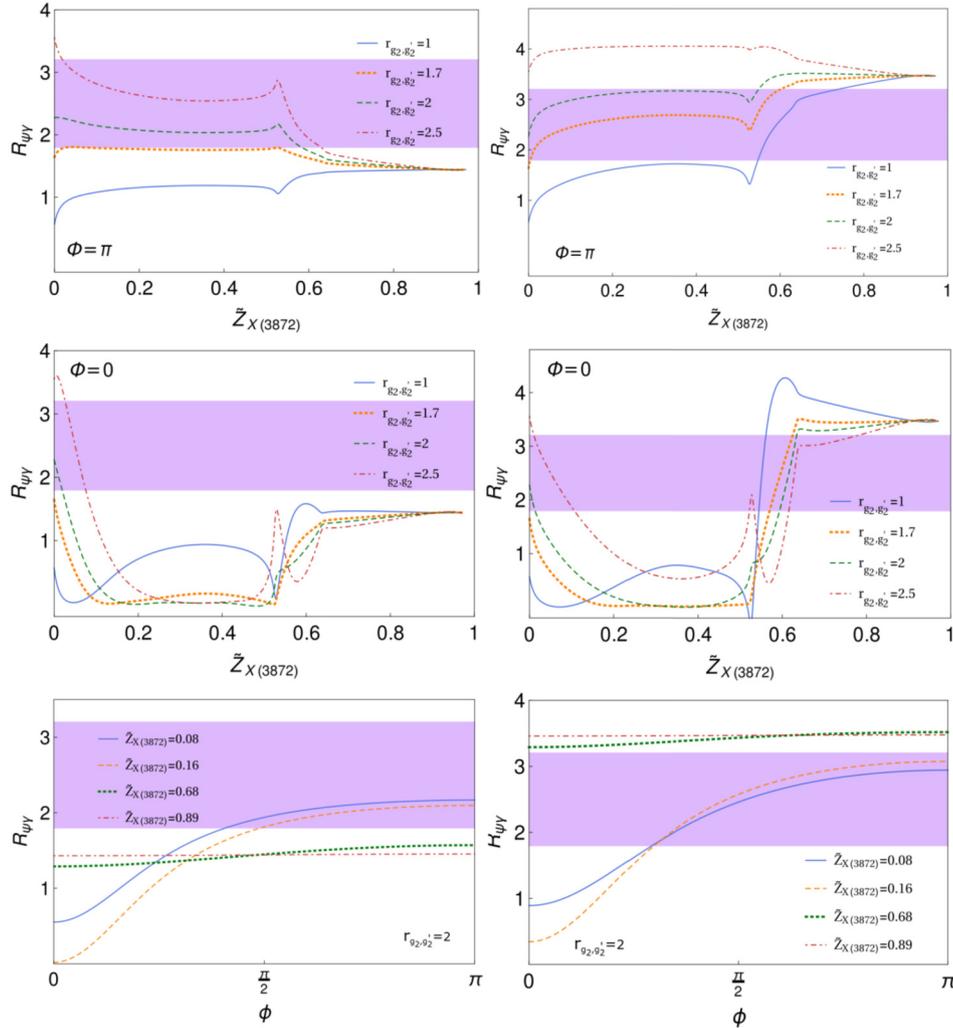| | Final state | $\langle \psi(nS)|r|\chi_{c1}(2P)\rangle$ [GeV$^{-1}$] | $\delta^{nS2P}$ [GeV$^{-1}$] | $\Gamma[X(3872) \to \psi(nS)\gamma]$ [keV] | $R_{\psi\gamma}$ |
|---|---|---|---|---|---|
| Set 1 | $J\psi\gamma$ | $0.389^{\dagger}$ | 0.045 | 60.87 | 1.48 |
| | $\psi(2S)\gamma$ | $3.26^{\dagger}$ | 0.38 | 87.25 | |
| Set 2 | $J\psi\gamma$ | 0.202 [29] | 0.024 | 16.06 | 3.52 |
| | $\psi(2S)\gamma$ | 2.63 [29] | 0.31 | 56.65 | |



**Fig. 2.** $R_{\psi\gamma}$ as a function of the charmonium probability $\tilde{Z}_{X(3872)}$ (first two rows) and $\phi$ (last row). See text for explanation of the legends.

be compensated by a corresponding variation in the counter-term contribution $\mathcal{A}_{\mu\sigma\lambda}$. Here, we have computed the full amplitude $\mathcal{M}^{full}$ using dimension regularization with the $\overline{MS}$ subtraction scheme, while the couplings of $X(3872)$ state to the charmonium and $D\bar{D}^*$ molecule were computed in Ref. [30] using an ultraviolet cut-off at the scale $\Lambda = 1$ GeV. In Ref. [30], both of the regularization schemes were compared considering the two meson-loop function and found that UV cut-off at the scale $\Lambda = 1$ GeV would correspond to a $\overline{MS}$ scale, $\mu$ of the order of 1 GeV. Therefore, all calculations have been carried out with $\overline{MS}$ scale, $\mu = 1$ GeV.

Finally, the importance of the relative signs of the coupling constants was stressed in Ref. [19]. To study the effects of this phase, along the lines of study in Ref. [19], the coupling $g_2$ ($g_2'$) is given an arbitrary phase $e^{i\phi}$ with $0 \le \phi \le \pi$.

## 3. Results and discussion

In Fig. 2, the ratio $R_{\psi\gamma}$ is shown as a function of the $X(3872)$ charmonium content, $\tilde{Z}_{X(3872)}$, and $\phi$. In the first two rows, the dependence of $R_{\psi\gamma}$ is depicted for four different values of the coupling constant ratio $r_{g_2, g_2'} = 1$, 1.7, 2 and 2.5, along with the experimental band. In the first row $\phi$ is set to $\phi = \pi$ (constructive interference) and in the second row $\phi = 0$ (destructive interference). The third row depicts the dependence of $R_{\psi\gamma}$ on the phase $\phi$ for $\tilde{Z}_{X(3872)} = 0.08$, 0.16, 0.68 and 0.89 with $r_{g_2, g_2'} = 2$. In the first column, the values of $\delta^{nS2P}$ are taken from Set 1, and in the second column, they are taken from Set 2. Note that, as can be seen from Table 1, $\delta^{2S2P}$ is almost the same in the two quark mod-
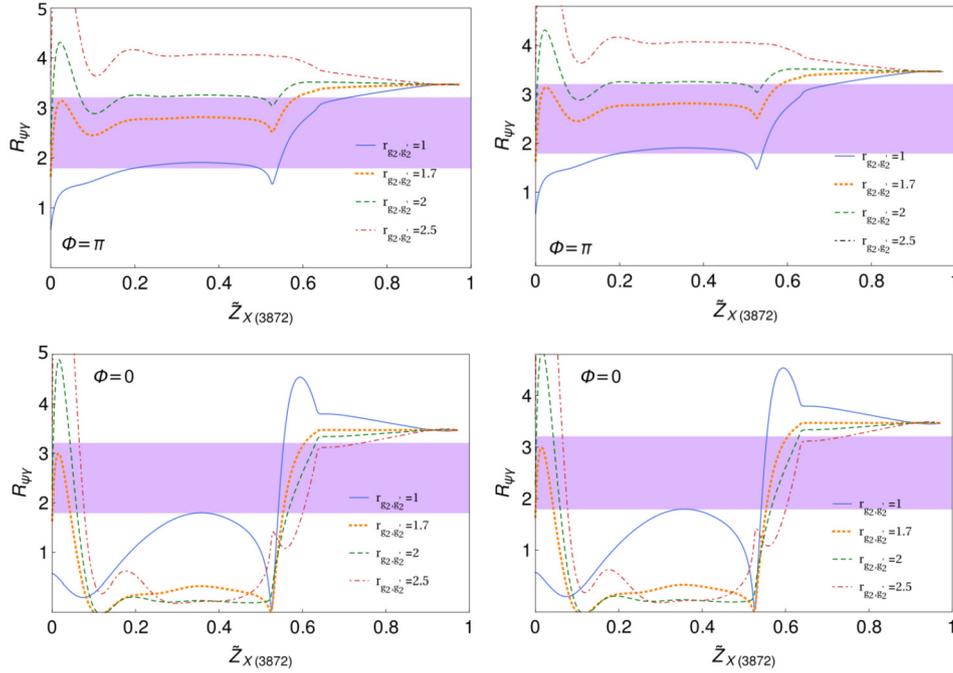
**Fig. 3.** Radiative branching fractions of the $X(3872)$ is shown as a function of the charmonium probability $\tilde{Z}_{X(3872)}$ for four different values of $r_{g_2, g_2'} = 1, 1.7, 2$ and 2.5, together with the experimental band $R_{\psi\gamma} = 2.5 \pm 0.7$ from Ref. [25]. Here, coupling of the $\psi(1S)$ to the charm meson-antimeson pair is fixed as $g_2 = 2$ GeV$^{-3/2}$. In the first column, 2P charmonium to $\psi(nS)\gamma$ couplings are taken from set 1 in Table 1 while the second column illustrates the results for set 2 in that table. The first row shows the effects of constructive interferences ($\phi = \pi$) between the meson-loop and the counter-term contributions, while the second one presents destructive effects ($\phi = 0$). All calculations have been carried out with $\overline{MS}$ scale $\mu = 1$ GeV.

els. Hence, the difference in the figures in the two rows are mainly due to the different values of $\delta^{1S2P}$.

As can be seen from the figure, contrary to the claim made in Ref. [19] that a non-trivial interplay between a possible charmonium and molecular components in the $X(3872)$ is necessary to explain the $R_{\psi\gamma}$ ratio, both a trivial and a non-trivial charmonium component of $X(3872)$ is consistent with the radiative decay ratio, independent of which quark model prediction is used for the coupling constants $\delta^{nS2P}$. The findings of that reference, where the approach is based on phenomenological hadron Lagrangians and relies on the inspired quark model result of Ref. [9], can be easily accommodated within our result.

It can be also seen from the figure that the behavior of the predictions of the ratio of radiative decays is different when $\tilde{Z}_{X(3872)} \lesssim 0.55$ and when $\tilde{Z}_X \gtrsim 0.55$. For large values of $\tilde{Z}_{X(3872)}$, the prediction of the ratio has a small dependence on the ratio $r_{g_2, g_2'}$ and $\phi$, and a large dependence on $\delta^{nS2P}$. This is the expected behavior, since in this range, $X(3872)$ is dominantly a charmonium state. Such large values of $\tilde{Z}_{X(3872)}$ can be consistent with the observed ratio, provided that $\delta^{nS2P}$ has a value in between the values of $\delta^{nS2P}$ used in this work. For smaller values of $\tilde{Z}_{X(3872)} \lesssim 0.55$, the predictions are less sensitive to $\delta^{nS2P}$, but more sensitive to $r_{g_2, g_2'}$ and $\phi$.[2] In the case of constructive interference, the first row of figures, the predicted ratio is consistent

with experimental values for both values of $\delta^{nS2P}$ for almost any value of $\tilde{Z}_{X(3872)} \lesssim 0.55$ if $r_{g_2, g_2'}$ is between 1.7 and 2. Larger values of $r_{g_2, g_2'}$ would become consistent with observations for larger values of $\delta^{nS2P}$, and smaller values of $r_{g_2, g_2'}$ are consistent with observations for smaller values of $\delta^{nS2P}$. In the case of destructive interference, the second row of figures, the allowed range of $\tilde{Z}_{X(3872)}$ is pushed to smaller values.

From the above considerations, it is concluded that a wide range of charmonium probability in the $X(3872)$ is consistent with the experimentally observed value of $R_{\psi\gamma}$. This confirms that this ratio is not in conflict with a predominantly molecular or charmonium nature of the $X(3872)$. In the case of the destructive interferences between the meson loops and the counter-term, a strong constraint on the $\chi_{c1}(2P)$ content in the $X(3872)$ is found.[3] A more precise knowledge of $\delta^{2S2P}$ coupling would put a stronger restriction on the charmonium admixture in the $X(3872)$.

### Acknowledgement

### Appendix A. $g_2 = 2$ GeV$^{-3/2}$ results

In this appendix, results for the radiative branching fractions $R_{\psi\gamma}$ as a function of the charmonium probability $\tilde{Z}_{X(3872)}$ are given for $g_2 = 2$ GeV$^{-3/2}$ (Fig. 3).

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[2] Note that, when $\tilde{Z}_{X(3872)} = 0$, there is no $\phi$ dependence. As $\tilde{Z}_{X(3872)}$ increases, the $\phi$ dependency also increases, up to $\tilde{Z}_{X(3872)} \lesssim 0.43$, the dressed $\chi_{c1}(2P)$ pole stays in the second Riemann sheet (SRS) above threshold with its width increasing rapidly. For the larger values of $\tilde{Z}_{X(3872)}$, $\tilde{Z}_{X(3872)} \gtrsim 0.55$, the contribution from the short-range interaction to the radiative branching ratio becomes dominant, and thus the phase dependency of $R_{\psi\gamma}$ decreases. In this latter case, $\chi_{c1}(2P)$ pole is located in the SRS below threshold and quite wide or it is close to the real axis. Since SRS and first Riemann sheet (FRS) are disconnected below threshold, it will not have any visible effects in scattering observables (see Table 1 of Ref. [30]).

[3] In the destructive interference case, as can be seen from the second row in Fig. 2, the predicted ratio is not consistent with experimental values for a charmonium probability in the interval ∼15-55%. However, one should bear in mind that larger values of $\tilde{Z}_{X(3872)}$ ($\gtrsim 0.55$) can be consistent with the observed ratio for a value in between the values of $\delta^{nS2P}$ used in this work.

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