METU Studies in Development, 43 (August), 2016, 497-506

Uniqueness of the reserve price with asymmetric bidders

Hikmet Günay[1]
University of Manitoba, Department of Economics, Winnipeg, Canada
e-mail: gunay@cc.umanitoba.ca

Xin Meng
Dongbei University of Finance and Economics, Center for Industrial and Business Organization
e-mail: mengxin@swufe.edu.cn

Mark Nagelberg
PRA Inc. Winnipeg, Manitoba, Canada
e-mail: nagelbergmark@gmail.com

Abstract

We analyze the optimal reserve price in a second price auction when there are N types of bidders whose valuations are drawn from different distribution functions. The seller cannot determine the specific "distribution type" of each bidder. In this paper, we give sufficient conditions for the uniqueness of the optimal reserve price. Then, we give sufficient conditions that ensure the seller will not use a reserve price; hence, the auction will be efficient.

Key words: Auction, Reserve (Reservation) Price, Asymmetric Bidders.

JEL classification: D44.

1. Introduction

E-bay auctions have been very popular among the general public. With the diffusion of the internet, the bidders of E-bay auctions come from many different

[1] We thank Krishnendu G. Dastidar, René Kirkegaard, Ryo Ogawa, Maxwell Pak, and anonymous referees for helpful discussions and suggestions. Part of this research has been done while H. Gunay was visiting Osaka University-ISER; their hospitality is much appreciated. H. Gunay and X. Meng acknowledge the financial support of Social Sciences and Humanities Research Council (SSHRC) and the University of Manitoba. X.Meng also acknowledges the financial support from the National Natural Science Foundation of China(NSFC) and Fundamental Research Funds for the Central Universities.

parts of the world and may have widely varying preferences; hence, expecting them to be symmetric bidders may be a strong assumption. In this paper, we relax this commonly-used assumption (e.g., Maskin and Laffont, 1980; Riley and Samuelson, 1981; Englebrecht-Wiggans, 1987; McAfee and McMillan, 1987; Levin and Smith, 1996; McAfee and Vincent, 1997) by allowing the independent private valuations to be drawn from different distribution functions. We then analyze how this will affect the optimal reserve prices.

In our model, there is one good that is auctioned off using a second price auction. The seller knows that there are *N* types of bidders whose valuations are drawn from different distribution functions. We will use the term "distribution type" to denote these different distribution functions. Specifically, we say that two agents have the same distribution type if their valuations are drawn from the same distribution function. The seller cannot determine the specific distribution type of each bidder,[2] or equivalently we assume that the seller has to use a unique reserve price. We give conditions that determine whether or not the seller will use a reserve price. If a reserve price is used, we analyze its properties for cases where the bidders' valuations are drawn from common supports, and find a sufficient condition for its uniqueness. Our assumptions differ from Myerson (1981) who studies the revenue maximizing mechanism under the assumption that seller can determine the distribution types. As a result, he finds a "discriminatory reserve price"; i.e., an optimal reserve price for each type. Our motivation comes from the fact that in practice, almost all auctions have a single reserve price (or no reserve price at all). We show that the optimal reserve price is determined when bidders' weighted average of virtual valuations is equal to the seller's valuation. In Myerson (1981), the optimal reserve price is found by equating the virtual valuation of each bidder to the seller's valuation; hence, there is a different reserve price for each distribution type of bidders.

Our main contribution is giving sufficient conditions for the uniqueness of the optimal reserve price. We prove this by showing that the revenue function is quasi-concave when there are bidders that can be ranked via hazard rate and reverse hazard rate dominance. Alternatively, one can assume that the bidders can be ranked via likelihood ratio dominance which implies both hazard rate and reverse hazard rate dominance.

Our second contribution, we believe, has real-world implications since we observe that the reserve price is not always used (e.g. Englmaier and Schmöller, 2010) in auctions. We show when the seller will use (effective) reserve prices, and when she will not. By the effective reserve price, we mean a reserve price strictly

---

2 This is an assumption commonly used in industrial organization. Firms are aware of the distribution of types across consumers but they cannot price discriminate because they do not know which specific consumers have each given type.

greater than the lower bound of buyer's valuations. We show that when the virtual valuation of each bidder at the lower bound of the support is greater than the seller's valuation and Myerson's regularity condition holds, then the seller will not use an effective reserve price. When we find conditions that the seller will use an effective reserve price, we do not need to assume Myerson's regularity condition. While there are other papers in the literature that find no effective reserve price (e.g. Englebrecht-Wiggans, 1987; McAfee and McMillan, 1987), they all assume that *symmetric* bidders have to pay some participation (or information) cost/fee before entering the auction or non quasi linear utility functions (Dastidar, 2015). In our model, *asymmetric* bidders do not have a cost of participation nor do they buy information.

There are a few papers other than Myerson (1981) in the literature that assume asymmetric bidders (e.g., Plum, 1992; Lebrun, 1999; Maskin and Riley, 2000). None of these papers analyzes the optimal reserve price. Also, our paper does not have information externalities, nor it uses mechanism design approaches unlike Kucuksenel (2013).

2. Asymmetric distributions with common support

Assume that there are $N \geq 2$ risk neutral bidders, and one risk-neutral seller with one object. Bidders' valuations are independently distributed with cumulative distribution functions F_i, i =1,2,…, $N$with support $[\underline{v}, \overline{v}]$ (with $\underline{v} \geq 0$). All distribution and density functions are continuously differentiable and density functions f_i are positive everywhere.[3] The distributions are common knowledge but the valuations are private information. The seller cannot determine the specific distribution type for each bidder.[4] We assume that we have at least one i such that $F_i \neq F_j$; hence, we have asymmetric bidders. The value of the object to the seller is $x_0 < \overline{v}$. If the inequality does not hold, the seller will never sell the object.

We will calculate the optimal reserve price that will be set by the seller in a second-price auction. It is well-known that the bidders bid truthfully in a second price auction even when they know the existence of asymmetric bidders (e.g. see Krishna, 2009). First, we have to calculate the seller's revenue function, $E\Pi$, when facing asymmetric bidders. For this, we will fix a bidder i, and calculate the first (i.e., highest) order statistic of the remaining bidders; $H_i(x) = \prod_{j \neq i} F_j(x)$. We let

3 This implies continuous differentiability of the expected revenue function.

4 This is analogous to a firm facing two or more different types that it cannot distinguish. One can also assume that the seller can distinguish the types but is required to use a unique reserve price by law. For example, a government agency cannot treat bidders differently and use different reserve **prices** for different bidders.

h_i denote the corresponding density function. The expected payoff function is given by:

$$E\Pi = x_0 \prod_i^N F_i(r) + \sum_i^N \left[\big(1 - F_i(r)\big) H_i(r) r + \int_r^{\bar{v}} \left(\int_r^{x_i} y\, h_i(y) dy \right) f_i(x_i) dx_i \right] \quad (1)$$

The first term with x_0 shows that the case where all bidders bid below the reserve price r. The first term in the brackets of summation shows the case where a given bidder i wins the auction and all other bidders bid below the reserve price r, or equivalently, the other bidders do not enter the auction. The second term is the expected payment of bidder *i* given that he wins the auction, and at least one other bidder bids above r. Finally, to get the expected revenue function, we have to sum over all N bidders.

By changing the order of integration, we can write this as:

$$E\Pi = x_0 \prod_i^N F_i(r) + \sum_i^N \left[\big(1 - F_i(r)\big) H_i(r) r + \int_r^{\bar{v}} y h_i(y) \big(1 - F_i(y)\big) dy \right] \quad (2)$$

By taking the derivative of this function with respect to *r*, we have the following equation:

$$E\Pi' = x_0 \sum_i^N F_i(r) H_i(r) + \sum_i^N H_i(r)(1 - F_i(r) - f_i(r) r) = \sum_i^N H_i(r)\big(1 - F_i(r) - f_i(r)(r - x_0)\big)$$

Using Myerson's virtual valuation which is $J_i(x) = x - \frac{1-F(x)}{f_i(x)}$ in the equation, we get:

$$E\Pi' = -[\textstyle\sum_i^N H_i(r) f_i(r)(J_i(r) - x_0)] \quad (3)$$

So we find an equation that the optimal reserve price must satisfy. Our first lemma summarizes this result.

Lemma 1 Any interior optimal reserve price must satisfy the implicit equation:

$$\textstyle\sum_i^N H_i(r)\big(1 - F_i(r) - f_i(r)(r - x_0)\big) = -(\sum_i^N H_i(r) f_i(r)(J_i(r) - x_0)) = 0 \quad (4)$$

The first order condition shows that bidders' weighted average of virtual valuations is x_0 at the optimal reserve price, since one can re-write the first order condition as $\frac{\sum_i^N H_i(r) f_i(r)(J_i(r) - x_0)}{\sum_i^N H_i(r) f_i(r)} = 0 \Rightarrow \frac{\sum_i^N H_i(r) f_i(r)\big(J_i(r)\big)}{\sum_i^N H_i(r) f_i(r)} = x_0$. In Myerson (1981), when bidders are asymmetric but their distributions are known, a different reserve price is determined for each bidder by setting their virtual valuation equal to the seller's valuation. As we discussed before, even if the seller knew the distribution type of each bidder, setting a different reserve price for different bidders is not practical. For example, government agencies cannot set different reserve price for different bidders since this may be labelled as discrimination against some bidders.

Now, we will find conditions under which the seller will set an effective reserve price $r > \underline{v}$, and when he will not. In the theorem below, note that we do not assume regularity condition in part a; i.e, we do not assume that J_i functions are increasing.

Proposition 2 Let $t_i(\underline{v}) = lim_{x\downarrow\underline{v}} \frac{H_i(x)f_i(x)}{\sum_i^N H_i(x)f_i(x)}$ *exist.*[5]

a) If $\sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) < 0$ then $r > \underline{v}$.

b) Assume J_i *is increasing for each* i*. If* $\sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) \geq 0$ *then there will be no effective reserve price.*

Proof Part a) Suppose $m = \sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) < 0$. Note that by the properties of the limit function and J is a continuous function (f and F are continuous and f is positive everywhere) we have

$$m = \sum_i^N t_i(\underline{v})(lim_{x\downarrow\underline{v}} J_i(x) - x_0) = \sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) < 0$$

Then, by definition of the limit, for all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x \in (\underline{v}\,, \underline{v} + \delta\,)$, $m - \varepsilon < \frac{\sum_i^N H_i(x)f_i(x)}{\sum_i^N H_i(x)f_i(x)}(J_i(\mathrm{x}) - x_0) < m + \varepsilon$. Since $m < 0$, we can pick ε small enough so that $\sum_i^N H_i(x)f_i(x)(J_i(x) - x_0) < 0$, and thus $-\sum_i^N H_i(x)f_i(x)(J_i(x) - x_0) > 0$ for all $x \in (\underline{v}\,, \underline{v} + \delta\,)$. But this last expression is the derivative of the revenue function, so a seller will strictly increase revenues by setting $r > \underline{v}$.

Part b) Assume that J_i is increasing for each i and $\sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) > 0$ and an effective reserve price $r > \underline{v}$ exists. By Lemma 1, this r must satisfy $\sum_i^N H_i(r)f_i(r)(J_i(r) - x_0) = 0$. But then for all $x \in (\underline{v}\,, r\,)$ we have $\sum_i^N H_i(x)f_i(x)(J_i(x) - x_0) < 0$ by the fact that J_i are increasing, and $H(.)$and $f(.)$are positive. But then the assumption $\sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) > 0$ cannot hold. This is a contradiction; hence, no effective reserve price exists.

Now suppose $\sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) = 0$ and an effective reserve price $r > \underline{v}$ exists. Again, by Lemma 1, this r must satisfy$\sum_i^N H_i(r)f_i(r)(J_i(r) - x_0) = 0$. But we have $\sum_i^N H_i(r)f_i(r)(J_i(r) - x_0) > 0$ by the fact that $\sum_i^N t_i(\underline{v})(J_i(\underline{v}) - x_0) =$

[5] We know that this condition exists for a large class of distributions given our examples. Even if it does not exist, our corollary below still gives the same result -despite with stronger conditions-.

0, J_i are increasing, and $H(.)$and $f(.)$are positive. Again, this is a contradiction; hence, no effective reserve price exists.

Before discussing the implications of the proposition, we give a corollary providing easier sufficient conditions to check.

Corollary 3 a) If $J_i(\underline{v}) < x_0$ for each i then $r > \underline{v}$.

b) Assume that J_i is increasing and $J_i(\underline{v}) > x_0$ for each i then $r = \underline{v}$. That is, effectively, there is no reserve price.

Proof Part a) If the seller values the object at x_0, the derivative of the expected revenue function becomes:

$$x_0 \sum_i^N f_i(r)H_i(r) + \sum_i^N H_i(r)(1 - F_i(r) - f_i(r)r)=$$

$$\sum_i^N H_i(r)\big(1 - F_i(r) - f_i(r)(r - x_0)\big) = -[\sum_i^N H_i(x)f_i(x)(J_i(x) - x_0)] \quad (5)$$

By the assumption $J_i(\underline{v}) < x_0$ and continuity of revenue function and the fact that derivative of the revenue function is zero exactly at $\underline{v}$, we can find a small $\varepsilon > 0$, $-\left[\sum_i^N \left(J_i(\underline{v} + \varepsilon) - x_0\right) f_i(\underline{v} + \varepsilon)\, H_i(\underline{v} + \varepsilon)\right]$>0 without making any assumption on the curvature of J_i. But then, by setting the reserve price slightly more than $\underline{v} + \varepsilon$ increases revenue.

Part b) The first order derivative of expected revenue function, $-\left[\sum_i^N H_i(\underline{v}) f_i(\underline{v})\left(J_i(\underline{v}) - x_0\right)\right]$, is zero. For any $r > \underline{v}$ in the support, the first order derivative is negative since $J(.)$is increasing and $J_i(\underline{v}) > x_0$, and H and f are positive. Since increasing the reserve price decreases the revenue, the optimal reserve price should be set at $r = \underline{v}$.[6] This completes the proof.

When will the seller not use the reserve price? $J_i(\underline{v}) > x_0$, implies that $1 < f_i(\underline{v})(\underline{v} - x_0)$ which in turn implies that $\underline{v} > x_0$. Hence, a necessary condition not to use reserve price is $\underline{v} > x_0$. Also, $f_i(\underline{v})$ should be big enough. This means, there has to be an enough mass of consumers with valuations close to $\underline{v}$. Our contribution is to explicitly state the sufficient conditions for not using **an** effective reserve price.

We believe that there are certain cases in which x_0 can be negative. For example, the seller may face a storage cost for keeping the object or a firm that is making a loss under the current management may have negative values to the seller. In these cases, the seller may not use the reserve price (or equivalently set the reserve price equal to $\underline{v}$).

Example (No reserve price): Assume two bidders. Their valuation is drawn from the support [4,5]. The first one has a uniform density function $f_1 = 1$. The

[6] The seller may set any reserve price less than or equal to $\underline{v}$ but this is effectively using no reserve price.

second one has a density function $f_2 = -3.5 + x$. The seller values the object at $x_0 = 1$. Then, the seller would not set an effective reserve price.

There are other papers in the literature that show the seller may use zero reserve price (e.g. Englebrecht-Wiggans (1987), McAfee and McMillan (1987) assume costly participation in terms of paying fee or buying information; Dastidar (2015) assume a model with non-quasi linear utility functions). Unlike all these papers, we find conditions when the seller uses (or does not use) the reserve price when she is dealing with asymmetric bidders. If the reserve price is not used, this implies that the auction is efficient. When using a reserve price is not optimal in a second price auction, we do not have to be concerned with the assumption of whether the seller can commit not to sell the object in a second auction.[7]

Now, we will find a sufficient condition for the uniqueness of the reserve price. We will introduce *N* distribution types that we will order from "strongest" to "weakest" in the sense of reverse hazard rate and hazard rate dominance. Of course, one could order them in the sense of likelihood ratio dominance, and this would imply both reverse hazard rate, hazard rate and first order stochastic dominance (Krishna, 2009:60,289). Specifically, we assume that $F_1/f_1 < F_2/f_2 < \cdots < F_N/f_N$ in the interior of the support.[8] In addition, we assume f_i nondecreasing which in turn implies that J_i are increasing. The proof of the proposition shows that the derivative of the revenue function is single-crossing zero from above -for the interior values-. This implies that the revenue function is quasi-concave, and hence, the reserve price is unique.

Proposition 4 Suppose that all f_i are non-decreasing and $J_i(\underline{v}) < x_0$ for each i (i.e., r> $\underline{v}$). There are N types of bidders ranked from strongest to weakest in the sense of both hazard rate dominance, $\frac{1-F_1(x)}{f_1(x)} > \frac{1-F_2(x)}{f_2(x)} > \cdots > \frac{1-F_N(x)}{f_N(x)}$ and reverse hazard rate dominance, $F_1(x)/f_1(x) < F_2(x)/f_2(x) < \cdots < F_N(x)/f_N(x)$ for $x \in (\underline{v}, \overline{v})$ and M_i bidders for each type. Then, there is a unique reserve price r that maximizes revenue which can be calculated from equation 2.

Proof of Proposition 4: From equation 2 in Lemma 1, the optimal reserve price satisfies: $\sum_{i=1}^{N} M_i H_i(r)\big(1 - F_i(r) - f_i(r)(r - x_0)\big) = 0$.

---

7 With the exception of McAfee and Vincent (1997) who assume the seller will re-auction the object if it is not sold, the literature generally assumes that the seller will make this "not re-auction" commitment credibly (e.g. Myerson, 1981).

8 Note that the way we write the inequalities are such that F_i dominates F_j for $i < j$ in terms of the reverse hazard rate. Also note that $F_i(\underline{v}) = 0$ and hence the whole term will be equal to zero for all i, so we define inequalities only in the interior of the support.

First note that hazard rate dominance (i.e. $\lambda_i(r) = \left(f_i(r)/\left(1 - F_i(r)\right) < \lambda_j(r)\right)$ implies first order stochastic dominance (i.e., $F_i(r) < F_j(r)$) in the interior of the support. This implies that $\left(1 - \lambda_i(r)(r - x_0)\right)\left(1 - F_i(r)\right) > \left(1 - \lambda_j(r)(r - x_0)\right)\left(1 - F_j(r)\right) \Leftrightarrow \left(1 - F_i(r) - f_i(r)(r - x_0)\right) > \left(1 - F_j(r) - f_j(r)(r - x_0)\right)$ for $i < j$. Therefore, there cannot be a unique r that makes $1 - F_i(r) - f_i(r)(r - x_0) = 0$ for all i.

It is clearly not the case that all terms in the first order condition sum, $\sum_{i=1}^{N} M_i H_i(r)\left(1 - F_i(r) - f_i(r)(r - x_0)\right)$, are positive or negative. So for some $j \in \{2, \dots, N-1\}$, $1 - F_k(r) - f_k(r)(r - x_0) > 0$ for all $1 \leq k \leq j - 1$ and $1 - F_k(r) - f_k(r)(r - x_0) < 0$ for $j \leq k \leq N$. From now on, j will denote that cutoff agent.

Take the derivative of this first order equation and get:

$$\sum_{i=1}^{N}\left[M_i h_i(r)\left(1 - F_i(r) - f_i(r)(r - x_0)\right) + M_i H_i(r)\left(-2f_i(r) - f_i'(r)(r - x_0)\right)\right].$$

Since the density functions are nondecreasing, we have $\sum_{i=1}^{N} M_i H_i(r)\left(-2f_i(r) - f_i'(r)(r - x_0)\right) < 0$, so we will ignore this part of the sum and focus on showing that $(*)$ $\sum_{i=1}^{N}\left[M_i h_i(r)\left(1 - F_i(r) - f_i(r)(r - x_0)\right)\right] < 0$.

We solve for $1 - F_i(r) - f_i(r)(r - x_0)$ from the first order condition and substitute this into $(*)$ to get:

$$\sum_{i \neq j}^{N} M_i h_i(r)\left(1 - F_i(r) - f_i(r)(r - x_0)\right) + M_j h_j(r)\left(-\sum_{i=2}^{N} M_i H_i(r)\left(1 - F_i(r) - f_i(r)(r - x_0)\right)/M_j H_j(r)\right)$$

We do not have to be concerned with division by zero since our assumptions ensure that $r > \underline{v}$ by proposition 3. After cancelling M_j in the right hand term and combining both terms, we are left with: $\sum_{i \neq j}^{N} M_i H_i(r)\left(h_i(r)/H_i(r) - h_j(r)/H_j(r)\right)\left(1 - F_i(r) - f_i(r)(r - x_0)\right)$

By using $h_k(r)/H_k(r) = \sum_{l \neq i} f_l(r)/F_l(r)$, we can re-write the term above as: $\sum_{i \neq j}^{N} M_i H_i(r)\left(f_j(r)/F_j(r) - f_i(r)/F_i(r)\right)\left(1 - F_i(r) - f_i(r)(r - x_0)\right)$

This is less than 0: The first $j - 1$ terms are less than 0 because for each i such that $1 \leq i \leq j - 1$, we have $f_j/F_j - f_i/F_i < 0$ (by reverse hazard rate assumption) and $1 - F_i(r) - f_i(r)(r - x_0) > 0$ (since j is the cutoff agent). The $j + 1$ to nth terms are less than 0 because for each i such that $j + 1 \leq i \leq$

$N, f_j(r)/F_j(r) - f_i(r)/F_i(r) > 0$ and $1 - F_i(r) - f_i(r)(r - x_0) < 0$. Therefore, any r that satisfies the first order condition must also satisfy the second order condition (concavity). This implies uniqueness of an optimal reserve price. To see why, suppose there are two distinct values r_1 and r_2 that maximize expected revenue and suppose without loss of generality that $r_1 < r_2$. Both of these points must satisfy the first order condition and hence the second order condition, as we have just shown. But by the differentiability (and hence, continuity) of the expected revenue function, this would imply that there is a point $r_3 \in (r_1, r_2)$ such that there is a local minimum at r_3 and the first order condition is therefore satisfied at r_3.[9] But this is a contradiction because we have just shown that any point that satisfies the first order condition must be a local maximum. Therefore, the optimal reserve price is unique. This completes the proof.

For example, let $F_N = x$ be uniform distribution function on $[0,1]$, $F_{N-1} = (F_N)^2 = x^2, \dots, F_1 = (F_N)^N = x^N$. This is the power distribution widely used in asymmetric bidders literature. (see Cantillon, 2008, for example). This example satisfies all conditions of our proposition 4; hence, the reserve price that satisfies equation 2 is the optimal unique reserve price. More examples can be created by replacing the uniform distribution in the example with any other distribution that has a non-decreasing density function.

3. Conclusion

We characterized the optimal reserve price for asymmetric bidders. In our model, we show that the unique reserve price is found by setting a weighted average of bidders' virtual valuations equal to the seller's valuation of the object.

Second, as a novel feature of the paper, we give sufficient conditions for the uniqueness of the optimal reserve price by ordering distribution types by hazard rate and reverse hazard rate dominance. One can order distribution types in the sense of likelihood ratio dominance, and this will suffice to have a unique optimal reserve price.

Finally, we showed when the seller will use a reserve price in an asymmetric auction (without assuming Myerson's regularity condition) and when she will not (by assuming Myerson's regularity condition). The conditions that make the seller abstain from using a reserve price are sufficient to make the auction efficient, since the object will always be sold to the bidder who values it most.

---

9 Since r_1 and r_2 satisfy the second order condition, the expected revenue function cannot be a line between r_1 and r_2.

References

CANTILLON, E. (2008) “The Effect of Bidders’ Asymmetries on Expected Revenue in Auctions”, *Games and Economic Behavior,* 62-1, 1-25.

DASTIDAR, K. G. (2015) “Basic Auction Theory Revisited”, *International Journal of Economic Theory,* 11, 89-106.

ENGLEBRECHT-WIGGANS, R. (1987) “On Optimal Reservation Price in Auctions”, *Management Science,* 33, 763-770.

ENGLEMAIER, F. AND SCHMÖLLER, A. (2010) “Determinants and Effects of Reserve Prices in Online Auctions”, *University of Munich Discussion Paper*. No: 326

KRISHNA, V. (2009) *Auction Theory,* Academic Press.

KUCUKSENEL, S. (2013) “The Effects of Outside Options on Optimal Auction Outcomes”, *METU Studies in Development,* 40-1,83-96.

LEBRUN, B. (1999) “First Price Auctions in the Asymmetric N Bidder Case”, *International Economic Review,* 40-1, 125-142.

LEVIN, D. and SMITH, J.L. (1996) “Optimal Reservation Prices in Auctions”, *Economic Journal,*106-438, 1271-1283.

MASKIN, E. and LAFFONT, J.J. (1980) “Optimal Reservation Price in the Vickery Auction”, *Economics Letters,* 6, 309-313.

MASKIN, E. and RILEY, J. (2000) “Asymmetric Auctions”, *Review of Economic Studies,* 67-3, 413-438.

MCAFEE, P. and MCMILLAN, J. (1987) “Auctions with Entry”, *Economics Letters,* 23, 343-347.

MCAFEE, P. and VINCENT, D. (1997) “Sequentially Optimal Auctions”, *Games and Economic Behavior,* 18-2, 246-276.

MYERSON, R.B. (1981) “Optimal Auction Design”, *Mathematics of Operation Research,* 6-1, 58-73.

PLUM, M. (1992) “Characterization and Computation of Nash-Equilibria for Auctions with Incomplete Information”, *International Journal of Game Theory,* 20-4, 393-418.

RILEY, J.G. and SAMUELSON, W.F. (1981) “Optimal Auctions”, *American Economic Review,* 71-3, 381-392.

Özet

Alıcıların asimetrik olduğu açık arttırmalarda rezerv fiyatın tekliği

İkinci fiyat müzayedelerinde, N sayıda alıcının özel değerlerinin değişik birikimli dağılım fonksiyonundan belirlendiği durumlarda, optimal rezerve fiyatını analiz ediyoruz. Satıcı, alıcıların özel değerlerinin hangi “birikimli dağılım fonksiyonundan” çekildiğini bilmemektedir. Bu durumda, optimal rezerv fiyatının tekliğini sağlayan yeterli koşulları gösteriyoruz. Daha sonra, satıcının rezerv fiyat kullanmayacağı durumlar için yeterli koşulları buluyoruz. Satıcı, rezerv fiyat kullanmadığı zaman müzayede etkin olmaktadır.

Anahtar kelimeler: Müzayede, rezerv (çekince) fiyatı, asimetrik teklif verenler.

JEL code: D44.