Supersymmetric solutions of $\mathcal{N} = (1,1)$ general massive supergravity

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We construct supersymmetric solutions of three-dimensional $\mathcal{N} = (1,1)$ general massive supergravity (GMG). Solutions with a null Killing vector are, in general, pp-waves. We identify those that appear at critical points of the model, some of which do not exist in $\mathcal{N} = (1,1)$ new massive supergravity (NMG). In the timelike case, we find that many solutions are common with NMG, but there is a new class that is genuine to GMG, two members of which are stationary Lifshitz and timelike squashed AdS spacetimes. We also show that in addition to the fully supersymmetric AdS vacuum, there is a second AdS background with a nonzero vector field that preserves $1/4$ supersymmetry.

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I. INTRODUCTION

Trying to understand quantum gravity in three-dimensional rather than four-dimensional spacetime is a technically more manageable problem. A reason for this is that a three-dimensional gravity theory on anti-de Sitter (AdS) space is dual to a two-dimensional conformal field theory (CFT), and such CFTs are much better understood compared to higher-dimensional ones. With this goal in mind, topologically massive gravity (TMG) [1] has been widely studied in recent years (see e.g., [2]), which is obtained by adding the gravitational Chern-Simons term to pure Einstein gravity with or without a cosmological constant. This model is unitary and propagates a single massive mode. Recently, a novel modification of this theory was achieved where a particular four-derivative curvature term was added to the TMG action, after which it remained unitary and there were two massive graviton states. Depending on whether the model contains the gravitational Chern-Simons term or not, it is called general massive gravity (GMG) or new massive gravity (NMG), respectively [3,4].

Since supersymmetry, in general, improves ultraviolet behavior, it is natural to consider supersymmetric extensions of these models, which was carried out in a series of papers. The fact that the isometry group of AdS_3 can be written as $SO(2,2) \simeq SO(2,1) \times SO(2,1)$ makes it possible to have $\mathcal{N} = (p,q)$ supergravities in three dimensions [5] with either on-shell or off-shell formulations. The off-shell $\mathcal{N} = 1$ versions of the TMG [1] and GMG [3,4] were constructed in [6,7] and [8,9], respectively. The off-shell action of $\mathcal{N} = 2$ conformal supergravity was obtained in [10], and the off-shell $\mathcal{N} = (1,1)$ TMG action was given in [11]. The most general off-shell formulations for $\mathcal{N} = 2$ supergravity were developed in [12,13] in superspace, and the corresponding $\mathcal{N} = 2$ TMG actions were given in [14], both in the superspace and the component settings. The bosonic action and supersymmetry transformations of off-shell $\mathcal{N} = 2$ GMG were obtained in [15]. A complete superspace action for off-shell $\mathcal{N} = 2$ GMG was given in [16] using earlier results in [13,14].

Identifying supersymmetric vacua of these theories is an important problem, and in this paper, we will study supersymmetric solutions of $\mathcal{N} = (1,1)$ GMG [15,16]. In [17] a general Killing spinor analysis was performed to classify supersymmetric solutions of $\mathcal{N} = (1,1)$ TMG, and some particular warped AdS solutions were found. A big advantage of working with off-shell supergravities is that such an analysis remains valid for any extension of the model; only field equations change. Later, supersymmetric solutions of $\mathcal{N} = (1,1)$ NMG were examined in [18], and it was found that some additional backgrounds are allowed compared to $\mathcal{N} = (1,1)$ TMG, such as static Lifshitz spacetime. We will show that for $\mathcal{N} = (1,1)$ GMG, even more configurations appear, such as stationary Lifshitz spacetime and a timelike warped AdS with no restriction on the norm of the warping. The latter can be turned into a black hole–like object when it is squashed after a proper identification of a coordinate[19]. Moreover, we find that in

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addition to a maximally supersymmetric AdS vacuum, there exists a $1/4$ supersymmetric AdS vacuum too. As usual, supersymmetric solutions can be grouped as null or timelike with respect to the norm of a Killing vector that is obtained as a Killing spinor bilinear [20]. We summarize our findings for the timelike case in Table I where we also make comparison with $\mathcal{N}=(1,1)$ NMG and TMG models. In all these papers the same ansatz for the auxiliary fields is assumed. For all versions of off-shell $\mathcal{N}=2$ supergravities, the general Killing spinor analysis was given in [14] and all maximally supersymmetric solutions were obtained. The superalgebras corresponding to these backgrounds were provided in [21], which also contains further observations about supersymmetric solutions of these theories.

The organization of our paper is as follows. In the next section we give a brief introduction of the $\mathcal{N}=(1,1)$ GMG model [15,16]. The two subsequent sections constitute our main results where we apply the Killing spinor analysis of [17] to our model. In Sec. III, we analyze a null Killing vector and show that only pp-waves are allowed. We also explicitly give new solutions that appear at critical points of the theory. In Sec. IV, we do an analogous investigation of the timelike case. Our ansatz for the auxiliary fields gives rise to three choices for the vector field components, and one of them leads to a new set of solutions that exists only in GMG and not in NMG or TMG. Additionally, we also find some solutions that were overlooked in earlier works [17,18]. We conclude in Sec. V with some comments and future directions. In Appendix A we verify that the new AdS vacuum with nonzero vector fields is $1/4$ supersymmetric, whereas the other one with vanishing vector fields preserves full supersymmetry. In Appendix B we give a preliminary analysis of the black hole–like geometry that one obtains from our timelike squashed AdS solution (4.23).

II. $\mathcal{N}=(1,1)$ MASSIVE SUPERGRAVITY

We begin with a summary of the $\mathcal{N}=(1,1)$ general massive supergravity which was constructed in [15,16]. The bosonic part of its Lagrangian is given by

$$e^{-1}\mathcal{L}_{\rm GMG}=\sigma(R+2V^2-2|S|^2)+4MA-\frac{\varepsilon^{\mu\nu\rho}}{4\mu}\left[R_{\mu\nu}{}^{ab}\omega_{\rho ab}+\frac{2}{3}\omega_\mu^{ab}\omega_{\nu b}{}^c\omega_{\rho ca}-4F_{\mu\nu}V_\rho\right]+\frac{1}{m^2}\left[R_{\mu\nu}R^{\mu\nu}-\frac{3}{8}R^2-R_{\mu\nu}V^\mu V^\nu\right.$$
$$\left.-F_{\mu\nu}F^{\mu\nu}+\frac{1}{4}R(V^2-B^2)+\frac{1}{6}|S|^2(A^2-4B^2)-\frac{1}{2}V^2(3A^2+4B^2)-2V^\mu B\partial_\mu A\right],\qquad(2.1)$$

where (σ,M,m^2,μ) are arbitrary real constants and the complex scalar field S is defined as $S=A+\mathrm{i}B$. The limit $m^2\to\infty$ corresponds to $\mathcal{N}=(1,1)$ TMG, and the limit $\mu\to\infty$ corresponds to $\mathcal{N}=(1,1)$ NMG models. Their supersymmetric solutions were studied in [17,18], respectively. The model can be truncated to $\mathcal{N}=1$ GMG [8,9] by setting the vector field V and the imaginary part of the scalar field, i.e., B, to zero.

Equations of motion for A, B, V_μ and $g_{\mu\nu}$ fields are given, respectively, as

$$0=4M-4\sigma A+\frac{1}{m^2}\left[\frac{2}{3}A^3-B^2A-3V^2A+2(\nabla\cdot V)B+2V^\mu\partial_\mu B\right],$$
$$0=4\sigma B+\frac{1}{m^2}\left[\frac{1}{2}RB+A^2B+\frac{8}{3}B^3+4V^2B+2V^\mu\partial_\mu A\right],$$
$$0=4\sigma V_\mu-\frac{1}{m^2}\left[2R_{\mu\nu}V^\nu+4\nabla^\nu F_{\mu\nu}+V_\mu\left(3A^2+4B^2-\frac{R}{2}\right)+2B\partial_\mu A\right]+\frac{2}{\mu}\varepsilon_{\mu\nu\rho}F^{\nu\rho},$$
$$0=\sigma\left(R_{\mu\nu}+2V_\mu V_\nu-\frac{1}{2}g_{\mu\nu}[R+2V^2-2(A^2+B^2)]\right)-2g_{\mu\nu}MA+\frac{1}{\mu}C_{\mu\nu}+\frac{1}{m^2}\left[\Box R_{\mu\nu}-\frac{1}{4}\nabla_\mu\nabla_\nu R+\frac{9}{4}RR_{\mu\nu}-4R_\mu^\rho R_{\nu\rho}\right.$$
$$-2F_\mu{}^\rho F_{\nu\rho}+\frac{1}{4}RV_\mu V_\nu-2R_{\rho(\mu}V_{\nu)}V^\rho-\frac{1}{2}\Box(V_\mu V_\nu)+\nabla_\rho\nabla_{(\mu}(V_{\nu)}V^\rho)+\frac{1}{4}R_{\mu\nu}(V^2-B^2)-\frac{1}{4}\nabla_\mu\nabla_\nu(V^2-B^2)$$
$$-\frac{1}{2}V_\mu V_\nu(3A^2+4B^2)-2BV_{(\mu}\partial_{\nu)}A-\frac{1}{2}g_{\mu\nu}\left(\frac{13}{8}R^2+\frac{1}{2}\Box R-3R_{\rho\sigma}^2-R_{\rho\sigma}V^\rho V^\sigma+\nabla_\rho\nabla_\sigma(V^\rho V^\sigma)-F_{\rho\sigma}^2\right.$$
$$\left.\left.+\frac{1}{4}R(V^2-B^2)-\frac{1}{2}\Box(V^2-B^2)+\frac{1}{6}(A^2+B^2)(A^2-4B^2)-\frac{1}{2}V^2(3A^2+4B^2)-2BV^\rho\partial_\rho A\right)\right],\qquad(2.2)$$

where the Cotton tensor is defined as

$$C^{\mu}{}_{\nu} = \varepsilon^{\mu\rho\sigma}\nabla_{\rho}\left(R_{\sigma\nu} - \frac{1}{4}g_{\sigma\nu}R\right). \tag{2.3}$$

Note that the gravitational Chern-Simons term has no contribution to the scalar field equations. The complete field equations of $\mathcal{N} = (1,1)$ GMG were first given in [14] where all maximally supersymmetric solutions were also obtained.

The $\mathcal{N} = (1,1)$ supersymmetry transformations are[1]

$$\begin{aligned}
\delta e_{\mu}{}^{a} &= \frac{1}{2}\bar{\epsilon}\gamma^{a}\psi_{\mu} + \text{H.c.},\\
\delta\psi_{\mu} &= \left(\partial_{\mu} + \frac{1}{4}\omega_{\mu}{}^{ab}\gamma_{ab}\right)\epsilon - \frac{\mathrm{i}}{2}V_{\nu}\gamma^{\nu}\gamma_{\mu}\epsilon - \frac{S}{2}\gamma_{\mu}\epsilon^{*},\\
\delta V_{\mu} &= \frac{\mathrm{i}}{8}\bar{\epsilon}\gamma^{\nu\rho}\gamma_{\mu}(2D_{[\nu}\psi_{\rho]} - \mathrm{i}V_{\sigma}\gamma^{\sigma}\gamma_{\nu}\psi_{\rho} - S\gamma_{\nu}\psi^{*}_{\rho}) + \text{H.c.},\\
\delta S &= -\frac{1}{4}\bar{\epsilon^{*}}\gamma^{\mu\nu}(2D_{[\mu}\psi_{\nu]} - \mathrm{i}V_{\sigma}\gamma^{\sigma}\gamma_{\mu}\psi_{\nu} - S\gamma_{\mu}\psi^{*}_{\nu}),
\end{aligned} \tag{2.4}$$

where ϵ is a complex Dirac spinor. These are off-shell transformations since the supersymmetry algebra closes without imposing the field equations (2.2).

The model has a fully supersymmetric AdS_3 vacuum when

$$A = -\frac{1}{\ell}, \qquad B = 0, \qquad V_{\mu} = 0, \qquad M = A\sigma - \frac{A^3}{6m^2}, \tag{2.5}$$

where the effective cosmological constant is $\Lambda = -1/\ell^2$, that is, $R_{\mu\nu} = 2\Lambda g_{\mu\nu}$. Linearizing the theory around this vacuum, one finds that, generically, the graviton has two massless modes with $\eta = 1$ and $\eta = -1$ and two massive modes with masses η_1 and η_2 given by

$$\eta_1\eta_2 = \frac{1}{\Omega}, \qquad \eta_1 + \eta_2 = \frac{\ell m^2}{\mu\Omega}, \tag{2.6}$$

where $\Omega = \sigma\ell^2 m^2 - \frac{1}{2}$. When mass values are repeated, logarithmic modes appear, and such points of the parameter space are labeled as critical. Assuming that $1/\mu \neq 0$, there are five possibilities [22]:

$$\begin{aligned}
&(\mathrm{i})\,\eta_1 = \eta_2 \neq \pm 1, \qquad (\mathrm{ii})\,\eta_1 = 1, \eta_2 \neq \pm 1,\\
&(\mathrm{iii})\,\eta_1 = -1, \eta_2 \neq \pm 1, \qquad (\mathrm{iv})\,\eta_1 = \eta_2 = 1,\\
&(\mathrm{v})\,\eta_1 = \eta_2 = -1.
\end{aligned} \tag{2.7}$$

The supermultiplet structure of this theory at these critical points, as well as at ordinary points, was studied in [23].

[1] In this paper, we follow the conventions of [18]. In [17], on the other hand, $S = -Z$ and $\sigma = 1$.

Now we would like to find supersymmetric solutions of this model. Since it is off shell, the Killing spinor analysis done in [17] is also valid here, which we summarize in the next two sections. As usual, assuming the existence of at least one Killing spinor, one finds that there is a Killing vector constructed as a spinor bilinear which is either null or timelike.

III. NULL KILLING VECTOR

The Killing spinor analysis of [17] shows that in the null case the vector field should be of the form $V_{\mu} = \partial_{\mu}\theta$ for some arbitrary function $\theta(u,x)$. Hence, the contribution of the gravitational Chern-Simons term to the vector field equation (2.2) vanishes automatically. Furthermore, if S is a real constant one finds that supersymmetry actually requires the vector field to vanish and the Killing spinor becomes a constant spinor [17,18]. Now, we choose

$$A = -1, \qquad B = 0, \qquad V_{\mu} = 0, \tag{3.1}$$

where the AdS radius $|\ell|$ is fixed to 1 in (2.5), which requires $M = \frac{1}{6m^2} - \sigma$, from the scalar field equation. The only remaining field, namely, the metric, has the form [17]

$$ds^2 = dx^2 + 2e^{2x}dudv + Q(x,u)du^2, \tag{3.2}$$

with the null Killing vector in the v direction. This generically describes a pp-wave; however when $Q(x,u) = \text{const}$ or $Q(x,u) = e^{2x}$, it is AdS_3. The metric field equation in (2.2) implies that

$$Q_{xxxx} - \left(4 + \frac{m^2}{\mu}\right)Q_{xxx} + \left(\frac{9}{2} + m^2\sigma + \frac{3m^2}{\mu}\right)Q_{xx} - \left(1 + 2m^2\sigma + \frac{2m^2}{\mu}\right)Q_x = 0. \tag{3.3}$$

When there is no degeneracy, the most general solution of this differential equation is

$$Q(x,u) = c_1(u) + c_2(u)e^{2x} + c_3(u)e^{\lambda_1 x} + c_4(u)e^{\lambda_2 x}, \tag{3.4}$$

where the functions $c_i(u), i = 1, \ldots, 4$, are arbitrary functions of u and

$$\lambda_{1,2} = 1 + \frac{m^2}{2\mu} \pm \sqrt{\left(1 + \frac{m^2}{2\mu}\right)^2 - \left(\frac{1}{2} + \frac{m^2}{\mu} + m^2\sigma\right)}. \tag{3.5}$$

One can show that functions $c_1(u)$ and $c_2(u)$ can be set to zero without loss of generality [18,24].

There are five special cases that must be analyzed separately.[2] They correspond precisely to the critical points that we listed in (2.7) since $\lambda_i = \Omega\eta_i + 1, (i = 1, 2)$. Now solutions at these critical points take the form

$$\text{(i)}\ \lambda_1 = \lambda_2 = 1 + \frac{m^2}{2\mu}, \qquad \mu^2 = \frac{m^4}{4m^2\sigma - 2},$$
$$Q(x,u) = c_1(u) + c_2(u)e^{2x} + [c_3(u) + c_4(u)x]e^{\lambda_1 x},$$
$$\text{(ii)}\ \lambda_1 = 2, \qquad \lambda_2 = \frac{m^2}{\mu} = \frac{1}{2} + m^2\sigma,$$
$$Q(x,u) = c_1(u) + [c_2(u) + c_3(u)x]e^{2x} + c_4(u)e^{\lambda_2 x},$$
$$\text{(iii)}\ \lambda_1 = 0, \lambda_2 = 2 + \frac{m^2}{\mu} = \frac{3}{2} - m^2\sigma,$$
$$Q(x,u) = c_1(u) + c_2(u)e^{2x} + c_3(u)x + c_4(u)e^{\lambda_2 x},$$
$$\text{(iv)}\ \lambda_1 = \lambda_2 = 2, \qquad m^2 = \frac{3}{2\sigma} = 2\mu,$$
$$Q(x,u) = c_1(u) + [c_2(u) + c_3(u)x + c_4(u)x^2]e^{2x},$$
$$\text{(v)}\ \lambda_1 = \lambda_2 = 0, \qquad m^2 = \frac{3}{2\sigma} = -2\mu,$$
$$Q(x,u) = c_1(u) + c_2(u)e^{2x} + c_3(u)x + c_4(u)x^2. \qquad (3.6)$$

Note that in the last two cases we have triple degeneracy, which does not occur in supersymmetric null solutions of $\mathcal{N} = (1,1)$ NMG [18].

In general, these solutions preserve $1/4$ supersymmetry, except the round AdS_3 which preserves full supersymmetry [18].

IV. TIMELIKE KILLING VECTOR

We now discuss the timelike case. We will employ the same ansatz for scalar and vector fields as in [17,18], namely,

$$A = \text{const}, \qquad B = 0, \qquad V_0 = \text{const},$$
$$V_1 = \text{const}, \qquad V_2 = 0, \qquad (4.1)$$

where tangent indices $\{0,1,2\}$ correspond to $\{t,x,y\}$ coordinates, respectively. The metric with a timelike Killing vector in the t-direction has the form [17]

$$ds^2 = -e^{2\varphi(y)}(dt + C(y)dx)^2 + e^{2\lambda(y)}(dx^2 + dy^2), \qquad (4.2)$$

where $\lambda(y)$, $\varphi(y)$ and $C(y)$ are arbitrary functions of y. For supersymmetry, metric functions should satisfy

$$e^{-\lambda}\varphi' = V_1 + A,$$
$$e^{-\lambda}\lambda' = A - V_1,$$
$$e^{-\lambda}C' = 2V_0 e^{\lambda-\varphi}, \qquad (4.3)$$

where a prime indicates differentiation with respect to y. Note that, for this differential equation system, the following choices are special:

$$\text{(i)}\ V_1 = A, \qquad \text{(ii)}\ V_1 = -A,$$
$$\text{(iii)}\ V_0 = 0, \qquad \text{(iv)}\ V_1 = 0, \qquad (4.4)$$

since they reduce the number of independent metric functions from three to two or one. The significance of the $V_1 = -A$ case was overlooked in [18], and hence the corresponding solutions were missed. Also, cases with $A = M = 0$ were not considered in [18,17] since this makes the effective cosmological constant zero. We will allow this option here.

Next, we solve the field equations of the model (2.2) with these constraints. The advantage of our ansatz (4.1) is that it makes scalar and vector field equations algebraic, and once they are satisfied, Einstein equations become automatic. From the two nontrivial vector field equations, one can show that the condition

$$\left(\frac{3V_0}{m^2} - \frac{1}{\mu}\right)(V_1 - A)(V_0^2 - V_1^2) = 0 \qquad (4.5)$$

has to be fulfilled, which puts a strong restriction on the possible supersymmetric solutions. After satisfying this condition, only one of the vector field equations in (2.2) remains free. Now we will follow these three possibilities and solve the constraint equations (4.3) together with the remaining vector field equation for each case, paying particular attention to the special choices (4.4).

Supersymmetric solutions in this section are $1/4$ supersymmetric [17], except for the round AdS_3 with no vector fields (4.18), which is fully supersymmetric as we show in Appendix A.

A. The $V_1 = A$ case

In this case the scalar field equation implies

$$M = A\sigma + \frac{A}{4m^2}\left(\frac{7A^2}{3} - 3V_0^2\right). \qquad (4.6)$$

The vector field equations are automatically satisfied when $V_1 = A = V_0 = 0$, and otherwise we get

$$\sigma = \frac{2V_0}{\mu} + \frac{7}{4m^2}(A^2 - 3V_0^2). \qquad (4.7)$$

There are two special cases for (4.4), namely, $V_0 = 0$ and $V_1 = A = 0$.

[2]We ignore the case $\lambda_1 = 2$ and $\lambda_2 = 0$ since that requires $1/\mu = 0$, which we do not allow.

1. The $V_1 = A \neq 0$ case

In this case, by solving Eq. (4.3) the metric (4.2) takes the form

$$ds^2 = \frac{V^2}{A^2}\left(dx + \frac{V_0 A}{V^2}e^{2Ay}dt\right)^2 - \frac{A^2}{V^2}e^{4Ay}dt^2 + dy^2, \quad (4.8)$$

where $V^2 \equiv -V_0^2 + V_1^2$.

When $V_0 \neq 0$ this corresponds to a warped AdS_3 space with warping parameter

$$\nu^2 = 1 - \frac{V^2}{A^2}. \quad (4.9)$$

Depending on if the norm of the vector field, i.e., V^2, is positive, negative or zero, it describes a spacelike squashed ($0 < \nu^2 < 1$), a timelike stretched ($\nu^2 > 1$) or a null warped AdS_3 spacetime, respectively (see [17,18] for details).

When $V_0 = 0$, note that (4.7) requires $\sigma \geq 0$ and $M = 7A^3/(3m^2)$. The metric (4.8) takes the form

$$ds^2 = -e^{4Ay}dt^2 + dy^2 + dx^2, \quad (4.10)$$

which corresponds to $AdS_2 \times \mathbb{R}$ or $AdS_2 \times S^1$ geometries. Note that the gravitational Chern-Simons term has no effect. Here, if we assume $\sigma \neq 0$, then this solution does not exist in $\mathcal{N} = (1,1)$ TMG.

2. The $V_1 = A = 0$ and $V_0 \neq 0$ case

For this case, $M = 0$ and (4.7) implies $V_0 = \frac{2m^2}{21}\left(\frac{2}{\mu} \pm \sqrt{\frac{4}{\mu^2} - \frac{21\sigma}{m^2}}\right)$ when $V_0 \neq 0$. Solving supersymmetry constraints (4.3) we get

$$ds^2 = -(dt + 2V_0 y dx)^2 + dx^2 + dy^2, \quad (4.11)$$

which represents a timelike warped flat space [25]. Note that if $V_0 = 0$, then (4.7) is no longer valid and the vector field equation is automatically satisfied. In this case (4.11) becomes Minkowski spacetime.

These are still solutions when $1/\mu = 0$ or $1/m^2 = 0$, but they were not considered in [18,17].

B. The $|V_1| = |V_0|, V_1 \neq A$ case

Let $V_0 = -\varepsilon V_1$, where $\varepsilon^2 = 1$. The scalar and remaining vector field equation give

$$M = A\sigma - \frac{A^3}{6m^2}, \quad (4.12)$$

$$0 = V_1\left[\frac{(A^2 + 4AV_1 + 2V_1^2)}{m^2} - \frac{2\varepsilon}{\mu}(A + V_1) + 2\sigma\right]. \quad (4.13)$$

Here the special cases (4.4) to be considered are $V_1 = 0$ and $V_1 = -A$.

1. The $V_1 \neq -A$ case

For this case, solving (4.3) gives the metric

$$ds^2 = -y^{2\alpha}dt^2 + \frac{2\varepsilon}{A - V_1}y^{\alpha - 1}dtdx + \frac{dy^2}{y^2(A - V_1)^2}, \quad (4.14)$$

where $\alpha = (V_1 + A)/(V_1 - A)$. Although, at first sight, $V_1 = A$ looks problematic, in this subsection that is not allowed. For $A \neq 0$, this solution corresponds to an AdS_3 pp-wave when $|V_1| = |V_0| \neq 0$ [17,18]. Supersymmetry requires $\varepsilon = -1$ [17].

When $A = 0$, from (4.12) we have $M = 0$, and

$$V_1 = -\frac{m^2\varepsilon}{2\mu} \pm \sqrt{m^2\sigma - \frac{m^4}{4\mu^2}}, \quad (4.15)$$

which requires $m^2 \leq 4\mu^2\sigma$. The metric (4.14) with $\alpha = 1$ and $V_1 \neq 0$ becomes

$$ds^2 = -y^2 dt^2 - \frac{2\varepsilon}{V_1}dtdx + \frac{dy^2}{V_1^2 y^2}. \quad (4.16)$$

After the coordinate transformations $u = (\ln y)/V_1$ and $z = -\varepsilon x/V_1$, we get

$$ds^2 = -e^{2uV_1}dt^2 + 2dtdz + du^2, \quad (4.17)$$

which describes a pp-wave in flat spacetime in Brinkmann coordinates [25]. This solution also exists in TMG and NMG but was not considered in [17,18].

On the other hand, when $A \neq 0$ but $|V_1| = |V_0| = 0$, the metric (4.14) takes the form

$$ds^2 = \frac{1}{A^2y^2}(-d\tau^2 + dx^2 + dy^2), \quad (4.18)$$

which is the round AdS_3 spacetime with radius $1/|A|$. Here we defined $\tau = At - \varepsilon x$. This is the only solution which is maximally supersymmetric, as we show in Appendix A.

2. The $V_1 = -A$ case

Note that in this case (4.13) and (4.12) imply $M = 2A\sigma/3$ and $A^2 = 2m^2\sigma$, which means $\sigma > 0$. Now, putting $\alpha = 0$ in (4.14) we obtain

$$ds^2 = -\left(dt - \frac{\varepsilon dx}{2Ay}\right)^2 + \frac{1}{4A^2y^2}(dx^2 + dy^2), \quad (4.19)$$

which is the round AdS_3 with radius $1/|A|$ written as a timelike Hopf fibration over a hyperbolic space without any warping [25]. Since AdS is conformally flat, its Cotton tensor vanishes. Therefore, this solution also exists in NMG, which was not noticed in [18]. It is not a solution of TMG.

In Appendix A we show that unlike our previous AdS_3 solution (4.18), this one preserves only 1/4 supersymmetry. This requires $\varepsilon = -1$, that is, $V_0 = V_1$, and the resulting Killing spinor is constant.

C. The $V_0 = \frac{m^2}{3\mu}, |V_0| \neq |V_1|, V_1 \neq A$ case

The main difference between the solutions of $\mathcal{N} = (1,1)$ GMG and NMG [18] shows up in this class since $1/\mu$ appears directly in V_0. Solutions here either do not exist in NMG or they exist as a special case of the more general form that is allowed in GMG. They are not solutions of $\mathcal{N} = (1,1)$ TMG [17].

The scalar and vector field equations become

$$M = A\left(\sigma + \frac{3V_1^2}{4m^2} - \frac{m^2}{12\mu^2} - \frac{A^2}{6m^2}\right), \tag{4.20}$$

$$0 = m^4 + 3\mu^2(5V_1^2 + 4AV_1 - 2A^2) - 12m^2\mu^2\sigma. \tag{4.21}$$

The cases $V_1 = -A$ and $V_1 = 0$ should be considered separately (4.4).

1. The $V_1 \neq 0$ case

For this case, solving (4.3) leads to the metric

$$ds^2 = -y^{2\alpha}dt^2 - \frac{V_0(\alpha+1)y^{\alpha-1}}{V_1^2}dtdx + \frac{V^2(\alpha+1)^2}{4V_1^4}\frac{dx^2}{y^2} + \frac{(\alpha+1)^2}{4V_1^2}\frac{dy^2}{y^2}, \tag{4.22}$$

where $\alpha = (V_1 + A)/(V_1 - A)$. Note that, since $V_1 \neq 0$, we have $\alpha \neq -1$.

First let us assume that $V_1 \neq -A$, which implies that $\alpha \neq 0$. Then, the metric (4.22) remains invariant under the rescalings $y \to \lambda y, x \to \lambda x, t \to \lambda^{-\alpha} t$, where λ is an arbitrary constant. Hence this solution corresponds to a stationary Lifshitz spacetime [26] with dynamical exponent $z = -\alpha$. Such a solution was obtained before for the minimal massive 3D gravity model [27] in [28]. The solution exists even when $A = 0$ with dynamical exponent $z = -1$, although $M = 0$. In the NMG limit, namely, $\mu \to \infty$, we have $V_0 = 0$ and (4.22) becomes static Lifshitz spacetime as in minimal massive 3D gravity [28].

When $V_1 = -A$, i.e., $\alpha = 0$ and $M = A^3/(3m^2)$, the metric (4.22) with $\tau = 2A^2 t/V_0$ becomes

$$ds^2 = \frac{1}{4A^2}\left[-\nu^2\left(d\tau + \frac{dx}{y}\right)^2 + \frac{1}{y^2}(dx^2 + dy^2)\right], \tag{4.23}$$

which is a timelike warped AdS_3 [25] with warping $\nu^2 = V_0^2/A^2$. The form of the metric is similar to our solution given in (4.19); however, here we have $V_0 \neq \pm A$, and therefore it is not round AdS_3. Moreover, unlike our previous timelike warped solution (4.8), there is no restriction on the warping; it can be squashed or stretched. When it is squashed, it becomes a self-dual-type solution [29] after an appropriate identification that is free from closed timelike curves and has a Killing horizon [19], which we study further in Appendix B. Note that in the NMG limit, that is, $1/\mu = 0$, V_0 vanishes and the metric (4.23) becomes $\mathbb{R}_t \times H_2$, where H_2 is a two-dimensional hyperbolic space. This case was overlooked in [18].

2. The $V_1 = 0$ case

In this case $M = -2A^3/(3m^2)$ and $A^2 = m^2(m^2 - 12\mu^2\sigma)/6\mu^2$; therefore, we need to have $m^2 > 12\mu^2\sigma$. Here we also assume $A \neq 0$ (that is, $m^2 \neq 12\mu^2\sigma$) since this was covered in Sec. IV A 2. Solving (4.3) the metric becomes

$$ds^2 = -\frac{1}{y^2}\left[dt + \frac{2m^2}{3\mu A^2}\ln y dx\right]^2 + \frac{1}{A^2 y^2}(dx^2 + dy^2). \tag{4.24}$$

This can be thought of as some "logarithmic" deformation of AdS_3 in Poincaré coordinates. We are not familiar with this metric. Two of its curvature invariants are

$$R = 2(V_0^2 - 3A^2), \qquad R_{\mu\nu}R^{\mu\nu} = 4(3V_0^4 - 4A^2V_0^2 + 3A^4). \tag{4.25}$$

V. DISCUSSION

In this paper we constructed a large number of supersymmetric backgrounds of $\mathcal{N} = (1,1)$ GMG theory [15,16]. Since supersymmetric solutions of $\mathcal{N} = (1,1)$ TMG and NMG were studied earlier [17,18] with the same ansatz for auxiliary fields, the picture is now complete, and one can see consequences of including separate off-shell invariant pieces from our Table I. In [18] it was found that $\mathcal{N} = (1,1)$ NMG allows more solutions in comparison to $\mathcal{N} = (1,1)$ TMG [17], and here we showed that $\mathcal{N} = (1,1)$ GMG is even richer. Since the difference between NMG and GMG is the presence of the gravitational Chern-Simons term in the latter, our findings highlight the effect of this term. In particular, we have seen that the static Lifshitz solution of NMG becomes stationary in GMG. This phenomenon was also observed for minimal massive gravity [27] in [28]. Similarly, comparing solutions of GMG with TMG in Table I, one can see the significance of including higher derivative terms.

Looking at supersymmetric solutions [25] of a closely related model, namely, $\mathcal{N} = (2,0)$ TMG [15,16], one realizes that many are common and almost all of them are homogeneous backgrounds [28]. It is desirable to

TABLE I. Supersymmetric timelike solutions of $\mathcal{N}=(1,1)$ GMG in comparison with $\mathcal{N}=(1,1)$ NMG and $\mathcal{N}=(1,1)$ TMG. For GMG we assume $\sigma\neq 0, 1/\mu\neq 0$ and $1/m^2\neq 0$. When $A=0$, then $M=0$.

Solution	Conditions	GMG	NMG	TMG
Round AdS (max. susy)	$V_1=V_0=0,\ A\neq 0$	(4.18)	✓	✓
Round AdS (1/4 susy)	$V_1=V_0=-A\neq 0$	(4.19)	✓	✗
Spacelike squashed AdS	$V_1=A\neq 0, V_0\neq 0, V^2>0$	(4.8)	✓	✓
Timelike stretched AdS	$V_1=A\neq 0, V_0\neq 0, V^2<0$	(4.8)	✓	✓
Null warped AdS	$V_1=A\neq 0, V^2=0$	(4.8)	✓	✓
$AdS_2\times\mathbb{R}$	$V_1=A\neq 0, V_0=0$	(4.10)	✓	✗
Timelike warped flat	$V_1=A=0, V_0\neq 0$	(4.11)	✓	✓
Minkowski	$V_1=A=V_0=0$	(4.11)	✓	✓
AdS pp-wave	$\lvert V_1\rvert=\lvert V_0\rvert\neq\lvert A\rvert,\ A\neq 0$	(4.14)	✓	✓
Flat space pp-wave	$\lvert V_1\rvert=\lvert V_0\rvert\neq 0,\ A=0$	(4.17)	✓	✓
Stationary Lifshitz	$V_0=\frac{m^2}{3\mu}, V_1\neq -A, V_1\neq 0$	(4.22)	Static	✗
Timelike warped AdS	$V_0=\frac{m^2}{3\mu}, V_1=-A\neq 0, V^2\neq 0$	(4.23)	$\mathbb{R}_t\times H_2$	✗
Deformation of AdS	$V_0=\frac{m^2}{3\mu}, V_1=0, A\neq 0$	(4.24)	✗	✗

understand the connection between supersymmetry and homogeneity better. As far as we know, (4.22) is the first example of a supersymmetric stationary Lifshitz spacetime [26], and its properties should be investigated further. In particular, one may look for some stationary, supersymmetric Lifshitz black holes since it is known that they do not exist in $\mathcal{N}=(1,1)$ NMG [18]. It would also be interesting to study thermodynamics and conserved charges [30] of our timelike warped AdS solution (4.23), which has a Killing horizon as shown in Appendix B. We were able to give geometric identification of all our solutions except (4.24). This is a deformation of AdS_3 that we have not seen before, and it deserves a more detailed examination.

The second AdS vacuum that we found (4.19) is worth investigating further. For example, one may try to obtain a renormalization group flow between the two AdS vacua using an appropriate string solution. For that purpose, constructing matter couplings or extensions with more supersymmetries of this theory might be necessary. Moreover, by making the x-coordinate periodic in (4.19), one obtains a massless, static BTZ black hole [31] with a nonzero charge. To check whether the model admits massive, rotating versions of this, we need a more elaborate investigation, which will require relaxing our ansatz (4.1). Searching supersymmetric flows between warped vacua of the model is also interesting [32].

Finally, connecting these three-dimensional models to higher dimensions as well as to three-dimensional on-shell supergravities is an important task. We hope to come back to these issues in the near future.

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APPENDIX A: SUPERSYMMETRY OF AdS_3 WITH VECTOR FIELDS

In this appendix, we show that our AdS_3 solution with the metric (4.19) and nonzero vector fields $V_1=-A$ and $V_0=-\varepsilon V_1$ preserves 1/4 supersymmetry when $\varepsilon=-1$.

The Killing spinor equation $\delta\psi_\mu=0$ can be written from (2.4) with $S=A$ as

$$d\epsilon+\frac{1}{4}\omega_{ab}\gamma^{ab}\epsilon-\frac{A}{2}\gamma_a e^a\epsilon^*-\kappa\frac{i}{2}(V_a\gamma^a)(\gamma_b e^b)\epsilon=0, \qquad \text{(A1)}$$

where we inserted the constant κ (which is actually 1) to be able to more easily compare with the AdS_3 solution that has no vector fields, (4.18).

We choose the orthonormal frame for the metric (4.19) as

$$e^0=dt-\frac{\varepsilon}{2Ay}dx,\quad e^1=\frac{\varepsilon}{2Ay}dx,\quad e^2=\frac{\varepsilon}{2Ay}dy, \qquad \text{(A2)}$$

whose spin connections are

$$\omega_{01}=\frac{dy}{2y},\quad \omega_{02}=-\frac{dx}{2y},\quad \omega_{12}=-\varepsilon A dt-\frac{dx}{2y}. \qquad \text{(A3)}$$

We take the γ-matrices as

$$\gamma_0=i\sigma_2,\qquad \gamma_1=\sigma_1,\qquad \gamma_2=\sigma_3, \qquad \text{(A4)}$$

where σ_i's are Pauli matrices, and we decompose the complex Dirac spinor ϵ as

$$\epsilon = \begin{pmatrix} \epsilon_1 + i\zeta_1 \\ \epsilon_2 + i\zeta_2 \end{pmatrix}. \tag{A5}$$

With these choices, for $\varepsilon = -1$ we get the following four equations from (A1):

$$\begin{aligned} d\epsilon_1 - A\epsilon_2 dt &= 0, \\ d\zeta_1 - \frac{\zeta_1}{2y}dy &= 0, \\ d\epsilon_2 + A\epsilon_1 dt + \kappa\left(\frac{\zeta_1}{2y}dy - A\zeta_2 dt\right) &= 0, \\ d\zeta_2 + \frac{\zeta_2}{2y}dy - \frac{\zeta_1}{y}dx - \kappa\left(\frac{\epsilon_1}{2y}dy - A\epsilon_2 dt\right) &= 0. \end{aligned} \tag{A6}$$

It is easy to see that for $\kappa = 0$ (that is, pure AdS_3) this differential equation system has the following four linearly independent solutions:

$$\begin{aligned} &\text{(i)}\ \zeta_1 = \zeta_2 = 0, \quad \epsilon_1 = \cos At, \quad \epsilon_2 = -\sin At, \\ &\text{(ii)}\ \zeta_1 = \zeta_2 = 0, \quad \epsilon_1 = \sin At, \quad \epsilon_2 = \cos At, \\ &\text{(iii)}\ \zeta_1 = 0, \quad \zeta_2 = y^{-1/2}, \quad \epsilon_1 = \epsilon_2 = 0, \\ &\text{(iv)}\ \zeta_1 = y^{1/2}, \quad \zeta_2 = xy^{-1/2}, \quad \epsilon_1 = \epsilon_2 = 0. \end{aligned} \tag{A7}$$

So, AdS_3 with no vector fields is fully supersymmetric as it should be. Now, for $\kappa = 1$ (so there is a contribution from the vector field) we see that Killing spinor equations (A6) admit only one solution, which is given as

$$\epsilon_0 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \qquad (\varepsilon = -1). \tag{A8}$$

The constant Killing spinor (A8) satisfies $\mathbb{P}\epsilon_0 = \epsilon_0$ and $\mathbb{P}^*\epsilon_0 = 0$, where $\mathbb{P} = \frac{1}{2}(\mathbb{I}_2 + i\gamma^0)$.

On the other hand, when $\varepsilon = 1$, the Killing spinor equations (A6) still admit four solutions for pure AdS_3 [just interchange $\zeta_i \leftrightarrow \epsilon_i$ in (A8)], as it should be, since ε can be absorbed by the x-coordinate in the metric (4.19). But there are no solutions with $\varepsilon = \kappa = 1$.

In summary, our AdS_3 solution with nonzero vector fields (4.19) preserves 1/4 supersymmetry when $\varepsilon = -1$. This result is in agreement with the Killing spinor analysis done in the appendix of [17], where it was shown that supersymmetry enhancement happens only for AdS_3 with no vector fields turned on.

APPENDIX B: TIMELIKE SQUASHED SELF-DUAL BLACK HOLES

The spacelike (4.8) and timelike squashed solutions (4.23) that we obtained are examples of the so-called self-dual-type solutions [29] after a proper identification. Although they do not have an event horizon or a singularity, they possess a Killing horizon and hence can be interpreted as black hole–like objects with similar thermodynamic properties [19]. The spacelike squashed self-dual solution appears as the near horizon region of the extremal Kerr black hole [33] and has been studied for TMG in [34–36] and for $\mathcal{N} = (1,1)$ extended TMG in [17]. However, so far the timelike version has only appeared in $\mathcal{N} = (2,0)$ TMG as a supersymmetric solution [25]. Here, we initiate its study by looking at its geometry more closely.

The timelike AdS_3 solution that we obtained (4.23) is squashed when $V_0^2 < A^2$, which requires $m^2 > 12\mu^2\sigma$. It is a self-dual-type solution [29] with no closed timelike curves when the x-coordinate is identified such that $x \sim x + 2\pi$. First, note that after the following coordinate changes,

$$\begin{aligned} y &= \frac{e^u}{\cosh\sigma}, \qquad x = e^u \tanh\sigma, \\ \tau &= t' - 2\tan^{-1}\left[\tanh\left(\frac{\sigma}{2}\right)\right], \end{aligned} \tag{B1}$$

it can be mapped into global AdS coordinates

$$ds^2 = \frac{1}{4A^2}[\cosh^2\sigma du^2 + d\sigma^2 - \nu^2(dt' + \sinh\sigma du)^2], \tag{B2}$$

where now the periodic coordinate is u. To see its causal structure more clearly, we now look at Schwarzschild-type coordinates in (4.23) with $\nu^2 < 1$ by defining[3]

$$y = \frac{1}{r - r_h}, \quad \tau = \frac{\tilde{t}}{\nu} - \frac{6(1-\nu)r_h}{\nu(4-\nu^2)}\theta, \quad x = \frac{6}{4-\nu^2}\theta, \tag{B3}$$

where $r_h \geq 0$ is a constant. Now, the metric (4.23) transforms into

$$\begin{aligned} ds^2 = \frac{1}{4A^2}\Bigg[&\frac{[36(r-r_h)^2}{(4-\nu^2)^2}d\theta^2 + \frac{dr^2}{(r-r_h)^2} \\ &- \left(d\tilde{t} + \frac{6(\nu r - r_h)}{4-\nu^2}d\theta\right)^2\Bigg], \end{aligned} \tag{B4}$$

where $\theta \sim \theta + 2\pi$. Note that there is a Killing horizon at $r = r_h$ where the Killing vector $\chi = \frac{6(\nu r - r_h)}{4-\nu^2}\partial_{\tilde{t}} - \partial_\theta$ null. Unlike the spacelike self-dual case [17], here the Killing horizon has Lorentzian signature.

[3]Going to Schwarzschild-type coordinates directly from (4.23) rather than (B2) leads to a much simpler set of transformations compared to those given in [19,17].

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