# Semileptonic $B \rightarrow \eta \ell \nu$ decay in light cone QCD 

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#### Abstract

We study semileptonic decay $B \rightarrow \eta \ell \nu$. The transition form factors for this decays are calculated by using light cone QCD sum rules.


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## 1 Introduction

The paradigm of CP violation related to the structure of CKM matrix in the Standard Model (and its beyond) is fueling an impressive experimental programme for the study of both exclusive and inclusive B-decays. The data from BaBar and Belle open new era of $B$ meson physics, and open a real possibility for detecting semileptonic decay modes of $B$ mesons. The semileptonic decay modes of B mesons are much cleaner samples then the nonleptonic decay modes since in these modes, there does not exist any problems connected with the presence of a third strongly interacting particle. For this reason, the study of semileptonic decays is one of the efficient ways for the determination of the CKM matrix elements. For example $V_{c b}$ has been determined from semileptonic meson decays [1]. CLEO collaboration [2] have measured the branching ratios of $B^{0} \rightarrow \pi^{-} \ell^{+} \nu$ and $B^{0} \rightarrow \rho^{-} \ell^{+} \nu$ which leads to $\left|V_{u b}\right|=\left(3.25 \pm 0.14_{-0.29}^{+0.21} \pm 0.55\right) 10^{-3}$. It is well known that for accurate determination of CKM matrix elements we need more reliable determination of transition form factors.

In this work we calculate the transition form factors for $B \rightarrow \eta \ell \nu$ decay in light cone QCD sum rules (LCQSR) method. The detailed description of the method and its applications can be found in $[3,4,5]$. It should be noted that interest to the $B \rightarrow \eta \ell \nu$ and $B \rightarrow \eta^{\prime} \ell \nu$ semileptonic decays is due to the fact that they also give information on the $\eta-\eta^{\prime}$ mixing (see for example $[6,7]$ ).

The work is organized as follows. In Sect. 2, we derive the sum rules for the transition form factors of $B \rightarrow \eta \ell \nu$ decay. Section 3 is devoted to the numerical analysis and contain a summary of the results and our conclusions.

## 2 Light Cone QCD Sum Rules for the $B \rightarrow \eta$ transition form factors

The $B \rightarrow \eta$ weak transition form factors $f_{+}\left(q^{2}\right)$ and $f_{-}\left(q^{2}\right)$ are defined as:

$$
\begin{equation*}
\langle\eta(p)| \bar{u} \gamma_{\mu}\left(1+\gamma_{5}\right) b|B(p+q)\rangle=2 f_{+} p_{\mu}+\left(f_{+}+f_{-}\right) q_{\mu} \tag{1}
\end{equation*}
$$

In this section, we calculate these form factors using light cone QCD sum rule method. We adopt the usual strategy of light cone sum rules method by considering the following correlator function:

$$
\begin{equation*}
\Pi_{\mu}(p, q)=i \int d^{4} x e^{i q x}\langle\eta(p)| T\left\{\bar{u}(x) \gamma_{\mu}\left(1+\gamma_{5}\right) b(x) \bar{b}(0) i\left(1+\gamma_{5}\right) u(0)\right\}|0\rangle \tag{2}
\end{equation*}
$$

The main reason for choosing the chiral $\bar{b} i\left(1+\gamma_{5}\right) u$ current instead of the $\bar{b} i \gamma_{5} u$ current, which have been used in the calculation of the $B \rightarrow \pi$ weak form factor [8], is that in this case, twist-3 wave functions which are the main inputs in LCQSR method and which bring the main uncertainty to the prediction, do not contribute (see below, see also [9]).

First let us consider the hadronic representation of the correlator. By inserting a complete set of states with the same quantum numbers as the operator $\bar{b} i\left(1+\gamma_{5}\right) u$ between the currents and isolating the pole term of the lowest pseudo scalar $B$ meson, we get the
following hadronic representation of the correlator:

$$
\begin{align*}
\Pi_{\mu}(p, q) & =\frac{\langle\eta| \bar{u} \gamma_{\mu} b|B\rangle\langle B| \bar{b} i \gamma_{5} u|0\rangle}{m_{B}^{2}-(p+q)^{2}}+\sum_{h} \frac{\langle\eta| \bar{u} \gamma_{\mu} b|h\rangle\langle h| \bar{b} i\left(1+\gamma_{5}\right) u|0\rangle}{m_{h}^{2}-(p+q)^{2}} \\
& =\Pi_{1}\left(q^{2},(p+q)^{2}\right) p_{\mu}+\Pi_{2}\left(q^{2},(p+q)^{2}\right) q_{\mu} \tag{3}
\end{align*}
$$

The sum in Eq. 3 takes into account the contributions of the higher states and continuum. Note that the intermediate states $h$ contain not only pseudo scalar resonances of masses greater the $m_{B}$, but also scalar resonances with $J^{P}=0^{+}$corresponding to the operator $\bar{b} u$.

For the invariant amplitudes $\Pi_{i}$, one can write a general dispersion relation in the $B$ meson momentum squared, $(p+q)^{2}$, as:

$$
\begin{equation*}
\Pi_{i}\left(q^{2},(p+q)^{2}\right)=\int d s \frac{\rho_{i}(s)}{s-(p+q)^{2}}+\text { subtractions } \tag{4}
\end{equation*}
$$

where subtractions are polynomials in $(p+q)^{2}$ and the spectral densities corresponding to Eq. (3) are given as:

$$
\begin{align*}
& \rho_{1}(s)=2 f_{+}\left(q^{2}\right) \frac{m_{B}^{2} f_{B}}{m_{b}} \delta\left(s-m_{B}^{2}\right)+\rho_{1}^{h}(s)  \tag{5}\\
& \rho_{2}(s)=\left(f_{+}+f_{-}\right) \frac{m_{B}^{2} f_{B}}{m_{b}} \delta\left(s-m_{B}^{2}\right)+\rho_{2}^{h}(s) \tag{6}
\end{align*}
$$

The first terms in Eqs. (5) and (6) represent the ground state B-meson contribution and are easily obtained from Eq. (3) using Eq. (1) and the definition

$$
\langle B| \bar{b} i \gamma_{5} d|0\rangle=\frac{m_{B}^{2} f_{B}}{m_{b}}
$$

whereas $\rho_{i}^{h}$ represent the spectral densities of the higher resonances and the continuum. The spectral densities $\rho_{i}^{h}$ can be approximated by invoking the quark-hadron duality ansatz:

$$
\begin{equation*}
\rho_{i}^{h}(s)=\rho_{i}^{Q C D} \theta\left(s-s_{0}\right) \tag{7}
\end{equation*}
$$

So for the hadronic representation of the invariant amplitudes $\Pi_{i}$ we get:

$$
\begin{align*}
& \Pi_{1}=\frac{2 f_{+}\left(q^{2}\right) m_{B}^{2} f_{B}}{m_{b}\left(m_{B}^{2}-(p+q)^{2}\right)}+\int_{s_{0}}^{\infty} d s \frac{\rho_{1}^{Q C D}(s)}{s-(p+q)^{2}}+\text { subtractions } \\
& \Pi_{2}=\frac{\left(f_{+}+f_{-}\right) m_{B}^{2} f_{B}}{m_{b}\left(m_{B}^{2}-(p+q)^{2}\right)}+\int_{s_{0}}^{\infty} d s \frac{\rho_{2}^{Q C D}(s)}{s-(p+q)^{2}}+\text { subtractions } \tag{8}
\end{align*}
$$

For obtaining sum rules for the form factors $f_{+}$and $f_{-}$we must calculate the correlator function in QCD. It can be done by using the light cone OPE method; i.e. by expanding the $T$ product of currents near the light cone $x^{2} \simeq 0$. After contracting the $b$-quark fields we get

$$
\begin{equation*}
\Pi_{\mu}(p, q)=i \int d^{4} x e^{i q x}\langle\eta(p)| \bar{u}(x) \gamma_{\mu}\left(1+\gamma_{5}\right) S_{b}(x)\left(1+\gamma_{5}\right) u(0)|0\rangle \tag{9}
\end{equation*}
$$

where $S_{b}$ is the full propagator of the $b$ quark. Its explicit expression is given by the following expression

$$
\begin{align*}
i S_{b}(x)= & i S_{b}^{0}(x)-i g s \int \frac{d^{4} k}{(2 \pi)^{4}} e^{-i k x} \int_{0}^{2} d v\left\{\frac{1}{2} \frac{\not k+m_{b}}{\left(m_{b}^{2}-k^{2}\right)^{2}} G^{\mu \nu}(v x) \sigma_{\mu \nu}\right. \\
& \left.+\frac{1}{m_{b}^{2}-k^{2}} v x_{\mu} G^{\mu \nu}(v x) \gamma_{\nu}\right\} \tag{10}
\end{align*}
$$

Here $S_{b}^{0}(x)$ is the free quark propagator of the massive $b$ quark:

$$
\begin{equation*}
S_{b}^{0}(x)=\frac{m_{b}^{2}}{4 \pi^{2}} \frac{K_{1}\left(m_{b} \sqrt{-x^{2}}\right)}{\sqrt{-x^{2}}}-i \frac{m_{b}^{2}}{4 \pi^{2}} \frac{\not x}{x^{2}} K_{2}\left(m_{b} \sqrt{-x^{2}}\right) \tag{11}
\end{equation*}
$$

where $K_{i}$ are the Bessel functions.
From Eqs. (9)-(11) it follows that, in order to calculate the theoretical part of the correlator function, the matrix elements of the nonlocal operators between $\eta$ meson and vacuum states are needed.

Here we would like to do following remark: in all the following calculations we neglect the $\eta-\eta^{\prime}$ mixing since it is very small [1], and therefore we choose as the interpolating current for the $\eta$ meson, the $\mathrm{SU}(3)$ octet axial vector current:

$$
\begin{equation*}
J_{\mu}=\frac{1}{\sqrt{6}}\left(\bar{u} \gamma_{\mu} \gamma_{5} u+\bar{d} \gamma_{\mu} \gamma_{5} d-2 \bar{s} \gamma_{\mu} \gamma_{5} s\right) \tag{12}
\end{equation*}
$$

In order to simplify the notation we use $\bar{q} \Gamma_{\mu} q$ to denote $\frac{1}{\sqrt{6}}\left(\bar{u} \gamma_{\mu} \gamma_{5} u+\bar{d} \gamma_{\mu} \gamma_{5} d-2 \bar{s} \gamma_{\mu} \gamma_{5} s\right)$ and introduce notation $F_{\eta}=\frac{f_{\eta}}{\sqrt{6}}$, where $f_{\eta}$ determined as $\langle 0| \bar{q}(0) \Gamma_{\mu} q(0)|\eta(q)\rangle=-i f_{\eta} q_{\mu}$.

An important observation is that the terms in $S_{b}(x)$ containing odd number of gamma matrices do not contribute, i.e. leading twist-3 terms do not give any contribution. Up to twist-4, the $\eta$ meson wave functions are defined in the following way [11] :

$$
\begin{align*}
\langle\eta(p)| \bar{q} \gamma_{\mu} \gamma_{5} q|0\rangle= & -i f_{\eta} p_{\mu} \int_{0}^{1} d u e^{-i u p x}\left[\varphi_{\eta}(u)+\frac{1}{16} m_{\eta}^{2} x^{2} A(u)\right] \\
& -\frac{i}{2} f_{\eta} m_{\eta}^{2} \frac{x_{\mu}}{p x} \int_{0}^{1} d u e^{-i u p x} B(u)  \tag{13}\\
\langle\eta(p)| \bar{q}(x) \gamma_{\mu} \gamma_{5} g_{s} G_{\alpha \beta}(v x) q(0)|0\rangle= & f_{\eta} m_{\eta}^{2}\left[p_{\beta}\left(g_{\alpha \mu}-\frac{x_{\alpha} p_{\mu}}{p x}\right)-p_{\alpha}\left(g_{\beta \mu}-\frac{x_{\beta} p_{\mu}}{p x}\right)\right] \\
& \times \int \mathcal{D} \alpha_{i} \varphi_{\perp}\left(\alpha_{i}\right) e^{-i p x\left(\alpha_{1}+u \alpha_{3}\right)} \\
& +f_{\eta} m_{\eta}^{2} \frac{p_{\mu}}{p x}\left(p_{\alpha} x_{\beta}-p_{\beta} x_{\alpha}\right) \int \mathcal{D} \alpha_{i} \varphi_{\|}\left(\alpha_{i}\right) e^{-i p x\left(\alpha_{1}+u \alpha_{3}\right)} \\
\langle\eta(p)| \bar{q}(x) g_{s} \tilde{G}_{\alpha \beta}(v x) \gamma_{\mu} q(0)|0\rangle= & i f_{\eta} m_{\eta}^{2}\left[p_{\beta}\left(g_{\alpha \mu}-\frac{x_{\alpha} p_{\mu}}{p x}\right)-p_{\alpha}\left(g_{\beta \mu}-\frac{x_{\beta} p_{\mu}}{p x}\right)\right]  \tag{14}\\
& \times \int \mathcal{D} \alpha_{i} \tilde{\varphi}_{\perp}\left(\alpha_{i}\right) e^{-i p x\left(\alpha_{1}+u \alpha_{3}\right)} \\
& +i f_{\eta} m_{\eta}^{2} \frac{p_{\mu}}{p x}\left(p_{\alpha} x_{\beta}-p_{\beta} x_{\alpha}\right) \int \mathcal{D} \alpha_{i} \tilde{\varphi}_{\|}\left(\alpha_{i}\right) e^{-i p x\left(\alpha_{1}+u \alpha_{3}\right)} \tag{15}
\end{align*}
$$

where $\tilde{G}_{\mu \nu}=\frac{1}{2} \epsilon_{\mu \nu \sigma \lambda} G^{\sigma \lambda}, \mathcal{D} \alpha_{i}=d \alpha_{1} d \alpha_{2} d \alpha_{3} \delta\left(1-\alpha_{1}-\alpha_{2}-\alpha_{3}\right)$. In Eqs. (13)-(15), the $\varphi_{\eta}(u)$ is the leading twist-2, $A(u)$ and part of $B(u)$ are two particle twist- $4, \varphi_{\|}\left(\alpha_{i}\right), \varphi_{\perp}\left(\alpha_{i}\right)$, $\tilde{\varphi}_{\|}\left(\alpha_{i}\right)$ and $\tilde{\varphi}_{\perp}\left(\alpha_{i}\right)$ are three particle twist- 4 wave functions.

Here we should note that that matrix element $\langle\eta(p)| \bar{u} \gamma_{\mu} G^{\alpha \beta}(u x) \sigma_{\alpha \beta} u|0\rangle=0$ due to the parity invariance of strong interactions and using the identity

$$
\gamma_{\mu} \sigma_{\alpha \beta}=i\left(g_{\mu \alpha} \gamma_{\beta}-g_{\mu \beta} \gamma_{\alpha}\right)+\epsilon_{\mu \alpha \beta \rho} \gamma^{\rho} \gamma_{5}
$$

Inserting Eqs. (10) and (11) into Eq. (9) and using definitions of $\eta$-meson wave functions (Eqs. (13)-(15)) for the theoretical part of the correlation function we get:

$$
\begin{align*}
\Pi_{\mu}^{t h}=\quad & i \sqrt{2} \int d^{4} x e^{i q x}\left[\int _ { 0 } ^ { 1 } d u e ^ { i u p x } \left\{-\frac{1}{32 \pi^{2}} F_{\eta} m_{b}^{2}\left(16 \varphi_{\eta}+A m_{\eta}^{2} x^{2}\right) \frac{K_{1}\left(m_{b} \sqrt{-x^{2}}\right)}{\sqrt{-x^{2}}} p_{\mu}\right.\right. \\
& \left.-\frac{1}{4 \pi^{2}} F_{\eta} m_{b}^{2} m_{\eta}^{2} B \frac{K_{1}\left(m_{b} \sqrt{-x^{2}}\right)}{\sqrt{-x^{2}}} \frac{x_{\mu}}{p x}\right\} \\
& +\int \mathcal{D} \alpha \int_{0}^{1} d u e^{i\left(\alpha_{2}+u \alpha_{3}\right) p x}\left\{\frac{1}{12 \pi^{2}} F_{\eta} m_{\eta}^{2} m_{b}\left(\varphi_{\|}+\tilde{\varphi}_{\|}\right) K_{0}\left(m_{b} \sqrt{-x^{2}}\right) \frac{x_{\mu} p^{2}-p_{\mu}(p x)}{p x}\right. \\
& \left.\left.+\frac{1}{12 \pi^{2}} F_{\eta} m_{\eta}^{2} m_{b}\left(\varphi_{\perp}+\tilde{\varphi}_{\perp}\right) K_{0}\left(m_{b} \sqrt{-x^{2}}\right) \frac{p^{2} x_{\mu}+2(p x) p_{\mu}}{p x}\right\}\right] \tag{16}
\end{align*}
$$

The next task is to carry out the fourier transformation and then to take the Borel transform with respect to the variable $(p+q)^{2}$ in order to suppress the contributions of the higher states and the continuum and also in order to eliminate the subtraction terms. Finally we identify the same structures both at the hadronic and quark gluon levels. Subtraction of the continuum contribution is done by employing the quark-hadron duality as mentioned before. This amounts to restricting the spectral integral to $s \leq s_{0}$. (for more details about this procedure see the appendix and also [8]). For the theoretical part of the correlator (2) function we get:

$$
\begin{align*}
\Pi_{\mu}^{t h}= & \sqrt{2} F_{\eta} \frac{m_{\eta}^{2}}{3} I_{1}^{1}\left(\varphi_{\|}+\tilde{\varphi}_{\|}-2 \varphi_{\perp}-2 \tilde{\varphi}_{\perp}\right) p_{\mu}+\sqrt{2} F_{\eta} \frac{m_{\eta}^{4}}{3 m_{b}} \tilde{I}_{2}^{2}\left(\varphi_{\|}+\tilde{\varphi}_{\|}+\varphi_{\perp}+\tilde{\varphi}_{\perp}\right) p_{\mu} \\
& +2 \sqrt{2} F_{\eta} m_{b} J_{1}^{0}\left(\varphi_{\eta}\right) p_{\mu}-\sqrt{2} \frac{F_{\eta}}{16} m_{b} m_{\eta}^{2} J_{1}^{2}(A) p_{\mu}-\sqrt{2} F_{\eta} m_{\eta}^{2} \tilde{J}_{2}^{1}(B) p_{\mu} \\
& -\sqrt{2} F_{\eta} m_{\eta}^{2} J_{2}^{1}(B) q_{\mu}+\frac{\sqrt{2}}{3 m_{b}} F_{\eta} m_{\eta}^{4} I_{2}^{2}\left(\varphi_{\|}+\tilde{\varphi}_{\|}+\varphi_{\perp}+\tilde{\varphi}_{\perp}\right) q_{\mu} \tag{17}
\end{align*}
$$

where the functions $I_{n}^{m}(\varphi), J_{n}^{m}(\varphi), \tilde{I}_{n}^{m}(\varphi)$, and $\tilde{J}_{n}^{m}(\varphi)$ are defined in the appendix.
Equating the coefficients of the corresponding $p_{\mu}$ and $q_{\mu}$ structures in Eqs. (3-6)) and (17) we get the following sum rules for the form factors $f_{+}$and $f_{+}+f_{-}$respectively:

$$
\begin{align*}
f_{+}= & \frac{m_{b} F_{\eta}}{\sqrt{2} m_{B}^{2} f_{B}} e^{\frac{m_{B}^{2}}{M^{2}}}\left[\frac{m_{\eta}^{2}}{3} I_{1}^{1}\left(\varphi_{\|}+\tilde{\varphi}_{\|}-2 \varphi_{\perp}-2 \tilde{\varphi}_{\perp}\right)\right. \\
& \left.+\frac{m_{\eta}^{4}}{3 m_{b}} \tilde{I}_{2}^{2}\left(\varphi_{\|}+\tilde{\varphi}_{\|}+\varphi_{\perp}+\tilde{\varphi}_{\perp}\right)+2 m_{b} J_{1}^{0}\left(\varphi_{\eta}\right)-\frac{m_{b} m_{\eta}^{2}}{16} J_{1}^{2}(A)-m_{\eta}^{2} \tilde{J}_{2}^{1}(B)\right]  \tag{18}\\
f_{+}+f_{-}= & \frac{\sqrt{2} F_{\eta} m_{b}}{m_{B}^{2} f_{B}} e^{\frac{m_{B}^{2}}{M^{2}}}\left[-m_{\eta}^{2} J_{2}^{1}(B)+\frac{m_{\eta}^{4}}{3 m_{b}} I_{2}^{2}\left(\varphi_{\|}+\tilde{\varphi}_{\|}+\varphi_{\perp}+\tilde{\varphi}_{\perp}\right)\right] \tag{19}
\end{align*}
$$

Eqs. (18) and (19) are our final results for the $B \rightarrow \eta$ transition form factors.

## 3 Numerical Analysis

From Eqs. (18) and (19) it follows that the main input parameters of these sum rules are the $\eta$ meson wave functions. The explicit expressions of the wave functions $\varphi_{\eta}(u), A(u)$, $B(u)$ and $\varphi_{\|}\left(\alpha_{i}\right), \varphi_{\perp}\left(\alpha_{i}\right), \tilde{\varphi}_{\|}\left(\alpha_{i}\right)$, and $\tilde{\varphi}_{\perp}\left(\alpha_{i}\right)$ are given in [11].

The next input parameter of the sum rule is leptonic decay constant $F_{\eta}$. It should be noted that for $\eta$ meson case, the situation is more complicated compared to the $\pi$ and $K$ meson cases, due to the mixing with $\eta^{\prime}$. As we already noted that we neglect mixing between $\eta$ and $\eta^{\prime}$. More recent analysis show that the coupling to the octet current $\eta$ is $f_{\eta}=159 \mathrm{MeV}$ [13] which we use in our numerical calculations.

In our calculations we neglect QCD radiative corrections. Therefore for consistency we neglect QCD corrections to the leptonic decay $f_{B}=160 \mathrm{MeV}$ [8] (see also [14]).

Having these input parameters, we can carry out our numerical calculation. According to QCD sum rules philosophy, first of all, a suitable range for the auxiliary Borel parameter $M^{2}$ should be found such that the numerical results are stable. The lower limit of $M^{2}$ is determined by the requirement that the terms $\sim M^{-2 n},(n>1)$ remain sub dominant. The upper bound of $M^{2}$ is determined by requiring that the contributions of the higher resonances and the continuum are less then $30 \%$ of the total result. Our numerical calculations leads that both requirements are satisfied in the region $10 \mathrm{GeV}^{2}<M^{2}<16 \mathrm{GeV}^{2}$. It should be noted that the light cone QCD sum rules predictions for the form factors are reliable at the region

$$
q^{2} \lesssim m_{b}^{2}-2 m_{b} \chi
$$

where $\chi$ is a typical hadronic scale of roughly 0.5 GeV . Then $q^{2} \lesssim 18 \mathrm{GeV}^{2}$.
In Figs. (1) and (2), the $M^{2}$ dependencies of the form factors $f_{+}\left(q^{2}\right)$ and $f_{-}\left(q^{2}\right)$ is depicted for three different values of $q^{2}=0,5,10 \mathrm{GeV}^{2}$ for two different values of the continuum threshold $s_{0}=35,40 \mathrm{GeV}^{2}$. From these figures, we see that $f_{+}\left(q^{2}\right)$ and $f_{-}\left(q^{2}\right)$ vary weakly as $M^{2}$ varies in the region $10 \mathrm{GeV}^{2}<M^{2}<16 \mathrm{GeV}^{2}$ up to $q^{2}=18 \mathrm{GeV}^{2}$.

Having the working region of $M^{2}$, our next problem is to find the dependence of the form factors $f_{+}\left(f_{-}\right)\left(q^{2}\right)$ on $q^{2}$ at given values of $M^{2}$, namely at $M^{2}=12 \mathrm{GeV}^{2}$ and $M^{2}=$ $16 \mathrm{GeV}^{2}$, at $s_{0}=35 \mathrm{GeV}^{2}$ and $s_{0}=40 \mathrm{GeV}^{2}$. We find that $f_{+}^{\eta}(0)=0.15 \pm 0.3$ and $f_{-}^{\eta}(0)=-0.16 \pm 0.2$. As we already noted, in the region $q^{2} \geq 18 \mathrm{GeV}^{2}$, applicability of the light cone QCD sum rules is questionable. In order to extend our results to the full physical region, we look some parameterization of the form factors in such a way that in the region, $0 \leq q^{2} \leq 18 \mathrm{GeV}^{2}$, this parameterization coincide with light cone QCD sum rules predictions. A good parameterization of the $q^{2}$ dependence can be given in terms of three parameters as:

$$
\begin{equation*}
f_{i}\left(q^{2}\right)=\frac{f_{i}(0)}{1-a_{F}\left(\frac{q^{2}}{m_{B}^{2}}\right)+b_{F}\left(\frac{q^{2}}{m_{B}^{2}}\right)^{2}} \tag{20}
\end{equation*}
$$

The values of these parameters for the $B \rightarrow \eta$ transition form factors $f_{+}\left(f_{-}\right)$are obtained as $a_{F}=1.07 \pm 0.08\left(a_{F}=1.1 \pm 0.1\right)$ and $b_{F}=0.19 \pm 0.16\left(b_{F}=0.28 \pm 0.19\right)$ where the quoted errors are only due to the variations of the continuum threshold, $s_{0}$, and the Borel mass, $M^{2}$.

It should be noted that the form factors of $B \rightarrow \eta$ transition can be related from $B \rightarrow \pi$ transition form factors using $S U(3)_{F}$ symmetry. For example, the value of $f_{+}\left(q^{2}=0\right)=$ 0.15 obtained via $S U(3)_{F}$ symmetry is consistent with our prediction of $f_{+}\left(q^{2}=0\right)$.

Finally, we would like to note that the background for the $B \rightarrow \eta \ell \nu$ decay would be much smaller than that for the $B \rightarrow \pi \ell \nu$ decay, due to the much lower multiplicity, since the background caused by $B \rightarrow \eta X$ is one order of magnitude smaller than that of $B \rightarrow \pi X$.

In conclusion, we calculate the transition form factors for $B \rightarrow \eta \ell \nu$ decay using the light cone QCD sum rules.

## A Appendix

In this appendix, the derivations of the explicit forms of the functions, $I_{i}$, and $J_{i}$, appearing in Eq. (17) are presented and the method to calculate the continuum subtractions is demonstrated. For this purpose, consider a general term of the correlation function :

$$
\begin{equation*}
\Pi(p, q)=\int d^{4} x e^{i q x} \int_{0}^{1} d u \int \mathcal{D} \alpha_{i} e^{i\left(\alpha_{2}+u \alpha_{3}\right) p x} \varphi\left(\alpha_{i}\right) f(u) \frac{K_{\nu}\left(m_{b} \sqrt{-x^{2}}\right)}{\left(\sqrt{-x^{2}}\right)^{\nu}} \tag{A1}
\end{equation*}
$$

where $\mathcal{D} \alpha_{i}=d \alpha_{1} d \alpha_{2} d \alpha_{3} \delta\left(1-\alpha_{1}-\alpha_{2}-\alpha_{3}\right)$ After switching to Euclidean space and carrying out the $x$ integration, one obtains:

$$
\begin{equation*}
\Pi(p, q)=-i \frac{2 \pi^{2}}{m_{b}^{2}} \int d u \int \mathcal{D} \alpha_{i} d t t^{1-\nu} \varphi\left(\alpha_{i}\right) f(u) e^{-\frac{t}{2 m_{b}}\left(Q^{2}+m_{b}^{2}\right)} \tag{A2}
\end{equation*}
$$

where $Q^{2}=(q+k p)^{2}=q^{2} \bar{k}-p^{2} k \bar{k}+k(q+p)^{2}$ and $k=\alpha_{2}+u \alpha_{3}, \bar{k}=1-k$, and all the appearing momenta are Euclidean. In order to obtain Eq. (A2), the following representation of the Bessel function $K_{\nu}(x)$ is used:

$$
\begin{equation*}
\frac{K_{\nu}\left(m_{b} \sqrt{x_{E}^{2}}\right)}{\left(\sqrt{x_{E}^{2}}\right)^{\nu}}=\frac{1}{2} \int_{0}^{\infty} \frac{d t}{t^{\nu+1}} \exp \left[-\frac{m_{b}}{2}\left(t+\frac{x_{E}^{2}}{t}\right)\right] \tag{A3}
\end{equation*}
$$

where $x_{E}$ is the Euclidean position vector. Carrying out the Borel transformation with respect to $(p+q)^{2}$ using the identity:

$$
\begin{equation*}
\mathcal{B} e^{-\alpha(p+q)^{2}}=\delta\left(\frac{1}{M^{2}}-\alpha\right) \tag{A4}
\end{equation*}
$$

where $\mathcal{B}$ stands for the Borel transformation and $M^{2}$ is the Borel parameter and carrying out the $t$ integral, one obtains:

$$
\begin{equation*}
\Pi\left(M^{2}\right)=-\frac{4 i \pi^{2}}{m_{b}} \int d u \int \mathcal{D} \alpha_{i}\left(\frac{2 m_{b}}{k M^{2}}\right)^{1-\nu} e^{-s(k)} \frac{\varphi\left(\alpha_{i}\right) f(u)}{k} \tag{A5}
\end{equation*}
$$

where

$$
\begin{equation*}
s(k)=\frac{m_{b}^{2}-q^{2} \bar{k}+p^{2} k \bar{k}}{k} \tag{A6}
\end{equation*}
$$

where we have switched back to Minkowskian spacetime. Note that one can write the factors $\frac{2 m_{b}}{k M^{2}}$ is the integrand as a differential operator acting on the exponential in the integrand and hence:

$$
\begin{equation*}
\Pi\left(M^{2}\right)=-\frac{4 i \pi^{2}}{m_{b}}\left(-\frac{\partial}{\partial m_{b}}\right)^{1-\nu} \int d u \mathcal{D} \alpha_{i} e^{-s(k)} \frac{\varphi\left(\alpha_{i}\right) f(u)}{k} \tag{A7}
\end{equation*}
$$

Note that

$$
\begin{equation*}
e^{-\frac{s(k)}{M^{2}}}=\int_{0}^{\infty} d s e^{-\frac{s}{M^{2}}} \delta(s-s(k)) \tag{A8}
\end{equation*}
$$

which is nothing but the spectral representation of the exponential. The contributions of the higher states and the continuum are subtracted by replacing a cutoff, $s_{0}$, instead of the infinity as the upper limit of the spectral integral. Hence the effect of subtraction of the continuum and the higher states is to restrict the integration region in Eq. (A7) to the regions of $k$ for which $0 \leq s(k) \leq s_{0}$ which implies that $\delta \leq k \leq 1$ where

$$
\begin{equation*}
\delta=\frac{m_{\eta}^{2}+q^{2}-s_{0}+\sqrt{\left(m_{\eta}^{2}+q^{2}-s_{0}\right)^{2}-4 m_{\eta}^{2}\left(q^{2}-m_{b}^{2}\right)}}{2 m_{\eta}^{2}} \tag{A9}
\end{equation*}
$$

where $p^{2}=m_{\eta}^{2}$. Hence the contributions of the term given in Eq. (A1) to the sum rules after Borel transformation and the continuum is subtracted is given by:

$$
\begin{equation*}
\Pi\left(M^{2}\right)=-\frac{4 i \pi^{2}}{m_{b}}\left(-\frac{\partial}{\partial m_{b}}\right)^{1-\nu} \int d u \mathcal{D} \alpha_{i} \frac{\varphi\left(\alpha_{i}\right) f(u)}{k} e^{-\frac{s(k)}{M^{2}}} \tag{A10}
\end{equation*}
$$

where the integration region is determined by the conditions:

$$
\begin{array}{r}
\delta \leq k \leq 1 \\
0 \leq u, \alpha_{i} \leq 1 \tag{A11}
\end{array}
$$

and the Dirac's delta function in the definition of $\mathcal{D} \alpha_{i}$ fixes $\sum_{i} \alpha_{i}=1$.
In order to obtain the contributions to the sum rules from terms containing additional powers of $x_{\mu}$ one can use the trick of replacing $x_{\mu}$ by differentiation with respect to $q_{\mu}$. For terms containing a factor of $p x$ in the denominator, one can use the following trick: in order not to have any singularity at $p x=0$, the integral of these wave functions in the absence of the exponential should cancel. Hence, for these terms only, one can write:

$$
\begin{equation*}
e^{i \alpha p x} \rightarrow e^{i \alpha p x}-1=i p x \int_{0}^{\alpha} d k e^{i k p x} \tag{A12}
\end{equation*}
$$

and the rest of the calculation is similar to the presented one. Note that the subtracted 1 does not contribute.

When the contributions of the continuum is subtracted and the Borel transformation is carried out for the other terms, the following functions which appear in Eq. (17) are
encountered:

$$
\begin{align*}
I_{1}^{n}(\varphi)= & \left(-\frac{\partial}{\partial m_{b}}\right)^{n}\left[\int_{0}^{1-\delta} d \alpha_{1} \int_{0}^{1-\delta-\alpha_{1}} d \alpha_{3} \int_{0}^{1} d u+\int_{0}^{1-\delta} d \alpha_{1} \int_{1-\delta-\alpha_{1}}^{1-\alpha_{1}} \int_{\frac{\delta-1+\alpha_{1}+\alpha_{3}}{\alpha_{3}}}^{1} d u\right] \\
& \frac{\varphi\left(\alpha_{1}, 1-\alpha_{1}-\alpha_{3}, \alpha_{3}\right)}{1-\alpha_{1}-\alpha_{3}+u \alpha_{3}} e^{-\frac{s\left(1-\alpha_{1}-\alpha_{3}+u \alpha_{3}\right)}{M^{2}}}  \tag{A13}\\
I_{2}^{n}(\varphi)= & \left(-\frac{\partial}{\partial m_{b}}\right)^{n}\left[\int_{0}^{1-\delta} d \alpha_{1} \int_{0}^{1-\delta-\alpha_{1}} d \alpha_{3} \int_{0}^{1} d u \int_{\delta}^{1-\alpha 1-\alpha_{3}+u \alpha_{3}} d k\right. \\
& \left.+\int_{0}^{1-\delta} d \alpha_{1} \int_{1-\delta-\alpha_{1}}^{1-\alpha_{1}} d \alpha_{3} \int_{\frac{\delta-1+\alpha_{1}+\alpha_{3}}{\alpha_{3}}}^{1} d u \int_{\delta}^{1-\alpha_{1}-\alpha_{3}+u \alpha_{3}} d k\right] \\
& \frac{\varphi\left(\alpha_{1}, 1-\alpha_{1}-\alpha_{3}, \alpha_{3}\right)}{1-\alpha_{1}-\alpha_{3}+u \alpha_{3}} e^{-\frac{s\left(1-\alpha_{1}-\alpha_{3}+u \alpha_{3}\right)}{M^{2}}}  \tag{A14}\\
& +\int_{0}^{1-\delta} d \alpha_{1} \int_{1-\delta-\alpha_{1}}^{1-\alpha_{1}} d \alpha_{3} \int_{\left.\frac{\delta-1+\alpha_{1}+\alpha_{3}}{\alpha_{3}} d u \int_{\delta}^{1-\alpha_{1}-\alpha_{3}+u \alpha_{3}} d k\right]}^{\tilde{I}_{2}^{n}(\varphi)=} \\
& \left(-\frac{\partial}{\partial m_{b}}\right)^{n}\left[\int_{0}^{1-\delta} d \alpha_{1} \int_{0}^{1-\delta-\alpha_{1}} d \alpha_{3} \int_{0}^{1} d u \int_{\delta}^{1-\alpha 1-\alpha_{3}+u \alpha_{3}} d k\right. \\
J_{1}^{n}(\varphi)= & \left.\left(-\frac{\partial}{\partial m_{b}}\right)^{n} \int_{\delta}^{\left.1-\alpha_{3}, \alpha_{3}\right) e^{-\frac{s\left(1-\alpha_{1}-\alpha_{3}+u \alpha_{3}\right)}{M^{2}}} d u \frac{\varphi(u)}{u} e^{-\frac{s(u)}{M^{2}}}} \begin{array}{rl}
J_{2}^{n}(\varphi)= & \left(-\frac{\partial}{\partial m_{b}}\right)^{n} \int_{\delta}^{1} d u \int_{\delta}^{u} d k \frac{\varphi(u)}{k} e^{-\frac{s(k)}{M^{2}}} \\
\tilde{J}_{2}^{n}(\varphi)= & \left(-\frac{\partial}{\partial m_{b}}\right)^{n} \int_{\delta}^{1} d u \int_{\delta}^{u} d k \varphi(u) e^{-\frac{s(k)}{M^{2}}}
\end{array}\right) \tag{A15}
\end{align*}
$$



Figure 1: The dependence of the form factor $f_{+}\left(q^{2}\right)$ on the Borel parameter $M^{2}$ at $q^{2}=0$, 5, and $10 \mathrm{GeV}^{2}$ for $s_{0}=35 \mathrm{GeV}^{2}$ and $s_{0}=40 \mathrm{GeV}^{2}$.


Figure 2: The same as Fig. (1) but for the form factor $f_{-}\left(q^{2}\right)$.


Figure 3: The dependence of the form factor $f_{+}\left(q^{2}\right)$ on $q^{2}$ at the continuum threshold $s_{0}=35 \mathrm{GeV}^{2}$ and $s_{0}=40 \mathrm{GeV}^{2}$ and at the Borel mass $M^{2}=12 \mathrm{GeV}^{2}$ and $M^{2}=16 \mathrm{GeV}^{2}$.


Figure 4: The same as Fig. (3) but for the form factor $f_{-}\left(q^{2}\right)$.

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