$\Sigma_Q \Lambda_Q$ transition magnetic moments in light cone QCD sum rules

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Abstract

Using the general form of Σ_Q and Λ_Q (Q = b or c) currents, $\Sigma_Q \Lambda_Q$ transition magnetic moments are calculated in framework of the light cone QCD sum rules method. In this approach nonperturbative effects are described by the photon wave functions and only two-particle photon wave functions are taken into account. Our predictions on transition magnetic moments are $\mu_{\Sigma_c\Lambda_c} = -(1.5 \pm 0.4)\mu_N$ and $\mu_{\Sigma_b\Lambda_b} =$ $-(1.6 \pm 0.4)\mu_N$. A comparison of our results with the ones existing in the literature is given.

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1 Introduction

Among all nonperturbative approaches, the QCD sum rules method [1], is an especially powerful one for calculating the fundamental parameters of hadrons. In this approach a deep connection is established between the hadronic parameters and the QCD vacuum via a few condensates. This method has been applied very successfully to the various problems of hadron physics and is discussed in detail in many review articles [2]-[7]. One of the important static characteristic parameters of hadrons is their magnetic moment. Nucleon magnetic moment was investigated in the framework of the traditional QCD sum rules method in [8, 9]. Within this method, the $\Sigma\Lambda$ transition magnetic moment was calculated in [10]. Later this method was used in determining magnetic moments of baryons containing heavy mesons [11]. In the present work our goal is to calculate $\Sigma_Q \Lambda_Q$ (Q = b or c) transition magnetic moment in the framework of the light cone QCD sum rules method (LCQSR) (more about this approach can be found in [7] and [12] and the references therein), which is an alternative approach to the traditional QCD sum rules method. Note that magnetic moments of the nucleons and decuplet baryons were studied in [13, 14] using the LCQSR approach. Moreover, magnetic moments of heavy Λ_Q baryons and $\Sigma\Lambda$ transition magnetic moment were investigated in [15] and [16], respectively, using the LCQSR method.

The paper is organized as follows: In section 2, the LCQSR for $\Sigma_Q \Lambda_Q$ transition magnetic moment is derived. We present numerical calculations and conclusion in section 3.

2 LCQSR for $\Sigma_Q \Lambda_Q$ transition magnetic moment

In order to calculate $\Sigma_Q \Lambda_Q$ transition magnetic moment we start by considering the following correlator function:

$$\Pi = i \int d^4x \, e^{ipx} \left\langle 0 | T\{\eta_{\Lambda_Q}(x)\bar{\eta}_{\Sigma_Q}(0)\} | 0 \right\rangle_{\mathcal{F}_{\alpha\beta}} \,, \tag{1}$$

where $\mathcal{F}_{\alpha\beta}$ is the external electromagnetic field, η_{Σ_Q} and η_{Λ_Q} are the interpolating currents with Σ_Q and Λ_Q quantum numbers, respectively. It is well known that there is a continuum of choices for the baryon interpolating currents. The general form of Σ_Q and Λ_Q currents can be written as [17, 18]

$$\eta_{\Lambda_Q} = 2 \left(\eta_{\Lambda_1} + b \eta_{\Lambda_2} \right) ,$$

$$\eta_{\Sigma_Q} = 2 \left(\eta_{\Sigma_1} + b' \eta_{\Sigma_2} \right) ,$$
(2)

where b and b' are arbitrary parameters and

$$\eta_{\Sigma_1} = \frac{1}{\sqrt{2}} \epsilon_{abc} \left[(u_a^T C Q_b) \gamma_5 d_c + (d_a^T C Q_b) \gamma_5 u_c \right] , \qquad (3)$$

$$\eta_{\Sigma_2} = \frac{1}{\sqrt{2}} \epsilon_{abc} \left[(u_a^T C \gamma_5 Q_b) d_c + (d_a^T C \gamma_5 Q_b) u_c \right] , \qquad (4)$$

$$\eta_{\Lambda_1} = \frac{1}{\sqrt{6}} \epsilon_{abc} \Big[2(u_a^T C d_b) \gamma_5 Q_c + (u_a^T C Q_b) \gamma_5 d_c - (d_a^T C Q_b) \gamma_5 u_c \Big] , \qquad (5)$$

$$\eta_{\Lambda_2} = \frac{1}{\sqrt{6}} \epsilon_{abc} \Big[2(u_a^T C \gamma_5 d_b) Q_c + (u_a^T C \gamma_5 Q_b) d_c - (d_a^T C \gamma_5 Q_b) u_c \Big] , \qquad (6)$$

with a, b, and c being the color indices. Note that the Ioffe current corresponds to the choice b = b' = -1.

Phenomenological part of the sum rules is obtained by inserting a complete set of states with Σ_Q and Λ_Q quantum numbers between the currents in Eq. (1), which can be expressed as

$$\Pi = \frac{\left\langle 0|\eta_{\Sigma_Q}|\Sigma_Q(p_1)\right\rangle}{p_1^2 - m_{\Sigma_Q}^2} \left\langle \Sigma_Q(p_1)|\Lambda_Q(p_2)\right\rangle_{\mathcal{F}_{\alpha\beta}} \frac{\left\langle \Lambda_Q(p_2)|\bar{\eta}_{\Lambda_Q}|0\right\rangle}{p_2^2 - m_{\Lambda_Q}^2} + \sum_{h_i, H_i} \frac{\left\langle 0|\eta_{\Sigma_Q}|h_i(p_1)\right\rangle}{p_1^2 - m_{h_i}^2} \left\langle h_i(p_1)|H_i(p_2)\right\rangle_{\mathcal{F}_{\alpha\beta}} \frac{\left\langle H_i(p_2)|\bar{\eta}_{\Lambda_Q}|0\right\rangle}{p_2^2 - m_{H_i}^2} ,$$

$$(7)$$

where $p_2 = p_1 + q$, and q is the photon momentum. The second term in Eq. (7) takes higher resonances and continuum contributions into account.

The coupling of currents with the corresponding baryon states is parametrized by the overlap amplitudes λ_i which are defined as

$$\left\langle 0|\eta_{\Sigma_Q}|\Sigma_Q \right\rangle = \lambda_{\Sigma_Q} u_{\Sigma_Q}(p) , \left\langle 0|\eta_{\Lambda_Q}|\Lambda_Q \right\rangle = \lambda_{\Lambda_Q} u_{\Lambda_Q}(p) .$$

$$(8)$$

It follows from Eq. (7) that in calculating the phenomenological part of the correlator, we need the expression of the matrix element $\langle \Sigma_Q(p_1) | \Lambda_Q(p_2) \rangle_{\mathcal{F}_{\alpha\beta}}$, which can be parametrized as

$$\langle \Sigma_Q(p_1) | \Lambda_Q(p_2) \rangle_{\mathcal{F}_{\alpha\beta}} = \bar{u}(p_1) \left[f_1 \gamma_\mu + i \frac{\sigma_{\mu\alpha} q^\alpha}{m_{\Sigma_Q} + m_{\Lambda_Q}} f_2 \right] u(p_2) \varepsilon^\mu ,$$

$$= \bar{u}(p_1) \left[(f_1 + f_2) \gamma_\mu + \frac{(p_1 + p_2)_\mu}{m_{\Sigma_Q} + m_{\Lambda_Q}} f_2 \right] u(p_2) \varepsilon^\mu ,$$

$$(9)$$

where $f_1(q^2)$ and $f_2(q^2)$ are the form factors which are functions of $q^2 = (p_2 - p_1)^2$, and ε^{μ} is the polarization vector of photon. In order to calculate $\Sigma_Q \Lambda_Q$ transition magnetic moment only the values of the form factors at $q^2 = 0$ are needed, since the photon is a real one. Using Eqs. (7)–(9), for the hadronic representation of the correlator function, we get

$$\Pi = -\lambda_{\Sigma_Q} \lambda_{\Lambda_Q} \varepsilon^{\mu} \frac{\not p_1 + m_{\Sigma_Q}}{p_1^2 - m_{\Sigma_Q}^2} \left[(f_1 + f_2) \gamma_{\mu} + \frac{(p_1 + p_2)_{\mu}}{m_{\Sigma_Q} + m_{\Lambda_Q}} f_2 \right] \frac{\not p_2 + m_{\Lambda_Q}}{p_2^2 - m_{\Lambda_Q}^2} .$$
(10)

Among a number of different structures which appear in the phenomenological part, we choose the structure $\not p_1 \notin \not p_2$ that contains transition magnetic form factor $f_1 + f_2$. This form factor gives transition magnetic moment in units of $e\hbar/(m_{\Sigma_Q} + m_{\Lambda_Q})$ at $q^2 = 0$. Choosing the above mentioned structure from the phenomenological part, we get

$$\Pi = -\lambda_{\Sigma_Q} \lambda_{\Lambda_Q} \frac{1}{p_1^2 - m_{\Sigma_Q}^2} \mu_{\Sigma_Q \Lambda_Q} \frac{1}{p_2^2 - m_{\Lambda_Q}^2} , \qquad (11)$$

where $\mu_{\Sigma_Q \Lambda_Q} = (f_1 + f_2)|_{q^2=0}$ is the transition magnetic moment.

In order to construct sum rules, theoretical part of the correlator function needs to be calculated. Calculation of the correlator function in QCD leads to the following result:

$$\begin{aligned} \Pi(p,q) &= -i\frac{2}{\sqrt{3}}\epsilon_{abc}\epsilon_{def} \int d^{4}x e^{ipx} \langle \gamma(q) | \\ & 2\gamma_{5}S_{d}^{be}S_{u}^{'ad}S_{Q}^{cf}\gamma_{5} + 2b\gamma_{5}S_{d}^{be}\gamma_{5}S_{u}^{'ad}S_{Q}^{cf} + 2b'S_{d}^{be}S_{u}^{'ad}\gamma_{5}S_{Q}^{cf}\gamma_{5} \\ &+ 2bb'S_{d}^{be}\gamma_{5}S_{u}^{'ad}\gamma_{5}S_{Q}^{cf} + \gamma_{5}S_{d}^{be}\gamma_{5}\mathrm{Tr}S_{u}^{ad}S_{Q}^{'cf} + b\gamma_{5}S_{d}^{be}\mathrm{Tr}S_{u}^{ad}\gamma_{5}S_{Q}^{'cf} \\ &+ b'S_{d}^{be}\gamma_{5}\mathrm{Tr}S_{u}^{ad}S_{Q}^{'cf}\gamma_{5} + bb'S_{d}^{be}\mathrm{Tr}S_{u}^{ad}\gamma_{5}S_{Q}^{'cf}\gamma_{5} - \gamma_{5}S_{d}^{be}S_{Q}^{'cf}S_{u}^{ad}\gamma_{5} \\ &- b\gamma_{5}S_{d}^{be}\gamma_{5}S_{Q}^{'cf}S_{u}^{ad} - b'S_{d}^{be}S_{Q}^{'cf}\gamma_{5}S_{u}^{ad}\gamma_{5} - bb'S_{d}^{be}\gamma_{5}S_{Q}^{'cf}\gamma_{5}S_{u}^{ad} \\ &- 2\gamma_{5}S_{u}^{ad}S_{d}^{'be}S_{Q}^{cf}\gamma_{5} - 2b\gamma_{5}S_{u}^{ad}\gamma_{5}S_{d}^{'cf}S_{Q}^{be}\gamma_{5} - bb'S_{u}^{ad}S_{d}^{'be}\gamma_{5}S_{Q}^{cf}\gamma_{5} \\ &- 2bb'S_{u}^{ad}\gamma_{5}S_{d}^{'cf}\gamma_{5}S_{u}^{cf} + \gamma_{5}S_{u}^{ad}S_{Q}^{'cf}S_{d}^{be}\gamma_{5} + b\gamma_{5}S_{u}^{ad}\gamma_{5}S_{Q}^{'cf}S_{d}^{be} \\ &+ b'S_{u}^{ad}S_{Q}^{'cf}\gamma_{5}S_{d}^{be}\gamma_{5} + bb'S_{u}^{ad}\gamma_{5}S_{Q}^{'cf}\gamma_{5}S_{d}^{be} - \gamma_{5}S_{u}^{ad}\gamma_{5}\mathrm{Tr}S_{d}^{'be}S_{Q}^{cf} \\ &- b\gamma_{5}S_{u}^{ad}\mathrm{Tr}\gamma_{5}S_{d}^{'be}S_{Q}^{cf} - b'S_{u}^{ad}\gamma_{5}\mathrm{Tr}S_{d}^{'be}\gamma_{5}S_{Q}^{cf} - bb'S_{u}^{ad}\mathrm{Tr}\gamma_{5}S_{d}^{'be}\gamma_{5}S_{Q}^{cf} |0\rangle , \qquad (12)$$

where $S'_q = CS^T_q C$ with C being the charge conjugation operator, T denotes transpose of the operator and S_q is the quark propagator with the subindices referring to the corresponding quarks.

Theoretical part of the correlator contains two pieces: perturbative and nonperturbative. Perturbative part corresponds to the case when photon is radiated from the freely propagating quarks. Its expression can be obtained by making the following replacement in each one of the quark propagators and keeping the other two as they are in Eq. (12)

$$S^{ab}_{\alpha\beta} \to -\frac{1}{2} \left(\int dy \, \mathcal{F}^{\mu\nu} y_{\nu} S^{free}(x-y) \gamma_{\mu} S^{free}(y) \right)^{ab}_{\alpha\beta} \,, \tag{13}$$

where the Fock–Schwinger gauge $x_{\mu}A^{\mu}(x) = 0$ has been used and S^{free} is the free quark operator. In *x*–representation the propagator of the free massive quark is

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2} \frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}} - i\frac{m_Q^2}{4\pi^2 x^2} K_2\left(m_Q\sqrt{-x^2}\right) , \qquad (14)$$

where m_Q is the heavy quark mass and K_i are Bessel functions. The expression for massless propagator $S_q^{free} = i / x/(2\pi^2 x^4)$ can be obtained from Eq. (14) by making use of the following expansions for Bessel functions

$$K_1(x) \sim \frac{1}{x} + \mathcal{O}(x) ,$$

 $K_2(x) \sim \frac{2}{x^2} - \frac{1}{2} + \mathcal{O}(x^2) ,$

and then formally setting $m_Q \to 0$.

The nonperturbative contributions can be obtained from Eq. (12) by making the following replacement in each one of the massless quark propagators:

$$S^{ab}_{\alpha\beta} \to -\frac{1}{4} \bar{q}^a A_j q^b \left(A_j\right)_{\alpha\beta} ,$$

where $A_j = \{1, \gamma_5, \gamma_{\alpha}, i\gamma_5\gamma_{\alpha}, \sigma_{\alpha\beta}/\sqrt{2}\}$ and sum over A_j is implied, and the other two propagators are the full propagators involving both perturbative and nonperturbative contributions. In order to calculate perturbative and nonperturbative contributions, the explicit expressions of the heavy and light quark propagators in the presence of external field are needed.

The complete light cone expansion of the propagator in external field is presented in [19]. It receives contributions from the nonlocal operators $\bar{q}Gq$, $\bar{q}GGq$, $\bar{q}q\bar{q}q$, where G is the gluon field strength tensor. Here we consider operators with only one gluon field and neglect terms with two gluons $\bar{q}GGq$, and four quarks $\bar{q}q\bar{q}q$. Neglecting these terms can be justified on the basis of an expansion in conformal spin [20]. In this approximation massive and massless quark propagators are given by the following expressions,

$$iS_{q}(x) = iS_{q}^{free}(x) - \frac{\langle \bar{q}q \rangle}{12} \left(1 + \frac{x^{2}m_{0}^{2}}{16} \right) - ig_{s} \int dv \left[\frac{\not}{16\pi^{2}x^{2}} G^{\mu\nu}(vx)\sigma_{\mu\nu} - \frac{i}{4\pi^{2}x^{2}} vx_{\mu}G^{\mu\nu}\gamma_{\nu} \right], \qquad (16)$$

where $S_Q^{free}(x)$ and $S_q^{free}(x)$ are the free propagators of the heavy and light quarks, respectively, and m_0 is defined from the relation

$$\langle ar{q} i g_s G_{\mu
u} \sigma^{\mu
u} q
angle = m_0^2 \langle ar{q} q
angle$$
 .

In Eq. (16) we neglect the operators with dimensions larger than five, since their contributions are negligible.

As can be seen from Eqs. (12)–(16), in order to calculate the theoretical part of the correlator function (1), the matrix elements $\langle \gamma | \bar{q} A_i q | 0 \rangle$ of the non–local operators between photon and vacuum states are needed. Up to twist–4 these matrix elements are defined in terms of the photon wave functions in the following way [20]–[22]:

$$\langle \gamma(q) | \bar{q} \gamma_{\alpha} \gamma_{5} q | 0 \rangle = \frac{f}{4} e_{q} \epsilon_{\alpha\beta\rho\sigma} \varepsilon^{\beta} q^{\rho} x^{\sigma} \int_{0}^{1} du \, e^{iuqx} \psi(u) , \langle \gamma(q) | \bar{q} \sigma_{\alpha\beta} q | 0 \rangle = i e_{q} \langle \bar{q}q \rangle \int_{0}^{1} du \, e^{iuqx} \bigg\{ (\varepsilon_{\alpha} q_{\beta} - \varepsilon_{\beta} q_{\alpha}) \big[\chi \phi(u) + x^{2} \big(g_{1}(u) - g_{2}(u) \big) \big] + \big[qx (\varepsilon_{\alpha} x_{\beta} - \varepsilon_{\beta} x_{\alpha}) + \varepsilon x (x_{\alpha} q_{\beta} - x_{\beta} q_{\alpha}) \big] g_{2}(u) \bigg\} .$$

$$(17)$$

In Eq. (17), χ is the magnetic susceptibility of the quark condensate, e_q is the quark charge, $\phi(u)$ and $\psi(u)$ are the leading twist-2 photon wave functions, while $g_1(u)$ and $g_2(u)$ are the twist-4 functions.

Theoretical part of the correlator (1) can be obtained by substituting photon wave functions and expressions for massive and massless quark propagators into Eq. (12). Sum rules for $\Sigma_Q \Lambda_Q$ transition magnetic moment are obtained by equating the phenomenological and theoretical parts of the correlator (1). Performing double Borel transformations on the variables $p_1^2 = p^2$ and $p_2^2 = (p+q)^2$ on both sides of the correlator, which suppress the continuum and higher state contributions (see [14, 23, 24] for details), $\Sigma_Q \Lambda_Q$ transition magnetic moment can be expressed as

$$\begin{split} &\sqrt{3}\lambda_{\Sigma_Q}\lambda_{\Lambda_Q}\mu_{\Sigma_Q\Lambda_Q}\,e^{-\left(m_{\Lambda_Q}^2/M_1^2+m_{\Sigma_Q}^2/M_2^2\right)} = (e_d - e_u) \bigg\{ \\ & \frac{3}{16\pi^4}(2+b+b'+2bb')M^6\Psi(2,-1,m_Q^2/M^2) \\ &+ \frac{m_Q}{8\pi^2}\langle\bar{q}q\rangle(-3b+b'+2bb')\chi\varphi(u_0)M^4\Psi(2,0,m_Q^2/M^2) \\ &- \frac{1}{8\pi^2}(2+b+b'+2bb')f\psi(u_0)M^4\Psi(1,-1,m_Q^2/M^2) \\ &- \frac{m_0^2}{9M^2}\langle\bar{q}q\rangle^2(-1+b)(1+b')[g_1(u_0) - g_2(u_0)] \\ &\times \Big\{ 3F_1(m_Q^2/M^2) - \Big[F_4(m_Q^2/M^2) - F_5(m_Q^2/M^2)\Big] \Big\} \\ &- \frac{m_Q}{48M^2}m_0^2\langle\bar{q}q\rangle f\psi(u_0) \\ &\times \Big\{ (-2-b'-b+4bb')\Big[F_4(m_Q^2/M^2) - F_5(m_Q^2/M^2)\Big] + 3(-1+bb')F_5(m_Q^2/M^2) \Big\} \\ &- \frac{m_Q}{8\pi^2}\langle\bar{q}q\rangle(-2-b'-b+4bb')M^2\Psi(1,0,m_Q^2/M^2) \\ &- \frac{m_Q}{2\pi^2}\langle\bar{q}q\rangle(b'-3b+2bb')(g_1(u_0) - g_2(u_0))M^2\Psi(1,0,m_Q^2/M^2) \\ &- \frac{\langle\bar{q}q\rangle^2}{3}(-1+b)(1+b')\chi\varphi(u_0)M^2F_5(m_Q^2/M^2) \\ &+ \frac{4}{3}\langle\bar{q}q\rangle^2(-1+b)(1+b')\chi\varphi(u_0)\Big[3F_4(m_Q^2/M^2) - F_5(m_Q^2/M^2)\Big] \\ &- \frac{m_Q}{12}(2+b+b'-4bb')\langle\bar{q}q\rangle f\psi(u_0)F_5(m_Q^2/M^2) \\ &+ \frac{m_Q}{32\pi^2}m_0^2\langle\bar{q}q\rangle \Big[(-2-b'-b+4bb')F_5(m_Q^2/M^2) + 3(-1+bb')\Psi(1,1,m_Q^2/M^2) \Big] \Big\} \,, \end{split}$$

where the functions $\Psi(\alpha, \beta, z)$ and $F_i(z)$ are defined as

$$\Psi(\alpha, \beta, z) = \frac{1}{\Gamma(\alpha)} \int_{1}^{\infty} dt \, e^{-tz} \, t^{\beta - \alpha - 1} (t - 1)^{\alpha - 1} \,, \qquad (\alpha > 0) \,,$$

$$F_{1}(z) = z \, (z - 2) \, e^{-z} \,,$$

$$F_{2}(z) = \left(z^{2} - 4z + 2\right) e^{-z} \,,$$

$$F_{3}(z) = \left(z^{2} - 6z + 6\right) e^{-z} \,,$$

$$F_4(z) = ze^{-z}$$
,
 $F_5(z) = e^{-z}$,

and

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2}$$
, $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$,

with M_1^2 and M_2^2 being the Borel parameters. Since masses of the initial and final baryons are very close to each other, we will set $m_{\Sigma_Q} = m_{\Lambda_Q}$ and $M_1^2 = M_2^2 = 2M^2$, from which it follows that $u_0 = 1/2$. It should be noted that Borel transformations of Bessel functions are given in [25].

It follows from Eq. (18) that determination of $\mu_{\Sigma_Q \Lambda_Q}$ transition magnetic moment requires a knowledge of the residues λ_{Σ_Q} and λ_{Λ_Q} . These residues can be determined from heavy baryons mass sum rules (for the Λ_b case the residue is calculated in [15])

$$\begin{split} m_{\Lambda_Q} \lambda_{\Lambda_Q}^2 e^{-m_{\Lambda_Q}^2/M^2} &= \frac{m_Q}{32\pi^4} (-13 + 2b + 11b^2) M^6 \Psi(3, 0, m_Q^2/M^2) \\ &+ \frac{\langle \bar{q}q \rangle}{12\pi^2} (1 + 4b - 5b^2) M^4 \Psi(1, -1, m_Q^2/M^2) \\ &- \frac{m_Q}{36M^2} m_0^2 \langle \bar{q}q \rangle^2 \Big\{ 3(5 + 2b + 5b^2) \Big[F_4(m_Q^2/M^2) - F_5(m_Q^2/M^2) \Big] \\ &+ (-1 + b)^2 F_5(m_Q^2/M^2) \Big\} \\ &+ \frac{\langle \bar{q}q \rangle}{96\pi^2} m_0^2 M^2 \Big[(-1 - 4b + 5b^2) F_5(m_Q^2/M^2) + (-5 + 4b + b^2) \Psi(1, 0, m_Q^2/M^2) \Big] \\ &+ \frac{m_Q}{6} \langle \bar{q}q \rangle^2 (5 + 2b + 5b^2) F_5(m_Q^2/M^2) , \end{split}$$

$$\lambda_{\Lambda_Q}^2 e^{-m_{\Lambda_Q}^2/M^2} = \frac{3}{32\pi^4} (5 + 2b + 5b^2) M^6 \Psi(3, -1, m_Q^2/M^2) - \frac{m_0^2}{72M^2} \langle \bar{q}q \rangle^2 \left[(-26 + 4b + 22b^2) F_4(m_Q^2/M^2) + (-1 + b)^2 F_5(m_Q^2/M^2) \right] + \frac{m_Q}{12\pi^2} \langle \bar{q}q \rangle (1 + 4b - 5b^2) M^2 \Psi(2, 0, m_Q^2/M^2) + \frac{\langle \bar{q}q \rangle^2}{18} (-13 + 2b + 11b^2) F_5(m_Q^2/M^2) + \frac{m_Q}{96\pi^2} m_0^2 \langle \bar{q}q \rangle \left[(-1 - 4b + 5b^2) \Psi(1, 0, m_Q^2/M^2) + 6(-1 + b^2) \Psi(2, 1, m_Q^2/M^2) \right] ,$$
(20)

$$m_{\Sigma_Q} \lambda_{\Sigma_Q}^2 e^{-m_{\Sigma_Q}^2/M^2} = \frac{3m_Q}{32\pi^4} (1-b')^2 M^6 \Psi(3,0,m_Q^2/M^2) + \frac{3}{4\pi^2} \langle \bar{q}q \rangle (1-b'^2) M^4 \Psi(1,-1,m_Q^2/M^2) - \frac{m_Q}{12M^2} m_0^2 \langle \bar{q}q \rangle^2 \Big\{ (5+2b'+5b'^2) \Big[F_4(m_Q^2/M^2) - F_5(m_Q^2/M^2) \Big] + (3+2b'+3b'^2) F_5(m_Q^2/M^2) \Big\}$$
(21)

$$+ \frac{m_0^2}{32\pi^2} \langle \bar{q}q \rangle (-1 + b'^2) M^2 \left[7F_5(m_Q^2/M^2) - \Psi(1, 0, m_Q^2/M^2) \right] + \frac{m_Q}{6} \langle \bar{q}q \rangle^2 (5 + 2b' + 5b'^2) F_5(m_Q^2/M^2) , \lambda_{\Sigma_Q}^2 e^{-m_{\Sigma_Q}^2/M^2} = \frac{3}{32\pi^4} (5 + 2b' + 5b'^2) M^6 \Psi(3, -1, m_Q^2/M^2) - \frac{m_0^2}{24M^2} \langle \bar{q}q \rangle^2 (1 - b')^2 \left[2F_4(m_Q^2/M^2) - F_5(m_Q^2/M^2) \right] - \frac{3m_Q}{4\pi^2} (-1 + b'^2) \langle \bar{q}q \rangle M^2 \Psi(2, 0, m_Q^2/M^2) + \frac{\langle \bar{q}q \rangle^2}{6} (1 - b')^2 F_5(m_Q^2/M^2) + \frac{m_Q}{32\pi^2} m_0^2 \langle \bar{q}q \rangle (-1 + b'^2) \left[7\Psi(1, 0, m_Q^2/M^2) + 6\Psi(2, 1, m_Q^2/M^2) \right] .$$

Eqs. (19) and (21) correspond to the structure proportional to the unit operator, while Eqs. (20) and (22) correspond to the structure $\not p$. Finally, we remark that subtraction of the continuum contribution in Eqs. (18)–(22) can be performed with the help of the following replacement

$$M^{2n}\Psi(\alpha,\beta,m_Q^2/M^2) \to \frac{1}{\Gamma(\alpha)} \frac{1}{\Gamma(n)} \int_{m_Q^2}^{s_0} ds e^{-s/M^2} \int_1^{s/m_Q^2} dt (s-tm_Q^2)^{n-1} t^{\beta-\alpha-1} (t-1)^{\alpha-1} , (23)$$

for $\alpha > 0$ and n > 0.

3 Numerical analysis

In this section we present our numerical calculation for $\mu_{\Sigma_Q \Lambda_Q}$ transition magnetic moment. It follows from Eq. (18) that the main input parameters of the LCQSR are the photon wave functions. The photon wave functions which we use in the present work are given as [20, 23]:

$$\phi(u) = 6u(1-u) , \qquad \psi(u) = 1 ,$$

$$g_1(u) = -\frac{1}{8}(1-u)(3-u) , \qquad g_2(u) = -\frac{1}{4}(1-u)^2 .$$

The values of the other input parameters entering to the sum rules are: $\chi(1GeV) = -4.4 \ GeV^{-2}$ [26] (in [27] this quantity is estimated to be $\chi = -3.3 \ GeV^{-2}$), $\langle \bar{q}q \rangle (1 \ GeV) = -(0.243)^3 \ GeV^3$, $m_0^2 = (0.8 \pm 0.2) \ GeV^2$ [22], $m_c = 1.3 \ GeV$, $m_b = 4.8 \ GeV$, and $f = 0.028 \ GeV^2$. Few word about the value of f are in order. The analysis of the magnetic susceptibility χ by the QCD sum rules (see [26] and [27]) shows that at an accuracy of the order of 30%, the dominant contribution to this quantity comes from ρ meson at $\mu = 1 \ GeV^2$. It has been suggested that on these argument that the vector dominance model works sufficiently well for electromagnetic properties of hadrons in the constant external field limit $q \to 0$. To the quoted accuracy, in this limit for the normalization constant is obtained [28]

$$f \simeq \frac{e_u}{g_\rho} f_\rho m_\rho \simeq 0.028 \pm 0.009$$

at $g_{\rho} = 5.5$ and $f_{\rho} = 0.2$.

The parameters b and b' are completely arbitrary and the physical quantities should not depend on the precise value of these parameters. To simplify further analysis, we will set b = b'. Since transition magnetic moment is a physical quantity it must be independent of the auxiliary parameters b, Borel mass square M^2 and continuum threshold s_0 . So, our problem reduces to determining the respective regions for which $\mu_{\Sigma_Q \Lambda_Q}$ transition magnetic moment is independent of the above-mentioned parameters.

For this aim we prefer to consider the following three steps. In the first step, we try to find the working region of M^2 where $\mu_{\Sigma_Q\Lambda_Q}$ is independent of the Borel parameter M^2 , at fixed values of b and s_0 . Along these lines, we present in Figs. (1) and (2) the dependence of $\mu_{\Sigma_b\Lambda_b}$ and $\mu_{\Sigma_c\Lambda_c}$ on M^2 , respectively. From both figures we observe that, except b = -1 case, transition magnetic moments seem to be almost independent for the different choices of b and s_0 . The working region for $\mu_{\Sigma_b\Lambda_b}$ transition magnetic moment is $15 \ GeV^2 < M^2 < 30 \ GeV^2$, while it is $2 \ GeV^2 \le M^2 \le 6 \ GeV^2$ for the $\mu_{\Sigma_c\Lambda_c}$ case.

The next step is to determine the working region for the parameter b. For this purpose we use the fact that both mass sum rules for Λ_Q and Σ_Q should be positive. Note also that the mass of the Λ_Q baryon can be obtained by dividing Eq. (19) with Eq. (20), and that of Σ_Q can be obtained by dividing Eq. (21) with Eq. (22). In order to see whether this requirement is fulfilled or not, in Figs. (3) and (4) we present the dependence of the sum rules for the masses of the above-mentioned baryons on $\cos \theta$, where θ is determined from the relation $\tan \theta = b$. It follows from these figures that the working region of b, which guarantees the positiveness of the sum rules, are in the intervals $-0.7 \leq \cos \theta \leq +0.7$ for Λ_b baryon and $-0.75 \leq \cos \theta \leq +0.7$ for Λ_c baryon. Similar analysis for the Σ_Q baryons leads to the following result for the working region of the parameter b: $-0.7 \leq \cos \theta \leq +0.7$ and $-0.8 \leq \cos \theta \leq +0.7$ for the Σ_b and Σ_c baryons, respectively. So, the common working region for the parameter b for both cases is $-0.7 \leq \cos \theta \leq +0.7$.

Having decided about the restriction of the parameter b, our third and final step is to determine the main objective of the present work, i.e., $\mu_{\Sigma_Q\Lambda_Q}$ transition magnetic moment. As before, we are supposed to find a region for the parameter b where transition magnetic moment $\mu_{\Sigma_Q\Lambda_Q}$ is independent of its variation. In the first step of our analysis we have decided on the working region of the Borel parameter M^2 for which $\mu_{\Sigma_Q\Lambda_Q}$ does not depend on its variation, and additionally, we have verified its insensitiveness to several different choices of the continuum threshold s_0 . Fig. (5) presents the dependence of transition magnetic moment $\mu_{\Sigma_b\Lambda_b}$ (in units of the nucleon magneton μ_N) on $\cos\theta$, at $M^2 = 25 \ GeV^2$ and at three fixed values of s_0 . Similarly, in Fig. (6) we present the dependence of $\mu_{\Sigma_c\Lambda_c}$ on $\cos\theta$, at $M^2 = 4 \ GeV^2$ and at three fixed values of s_0 . From both figures we deduce that $\mu_{\Sigma_Q\Lambda_Q}$ is quite stable in the region $-0.5 \leq \cos\theta \leq +0.08$, and can be said to be practically independent of the parameter b and the continuum threshold s_0 . It follows from all these arguments that

$$\mu_{\Sigma_c \Lambda_c} = -(1.5 \pm 0.4)\mu_N , \mu_{\Sigma_b \Lambda_b} = -(1.6 \pm 0.4)\mu_N ,$$

where the uncertainty in the results can be mainly attributed to the variation of the continuum threshold s_0 , Borel parameter M^2 and the twist-3 photon wave functions which are neglected, since we estimate that their contribution to the transition magnetic moment is less than 5%, as well as to the uncertainties in the values of the parameters f, χ , and m_0^2 .

Finally, we compare our results on $\mu_{\Sigma_Q \Lambda_Q}$ transition magnetic moment with the ones predicted by other methods in literature. Transition magnetic moment $\mu_{\Sigma_c \Lambda_c}$ is calculated in the LCQSR to leading order in heavy quark effective theory which predicts $\mu_{\Sigma_c \Lambda_c} =$ $(1.0 \pm 0.2)\mu_N$ [29]. This result is calculated using Ioffe currents for Σ_c and Λ_c . When we compare our results with the results given in [29] we see that the discrepancy between them is about 50%. In our opinion this discrepancy can be attributed to the fact that the result presented in [29] is calculated for the choice b = -1 (Ioffe current), which is unphysical in our case.

In summary, transition magnetic moments $\mu_{\Sigma_Q \Lambda_Q}$ are calculated in the framework of the LCQSR, using the general form of the interpolating currents for Σ_Q and Λ_Q mesons. In our analysis, only two-particle photon wave functions are taken into account while threeparticle photon wave functions are neglected, since we estimate that their contribution to the transition magnetic moments, as has already been mentioned above, is less than 5%.

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Figure captions

Fig. (1) The dependence of the transition magnetic moment $\mu_{\Sigma_b\Lambda_b}$ on M^2 at two different values of the continuum threshold $s_0 = 35 \ GeV^2$ and $s_0 = 45 \ GeV^2$, for several fixed values of the parameter *b*. Here in this figure and Figs. (2), (5) and (6) the transition magnetic moments are given in units of the nucleon magneton μ_N .

Fig. (2) The dependence of the transition magnetic moment $\mu_{\Sigma_c\Lambda_c}$ on M^2 at two different values of the continuum threshold $s_0 = 8 \ GeV^2$ and $s_0 = 12 \ GeV^2$, for several fixed values of the parameter b.

Fig. (3) The dependence of the mass sum rule m_{Λ_b} on $\cos \theta$, at $M^2 = 25 \ GeV^2$ and at three different values of the continuum threshold $s_0 = 35 \ GeV^2$, $40 \ GeV^2$ and $45 \ GeV^2$.

Fig. (4) The dependence of the mass sum rule m_{Λ_c} on $\cos \theta$, at $M^2 = 4 \ GeV^2$ and at three different values of the continuum threshold $s_0 = 8 \ GeV^2$, 10 GeV^2 and 12 GeV^2 .

Fig. (5) The dependence of the transition magnetic moment $\mu_{\Sigma_b\Lambda_b}$ on $\cos\theta$, at the fixed value $M^2 = 25 \ GeV^2$ of the Borel parameter, and at three different values of the continuum threshold $s_0 = 35 \ GeV^2$, 40 GeV^2 and 45 GeV^2 .

Fig. (6) The dependence of the transition magnetic moment $\mu_{\Sigma_c \Lambda_c}$ on $\cos \theta$, at the fixed value $M^2 = 4 \ GeV^2$ of the Borel parameter, and at three different values of the continuum threshold $s_0 = 8 \ GeV^2$, 10 GeV^2 and 12 GeV^2 .



Figure 1:



Figure 2:



Figure 3:



Figure 4:



Figure 5:



Figure 6: