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Energy Density Associated with the Bianchi Type-II Space-Time

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To calculate the total energy distribution (due to both matter and fields including gravitation) associated with locally rotationally symmetric (LRS) Bianchi type-II space-times. We use the Bergmann-Thomson energy-momentum complex in both general relativity and teleparallel gravity. We find that the energy density in these different gravitation theories is vanishing at all times. This result is the same as that obtained by one of the present authors who solved the problem of finding the energy-momentum in LRS Bianchi type-II by using the energy-momentum complexes of Einstein and Landau and Lifshitz. The results of this paper also are consistent with those given in the previous works of Cooperstock and Israelit, Rosen, Johri et al., Banerjee-Sen, Vargas, and Saltı et al. In this paper, we perform the calculations for a non-diagonal expanding space-time to determine whether the Bergmann-Thomson energy momentum prescription is consistent with the other formulations. (We previously considered diagonal and expanding space-time models.) Our result supports the viewpoints of Albrow and Tryon.

§1. Introduction

Energy-momentum localization is one of the oldest and most debated problems in both Einstein's theory of general relativity and teleparallel theory of gravity. Meisner, Thorne and Wheler[1)] claimed that the energy is localizable only for spherical systems. However, Cooperstock and Sarracino[2)] argued that if the energy is localizable in spherical systems, it is localizable in all systems. This problem has remained unsolved since the advent of Einstein's theory of general relativity and the teleparallel theory of gravity because of its unusual nature and various points of view. To solve the problem of energy momentum localization, many energy-momentum definitions have been proposed since that of Einstein based on the canonical approach: e.g., Einstein,[3)] Tolman,[4)] Landau and Lifshitz,[5)] Papapetrou,[6)] Bergmann and Thomson,[7)] Weinberg,[8)] Quadir and Sharif,[9)] and Møller[10)] and the teleparallel gravity analogs of Einstein, Landau and Lifshitz, Bergman and Thomson[11)] and Møller.[12)] Except for the Møller energy-momentum complex, all energy-momentum formulations are coordinate dependent. Therefore, if the calculations are carried out in Cartesian coordinates, these complexes can give a reasonable and meaningful result despite the coordinate dependence. To treat this problematic issue, a quasi-local approach has been used for a long time.

In the literature, the energy-momentum localization problem was reviewed by Virbhadra and his collaborators, using various energy-momentum prescriptions. They

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have found that several energy-momentum complexes give the same reasonable results for a given space-time.[13)–16)] Virbhadra[16)] showed that the complexes of Einstein, Landau and Lifshitz, Papapetrou and Weinberg give the same energy density as the Penrose energy-momentum complex in a general non-static spherically symmetric metric of the Kerr-Schild class. Subsequently, Xulu,[17)] Radinschi[18)] and Saltı and Havare[19)] considered the Bergmann-Thomson energy and/or momentum formulation for different space-time models and showed that this prescription is consistent with the other energy-momentum complexes. Xulu carried out the calculations using the Kerr-Schild Cartesian coordinates, and the Bergmann-Thomson definition provides for a given metric the same energy expression for the energy-momentum distributions as the Einstein, Landau and Lifshitz, Papapetrou and Weinberg energy-momentum definitions.

Albrow[20)] and Tryon[21)] assumed that the total energy of the universe can be equal to zero. The energy of a closed homogeneous isotropic universe described by the Friedman-Robertson-Walker (FRW) metric was computed by Rosen.[22)] He found that the total energy is zero everywhere. Johri et al.[23)] calculated the total energy of a FRW spatially closed universe and found it to be zero at all times, using the Landau-Lifshitz energy-momentum complex. Furthermore, they showed that the total energy enclosed within any finite volume of a spatially flat FRW universe is zero at all times. Banerjee and Sen,[24)] using the Einstein energy-momentum complex, found that the total energy density of Bianchi type-I universes is zero everywhere. Using the definitions of Landau and Lifshitz, Papapetrou and Weinberg, Xulu[25)] calculated the energy of the universe in the case of the Bianchi type-I model and found it to be zero. The energy distribution of a Bianchi type-VI0 universe, with the energy-momentum complexes of Tolman, Bergman and Thomson and Møller, was computed by Radinschi.[26)] She found that the energy is equal to zero. Vargas,[11)] using the definitions of Einstein and Landau-Lifshitz complexes in teleparallel gravity, found that the total energy is zero in closed FRW space-times. The result obtained by Vargas is consistent with the work of Rosen and Johri et al. Following that interesting work, Saltı et al.[19),27),28)] considered several energy-momentum definitions in both general relativity and teleparallel gravity for several diagonal and expanding space-time models, and they found that the total energy is zero at all times for a given closed space-time. Finally, the present authors,[29)] using the Møller definition of the energy-momentum in teleparallel gravity for the Bianchi type-I metric, found the total energy to be zero everywhere.

The aim of this paper is to determine the total energy of the universe on the basis of the LRS Bianchi type-II models, using the energy-momentum complex of Bergman and Thomson in Einstein's theory of general relativity and the teleparallel theory of gravity and show that this definition is consistent with the Einstein and Landau-Lifshitz formulations for a non-diagonal and expanding space-time. In the next section, we introduce the LRS Bianchi type-II space-time model and carry out some necessary calculations for this model. In §3, we give the Bergman-Thomson energy momentum complex in both general relativity and teleparallel gravity and then compute the energy-momentum density. The last section is devoted to discussion and a conclusion. Throughout this paper, Latin indices $(i, j, k, \cdots)$ denote the vector number and Greek indices $(\mu, \nu, \lambda, \cdots)$ represent the vector components. All indices run from 0 to 3 and we use units in which $G = 1$, and $c = 1$ units.

§2. LRS Bianchi type-II cosmological models

The FRW models have played a significant role in cosmology. Whether or not these models correctly represent the universe known, but it is believed that they provide good global approximations of the present universe. These models are characterized by spatial homogeneity and isotropy. In recent decades, theoretical interest in anisotropic cosmological models has increased. In modern cosmology, the spatially homogeneous and anisotropic Bianchi models, which are in some sense between FRW models and completely inhomogeneous and anisotropic universes, play an important role. Here, we consider the LRS model of Bianchi type-II. The metric for Bianchi type-II in the LRS case is given by[30)]

$$ds^2 = dt^2 - D(t)^2 dx^2 - H(t)^2 dy^2 - [D(t)^2 + x^2 H(t)^2] dz^2 - 2xH(t)^2 dydz, \quad (2\cdot1)$$

where $D(t)$ and $H(t)$ are expansion factors, can be obtained with Einstein's field equations. The non-vanishing components of the Einstein tensor $G_{\mu\nu}$ ($\equiv 8\pi T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor for the matter field described by a perfect fluid of density ρ and pressure p) are

$$G_{11} = D\ddot{D} + D^2 \frac{\ddot{H}}{H} + \frac{\dot{D}}{D}\frac{\dot{H}}{H} + \frac{H^2}{4D^2}, \quad (2\cdot2)$$

$$G_{22} = 2H^2 \frac{\ddot{D}}{D} + H^2 \frac{\dot{D}^2}{D^2} - \frac{3H^4}{4D^4}, \quad (2\cdot3)$$

$$G_{33} = (2x^2H^2 + D^2)\frac{\ddot{D}}{D} + D^2 \frac{\ddot{H}}{H} + x^2 H^2 \frac{\dot{D}^2}{D^2} + \frac{D}{H}\dot{D}\dot{H} + \frac{H^2}{4D^2} - x^2 \frac{3H^2}{4D^4}, \quad (2\cdot4)$$

$$G_{00} = \frac{\dot{D}^2}{D^2} - 2\frac{\dot{D}}{D}\frac{\dot{H}}{H} + \frac{H^2}{4D^4}, \quad (2\cdot5)$$

$$G_{23} = H^2 \left(2\frac{\ddot{D}}{D} + \frac{\dot{D}^2}{D^2} - \frac{3H^2}{4D^4} \right), \quad (2\cdot6)$$

where the dot represents differentiation with respect to time. For the line element (2·1), the metric tensor is written

$$g_{\mu\nu} = \delta^0_\mu \delta^0_\nu - D^2 \delta^1_\mu \delta^1_\nu - H^2 \delta^2_\mu \delta^2_\nu - (D^2 + x^2 H^2)\delta^3_\mu \delta^3_\nu - xH^2(\delta^3_\mu \delta^2_\nu + \delta^2_\mu \delta^3_\nu), \quad (2\cdot7)$$

and its inverse is

$$g^{\mu\nu} = \delta^\mu_0 \delta^\nu_0 - D^{-2}\delta^\mu_1 \delta^\nu_1 - \frac{D^2 + x^2 H^2}{H^2 D^2}\delta^\mu_2 \delta^\nu_2 - D^{-2}\delta^\mu_3 \delta^\nu_3 + xD^{-2}(\delta^\mu_3 \delta^\nu_2 + \delta^\mu_2 \delta^\nu_3). \quad (2\cdot8)$$

The Riemannian metric takes the form

$$g_{\mu\nu} = \eta_{ij} h^i_\mu h^j_\nu. \quad (2\cdot9)$$

The non-trivial tetrad field induces a teleparallel structure on space-time which is directly related to the presence of the gravitational field, and the above Riemannian metric results. Using this relation, we obtain the tetrad components

$$h^i_\mu = \delta^i_0\delta^0_\mu + D\delta^i_1\delta^1_\mu + H\delta^i_2\delta^2_\mu + D\delta^i_3\delta^3_\mu + xH\delta^i_2\delta^3_\mu, \tag{2·10}$$

and their inverses are given by

$$h_i^\mu = \delta^0_i\delta^\mu_0 + D^{-1}\delta^1_i\delta^\mu_1 + H^{-1}\delta^2_i\delta^\mu_2 + D^{-1}\delta^3_i\delta^\mu_3 - \frac{x}{D}\delta^3_i\delta^\mu_2. \tag{2·11}$$

§3. Energy distribution in the LRS Bianchi Type-II space-time

In this part of the paper, we compute energy-momentum distributions associated with the LRS Bianchi type-II universe models, using the Bergmann-Thomson definition in general relativity and its teleparallel gravity analog.

3.1. *Energy in general relativity*

The Bergmann-Thomson[7)] energy-momentum complex in general relativity is given by

$$\varXi^{\mu\nu} = \frac{1}{16\pi}\varPi^{\mu\nu\beta}_{\ \ \ ,\beta}, \tag{3·1}$$

where

$$\varPi^{\mu\nu\beta} = g^{\mu\alpha}V_\alpha^{\nu\beta}, \tag{3·2}$$

with

$$V_\mu^{\nu\beta} = \frac{g_{\mu\alpha}}{\sqrt{-g}}\left[-g(g^{\nu\alpha}g^{\beta\sigma} - g^{\beta\alpha}g^{\nu\sigma})\right]_{,\sigma}. \tag{3·3}$$

Here, $\varXi^{00}$ is the energy density, $\varXi^{\alpha 0}$ are the momentum density components, and $\varXi^{0\alpha}$ are the components of the energy current density. The Bergmann-Thomson energy-momentum density satisfies the local conservation laws

$$\partial_\nu \varXi^{\mu\nu} = 0. \tag{3·4}$$

The energy and momentum components are given by

$$P^\nu = \int\int\int \varXi^{\nu 0} dxdydz. \tag{3·5}$$

Further, Gauss's theorem provides the relation

$$P^\nu = \frac{1}{16\pi}\int\int \varPi^{\nu 0\alpha}\mu_\alpha dS. \tag{3·6}$$

Here, μ_α represents the 3 components of the unit vector normal to an infinitesimal surface element dS. The quantities P^i for $i = 1, 2, 3$ are the momentum components, while P^0 is the energy.

From Eq. (3·2) with Eqs. (3·3), (2·7) and (2·8), we find that the $\varPi^{\mu\nu\alpha}$ component that must be non-vanishing is

$$\varPi^{101} = \frac{2}{D}(H\dot{D} + D\dot{H}). \tag{3·7}$$

Using the above result, we find

$$\Xi^{00} = \Xi^{\mu 0} = 0. \tag{3·8}$$

From this result, we easily see that the Bergmann-Thomson energy in the LRS Bianchi type II space-time is

$$E_{GR} = 0. \tag{3·9}$$

3.2. *Energy in teleparallel gravity*

Teleparallel gravity (the tetrad theory of gravitation), which corresponds to a gauge theory for the translation group based on the Weitzenböck geometry,[31] is an alternative approach to Einstein gravitation.[32] In this theory, gravitation is attributed to torsion,[33] which plays the role of a force,[34] whereas the curvature tensor vanishes identically. The fundamental field is a nontrivial tetrad field, which gives rise to the metric as a by-product. The last translational gauge potentials appear as the nontrivial part of the tetrad field, and thus they induce on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting point of teleparallel gravity is that it can reveal a more appropriate approach to considering the same specific problem due to gauge structure. This is the case, for example, for the energy-momentum problem, which becomes more transparent when considered from the teleparallel point of view.

The energy-momentum complex of Bergmann and Thomson in teleparallel gravity[11] is defined as

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_\xi (g^{\mu\kappa} U_\kappa{}^{\nu\xi}), \tag{3·10}$$

where $h = \det(h^i{}_\mu)$, and $U_\kappa{}^{\nu\xi}$ is the Freud super-potential, which is given by

$$U_\kappa{}^{\nu\xi} = hS_\kappa{}^{\nu\xi}. \tag{3·11}$$

Here, $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = k_1 T^{\mu\nu\lambda} + \frac{k_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{k_3}{2}(g^{\mu\lambda}T^{\beta\nu}{}_\beta - g^{\nu\mu}T^{\beta\lambda}{}_\beta), \tag{3·12}$$

with k_1, k_2 and k_3 the three dimensionless coupling constants of teleparallel gravity.[33] For the teleparallel equivalent of general relativity, the specific choice of these three constants is

$$k_1 = \frac{1}{4}, \qquad k_2 = \frac{1}{2}, \qquad k_3 = -1. \tag{3·13}$$

To calculate this tensor, firstly, we must compute the Weitzenböck connection,

$$\Gamma^\sigma{}_{\zeta\beta} = h_i{}^\sigma \partial_\beta h^i{}_\zeta. \tag{3·14}$$

Next, we obtain the torsion of the Weitzenböck connection,

$$T^\mu{}_{\nu\lambda} = \Gamma^\mu{}_{\lambda\nu} - \Gamma^\mu{}_{\nu\lambda}. \tag{3·15}$$

For the Bergmann-Thomson definition, the energy and momentum components are given by

$$P^\mu = \int_\Sigma hB^{\mu 0} dxdydz, \tag{3·16}$$

where P^i for $i = 1, 2, 3$ are the momentum components, while P^0 is the energy. The integration hypersurface Σ is characterized by $x^0 = t =$constant.

From Eq. (3·14), the non-vanishing Weitzenböck connection components are obtained as

$$\Gamma^1{}_{10} = \Gamma^3{}_{30} = \frac{\dot{D}}{D}, \qquad \Gamma^2{}_{20} = \frac{\dot{H}}{H},$$
$$\Gamma^2{}_{30} = \frac{x}{DH}(D\dot{H} - H\dot{D}), \qquad \Gamma^2{}_{31} = 1. \tag{3·17}$$

The corresponding non-vanishing torsion components are obtained as

$$T^1{}_{01} = -T^1{}_{10} = T^3{}_{03} = -T^3{}_{30} = \frac{\dot{D}}{D}, \qquad T^2{}_{02} = -T^2{}_{20} = \frac{\dot{H}}{H},$$
$$T^2{}_{13} = -T^2{}_{31} = 1, \qquad T^2{}_{03} = -T^2{}_{30} = \frac{x}{DH}(D\dot{H} - H\dot{D}). \tag{3·18}$$

Using these results with Eq. (3·12), the non-vanishing components of the tensor $S^{\mu\nu\lambda}$ are obtained as

$$S^{023} = -\frac{x}{4D^2}\left(\frac{\dot{H}}{H} - \frac{\dot{D}}{D}\right), \qquad S^{101} = -\frac{x}{2D^2}\left(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}\right), \qquad S^{123} = \frac{1}{4D^4}, \tag{3·19}$$

$$S^{202} = \frac{1}{H^2 D} + \frac{x^2 \dot{R}}{D^3}, \qquad S^{203} = \frac{3x}{4D^2}\left(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}\right), \qquad S^{213} = \frac{1}{4D^4}, \tag{3·20}$$

$$S^{303} = \frac{1}{2D^2}\left(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}\right), \qquad S^{302} = -\frac{-x}{2D^2}\left(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}\right), \qquad S^{312} = \frac{1}{4D^4}. \tag{3·21}$$

From Eq. (3·11), the required components of Freud's super-potential are calculated as

$$U_0{}^{01} = 0, \qquad U_1{}^{01} = H^2 D^2 \left(\frac{\dot{D}}{D} + \frac{\dot{H}}{H}\right). \tag{3·22}$$

These results, together with Eq. (3·10), immediately imply that the total energy associated with LRS Bianchi-type II space-time in teleparallel gravity is

$$E_{TP} = 0. \tag{3·23}$$

§4. Summary and discussion

The subject of energy-momentum localization has generated great deal of interest in both general relativity and teleparallel gravity, although it has been the focus of some debate. Recently, a large number of studies have investigated the energy density of the universe in various models. Rosen calculated the total energy of a FRW metric and found it to be zero, using Einstein's energy-momentum definitions. The total energy of the same universe is obtained by Johri et al. with the Landau-Lifshitz energy-momentum complex. They found that it is zero at all times.

Moreover, they showed that the total energy enclosed within any finite volume of a spatially flat FRW universe is vanishing. Banerjee and Sen, who considered Bianchi type-I space-times, showed that the energy-momentum density is zero everywhere, with the energy-momentum definition of Einstein. Recently, Vargas and Saltı et al. considered the same problems also in teleparallel gravity, and they found the same results.

In this paper, we used the LRS Bianchi type-II metric and calculated the energy-momentum density for this space-time model with the Bergmann-Thomson energy-momentum definitions in general relativity and its teleparallel gravity version. We found that these two gravitational theories give the same result for the total energy:

$$E_{GR} = E_{TP} = 0. \tag{4·1}$$

Although Einstein's energy-momentum tensor has non-vanishing components, the total energy for the LRS Bianchi type-II space-time is zero. This is because the energy-momentum contributions from the matter and field inside two arbitrary surfaces, (in the case of the anisotropic model based on the LRS Bianchi type-II metric) cancel each other.

In a previous work, Aydogdu[35] solved the same problem by using the energy-momentum definitions of Einstein and Landau and Lifshitz both in general relativity and teleparallel gravity. It is found that the results of these papers are consistent. A work of Virbhadra and Radinschi shows that the energy-momentum definitions of the Einstein, Landau-Lifshitz, Papapetrou, Weinberg, Penrose and Bergmann-Thomson complexes give the same energy expression in general relativity. That paper demonstrates the important point that these energy momentum definitions are identical not only in general relativity but also in teleparallel gravity and supports the results of the present paper.

We end by noting the following points. (a) The result of this paper extends the previous works of Cooperstock and Israelit,[36] Rosen, Johri et al., Banerjee and Sen, Vargas, Saltı et al. and Aydogdu. We performed calculations for a non-diagonal expanding space-time to find out whether the Bergmann-Thomson energy momentum prescription is consistent with the other formulations. We previously considered diagonal and expanding space-time models. (b) Our result supports the viewpoints of Albrow and Tryon. (c) Finally, the result that the total energy density is vanishing everywhere elucidates the importance of the energy-momentum definitions.

Appendix
—— *Kinematical Quantities* ——

The idea of the global rotation of the universe has been considered in general relativistic calculations since Gamov[37] and Gödel's[38] works. For the LRS Bianchi type-II metric, one can introduce the tetrad basis as follows:

$$\Theta^0 = dt, \qquad \Theta^1 = Ddx, \qquad \Theta^2 = Hdy + xHdz, \qquad \Theta^3 = Ddz. \tag{A·1}$$

Using the co-moving tetrad formalism, the kinematic variables of this model can be expressed solely in terms of the structure coefficients of the tetrad basis, defined

Table I. List of the kinematic quantities with their formulas.

Four-acceleration vector	$a_\gamma = \varDelta^0_{\gamma 0}$
Vorticity tensor	$w_{\gamma\beta} = \frac{1}{2}\varDelta^0_{\gamma\beta}$
Expansion tensor	$\chi_{\alpha\beta} = \frac{1}{2}(\varDelta_{\alpha 0\beta} + \varDelta_{\beta 0\alpha})$
Expansion scalar	$\chi = \varDelta^1_{01} + \varDelta^2_{02} + \varDelta^3_{03}$
Vorticity vector	$w^1 = \frac{1}{2}\varDelta^0_{23}, \quad w^2 = \frac{1}{2}\varDelta^0_{31}, \quad w^3 = \frac{1}{2}\varDelta^0_{12}$
Vorticity scalar	$w^2 = \frac{1}{4}[(\varDelta^0_{23})^2 + (\varDelta^0_{31})^2 + (\varDelta^0_{12})^2]$
Shear tensor	$\sigma_{\mu\nu} = \chi_{\mu\nu} - \frac{1}{3}\chi\delta_{\mu\nu}$

as

$$d\Theta^\xi = \frac{1}{2}\varXi^\xi_{\mu\nu}\Theta^\mu \wedge \Theta^\nu. \tag{A·2}$$

We calculated the following kinematic quantities for the line element (2·1), taking the exterior derivatives of the tetrad basis and using the kinematic[39)] formulas given in Table I.

$$a_\gamma = w^i = w_{\gamma\beta} = w = 0, \tag{A·3}$$

$$\chi_{11} = \frac{\dot{D}}{D}, \qquad \chi_{22} = \frac{\dot{H}}{H}, \qquad \chi_{33} = \frac{\dot{D}}{D}, \qquad \chi = 2\frac{\dot{D}}{D} + \frac{\dot{H}}{H}, \tag{A·4}$$

$$\sigma_{11} = \frac{1}{3}\left(\frac{\dot{D}}{D} - \frac{\dot{H}}{H}\right), \qquad \sigma_{22} = \frac{2}{3}\left(\frac{\dot{H}}{H} - \frac{\dot{D}}{D}\right), \qquad \sigma_{33} = \frac{1}{3}\left(\frac{\dot{D}}{D} - \frac{\dot{H}}{H}\right). \tag{A·5}$$

From the above result, we see that the model given in Eq. (2·1) has non-vanishing shear expansion. This is a cosmological model with vanishing vorticity and four-acceleration. We also note that the LRS Bianchi type-II universe has vanishing shear expansion when we set $D = H$.

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