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**Sıralama ve Klasifikasyon Problemlerinde Çok Amaç
Gözeterek Değerlendirme**

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ÖZET

Bu projede, deęişik kriterler kullanılarak deęerlendirilen seeneklerin en ok tercih edilenden en az tercih edilene doęru sıralanmıř kategorilere atanma problemi zerinde alıřtık. Bu problem ok kriterli sıralama problemi olarak bilinmektedir ve lkelerin ekonomik ve sosyo-politik kriterlere gre deęişik risk seviyelerine ayrılması, banka mřterilerinin kredi bařvurularının deęerlendirilmesi gibi durumlarda karřımıza ıkmaktadır.

stn sıralama (outranking) iliřkileri zerine yapılandırılmıř sıralama yntemlerinden Electre Tri metodu ardıřık kategorileri ayıran sınırları tanımlayan ok boyutlu profillerin belirlenmesini gerektirmekte ve seenekleri bu profillerin her biriyle karřılařtırıp kategorilere atamaktadır.

Ardıřık kategoriler iin profillerin aıka belirlenmesi karar vericiler iin zor olabilmektedir. Profillerin aıka belirlenmesini gerektirmeyen yeni bir stn sıralama tabanlı sıralama metodu geliřtirdik. Bunun yerine karar vericiden referans seeneęi adı verilen bir grup seeneęi kategorilere atamasını istiyoruz. Sonraki ařamada, bir tamsayılı program yardımıyla, geride kalan seenekleri bu referans seeneklerle karřılařtırarak kategorilere atıyoruz.

Anahtar Kelimeler: ok Kriterli Karar Verme, Sıralama, Tam Sayılı Programlama

ABSTRACT

We consider the problem of assigning alternatives evaluated on several criteria into ordered categories C_1, C_2, \dots, C_p . This problem is known as the multi-criteria sorting problem and arises in many situations such as classifying countries into different risk levels based on economical and socio-political criteria, evaluating credit applications of bank customers.

We are interested in sorting methods that are grounded on the construction of outranking relations. Among these, the Electre Tri method requires defining multidimensional profiles that represent the “frontier” separating consecutive categories C_h and C_{h+1} , and assigns an alternative to categories according to how it compares to each of the profiles.

The explicit specification of the profiles for consecutive categories can be difficult for decision makers. We develop a new outranking based sorting method that does not require the explicit definition of profiles. We instead require the decision maker to assign a subset of reference alternatives to the categories. To assign the remaining alternatives, each such alternative is compared to reference alternatives, and assigned to categories accordingly. The assignment of the remaining alternatives is done simultaneously via an integer program.

Keywords: Multiple Criteria Decision Making, Sorting, Integer Programming

1. GİRİŞ

Çok amaçlı karar problemlerinde, seçeneklerin değişik ve sıralı gruplar altına yerleştirilmesi probleminin çok çeşitli uygulamaları vardır. Kredi riski açısından ülkeler, şirketler, kişiler çeşitli kriterler kullanılarak gruplanmaktadır. Üniversiteler ve akademik programlar çeşitli yayın organlarıncı sıralanmakta veya gruplanmaktadır. Oteller ve restoranlara, kalitesini ölçmek için yıldızlar verilmektedir. Bu gruptama işlemini yaparken çeşitli teknikler kullanılmakta ve yeni teknikler geliştirilmektedir. Bu projede, mevcut tekniklerin bazı sakıncalarını giderecek yeni bir yaklaşım geliştirilmiş ve çeşitli verilerle denenmiştir.

2. GENEL BİLGİLER

Seçeneklerin gruplanması için geliştirilen bazı teknikler durumu fazla basite indirgemekte, bazıları ise karar vericiden (KV) kapsamlı ve zor tercih bilgileri istemektedir. Bu projede her iki yaklaşımın dezavantajlarını ortadan kaldırmak amacıyla bir yaklaşım geliştirilmiştir.

3. GEREÇ VE YÖNTEM

Bu projede, oldukça genel varsayımlar kullanılarak KV'nin tercihleri temsil edilmeye çalışılmıştır. KV az sayıda seçeneği gruplara yerleştirdikten sonra, geri kalan seçeneklerin yerleştirilmesinin modeller aracılığı ile yapılmasına çalışılmaktadır. KV'nin tercihlerinin "outranking" denilen, "çoğunluk" ve "veto" temelli genel bir tercih yapısına uyduğu varsayılmakta, bu yapının parametreleri matematiksel modeller aracılığıyla kestirilmeye çalışılmaktadır. Bu parametreler kestirildikten sonra, geri kalan seçeneklerin yerleştirilmesi yapılmaktadır. Matematiksel modellerde ortaya çıkan hesaplama zorluklarının giderilmesi için çeşitli yaklaşımlar geliştirilmiş ve kullanılmıştır.

3.1. Atama kurallarına uygun referans seçeneklerinin tanımlanması

Başlangıç atamalarını elde etmek için, karar vericiden bazı seçenekleri kategorilere yerleştirmesini isteriz. Bu seçeneklerin oluşturduğu küme A^* olsun. Karar verici gerekli yerleştirmeleri yaptıktan sonra yapısal atamalardan çevrim içermeyen bir alt küme elde etmeye çalışırız. Çevrim olmaması için, $C(a_q) < C(a_r)$ ise, a_q seçeneği a_r seçeneğinden üstün olmamalıdır. (P1) modelini kullanarak, varolan çevrimlere sebep olan seçenekleri referans seçenek kümesinden eleriz. Sonuçta kalan referans seçenek sayısını en çoklamayı çalışıyoruz.

Karar değişkenleri:

α : bir gruptaki minimum alternatif sayısı

λ : uyum eşiği

w_j : j kriterinin ağırlığı

$y_i = \begin{cases} 1 & \text{referans alternatifi } a_i \text{ silinmediyse} \\ 0 & \text{diğer hallerde} \end{cases}$

$$(P1) \text{ Enfazla } \alpha + \varepsilon \sum_{i: a_i \in A^*} y_i \quad (1)$$

$$\alpha \leq \sum_{i: C(a_i) = C_h} y_i, \forall h \in H \quad (2)$$

$$S(a_q, a_r) = \sum_{j=1}^n w_j c_j(a_q, a_r), \forall a_q, a_r \in A^* \quad (3)$$

$$S(a_q, a_r) \leq \lambda - \varepsilon + M(1 - y_q) + M(1 - y_r), \forall a_q, a_r \in A^* \ni C(a_q) < C(a_r) \quad (4)$$

$$y_i \in \{0, 1\}, \forall i \quad (5)$$

$$w \in \Omega \quad (6)$$

$$\lambda \in [0.5, 1] \quad (7)$$

ε küçük bir pozitif tamsayı ve M büyük bir pozitif tamsayıdır. Ω kümesini tanımlayan, w üzerindeki kısıtlar

$$\sum_{j=1}^n w_j = 1 \quad (8)$$

$$\varepsilon \leq w_j \leq 0.5, \forall j \quad (9)$$

Karar verici tarafından tanımlanan başka tercih bilgileri olduğu durumda modele yeni kısıt olarak eklenebilir.

(2) numaralı kısıt α 'nın her gruptaki silinmemiş seçenek sayısından küçük veya eşit olmasını sağlar. Bu kısıt ve amaç fonksiyonundaki α , karar verici tarafından gruplara atanmış seçenek sayısına göre minimum grup büyüklüğünü maksimize eder. (3) numaralı kısıt her kriterdeki bileşenlerinin ağırlıklı toplamları yoluyla toplam uyum ölçütü hesaplar. (4) numaralı kısıt a_q ve a_r seçenekleri arasında çevrim varsa bunu tesbit eder. Eğer çevrim varsa, a_q ve a_r seçeneklerinden birisi, amaç fonksiyonundaki etkilerine göre seçilip elenecek.

3.2. Seçeneklerin kategorilere atanması

Çevrimleri engellemek için gerekli referans seçeneklerinin elimine edilmesinin (A_1^* kümesinin oluşturulması) ardından geride kalan seçeneklerin ataması yapılır. Bunun için profillerden değil, karar verici tarafından atanmış referans seçeneklerden faydalanılır (A_1^* kümesindeki seçenekler). Karar verici tarafından atanmış seçenekler a_k , bizim atamaya çalıştığımız seçenekler a_i ile gösterilmiştir.

Karar değişkenleri:

$$y_{ih} = \begin{cases} 1 & a_i \text{ alternatifi } C_h \text{ kategorisine atandıysa} \\ 0 & \text{diğer hallerde} \end{cases}$$

$u_{ihk} = a_i$ seçeneği C_h kategorisine atandığında, a_i ve a_k seçenekleri arasında üstün sıralama ilişkisinin bozulma miktarı

Parametreler: ε küçük bir pozitif tamsayı ve M büyük bir pozitif tamsayıdır.

$$(P2) \text{ Enazla } \sum_i \sum_h \sum_k u_{ihk} - \varepsilon \lambda \quad (10)$$

$$S(a_i, a_k) \leq \lambda - \varepsilon + M(1 - y_{ih}), \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (11)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon + M(1 - y_{ih}), \forall a_k \in C_{<h}, h = 2, \dots, ncat, \forall i \quad (12)$$

$$S(a_i, a_k) \geq \lambda - M(1 - y_{ih}) - u_{ihk}, \forall a_k \in C_{<h}, h = 2, \dots, ncat, \forall i \quad (13)$$

$$S(a_k, a_i) \geq \lambda - M(1 - y_{ih}) - u_{ihk}, \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (14)$$

$$\sum_{h=1}^{ncat} y_{ih} = 1, \forall i \quad (15)$$

$$S(a_q, a_r) \leq \lambda - \varepsilon, \forall a_q, a_r \in A_i \ni C(a_q) < C(a_r) \quad (16)$$

$$S(a_i, a_k) = \sum_{j=1}^n w_j c_j(a_i, a_k), \forall i, k \quad (17)$$

$$\lambda \in [0.5, 0.8] \quad (18)$$

$$y_{ih} \in \{0, 1\}, u_{ihk} \geq 0 \quad (19)$$

$$w \in \Omega \quad (20)$$

(P2) modeli, atadığı seçenek a_i 'nin karar verici tarafından daha üst sınıflardan birine atanmış bir seçenekten daha üstün olmamasını ve karar verici tarafından daha alt sınıflardan birine atanmış bir seçeneğin a_i 'den daha üstün olmamasını garantilemektedir ((11) ve (12) numaralı kısıtlar). Ayrıca, model üst sınıflara atanan seçeneklerin alt sınıflara atanan seçeneklerden üstün olmasını sağlamaya çalışır; bu sağlanamazsa, uyumsuzluğun en az olmasını sağlar ((13) ve (14) numaralı kısıtlar). Amaç fonksiyonundaki ikinci terim de λ değerini maksimize etmek içindir. (15) numaralı kısıt her seçeneğin bir gruba atanmasını sağlar. (16) numaralı kısıt alt sınıflara atanan referans seçeneklerinin üst sınıflardaki referans seçeneklerinden üstün olmamasını garanti eder. (18) numaralı kısıt λ değeri için sınır belirler.

(P2) modelinin yapılamaz olması durumunda, atamaların (11) ve (12) numaralı kısıtlarda en az uyumsuzluk ile yapılması tercih edilebilir. Bu durumda aşağıdaki model kullanılabilir.

$$z_{ik} = \begin{cases} 1 & a_i \text{ ve } a_k \text{ arasındaki üstün sıralama ilişkisine uyulmadığı hallerde} \\ 0 & \text{diğer hallerde} \end{cases}$$

$$\text{Enazla } \sum_i \sum_k z_{ik} + \varepsilon \sum_i \sum_h \sum_k u_{ihk} \quad (21)$$

$$S(a_i, a_k) \leq \lambda - \varepsilon + M(1 - y_{ih}) + Mz_{ik}, \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (22)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon + M(1 - y_{ih}) + Mz_{ik}, \forall a_k \in C_{<h}, h = 2, \dots, ncat, \forall i \quad (23)$$

Kısıtlar (13)-(20)

z_{ik} değişkeninin 1 değerini alması halinde, model, (22) veya (23) numaralı kısıtların z_{ik} ile ilgili olanlara uyulmamasına izin verir. Amaç fonksiyonu öncelikli olarak bu gibi uyulmama durumlarının adedini en azlar. İkincil amaç ise u_{ihk} değişkenlerinin toplam değerlerinin en azlanmasıdır.

Diğer taraftan, uyulmama durumlarının adedi yerine uyulmama miktarlarının toplamını en azlamak için aşağıda verilen model kullanılabilir. Bu modelde u'_{ik} değişkeni (a_i, a_k) seçenek ikilisi arasındaki üstün sıralama ilişkisine uyulmama miktarına karşılık gelmektedir.

$$\text{Enazla } \sum_i \sum_k u'_{ik} + \varepsilon \sum_i \sum_h \sum_k u_{ihk} \quad (24)$$

$$S(a_i, a_k) \leq \lambda - \varepsilon + M(1 - y_{ih}) + u'_{ik}, \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (25)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon + M(1 - y_{ih}) + u'_{ik}, \forall a_k \in C_{<h}, h = 2, \dots, ncat, \forall i \quad (26)$$

Kısıtlar (13)-(20)

Kısıt (24) uyulmama durumlarının toplam değerini en azlar. Toplam uyulmama durumları yerine en büyük uyulmama durumunun değerini en azlamak istenirse, model aşağıdaki şekilde yazılabilir.

$$\text{Enazla } u + \varepsilon \sum_i \sum_h \sum_k u_{ihk} \quad (27)$$

$$u \geq u'_{ik}, \forall i, k \quad (28)$$

Kısıtlar (25), (26), (13)-(20)

3.3. Seçeneklerin atanabileceği kategori aralığı bulmak

(P1) modelinden elde ettiğimiz referans seçenekleri kümesi A_1^* kullanılarak elde edebileceğimiz bir diğer bilgi de her seçeneğin atanabileceği kategori aralığıdır. Seçenek i 'nin kategori h 'ye atanıp atanamayacağını bulmak için (P2) modelinin değişik bir varyasyonu kullanılabilir.

(P2) modeline $y_{th} = 1$ kısıtını ekleyerek t seçeneğinin h ' kategorisine atanmasının mümkün olup olmadığını belirleyebiliriz. Bu model $(P2_{th})$ olsun. Eğer bu problemin çözümü varsa, diğer bütün seçenekleri kategorilere yerleştirerek t seçeneğini h ' kategorisine atayabiliriz demektir. $(P2_{th})$ modelini bütün $h=1, \dots, ncat$ için çözerek t seçeneğinin atanabileceği bütün kategorileri bulabiliriz. Bu prosedürü bütün seçenekler için tekrar edebiliriz.

$(P2_{th})$ modelinin gevşetilmesiyle $(P3_{ih})$ doğrusal modeli elde edilebilir.

$$(P3_{ih}) \text{ Enazla } 0 \quad (29)$$

s.t.

$$S(a_i, a_k) \leq \lambda - \varepsilon, \forall a_k \in C_{>h} \quad (30)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon, \forall a_k \in C_{<h} \quad (31)$$

$$S(a_q, a_r) \leq \lambda - \varepsilon, \forall a_q, a_r \in A_1^* \ni C(a_q) < C(a_r) \quad (32)$$

$$S(a_i, a_k) = \sum_{j=1}^n w_j c_j(a_i, a_k), \forall k \quad (33)$$

$$\lambda \in [0.5, 1] \quad (34)$$

$$w \in \Omega \quad (35)$$

$(P3_{ih})$ modelinin kısıtları önceki modellerle benzerlik gösterir. Amacımız yalnızca yapılabilir bir çözüm bulmak olduğu için amaç fonksiyonumuz bir sabit sayıdan oluşuyor. Her a_i için, $(P3_{ih})$ modelini bütün kategoriler için çözüyoruz. Yapılabilir bir çözüm, a_i seçeneğini h kategorisine atanmanın mümkün olduğunu gösterir. $(P3_{ih})$ modelinin

yapılabilir bir çözümlü yoksa, a_i seçeneğini h kategorisine atamak karar vericinin yerleştirdiği referans seçenekleri ile uyumlu olmayacaktır.

(P3_{ih}) modeli ayrıca (P2) ve (P2_{th}) modellerindeki tamsayı değişkenlerinin sayısını azaltmak için de kullanılabilir. Bu modellerde, yalnızca (P3_{ih}) modelinin yapılabilir sonuç verdiği y_{ih} değişkenlerini tanımlarız.

3.4. Kategori büyüklüklerinin kısıtlanması

Bazı durumlarda kategorilere atanması düşünülen seçenek sayısında kısıtlamalar olabilir. h kategorisine atanabilecek seçenek sayısını cap_h ile kısıtlamak istersek, bunu (P2) modeline

$$\sum_{i=1} y_{ih} = cap_h, \forall h$$

kısıtlarını ekleyerek gerçekleştirebiliriz.

4. YAKLAŞIMIN DENENMESİ

Geliştirilen yaklaşım çeşitli problemler üstünde denenmiştir.

4.1. Lisansüstü İşletme (MBA) programlarının sıralanması

Financial Times (FT) tarafından yapılan sıralama verileri kullanılarak denemeler yapılmıştır. FT her yıl 20 kriter kullanarak ilk 100 MBA programını sıralamaktadır. Biz yaklaşımımızın özelliklerini göstermek için 20 kriteri üç ana başlık altında grupladık. Yaklaşımımızı, FT listesindeki 30 programı 5 kategoriye atamak için çalıştırdık. Önce seçtiğimiz 8 programı referans olarak kullanarak (P1) modelini çözdük. Referans programlarının hiçbir çifti birbiriyle çevrim yaratmadığı için referans kümesinden çıkartmamız gerekmedi. Diğer programları kategorilere atamak için önce, çözmesi daha kolay olan, (P3_{ih}) modellerini çözerek programları atamanın mümkün olduğu kategorileri bulduk. Daha sonra, bu çözümlerin sağladığı kolaylıktan yararlanarak, göreceli olarak

daha zor olan, karışık tamsayılı model (P2_{ih})'leri çözerek de aynı işi yaptık. Bu model az sayıda ek bilgi kazandırdı. En son olarak (P2) modelini çözerek programları kategorilere yerleştirdik. Bu modellerle elde edilen çözümleri Tablo 1'de veriyoruz.

4.2. Büyük Örnek

Bankacılık sektöründen alınmış 100 seçenek ve 7 kriterli bir problem çözdük. Referans seçenek sayısını, her kategoriye 2, 3, ve 4 seçenek ekleyerek denemeler yaptık. Bu problemlerde de (P1) bir çevrime yol açmadı. (P2) sonuçları referans seçenek sayısına pek duyarlı çıkmadı. Değişik referans sayıları ile çözüldüğünde, büyük ölçüde benzer atamalar yapıldı.

Tablo 1: P3_{ih}, P2_{ih} and P2 Modellerinin Sonuçları

Program Name	P3 _{ih}	P2 _{ih}	P2
Columbia Business School	4,5	4,5	5
Harvard Business School	4,5	4,5	5
New York University: Stern	3,4,5	3,4,5	5
Northwestern University: Kellogg	3,4,5	3,4,5	5
Stanford University GSB	3,4,5	3,4,5	5
University of Chicago GSB	3,4,5	3,4,5	5
University of Pennsylvania: Wharton	4,5	4,5	5
Duke University: Fuqua	1,2,3,4,5	1,2,3,4,5	4
Insead	2,3,4,5	2,3,4,5	4
UC Berkeley: Haas	2,3,4,5	2,3,4,5	4
University of Michigan: Ross	1,2,3,4,5	1,2,3,4,5	4
Iese Business School	2,3,4,5	2,3,4,5	3
Cornell University: Johnson	1,2,3,4,5	1,2,3	2
University of Iowa: Tippie	1,2,3	1,2	2
University of Rochester: Simon	1,2,3	1,2,3	2
Vanderbilt University: Owen	1,2,3,4	1,2,3,4	2
York University: Schulich	2,3,4,5	2,3,4,5	2
Carnegie Mellon: Tepper	1,2	1,2	1
Esade Business School	1,2,3,4,5	1,2,3,4,5	1

Rotterdam School of Management	1,2,3,4,5	1,2,3,4,5	1
University of Virginia: Darden	1,2,3,4	1,2,3,4	1
University of Western Ontario: Ivey	1,2,3,4,5	1,2,3,4,5	1

5. PROJEDE GERÇEKLEŞTİRİLEN AŞAMALAR

Proje önerisinde, proje süresinde gerçekleştirilmesi planlanan aşamalar ve proje bitiminde alınması beklenen sonuçlardan bahsedilmiştir. Bu bölümde gerçekleştirilen aşamalar ve alınan sonuçlardan bahsedilmekte ve önerilen-gerçekleşen karşılaştırması yapılmaktadır.

Proje önerisinde önerilen aşamaların nasıl gerçekleştirildiği aşağıda maddeler halinde belirtilmiştir.

- *Profil bilgisi gerektirmeyen çok kriterli sıralama metodu geliştirilmesi:* Projenin ilk aşaması olarak belirtilen metot, proje kapsamında geliştirilmiştir. Metotla ilgili ayrıntılar raporun 3. ve 4. bölümlerinde detaylı olarak açıklanmıştır.
- *Kategori büyüklüklerine göre atama yapma:* Bu problem, üzerinde çalışılan problem ile benzerlikler göstermektedir. Üzerinde çalışılan problem, bu problemin bir özel durumu olarak düşünülebilir. Kategori büyüklüklerinin önceden verilmiş olması durumu yapı itibariyle probleme yeni bir boyut katmaktadır. Bu durum, raporun 3.3 numaralı bölümünde ele alınmıştır.

- *Yazılım geliştirilmesi:* 3. ve 4. bölümlerde belirtilen modelin uygulaması olan yazılım ODTÜ grubu tarafından gerçekleştirilmiştir. Yazılım, modelin girdileri verildikten sonra tüm aşamaları uygulayıp kullanıcıya alternatiflerin atandıkları sınıfları vermektedir. Ayrıca girdi ve çıktı dosyalarının standart olarak tanımlanması ile yazılım genel amaçlı olarak kullanılabilir duruma getirilmiştir.
- *Gerçek hayat uygulaması:* Geliştirilen metot işletme yüksek lisans (MBA) programlarının sıralanması problemine uygulanmış ve Bölüm 4.1'de ayrıntılı olarak tartışılmıştır. Ayrıca bankacılık sektöründen alınmış verilerle daha büyük bir problemde de uygulaması yapılmış (Bölüm 4.2) ve sonuçları alınmıştır.
- *Sıralanmamış kategorilere atama problemi:* Bu problem, ele alınan problemde oldukça önemli farklılıklar göstermektedir. Karşılıklı seyahatlerdeki (her iki tarafın yoğun programları nedeniyle) aksamalardan dolayı öngörülenin çok altında biraraya gelme fırsatı olmuştur. Bu nedenle, söz konusu problemin daha ileri aşamalarda ele alınması planlanmıştır.

Proje önerisinde projeden beklenen sonuçlardan bahsedilmiştir. Beklenen sonuçlar ve bunların gerçekleşmeleri aşağıdaki gibi olmuştur.

- *Uluslararası bilimsel toplantılarda sunular:* Çalışmamız aşağıda belirtilen bilimsel toplantılarda sunulmuştur.

Selin Bilgin, Murat Köksalan, Vincent Mousseau, Özgür Özpeynirci, "A New Outranking-Based Approach for Assigning Alternatives to Ordered Classes", The 18th International Conference on Multiple Criteria Decision Making, Chania, Yunanistan, Haziran 2006.

Selin Bilgin, Murat Köksalan, Vincent Mousseau, Özgür Özpeynirci, "A New Outranking-Based Approach for Assigning Alternatives to Ordered Classes",

Meeting of the FNRS Contact Group “Méthodes quantitatives de Gestion”, Mons, Belçika, Ekim 2006.

- *Modelin gerçek hayat problemlerine uygulanması:* 4. bölümde belirtildiği gibi modelimiz MBA programlarının sıralanması problemi ve bankacılık sektöründen bir problem üzerinde uygulanmıştır.
- *Yazılım geliştirilmesi:* Modelimizin uygulamasını gerçekleştiren bir bilgisayar programı geliştirilmiştir.
- *Uluslararası bilimsel dergilerde yayın yapılması:* Çalışmamız *Naval Research Logistics* isimli dergiye değerlendirme için gönderilmiş ve hakemlerin isteği doğrultusunda iki defa revize edilmiştir.
- *Diğer araştırmacıların konu üzerinde çalışmaya teşvik edilmesi:* Bu amacımızın bilimsel toplantılarda yaptığımız sunumlar ve çıkacak yayınumuzla uzun vadede gerçekleşmesini bekliyoruz.

Projede önerisinde belirtilen iki yıllık çalışma planı aşağıdaki gibi zamanında gerçekleştirilmiştir.

1. (0 - 6 ay) Kavramsal çalışmalar.
2. (6 - 12 ay) Algoritmik çalışmalar.
3. (12 - 18 ay) Yazılım çalışmaları.
4. (18 – 24 ay) lik uygulamalar.

6. SONUÇ

Proje kapsamında geliştirilen yaklaşım örnekler üzerinde denenerek uygunluğu gösterilmiştir. Bu yaklaşım makul büyüklükteki problemlerde fazla hesaplama zorluğuyla

karşılaşmadan uygulanabilir. Yaklaşımın daha da geliştirilmesi için çalışmalar yararlı olacaktır.

Bu aşamada, projede yapılan çalışmaları içeren bir makale hazırlanmış ve "Naval Research Logistics" adlı dergiye değerlendirme için gönderilmiştir. Bu makalenin son hali ekte verilmiştir.

TÜBİTAK

PROJE ÖZET BİLGİ FORMU

Proje No: MİSAG-CNRS-5
Proje Başlığı: Sıralama ve Klasifikasyon Problemlerinde Çok Amaç Gözeterek Değerlendirme
Proje Yürütücüsü ve Araştırmacılar: Proje Yürütücüsü: Prof. Dr. Murat Köksalan Araştırmacılar: Selin Bilgin Özpeynirci, Nail Özgür Özpeynirci
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Destekleyen Kuruluş(ların) Adı ve Adresi: Türkiye Bilimsel ve Teknolojik Araştırma Kurumu (TÜBİTAK) Tunus Caddesi No:80 06100 Kavaklıdere/Ankara Centre National de la Recherche Scientifique (CNRS) Headquarters: 3, rue Michel-Ange 75794 Paris cedex 16 - France
Projenin Başlangıç ve Bitiş Tarihleri: 01 Nisan 2005 – 31 Mart 2007
Öz (en çok 70 kelime): Değişik kriterler kullanılarak değerlendirilen seçeneklerin sıralanmış kategorilere atanma problemi üzerinde çalıştık. Bu problem çok kriterli sıralama problemi olarak bilinmektedir. Üstün sıralama ilişkileri üzerine yapılandırılmış bazı sıralama yöntemleri ardışık kategorileri ayıran sınırları ifade eden profillerin belirlenmesini gerektirmekte ve seçenekleri profillerle karşılaştırıp kategorilere atamaktadır. Profillerin açıkça belirlenmesi karar vericiler açısından zor olabileceği için bunu gerektirmeyen bir metod geliştirdik. Karar vericiden referans seçenekleri kategorilere atmasını istiyoruz ve diğer seçenekleri referans seçeneklerle karşılaştırarak atıyoruz.
Anahtar Kelimeler: Çok Kriterli Karar Verme, Sıralama, Tamsayılı Programlama
Projeden Yapılan Yayınlar: Murat Köksalan, Vincent Mousseau, Özgür Özpeynirci, Selin Bilgin Özpeynirci, "A new outranking-based approach for assigning alternatives to ordered classes", Naval Research Logistics, değerlendirme sürecinde.

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A new outranking-based approach for assigning alternatives to ordered classes

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A new outranking-based approach for assigning alternatives to ordered classes

Abstract

We consider the problem of assigning alternatives evaluated on several criteria into ordered categories C_1, C_2, \dots, C_p . This problem is known as the multi-criteria sorting problem and arises in many situations such as classifying countries into different risk levels based on economical and socio-political criteria, evaluating credit applications of bank customers.

We are interested in sorting methods that are grounded on the construction of outranking relations. Among these, the Electre Tri method requires defining multidimensional profiles that represent the "frontier" separating consecutive categories C_k and C_{k+1} , and assigns an alternative to categories according to how it compares to each of the profiles. The explicit specification of the profiles of consecutive categories can be difficult for decision makers. We develop a new outranking based sorting method that does not require the explicit definition of profiles. We instead require the decision maker to assign a subset of reference alternatives to the categories. To assign the remaining alternatives, each such alternative is compared to reference alternatives, and assigned to categories accordingly.

Keywords: *Multiple criteria sorting, Outranking relations.*

1 Introduction

When modeling a real world decision problem using multiple criteria decision aid, several problematics (or problem formulations) can be considered. [16] distinguishes three basic problematics: choice, sorting and ranking.

Given a set A of alternatives (or actions), the choice (or selection) problematic consists in a choice of a subset $A' \subset A$, as small as possible, composed of alternatives being judged as the most satisfying. Optimization problems are particular cases of a choice problematic where A' is restricted to a single alternative. Purchasing a car or a house are such problems. The ranking problematic consists in establishing a preference pre-order (either partial or complete) in the set of alternatives A . Ranking academic programs is a typical example. Choice and ranking are comparative problem statements in that comparisons among alternatives are required to reach the results.

The sorting problematic consists in formulating the decision problem in terms of a classification so as to assign each alternative from A to one of the predefined categories. The assignment of an alternative a to the appropriate category should rely on the intrinsic value of a (and not on the comparison of a to other alternatives from A). Typical sorting examples arise in situations such as classifying countries into different risk levels based on economical and socio-political criteria and evaluating credit applications of bank customers. For example, credit rating agencies like Standard & Poor's (S&P) regularly evaluate countries based on nine criteria (<http://www2.standardandpoors.com/servlet/Satellite?pagename=sp/Page/HomePg>). S&P currently classifies 113 sovereign governments into one of a number of categories between "AAA" and "CC". AAA corresponds to S&P's estimate of the group of governments having the least risk of not fulfilling their financial obligations. Naturally, these estimates have a huge impact on the potentials of these countries to borrow and on their borrowing conditions. S&P indicates that they don't use a fixed formula or weights to establish the ratings.

Ranking academic programs can also be treated as a sorting problem. [6] and [5] argue that characteristics of this problem makes it more suitable as a sorting problem and they mention various shortcomings of the existing procedures. Whether it is treated as a ranking or a sorting problem, the resulting evaluations are very important for the academic programs, especially in terms of the recruitment of students.

The literature proposes various multiple criteria sorting methods: Trichotomic Segmentation [10], N-TOMIC [9], ORCLASS [8], ELECTRE TRI [20], PROAFTN [1], UTADIS [3], a general class of filtering methods [14], rough sets [4] or Koksalan and Ulu methods [19],[17].

In this paper, we are interested in the multiple criteria sorting problems when the preference model is grounded on an outranking relation. More precisely, our work is related to a well known existing method called ELECTRE TRI (see [20], [12]). This method uses a majority rule and a veto rule to compare alternatives to the so called profiles that are considered as representative vectors defining categories. Roughly speaking, when an alternative "outperforms" a category profile in terms of the majority of criteria, and is not "inferior" to that profile by a "large margin", then that alternative cannot be assigned to a worse category than the category represented by that profile. The DM has to define weights, thresholds and profiles in order to operationalize the method. In practice, it is not realistic to assume that the DM would be able to give explicitly the values of these parameters; the natural terms in which the DM usually expresses his/her preferences and expertise are by making assignments of alternatives to typical categories. Inferring the model parameters of

ELECTRE TRI through an analysis of assignment examples given by the DM has already been studied either inferring all parameters simultaneously (see [11]) or a subset of parameters only ([13] and [2]). Nevertheless, one difficulty that has not been addressed when implementing ELECTRE TRI consists in defining (either directly or by inference) the profiles representing the categories. In practice, it is not even reasonable to assume the existence of such profiles. Requiring a DM to specify such profiles is not realistic. However, these profiles have played a central role for methods that are based on ELECTRE TRI.

We propose a new method that avoids the use of such profiles, the categories being implicitly defined by assignment examples. The paper is organized as follows. Section 2 briefly reviews the ELECTRE TRI method and studies properties for outranking based sorting methods. We introduce, in section 3, our new outranking based sorting method. Section 4 provides a demonstration of the approach through illustrative examples. Last section presents conclusions and issues for further research.

2 Electre Tri and Outranking based sorting

2.1 Notation and definitions

We consider the following notation:

- $A = \{a_1, a_2, \dots, a_i, \dots, a_m\}$ - a finite set of m alternatives to be assigned to categories,
- $g_1, g_2, \dots, g_j, \dots, g_n$ - n evaluation criteria, $g_j : A \rightarrow \mathbb{R}$ for all $j \in G = \{1, 2, \dots, n\}$, we will assume, without any loss of generality, that preferences increase with the value on each criterion.
- Considering these n criteria, we define the dominance relation Δ as $a\Delta b \Leftrightarrow g_j(a) \geq g_j(b), \forall j \in G$, one of the inequalities at least being strict.
- We consider an outranking relation S on A , whose semantic is "at least as good as". We denote indifference relation I and the preference relation P, I and P correspond to the symmetric and asymmetric parts of S ($I = S \cap S^{-1}$ and $P = S \cap \neg S^{-1}$). We also denote R the incomparability relation on A defined as $R = \neg S \cap \neg S^{-1}$, i.e., $aRb \Leftrightarrow \neg(aSb)$ and $\neg(bSa)$.
- C_1, C_2, \dots, C_p - p predefined ordered classes, where $C_{h+1} \gg C_h (\gg)$ a complete order on the set of classes), $h = 1, \dots, p-1$, moreover, $H = \{1, \dots, p\}$,
- $A^* \subseteq A$ - a set of reference alternatives for which the decision maker (DM) defines a desired assignment. We denote $C_h^* \subseteq A^*$ the set of reference alternatives assigned by the DM to category C_h , $h \in H$ ($C^*(a) = h, \forall a \in C_h^*$),
- $C_{>h}^* = \bigcup_{\mu > h} C_\mu^*$ and $C_{\leq h}^* = \bigcup_{\mu \leq h} C_\mu^*$ the set of reference alternatives that are assigned by the DM to category higher (lower, respectively) than C_h .
- $X_j = \{x_j \in \mathbb{R} : g_j(a_i) = x_j, a_i \in A\}$ - the set of all different evaluations on $g_j, j \in G$,
- $X = \prod_{j \in G} X_j$ the cartesian product of all evaluation scales, $x \in X$ is called an evaluation vector,

- P denotes a strict preference relation on A (asymmetric and irreflexive), I denotes an indifference relation on A (symmetric and reflexive), R denotes an incomparability relation on A (symmetric and irreflexive). These relations are defined such that: $P \cap I = P \cap R = I \cap R = \emptyset$ and $P \cup I \cup R$ is complete,
- S denotes an outranking relation (weak preference relation) defined by $S = P \cup I$.

2.2 Brief review of Electre Tri

In the Electre Tri method (see [20], [12]), the assignment of an alternative a results from the comparison of a with multidimensional profiles defining the limits of the categories. Let us denote b_1, b_2, \dots, b_{p-1} the profiles delimiting the p categories ($B = \{1, 2, \dots, p-1\}$), b_h being the upper limit of category C_h , and the lower limit of category C_{h+1} , $h = 1, 2, \dots, p-1$ (see Figure 1, where the profiles b_p and b_0 correspond to the ideal and anti-ideal alternatives, respectively).

Schematically, Electre Tri assigns alternatives to categories following two consecutive steps:

- construction of an outranking relation S that characterizes how alternatives compare to the limits of categories,
- exploitation of the relation S in order to assign each alternative to a specific category.

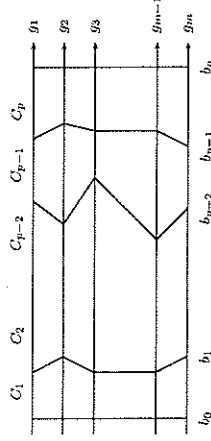


Figure 1: Definition of categories using limit profiles

Electre Tri builds a valued outranking relation S whose meaning is "a is at least as good as b_h ", i.e., defines an index $S(a, b_h) \in [0, 1]$ ($S(b_h, a)$, resp.) that represents the degree of credibility of the assertion aSb_h (b_hSa , resp.), $\forall a \in A, \forall h \in B$. The assertion aSb_h (b_hSa , resp.) is considered to be valid if $S(a, b_h) \geq \lambda$ ($S(b_h, a) \geq \lambda$, resp.), λ being a "cutting level" such that $\lambda \in [0.5, 1]$.

Determining $S(a, b_h)$ is done as follows (the value of $S(b_h, a)$ is computed analogously):

- 1 - compute the partial concordance index $c_j(a, b_h), \forall j \in G$:

$$c_j(a, b_h) = \begin{cases} 0 & \text{if } g_j(b_h) - g_j(a) \geq p_j(b_h) \\ 1 & \text{if } g_j(b_h) - g_j(a) \leq q_j(b_h) \\ \frac{p_j(b_h) + g_j(a) - g_j(b_h)}{p_j(b_h) - q_j(b_h)} & \text{otherwise} \end{cases} \quad (1)$$

where $q_j(b_h)$ and $p_j(b_h), j \in G, h \in H$, denote the indifference and preference thresholds that account for the imprecise nature of the evaluations $g_j(a)$ (see [18]). $q_j(b_h)$ specifies the largest difference

$g_j(a) - g_j(b_h)$ that preserves indifference between a and b_h on criterion g_j ; $p_j(b_h)$ represents the smallest difference $g_j(a) - g_j(b_h)$ compatible with a preference in favor of a on criterion g_j .

2 - compute the comprehensive concordance index $c(a, b_h)$:

$$c(a, b_h) = \frac{\sum_{j \in F} w_j c_j(a, b_h)}{\sum_{j \in F} w_j} \quad (2)$$

3 - compute the discordance indices $d_j(a, b_h), \forall j \in F$:

$$d_j(a, b_h) = \begin{cases} 0 & \text{if } g_j(b_h) \leq g_j(a) + p_j(b_h) \\ 1 & \text{if } g_j(b_h) > g_j(a) + p_j(b_h) \\ \in [0, 1] & \text{otherwise} \end{cases} \quad (3)$$

where $v_j(b_h), j \in G, h \in H$, denote the veto thresholds: an alternative a such that $g_j(a) \leq g_j(b_h) - v_j(b_h)$ cannot outrank b_h , and an alternative a' such that $g_j(a') \geq g_j(b_h) + v_j(b_h)$ cannot be outranked by b_h .

4 - compute the credibility index $S(a, b_h)$ of the outranking relation:

$$S(a, b_h) = c(a, b_h) \prod_{j \in F} \frac{1 - d_j(a, b_h)}{1 - c(a, b_h)}, \quad \text{where } \bar{G} = \{j \in G : d_j(a, b_h) > c(a, b_h)\} \quad (4)$$

The values of $S(a, b_h), S(b_h, a)$ and λ determine how a and b_h compare:

- $S(a, b_h) \geq \lambda$ and $S(b_h, a) \geq \lambda \Rightarrow aSb_h$ and $b_hSa \Rightarrow a$ is indifferent to b_h ,
- $S(a, b_h) \geq \lambda$ and $S(b_h, a) < \lambda \Rightarrow aSb_h$ and not $b_hSa \Rightarrow a$ is preferred to b_h ,
- $S(a, b_h) < \lambda$ and $S(b_h, a) \geq \lambda \Rightarrow \text{not } aSb_h$ and $b_hSa \Rightarrow b_h$ is preferred to a ,
- $S(a, b_h) < \lambda$ and $S(b_h, a) < \lambda \Rightarrow \text{not } aSb_h$ and not $b_hSa \Rightarrow a$ is incomparable to b_h .

Two assignment procedures are then available:

Pessimistic procedure :

- compare a successively to b_i , for $i=p, p-1, \dots, 0$,
- b_h being the first profile such that aSb_h , assign a to category C_{h+1} ($C(a) = h + 1$).

Optimistic procedure :

- compare a successively to $b_i, i=1, 2, \dots, p$,
- b_h being the first profile such that b_hPa , assign a to category C_h ($C(a) = h$).

2.3 Properties and issues for outranking based sorting

The assignment principle we intend to study is the following:

$$aSb \Rightarrow C(a) \geq C(b), \forall a, b \in A \quad (5)$$

This principle states that "if an alternative a is as good as an alternative b , a should be assigned to a category at least as good as b ". In other words, this principle forbids that b is assigned to a higher category than a whenever aSb . Figure 2, in which no alternative outranks another alternative assigned to a higher category, illustrates this assignment principle. Such assignment principle has been used in [15].

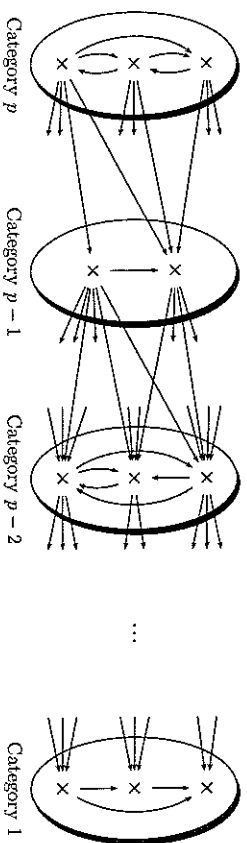


Figure 2: Outranking relations between categories

Consider the following proposition:

Proposition 1. *Considering the assignment principle (5) holds: If $a_1Sa_2, a_2Sa_3, \dots, a_{k-1}Sa_k$ and a_kSa_1 , for $a_1, a_2, \dots, a_k \in A$, then $C(a_1) = C(a_2) = \dots = C(a_k)$.*

Proof. Consider k alternatives a_1, a_2, \dots, a_k that are involved in a circuit of the outranking relation S , i.e., such that $a_iSa_{i+1}, i = 1, \dots, k-1$ and a_kSa_1 . According to the assignment principle (5) we can deduce that $C(a_1) \geq C(a_2) \geq \dots \geq C(a_k) \geq C(a_1)$. The only possible assignments respecting these inequalities are: $C(a_1) = C(a_2) = \dots = C(a_k)$. \square

This proposition states that the assignment principle (5) implies all alternatives that are in a circuit of the outranking relation S should be assigned to the same category. Hence, the absence of circuits among alternatives assigned to different categories is a necessary condition for the assignment principle (5) to be fulfilled.

It should be noted that the Electre TRI method does not respect such property of absence of circuit among alternatives assigned to different categories. A simple counter-example can be built, considering three alternatives involved in a circuit resulting from a so-called Condorcet effect: 3 alternatives evaluated on 3 criteria (to be maximized) are to be assigned to two categories C_1 and C_2 . The evaluations of alternatives are given in Table 1 below:

Table 1: Condorcet effect

	g_1	g_2	g_3
a_1	2	0	1
a_2	1	2	0
a_3	0	1	2

The criteria weights are equal ($w_1 = w_2 = w_3 = \frac{1}{3}$) and λ is set to any value between 0.5 and 1. Moreover, we consider true-criteria ($g_j(b_1) = p_j(b_1) = 0, j \in G$), and veto thresholds are set so that

no veto phenomenon occur. It is easy to observe that, due to a Condorcet effect, a_1Pa_2 , a_2Pa_3 and a_3Pa_1 holds. Moreover the profile delimiting the categories C_1 and C_2 is $b_1 = (2, 0, 1)$. Note that b_1 is the same as a_1 . Hence a_1Ib_1 , b_1Pa_2 , a_3Pb_1 holds. Therefore both Electre Tri assignment rules will assign a_1 and a_3 to C_2 , but a_2 to C_1 .

However, such statements should not be analyzed as shortcomings for the Electre Tri method. Indeed, the principle underlying any sorting method is not to compare alternatives to each other, but rather to analyze each alternative individually so as to define its assignment to a category. In the Electre Tri method, this is performed by comparing each alternative a to reference alternatives that represent the limits of categories.

Hence, it is reasonable to weaken the requirements of the assignment principle (5), so that the implication holds for a subset of alternatives $A' \subseteq A$:

$$aSb \Rightarrow C(a) \geq C(b), \forall a, b \in A' \subseteq A \quad (6)$$

In Electre Tri, as the ordered categories are defined using profiles b_h corresponding to the frontier between category C_h and C_{h+1} , these b_h are such that $b_{h+1}\Delta b_h$, i.e., $g_j(b_{h+1}) \geq g_j(b_h)$, $\forall j \in G$, one of the inequalities at least being strict, $h = 1, \dots, p-1$. Moreover, [17] imposes an even stronger condition ($g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h)$), $\forall j \in G$, $\forall h \in H$).

Lemma 2.1. *In the Electre Tri method, when categories are defined such that $g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h)$, $\forall j \in G$, $\forall h \in H$, no circuits in the outranking relation S can occur involving only a subset of profiles $B' \subseteq B = \{b_h, h = 1, \dots, p-1\}$.*

Proof. Consider the $p-1$ profiles b_1, b_2, \dots, b_{p-1} defining the categories in Electre Tri. These profiles ordered by dominance, i.e., defined such that $b_{p-1}\Delta b_{p-2}\Delta \dots \Delta b_1$. As the outranking relation S contains the dominance relation Δ , $b_hSb_{h'}$, for all h, h' such that $h > h'$. Moreover, the definition of categories imposes that $g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h)$, $\forall j \in G$, $\forall h \in H$, which makes it impossible to have $b_{h'}Sb_h$, for h, h' such that $h > h'$. Hence, no circuit can occur in S involving only profiles in $B = \{b_h, h = 1, \dots, p-1\}$. \square

Lemma 2.2. *In the Electre Tri method, when categories are defined such that $g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1})$, $\forall j \in G$, $\forall h \in H$, no circuits in the outranking relation S can occur involving only one alternative $a \in A$ and a subset of profiles $B' \subseteq B \cup \{a\}$ containing more than one profile ($|B'| \geq 2$).*

Proof. The profiles separating the categories are defined such that $b_{p-1}\Delta b_{p-2}\Delta \dots \Delta b_1$. Moreover, these profiles are defined such that no alternative $a \in A$ can be indifferent to two consecutive profiles $b_h, b_{h+1}, h \in B$, i.e.:

$$\forall a \in A, \text{ it holds: } aIb_h \Rightarrow \neg[aIb_{h-1}] \text{ and } \neg[aIb_{h+1}] \quad (7)$$

Usually imposing (7) is done by posing the following sufficient condition:

$$g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1}), \forall j \in G, \forall b_h \in B \quad (8)$$

Considering an alternative $a \in A$ and a profile $b_h \in B$, four situations can occur: aIb_h , b_hPa , aPb_h or aRb_h .

- Suppose aIb_h , i.e., aSb_h and b_hSa . Consider $b_{h'} \in B$, $h' > h$. (8) and aIb_h implies that $b_{h'}Pa$. As $b_{h'}Sb_h$ (because $b_{h'}\Delta b_h$), alternative a can not be included in a circuit of S involving b_h and $b_{h'}$. Consider $b_{h''} \in B$, $h'' < h$. (8) and aIb_h implies that $aPb_{h''}$. As $b_{h''}Sb_{h'}$ (because $b_{h''}\Delta b_{h'}$), alternative a can not be included in a circuit of S involving b_h and $b_{h''}$.

- Suppose $b_{h'}Pa$ and consider $b_{h''} \in B$, $h'' > h$. As $b_{h''}\Delta b_h$, it holds $b_{h''}Pa$. Hence, alternative a can not be included in a circuit of S involving b_h and $b_{h''}$. Consider $b_{h'''} \in B$, $h''' < h$. As $b_h\Delta b_{h'''}$, it holds $b_hPb_{h'''}$. Hence, alternative a can not be included in a circuit of S involving b_h and $b_{h'''}$.

- Suppose aPb_h . An argument analogous to the preceding case proves that alternative a can not be included in a circuit of S involving b_h and another profile.

- Suppose aRb_h ; alternative a can obviously not be included in a circuit of S involving b_h as neither aSb_h nor b_hSa holds.

Therefore no circuits in the outranking relation S can occur involving only one alternative $a \in A$ and a subset of profiles $B' \subseteq B \cup \{a\}$ containing more than one profile. \square

Proposition 2. *When categories are defined such that $g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1})$, $\forall j \in G$, $\forall h \in H$, Electre Tri method's assignments are compatible with the assignment principle (6) when considering the limit profiles only, i.e., when $A' = B$.*

Proof. Due to the dominance among profiles, S constitutes a strict total order on B : $b_{h-1}Sb_{h-2}S\dots Sb_1$. Electre Tri assigns b_h to category C_{h+1} . Hence, Electre Tri method's assignments are compatible with the assignment principle (6), when considering the limit profiles only. \square

Proposition 3. *When categories are defined such that $g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1})$, $\forall j \in G$, $\forall h \in H$, Electre Tri method's assignments are compatible with the assignment principle (6) when considering the limit profiles and any single alternative, i.e., when $A' = B \cup \{a\}$, $a \in A$.*

Proof. According to proposition 2, Electre Tri method's assignments are compatible with the assignment principle (6), when considering the limit profiles only. In order to prove that it remains true when considering $A' = B \cup \{a\}$, $a \in A$, we must consider how a compares to the profiles b_h and its comparative assignment, i.e., we need to prove that:

$$\text{if } aSb_h \text{ then } C(a) \geq C(b_h), \forall b_h \in B \quad (9)$$

$$\text{if } b_hSa \text{ then } C(b_h) \geq C(a), \forall b_h \in B \quad (10)$$

- Electre Tri pessimistic rule assigns a to the highest class for which a outranks the lower profile. Let $b_{h'}$ denote the highest profile for which $aSb_{h'}$. Then a and $b_{h'}$ are both assigned to $C_{h'+1}$. As $b_{h'}$ is the highest profile for which $aSb_{h'}$, a does not outrank any higher profile. Moreover, due to dominance among the ordered profiles, a outranks all profiles lower than $b_{h'}$; however each of them are assigned to a category lower than $C_{h'+1}$. The optimistic assignment rule assigns a to a category that is higher or equal to the pessimistic assignment. Therefore (9) holds for both assignment rules.

- Electre Tri optimistic rule assigns a to the lowest class for which the higher profile is preferred to a . Let $b_{h''}$ denote the lowest profile for which $b_{h''}Pa$, i.e., $b_{h''}Sa$ and not $aSb_{h''}$. Then a is assigned to $C_{h''}$ whereas $b_{h''}$ is assigned to $C_{h''+1}$. It is possible that a is indifferent to $b_{h''-1}$ (but only $b_{h''-1}$ as a can not be indifferent to more than one profile) i.e., $b_{h''-1}Sa$ and $aSb_{h''-1}$; in this case a and $b_{h''-1}$ are both assigned to $C_{h''}$. The pessimistic assignment rule assigns a to a category that is lower or equal to the optimistic assignment. Therefore (10) holds for both assignment rules. \square

Proposition 4. *Even when categories are defined such that $g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1})$, $\forall j \in G, \forall h \in H$, Electre Tri method's assignments are not necessarily compatible with the assignment principle (6) when considering all alternatives, i.e., when $A^* = A$.*

Proof. The example presented in Table 1 provides a counter-example that proves that Electre Tri method's assignments are not necessarily compatible with the assignment principle (6) when considering all alternatives. \square

3 A new outranking-based sorting method

Specifying the category limits through profiles can be a difficult task for the DM. On the other hand, in many decision situations, DMs can easily specify alternatives that are typical examples of a category, say reference alternatives. Therefore, we propose a new outranking-based sorting method that does not require explicit profiles characterizing the multidimensional limits of consecutive categories, but rather represents categories implicitly using reference alternatives.

In this section, we first develop a procedure that captures the circuits, if any, among alternatives assigned by the DM to different categories. We detect the alternatives that cause these circuits and determine which assignments to disregard in order to eliminate the circuits. We then try to assign the remaining alternatives considering the available preference information. We assume that preference and indifference thresholds, therefore partial concordance indices of alternatives are known. We treat criteria weights as unknown.

3.1 Identifying assignment examples compatible with the assignment principle

To obtain the initial assignments, we ask the DM to assign some alternatives to categories. Let A^* be the set of these alternatives. We would like to have approximately equal number of initially assigned alternatives in each category. To achieve this, different approaches can be employed in selecting the alternatives to be presented to the DM for initial assignments. For example, we could use a simplified version of Electre Tri without the veto property. We could use equal weights and suitably chosen profiles and thresholds to come up with some rough assignments. We can then select alternatives from these categories with the expectation that the DM's assignments will roughly be consistent with Electre Tri's assignments. Another approach would be to select the alternatives sequentially utilizing the information obtained from the previous assignments of the DM. We can keep presenting new alternatives to the DM until a desired number of assignments are made to each category. We should emphasize that these are some possible ways of choosing the alternatives in A^* . The chosen alternatives will be presented to the DM who will make the actual placement of them into categories.

After the DM assigns the given alternatives to categories, we would like to find a large subset of the assignments that do not have circuits. A circuit occurs when two alternatives from different categories outrank each other. There may also be circuits between more than two alternatives. Since there are no profiles for categories, circuits between different categories may exist. We are not concerned about circuits within the same category but we do not allow circuits between categories. In order not to have such circuits, whenever $C(a_q) < C(a_r)$, alternative a_q should not outrank alternative a_r . That is, if an alternative placed in a lower level category outranks an alternative placed in a higher-level category, there is a circuit.

Using model (P1), we eliminate these circuits (if there exists any) from the reference alternative set. We remove some of the assigned alternatives in order to eliminate these circuits. (P1) achieves this by maximizing the minimum number of alternatives assigned to any category. This helps in maintaining an approximately equal number of alternatives in each of the categories. After the necessary eliminations from set A^* , set $A_1^* \subseteq A^*$ is obtained so that there are no circuits between the alternatives in A_1^* .

Decision variables:

- α : minimum number of alternatives in a category
- λ : concordance threshold
- w_j : weight of criterion j

$$y_i = \begin{cases} 1 & \text{if reference alternative } a_i \text{ is not deleted} \\ 0 & \text{otherwise} \end{cases}$$

$$(P1) \text{ Max } \alpha + \rho \sum_{i: a_i \in A^*} y_i \quad (11)$$

s.t.

$$\alpha \leq \sum_{i: C(a_i) = C_k} y_i, \forall k \in H \quad (12)$$

$$S(a_{q_i}, a_r) = \sum_{j=1}^n w_j c_j(a_{q_i}, a_r), \forall a_{q_i}, a_r \in A^* \quad (13)$$

$$S(a_{q_i}, a_r) \leq \lambda - \epsilon + M(1 - y_{q_i}) + M(1 - y_r), \forall a_{q_i}, a_r \in A^* \ni C(a_{q_i}) < C(a_r) \quad (14)$$

$$y_i \in \{0, 1\} \forall i: a_i \in A^* \quad (15)$$

$$w \in \Omega \quad (16)$$

$$\lambda \in [0.5, 1] \quad (17)$$

where ρ and ϵ are arbitrarily small positive constants, and M (big- M) is an arbitrarily large positive constant. The minimal restrictions on w that define set Ω are

$$\sum_{j=1}^n w_j = 1 \quad (18)$$

$$\epsilon \leq w_j \leq 0.5, \forall j \quad (19)$$

If we have additional preference information of the DM, we can incorporate these as additional restrictions in Ω .

Constraints (12) assure that α is less than or equal to the number of assigned alternatives that are not eliminated in each class. These constraints, together with the term, α , in the objective function maximize the minimum category size in terms of the retained alternatives assigned by the DM. Constraints (13) define the overall concordance measure in terms of the weighted sum of its components in each criterion. Constraints (14) determine if a circuit exists based on the assignments of a_q and a_r . If a circuit exists, either a_{q_i} or a_r will be eliminated depending on their effects on the objective function.

3.2 Assigning alternatives to categories

After we eliminate some of the reference alternatives in order to prevent circuits (i.e. after we construct set A_1^*) we try to place the unassigned alternatives. In making these assignments, we do not use any profiles. Instead, we utilize the reference alternatives assigned to categories by the DM (i.e. the alternatives in A_1^*) as benchmarks to place the remaining alternatives. We denote the alternatives

already assigned by the DM as a_k and the alternatives we try to place as a_i .

Decision variables:

$$y_{ih} = \begin{cases} 1 & \text{if alternative } a_i \text{ is assigned to category } C_h \\ 0 & \text{otherwise} \end{cases} \quad (32)$$

u_{ihk} = Amount of violation in the outranking relation between alternatives a_i and a_k when a_i is assigned to C_h

The parameters, ρ , ε , and M are as defined before.

$$(P2) \quad \text{Min} \sum_i \sum_h \sum_k u_{ihk} - \rho \lambda \quad (20)$$

s.t.

$$S(a_i, a_k) \leq \lambda - \varepsilon + M(1 - y_{ih}), \quad \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (21)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon + M(1 - y_{ih}), \quad \forall a_k \in C_{<h}, h = 2, \dots, ncat, \forall i \quad (22)$$

$$S(a_i, a_k) \geq \lambda - M(1 - y_{ih}) - u_{ihk}, \quad \forall a_k \in C_{<h}, h = 2, \dots, ncat, \forall i \quad (23)$$

$$S(a_k, a_i) \geq \lambda - M(1 - y_{ih}) - u_{ihk}, \quad \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (24)$$

$$\sum_{h=1}^{ncat} y_{ih} = 1, \quad \forall i \quad (25)$$

$$S(a_q, a_r) \leq \lambda - \varepsilon, \quad \forall a_q, a_r \in A_1^* \ni C(a_q) < C(a_r) \quad (26)$$

$$S(a_i, a_k) = \sum_{j=1}^n w_j c_j(a_i, a_k), \quad \forall i, k \quad (27)$$

$$\lambda \in [0.5, 1] \quad (28)$$

$$y_{ih} \in \{0, 1\}, u_{ihk} \geq 0 \quad (29)$$

$$w \in \Omega \quad (30)$$

The model, (P2), makes sure that any alternative, a_i , placed by the model does not outrank any alternative, a_k , placed by the DM to higher-level categories, and is not outranked by any alternative a_k placed by the DM to lower-level categories (constraints (21) and (22)). Furthermore, the model tries to have an alternative in a higher-level category to outrank an alternative in a lower-level category between all pairs, and tries to minimize the total magnitude of violations when it is not possible to maintain these relations (constraints (23) and (24)). That is, the u_{ihk} variable measures the magnitude of violation of this outranking relation and the objective function tries to minimize the sum of these violations. The second term in the objective function maximizes the λ value as a secondary objective due to its sufficiently small positive coefficient. The constraints (25) make sure that each a_i is assigned to exactly one category. (26) force the reference alternatives assigned to lower-level categories not to outrank those assigned to higher-level categories. Constraint (28) specifies the allowable values for λ . Small values of λ are not allowed since it would be too easy for solutions to outrank each other.

In order to compute the assignment of the non-reference alternatives, we can consider another objective function which minimizes the maximum variable u_{ihk} instead of the sum of these variables. This leads to a program corresponding to an interesting alternative formulation in which the worst deviation is considered. This can be achieved by replacing the objective function of (P2) by:

$$\text{Min } u - \rho \lambda \quad (31)$$

and adding constraints

where u represents the maximum violation.

If (P2) turns out to be infeasible, we may wish to find the placements with minimum number of violations of constraints (21) and (22). Let

$$z_{ik} = \begin{cases} 1 & \text{if outranking relation between } a_i \text{ and } a_k \text{ is violated} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Min} \sum_i \sum_k z_{ik} + \rho \sum_i \sum_h \sum_k u_{ihk} \quad (33)$$

s.t.

$$S(a_i, a_k) \leq \lambda - \varepsilon + M(1 - y_{ih}) + Mz_{ik}, \quad \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (34)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon + M(1 - y_{ih}) + Mz_{ik}, \quad \forall a_k \in C_{<h}, h = 1, \dots, ncat, \forall i \quad (35)$$

$$\text{Constraints (23) - (30)}$$

When z_{ik} takes a value of 1, the above model allows violating the corresponding constraint (34) or (35). The objective function primarily minimizes the number of such violations. Minimizing sum of u_{ihk} variables becomes a secondary objective.

On the other hand, if we want to minimize the total magnitude of violation, or minimize the maximum violation, we can do without the z_{ik} variables. Let u'_{ik} be the violation in the outranking relation for the pair (a_i, a_k) . In this case, we can write the following model.

$$\text{Min} \sum_i \sum_k u'_{ik} + \rho \sum_i \sum_h \sum_k u_{ihk} \quad (36)$$

s.t.

$$S(a_i, a_k) \leq \lambda - \varepsilon + M(1 - y_{ih}) + u'_{ik}, \quad \forall a_k \in C_{>h}, h = 1, \dots, ncat - 1, \forall i \quad (37)$$

$$S(a_k, a_i) \leq \lambda - \varepsilon + M(1 - y_{ih}) + u'_{ik}, \quad \forall a_k \in C_{<h}, h = 1, \dots, ncat, \forall i \quad (38)$$

$$\text{Constraints (23) - (30)}$$

(36) minimizes the total violation u'_{ik} . If we wish to minimize the maximum violation, instead, we then use the following model.

$$\text{Min } u + \rho \sum_i \sum_h \sum_k u_{ihk} \quad (39)$$

s.t.

$$u \geq u'_{ik}, \quad \forall i, k \quad (40)$$

$$\text{Constraints (37), (38), (23) - (30)}$$

3.3 Finding a range of categories

Another useful information is the range of categories each alternative can be assigned to in light of the set of reference alternatives, A_i^* , obtained from (P1). To find if alternative i can be assigned to category h , we solve a slight variation of (P2).

Letting $y_{h'c} = 1$ in (P2), we can find if it is possible to assign a specific alternative i to a specific category h' . Let this model be (P2_{h'}). If there exists a feasible solution, then it is possible to assign t to h' while finding feasible assignments to all remaining alternatives. By solving (P2_{h'}) for all $h' = 1, \dots, ncat$, we can find all possible categories of alternative i . We can repeat the procedure for all alternatives.

A relaxed version of (P2_{h'}) can be obtained as a linear program given as (P3_{h'}).

$$(P3_{h'}) \text{ Min } 0 \tag{41}$$

s. t.

$$S(\alpha_i, \alpha_k) \leq \lambda - \epsilon, \quad \forall \alpha_k \in C_{>h} \tag{42}$$

$$S(\alpha_k, \alpha_i) \leq \lambda - \epsilon, \quad \forall \alpha_k \in C_{<h} \tag{43}$$

$$S(\alpha_{q_i}, \alpha_i) \leq \lambda - \epsilon, \quad \forall \alpha_{q_i}, \alpha_i \in A_i^* \ni C(\alpha_{q_i}) < C(\alpha_i) \tag{44}$$

$$S(\alpha, b) = \sum_{j=1}^n w_j c_j(\alpha, b), \tag{45}$$

$$\lambda \in [0.5, 1] \tag{46}$$

$$w \in \Omega \tag{47}$$

The constraints of (P3_{h'}) are similar to those in previous problems. Since we need to find whether a feasible solution exists or not, the objective function is not relevant and we just set it to a constant value, zero. For each α_i , we solve (P3_{h'}) for all categories. A feasible solution indicates that it might be possible to assign α_i to category h . When (P3_{h'}) does not have a feasible solution, assigning α_i to category h would not be consistent with the DM's assignments of reference alternatives.

(P3_{h'}) can also be used to reduce the number of integer variables in (P2) and (P2_{h'}). We may define only those y_{hi} variables for which (P3_{h'}) indicates a possible assignment.

4 Demonstration of the Approach

In this section we demonstrate the approach using several problems. We first consider the evaluation problem of MBA programs using actual data. We then use a problem from the banking sector to further demonstrate various aspects of our approach.

4.1 An Application to Global MBA rankings

Here, we demonstrate our method on the MBA program rankings using the Financial Times (FT) 2005 data. Every year, FT announces top 100 global MBA programs. FT uses 20 criteria grouped under three main criteria: alumni career progress, diversity and idea generation.

[6] studied the MBA program ranking problem for 81 programs of the FT data. We use 30 programs of their list to demonstrate our approach. They create numerical scores that are consistent with the criteria information that are provided in ordinal scale by FT. We use this data set to assign

the 30 MBA programs to five categories using three criteria.

We assume that eight programs (reference alternatives) have already been assigned to categories. The scores and categories of the reference alternatives are given in Table 2. Table 3 shows the data for the remaining alternatives that are to be assigned to categories by our models. We use indifference and preference thresholds given in Table 4. When we solve (P1), none of the reference alternatives is removed from set A^* .

Table 2: Reference Alternatives

Program Name	Alumni Career Progress	Diversity	Idea Generation	Category of Reference Alternatives
London Business School	68.78	62.03	59.87	5
MIT: Sloan	60.01	24.10	85.81	4
Yale School of Management	79.01	25.98	51.84	4
University of North Carolina: Kenan-Flagler	67.80	22.45	62.40	3
University of Toronto: Rotman	59.34	36.68	60.79	3
UCLA: Anderson	55.46	21.52	74.54	2
University of Oxford: Saïd	57.04	43.49	47.12	2
University of Maryland: Smith	55.33	25.28	64.13	1

We first determine the categories each alternative can be assigned to without causing any circuits. For this purpose, we solve the linear programming model (P3_{h'}) for each alternative-category pair. We then solve the mixed integer programming model (P2_{h'}). The possible categories of both models for each alternative are given in Table 5. (P2_{h'}) reduces the possible number of categories for Cornell University and University of Iowa compared to (P3_{h'}). Note that, the size of (P2_{h'}) can be reduced by defining binary variables for the assignment of program i only for the possible categories found by (P3_{h'}).

Then we solve (P2) and obtain the assignments given in the last column of Table 5. Optimal weight set is found as 0.362, 0.276 and 0.362 for the three criteria, and λ is found to be 0.638. The size of (P2) can also be reduced like (P2_{h'}) using the results of (P3_{h'}).

(P2) places each non-reference alternative a to a category such that a does not outrank a reference alternative of a better category. For example, consider two programs: University of Pennsylvania: Wharton (W) and London Business School (L). According to the optimal criteria weights and the λ value found in (P2), the concordance and outranking relations of these two programs can be calculated as follows:

Table 3: Alternative Scores

Program Name	Alumni Career Progress	Diversity	Idea Generation
Carnegie Mellon: Tepper	54.02	18.69	71.93
Columbia Business School	83.44	32.94	71.63
Cornell University: Johnson	60.54	30.31	55.99
Duke University: Fuqua	64.05	27.25	64.68
Esade Business School	54.61	60.15	14.45
Harvard Business School	77.84	29.05	93.91
Iese Business School	61.99	54.61	49.04
Insead	56.59	70.74	65.45
New York University: Stern	68.24	26.74	80.5
Northwestern University: Kellogg	68.68	24.52	72.43
Rotterdam School of Management	51.2	52.91	42.98
Stanford University GSB	76.77	28.52	81.8
UC Berkeley: Haas	61.26	31.04	73.69
University of Chicago: GSB	75.64	25.37	77.73
University of Iowa: Tippie	58.25	26.25	46.81
University of Michigan: Ross	63.23	28.36	69.61
University of Pennsylvania: Wharton	77.11	32.51	91.59
University of Rochester: Simon	60.25	29.6	49.25
University of Virginia: Darden	76.49	19.05	37.68
University of Western Ontario: Ivey	50.04	40.55	53.55
Vanderbilt University: Owen	64.67	26.33	35.14
York University: Schulich	57.03	50.73	47.55

Table 4: Preference and Indifference Thresholds

	Alumni Career Progress	Diversity	Idea Generation
Indifference threshold	0.35	0.7	0.5
Preference threshold	3	3	3

$$\begin{aligned}
 g_1(L) - g_1(W) &= 68.78 - 77.11 = -8.33 < q_1 = 0.35 \Rightarrow c_1(W, L) = 1 \\
 g_2(L) - g_2(W) &= 62.03 - 32.51 = 29.52 > p_2 = 3 \Rightarrow c_2(W, L) = 0 \\
 g_3(L) - g_3(W) &= 59.87 - 91.59 = -31.72 < q_3 = 0.5 \Rightarrow c_3(W, L) = 1 \\
 S(W, L) &= \sum_{i=1}^3 w_i c_i(W, L) = 0.362 + 0 + 0.362 = 0.724 \\
 g_1(W) - g_1(L) &= 77.11 - 68.78 = 8.33 > p_1 = 3 \Rightarrow c_1(L, W) = 0 \\
 g_2(W) - g_2(L) &= 32.51 - 62.03 = -29.52 < q_2 = 0.7 \Rightarrow c_2(L, W) = 1 \\
 g_3(W) - g_3(L) &= 91.59 - 59.87 = 31.72 > p_3 = 3 \Rightarrow c_3(L, W) = 0 \\
 S(L, W) &= \sum_{i=1}^3 w_i c_i(L, W) = 0 + 0.276 + 0 = 0.276 \\
 S(W, L) &= 0.724 \geq \lambda \Rightarrow W \text{ outranks } L \\
 S(L, W) &= 0.276 < \lambda \Rightarrow L \text{ does not outrank } W
 \end{aligned}$$

Table 5: Results of Models $P3_{it}$, $P2_{it}$ and $P2$

Program Name	$P3_{it}$	$P2_{it}$	$P2$
Columbia Business School	4,5	4,5	5
Harvard Business School	4,5	4,5	5
New York University: Stern	3,4,5	3,4,5	5
Northwestern University: Kellogg	3,4,5	3,4,5	5
Stanford University GSB	3,4,5	3,4,5	5
University of Chicago: GSB	3,4,5	3,4,5	5
University of Pennsylvania: Wharton	4,5	4,5	5
Duke University: Fuqua	1,2,3,4,5	1,2,3,4,5	4
Insead	2,3,4,5	2,3,4,5	4
UC Berkeley: Haas	2,3,4,5	2,3,4,5	4
University of Michigan: Ross	1,2,3,4,5	1,2,3,4,5	4
Iese Business School	2,3,4,5	2,3,4,5	3
Cornell University: Johnson	1,2,3,4,5	1,2,3	2
University of Iowa: Tippie	1,2,3	1,2	2
University of Rochester: Simon	1,2,3	1,2,3	2
Vanderbilt University: Owen	1,2,3,4	1,2,3,4	2
York University: Schulich	2,3,4,5	2,3,4,5	2
Carnegie Mellon: Tepper	1,2	1,2	1
Esade Business School	1,2,3,4,5	1,2,3,4,5	1
Rotterdam School of Management	1,2,3,4,5	1,2,3,4,5	1
University of Virginia: Darden	1,2,3,4	1,2,3,4	1
University of Western Ontario: Ivey	1,2,3,4,5	1,2,3,4,5	1

Therefore, W cannot be assigned to a category worse than that of L . Since L is a reference alternative placed in the best category, W should also be assigned to this category in order not to violate assignment principle (5). It should however be noted that the solution provided by (P2) guarantees principle (5) when considering the comparison of a single non-reference alternative to reference alternatives. As mentioned before, there may be circuits among non-reference alternatives.

Although it is neither necessary nor possible to guarantee preventing circuits among non-reference alternatives with an outranking-based methodology, it is easy to identify the existence of such circuits. A variation of our approach would be to identify those circuits and present them to the DM as feedback. If the DM wishes to prevent some of the circuits, the corresponding alternatives may be added to A^* and the whole process can be repeated. In this problem the assignment principle (5) is violated in five of the 231 pairs of non-reference alternatives, leading to circuits involving these five pairs.

In this problem there are 22 schools, each of which is assigned to one of the 5 categories. Hence (P2) has 110 binary variables. When we solve (P2) directly, it takes 1.23 CPU seconds with CPLEX 8.1 on a Pentium 4, 2.80 GHz computer with 520 MB RAM. As Table 5 indicates, ($P3_{it}$) narrows down the possible assignments and we see that 30 of the 110 assignments are not feasible. When we incorporate this information into (P2), the total CPU time of ($P3_{it}$) and (P2) turns out to be 0.66 seconds.

4.2 Some Computational Results

In order to further demonstrate the performance of our approach, we consider a sorting problem which corresponds to a real-world decision problem in the banking sector, see [20]. However, due to the confidentiality of the data, the precise context of the problem had not been explained. In this application, the DM considers each day a new set of alternatives. The sorting model aims at distinguishing alternatives so as to eliminate those considered as unsuitable with respect to some predefined norm; at a second stage, a preference ranking of the remaining alternative was built; the final choice was made on the basis of this ranking. We are interested here in the first stage, which consists in the elaboration of a sorting model.

This example problem contains 100 alternatives and seven criteria. We consider placing alternatives into three categories, we simulate DM's responses using Electre TRI with parameters given in Table 6, and we try to choose the reference alternatives to represent categories well.

Table 6: Indifference and preference thresholds and weights of criteria for Electre TRI

	Criteria						
	g_1	g_2	g_3	g_4	g_5	g_6	g_7
Indifference threshold q_i	0.64	1.56	1.56	1.56	1.56	1.56	2.04
Preference threshold p_j	1.28	3.17	3.17	3.17	3.17	3.17	4.15
Weights w_j	0.24	0.12	0.12	0.12	0.12	0.12	0.16

We first tried our approach by choosing two reference alternatives in each category. In this case, the remaining 94 alternatives were placed by our models. We next added one more reference alternative to each category and ran the models again to place 91 alternatives. Finally, we repeated the procedure adding one more reference alternative to each category and placing the 88 non-reference alternatives.

In none of the cases did Model ($P1$) encounter any circuits. Hence we did not need to eliminate any of the reference alternatives. The results of Model ($P2$) turned out to be very robust. Regardless of the number of reference alternatives, ($P2$) placed the alternatives into the same category, with the exception of one alternative. This alternative was placed to category one when using two and four reference alternatives, but to category two when using three reference alternatives. When we compare the results of the assignments with those of Electre TRI, we observe that a vast majority of alternatives are assigned to the same categories in all cases. There were at most 6 alternatives assigned to different categories by Electre TRI and our approach. Of course, in theory there could be more disagreement since these are two different approaches. In all solutions, the λ value turned out to be very close to its lower bound. This is expected since minimizing u_{ijk} values is in conflict with larger λ values. Although the assignments to categories were almost the same with our approach using different number of reference alternatives, there was some variation in the resulting weight values.

We also solved the variation of ($P2$) that minimizes the maximum violation discussed in Section 3.2. In this case we minimized the maximum u_{ijk} value, rather than the sum of u_{ijk} values. The placements made by the model were identical to the previous case except for one alternative. The alternative that was an exception in the previous case turned out to be an exception here as well. It was placed to category two when using two and three reference alternatives, but to category one

when using four reference alternatives.

Another observation we have is regarding the objective function value of the ($P2$) problem. It turned out to be very small (around 0.001), indicating that non-reference alternatives in better categories could outrank reference alternatives in worse categories, and reference alternatives in better categories could outrank non-reference alternatives in worse classes, in general. Looking more closely, only around 10 u_{ijk} variables turned out to be nonzero among 1000 to 2000 possible u_{ijk} values for different cases.

The computational difficulty of ($P2$) increases with the number of binary variables. Number of binary variables is equal to the number of alternatives to be assigned by the model times the number of categories. In order to see the computational performance of our algorithm, we conducted a small experiment. We tried 100 and 300 alternatives. For the 300-alternative case we randomly generated 200 alternatives in addition to the original 100 alternatives. We selected 5 reference alternatives in each category and solved each problem twice; once using only ($P2$) and once preprocessing with ($P3_M$) first and then using ($P2$) for the simplified problem. We present the CPU times we obtained using CPLEX 8.1 solver on a Pentium 4, 2.80 GHz computer with 520 MB RAM in Table 7. The results indicate that the CPU times increase substantially with the number of alternatives, as expected. The effectiveness of ($P3_M$) also increases substantially with the number of alternatives. The problem is manageable for reasonably large number of alternatives using the CPLEX solver directly. It may be possible to further improve the computations by studying the features of the integer programs solved. This is an issue we consider for future research.

Table 7: Computational results

Number of alternatives	CPU seconds	
	Using ($P2$) only	Using ($P3_M$) and ($P2$)
100	3.63	3.11
300	783.63	180.95

5 Conclusions and further research

In this paper, we developed an outranking based approach to place alternatives into preference-ordered categories. Our approach aims to avoid asking difficult preference information from the DM such as weights or category limits (profiles) or the λ threshold. We only require the DM to place a small number of reference alternatives into suitable categories. We then place the remaining alternatives with the aid of an integer program that maintains desired relationships between the reference alternatives and the alternatives that are placed by the model. We demonstrate our approach on a small example problem and observe that it performs well on larger problems as well. Studying various aspects of the integer programs is a future research direction that might prove useful to further improve the computation times.

We believe that there is need for further research to simplify the information requirements of outranking-based approaches. Further developments and experimentation will lead to more realistic outranking-based approaches.

Using the veto concept is not straightforward when parameters are not directly specified by the DM. This is a future research area. Extending our approach to the case of multiple decision makers is another future work we intend to do.

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