# Analysis of rare $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay within QCD sum rules 

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#### Abstract

Form factors of rare $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay are calculated within three-point QCD sum rules, with $K_{0}^{*}(1430)$ being the p-wave scalar meson. The branching ratios are estimated when only short, as well as short and long distance effects, are taken into account. It is obtained that the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}(\ell=e, \mu)$ decay is measurable at LHC. Measurement of these branching ratios for the semileptonic rare $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$can give valuable information about the nature of scalar meson $K_{0}^{*}(1430)$.


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## 1 Introduction

The flavor changing neutral current (FCNC) processes induced by $b \rightarrow s(d)$ transitions provide the most sensitive and stringiest test for the standard model (SM) at one loop level, since they are forbidden in SM at tree level [1,2]. For this reason these decays are very sensitive to the physics beyond the SM via the influence of new particles in the loops.

Despite the branching ratios of FCNC decays are small in the SM, quite intriguing results are obtained in ongoing experiments. The inclusive $B \rightarrow X_{s} \ell^{+} \ell^{-}$decay is observed in BaBaR [3] and Belle collaborations. These collaborations also measured exclusive modes $B \rightarrow K \ell^{+} \ell^{-}[4-6]$ and $B \rightarrow K^{*} \ell^{+} \ell^{-}[7]$. The experimental results on these decays are in a good agreement with theoretical estimations [8-10].

There is another class of rare decays induced by $b \rightarrow s$ transition, such as $B \rightarrow$ $K_{02}^{*}(1430) \ell^{+} \ell^{-}$in which B meson decays into p -wave scalar meson. The decays $B \rightarrow$ $K_{2}^{*}(1430) \ell^{+} \ell^{-}$and $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$are studied in [11] and [12], respectively. The main problem in these studies is the calculation of the transition form factors. Transition form factors of these decays in the framework of light front quark model [12] are estimated in [13] and [14], respectively.

In the present work we calculate the transition form factors of $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$ decay in 3 -point QCD sum rules method [15] (for a review on the QCD sum rules method, see [16]), where $K_{0}^{*}(1430)$ is the p -wave scalar meson. The main reason for studying $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay form factors in the framework of 3 -point QCD sum rules method is that part of the form factors, namely, those appearing in calculation of the matrix element of the axial vector current between initial and final meson states, can easily be obtained from the results for the $B_{s} \rightarrow D_{s}(2317)$ transition form factors [21], which are calculated within the same framework, with the help of appropriate replacements (see section 2). The quark structure of the scalar mesons have not been unambiguously determined yet and is still under discussion. In order to establish the inner structure of scalar mesons much more effort from theoretical and experimental sides are needed. This method is employed to calculate the form factors of heavy meson semileptonic decays, $D \rightarrow K^{0} e \bar{\nu}_{e}$ [17], $B \rightarrow D\left(D^{*}\right) \ell \bar{\nu}_{\ell}[18], D \rightarrow K\left(K^{*}\right) e \bar{\nu}_{e}$ [19], $D \rightarrow \pi e \bar{\nu}_{e}[20], B \rightarrow K\left(K^{*}\right) \ell^{+} \ell^{-}$(first reference in [8]), etc.

The work is organized as follows: In section 2, we calculate the relevant hadronic form factors of the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay with the help of $3-$ point QCD sum rules. Section 3 is devoted to the numerical analysis and discussion of the considered decay and our conclusions.

## 2 Form factors of the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay

The exclusive $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay is described at quark level by $b \rightarrow s \ell^{+} \ell^{-}$transition. The effective Hamiltonian responsible for the $b \rightarrow s \ell^{+} \ell^{-}$transition in the standard model is

$$
\mathcal{H}_{e f f}=\frac{G_{F} \alpha V_{t b} V_{t s}^{*}}{2 \sqrt{2} \pi}\left[C_{9}^{e f f}\left(m_{b}\right) \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\ell} \gamma^{\mu} \ell+C_{10}\left(m_{b}\right) \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right.
$$

$$
\begin{equation*}
\left.-2 m_{b} C_{7}\left(m_{b}\right) \frac{1}{q^{2}} \bar{s} i \sigma_{\mu \nu}\left(1+\gamma_{5}\right) b \bar{\ell} \gamma^{\mu} \ell\right] \tag{1}
\end{equation*}
$$

where $C_{7}, C_{9}^{e f f}$ and $C_{10}$ are the Wilson coefficients. It is well known that the Wilson coefficient $C_{9}^{\text {eff }}$ has a perturbative part as well as a resonant part which comes from the long-distance effects due to the conversion of the real $\bar{c} c$ into lepton pair (see, for example the second reference in [8]) which can be written as

$$
C_{9}^{e f f}=C_{9}^{p e r}+C_{9}^{r e s}
$$

The explicit expressions of $C_{7}, C_{9}^{\text {per }}$ and $C_{10}$ can be found in [1]. $C_{9}^{r e s}$ is usually parametrized by using Breit-Wigner ansatz,

$$
C_{9}^{r e s}=\frac{3 \pi}{\alpha^{2}} C^{(0)} \sum_{V_{i}=\psi(1 s) \cdots \psi(6 s)} æ_{i} \frac{\Gamma\left(V_{i} \rightarrow \ell^{+} \ell^{-}\right) m_{V_{i}}}{m_{V_{i}}^{2}-q^{2}-i m_{V_{i}} \Gamma_{V_{i}}},
$$

where $\alpha$ is the fine structure constant and $C^{(0)}=0.362$.
The phenomenological factors $æ_{i}$ for the $B \rightarrow K\left(K^{*}\right) \ell^{+} \ell^{-}$decay can be determined from the condition that they should reproduce correct branching ratio relation

$$
\mathcal{B}\left(B \rightarrow J / \psi K\left(K^{*}\right) \rightarrow K\left(K^{*}\right) \ell^{+} \ell^{-}\right)=\mathcal{B}\left(B \rightarrow J / \psi K\left(K^{*}\right)\right) \mathcal{B}\left(J / \psi \rightarrow \ell^{+} \ell^{-}\right),
$$

where the right-hand side is determined from experiments. Using the experimental values of the branching ratios for the $B \rightarrow V_{i} K\left(K^{*}\right)$ and $V_{i} \rightarrow \ell^{+} \ell^{-}$decays, for the lowest two $J / \psi$ and $\psi^{\prime}$ resonances, the factor æ takes the values: $æ_{1}=2.7, æ_{2}=3.51$ (for $K$ meson), and $æ_{1}=1.65, æ_{2}=2.36$ (for $K^{*}$ meson). The values of $æ_{i}$ used for higher resonances are usually the average of the values obtained for the $J / \psi$ and $\psi^{\prime}$ resonances. But, unfortunately the mode $B \rightarrow J / \psi K_{0}^{*}(1430)$ for the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay has not been seen yet and therefore the right-hand side of the above-mentioned equation for the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$ decay is unknown. For this reason we are not able to determine the factors $æ_{i}$ for the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay.

As we have already mentioned above, the values of the factors $x_{i}$ are of order $æ \sim 1$ or $æ \sim 2$, for the $B \rightarrow K \ell^{+} \ell^{-}$and $B \rightarrow K^{*} \ell^{+} \ell^{-}$decays. For this reason, even though qualitatively, in order to determine the branching ratio for the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay with the inclusion of long distance effects, the values of $æ_{i}$ we use are borrowed from $B \rightarrow K \ell^{+} \ell^{-}$and $B \rightarrow K^{*} \ell^{+} \ell^{-}$decays, namely, we choose $æ=1$ and $æ=2$ and performed numerical calculations with these values.

The amplitude for the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay can be obtained after evaluating the matrix elements of the quark operators in Eq. (1) between initial $B$ and final $K_{0}^{*}(1430)$ meson states. It follows from Eq. (1) that in order to obtain the amplitude for the $B \rightarrow$ $K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay the matrix elements $\left\langle K_{0}^{*}\right| \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right)|B\rangle$ and $\left\langle K_{0}^{*}\right| \bar{s} i \sigma_{\mu \nu} q^{\mu}\left(1+\gamma_{5}\right)|B\rangle$ are needed. These matrix elements are parametrized in terms of the form factors as follows:

$$
\begin{align*}
\left\langle K_{0}^{*}(1430)\left(p^{\prime}\right)\right| \bar{s} \gamma_{\mu} \gamma_{5} b|B(p)\rangle & =f_{+}\left(q^{2}\right) \mathcal{P}_{\mu}+f_{-}\left(q^{2}\right) q_{\mu},  \tag{2}\\
\left\langle K_{0}^{*}(1430)\left(p^{\prime}\right)\right| \bar{s} i \sigma_{\mu \nu} q^{\nu} \gamma_{5} b|B(p)\rangle & =\frac{f_{T}\left(q^{2}\right)}{m_{B}+m_{K_{0}^{*}}}\left[\mathcal{P}_{\mu} q^{2}-\left(m_{B}^{2}-m_{K_{0}^{*}}^{2}\right) q_{\mu}\right], \tag{3}
\end{align*}
$$

where $\mathcal{P}_{\mu}=\left(p+p^{\prime}\right)_{\mu}$ and $q_{\mu}=\left(p-p^{\prime}\right)_{\mu}$. For calculation of these form factors in the framework of the QCD sum rules method, we consider the following three-point correlators

$$
\begin{align*}
& \Pi_{\mu}\left(p, p^{\prime}, q\right)=-\int d^{4} x d^{4} y e^{i\left(p^{\prime} y-p x\right)}\langle 0| \mathrm{T} J^{K_{0}^{*}}(y) J_{\mu}^{A}(0) J_{5}^{B}(x)|0\rangle  \tag{4}\\
& \Pi_{\mu}\left(p, p^{\prime}, q\right)=-\int d^{4} x d^{4} y e^{i\left(p^{\prime} y-p x\right)}\langle 0| \mathrm{T} J^{K_{0}^{*}}(y) J_{\mu \nu}(0) J_{5}^{B}(x)|0\rangle \tag{5}
\end{align*}
$$

where $J^{K_{0}^{*}}=\bar{d} s, J_{5}^{B}=\bar{b} i \gamma_{5} d$ and $J_{\mu \nu}=\bar{s} i \sigma_{\mu \nu} b$ are the interpolating currents of the scalar $K_{0}^{*}(1430)$, and $B$ mesons and weak flavor changing quark currents, respectively. The expressions of the form factors $f_{+}\left(q^{2}\right)$ and $f_{-}\left(q^{2}\right)$ can be obtained from the results for the $B_{s} \rightarrow D_{s_{0}}(2317)$ transition form factors given in [21], with the help of the replacements $f_{D_{s_{0}}} \rightarrow f_{K_{0}^{*}}, m_{D_{s_{0}}} \rightarrow m_{K_{0}^{*}}, m_{c} \rightarrow m_{s}$, and $m_{s} \rightarrow m_{d}=0$. For completeness we also present the sum rules for the form factors $f_{+}\left(q^{2}\right)$ and $f_{-}\left(q^{2}\right)$ below.

For this reason we proceed by describing in full detail the calculation of $F_{T}$ by considering the correlator given in Eq. (5). This correlator can be decomposed into a set of the following independent Lorentz structures:

$$
\begin{equation*}
\Pi_{\mu \nu}=\left(p_{\mu} p_{\nu}^{\prime}-p_{\nu} p_{\mu}^{\prime}\right) \Pi_{T}+\sum_{n} a_{\mu \nu}^{(n)} \Pi^{(n)}, \tag{6}
\end{equation*}
$$

where $\Pi$ and $\Pi^{(n)}$ are functions of $p^{2}, p^{2}$ and $q^{2}$, and $a_{\mu \nu}^{(n)}$ are the other tensors which can be built using the vectors $p$ and $p^{\prime}$ and the metric tensor $g_{\mu \nu}$.

In further analysis we will be interested only on the invariant amplitude $\Pi_{T}$, for whose amplitude we write the dispersion relation

$$
\begin{equation*}
\Pi_{T}\left(p^{2}, p^{\prime 2}, q^{2}\right)=-\frac{1}{(2 \pi)^{2}} \int_{m_{b}^{2}}^{\infty} d s \int_{m_{s}^{2}}^{\infty} d s^{\prime} \frac{\rho\left(s, s^{\prime}, Q^{2}\right)}{\left(s-p^{2}\right)\left(s^{\prime}-p^{\prime 2}\right)}+\text { subtraction terms } \tag{7}
\end{equation*}
$$

where $\rho\left(s, s^{\prime}, Q^{2}\right)$ is the spectral density, $Q^{2}=-q^{2}$. According to the main idea of the QCD sum rules method, left hand side of Eq. (7) must be calculated at large Euclidean momenta $p^{2}$ and $p^{\prime 2}$ with the help of operator product expansion (OPE). The right hand side of Eq. (7) is determined by saturating it with the lowest mesonic states. Applying then the Borel transformation on the variables $p^{2}$ and $p^{\prime 2}$ suppresses the higher resonance contributions and higher power corrections, as a result of which we get the sum rules for the corresponding quantities.

Let us start by calculating the hadronic part of the correlator (5). Saturating this correlator with $B$ and $K_{0}^{*}(1430)$ mesons and selecting the structure $\left(p_{\mu} p_{\nu}^{\prime}-p_{\nu} p_{\mu}^{\prime}\right)$, we get

$$
\begin{equation*}
\Pi_{T}=-\frac{2 f_{K_{0}^{*}} m_{K_{0}^{*}}}{m_{B}+m_{K_{0}^{*}}} \frac{f_{B} m_{B}^{2}}{m_{b}} \frac{f_{T}\left(q^{2}\right)}{\left(m_{K_{0}^{*}}^{2}-p^{\prime 2}\right)\left(m_{B}^{2}-p^{2}\right)} . \tag{8}
\end{equation*}
$$

In deriving Eq. (8) we use

$$
\begin{align*}
\langle 0| J^{K_{0}^{*}}\left|K_{0}^{*}\right\rangle & =f_{K_{0}^{*}} m_{K_{0}^{*}}  \tag{9}\\
\langle B| J_{5}^{B}|0\rangle & =\frac{f_{B} m_{B}^{2}}{m_{b}},  \tag{10}\\
\left\langle K_{0}^{*}\right| \bar{s} i \sigma_{\mu \nu} b|B\rangle & =-2\left(p_{\mu} p_{\nu}^{\prime}-p_{\nu} p_{\mu}^{\prime}\right) \frac{f_{T}\left(q^{2}\right)}{m_{B}+m_{K_{0}^{*}}} . \tag{11}
\end{align*}
$$

The theoretical part of the correlator (5) can be calculated with the help of OPE at short distance. Up to operators having dimension 6 , it is determined by the bare loop and power corrections from operators with $d=3\langle\bar{q} q\rangle, d=4\left\langle G^{2}\right\rangle, d=5 m_{0}^{2}\langle\bar{q} q\rangle$ and $d=6\langle\bar{q} q\rangle^{2}$. After calculating all power corrections, we obtain that $\left\langle G^{2}\right\rangle$ and $\langle\bar{q} q\rangle^{2}$ give negligibly small correction and for this reason we do not present their explicit expressions.

The spectral density for bare loop in Eq. (7) can be obtained by replacing the denominator $1 /\left(p^{2}-m^{2}\right)$ of the quark propagators by $-2 \pi i \delta\left(p^{2}-m^{2}\right)$ in the initial Feynman integral. After standard calculation we get

$$
\begin{equation*}
\rho_{T}\left(p^{2}, p^{\prime 2}, Q^{2}\right)=\frac{N_{c}}{\lambda^{3 / 2}}\left\{m_{b}\left(2 s^{\prime} \Delta-\Delta^{\prime} u\right)-m_{s}\left(2 s \Delta^{\prime}-\Delta u\right)\right\}, \tag{12}
\end{equation*}
$$

where $N_{c}=3$ is the color factor, $\Delta=s-m_{b}^{2}, \Delta^{\prime}=s^{\prime}-m_{s}^{2}, u=s+s^{\prime}+Q^{2}, \lambda\left(p^{2}, p^{\prime 2}, Q^{2}\right)=$ $u^{2}-4 s s^{\prime}$.

The integration region for the perturbative contribution in Eq. (7) is determined from the condition that the argument of all three $\delta$ functions might vanish simultaneously. the bounds of $s$ and $s^{\prime}$ are determined from the following inequality:

$$
\begin{equation*}
-1 \leq \frac{u \Delta-2 s \Delta^{\prime}}{\lambda^{1 / 2}\left(s, s^{\prime}, Q^{2}\right) \Delta} \leq+1 \tag{13}
\end{equation*}
$$

The contributions of higher states is parametrized as the corresponding spectral density starting from $s>s_{0}$ and $s^{\prime}>s_{0}^{\prime}$ invoking quark hadron duality.

We now present the results of calculations of dimension-3 and -5 operators:

$$
\begin{align*}
\Pi_{T}^{(3)} & =-\frac{\langle\bar{q} q\rangle}{r r^{\prime}}  \tag{14}\\
\Pi_{T}^{(5)} & =-m_{0}^{2}\langle\bar{q} q\rangle\left[\frac{1}{3 r^{2} r^{\prime}}+\frac{1}{3 r^{\prime 2} r}-\frac{m_{b}^{2}}{2 r^{3} r^{\prime}}-\frac{m_{s}^{2}}{2 r r^{\prime 3}}-\frac{2 m_{b}^{2}+2 m_{s}^{2}-2 m_{b} m_{s}+2 Q^{2}}{12 r^{2} r^{\prime 2}}\right] \tag{15}
\end{align*}
$$

where $r=p^{2}-m_{b}^{2}, r^{\prime}=p^{\prime 2}-m_{s}^{2}$.
In our calculations we do not calculate $\mathcal{O}\left(\alpha_{S}\right)$ corrections to the bare loop, and for consistency we also neglect $\mathcal{O}\left(\alpha_{S}\right)$ corrections in leptonic decay constants $f_{K_{0}^{*}}$ and $f_{B}$.

Substituting Eqs. (8), (14) and (15) into Eq. (7) and performing double Borel transformation in the variables $p^{2}$ and $p^{\prime 2}$, we get the following sum rule for the form factor $f_{T}$ :

$$
\begin{align*}
f_{T} & =-\frac{m_{B}+m_{K_{0}^{*}}}{2 f_{K_{0}^{*}} m_{K_{0}^{*}}} \frac{m_{b}}{f_{B} m_{B}^{2}} e^{\left(m_{B}^{2} / M_{1}^{2}+m_{K_{0}^{*}}^{2} / M_{2}^{2}\right)}\left\{-\frac{1}{(2 \pi)^{2}} \int d s d s^{\prime} \rho_{T}\left(s, s^{\prime}, Q^{2}\right) e^{-\left(s / M_{1}^{2}+s^{\prime} / M_{2}^{2}\right)}\right. \\
& +e^{-\left(m_{b}^{2} / M_{1}^{2}+m_{s}^{2} / M_{2}^{2}\right)}\left[-\langle\bar{q} q\rangle+m_{0}^{2}\langle\bar{q} q\rangle\left(\frac{1}{M_{1}^{2}}+\frac{1}{3 M_{2}^{2}}+\frac{m_{b}^{2}}{4 M_{1}^{4}}+\frac{m_{s}^{2}}{4 M_{2}^{4}}\right.\right. \\
& \left.\left.\left.+\frac{2 m_{b}^{2}-2 m_{b} m_{s}+2 m_{s}^{2}+2 Q^{2}}{12 M_{1}^{2} M_{2}^{2}}\right)\right]\right\} \tag{16}
\end{align*}
$$

For completeness we also present sum rules for the form factors $f_{+}$and $f_{-}$.

$$
f_{+}=-\frac{m_{b}}{f_{B} m_{B}^{2}} \frac{1}{f_{K_{0}^{*}} m_{K_{0}^{*}}} e^{m_{B}^{2} / M_{1}^{2}+m_{K_{0}^{*}}^{2} / M_{2}^{2}}\left\{-\frac{1}{(2 \pi)^{2}} \int d s d s^{\prime} \rho_{+} e^{-\left(s / M_{1}^{2}+s^{\prime} / M_{2}^{2}\right)}\right.
$$

$$
\begin{align*}
& +e^{-\left(m_{b}^{2} / M_{1}^{2}+m_{s}^{2} / M_{2}^{2}\right)}\left[\frac{1}{2}\langle\bar{q} q\rangle\left(m_{b}-m_{s}\right)-\frac{1}{12} m_{0}^{2}\langle\bar{q} q\rangle\left(\frac{3 m_{b}^{2}\left(m_{b}-m_{s}\right)}{2 M_{1}^{4}}+\frac{3 m_{s}^{2}\left(m_{b}-m_{s}\right)}{2 M_{2}^{4}}\right.\right. \\
& \left.\left.\left.-\frac{2\left(m_{b}-2 m_{s}\right)}{M_{2}^{2}}-\frac{2\left(2 m_{b}-m_{s}\right)}{M_{1}^{2}}+\frac{\left(m_{b}-m_{s}\right)\left(2 m_{b}^{2}+m_{b} m_{s}+2 m_{s}^{2}+2 Q^{2}\right)}{M_{1}^{2} M_{2}^{2}}\right)\right]\right\},  \tag{17}\\
f_{-} & =-\frac{m_{b}}{f_{B} m_{B}^{2}} \frac{1}{f_{K_{0}^{*}} m_{K_{0}^{*}}} e^{m_{B}^{2} / M_{1}^{2}+m_{K_{0}^{*}}^{2} / M_{2}^{2}}\left\{-\frac{1}{(2 \pi)^{2}} \int d s d s^{\prime} \rho_{-} e^{-\left(s / M_{1}^{2}+s^{\prime} / M_{2}^{2}\right)}\right. \\
& +e^{-\left(m_{b}^{2} / M_{1}^{2}+m_{s}^{2} / M_{2}^{2}\right)}\left[-\frac{1}{2}\langle\bar{q} q\rangle\left(m_{b}+m_{s}\right)+\frac{1}{12} m_{0}^{2}\langle\bar{q} q\rangle\left(\frac{3 m_{b}^{2}\left(m_{b}+m_{s}\right)}{2 M_{1}^{4}}\right.\right. \\
& +\frac{3 m_{s}^{2}\left(m_{b}+m_{s}\right)}{2 M_{2}^{4}}-\frac{2\left(m_{b}+3 m_{s}\right)}{M_{2}^{2}}-\frac{2\left(3 m_{b}+m_{s}\right)}{M_{1}^{2}} \\
& \left.\left.\left.+\frac{\left(m_{b}+m_{s}\right)\left(2 m_{b}^{2}+m_{b} m_{s}+2 m_{s}^{2}+2 Q^{2}\right)}{M_{1}^{2} M_{2}^{2}}\right)\right]\right\} \tag{18}
\end{align*}
$$

where

$$
\begin{align*}
& \rho_{+}=\frac{N_{c}}{4 \lambda^{1 / 2}}\left\{\left(\Delta^{\prime}+\Delta\right)(1+A+B)+\left[\left(m_{b}+m_{s}\right)^{2}+Q^{2}\right](A+B)\right\}  \tag{19}\\
& \rho_{-}=\frac{N_{c}}{4 \lambda^{1 / 2}}\left\{\left[\Delta^{\prime}+\Delta+\left(m_{b}+m_{s}\right)^{2}+Q^{2}\right](A-B)+\Delta^{\prime}-\Delta\right\}, \tag{20}
\end{align*}
$$

and

$$
\begin{align*}
A & =\frac{1}{\lambda}\left[2 s^{\prime} \Delta-u \Delta^{\prime}\right] \\
B & =\frac{1}{\lambda}\left[2 s \Delta^{\prime}-u \Delta\right] \tag{21}
\end{align*}
$$

Using Eqs. (1)-(3), we get the following expression for the differential decay width:

$$
\begin{align*}
\frac{d \Gamma\left(B \rightarrow K_{0}^{*} \ell^{+} \ell^{-}\right)}{d \hat{s}} & =\frac{G^{2} \alpha^{2} m_{B}^{5}}{3072 \pi^{5}}\left|V_{t b} V_{t s}^{*}\right|^{2} v \sqrt{\lambda\left(1, \hat{m}_{K_{0}^{*}}^{2}, \hat{s}\right)}\left\{\left[\left|C_{9}^{e f f} f_{+}+\frac{2 \hat{m}_{b}}{1+\hat{m}_{K_{0}^{*}}} C_{7} f_{T}\right|^{2}\right.\right. \\
& \left.+\left|C_{10} f_{+}\right|^{2}\right]\left(3-v^{2}\right) \lambda\left(1, \hat{m}_{K_{0}^{*}}^{2}, \hat{s}\right)+12 \hat{m}_{\ell}^{2}\left[\left(2+2 \hat{m}_{K_{0}^{*}}^{2}-\hat{s}\right)\left|f_{+}\right|^{2}\right. \\
& \left.\left.+2\left(1-\hat{m}_{K_{0}^{*}}^{2}\right) \operatorname{Re}\left[f_{+} f_{-}^{*}\right]+\hat{s}\left|f_{-}\right|^{2} \mid\right]\left. C_{10}\right|^{2}\right\} \tag{22}
\end{align*}
$$

where

$$
\hat{s}=\frac{q^{2}}{m_{B}^{2}}, \quad v=\sqrt{1-\frac{4 \hat{m}_{\ell}^{2}}{\hat{s}}}, \quad \hat{m}_{b}=\frac{m_{b}}{m_{B}}, \quad \hat{m}_{\ell}=\frac{m_{\ell}}{m_{B}}, \quad \hat{m}_{K_{0}^{*}}=\frac{m_{K_{0}^{*}}}{m_{B}}
$$

and $\lambda\left(1, m_{K_{0}^{*}}^{2}, \hat{s}\right)=1+m_{K_{0}^{*}}^{4}+\hat{s}^{2}-2 \hat{s}-2 m_{K_{0}^{*}}^{2}(1+\hat{s})$.

## 3 Numerical results of the sum rules

In this section we present our numerical study for the form factors $f_{+}\left(q^{2}\right), f_{-}\left(q^{2}\right)$, and $f_{T}\left(q^{2}\right)$ for the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay. From the expressions of the sum rules for the form factors we see that the main input parameters of all form factors are quark condensates, leptonic decay constants $f_{B}$ and $f_{K_{0}^{*}}$ of $B$ and $K_{0}^{*}$ mesons, respectively, Borel parameters $M_{1}^{2}$ and $M_{2}^{2}$, as well as continuum thresholds $s_{0}$ and $s_{0}^{\prime}$.

The value of the quark condensate at $\mu=1 \mathrm{GeV}$ is $\langle\bar{q} q\rangle=-(240 \pm 10 \mathrm{MeV})^{3}, m_{c}(\mu=$ $\left.m_{c}\right)=(1.275 \pm 0.015) \mathrm{GeV}, m_{b}=(4.7 \pm 0.1) \mathrm{GeV}$ [22]. The leptonic decay constant of $B$ and the continuum thresholds $s_{0}$ and $s_{0}^{\prime}$ are determined from the analysis of twopoint sum rules which have the values: $f_{B}=180 \mathrm{MeV}[16], s_{0}=(35 \pm 2) \mathrm{GeV}^{2}$ and $s_{0}^{\prime}=(4.4 \pm 0.4) \mathrm{GeV}^{2}[23]$. Few words about the value of the leptonic decay constant $f_{K_{0}^{*}}$ are in order. This constant is calculated within the $2-$ point QCD sum rules method in [23], including $\mathcal{O}\left(\alpha_{s}\right)$ corrections. As has already been noted, we neglect $\mathcal{O}\left(\alpha_{s}\right)$ corrections in the bare loop calculations, and hence, for consistency, we shall neglect them in calculation of the leptonic decay constant $f_{K^{*}}$. Neglecting $\mathcal{O}\left(\alpha_{s}\right)$ corrections from the result of [23], we obtain for the leptonic decay constant $f_{K^{*}}=340 \mathrm{MeV}$, which we shall use in further numerical calculations.

The Borel parameters $M_{1}^{2}$ and $M_{2}^{2}$ are not physical quantities. The results for the physically measurable quantities should be independent of them if the OPE can be performed up to infinite order. But, in sum rules, OPE is truncated to some finite order and for this reason the Borel parameters have to be chosen in such a working region that the physical results are practically independent on them. In choosing the working region of $M_{1}^{2}$ and $M_{2}^{2}$ the conditions must be satisfied: 1) the contribution of the excited should be small, and, $2)$ the power corrections should converge.

Our numerical analysis shows that both conditions are satisfied and the best stabilities of all form factors are achieved when $M_{1}^{2}$ and $M_{2}^{2}$ vary in the regions $8 \mathrm{GeV}^{2} \leq M_{1}^{2} \leq 15 \mathrm{GeV}^{2}$ and $2.5 \mathrm{GeV}^{2} \leq M_{2}^{2} \leq 4.5 \mathrm{GeV}^{2}$, respectively.

The values of the form factors at $q^{2}=0$ are

$$
\begin{align*}
& f_{+}(0)=0.31 \pm 0.08 \\
& f_{-}(0)=-0.31 \pm 0.07 \\
& f_{T}(0)=-0.26 \pm 0.07 \tag{23}
\end{align*}
$$

where the errors are due to the variation of Borel parameters, the continuum thresholds $s_{0}$ and $s_{0}^{\prime}$, the uncertainty in the condensate parameters, the variation of quark mass and meson decay constants.

Here we would like to make the following cautionary note. From the sum rules expressions for the form factors it is seen that they are very sensitive to the magnitude of the leptonic decay constant $f_{K_{0}^{*}}$ of the $K_{0}^{*}(1430)$ meson. The above-mentioned results for $f_{i}(0)$ are obtained at $f_{K_{0}^{*}}=340 \mathrm{MeV}$. But, if we use $f_{K_{0}^{*}}=300 \mathrm{MeV}$ which is given in [14] (after normalizing it to our definition) we obtain

$$
\begin{align*}
& f_{+}(0)=0.34 \pm 0.08 \\
& f_{-}(0)=-0.34 \pm 0.07 \\
& f_{T}(0)=-0.29 \pm 0.07 \tag{24}
\end{align*}
$$

When compared with our results, $f_{+}(0)$ and $f_{T}(0)$ are in good agreement within the error limits, while the value of $f_{-}(0)$ is $50 \%$ larger compared to the prediction of [14].

In calculating the total width of the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay, we need to know the $q^{2}$ dependence of the form factors $f_{+}\left(q^{2}\right), f_{-}\left(q^{2}\right)$ and $f_{T}\left(q^{2}\right)$ in the physical region $4 m_{\ell}^{2} \leq$ $q^{2} \leq\left(m_{B}-m_{K_{0}^{*}}\right)^{2}$. The $q^{2}$ dependence of the form factors in the physical region can be calculated directly from sum rules, which has comprehensively been discussed in [19, 20] and [24]. We are restricted to consider a region of $q^{2}$, calculated from the QCD side, where the correlator can be reliable. Our numerical analysis shows that this region is bounded as $0 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$. In order to extend the results to full physical region, we look such a parametrization of the form factors that they coincide with the sum rule predictions in the above-mentioned region of $q^{2}$.

Our analysis shows that the best fit for the $q^{2}$ dependence of the form factors can be written in the following form:

$$
\begin{equation*}
f_{i}(\hat{s})=\frac{f_{i}(0)}{1-a_{i} \hat{s}+b_{i} \hat{s}^{2}}, \tag{25}
\end{equation*}
$$

where $i=+,-$ or $T$ and $\hat{s}=q^{2} / m_{B}^{2}$. The values of the parameters $f_{i}(0), a_{i}$ and $b_{i}$ are given in table 1.

|  | $f_{i}(0)$ | $a_{i}$ | $b_{i}$ |
| :---: | ---: | :---: | :---: |
| $f_{+}$ | $0.31 \pm 0.08$ | 0.81 | -0.21 |
| $f_{-}$ | $-0.31 \pm 0.07$ | 0.80 | -0.36 |
| $f_{T}$ | $-0.26 \pm 0.07$ | 0.41 | -0.32 |

Table 1: Form factors for $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay in a three-parameter fit for $f_{K_{0}^{*}}=$ 340 MeV .

Using the equation of motion the form factor $f_{T}$ can be related to $f_{-}$as follows:

$$
\begin{equation*}
f_{T}\left(q^{2}\right)=\frac{\left(m_{b}-m_{s}\right)}{\left(m_{B}-m_{K^{*}}\right)} f_{-} . \tag{26}
\end{equation*}
$$

We see that, within the errors Eq. (26) is in agreement with the computed form factor $f_{T}$ from QCD sum rules.

The kinematical interval of the dilepton invariant mass $q^{2}$ is $4 m_{\ell}^{2} \leq q^{2} \leq\left(m_{B}-m_{K_{0}^{*}}\right)^{2}$ in which the long distance effects (the charmonium resonances) can give substantial contribution. The dominant contribution to the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$decays comes from the two low lying resonances $J / \psi$ and $\psi^{\prime}$, in the interval of $8 \mathrm{GeV}^{2} \leq q^{2} \leq 14 \mathrm{GeV}^{2}$. In order to minimize the hadronic uncertainties we discard this subinterval by dividing the kinematical region of $q^{2}$ as follows:

$$
\begin{aligned}
& \text { I } \quad 4 m_{\ell}^{2} \leq q^{2} \leq\left(m_{J \psi}-0.02 G e V\right)^{2} \\
& \text { II } \quad\left(m_{J \psi}+0.02 \mathrm{GeV}\right)^{2} \leq q^{2} \leq\left(m_{\psi^{\prime}}-0.02 \mathrm{GeV}\right)^{2} \\
& \text { III } \quad\left(m_{\psi^{\prime}}+0.02 \mathrm{GeV}\right)^{2} \leq q^{2} \leq\left(m_{B}-m_{K_{0}^{*}}\right)^{2}
\end{aligned}
$$

In Fig. (1) we present the dependence of the differential branching ratio for the $B \rightarrow$ $K_{0}^{*}(1430) e^{+} e^{-}$decay on $q^{2}$, as well as its dependence on $q^{2}$ due solely to short distance effects (æ = 0 case).

Taking into account the $q^{2}$ dependence of the form factors given in Eq. (25), performing integration over $\hat{s}$, and using the total life time $\tau_{B}=1.53 \times 10^{-12} s$ [25], we get the following for the branching ratios when only short distance contribution is taken into account:

$$
\begin{aligned}
\mathcal{B}\left(B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}\right) & =2.09 \times 10^{-7}\left(2.68 \times 10^{-7}\right), \\
\mathcal{B}\left(B \rightarrow K_{0}^{*}(1430) \mu^{+} \mu^{-}\right) & =2.07 \times 10^{-7}\left(2.66 \times 10^{-7}\right), \\
\mathcal{B}\left(B \rightarrow K_{0}^{*}(1430) \tau^{+} \tau^{-}\right) & =1.70 \times 10^{-9}\left(2.20 \times 10^{-9}\right),
\end{aligned}
$$

where the values in the parenthesis correspond to the choice of $f_{K_{0}^{*}}=300 \mathrm{MeV}$.
Taking long distance effects into account in the above-mentioned kinematical regions, we get the following for the branching ratios:

$$
\mathcal{B}\left(B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}\right)= \begin{cases}1.97 \times 10^{-7} & \text { region I }, \\ 1.66 \times 10^{-8} & \text { region II }, \quad \text { at } æ=1, \\ 3.90 \times 10^{-10} & \text { region III }\end{cases}
$$

and

$$
\mathcal{B}\left(B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}\right)= \begin{cases}2.13 \times 10^{-7} & \text { region I }, \\ 1.63 \times 10^{-8} & \text { region II }, \quad \text { at } æ=2, \\ 3.38 \times 10^{-10} & \text { region III },\end{cases}
$$

at $f_{K_{0}^{*}}=340 \mathrm{MeV}$. Note that the branching ratios of the $B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}$and $B \rightarrow K_{0}^{*}(1430) \mu^{+} \mu^{-}$decays are practically the same.

It follows from these results that the dominant contribution comes from region I (low invariant mass region), and this can be attributed to the existence of the factor $1 / q^{2}$. Since at LHC-b $10^{11}-10^{12}$ pairs are hoped to be produced, the expected number of events for the $B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}$decay in the low invariant mass region is of the order of $10^{4}-10^{5}$. Therefore, this mode sounds to be quite measurable at LHC-b. Comparing the value of the branching ratio for the $B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}$decay for the case when only short distance effects are taken into account, with the value calculated in region I, it can be said that
they practically coincide. Hence, in the light of above results, we can conclude that long distance effects do not give significant contribution in region I, and therefore, measurement of branching ratio in this region allows us to check the short distance structure of the effective Hamiltonian.

The smallness of the value of $\mathcal{B}\left(B \rightarrow K_{0}^{*}(1430) \tau^{+} \tau^{-}\right)$can be attributed to the small phase volume of this decay. Taking into account the efficiency for detecting $\tau$ leptons, the measurement of the branching ratio for $B \rightarrow K_{0}^{*}(1430) \tau^{+} \tau^{-}$is very difficult, even at LHC.

As the concluding remark we can state that, we study the semileptonic rare $B \rightarrow$ $K_{0}^{*}(1430) \ell^{+} \ell^{-}$decay. The transition form factors of the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}$are calculated in the framework of three-point QCD sum rules. The branching ratios of the relevant decay for $\ell=e, \mu, \tau$ leptons, when short and long distance effects are taken into account, are estimated. From these results we conclude that the $B \rightarrow K_{0}^{*}(1430) \ell^{+} \ell^{-}(\ell=e, \mu)$ decay can be measured in future planned experiments at LHC.

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## Figure captions

Fig. (1) The dependence of the differential branching ratio for the $B \rightarrow K_{0}^{*}(1430) e^{+} e^{-}$ decay on $q^{2}$, for the values of the fudge factor $æ=0$ (corresponding to short distance effects), $æ=1$ and $æ=2$.


Figure 1:


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