

A Systematic Analysis of the Exclusive $B \rightarrow K^* \ell^+ \ell^-$ Decay

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Abstract

A model-independent analysis for the exclusive, rare $B \rightarrow K^* \ell^+ \ell^-$ decay is presented. Systematically studied are the experimentally measured quantities, such as, branching ratio, forward-backward asymmetry, longitudinal polarization of the final leptons, and the ratio Γ_L/Γ_T of the decay widths when K^* meson is longitudinally and transversally polarized. The dependence of the asymmetry parameter $\alpha = 2\Gamma_L/\Gamma_T - 1$ on the new Wilson coefficients is also studied in detail. It is found that the afore-mentioned physical observables are quite sensitive to the new Wilson coefficients. Therefore, once we have the experimental data with high statistics and a deviation from the Standard Model, we can interpret the source of such discrepancy.

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1 Introduction

Experimental discovery of the inclusive and exclusive $B \rightarrow X_s \gamma$ and $B \rightarrow K^* \gamma$ decays [1] stimulated the study of the rare B decays in a new manner. These decays take place via flavor-changing neutral current (FCNC) transition of $b \rightarrow s$, which occur only through loops in the Standard Model (SM). For this reason the study of the FCNC decays can provide sensitive test for investigation of the gauge structure of the SM at loop level. At the same time these decays constitute quite a suitable tool in looking for new physics beyond the SM. New physics can appear in rare decays through the Wilson coefficients which can take values distinctly different from their SM counterparts or through the new structure in effective Hamiltonian (see for example Refs. [2]–[11]).

Currently the main interest is focused on the rare meson decays for which the SM predicts “large” branching ratios, and which can be potentially measurable in the near future. The rare $B \rightarrow K^* \ell^+ \ell^-$ ($\ell = e, \mu, \tau$) decays are such decays. For these decays the experimental situation is very promising [12] with $e^+ e^-$ and hadron colliders focusing only on the observation of exclusive modes with $\ell = e, \mu$ and τ as the final states. At the quark level, the decay $B \rightarrow K^* \ell^+ \ell^-$ is described by $b \rightarrow s \ell^+ \ell^-$ transition. The inclusive $b \rightarrow s \ell^+ \ell^-$ transition in framework of the specific extended models were investigated in many papers (see for example [5, 11, 13, 14]). Note that the most general model independent analysis of the $b \rightarrow s \ell^+ \ell^-$ decay, in terms of 10 types of local four-Fermi interactions, was performed in Ref. [9], which has been extended to include two more non-local interactions in Ref. [10]. New physics effects in the exclusive rare decays, $B \rightarrow K^{(*)} \nu \nu$, have been systematically analyzed also in Ref. [15].

It is well known that theoretical analysis of the inclusive decays is easy but their experimental detection is difficult. For exclusive decays the situation is reversed, *i.e.*, these decays can easily be studied in experiments, but theoretically they have drawbacks and predictions are model dependent. This is due to the fact that in calculating the branching ratios and other observables for exclusive decays, we face the problem of computing the matrix element of the effective Hamiltonian responsible for exclusive decays, between initial and final hadron states. This problem is related to the non-perturbative sector of QCD and can be solved only by means of a non-perturbative approach. These matrix elements have been investigated in framework of different approaches such as chiral theory [16], three point QCD sum rules [17], relativistic model by using light-front formalism [18], effective heavy quark theory [19] and light cone QCD sum rules [20, 21].

The present paper is organized as follows: In Section 2 we give the most general form

of the effective Hamiltonian. Then, using this Hamiltonian and the helicity amplitude formalism, we calculate the differential decay width, including the lepton mass effects. In this Section we also present the expressions of the other physical observables, such as forward-backward asymmetry, and the ratio of the decay widths when K^* meson is polarized longitudinally and transversally. Section 3 is devoted to the numerical analysis, and concluding remarks are also in Section 3.

2 Theoretical Background

The matrix element of the $B \rightarrow K^* \ell^+ \ell^-$ decay at the quark level is described by $b \rightarrow s \ell^+ \ell^-$ transition. Following the work [9, 10], we write the matrix element of the $b \rightarrow s \ell^+ \ell^-$ transition as a sum of the SM and new physics contributions,

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{new}} , \quad (1)$$

where \mathcal{M}_{SM} is the SM part and is given by

$$\begin{aligned} \mathcal{M}_{\text{SM}} = & \frac{G\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ (C_9^{eff} - C_{10}) \bar{s}_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \ell_L + (C_9^{eff} + C_{10}) \bar{s}_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_R \right. \\ & \left. - 2C_7^{eff} \bar{s}_i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_s L + m_b R) b \bar{\ell} \gamma^\mu \ell \right\} , \end{aligned} \quad (2)$$

where $R = (1 + \gamma_5)/2$ and $L = (1 - \gamma_5)/2$, and all of the Wilson coefficients are evaluated at the scale $\mu = m_b = 4.8$ GeV.

In Ref. [9], it has been shown that there are ten independent local four-Fermi interactions which may contribute to the process, and the explicit form of \mathcal{M}_{new} can be written as

$$\begin{aligned} \mathcal{M}_{\text{new}} = & \frac{G\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ C_{LL} \bar{s}_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \ell_L + C_{LR} \bar{s}_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_R + C_{RL} \bar{s}_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \ell_L \right. \\ & + C_{RR} \bar{s}_R \gamma_\mu b_R \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR} \bar{s}_L b_R \bar{\ell}_L \ell_R + C_{RLLR} \bar{s}_R b_L \bar{\ell}_L \ell_R + C_{LRRL} \bar{s}_L b_R \bar{\ell}_R \ell_L \\ & \left. + C_{RLRL} \bar{s}_R b_L \bar{\ell}_R \ell_L + C_T \bar{s} \sigma_{\mu\nu} b \bar{\ell} \sigma^{\mu\nu} \ell + i C_{TE} \bar{s} \sigma_{\mu\nu} b \bar{\ell} \sigma_{\alpha\beta} \ell \epsilon^{\mu\nu\alpha\beta} \right\} \end{aligned} \quad (3)$$

It should be noted that in the present analysis we will neglect the tensor type interactions (*i.e.*, terms with coefficients C_T and C_{TE}) since the numerical analysis which is carried in Ref. [9] shows that the physical observables are not sensitive to the presence of the tensor interactions.

From Eq. (1), in order to calculate the decay width for the exclusive $B \rightarrow K^* \ell^+ \ell^-$ decay, the following matrix elements

$$\langle K^* | \bar{s} \gamma_\mu (1 \pm \gamma_5) b | B \rangle ,$$

$$\langle K^* | \bar{i}s\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b | B \rangle, \quad (\text{strange quark mass is neglected})$$

$$\text{and} \quad \langle K^* | \bar{s}(1 \pm \gamma_5)b | B \rangle$$

have to be calculated. These matrix elements can be written in terms of the form factors in the following way

$$\begin{aligned} \langle K^*(p_{K^*}, \varepsilon) | \bar{s}\gamma_\mu(1 \pm \gamma_5)b | B(p_B) \rangle = & \\ & -\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_{K^*}^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \pm i\varepsilon_\mu^*(m_B + m_{K^*})A_1(q^2) \mp i(p_B + p_{K^*})_\mu(\varepsilon^*q) \frac{A_2(q^2)}{m_B + m_{K^*}} \\ & \mp iq_\mu \frac{2m_{K^*}}{q^2}(\varepsilon^*q) [A_3(q^2) - A_0(q^2)] , \end{aligned} \quad (4)$$

$$\begin{aligned} \langle K^*(p_{K^*}, \varepsilon) | \bar{s}i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b | B(p_B) \rangle = & \\ & 4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_{K^*}^\rho q^\sigma T_1(q^2) + 2i \left[\varepsilon_\mu^*(m_B^2 - m_{K^*}^2) - (p_B + p_{K^*})_\mu(\varepsilon^*q) \right] T_2(q^2) \\ & + 2i(\varepsilon^*q) \left[q_\mu - (p_B + p_{K^*})_\mu \frac{q^2}{m_B^2 - m_{K^*}^2} \right] T_3(q^2) , \end{aligned} \quad (5)$$

where ε is the polarization vector of K^* meson, and $q = p_B - p_{K^*}$ is the momentum transfer. In order to ensure finiteness of (4) at $q^2 = 0$, we demand that $A_3(q^2 = 0) = A_0(q^2 = 0)$. For calculation of the matrix element $\langle K^* | \bar{s}(1 \pm \gamma_5)b | B \rangle$, we multiply both sides of Eq. (4) by q_μ and use equation of motion. Neglecting the strange quark mass, we get

$$\begin{aligned} \langle K^*(p_{K^*}, \varepsilon) | \bar{s}(1 \pm \gamma_5)b | B(p_B) \rangle = & \\ & \frac{1}{m_b} \left\{ \mp i(\varepsilon^*q)(m_B + m_{K^*})A_1(q^2) \pm i(m_B - m_{K^*})(\varepsilon^*q)A_2(q^2) \right. \\ & \left. \pm 2im_{K^*}(\varepsilon^*q) [A_3(q^2) - A_0(q^2)] \right\} . \end{aligned} \quad (6)$$

Using the equation of motion, the form factor A_3 can be written as a linear combination of the form factors $A_1(q^2)$ and $A_2(q^2)$ (see Ref. [17])

$$A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}}A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}}A_2(q^2) .$$

Substituting this relation in the matrix element $\langle K^* | \bar{s}(1 \pm \gamma_5)b | B \rangle$, we get

$$\langle K^*(p_{K^*}, \varepsilon) | \bar{s}(1 \pm \gamma_5)b | B(p_B) \rangle = \frac{1}{m_b} \left\{ \mp 2im_{K^*}(\varepsilon^*q)A_0(q^2) \right\} . \quad (7)$$

Finally, for the matrix elements of $B \rightarrow K^*\ell^+\ell^-$ decay we have

$$\begin{aligned} \mathcal{M} = \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^* \left\{ \left[-\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_{K^*}^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} - i\varepsilon_\mu^*(m_B + m_{K^*})A_1(q^2) \right. \right. \\ \left. \left. + i(p_B + p_{K^*})_\mu(\varepsilon^*q) \frac{A_2(q^2)}{m_B + m_{K^*}} + iq_\mu \frac{2m_{K^*}}{q^2}(\varepsilon^*q)(A_3 - A_0) \right] \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left[(C_9^{eff} - C_{10} + C_{LL})\bar{\ell}\gamma_\mu(1 - \gamma_5)\ell + (C_9^{eff} + C_{10} + C_{LR})\bar{\ell}\gamma_\mu(1 + \gamma_5)\ell \right] \\
& - 4\frac{C_7^{eff}}{q^2}m_b \left[4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_{K^*}^\rho q^\sigma T_1(q^2) + 2i(\varepsilon_\mu^*(m_B^2 - m_{K^*}^2) + (p_B + p_{K^*})_\mu(\varepsilon^*q))T_2(q^2) \right. \\
& \quad \left. + 2i(\varepsilon^*q)\left(q_\mu - (p_B + p_{K^*})_\mu\frac{q^2}{m_B^2 - m_{K^*}^2}\right)T_3(q^2) \right] \bar{\ell}\gamma_\mu\ell \tag{8} \\
& + \left[-\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_{K^*}^\rho q^\sigma\frac{2V(q^2)}{m_B + m_{K^*}} + i\varepsilon_\mu^*(m_B + m_{K^*})A_1(q^2) - i(p_B + p_{K^*})_\mu(\varepsilon^*q)\frac{A_2(q^2)}{m_B + m_{K^*}} \right. \\
& \quad \left. - iq_\mu\frac{2m_{K^*}}{q^2}(\varepsilon^*q)(A_3(q^2) - A_0(q^2)) \right] [C_{RL}\bar{\ell}\gamma_\mu(1 - \gamma_5)\ell + C_{RR}\bar{\ell}\gamma_\mu(1 + \gamma_5)\ell] \\
& + \frac{1}{m_b} \left[-2im_{K^*}(\varepsilon^*q)A_0(q^2) \right] [(C_{LRLR} - C_{RLLR})\bar{\ell}(1 + \gamma_5)\ell + (C_{LRRL} - C_{RLRL})\bar{\ell}(1 - \gamma_5)\ell] \Big\} .
\end{aligned}$$

Using the matrix element of $B \rightarrow K^*\ell^+\ell^-$ decay (see Eq. (8)) and the helicity amplitude formalism (for more detail see Refs. [22, 23]) for the differential decay rate width, we get

$$\begin{aligned}
\frac{d\Gamma}{dq^2 dx} &= \frac{G^2\alpha^2}{2^{14}\pi^5 m_B^3} |V_{tb}V_{ts}^*|^2 v\lambda^{1/2}(m_B^2, q^2, m_{K^*}^2) \\
& \times \left\{ |\mathcal{M}_+^{+-}|^2 + |\mathcal{M}_-^{+-}|^2 + |\mathcal{M}_-^{++}|^2 + |\mathcal{M}_+^{++}|^2 + |\mathcal{M}_+^{-+}|^2 + |\mathcal{M}_-^{-+}|^2 + |\mathcal{M}_+^{--}|^2 \right. \\
& \quad \left. + |\mathcal{M}_-^{--}|^2 + |\mathcal{M}_0^{++}|^2 + |\mathcal{M}_0^{+-}|^2 + |\mathcal{M}_0^{-+}|^2 + |\mathcal{M}_0^{--}|^2 \right\} , \tag{9}
\end{aligned}$$

where superscripts denote helicities of the leptons and subscripts correspond to the helicity of the K^* meson. In Eq. (9),

$$\begin{aligned}
\lambda(m_B^2, q^2, m_{K^*}^2) &= m_B^4 + m_{K^*}^4 + q^4 - 2m_B^2q^2 - 2m_B^2m_{K^*}^2 - 2m_{K^*}^2q^2 , \\
q^2 &= (p_B - p_{K^*})^2 , \\
v &= \sqrt{1 - 4m_\ell^2/q^2} , \quad (\text{velocity of the lepton}), \text{ and} \\
x &= \cos\theta , \quad (\theta = \text{angle between } K^* \text{ and } \ell^-).
\end{aligned}$$

The explicit forms of $\mathcal{M}_{\lambda_\nu}^{\lambda_\ell \lambda_\ell}$ are as follows:

$$\mathcal{M}_\pm^{++} = \pm\sqrt{2}m_\ell \sin\theta \left\{ (2C_9^{eff} + C_{LL} + C_{LR})H_\pm + 4C_7^{eff}\frac{m_b}{q^2}\mathcal{H}_\pm + (C_{RR} + C_{RL})h_\pm \right\} , \tag{10}$$

$$\begin{aligned}
\mathcal{M}_\pm^{+-} &= (-1 \pm \cos\theta)\sqrt{\frac{q^2}{2}} \left\{ [2C_9^{eff} + C_{LL} + C_{LR} + v(2C_{10} + C_{LR} - C_{LL})]H_\pm \right. \\
& \quad \left. + 4C_7^{eff}\frac{m_b}{q^2}\mathcal{H}_\pm + [C_{RL} + C_{RR} + v(C_{RR} - C_{RL})]h_\pm \right\} , \tag{11}
\end{aligned}$$

$$\mathcal{M}_\pm^{-+} = (1 \pm \cos\theta)\sqrt{\frac{q^2}{2}} \left\{ [2C_9^{eff} + C_{LL} + C_{LR} + v(-2C_{10} + C_{LL} - C_{LR})]H_\pm \right.$$

$$+4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_\pm + [C_{RL} + C_{RR} + v(C_{RL} - C_{RR})] h_\pm \} , \quad (12)$$

$$\begin{aligned} \mathcal{M}_\pm^{--} &= (\mp \sqrt{2} m_\ell \sin \theta) \left\{ (2C_9^{eff} + C_{LL} + C_{LR}) H_\pm + 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_\pm \right. \\ &\quad \left. + (C_{RL} + C_{RR}) h_\pm \right\} , \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{M}_0^{++} &= 2m_\ell \cos \theta \left\{ (2C_9^{eff} + C_{LL} + C_{LR}) H_0 - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + (C_{RL} + C_{RR}) h_0 \right\} \\ &\quad + 2m_\ell \left\{ (2C_{10} - C_{LL} + C_{LR}) H_S^0 + (-C_{RL} + C_{RR}) h_S^0 \right\} \\ &\quad + \frac{2}{m_b} \sqrt{q^2} \left\{ \left[\sqrt{q^2} (1-v) (C_{LRLR} - C_{RLLR}) - \sqrt{q^2} (1+v) (C_{LRRL} - C_{RLRL}) \right] H_S^0 \right\} , \end{aligned} \quad (14)$$

$$\begin{aligned} \mathcal{M}_0^{+-} &= -\sqrt{q^2} \sin \theta \left\{ [(C_9^{eff} - C_{10} + C_{LL})(1-v) + (C_9^{eff} + C_{10} + C_{LR})(1+v)] H_0 \right. \\ &\quad \left. - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + [C_{RL}(1-v) + C_{RR}(1+v)] h_0 \right\} , \end{aligned} \quad (15)$$

$$\begin{aligned} \mathcal{M}_0^{-+} &= -\sqrt{q^2} \sin \theta \left\{ [(C_9^{eff} - C_{10} + C_{LL})(1+v) + (C_9^{eff} + C_{10} + C_{LR})(1-v)] H_0 \right. \\ &\quad \left. - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + [C_{RL}(1+v) + C_{RR}(1-v)] h_0 \right\} , \end{aligned} \quad (16)$$

$$\begin{aligned} \mathcal{M}_0^{--} &= -2m_\ell \cos \theta \left\{ (2C_9^{eff} + C_{LL} + C_{LR}) H_0 - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + (C_{RL} + C_{LL}) h_0 \right\} \\ &\quad + 2m_\ell \left\{ (2C_{10} - C_{LL} + C_{LR}) H_S^0 + (C_{RR} - C_{RL}) h_S^0 \right\} \\ &\quad + \frac{2}{m_b} \sqrt{q^2} \left\{ \left[\sqrt{q^2} (1+v) (C_{LRLR} - C_{RLLR}) - \sqrt{q^2} (1-v) (C_{LRRL} - C_{RLRL}) \right] H_S^0 \right\} , \end{aligned} \quad (17)$$

where

$$H_\pm = \pm \lambda^{1/2} \frac{V(q^2)}{m_B + m_{K^*}} + (m_B + m_{K^*}) A_1(q^2) , \quad (18)$$

$$H_0 = \frac{1}{2m_{K^*} \sqrt{q^2}} \left[-(m_B^2 - m_{K^*}^2 - q^2) (m_B + m_{K^*}) A_1(q^2) + \lambda \frac{A_2(q^2)}{m_B + m_{K^*}} \right] , \quad (19)$$

$$\begin{aligned} H_S^0 &= \frac{\lambda^{1/2}}{2m_{K^*} \sqrt{q^2}} \left[-(m_B + m_{K^*}) A_1(q^2) + \frac{A_2(q^2)}{m_B + m_{K^*}} (m_B^2 - m_{K^*}^2) \right. \\ &\quad \left. + 2m_{K^*} [A_3(q^2) - A_0(q^2)] \right] , \\ &\equiv \frac{\lambda^{1/2}}{2m_{K^*} \sqrt{q^2}} \left[-2m_{K^*} A_0(q^2) \right] \end{aligned} \quad (20)$$

$$\mathcal{H}_\pm = 2 \left[\pm \lambda^{1/2} T_1(q^2) + (m_B^2 - m_{K^*}^2) T_2(q^2) \right] , \quad (21)$$

$$\begin{aligned} \mathcal{H}_0 &= \frac{1}{m_{K^*} \sqrt{q^2}} \left\{ (m_B^2 - m_{K^*}^2)(m_B^2 - m_{K^*}^2 - q^2) T_2(q^2) \right. \\ &\quad \left. - \lambda \left[T_2(q^2) + \frac{q^2}{m_B^2 - m_{K^*}^2} T_3(q^2) \right] \right\}, \end{aligned} \quad (22)$$

$$h_{\pm} = H_{\pm}(A_1 \rightarrow -A_1, A_2 \rightarrow -A_2), \quad (23)$$

$$h_0 = H_0(A_1 \rightarrow -A_1, A_2 \rightarrow -A_2). \quad (24)$$

In the present paper, we study the dependence of the following measurable physical quantities, such as

(i) Γ_+/Γ_- ,

(ii) $\Gamma_L/\Gamma_T = \Gamma_0/(\Gamma_+ + \Gamma_-)$,

(iii) the polarization parameter $[2\Gamma_0/(\Gamma_+ + \Gamma_-) - 1]$, and

(iv) the lepton forward–backward asymmetry and the longitudinal lepton polarization,

on the different “new” Wilson coefficients. Here the subscripts in the decay width denotes the helicities of the K^* meson. From Eq. (9), we can easily obtain the explicit expressions for Γ_+ , Γ_- and Γ_0 as

$$\begin{aligned} \Gamma_{\pm} &= \frac{G^2 \alpha^2}{2^{14} \pi^5 m_B^3} |V_{tb} V_{ts}^*|^2 \int dq^2 \int dx v \lambda^{1/2} \left\{ |\mathcal{M}_{\pm}^{+-}|^2 + |\mathcal{M}_{\pm}^{++}|^2 \right. \\ &\quad \left. + |\mathcal{M}_{\pm}^{-+}|^2 + |\mathcal{M}_{\pm}^{--}|^2 \right\}, \end{aligned} \quad (25)$$

where the upper(lower) subscript in Γ corresponds to $\mathcal{M}_+(\mathcal{M}_-)$ and

$$\begin{aligned} \Gamma_0 &= \frac{G^2 \alpha^2}{2^{14} \pi^5 m_B^3} |V_{tb} V_{ts}^*|^2 \int dq^2 \int dx v \lambda^{1/2} \left\{ |\mathcal{M}_0^{+-}|^2 + |\mathcal{M}_0^{++}|^2 \right. \\ &\quad \left. + |\mathcal{M}_0^{-+}|^2 + |\mathcal{M}_0^{--}|^2 \right\}. \end{aligned} \quad (26)$$

From Eqs. (25) and (26), the expressions for the ratios Γ_+/Γ_- , $\Gamma_L/\Gamma_T = \Gamma_0/(\Gamma_+ + \Gamma_-)$ and the polarization parameter, which is equal to $\alpha \equiv 2\Gamma_L/\Gamma_T - 1$, can easily be obtained. These quantities are separately measurable from the experiments. In further analysis we will study the dependence of the branching ratio on new Wilson coefficients which are related to the decay width by the relation $\mathcal{BR}(B \rightarrow K^* \ell^+ \ell^-) = \Gamma(B \rightarrow K^* \ell^+ \ell^-) \tau_B$, where τ_B is the life time of the B meson.

The lepton forward–backward asymmetry, A_{FB} , is one of the most useful tools in search of new physics beyond the SM. Especially the determination of the position of the zero value for A_{FB} can predict possibly new physics contributions. Indeed, existence of the new

physics can be confirmed by the shift in the position of the zero value of the forward-backward asymmetry [7]. Therefore, in the present work we analyze with special emphasis the dependence of A_{FB} on the different “new” Wilson coefficients. The lepton forward-backward asymmetry is defined in the following way

$$\frac{d}{dq^2}A_{FB}(q^2) = \frac{\int_0^1 dx \frac{d\Gamma}{dq^2 dx} - \int_{-1}^0 dx \frac{d\Gamma}{dq^2 dx}}{\int_0^1 dx \frac{d\Gamma}{dq^2 dx} + \int_{-1}^0 dx \frac{d\Gamma}{dq^2 dx}}. \quad (27)$$

Another very informative quantity in search of new physics is the final lepton polarization, as shown in Ref. [10]. Here we restrict ourselves only to the study of the longitudinal polarization of the τ -lepton. The expression for longitudinal polarization can be calculated from Eq. (9),

$$P_L =$$

$$\frac{\int_0^1 dx \left\{ \left[|\mathcal{M}_{\pm^+}^-|^2 + |\mathcal{M}_{\pm^-}^-|^2 + |\mathcal{M}_0^+|^2 + |\mathcal{M}_0^-|^2 \right] - \left[|\mathcal{M}_{\pm^+}^+|^2 + |\mathcal{M}_{\pm^-}^+|^2 + |\mathcal{M}_0^{+-}|^2 + |\mathcal{M}_0^{++}|^2 \right] \right\} v\lambda^{1/2}}{\int_0^1 dx \left\{ \left[|\mathcal{M}_{\pm^+}^-|^2 + |\mathcal{M}_{\pm^-}^-|^2 + |\mathcal{M}_{\pm^+}^+|^2 + |\mathcal{M}_{\pm^-}^+|^2 + |\mathcal{M}_0^+|^2 + |\mathcal{M}_0^-|^2 + |\mathcal{M}_0^{+-}|^2 + |\mathcal{M}_0^{++}|^2 \right] \right\} v\lambda^{1/2}}.$$

3 Numerical Analysis and Conclusions

Having the explicit expressions for the physically measurable quantities, in this Section we will study the dependence of these quantities on the new Wilson coefficients in \mathcal{M}_{new} , Eq. (3). The values of the main input parameters, which appear in the expression for the decay widths Γ_0 , Γ_+ , Γ_- , A_{FB} and the polarization parameter α , are:

$$\begin{aligned} m_b &= 4.8 \text{ GeV}, & m_c &= 1.35 \text{ GeV}, & m_\tau &= 1.78 \text{ GeV}, \\ m_\mu &= 0.105 \text{ GeV}, & m_B &= 5.28 \text{ GeV}, & m_{K^*} &= 0.892 \text{ GeV}. \end{aligned}$$

We use the following values for the Wilson coefficients of the SM:

$$C_9^{\text{NDR}} = 4.153, \quad C_{10} = -4.546, \quad C_7 = -0.311,$$

which correspond to the next-to-leading QCD corrections [24, 25]. The renormalization point μ and the top quark mass are set to be

$$\mu = m_b = 4.8 \text{ GeV}, \quad m_t = 175 \text{ GeV}.$$

(We follow Refs. [26]–[30] in taking into account the long–distance effects of the charmonium states). For the form factors, we have used the results of the works [20, 21]. Here we would like to stress that, throughout numerical analysis the central values of the input parameters are used and their theoretical errors, especially the ones related to the form factors, might be sizeable, but are not taken into account in the present work.

Let us first study the change in the differential decay rate when the corresponding Wilson coefficients change. We assume that all new Wilson coefficients C_X are real, *i.e.*, we do not introduce any new physics phase in addition to the one present in the SM. In Figs. 1–3 (Figs. 4–6), we change C_{LL} , C_{LR} , C_{RR} , C_{RL} , C_{LRLR} and C_{LRRL} for the $B \rightarrow K^* \mu^+ \mu^-$ ($B \rightarrow K^* \tau^+ \tau^-$) decays. From these Figures, we can easily see that, far from resonance regions, $d\mathcal{BR}/dq^2$ is more strongly dependent on C_{LL} and also on C_{RL} than on the other C_X 's. This behavior can be explained as follows:

(i) Considering $B \rightarrow K^* \mu^+ \mu^-$ decay, and neglecting the terms proportional to the lepton mass, the terms coming from C_{LL} and C_{RL} are (see Eqs. (10)–(17))

$$\begin{aligned}
|\mathcal{M}_{C_{LL}}|^2 &= (1 \pm \cos \theta)^2 \frac{q^2}{2} \left| 2(C_9^{eff} - C_{10} + C_{LL})H_{\pm} + 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_{\pm} \right|^2 \\
&+ \sin^2 \theta q^2 \left| 2(C_9^{eff} - C_{10} + C_{LL})H_0 - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 \right|^2, \quad (28)
\end{aligned}$$

$$\begin{aligned}
|\mathcal{M}_{C_{RL}}|^2 &= (1 \pm \cos \theta)^2 \frac{q^2}{2} \left| 2(C_9^{eff} - C_{10})H_{\pm} + 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_{\pm} + 2C_{RL}h_{\pm} \right|^2 \\
&+ \sin^2 \theta q^2 \left| 2(C_9^{eff} - C_{10})H_0 - 4C_7^{eff} \frac{m_b}{q^2} \mathcal{H}_0 + 2C_{RL}h_0 \right|^2. \quad (29)
\end{aligned}$$

Far from the resonance region, for example $q^2 \simeq 5 \text{ GeV}^2$, $\text{Re}(C_9^{eff} - C_{10}) \simeq 9.5$ and $\text{Re}(C_9^{eff} + C_{10}) \simeq 0.4$. Therefore, the interference terms between the terms proportional to $(C_9^{eff} - C_{10})$ and C_{LL} (C_{RL}) are large and for this reason the contributions coming from C_{LL} and C_{RL} are large. From these Figures we also see that the contribution of C_{LL} is constructive (destructive) when $C_{LL} = |C_{10}|$ ($C_{LL} = -|C_{10}|$). The situation for C_{RL} is opposite to the previous case, *i.e.*, its contribution is constructive (destructive) when $C_{LL} = -|C_{10}|$ ($C_{LL} = |C_{10}|$).

(ii) For the $B \rightarrow K^* \tau^+ \tau^-$ decay the situation is similar to the $B \rightarrow K^* \mu^+ \mu^-$ transition, but slightly different. Namely, in this case the largest contribution comes from C_{LL} and the contribution of the C_{RL} becomes equal to the contributions that come from C_{RR} , C_{LR} , and *etc.* This situation can be explained by the fact the term $\sim (1 - v^2)$, which is very small for the muon case, gives destructive contribution in the SM.

In Fig. 7, we investigate the dependence of the partially integrated branching ratio \mathcal{BR} on the new Wilson coefficients. The range for the integration is chosen $1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2$ for the $B \rightarrow K^* \mu^+ \mu^-$ decay and $15 \text{ GeV}^2 < q^2 < 20 \text{ GeV}^2$ for $B \rightarrow K^* \tau^+ \tau^-$ channel, in order to avoid the long distance contributions due to the J/ψ and its excitations. For the $B \rightarrow K^* \mu^+ \mu^-$ case, it follows from Fig. 7 that the partially integrated branching ratio \mathcal{BR} depends strongly on C_{LL} and C_{RL} , but for the $B \rightarrow K^* \tau^+ \tau^-$ decay it depends strongly only on C_{LL} , which is consistent with the previous results for $d\mathcal{BR}/dq^2$. Dependence on the other coefficients is rather weak. From these Figures it follows that the contributions of C_{LL} and C_{RL} to \mathcal{BR} are positive for $C_{LL} > 0$ and $C_{RL} < 0$, and negative for $C_{LL} < 0$ and $C_{RL} > 0$.

In Figs. 8–10 (Figs. 11–13) we plot the dependence of the lepton forward–backward asymmetry on the new Wilson coefficients, within the range $-|C_{10}| \leq C_X \leq |C_{10}|$, for the $B \rightarrow K^* \mu^+ \mu^-$ ($B \rightarrow K^* \tau^+ \tau^-$) decay. The experimental bounds on the branching ratio of the $B \rightarrow K^* \mu^+ \mu^-$ and the $B_s \rightarrow \mu^+ \mu^-$ decays [31] suggest that this is the right order of magnitude range for the vector and scalar Wilson coefficients. For the $B \rightarrow K^* \mu^+ \mu^-$ case, it follows from Figs. 8–10 that the lepton forward-backward asymmetry is more sensitive to the C_{LL} , C_{LR} and C_{RL} than to the other C_X 's. We emphasize that when C_{LL} and C_{LR} are positive then the zero point of dA_{FB}/dq^2 is shifted to the right, and when C_{LL} and C_{LR} are negative, it shifts to the left from its corresponding SM value. In other words, the determination of the zero point of the differential asymmetry tells us not only about the existence of new physics, but it also can fix the sign of the new Wilson coefficients. From these Figures, we also see that the lepton forward–backward asymmetry has a weak dependence on the other Wilson coefficients. From Figs. 11–13, we can deduce the following results for the $B \rightarrow K^* \tau^+ \tau^-$ decay:

- (i) Position of the zero value of the dA_{FB}/dq^2 for the $B \rightarrow K^* \tau^+ \tau^-$ decay can be useful for extracting only C_{LR} .
- (ii) The value of the dA_{FB}/dq^2 is very sensitive (excluding the resonance region) to C_{RR} and C_{LRR} . In other words, analyzing the zero point and magnitude of the dA_{FB}/dq^2 allows us in principle to determine different C_X 's.

As we have noted earlier, the experimentally measurable quantities, Γ_+/Γ_- , Γ_L/Γ_T and P_L , can be useful for distinguishing the effects of new physics from the ones of the SM. In Figs. 14–15, we present the dependence of the ratios Γ_+/Γ_- and Γ_L/Γ_T on C_X 's for the $B \rightarrow K^* \mu^+ \mu^-$ and $B \rightarrow K^* \tau^+ \tau^-$ decays, respectively. The main difference compared to the previous analysis is that the values Γ_+/Γ_- and Γ_L/Γ_T are more sensitive to the coefficient C_{RL} . (The result for the SM can be obtained by substituting $C_X = 0$.) From these Figures,

we observe that the ratio Γ_L/Γ_T , when C_{RL} is varied between -4 and 4, changes between 1 and 4.5. Therefore, the measurement of this ratio in experiments can yield unambiguous information about the existence of new physics. In the $B \rightarrow K^*\tau^+\tau^-$ decay, the ratio Γ_+/Γ_- is again more sensitive to the coefficient C_{RL} , while the ratio Γ_L/Γ_T is more sensitive to the coefficients C_{LRLR} and C_{LRRL} .

Finally, in Fig. 16 we present the dependence of the longitudinal polarization P_L of τ on the new coefficients C_X 's. We see that P_L is sensitive to all the coefficients except the coefficient C_{LRLR} . The dependence of P_L on different coefficients is not the same. For example, P_L always increases when C_{RL} and C_{LRRL} change in the region $(-4, 4)$. However, P_L first decreases when C_{LL} , C_{LR} and C_{RR} increase from -4 to 0, and then increases when the coefficients increase from 0 to 4.

To summarize, in the present work the most general model independent analysis of the exclusive $B \rightarrow K^*\ell^+\ell^-$ decay is presented. This exclusive decay is known to be very clean experimentally and will be measured at the present asymmetric B factories and future hadronic B factories, HERA-B, B-TeV and LHC-B. Moreover, the $B \rightarrow K^*\ell^+\ell^-$ decay is very sensitive to the various extensions of the Standard Model. We have studied the $B \rightarrow K^*\ell^+\ell^-$ decay in a model independent manner. The sensitivity to the new coefficients of the differential and partially integrated branching ratios, and forward-backward asymmetries are systematically studied. It is observed that the differential and partially integrated branching ratio for $B \rightarrow K^*\mu^+\mu^-$ decay is more strongly dependent on C_{LL} and C_{RL} than on the other C_X 's. The reason for such a strong dependence can be explained by the large interference between the terms proportional to $(C_9^{eff}-C_{10})$ and C_{LL} (C_{RL}). For $B \rightarrow K^*\tau^+\tau^-$ case, the partially integrated differential branching ratio is most sensitive to C_{LL} . This situation can be explained by the fact that the terms $\sim (1-v^2)$ give destructive contribution and, therefore, the contributions of the terms $\sim C_{RL}$ practically become equal to the contributions from the other coefficients. From an analysis of the position of the zero value of the lepton forward-backward asymmetry we can determine not only the magnitude, but also the sign of the new Wilson coefficients.

The other experimentally measurable quantities, Γ_L/Γ_T and Γ_+/Γ_- , have also been studied. It is found that Γ_+/Γ_- and Γ_L/Γ_T are sensitive to the C_{RL} for the $B \rightarrow K^*\mu^+\mu^-$ decay. On the other hand, for the $B \rightarrow K^*\tau^+\tau^-$ decay, Γ_+/Γ_- is more strongly dependent on C_{RL} as in the $B \rightarrow K^*\mu^+\mu^-$ case, while Γ_L/Γ_T is more sensitive to the coefficients C_{LRLR} or C_{LRRL} . As the final concluding remark, we state that, from the combined analyses of partially integrated differential branching ratio, lepton forward-backward asymmetry and ratios

of Γ_+/Γ_- and Γ_L/Γ_T for the $B \rightarrow K^*\mu^+\mu^-$ and $B \rightarrow K^*\tau^+\tau^-$ decays, we can unequivocally determine the existence of new physics beyond the Standard Model, and in particular we can obtain information about the various new Wilson coefficients.

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Figure captions

Fig. 1. Differential branching ratio, $d\mathcal{BR} / dq^2$ for $B \rightarrow K^* \mu^+ \mu^-$. The thick solid lines indicates standard model case, *i.e.*, $C_X = 0$. The thin solid, dashed, dotted and dot-dashed line correspond to $C_X = -C_{10}, -0.7 \times C_{10}, 0.7 \times C_{10}, C_{10}$ cases, respectively. Here (a) $C_X = C_{LL}$ and (b) $C_X = C_{LR}$.

Fig. 2. Same as Fig. 1. Here (a) $C_X = C_{RR}$ and (b) $C_X = C_{RL}$.

Fig. 3. Same as Fig. 1. Here (a) $C_X = C_{LRLR}$ and (b) $C_X = C_{LRRL}$.

Fig. 4. Differential branching ratio, $d\mathcal{BR} / dq^2$ for $B \rightarrow K^* \tau^+ \tau^-$. The thick solid lines indicates standard model case, *i.e.*, $C_X = 0$. The thin solid, dashed, dotted and dot-dashed line correspond to $C_X = -C_{10}, -0.7 \times C_{10}, 0.7 \times C_{10}, C_{10}$ cases, respectively. Here (a) $C_X = C_{LL}$ and (b) $C_X = C_{LR}$.

Fig. 5. Same as Fig. 4. Here (a) $C_X = C_{RR}$ and (b) $C_X = C_{RL}$.

Fig. 6. Same as Fig. 4. Here (a) $C_X = C_{LRLR}$ and (b) $C_X = C_{LRRL}$.

Fig. 7. The dependence of the partially integrated branching ratio on the new Wilson coefficients. The range for the integration is chosen (a) $1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2$ for the $B \rightarrow K^* \mu^+ \mu^-$ decay and (b) $15 \text{ GeV}^2 < q^2 < 20 \text{ GeV}^2$ for the $B \rightarrow K^* \tau^+ \tau^-$ decay. The thick solid, thin solid, thick dashed, thin dashed, dotted and dot-dashed line correspond to $C_X = C_{LL}, C_{LR}, C_{RL}, C_{RR}, C_{LRLR}$ and C_{LRRL} , respectively.

Fig. 8. Differential forward-backward asymmetry, dA_{FB} / dq^2 for $B \rightarrow K^* \mu^+ \mu^-$. The thick solid lines indicates standard model case, *i.e.*, $C_X = 0$. The thin solid, dashed, dotted and dot-dashed line correspond to $C_X = -C_{10}, -0.7 \times C_{10}, 0.7 \times C_{10}, C_{10}$ cases, respectively. Here (a) $C_X = C_{LL}$ and (b) $C_X = C_{LR}$.

Fig. 9. Same as Fig. 8. Here (a) $C_X = C_{RR}$ and (b) $C_X = C_{RL}$.

Fig. 10. Same as Fig. 8. Here (a) $C_X = C_{LRLR}$ and (b) $C_X = C_{LRRL}$.

Fig. 11. Differential forward-backward asymmetry, dA_{FB} / dq^2 for $B \rightarrow K^* \tau^+ \tau^-$. The thick solid lines indicates standard model case, *i.e.*, $C_X = 0$. The thin solid, dashed, dotted and dot-dashed line correspond to $C_X = -C_{10}, -0.7 \times C_{10}, 0.7 \times C_{10}, C_{10}$ cases, respectively. Here (a) $C_X = C_{LL}$ and (b) $C_X = C_{LR}$.

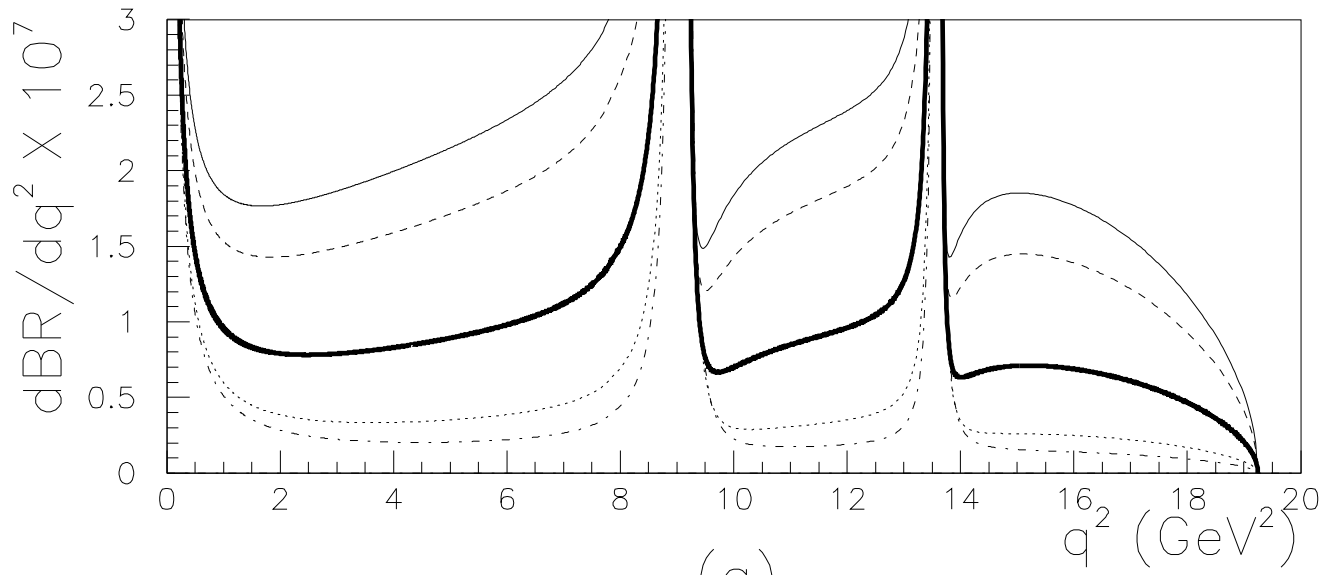
Fig. 12. Same as Fig. 11. Here (a) $C_X = C_{RR}$ and (b) $C_X = C_{RL}$.

Fig. 13. Same as Fig. 11. Here (a) $C_X = C_{LRLR}$ and (b) $C_X = C_{LRRL}$.

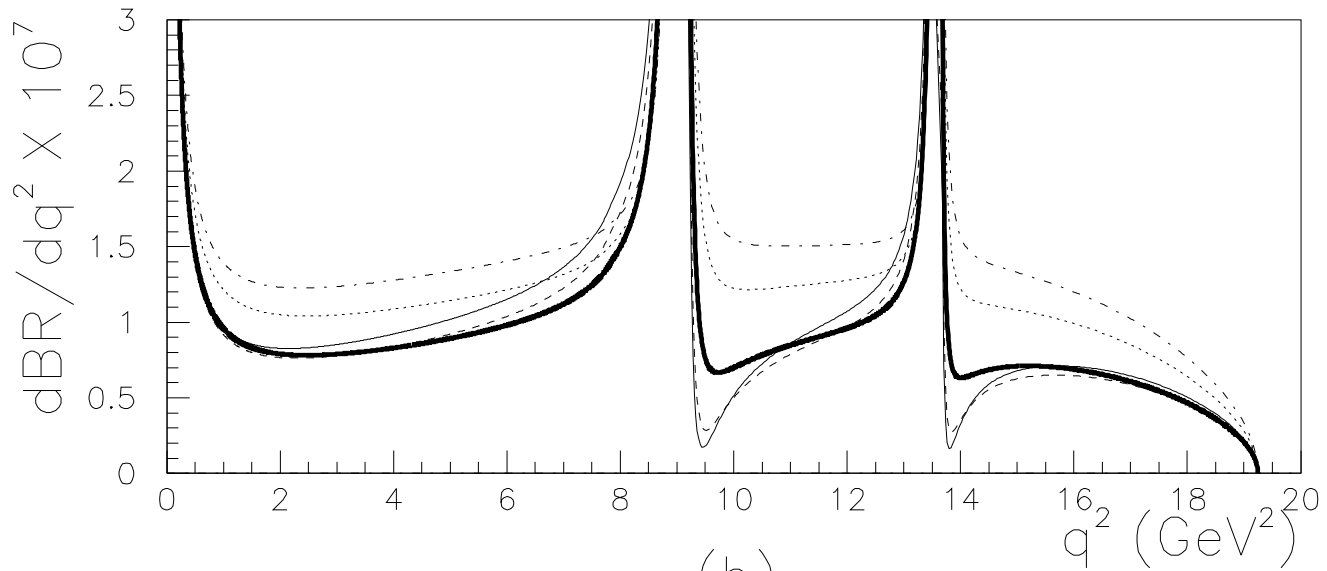
Fig. 14. The dependence of (a) Γ_+ / Γ_- and (b) Γ_L / Γ_T on the new Wilson coefficients for $B \rightarrow K^* \mu^+ \mu^-$ decay. The thick solid, thin solid, thick dashed, thin dashed, dotted and dot-dashed line correspond to $C_X = C_{LL}, C_{LR}, C_{RL}, C_{RR}, C_{LRLR}$ and C_{LRRL} cases.

Fig. 15. The dependence of (a) Γ_+ / Γ_- and (b) Γ_L / Γ_T on the new Wilson coefficients for $B \rightarrow K^* \tau^+ \tau^-$ decay. The thick solid, thin solid, thick dashed, thin dashed, dotted and dot-dashed line correspond to $C_X = C_{LL}, C_{LR}, C_{RL}, C_{RR}, C_{LRLR}$ and C_{LRRL} cases.

Fig. 16. The dependence of τ polarization on the new Wilson coefficients C_X for $B \rightarrow K^* \tau^+ \tau^-$ decay. The thick solid, thin solid, thick dashed, thin dashed, dotted and dot-dashed line correspond to $C_X = C_{LL}, C_{LR}, C_{RL}, C_{RR}, C_{LRLR}$ and C_{LRRL} cases.



(a)



(b)

Figure 1:

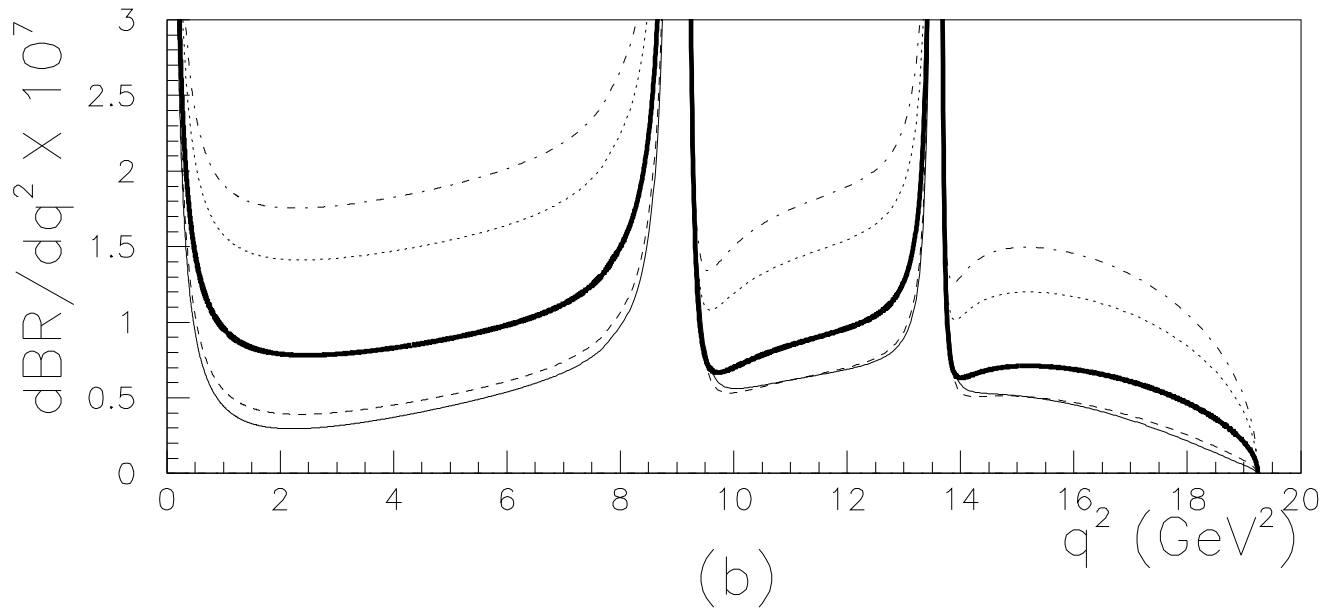
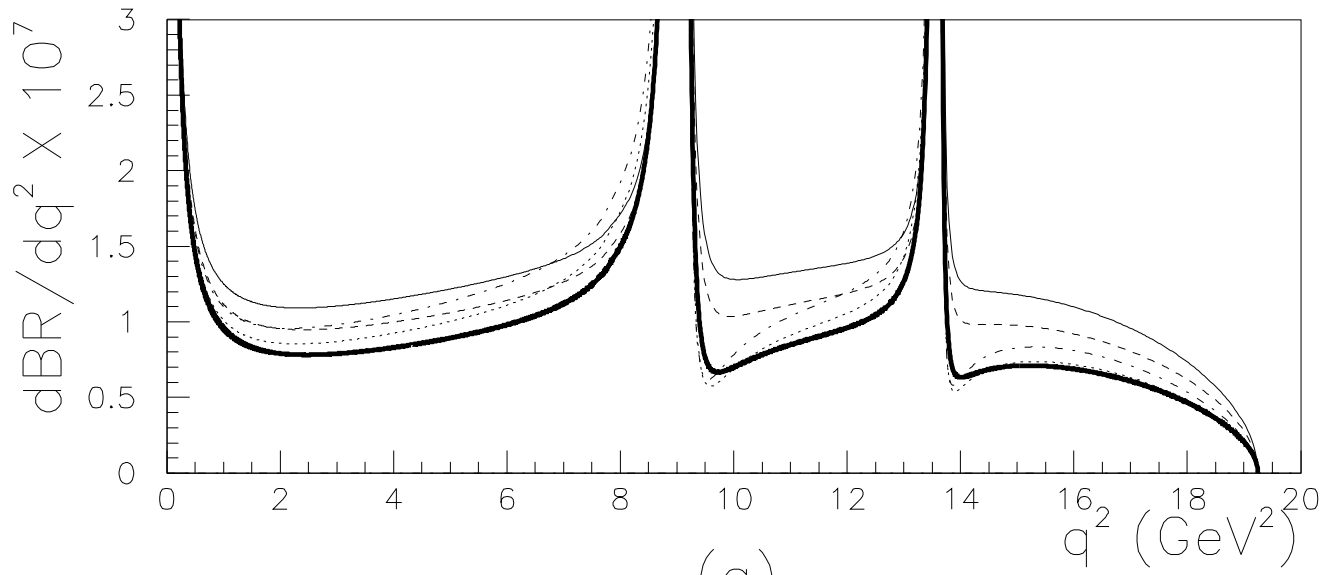


Figure 2:

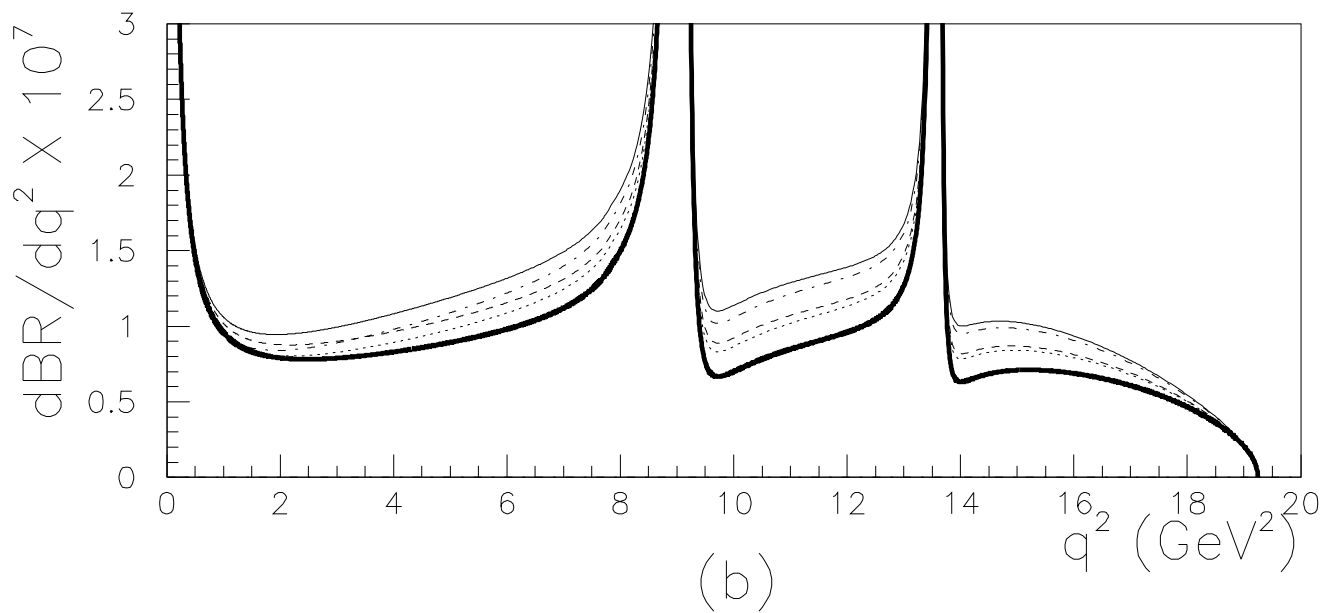
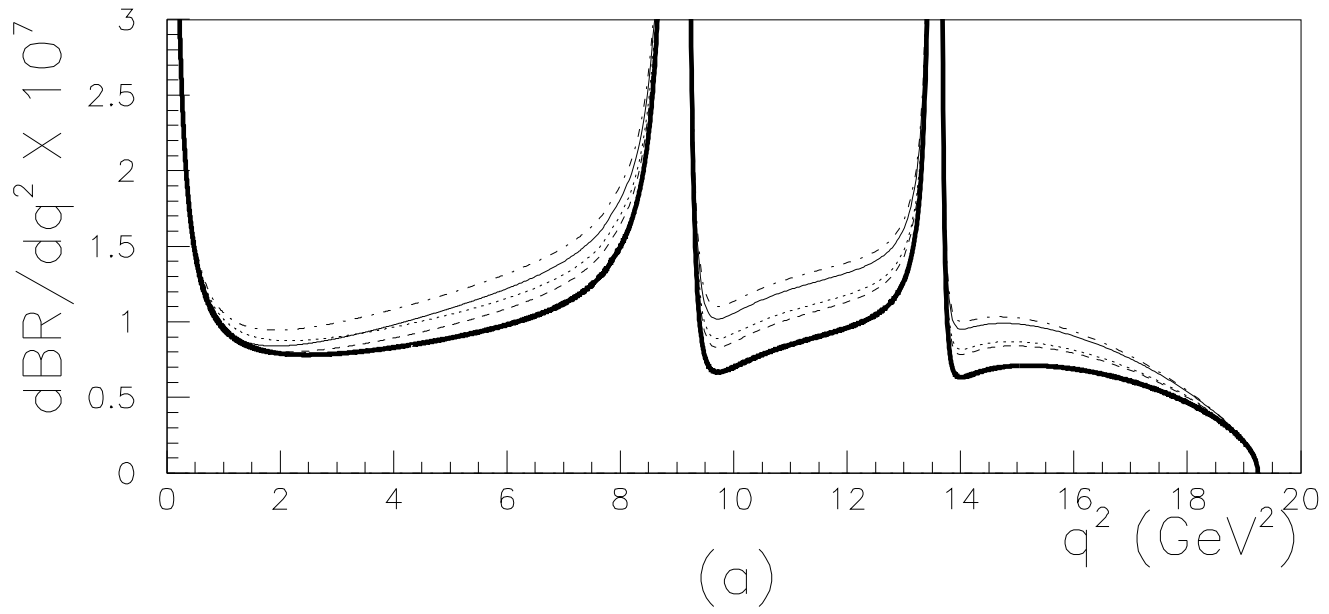
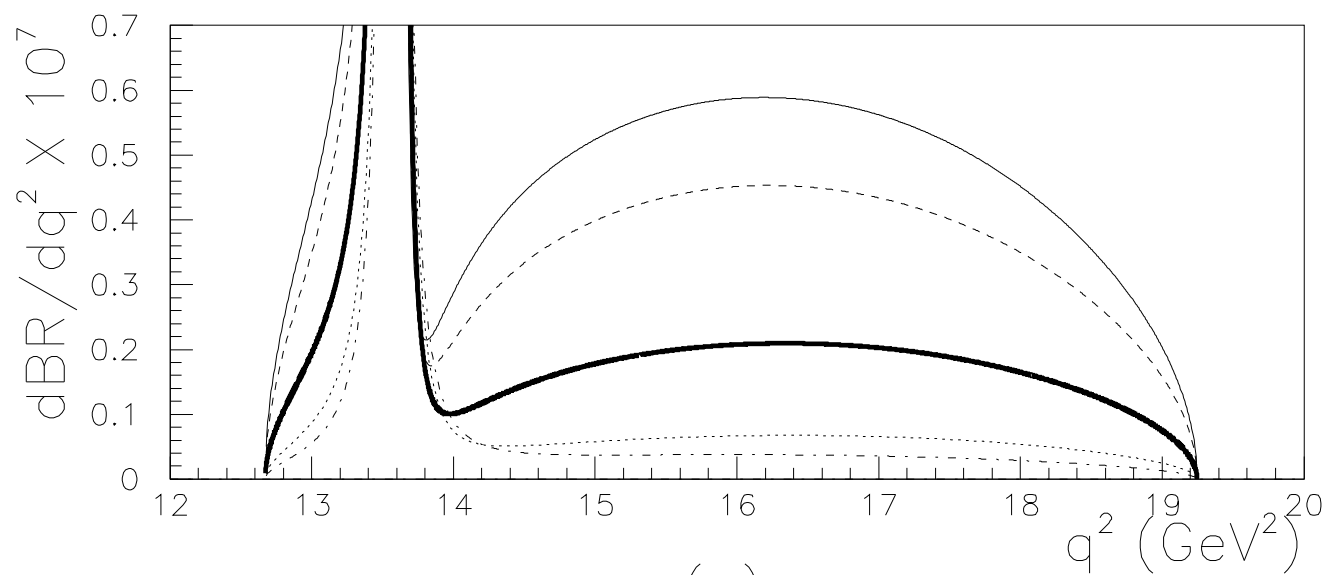
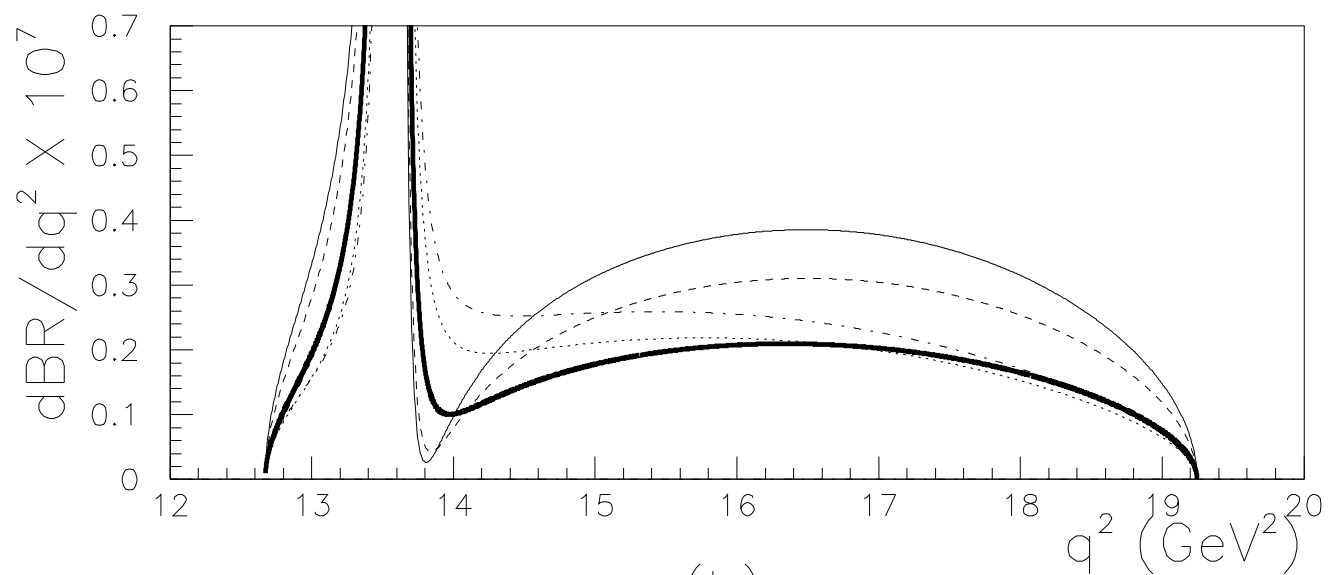


Figure 3:



(a)



(b)

Figure 4:

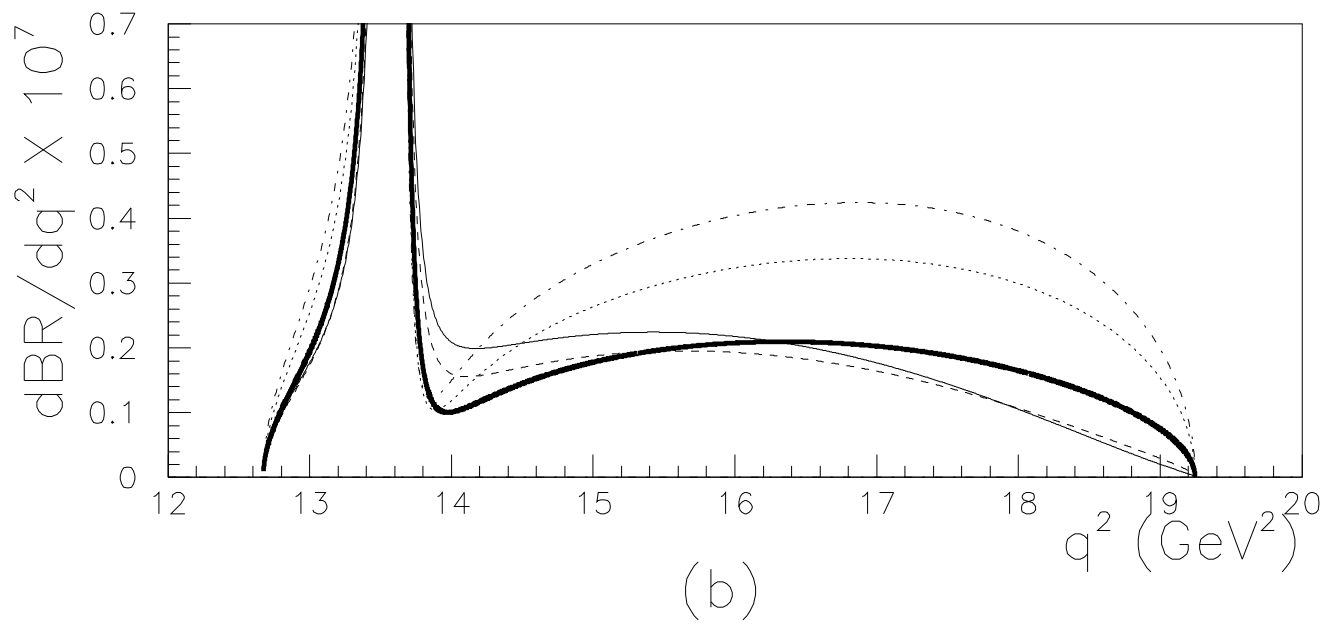
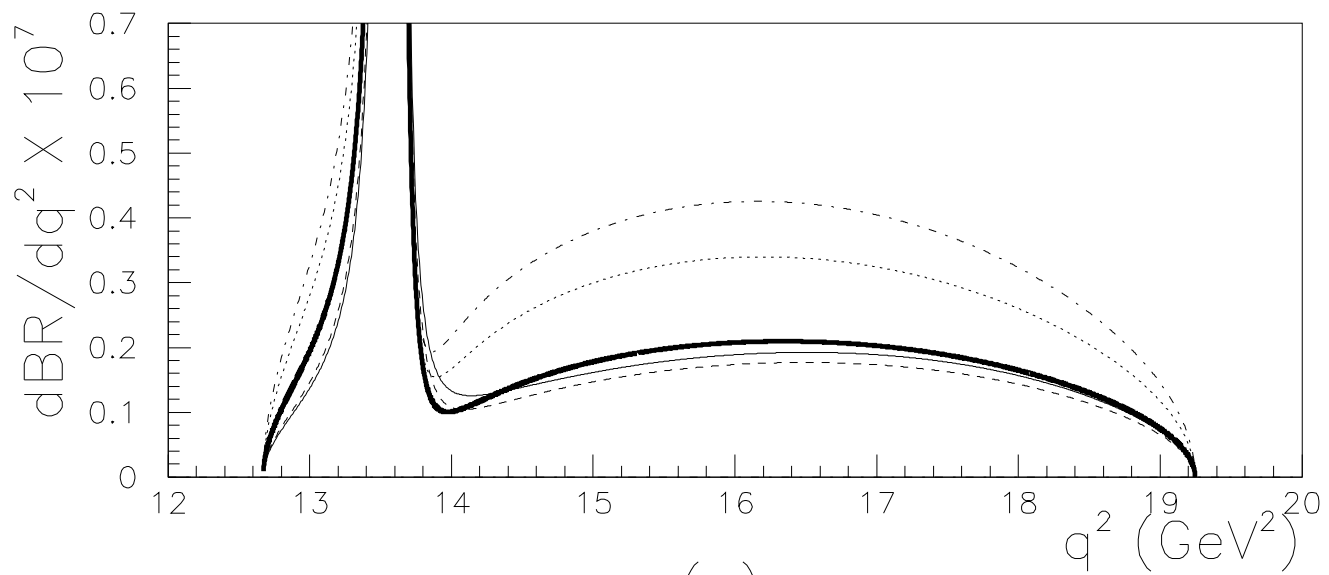


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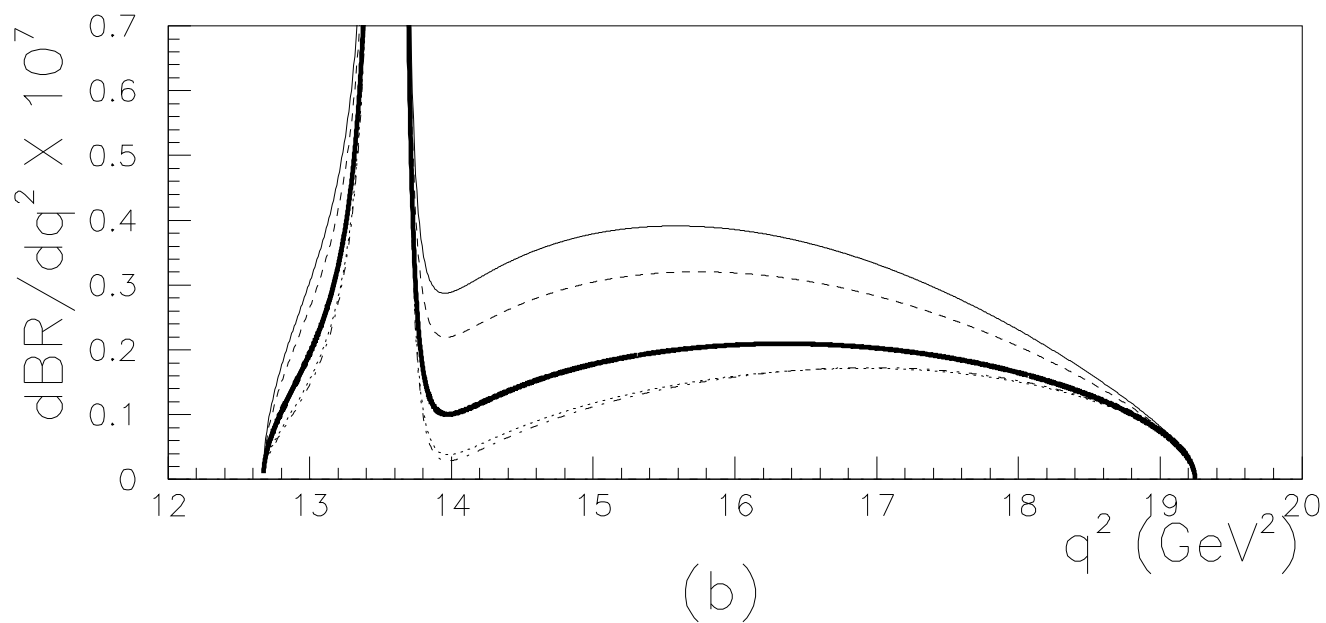
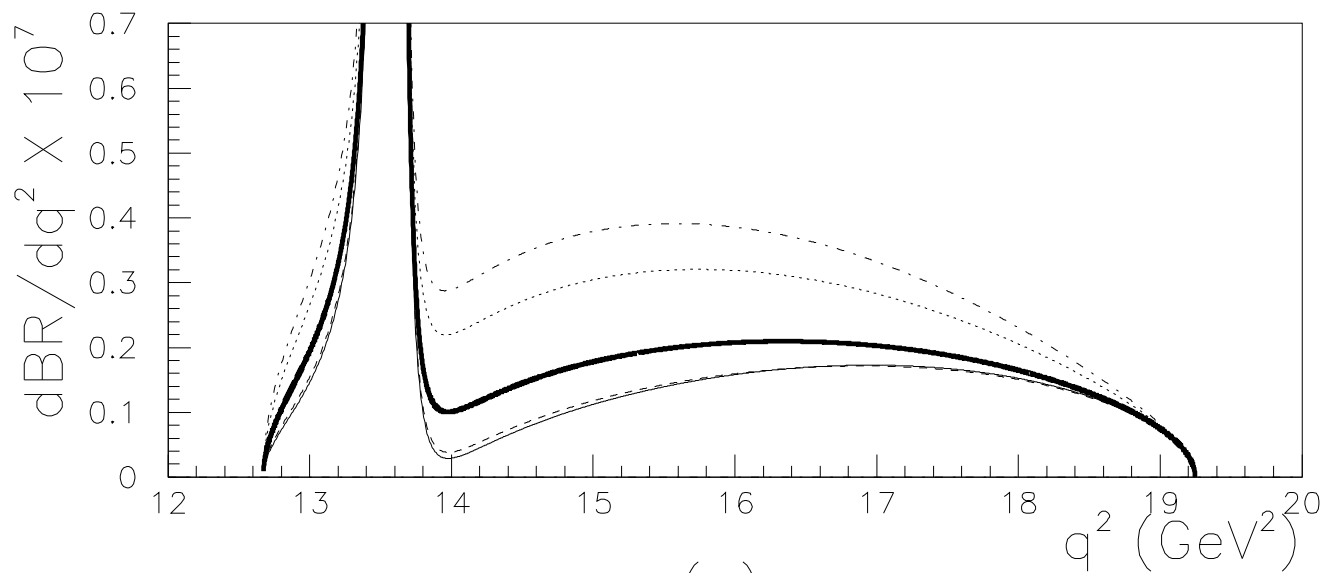
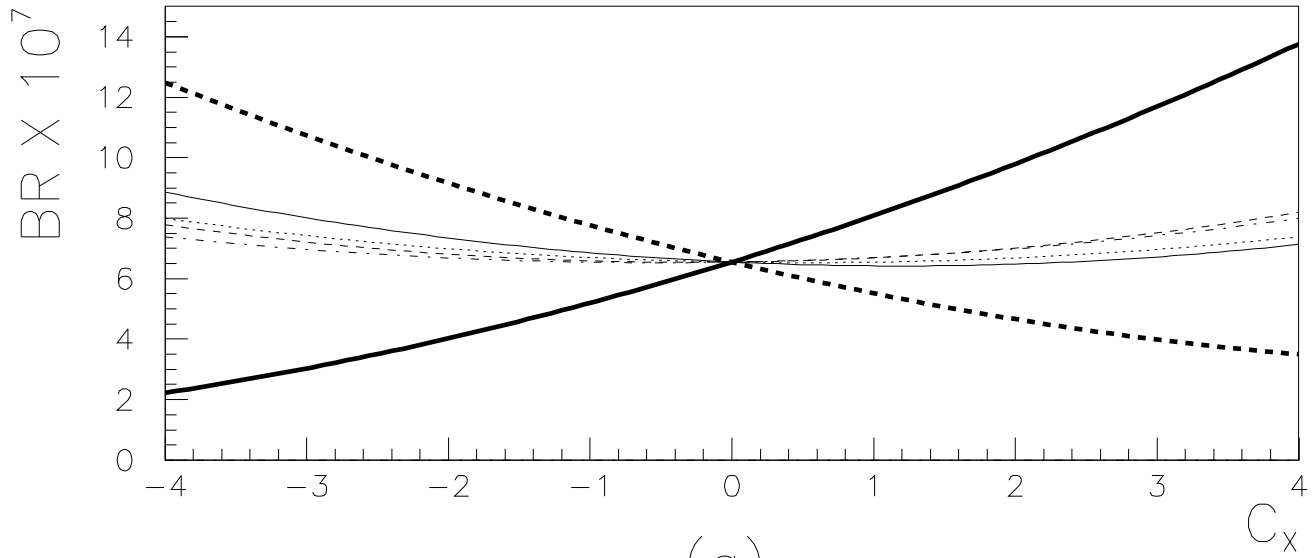
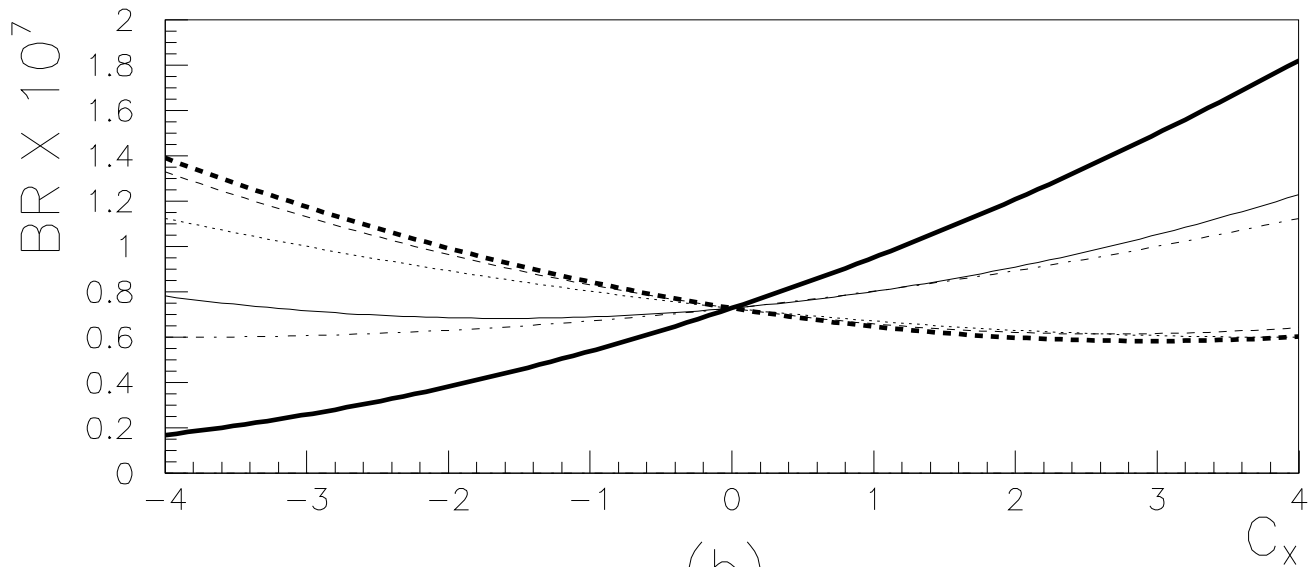


Figure 6:



(a)



(b)

Figure 7:

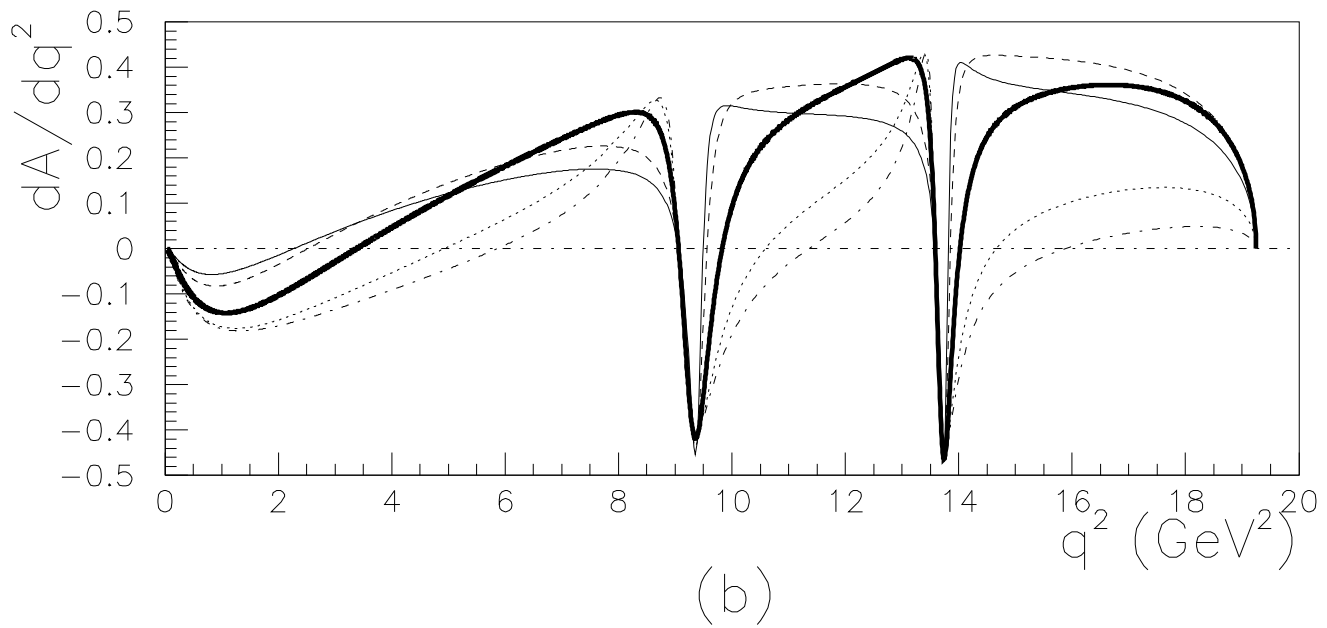
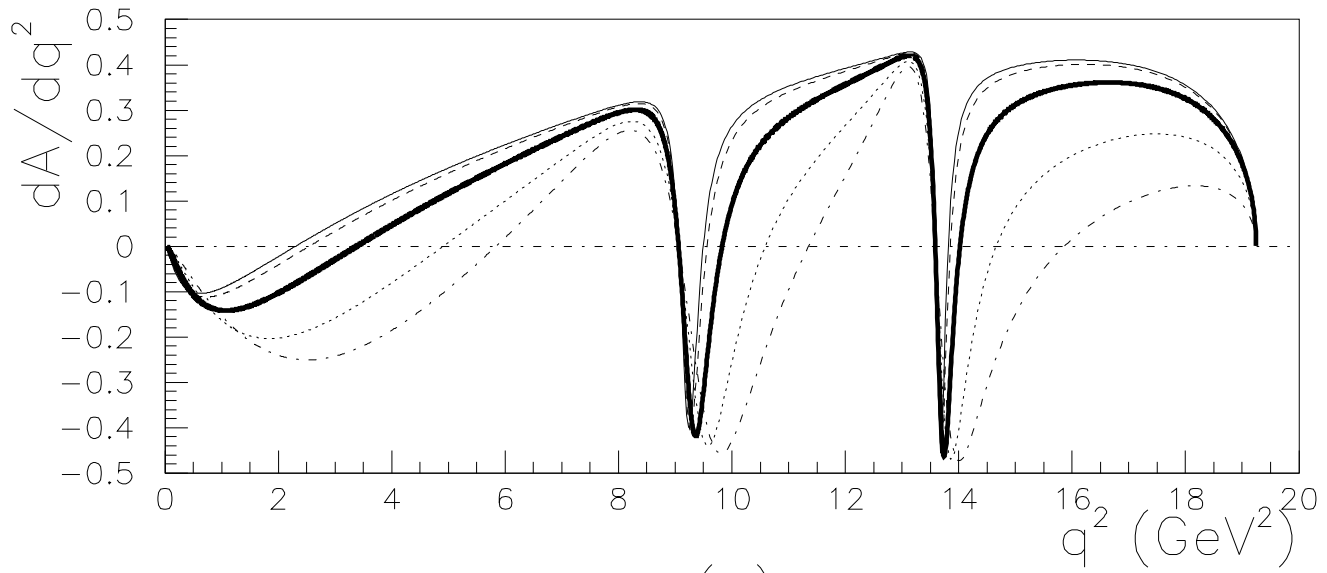


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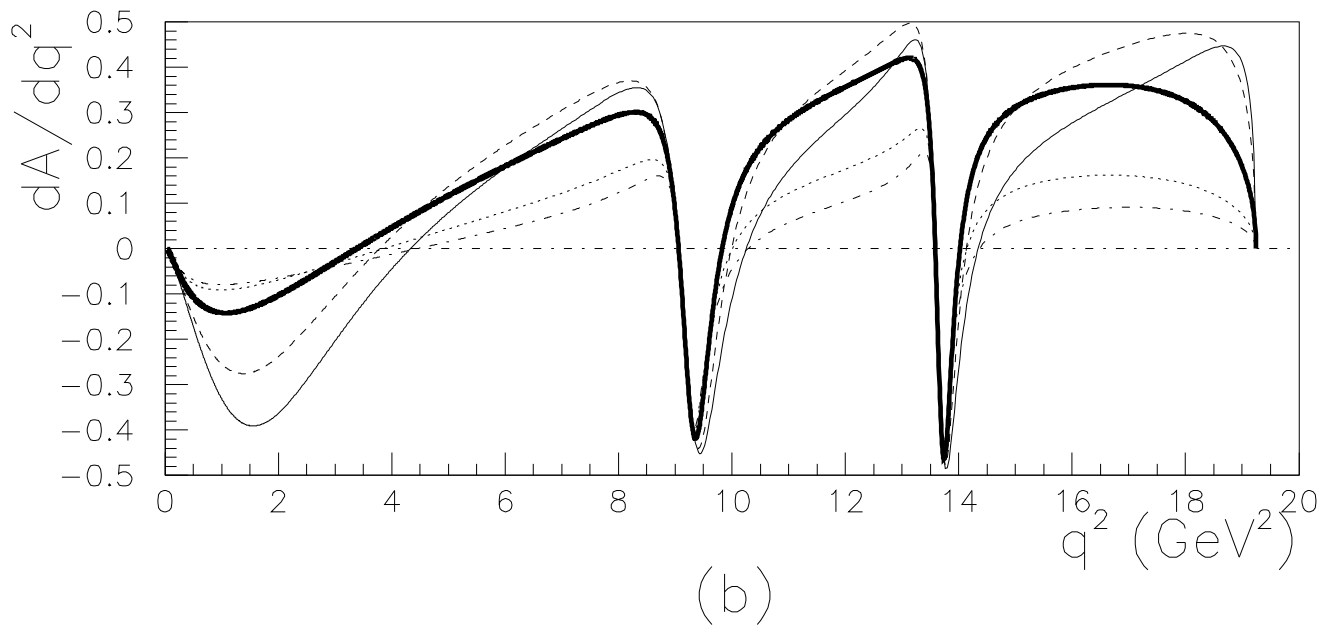
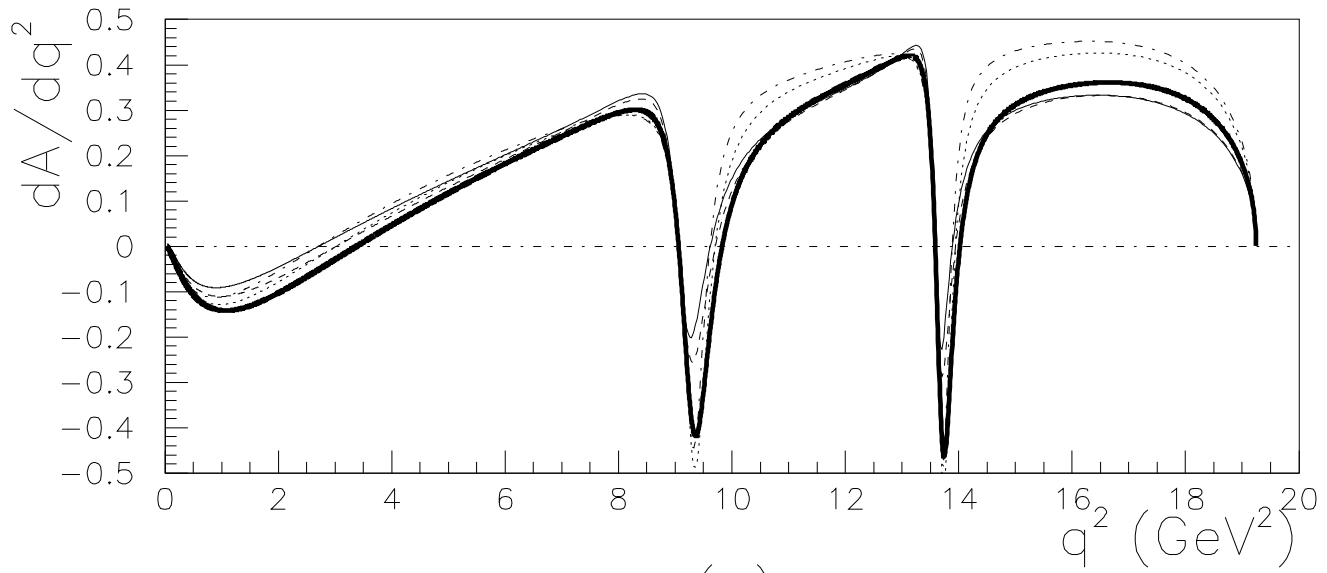
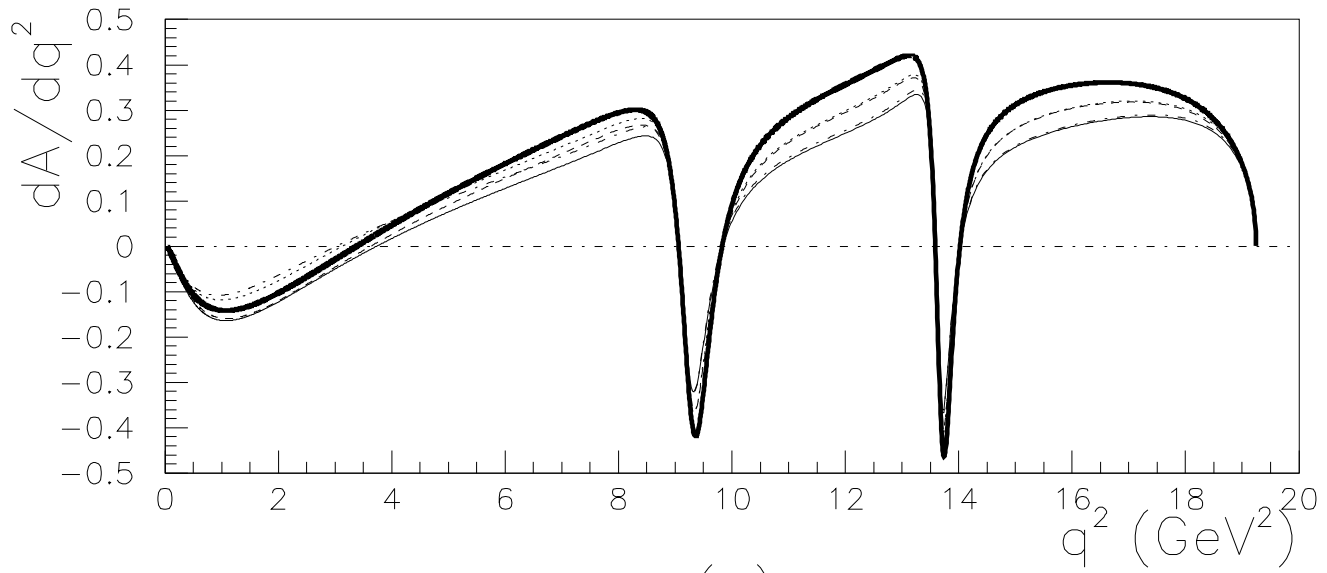
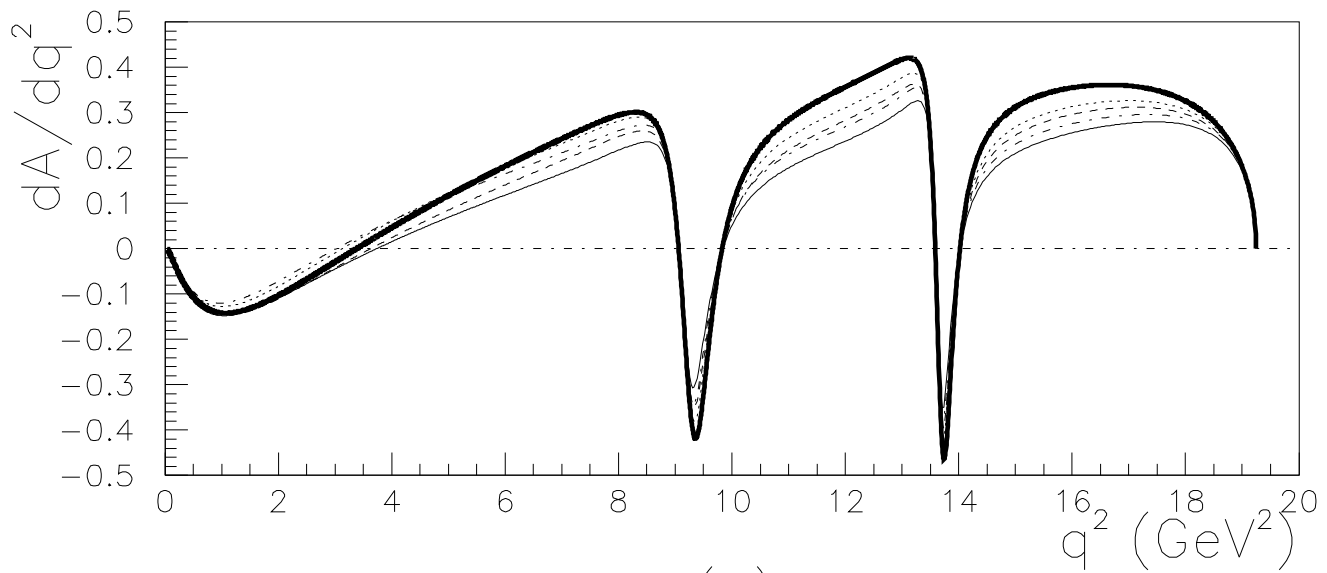


Figure 9:

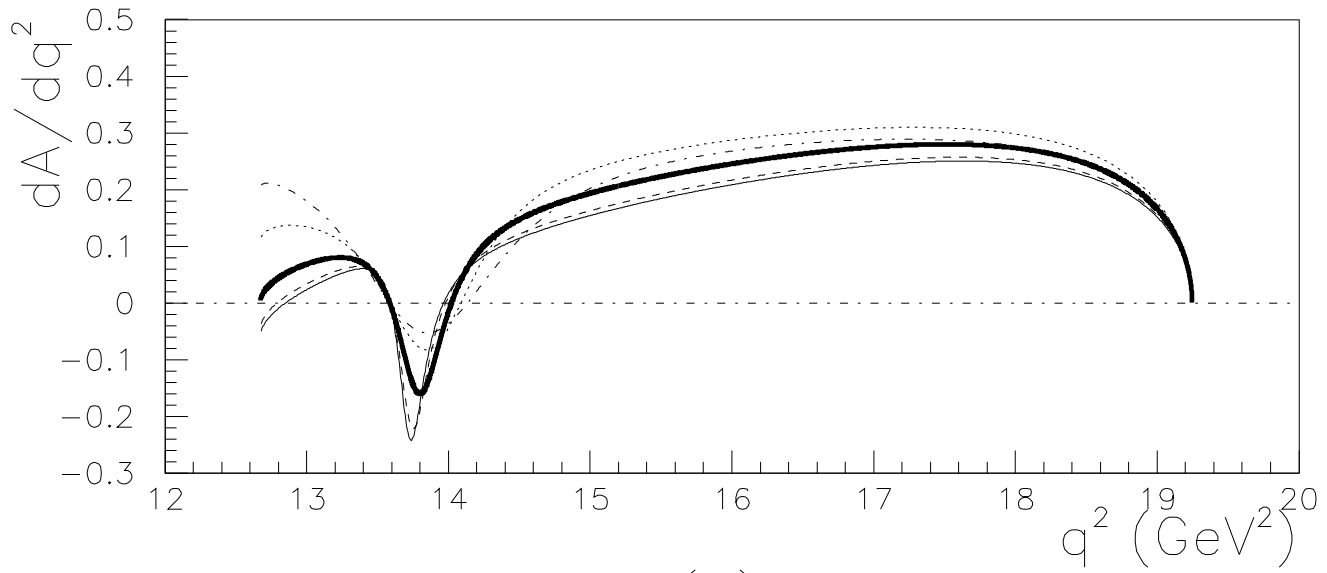


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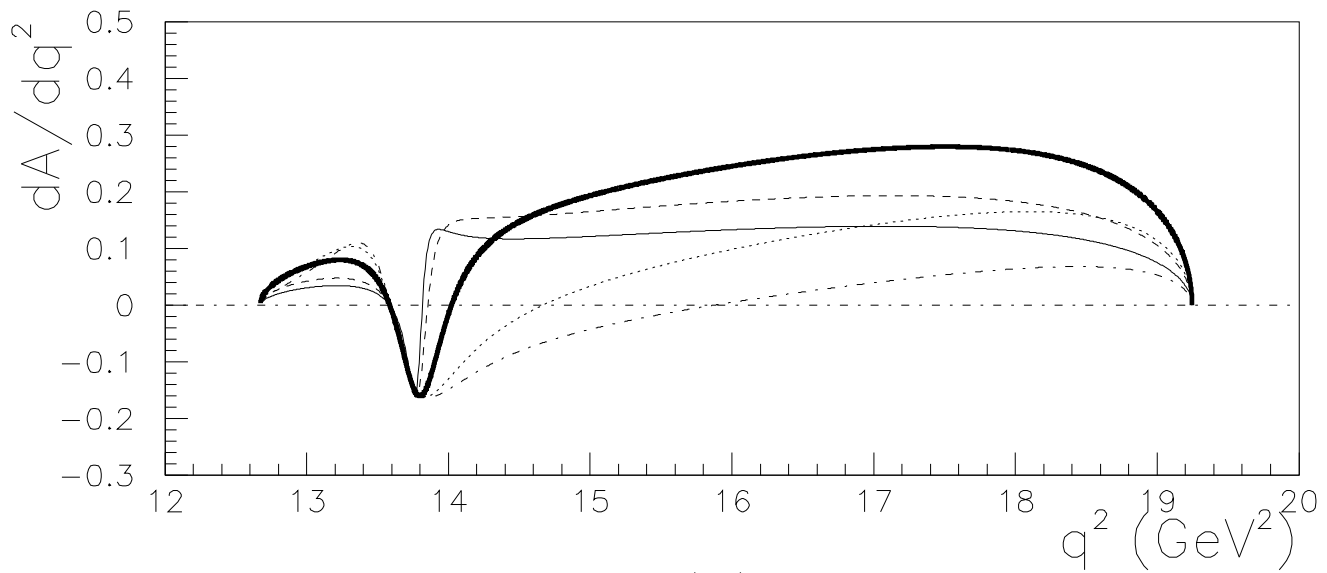


(b)

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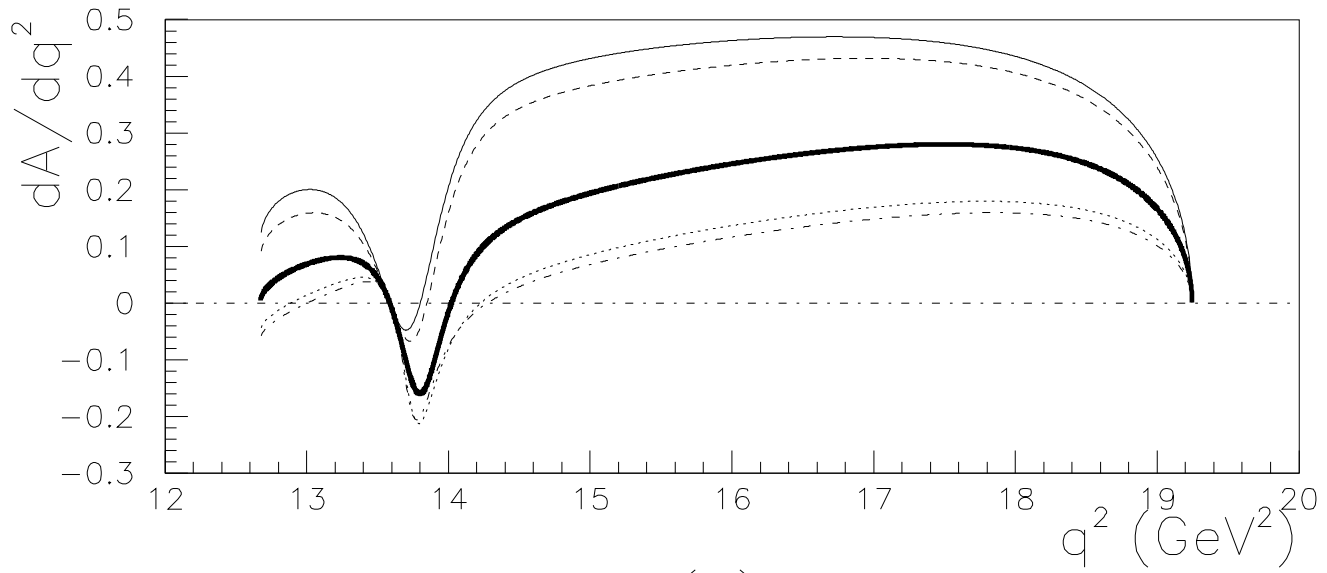


(a)

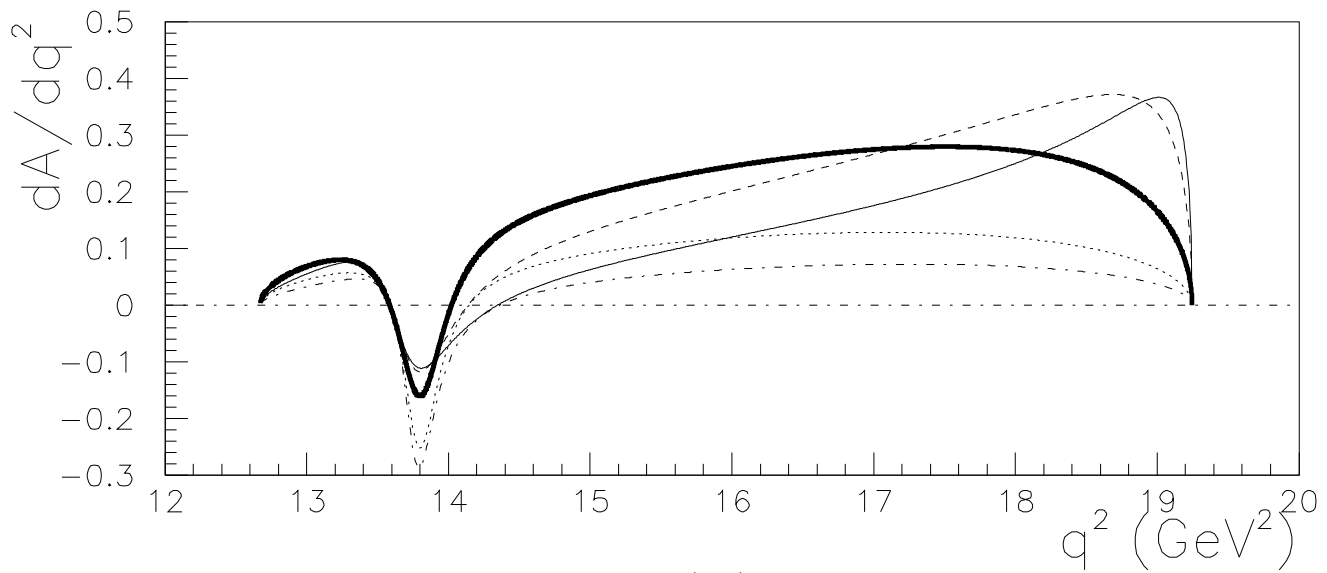


(b)

Figure 11:

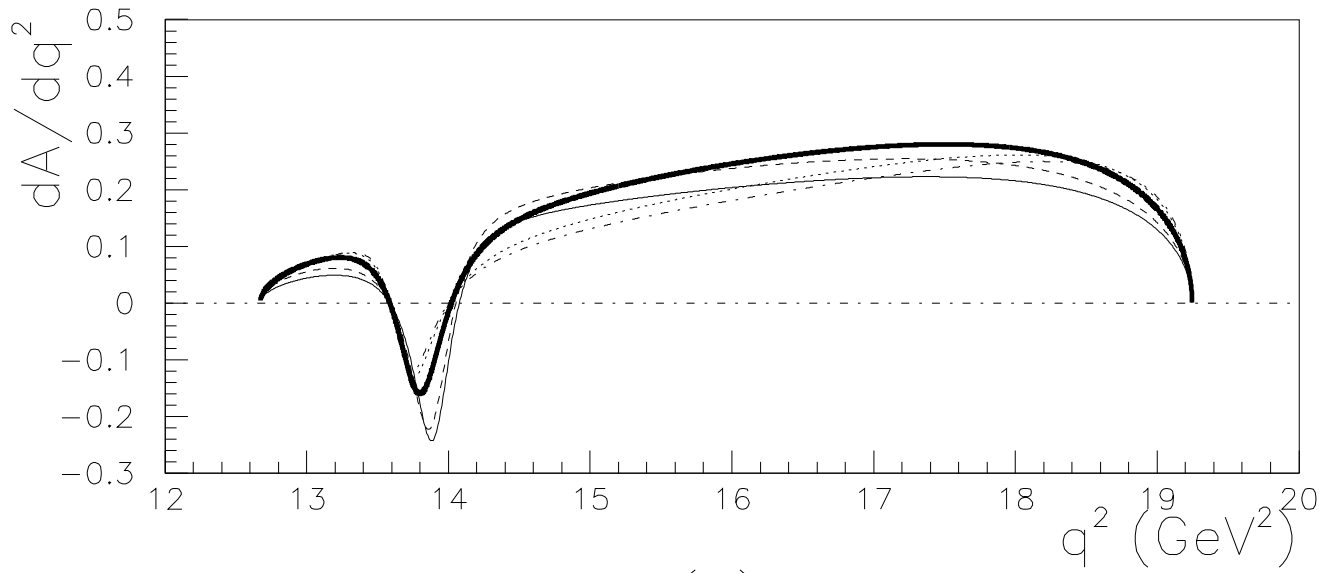


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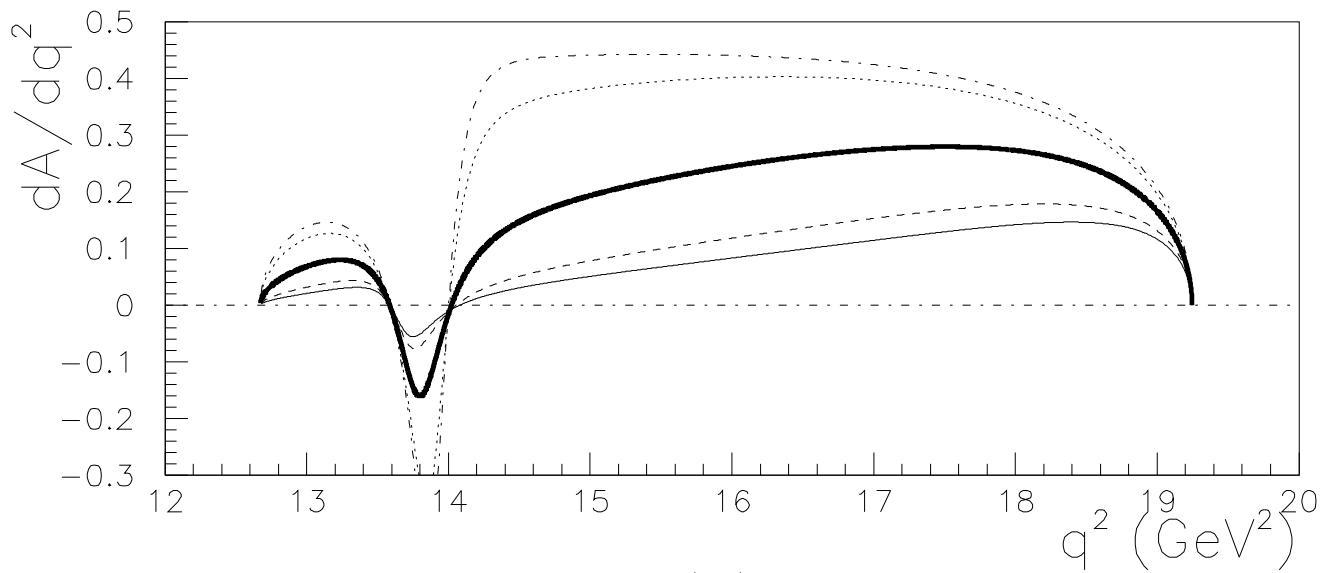


(b)

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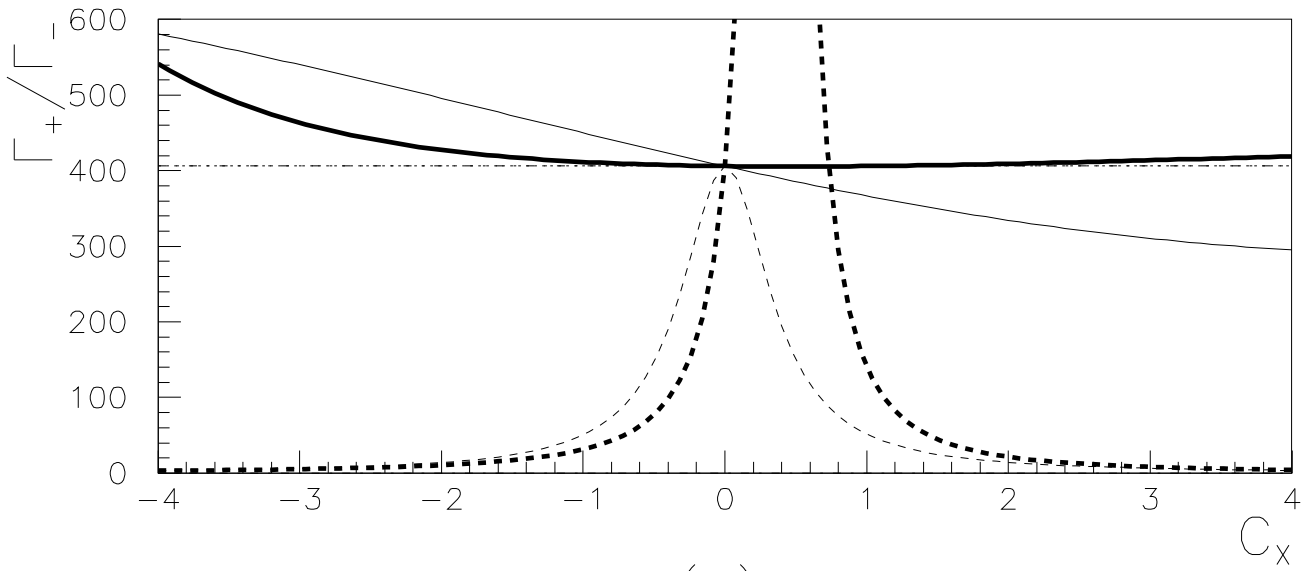


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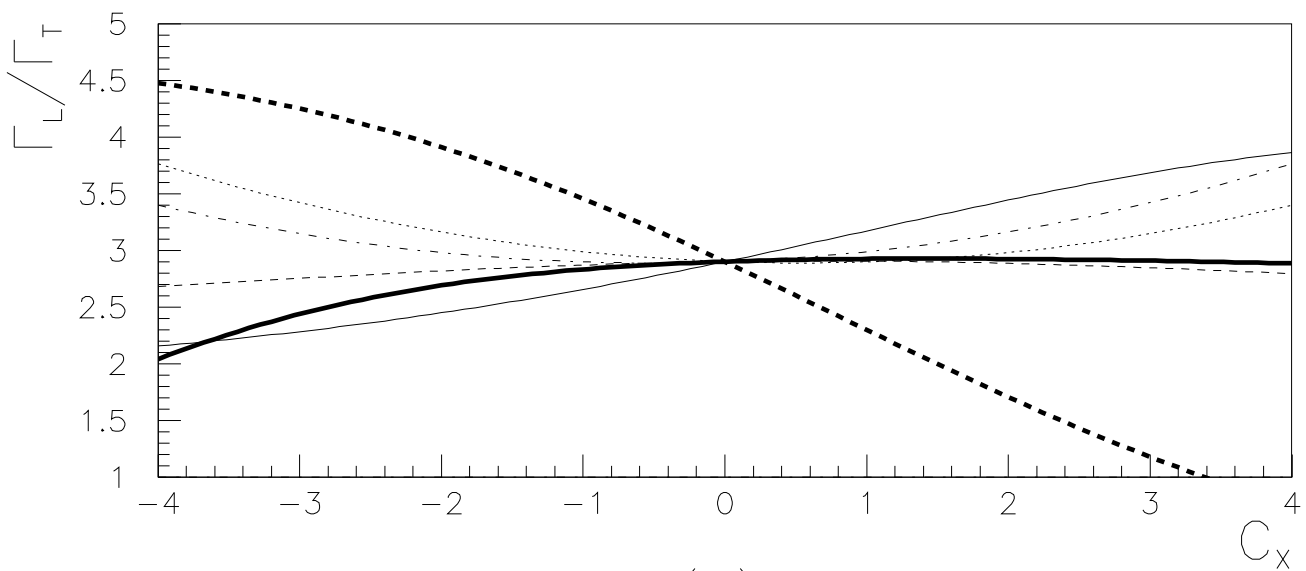


(b)

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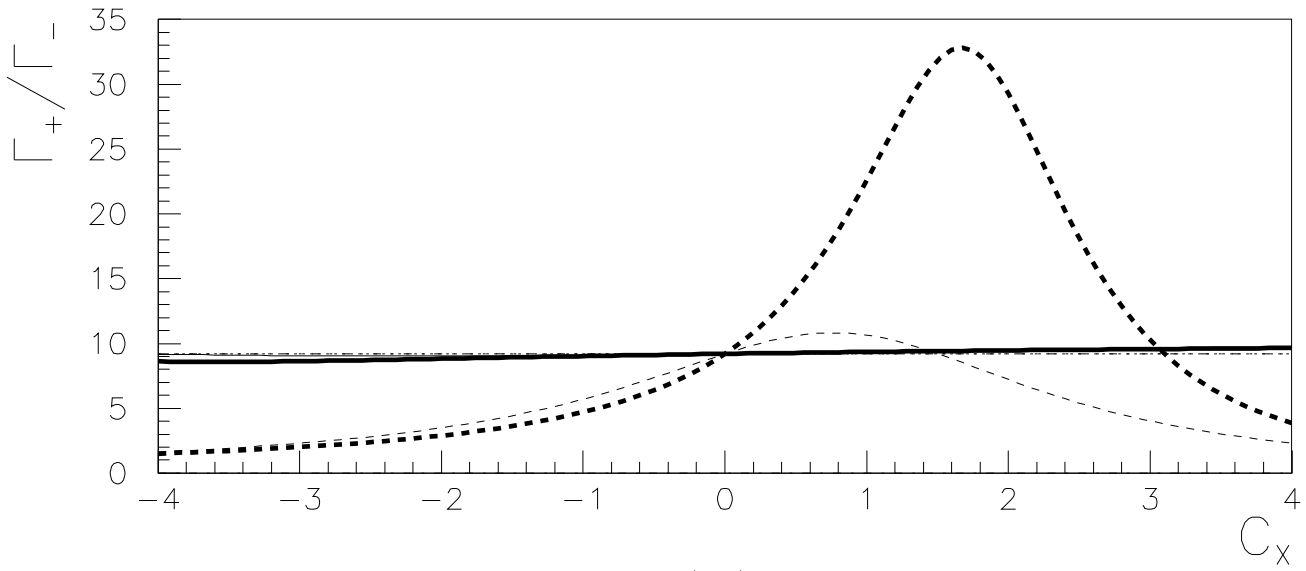


(a)

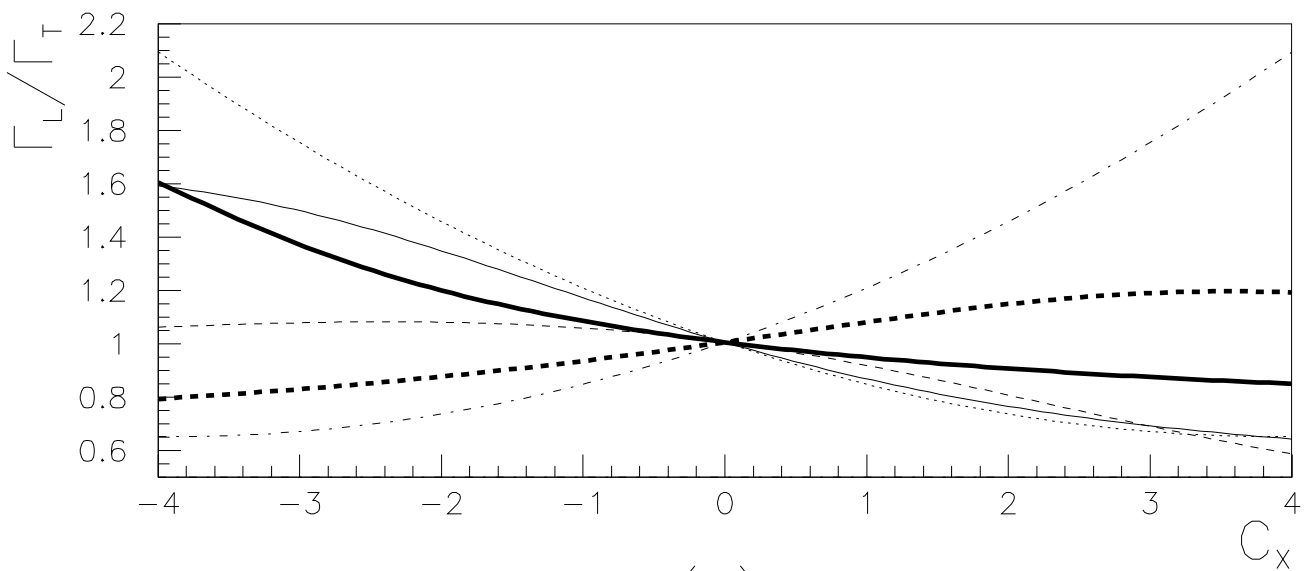


(b)

Figure 14:



(a)



(b)

Figure 15:

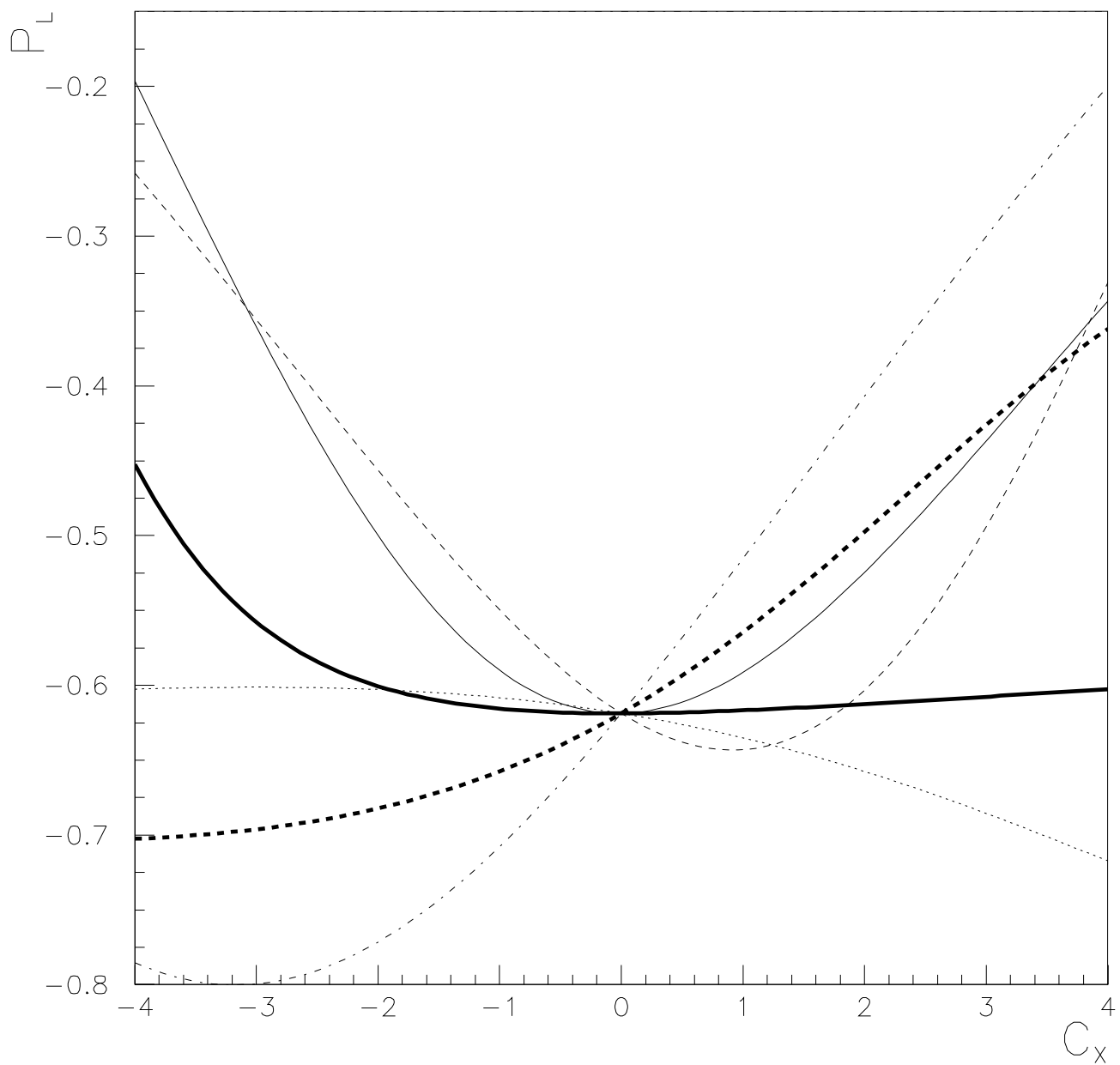
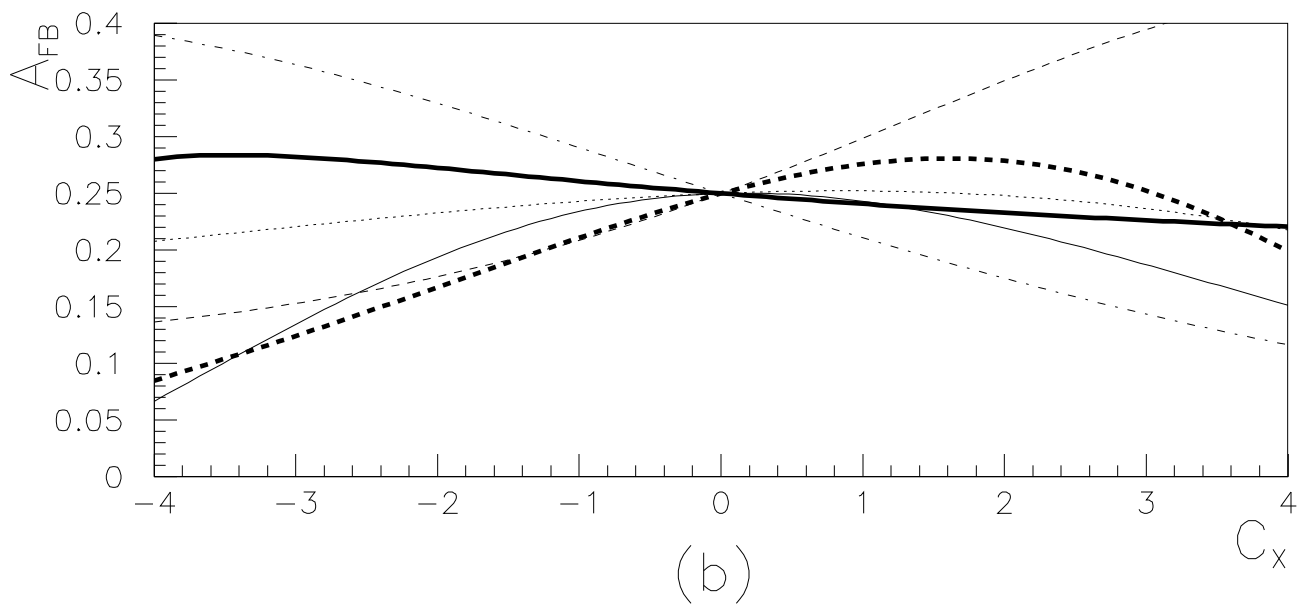
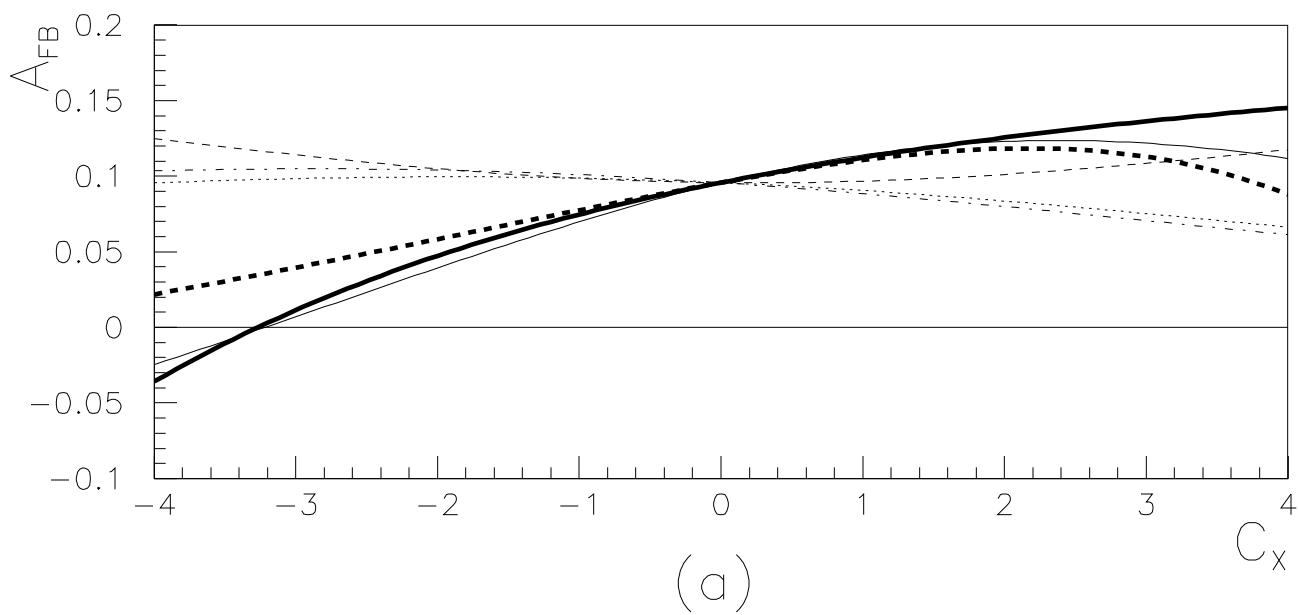
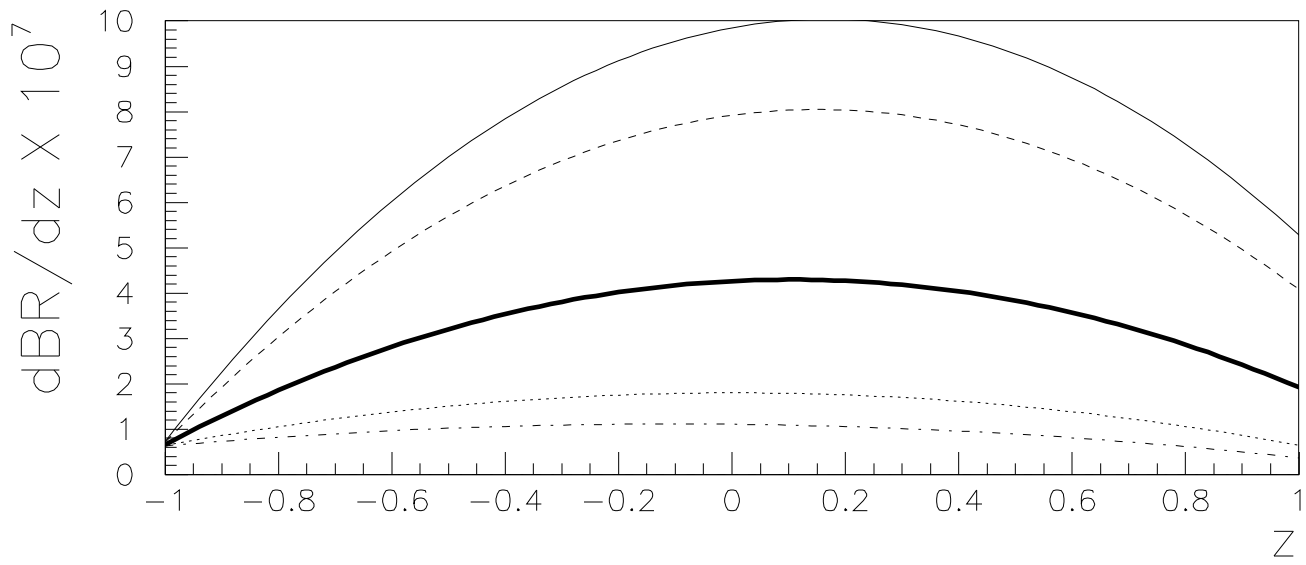
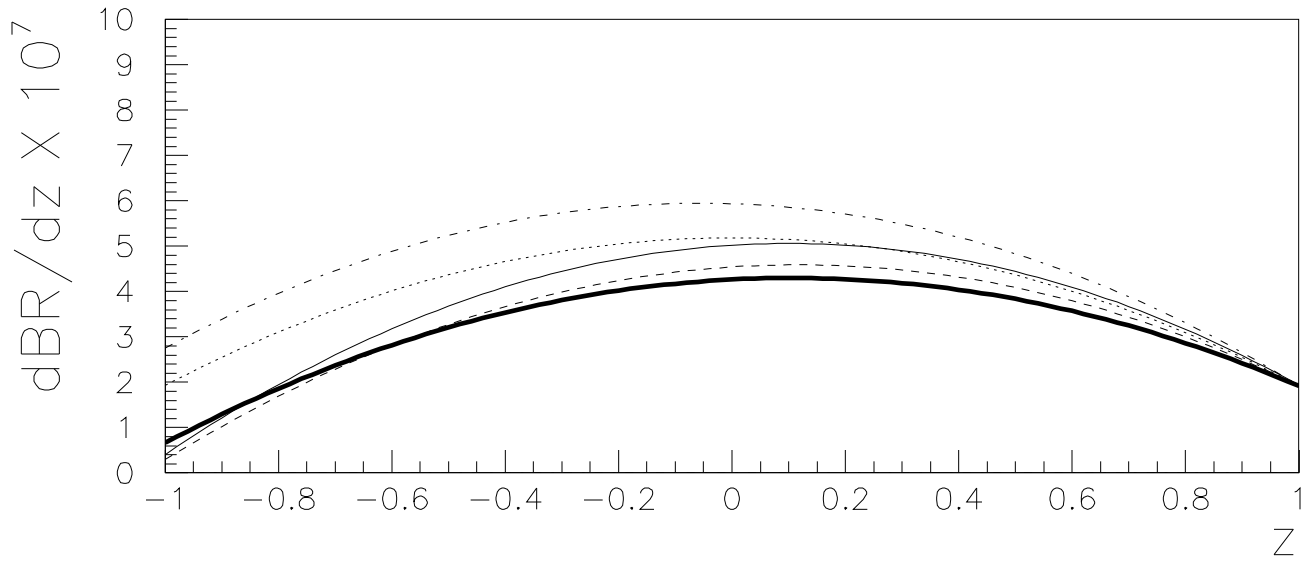


Figure 16:

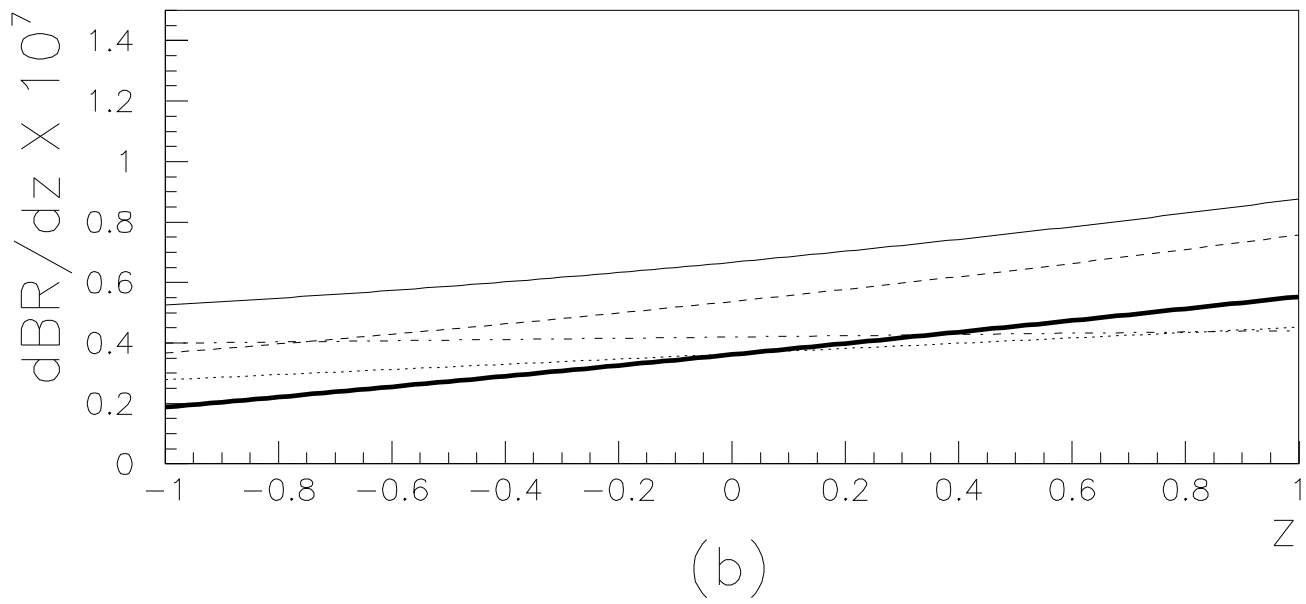
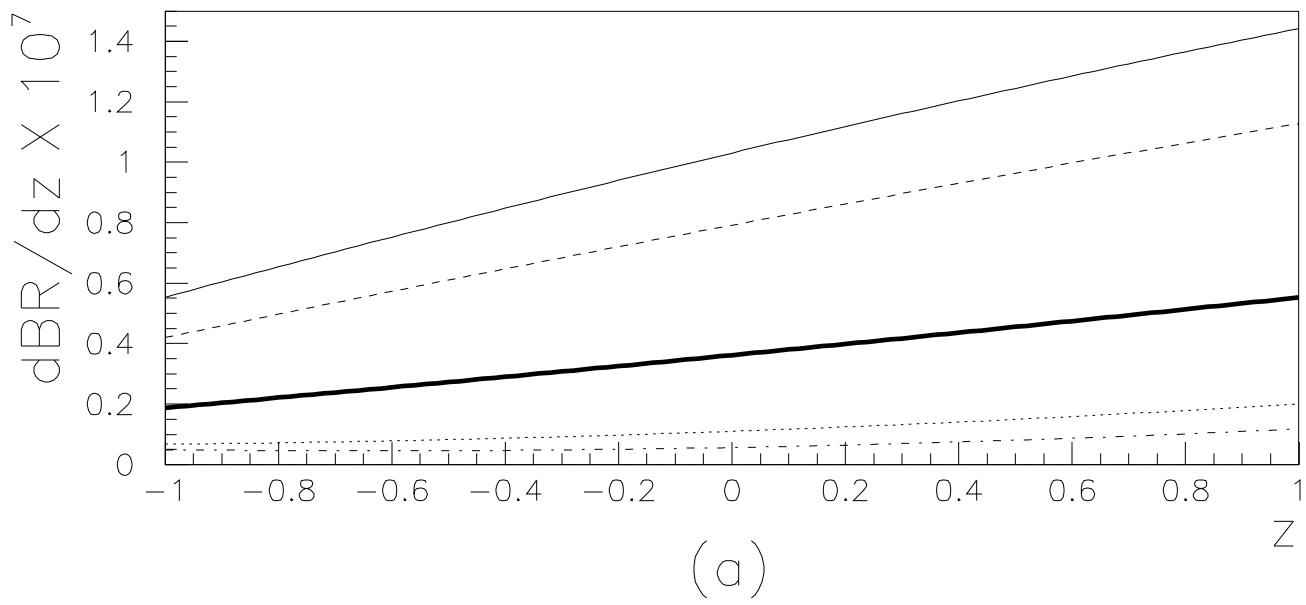


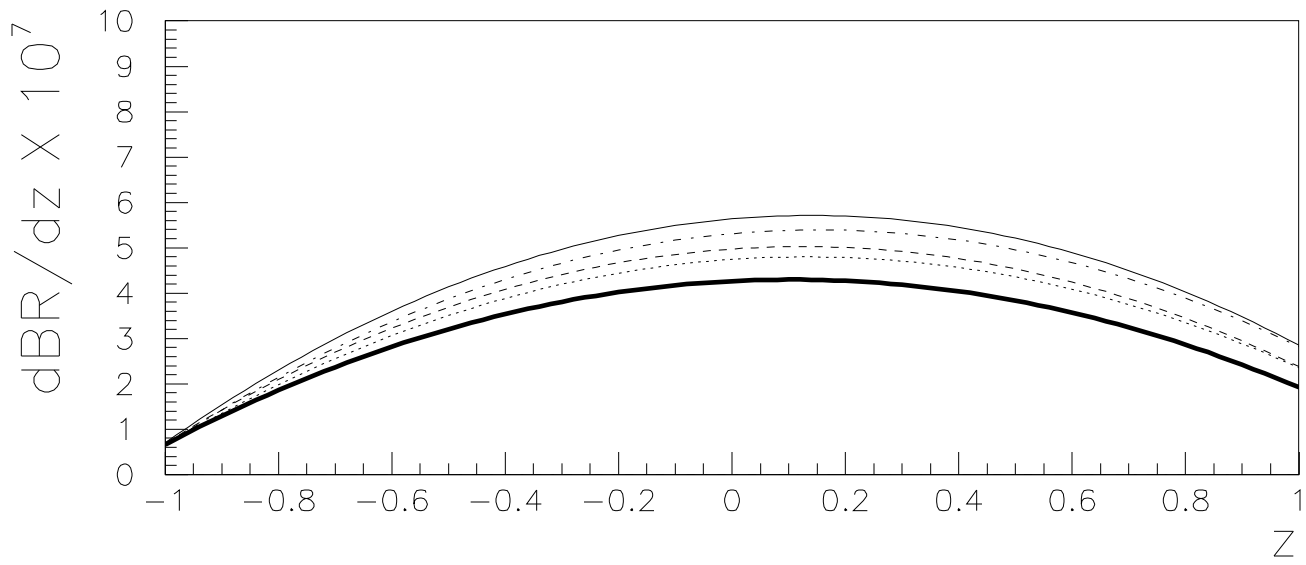


(a)

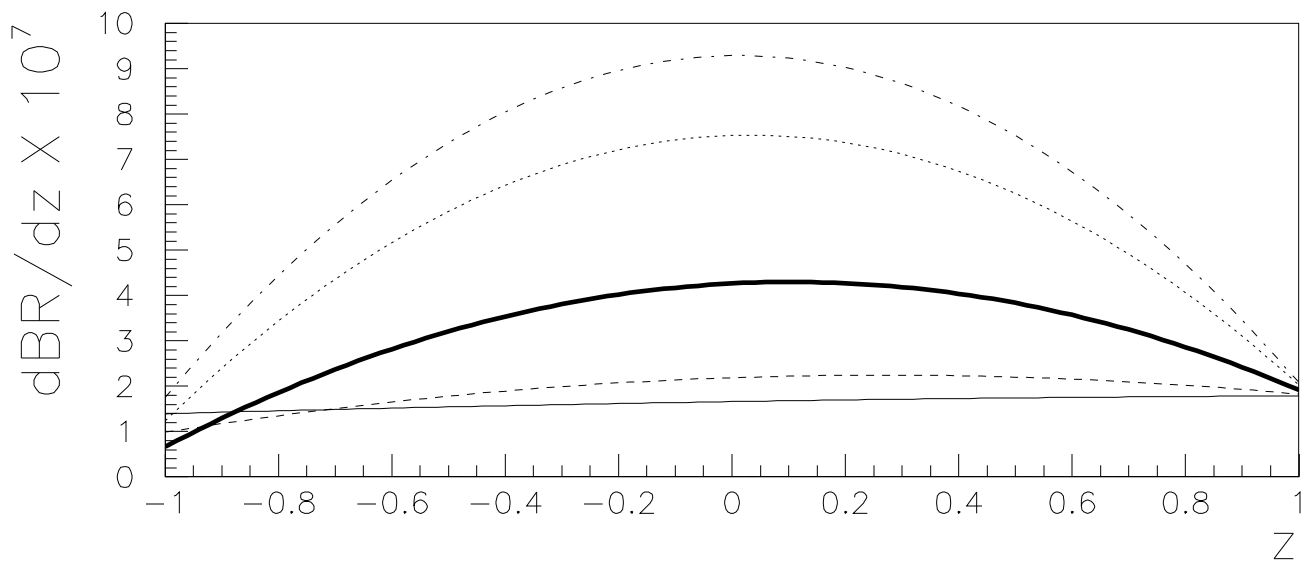


(b)

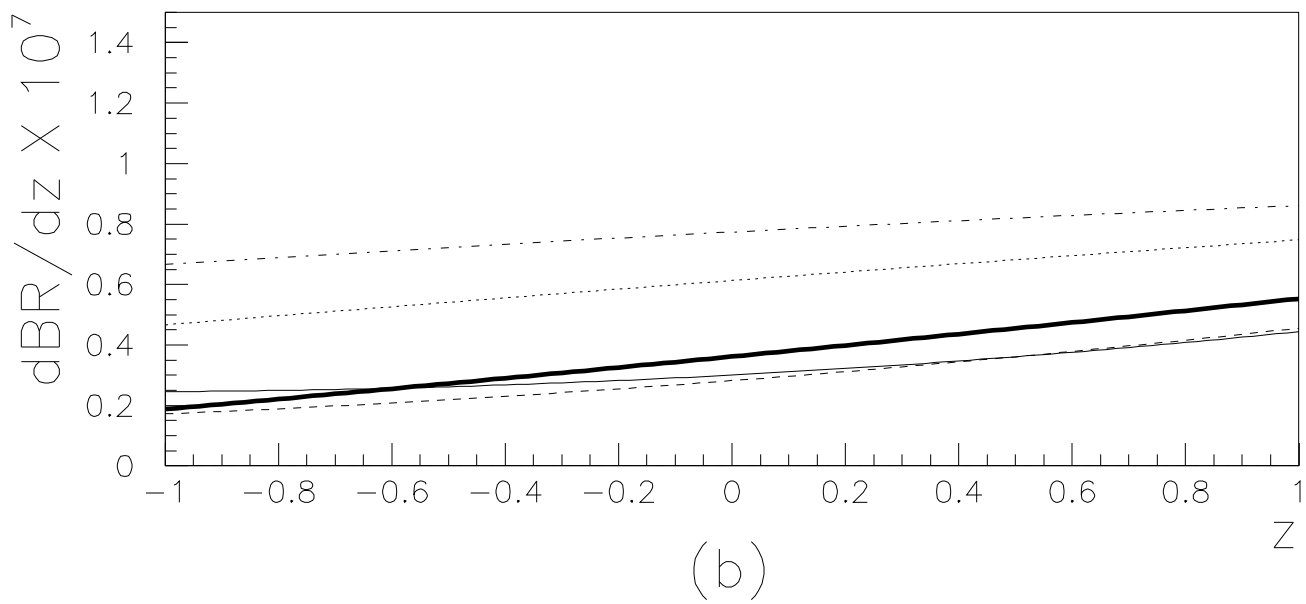
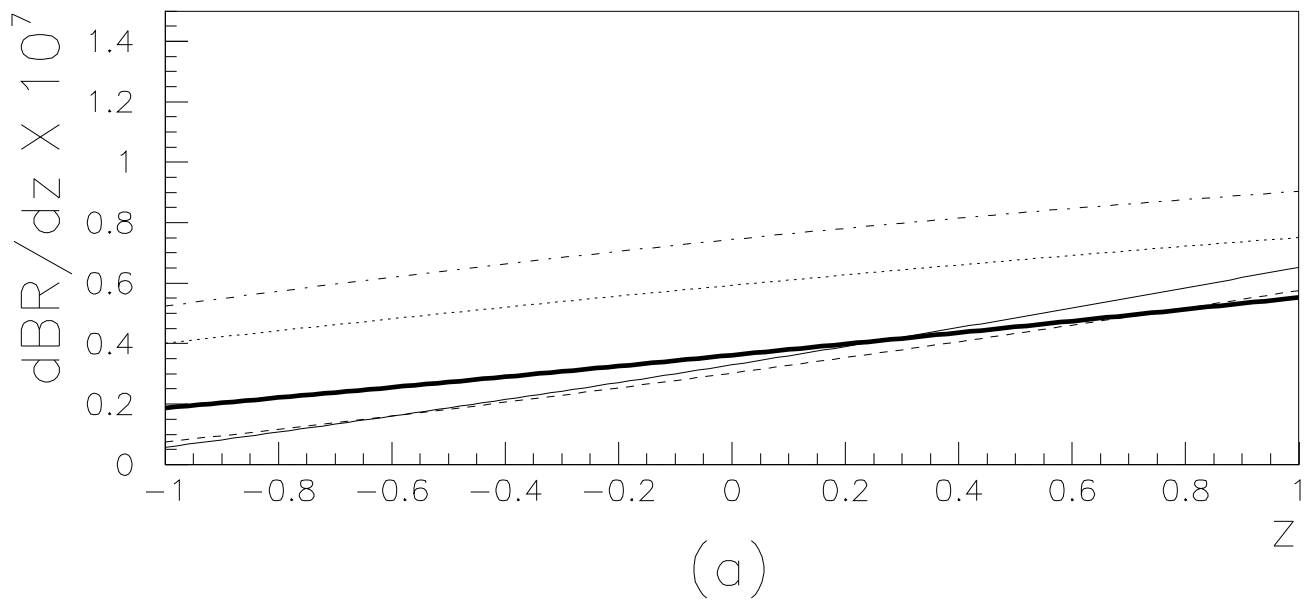


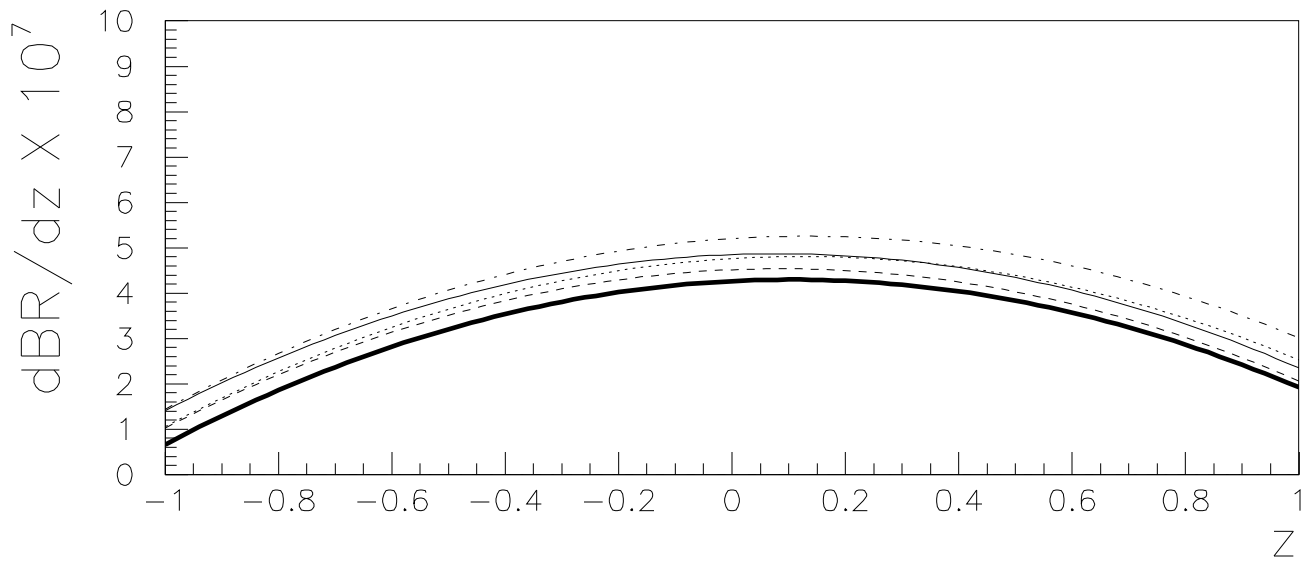


(a)

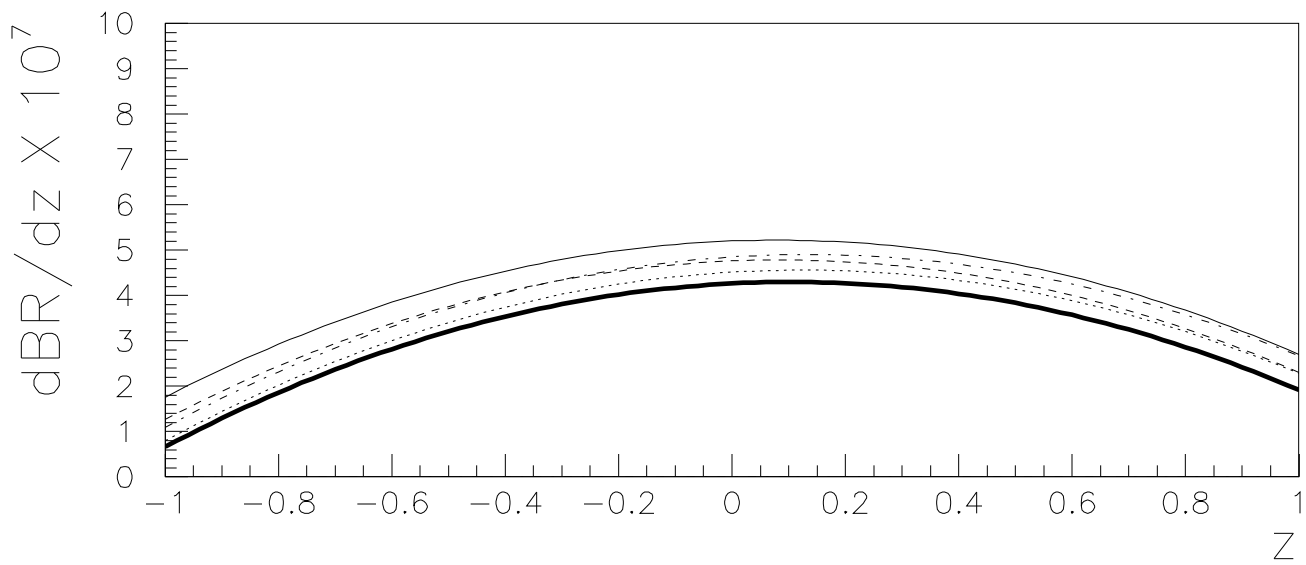


(b)

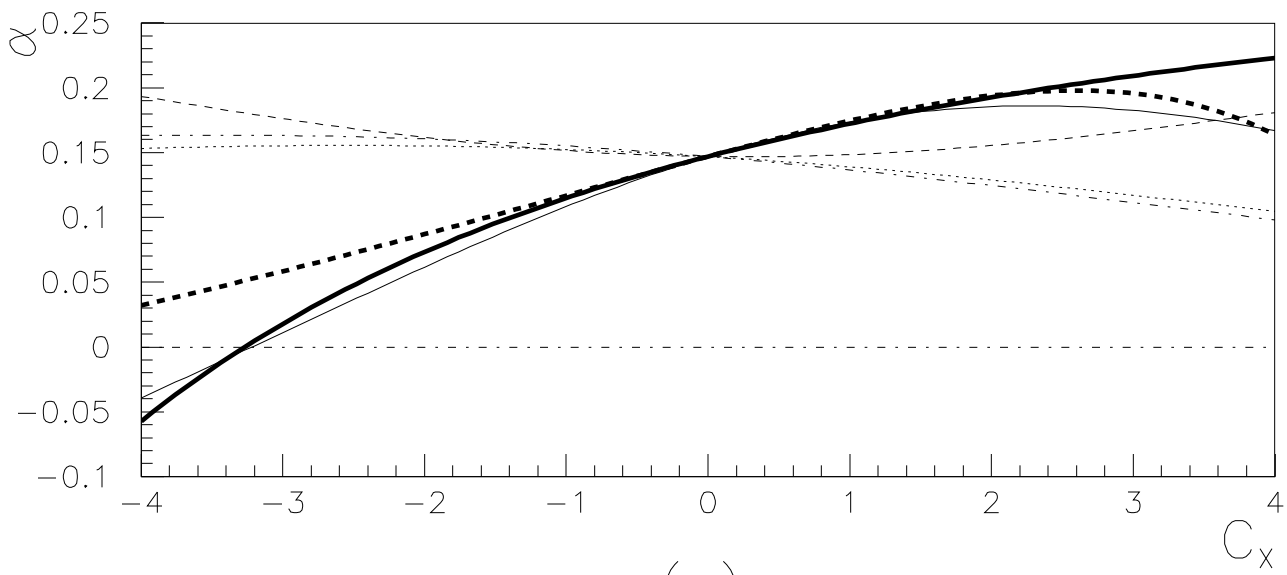




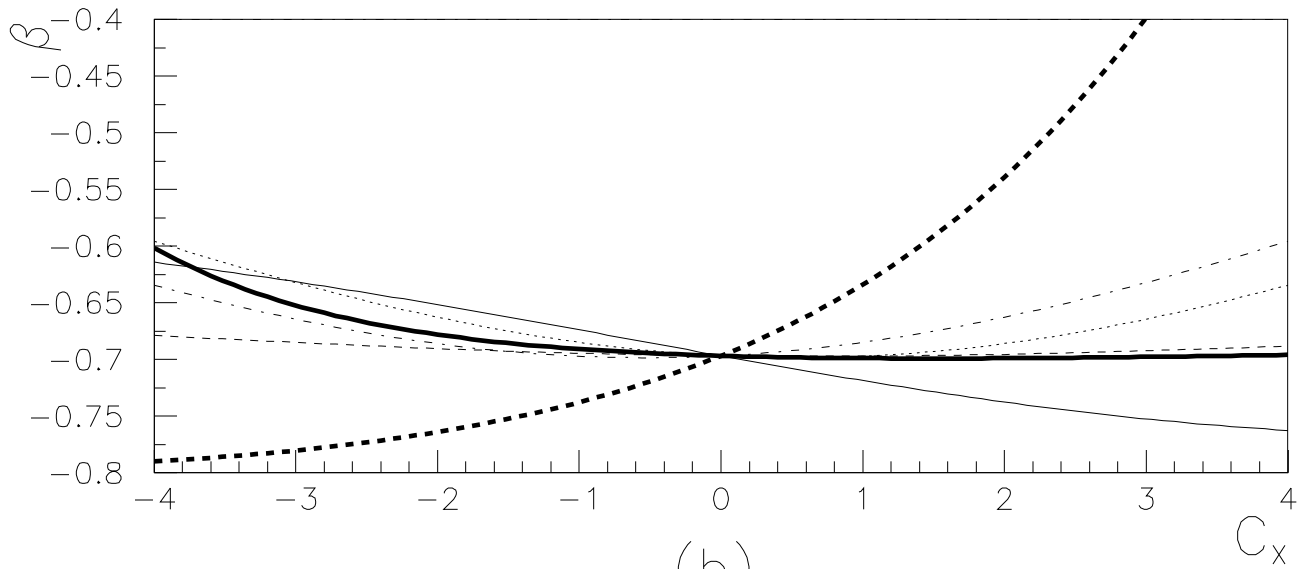
(a)



(b)



(a)



(b)

