

# Originating long-term fixed-rate Mortgages in developing economies: New evidence from Turkey

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## Abstract

This paper intends to analyze the recent developments to introduce and integrate mortgage markets into capital markets of Turkey. The Capital Market Board has recently prepared a legal framework not only for a proper mortgage system, but also for the eventual securitization of these mortgages. Turkish banks started to contract, for the first time ever, long-term fixed rate mortgages. The paper uses traditional option-pricing model to evaluate the current 10-year fixed rate mortgage (FRM) contracts with embedded default and prepayment options in Turkey. Our study is the very first attempt to use an option pricing model to price the FRM contracts in an emerging economy with its different and unique dynamics. Our findings show that, in 2007 almost every bank, except for Is Bank, offered mortgage interest rates that were significantly below the equilibrium coupon rates, involving arbitrage profit for the borrowers. We also conclude that even if the prevailing mortgage interest rates are below the equilibrium rates, these rates are extremely high for establishing a well-functioning primary mortgage market in any economy. Finally, the effects of the global financial crisis are started to be felt in Turkish mortgage market as the banks have increased their mortgage coupon rates and shortened the contract maturities drastically over a very short time period, from September 2007 to November 2008.

*Key words:* FRMs, emerging economies, option pricing model, default option, prepayment option, Turkish mortgage market, explicit finite difference method.

*JEL classification:* G01 ; G13 ; G21 ; C63.

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## 1. Introduction

The nature of the housing market, the allocation of interest rate risk, and the economic and institutional structure all contribute to the diverse mortgage contract designs that are found worldwide. Because of the volatile macroeconomic environment and the lack of legal and regulatory framework that supports collateralized lending, the provision of housing finance in developing countries is often problematic.

Mortgage market development is likely to be a key factor in overall financial market development. In particular, an efficient mortgage market acts as a positive externality for the other capital markets, creating pressure for higher efficiency in these markets. On the other hand, a poorly functioning mortgage market is likely to pollute other financial markets with its inefficiency (Jaffe and Renaud, 1997). Efficient mortgage markets in developed economies require that the lending risk (credit, liquidity, and interest rate risk) be allocated to the long-term investors who are best able to handle them. In emerging economies, housing finance has remained in a primitive state compared to the rapid development of banking and other financial markets. Although foreign banks operate in emerging markets, they are mostly in the government bond business. The financial sector is far deeper than trading government bond and actual financial sector development requires lending to businesses, lending to housing, and lending to construction. Developing mortgage financing is a fundamental part of the financial sector development, because it stimulates growth through the construction sector and provides access to credit for more people.

In spite of the crucial role of mortgage financing in developing the financial sector of the emerging economies, there are not many published studies focused on examining the mortgage markets in developing economies and the performance of their mortgage products. Existing academic studies mainly give importance to the Latin American countries, especially the mortgage markets in Mexico and Chile.<sup>1</sup> A few studies analyze the mortgage markets in transition economies<sup>2</sup> and others examine development of housing finance in East Asia. Although East Asia is a young market, Hong Kong, Singapore, and even China, have deep primary residential mortgage markets in comparison to other emerging markets,

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<sup>1</sup> Siembieda and Moreno (1997), Lipscomb and Hunt (1999), Pickering (2000a-b), and Lipscomb et al. (2003) all examine the structure of the Mexican mortgage market and the performance of different loan contracts such as dual index mortgages and inflation-indexed mortgages. Alvaay and Schwartz (1996) analyse the housing finance policies in Chile. Ortega (2000) studies the development of Chilean mortgage market and focuses on the risk management in inflation-indexed mortgages, namely Unidad de Fomento (UF) indexed mortgage loans.

<sup>2</sup> Jaffe and Renaud (1997) discuss the main factors that hinder the development of mortgage markets in economies that are in transition from central planning to a market system. Chiquier (1998) evaluates the performance of dual index mortgages in Poland in comparison to the standard mortgage contracts.

which can be grouped as the Latin American countries, transition economies, and Middle Eastern and North African (MENA) countries.

This paper, firstly, intends to analyze the recent developments in Turkish mortgage market in line with the macro-economic progress in the country over the past few years. Turkey is a G-20 member Eurasian country and commands the youngest population in Europe.<sup>3</sup> Turkish economy has been growing by around 6 per cent a year for the last five years, which is faster than many developed economies and most emerging markets. The economic measures, taken after the financial crisis of 2001, have been very effective in subduing immensely the inflation, building investor confidence and attracting substantial and record amounts of foreign investments, and dampening the unemployment<sup>4</sup>. The recent improvements in the Turkish economy, especially the drop in the inflation rate has led the Capital Market Board to work on a draft of regulatory changes that would facilitate the legal environment for the establishment of the mortgage system. The efforts for the development of the mortgage system have attracted the construction sector and the related financial sectors. The result was the increase in the construction of the new housing units, the development in the mortgage products, and the significant decline in mortgage interest rates. We believe that a better understanding of this particular economy with its developing mortgage market may shed light on other countries at the similar stage of economic development.

Secondly, this paper uses traditional option-pricing model to evaluate the current 10-year fixed rate mortgage (FRM) contracts with embedded default and prepayment options in Turkey. Indeed, option pricing models have been extensively used for pricing the fixed-rate and adjustable-rate mortgages with prepay and default options.<sup>5</sup> It is important to point out that theoretical and empirical research on mortgage design and pricing has been conducted on the fixed and adjustable rate mortgages used in the United States and United Kingdom<sup>6</sup>. A key and one of the first questions for any aspect of research on emerging markets is whether the models and theories put forth and tested several times in developed financial markets also describe the realities observed in emerging markets (see, for a recent

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<sup>3</sup> 66.5% of the population in Turkey is within the 15–64 age group, the 0–14 age group corresponds 26.4% of the population, while 65 years and higher of age correspond to 7.1% of the total population, 2007 Census -population statistics, Turkish Statistical Institute.

<sup>4</sup> Turkey has steadily opened up its markets through economic reforms by reducing government controls on foreign trade and investments and the liberalization of many sectors to private and foreign participation has continued.

<sup>5</sup> McConnell and Muller (1988) provide an overview of mortgage pricing techniques. Hendershott and Van Order (1987) survey the development of the option-pricing approach to mortgage valuation and Kau and Keenan (1995) provide a later review of the academic literature.

<sup>6</sup> Recently, some researchers have focused on evaluating the mortgage contracts in Korea and Singapore by using the standard contingent claims approach. Chu *et al.* (2003) model the default risk of Singapore mortgages using the classical option pricing theory, and Ambrose and Kim (2003) use a contingent claims model for modeling Korean Chonseil lease contract.

example, Demirguc-Kunt et al leverage papers in WBER and JFE). To our knowledge, this paper is the very first attempt to use an option pricing model to price the FRM contracts in an emerging economy with its different and unique dynamics.

In particular, we use the structural approach (option pricing model) in order to answer the following interrelated questions:

1. Is Turkish economy ready for originating long-term FRMs?  
More specifically,

do the currently observed FRM coupon rates represent a fair transaction between the banks and the borrowers in Turkey?

Mortgage coupon rates and contract provisions vary widely over time. The economic environment changes continuously and contract specifications are also subject to frequent adjustments. In line with the literature, the equilibrium framework proposed in this paper claims that a contract can only be acceptable if it represents a fair deal. Specifically, we examine if the current mortgage coupon rates are below or above the equilibrium coupon rates which avoid arbitrage.

2. What will be the value of bank's (lender's) position if they have mortgage default insurance policy, especially for the high loan-to-value (LTV) housing loans?

In developed economies, the law requires lenders to obtain mortgage insurance (or mortgage default insurance) on loans where homebuyers make down payments of less than 20 per cent of the purchase price of the home (for above 80% LTV loans). Currently, Turkish banks do not require mortgage default insurance for high LTV mortgages. This paper aims to price mortgage insurance product as a potential financial derivative in the Turkish capital markets.

Our findings show that almost every bank, with the exception for Is Bank, offers mortgage interest rates that are significantly below the equilibrium coupon rates, involving arbitrage profit for the borrowers. Thus, we conclude that these contracts do not represent a fair deal between the lenders and borrowers in Turkey. We also argue that even if the prevailing mortgage coupon rates are below the equilibrium rates, 20%-25% mortgage interest rates per annum are extremely high for establishing a well-functioning primary mortgage market in any economy. In addition, our findings show that it is highly beneficial for the lenders to have mortgage default insurance, especially for the high LTV ratio mortgages.

The rest of this paper is organized as follows. Section 2 reviews option pricing literature for evaluating the default and prepayment options of mortgage contracts. Section 3 provides information on the recent developments in Turkish mortgage market. Section 4 presents the classical option pricing model for the valuation of FRM contract with embedded default and prepayment options. Section 5 presents the discussion of the numerical results. Finally, section 6 offers concluding remarks.

## 2. Review of the literature

The extant literature shows that two main approaches have emerged on modeling mortgage termination either by prepayment or default: reduced-form and structural (options-pricing) models.

Termination may occur for non-financial (personal) reasons, such as divorce, a new job, or death in the family, causing a borrower to change the residence. Reduced-form approach treats non-financial termination behavior by modeling the termination decision as a function of a set of exogenous variables representing the factors that influence the likelihood of mortgage termination. Well-known reduced-form mortgage termination models include Schwartz and Torous (1989), Deng *et al.* (2000) and Deng (1997). The main advantages of this approach are flexibility and the ability to closely mimic the historical data record of mortgage terminations. However, reduced-form approach has low out-of-sample forecasting power. In addition, these models are often not well suited for valuation, to the extent that mortgage prices for the prepayment and default option values are included in the set of exogenous variables used to predict terminations (Downing *et al.*, 2005).

There are also financial reasons to terminate a mortgage contract that apply equally to all individuals. These are the terminations that lower the market cost of the mortgage contract for the borrower. The structural approach or the traditional option-pricing approach treats mortgage termination as the optimal response of a rational borrower to changes in interest rates and house prices, in order to minimize the market value of the loan. This modeling approach was first applied to mortgages by Dunn and McConnell (1981a, b) who modeled the optimal termination behavior of borrowers who could costlessly prepay, but not default.

According to the structural approach, well-informed borrowers in a perfectly competitive market will exercise either of the two options when they can increase their wealth. Absent either transaction costs or reputation costs that reduce credit ratings, these individuals can increase their wealth by defaulting on a mortgage when the market value of mortgage exceeds the value of the property. Similarly, by prepaying the mortgage when the market interest rate is below the contract rate, they can increase wealth by refinancing. However, a borrower who chooses to prepay the mortgage gives up the opportunity to exercise either prepayment or default option in the future. Likewise, a borrower who decides to default on the mortgage also forfeits the opportunity to exercise the prepayment or default option in the future.

Titman and Torous (1989) and Kau *et al.* (1992, 1995) examine structural approach and emphasize the importance of the joint-ness of prepayment and default options. In a mortgage contract, borrowers have the right to prepay a mortgage, but to rule out the possibility of default, or to consider default while ruling out the possibility of prepayment. Since

prepayment and default substitute for one another, contracts with only one of these options lead the borrower to behave differently from when both are present. This substitution effect means that one cannot accurately value either the individual provisions or their interaction without both options being present. A series of papers by Kau *et al.* (1993) and Azevedo-Pereira *et al.* (2002, 2003) evaluate the US ARMs and the UK FRMs, respectively, with the embedded default and prepayment options. Without using loan-base data to decide the pattern of termination, these studies provide theoretical valuation models, which impose termination decision endogenously, and use numerical methods in order to price the jointly exercise of the embedded options.

Well-specified structural models should perform well out of sample because termination behavior arises from borrowers' optimizing behavior. Downing *et al.* (2005) claim that the basic problem with these models is that, they predict that a mortgage can never trade above par. This is because borrowers will exercise their prepayment option the instant that the mortgage value exceeds par –what is often referred to as “ruthless option exercise. On the other hand, Kau *et al.*, (1992) state that while people prepay mortgages for personal reasons, they seldom default for this reason. Default is unlikely to occur unless the house value is less than the market value of the loan. Hence, financially induced termination seems particularly important in the case of default.

Over the past two decades, option pricing models (structural approach) have (has) been mainly used for pricing the fixed- and adjustable rate mortgages in developed economies, especially in the US and UK. This paper uses the traditional option-based pricing model to price FRM contracts in an emerging economy. The main objective of this study is to price both the default risk and the prepayment risk of the FRMs, from the lenders' perspective, using the well-known option pricing model.

### 3. Recent developments in Turkish mortgage market

Real estate is one of the most important sectors of the Turkish economy. In recent years, this sector has made up of about 10 percent of the country's Gross National Product (GNP), which has grown to 539.9 Billion TRY (US\$ 381 Billion) in 2006. One of the reasons for this significant increase is that Turkey is a dynamic and recently strongly performing emerging market economy. In spite of the historically high demand for real estate assets, a well-organized and deep enough mortgage market did not exist in Turkey until quite recently. The absence of an efficient mortgage market was mainly due to a long-running process of persistently high inflation<sup>7</sup>, the inability of the banks to fund mortgages from their deposit base, and the lack of standardization within the title and appraisal systems.

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<sup>7</sup> See Erol and Patel (2005) for failed attempts to introduce mortgages during the high inflation era.

In fact, the inflation rate in Turkey has stabilized within a band of 15%-20% within the last few years. The economy has been growing by around 6 per cent a year for the last five years, which is faster than many developed economies and most emerging markets. These recent improvements in the macro-economy enabled the Turkish finance sector to offer long-term funding at relatively cheap prices for the first time in history. According to the Banks Association of Turkey, mortgages represented only 7.64 per cent of the overall consumer loan portfolio in 2003, whereas over the first nine months of 2008 mortgages have a share of 29.5 per cent.<sup>8</sup>

Under the stable economic conditions of the recent years, the current government has recently prepared a legal framework not only for a proper mortgage system, designed especially for financing residential real estate for the middle-income households, but also for the eventual securitization of these mortgages. The Turkish Parliament ratified recently (March, 2007) this legal framework into law. Hence, the current government and the Capital Market Board initiated a legal framework to contract, for the first time ever, long-term fixed rate mortgages.

Turkey experienced strong growth in mortgage debt over the last few years. From 2002 to 2006, there has been continuous growth in the level of outstanding balances of mortgages at a compounded average growth rate of 191.5%. ([www.datamonitor.com](http://www.datamonitor.com)). Table 1 presents the mortgage debt as a percentage of GDP both for European Union (EU) new member and candidate countries at the end of 2006. As of December 2006, the value of mortgage debt in Turkey was approximately 12.24 € billion, which made it the second largest mortgage market among the new members and candidate members of the EU<sup>9</sup>. Mortgage debt-to-GDP ratio for Turkey (3.8%) was greater than those for Romania, Serbia, and Ukraine, and Turkey has experienced 65.7% growth in mortgage debt in December 2006 compared to the previous year.<sup>10</sup>

Almost all of the mortgages are originated from deposit banks in September 2007.<sup>11</sup> While private deposit banks originate 61% of the mortgages, public deposit banks initiate only 15% of the mortgages. And the remaining 24% of the mortgages are originated by the foreign banks. Examining the percentage shares of the banks in mortgage lending activity,

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<sup>8</sup> In 2003, the mortgage loans granted were 800.6 million TL while total consumer credits granted were 10,478 million TL. Over the first nine months of 2008, total consumer credits and mortgage loans granted were 46,727 million TL and 13,779 million TL, respectively ([www.tbb.org.tr](http://www.tbb.org.tr)).

<sup>9</sup> According to the Banks Association of Turkey, the market value of the mortgages was approximately 20.67 billion TRY as of September 2007

<sup>10</sup> Mortgage debt outstanding was a mere 0.224% of the GDP in 2003 ([www.tba.gov.tr](http://www.tba.gov.tr))

<sup>11</sup> At the end of September 2007 the total value of mortgages was 23,031,949 Thousand TL. Deposit banks originated 23,002,309 Thousand TL, which makes 99.87% of the overall mortgage loans granted. The remaining 0.13% of the mortgages was originated by Investment and development banks ([www.tbb.ogov.tr](http://www.tbb.ogov.tr)).



we observe that with a 15.2% share of the total mortgage debt Akbank stands out as the mortgage industry leader. Garanti Bank, which has the second largest share of the overall mortgage lending activity with a 14.7% share, is followed closely by Is Bank with 14% share of mortgage lending. Finans Bank, Vakıflar Bankası, and Yapi Kredi Bankası have 11.9%, 9.4%, and 7.9%, respectively. It is worth to note that with the exception of Vakıf Bank, a public deposit bank, all banks are private deposit banks in Turkey.

**Table 1**  
Mortgage Debt-to-GDP (%), End of 2006

	Value of Mortgage Debt, € million	Growth in Mortgage Debt	Residential Debt to GDP Ratio	Per Capita Mortgage Debt, € thousand
Bulgaria (2007)	1,745	73.5%	7.0%	0.2
Czech Republic (2004)	8,055	33.9%	7.1%	0.8
Estonia (2004)	4,278	63.4%	32.7%	3.2
Cyprus (2004)	3,077	43.5%	21.2%	4.0
Latvia (2004)	4,680	86.5%	28.9%	2.0
Lithuania (2004)	2,997	32.1%	12.6%	0.9
Hungary (2004)	10,215	11.0%	11.4%	1.0
Poland (2004)	22,514	53.7%	8.3%	0.6
Romania (2007)	2,276	57.2%	2.3%	0.1
Slovenia (2004)	1,956	43.0%	6.6%	1.0
Slovakia (2004)	4,209	36.7%	9.6%	0.8
Croatia (candidate)	5,219	37.2%	15.3%	1.2
Serbia (candidate)	650	111.7%	2.9%	0.1
Turkey	12,237	65.7%	3.8%	0.2
Ukraine (candidate)	4,301	157.6%	2.6%	0.1

Source: Eurostat, National Central Banks, EMF.

Deposit banks, as the main mortgage lenders, extend loans to borrowers who wish to purchase a single-family detached/semi-detached/apartment style houses. While the lenders generally rely on the appraisal company's determination of the eligibility of the property subject to transaction, some lenders have their own staff to do the appraisal work. Currently, Turkish banks offer a variety of mortgage products including Turkish Lira (TRY)-denominated fixed-rate, adjustable rate, and graduated payment mortgages and US Dollar-, and, Euro-denominated mortgages. The most popular mortgage products are fixed rate mortgages (FRMs) with 60 to 120-month contract maturity, and the prevailing mortgage coupon rates range from 1.2 to and 1.53 percent in September, 2007.

As the FRMs are popular mortgage products over the past few years, this paper concentrates on pricing the typical constant-payment mortgages, or fixed-rate mortgages based on structural option pricing models. In order



to determine the basic FRM contract to price in our study, we collect information on the FRM contract details of eight deposit banks with the largest mortgage portfolios. Namely; Oyak Bank, Vakıf Bank, Akbank, İş Bank, HSBC Bank, Finans Bank, Yapı Kredi Bank, and Garanti Bank. More specifically, we collect data for the contract maturity, coupon rate, Loan-to-Value (LTV) ratio, arrangement fee, prepayment penalty, and the available insurance policies of these deposit banks. It is important to note that market value of these banks' mortgage portfolio consists of 91.2% of the overall mortgage portfolio in Turkey as of September, 2007.

Table 2 illustrates that, with the exception of Finansbank and Yapı Kredi Bank, the maximum Loan-to-Value (LTV) ratio is 75% for the FRMs. Finansbank and Yapı Kredi Bank originate FRMs with a maximum LTV of 95% to 100%. The amount of upfront arrangement fee significantly varies among the banks. While Finansbank does not charge any arrangement fee, other banks may charge 1% to 5% of the loan amount as the arrangement and service fee. All the banks except for Yapı Kredi Bank charge a prepayment penalty of 2% of the outstanding loan balance at the time of prepayment.

In terms of insurance policies, hazard and earthquake insurance is required by all lenders. This has been a requirement since 1999 and is provided by Turkish Catastrophe Insurance Pool (TCIP). TCIP takes the first loss position and private insurers take the second loss position. The annual premiums due to TCIP are collected by private insurance companies from the home owners and then forwarded to TCIP. Earthquake insurance rates are not fixed. They are determined according to the type of dwelling and the earthquake zone it is in. Most of the lenders also require a life insurance policy that would remain in effect over the term of the mortgage. Such a policy would help to cover the full repayment of the loan in the event of borrower's death. Borrowers are required to renew their policy annually (at least during the term of the loan). Mortgage default insurance products are not prevalent in Turkey.<sup>12</sup> The existing sectoral studies suggest that there is no urgent need for mortgage insurance as this will increase the cost of funds for borrowers.

#### 4. Option pricing model (structural approach) for the valuation of fixed rate mortgage (FRM) contracts

It is widely accepted that a successful way of pricing mortgages is to view them as ordinary debt instruments with specific options attached to them. Default in the mortgage contract can be viewed as a put option since

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<sup>12</sup> Recently, a number of banks (Finansbank, İş Bank and Vakıfbank) have started to ask for mortgage payment protection insurance from the borrowers in the case of being unemployed or injured. The insurance policy generally pays up to six monthly payments to the bank. However, this product is different from mortgage default insurance that is widely used in the US and UK mortgage markets.

**Table 2**  
**Contract Details for FRMs Originated by the Largest Deposit Banks in Turkey (September, 2007)**

Bank	Contract maturity (months)	Coupon rate (%)	LTV Ratio	Up-front arrangement fee	Prepayment penalty	Appraisal of the property	Insurance services
Finans Bank	60 – 360	1.24 – 1.29	Min: 75% Max: 95% to 100%	No fee	2%	Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: Yes Accident insurance: Yes Mortgage insurance: No
Oyak Bank	60 -144	1.35	Max: 75%	Included in coupon rate	2%	Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: Yes Accident insurance: Yes Mortgage insurance: No
HSBC Bank	60 – 360	1.30 -1.44	Min: 5,000 YTL Max: 75%	1%-2% of Loan Amount	2%	Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: No Accident insurance: Yes Mortgage insurance: No
Akbank	60 -240	1.21 -1.31	Max: 75%	2%-3% of Loan Amount	2%	Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: Yes Accident insurance: Yes Mortgage insurance: No
Yapı Kredi Bank	60 -240	1.16 -1.33	Max: 100%	1%-5% of Loan Amount	1.5%	Bank (portfolio managers)	Hazard -Earthquake insurance: Yes Life insurance: Yes Accident insurance: Yes Mortgage insurance: No
Garanti Bank	60 -240	1.34	Max: 75%	1000 YTL	2%	Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: No Accident insurance: No Mortgage insurance: No
Vakıf Bank	60 -240	1.30	Max: 75%	2% of Loan Amount	2%	Bank's own Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: Yes Accident insurance: No Mortgage insurance: No
İş Bank	48 -120	1.46 -1.53	Max: 75%	1% -1.5% of Loan Amount	2%	Bank's own Appraisal firm	Hazard -Earthquake insurance: Yes Life insurance: Yes Accident insurance: Yes Mortgage insurance: No

Source: The web sites of the Deposit Banks.

by defaulting the borrower disposes of the housing asset. In other words, the borrower sells his housing property back to the lender in exchange for eliminating the mortgage obligation. The default option to terminate the mortgage contract fits into a European option framework because no rational borrower would ever choose to default until a payment is due. Prepayment can be considered as an American-style call option, in which the borrower has the right to gain the house at any time by paying off the mortgage loan. The borrower exchanges the unpaid balance on debt instrument for a release from further obligation. Thus, the analogy between a mortgage on a property and an option on a stock is quite close.

Pricing these options and also determining when a borrower exercises either option require specifying the underlying state variables and parameters that determine the value of the contract, and then deducing the decision rule that maximizes borrower wealth. The structural approach assumes that uncertainty about the returns from a mortgage can be summarized by two state variables: the value of the mortgaged housing property,  $H$ , whose dynamics follow a lognormal diffusion process (Equation 1) and the instantaneous risk-free interest rate,  $r$ , which evolves according to a mean-reverting square root diffusion process, Cox, Ingersoll and Ross, 1985b.

$$\frac{dH}{H} = (\mu - s)dt + \sigma_H dz_H \tag{1}$$

The return to owning the housing property consists both of price appreciation and of a service flow. Since the householder receives benefit from living in the house, the term  $s$  is included to denote the constant rate of service flow, or value of implicit rent, from the house. The instantaneous average rate of house price appreciation is denoted by  $\mu$ , and  $\sigma_H$  represents the volatility of disturbances in actual house price appreciation around the trend rate  $(\mu - s)$ , and  $z_H$  is the standardized Wiener process that drives the uncertainty in house prices.

The stochastic process for the instantaneous default-free nominal interest rate, which follows the Cox-Ingersoll-Ross (1985b) model, known as CIR model, can be written as

$$dr = \kappa(\theta - r)dt + \sigma_r \sqrt{r} dz_r \tag{2}$$

where  $\theta$  represents the long-term mean value for the interest rate  $r$ , and  $\kappa$  is the speed of adjustment in the mean reverting process. The standard deviation of the interest rate disturbance is denoted by  $\sigma_r$ , and  $z_r$  is the standardized Wiener process.

The unexpected changes in the value of the housing property are assumed to be correlated with unanticipated changes in the instantaneous

risk free interest rate,  $dz_H(t)dz_r(t) = \rho dt$  where  $\rho$  denotes the instantaneous correlation coefficient. With stochastic processes specified by Equations (1) and (2), the fundamental partial differential equation (PDE) for the valuation of mortgages as a function of time and of the stochastic variables of housing price and the interest rate takes the form<sup>13</sup>

$$\begin{aligned} \frac{1}{2}H^2\sigma_H^2\frac{\partial^2V}{\partial H^2} + \rho H\sqrt{r}\sigma_H\sigma_r\frac{\partial^2V}{\partial H\partial r} + \frac{1}{2}r\sigma_r^2\frac{\partial^2V}{\partial r^2} + \kappa(\theta - r)\frac{\delta V}{\delta r} \\ + (r - s)H\frac{\delta V}{\delta H} + \frac{\delta V}{\delta t} - rV = 0 \end{aligned} \quad (3)$$

where  $V$  represents the mortgage value. Equation (3) implies that with a continuous time, a standard arbitrage argument is sufficient to derive an equilibrium condition for the value of mortgage (a second order partial differential equation) such that the value of the mortgage equals the risk-adjusted expected present value of its net cash flows.

The tendency to price derivative assets, such as mortgage on a property, and to relax the strongest assumptions in order to make the models approach to the reality leads to the development of valuation frameworks of enlarged complexity for which no closed-form solutions are available. Thus, in many complex but realistic problems the analyst must resort to other methods to approximate the value of the asset. Three basic methods are; the Monte Carlo method (forward-pricing method) advocated by Boyle (1977), and finite difference approximation to the differential equation (backward-pricing method) suggested by Schwartz (1977) and employed extensively by Brennan and Schwartz (1976, 1977, and 1978), and lattice (or tree) approach suggested by Cox et al. (1979) and extended by Rendleman and Bartter (1979), Boyle (1986, 1988), and Hull and White (1988). As a result, there is considerable amount of research employing numerical methods to approximate solutions of the valuation of contingent contracts, when analytic solutions do not exist.

Despite the recent advances in forward pricing methods for pricing American options, backward pricing method is well established, and so has been used more extensively. Although it is computationally more complex by including dynamic programming, many researchers adopt backward pricing approach as the appropriate procedure to valuing mortgages with embedded default and prepayment options. Kierkegaard (1967) said that to understand life we must look backward but that we are doomed to live life forward. Much the same is true of mortgages. While time undoubtedly marches forward, to value a mortgage with termination, we must begin at the end and work back. When borrowers at a point of time consider whether to terminate a mortgage, they look toward future values, but because we are

<sup>13</sup> The derivation of fundamental partial differential equation follows from standard arguments in finance. See Cox, Ingersoll, and Ross 1985a, 1985b; Epperson *et al.* 1985, and Kau *et al.* 1992, 1993, 1995.

working backward, we will have the needed values available when we reach that point (Kau and Keenan, 1995).

Given the specific details of the contract, the values of the financial assets embedded in a mortgage (default option, prepayment option, insurance product) are known at the expiry. Using appropriately small time steps, equation (3) can be used to work backwards from the final mortgage payment, calculating the asset values sequentially to the previous mortgage payment, then using that new set of terminal conditions to work back to a still earlier payment until eventually the origination of the contract is reached (Azevedo-Pereira et al. (2002, 2003).

*4.1. Components of the mortgage contract*

The value of a mortgage to the borrower is composed not only of the present value of promised future monthly payments to the lender but also of options to prepay or to default. These options are valuable to the borrower and reduce the absolute value of the outstanding mortgage. At some point in time,  $t$ , the borrower’s joint option value,  $J$ , is equal to the summation of the value of call option to prepay,  $C$ , and the value of default option,  $D$ .

$$J(H, r, t) = C(H, r, t) + D(H, r, t) \tag{4}$$

The value of the mortgage contract to borrower is given by equation (5):

$$\begin{aligned} V_B(H, r, t) &= A(r, t) - C(H, r, t) - D(H, r, t) \\ &= A(r, t) - J(H, r, t) \end{aligned} \tag{5}$$

where  $A(r, t)$  is the present value of remaining mortgage payments. If the lender (bank) has mortgage default insurance, and circumstances arise in which a rational borrower chooses to default, then mortgage default insurance product only benefits the lender. The value of the contract for the lender, therefore, is the sum of its value to the borrower and the value of the mortgage insurance.

$$V_L(H, r, t) = V_B(H, r, t) + I(H, r, t) \tag{6}$$

where;  $I(H,r,t)$  =the value of the mortgage default insurance at time  $t$ .

In a typical constant payment, fixed rate mortgage contract, the value of each monthly payment,  $MP$ , is determined in order to allow the principal to be paid in full by the end of the contract. That is;

$$MP = \frac{\left(\frac{c}{12}\right) \left[1 + \left(\frac{c}{12}\right)\right]^n OB(0)}{\left[1 + \left(\frac{c}{12}\right)\right]^n - 1} \tag{7}$$

where  $OB(0)$  represents the amount of debt at the origination of the loan, and  $c$  is the annual mortgage coupon (interest) rate. The outstanding loan balance after each payment date,  $OB(t)$ , is given by the following equation:

$$OB(t) = \frac{\left[1 + \left(\frac{c}{12}\right)\right]^n - \left[1 + \left(\frac{c}{12}\right)\right]^t}{\left[1 + \left(\frac{c}{12}\right)\right]^n - 1} OB(0) \quad (8)$$

At payment dates, a distinction will be made between the value of an asset immediately before and immediately after each payment. In line with the literature we use the following notation:

$F^-(H,r,t)$  = Value of the asset  $F$  immediately before a payment is made;

$F^+(H,r,t)$  = Value of the asset  $F$  immediately after a payment is made.

In backward pricing methods, the valuation of the mortgage begins at the maturity of the mortgage. At the maturity, the terminal condition requires that the value of remaining payments must be equal to the final monthly payment due,  $MP$ .

$$A^-(r,t) = MP \quad \text{for } t = \eta(n) \quad (9)$$

where  $n$  = life of the mortgage in months. Moving backwards in time, as each monthly payment date is reached, the borrower's debt,  $A$ , changes immediately by the amount  $MP$ . This leads to solution of equation (3) by a finite difference method, starting with the terminal condition at maturity, working backwards in time until the next monthly payment date is reached. Then, a new boundary condition, equation (10), is applied and the backwards process is continued until valuation has been completed, at the origination time of the mortgage contract.

$$A^-(r,t) = A^+(r,t) + MP \quad \text{for } t = \eta(1), \dots, \eta(n-1) \quad (10)$$

During this valuation process, we also apply boundary conditions for the options held by the borrower. The value of default option depends directly on the house price. If the house price is different from the value of the remaining payments, the financially rational borrower either does nothing, or sells, or defaults and gives up the house to the lender if that proves to be the most advantageous solution from a financial point of view. The value of the prepayment option depends on the prevailing term structure of interest rates but not directly on the house price. However, there is an indirect relationship, since the exercise of the option to default automatically causes the prepayment option to expire worthless. Thus, the two options interact and cannot be separately valued and added. At expiry of the mortgage, the borrower holds the house and has an obligation to make the last mortgage payment but she also has a put option on the house

$D(H, w, t)$  allowing her to default and give up the house if she wishes. Therefore, the position of the borrower at maturity is,  $H + D(H, w, t) - MP$  with the following mortgage value:

$$V_B^-(H, r, t) = \min(MP, H) \quad \text{for } t = \eta(n) \tag{11}$$

Similarly, at any other payment date,  $t$ , the value of the mortgage to the borrower is given in equation (12).

$$V_B^-(H, r, t) = \min[V_B^+(H, r, t) + MP, H] \quad \text{for } t = \eta(1), \dots, \eta(n-1) \tag{12}$$

If the borrower prepays the mortgage, the amount to be paid is calculated from the outstanding balance and the accrued interest since the most recent scheduled payment. In Turkey, similar to the UK mortgage market, there is an additional penalty payment required in the terms of the fixed-rate mortgage contracts. Since the Turkish banks, in general, charge 2% of the loan balance as the prepayment penalty, in this paper penalty is modeled as a percentage of the outstanding balance plus accrued interest at the time of early termination (See Azevedo-Peraira, 2002 and 2003).

$$TD_t = \left\{ (1 + \pi) [1 + c(t - \eta(i))] OB(i) \right\} \quad \text{for } \eta(i) \leq t \leq \eta(i+1) \tag{13}$$

where  $TD(t)$  is the total outstanding debt,  $\pi$  represents the early termination penalty charged by the lender and  $c$  is the fixed coupon rate.

The default decision is assumed not to be simply triggered if the present value of the remaining payments exceeds the current market value of the house, but rather if the value of the mortgage to the borrower including options, exceeds the house value. Thus, a rational borrower defaults if the following condition is arises:

$$A(r, t) - [D(H, r, t) + C(H, r, t)] > H \tag{14}$$

At the maturity of the mortgage, when the borrower decides on whether or not to make the final mortgage payment, the default option will be worthless if the house is worth more than the final payment and otherwise equal to the difference between the two. That is;

$$D^-(H, r, t) = \max[0, (MP - H)] \quad \text{for } t = \eta(n) \tag{15}$$

On monthly payment dates other than the maturity, the default option value is adjusted for the difference between value of the remaining payments and the house price when there is default, and remains unchanged by the payment under conditions of no default.

$$D^-(H, r, t) = D^+(H, r, t) \quad \text{if } V_B^-(H, r, t) = V_B^+(H, r, t) + MP \quad (\text{no default})$$



$$A^-(r,t) - H \quad \text{if } V_B^-(H,r,t) \leq H \quad (\text{default}) \quad \text{for} \\ t = \eta(1), \dots, \eta(n-1) \quad (16)$$

The terminal condition for the prepayment option at maturity is unimportant. This is because exercising the prepayment option cannot have any value for the borrower. In addition, at any other payment date, prepayment can only have value in the absence of default and so the terminal conditions for the prepayment option are as follows:

$$C^-(H,r,t) = \\ C^+(H,r,t) \quad \text{if } V_B^-(H,r,t) = V_B^+(H,r,t) + MP \quad (\text{no default}) \\ 0 \quad \text{if } V_B^-(H,r,t) = H \quad (\text{default}) \quad \text{for} \\ t = \eta(1), \dots, \eta(n-1) \quad (17)$$

Alternatively, prepayment option value can be calculated as;  $C = A - V_B - D$ . Lastly, we model the mortgage default insurance as another component of the FRM contract. We basically follow the US mortgage insurance system in order to evaluate the insurance product as another financial derivative asset. At the contract maturity, the value of the mortgage insurance is given in equation (18).

$$I^-(H,r,t) = \max \left[ 0, \min (MP - H, \phi MP) \right] \quad (18)$$

If the borrower chooses to default on the mortgage and does not repay the last monthly payment, mortgage insurance company pays to the lender either the difference between the house price and the last monthly payment ( $MP-H$ ) or a specific proportion ( $\phi$ ) of the outstanding loan balance. In general, insurance companies pay the minimum amount of these two values. Thus, in the US system, the insurance coverage seems to be a simple pre-determined percentage of the value of the outstanding debt (see Kau *et al.*, 1993). The US insurance companies have, in general, a minimum insurance coverage of 25% of the outstanding balance.

At other payment dates during the contract term, the value of mortgage insurance is given in equation (19).

$$I^-(H,r,t) = \begin{cases} I^+(H,r,t) & \text{if } V_B^-(H,r,t) = V_B^+(H,r,t) + MP \quad (\text{no default}) \\ \max \left[ 0, \min (TD^-(t) - H, \phi TD^-(t)) \right] & \text{if } V_B^-(H,r,t) = H \quad (\text{default}) \end{cases}$$

$$\text{for } t = \eta(1), \dots, \eta(n-1) \quad (19)$$

where

$TD(t)$  = the total outstanding debt at time  $t$

$\phi$  = minimum insurance coverage ratio – generally 25% in the US (see Kau *et al.* 1993, Journal of Business, pp: 595-618).

Although the Turkish banks do not have a mortgage default insurance product, they usually ask borrowers to get a life insurance before granting any mortgage. Banks provide life insurance to the borrowers either by increasing the loan amount by the insurance expense and originate a higher LTV mortgage or by taking part of the loan as an insurance expense at the loan origination. The latter increases the up-front arrangement fee and provides a higher yield to the bank.<sup>14</sup> In the next section, we analyze the effect of life insurance policy on the lender’s position within 1% to 5% arrangement fee applications. We find that the banks have higher yields or offer lower mortgage interest rates by charging higher arrangement fees at the loan origination.

4.2. *No arbitrage condition*

Mortgage coupon rates and contract terms vary widely, especially in volatile economic environments, over time. In particular, the economic conditions change continuously and contract specifications are also subject to frequent readjustments. Following the extant literature, equilibrium framework proposed in this paper claims that a contract can only be acceptable if it represents a fair deal. It is necessary to ensure that the borrower is not able to make an instantaneous profit by prepaying the loan and, similarly that the contract is not structured in such a way that allows the lender to make any immediate profit. This is called “no arbitrage” condition.

The values of the two state variables in the model,  $r(0)$  and  $H(0)$ , are known at the origination of the mortgage; therefore, the determination of an equilibrium coupon rate is an iterative exercise in which, starting with the initial values for the state variables and the functional form specification for the contract, a search is done to find a coupon rate capable of allowing the mortgage contract to meet the condition of no arbitrage (Azevedo, et al. 2002, 2003). At the origination of the mortgage, the equilibrium condition which avoids arbitrage is shown in equation (20), where  $\xi$  represents arrangement fee,  $\Pi$  gives early termination fee, and  $L$  is the amount of the loan. Following Azevedo-Pereira et al. (2002, 2003), in order to find the equilibrium coupon rate, a secant iteration technique was used,

$$V_B(H(0), r(0), t(0), c, \pi) + I(H(0), r(0), t(0), c, \pi) = (1 - \xi)L \quad (20-a)$$

or

$$V_B(c, \pi) + I(c, \pi) - (1 - \xi)L = 0 \quad (20-b)$$

---

<sup>14</sup> The yield on a mortgage will not always equal its interest rate. Suppose the lender charges the borrower 1% of the 100,000 TL loan amount up front, just to grant the loan. With 1% origination fee (as arrangement fee or life insurance expense), the lender will actually disburse to the borrower only 99,000 TL, even though the contractual principal and the initial outstanding loan balance is 100,000 TL. Thus, the effect of the origination fee is to increase the mortgage yield to maturity over the stated contract interest rate in the loan (See Geltner and Miller, 2001; p: 420).

## 5. Results of the numerical analysis

For the numerical analysis of FRM valuation model, we specify the basic set of economic parameters and the contract provisions as presented in Table 3. A 10-year FRM contract originated for a 100,000 TRY house is evaluated in our analysis. We present numerical results for two contract specifications. First, a mortgage with an arrangement fee but no early termination penalty, secondly, one with an arrangement fee and an early termination penalty. We use daily 3-Month Turkish Treasury-Bill yields obtained from the secondary market to estimate the parameters of the CIR process. Data set covers the period between February, 2002 and November, 2007 (See Appendix for the estimation of CIR process for the Turkish economy). We calculate the historical volatility of house price using the monthly House Price Index published by the Turkish Statistics Institute between January 2002 and December 2007. Finally, the spot rate is 15% for the base case economic environment.

**Table 3**  
Basic Set of Economic Parameters and Contract Provisions

Base Case Parameter Values	CONTRACT	
	Fixed-rate Mortgage With Arrangement Fee and Without Early Termination Penalty	Fixed-rate Mortgage With Arrangement Fee and Early Termination Penalty
<b>Economic Environment</b>		
Spot interest rate, $r(0)$	15%	15%
Long term average of interest rate, $\theta$	24%	24%
Speed of reversion, $\kappa$	56%	56%
House service flow, $\delta$	4%	4%
Correlation coefficient, $\rho$	0	0
<b>Contract Provisions</b>		
Maturity, $\eta$	120 months	120 months
Value of house at origination, $H$	100 000 TRY	100 000 TRY
Arrangement fee, $\xi$	2%	2%
Early termination penalty, $\pi$	-	2%

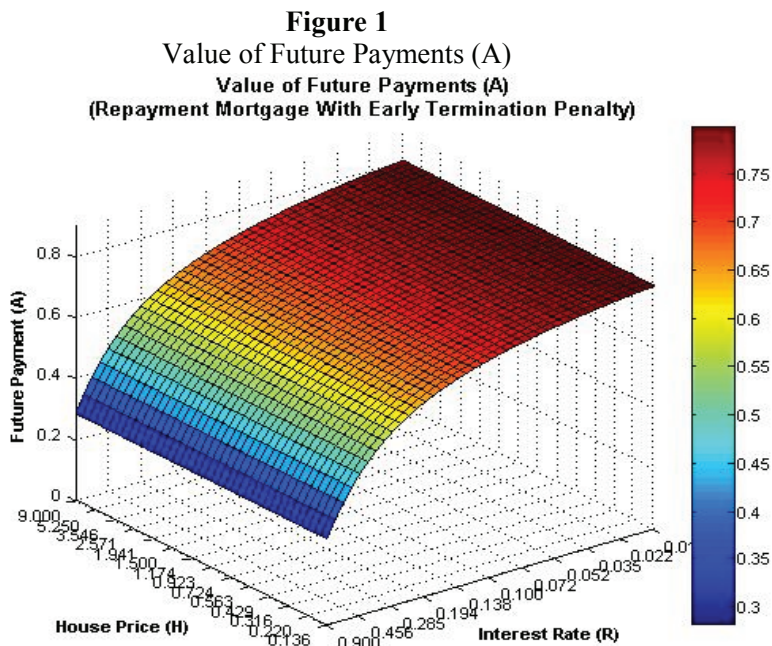
\* Parametes of Econ Environment are estimated by using daily 3 Month treasury yields (see Appendix 1).

\* Contact provisions are obtained from websites of the banks.

Figures 1 to 5 demonstrate the mortgage component values at the origination of contracts with early termination penalty. That is, the value of remaining mortgage payments,  $A$ , value of the option to default on the mortgage,  $D$ , value of the option to prepay the loan,  $C$ , and the total effect of  $A$ ,  $D$ , and  $C$ , which gives the entire contract value  $V$  at the origination of the loan. The main reason for presenting these 3-dimensional figures is to

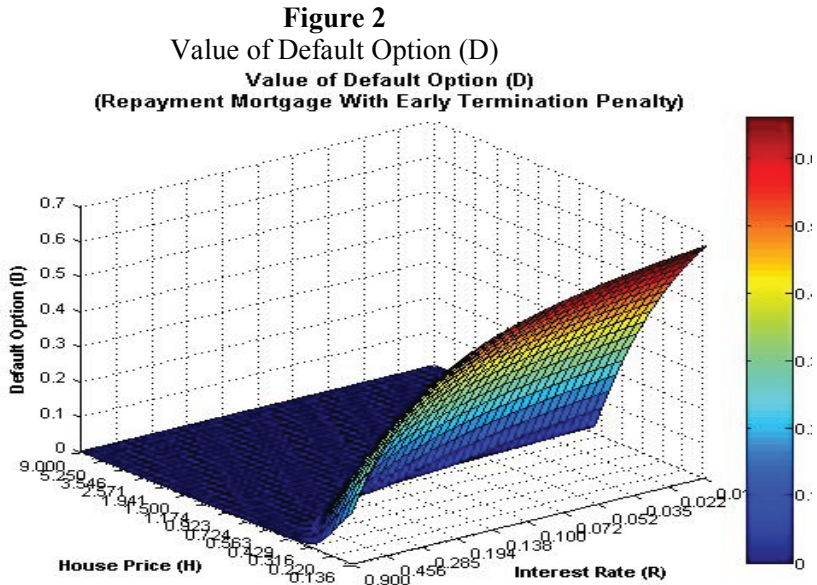
demonstrate both the smoothness of the numerical solution over the grid and economic consistency of the results.

The value of remaining mortgage payments,  $A$ , depends only on the interest rate (discount rate),  $r$ , therefore values parallel to the H-axis are constant (see Figure 1). As would be expected, the value of remaining mortgage payments shows an inverse relationship with the interest rate or the discount rate.



Source: Calculated by the authors.

Figure 2 shows the value of the default option,  $D$ . The relationship between the level of house price,  $H$  and the value of the mortgage contract,  $V$ , is the greatest influence on the value of default option. The value of  $D$  is positive in almost all of the subset of the state space where  $H < H(0)$ . As the increase in interest rate,  $r$  leads to decreases in the value of future payments,  $A$  and the mortgage value  $V$ , the value of default option, whenever positive, tends to be inversely related to interest rate. The combined effect of low house prices with low level of interest rate result in extremely high default value for the FRM contracts as seen from Figure 2.



*Source:* Calculated by the authors.

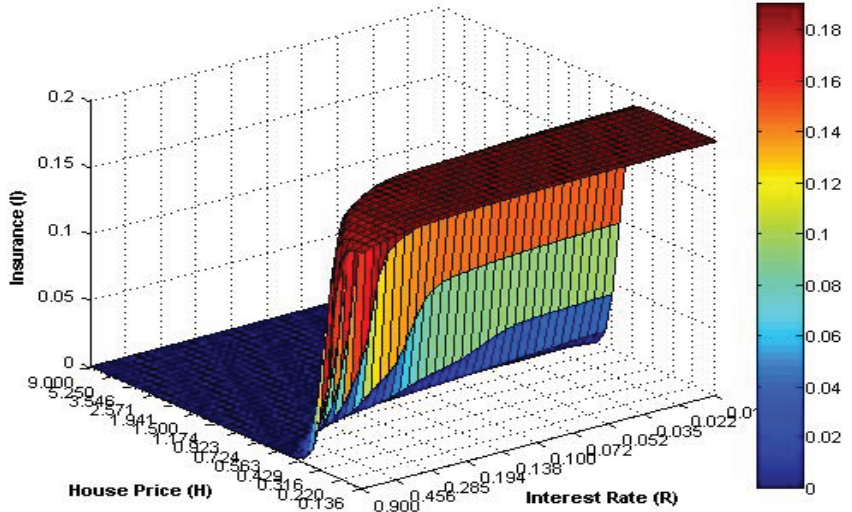
It is worthwhile to note that the US “sub-prime mortgage crisis” began with high default rates on sub-prime and adjustable-rate mortgages (ARMs) in 2005-2006. Mortgage defaults and foreclosure activity increased dramatically as the interest rates began to rise and ARM interest rates reset higher, and as house prices dropped continuously, after a long-term rising trend, in many parts of the US. Since most of the defaulted mortgages were ARMs, low house prices in combination with high interest rates resulted in significant amount of defaults. As we evaluate the default option for the FRM contracts in this paper, low house prices combined with low market interest rates result in extremely high default values for the borrowers.

Figure 3 presents the value of the mortgage default insurance, which is directly related to the evolution of the default option. At low house price levels, borrowers tend to default on their mortgages and the insurance policy is highly expected to be exercised. The value of the prepayment option,  $C$ , which primarily depends on the level of interest rate, is illustrated in Figure 4. Prepayment option has high values for low levels of interest rate,  $r$ , coinciding with high levels of house prices. This is because at low house prices borrowers tend to default on their loans and, of course, a defaulted mortgage cannot be prepaid.

Lastly, the value of mortgage contract,  $V$ , is a complex function of the remaining mortgage payments,  $A$ , the default option,  $D$ , and the prepayment option,  $C$ , is exhibited in Figure 5. At low levels of house prices, it is so valuable for the borrower to default on his mortgage, and high values for the

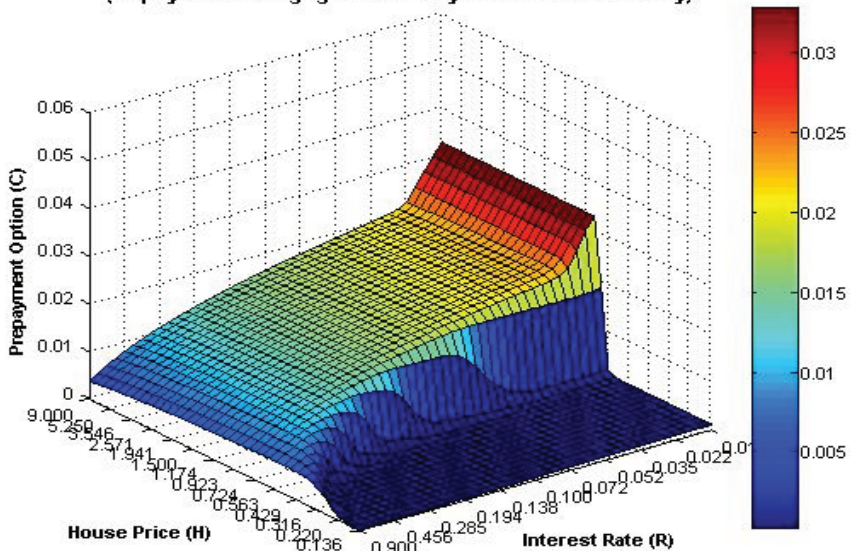


**Figure 3**  
Value of Insurance (I)  
Value of Insurance (I)  
(Repayment Mortgage With Early Termination Penalty)



Source: Calculated by the authors.

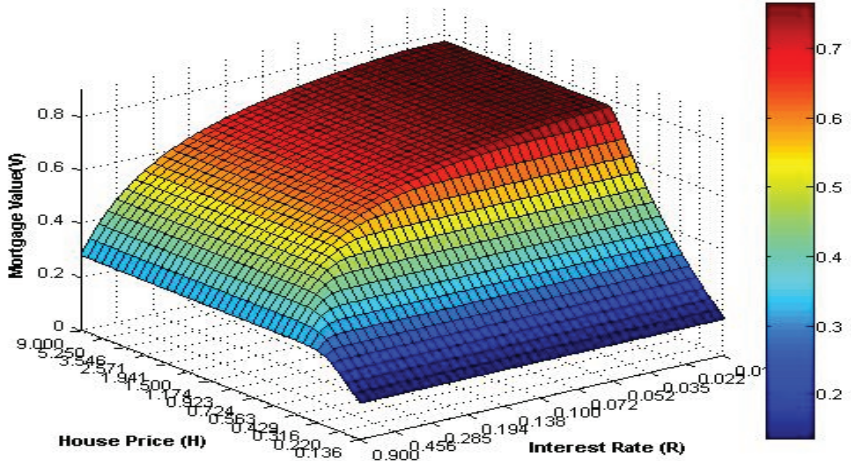
**Figure 4**  
Value of Prepayment Option (C)  
Value of Prepayment Option (C)  
(Repayment Mortgage With Early Termination Penalty)



Source: Calculated by the authors.

default option reduce the value of the mortgage contract. At higher levels of house prices, prepayment option seems to be the significant option. Changes in interest rates affect both A and C inversely but these two components produce opposite effects on the value of the contract, V. For instance, increases in A increases the value of the contract, V, whereas increases in C reduce the value of the contract. Since the value of the prepayment option cannot be bigger than A, the relationship between interest rates and the value of the mortgage contract tends to be dominated by the effect of the interest rate on A. An exception occurs when the combination of low interest rates and high house prices creates an environment in which it becomes preferable for the borrower to prepay the loan. This condition matches with the top section of the graph presented in Figure 5.

**Figure 5**  
Value of Mortgage Contract (V)  
Value of Mortgage Contract (V)  
(Repayment Mortgage With Early Termination Penalty)



Source: Calculated by the authors.

### 5.1. Equilibrium mortgage coupon rates for different contract specifications: No arbitrage condition

In this section, we analyze the equilibrium mortgage coupon rates for a typical fixed-rate mortgage (details given in Table 3) with early termination penalty of 2% by using the no arbitrage condition. We calculate the equilibrium coupon rates for 75%, 95%, and 100% LTV-ratio mortgage contracts separately. For each LTV-ratio, first, we analyze a FRM contract in which the arrangement fee and mortgage default insurance do not exist. Under these circumstances, no arbitrage profit condition can be written by the following equation.



$$V_B(c) - L = 0 \quad (21-a)$$

In order for this contract to be feasible, it is necessary that the value of the mortgage to the borrower,  $V_B$ , is equal to the loan amount at the mortgage origination,  $L$ . Any coupon rate,  $c$ , that corresponds to this no arbitrage condition is the equilibrium coupon rate that is capable of generating fair deal for both borrower and lender. Next, we analyze the value of lender's position with the inclusion of arrangement fees. No arbitrage condition can be modified as follows:

$$V_B(c) - (1 - \xi)L = 0 \quad (21-b)$$

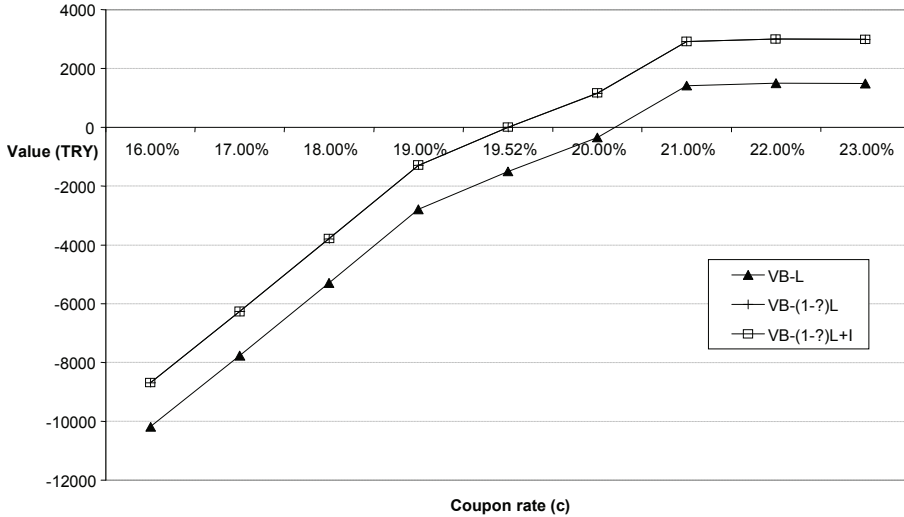
Finally, equation (21-c) illustrates the equilibrium coupon rates for the full mortgage contract with arrangement fee and mortgage default insurance.

$$V_B(c) - (1 - \xi)L + I = 0 \quad (21-c)$$

Figure 6 illustrates the equilibrium mortgage coupon rates for a 75% LTV mortgages under three different contract specifications described in Equations 21-a, 21-b, and 21-c. For a representative lender who does not charge any arrangement fee and does not have a mortgage default policy,  $[V_B(c) - L]$ , the equilibrium coupon rate is approximately 20% per annum. When we compare the value of lender's position who originates FRM contract with 2% arrangement fee,  $[V_B - (1 - \xi)L]$  with another lender who originates FRM contract with 2% arrangement fee and mortgage default insurance,  $[V_B - (1 - \xi)L + I]$ , we see that equilibrium coupon rates for both of the lenders are approximately 19.52% per annum. This result implies that 75% LTV mortgages do not have high default risk and, correspondingly, adding a mortgage insurance policy to the contract does not add any value to the lender's position. Thus, our results show that it is not beneficial for the lenders to have mortgage default insurance for the 75% LTV loans.

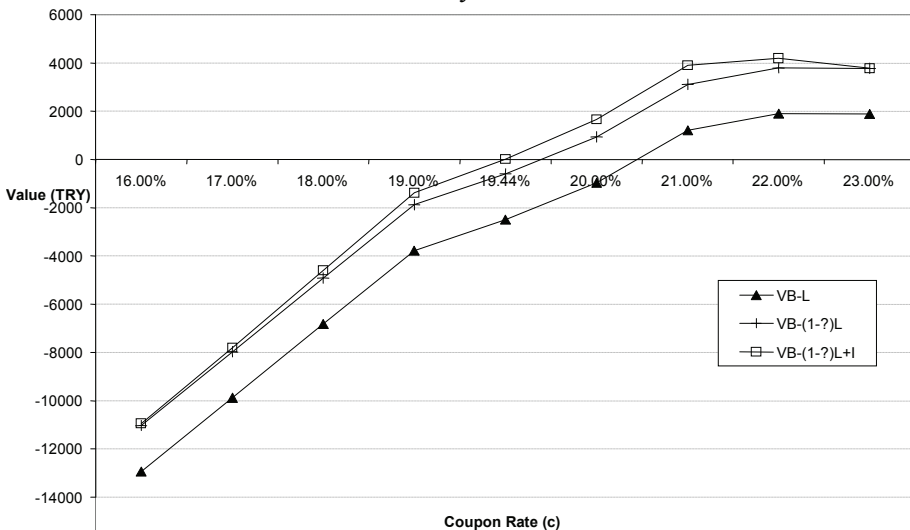
Figure 7 exhibits the equilibrium mortgage coupon rates for the 95% LTV mortgages. Although most of the banks in Turkey originate 75% LTV mortgages, Finansbank and Yapi Kredi Bank originate 95% and even 100% LTV mortgages (see Table 2). The equilibrium coupon rate for a typical FRM without an arrangement fee and mortgage default insurance,  $[V_B(c) - L]$ , is 20%. Next, we compare the equilibrium coupon rate for a FRM contract with 2% arrangement fee but without mortgage insurance,  $[V_B - (1 - \xi)L]$  and the full mortgage contract with 2% arrangement fee and mortgage default insurance,  $[V_B - (1 - \xi)L + I]$ . We find that while equilibrium coupon rate for the lender with a mortgage insurance policy ranges between 19% and 19.44%, it ranges from 19.44% to 20% for the lender without default insurance policy. This result indicates that the lenders with mortgage default insurance are able to offer lower coupon rates for high LTV mortgages. More specifically, it is beneficial for the lenders to have mortgage default insurance, especially for the high loan-to-value mortgages.

**Figure 6**  
 Mortgage Value for the Lender (75% LTV Mortgage Contract without Default Insurance and Arrangement Fee) - FRM Contract with Prepayment Penalty of 2%



Source: Calculated by the authors.

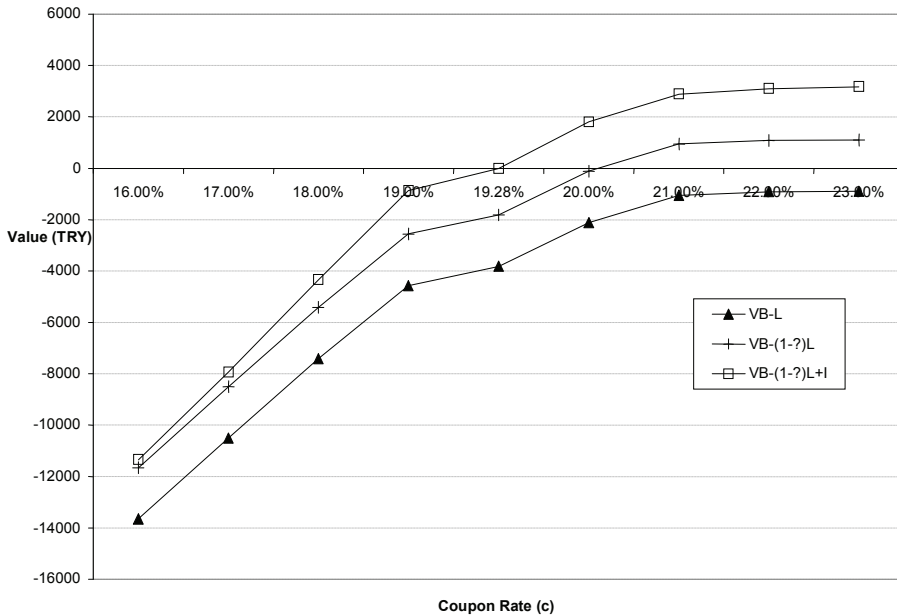
**Figure 7**  
 Mortgage Value for the Lender (95% LTV Mortgage Contract without Default Insurance, with Arrangement Fee) - FRM Contract with Prepayment Penalty of 2%



The following parameters were used in the construction of this table: the arrangement fee ( $\xi$ ) is 2%; the early termination penalty ( $\pi$ ) is 2%; the spot interest rate ( $r(0)$ ) is 15%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the interest rate volatility ( $\sigma$ ) is 12%; the house price volatility ( $v$ ) is 9%; the house service flow ( $\delta$ ) is 4% and the correlation coefficient ( $\rho$ ) is 0.

Finally, Figure 8 shows the value of lender’s position and the corresponding equilibrium mortgage coupon rates for a 100% LTV FRM contract. The important result is that there is no equilibrium coupon rate for the lender without arrangement fee and default insurance policy,  $[V_{B-L}]$ . It is noteworthy that if the loan-to-value ratio is 100%, attainment of equilibrium combinations would be impossible. On the other hand, if the lender has both 2% arrangement fee and mortgage insurance policy,  $[V_{B-(1-\xi)L+I}]$ , the equilibrium coupon rate is 19.28% per annum.

**Figure 8**  
 Mortgage Value for the Lender (100% LTV Mortgage Contract with Default Insurance and Arrangement Fee) - FRM Contract with Prepayment Penalty of 2%



The following parameters were used in the construction of this table: the arrangement fee ( $\xi$ ) is 2%; the early termination penalty ( $\pi$ ) is 2%; the spot interest rate ( $r(0)$ ) is 15%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the interest rate volatility ( $\sigma$ ) is 12%; the house price volatility ( $\nu$ ) is 9%; the house service flow ( $\delta$ ) is 4% and the correlation coefficient ( $\rho$ ) is 0.

All these contractual features, such as the prepayment penalty, arrangement fee, and mortgage default insurance, generate a benefit to the lender and, consequently, the equilibrium coupon rates are reached at slightly lower levels of the coupon rate. Table 4 summarizes the trade-off between arrangement fee, prepayment penalty and the equilibrium coupon rate. It is obvious that as the lender charges higher prepayment penalty and arrangement fee, it can offer lower mortgage coupon rates.

**Table 4**  
Trade-Off between Arrangement Fee, Early Termination Penalty and  
Equilibrium Contract Rates

Prepayment Penalty ( $\pi$ )	Arrangement Fee ( $\xi$ )				
	0.00	0.01	0.02	0.03	0.05
0.00	21.93%	19.83%	19.47%	19.15%	18.91
0.01	20.14%	19.77%	19.45%	19.14%	18.91
0.02	20.09%	19.75%	19.44%	19.13%	18.89
0.05	20.05%	19.73%	19.43%	19.13%	18.89

The following parameters were used in the construction of this table: the spot interest rate ( $r(0)$ ) is 15%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the interest rate volatility ( $\sigma$ ) is 12%; the house price volatility ( $v$ ) is 9%; the house service flow ( $\delta$ ) is 4%; LTV ratio is 95% and the correlation coefficient ( $\rho$ ) is 0.

Source: Calculated by the authors.

It is important to note that, some Turkish banks ask borrowers to get a life insurance before granting any mortgage. Banks usually provide this insurance to the borrowers by taking part of the loan as an insurance expense. Hence, they do not allow the borrowers to use the full amount of the loan. By increasing the origination fee further for covering the life insurance expenses, the banks may either increase the yield earned from the loan or allow lower regular loan payments (lower interest rate) for the same yield. Hence, the arbitrage opportunity observed in the Turkish banks may be attributable to higher arrangement fees with life insurance products.<sup>15</sup>

### 5.2. The effect of changes in the economic environment

As noted earlier, the economic environment is characterized in the present work through the set of parameters given in Table 3. In this section we present an analysis of the effects induced by changes in the base case parameters; particularly, the changes in house price volatility, interest rate volatility, and spot interest rate.

Table 5 presents the effect of house price volatility on the value of mortgage components for a 95% LTV mortgage with 2% early prepayment penalty. As the house price volatility increases from 3% to 15%, the value of default option rises significantly. Since the value of mortgage default insurance increases correspondingly, the value of mortgage contract for the lender,  $V$ , increases as well. The effect of interest rate volatility on the value of mortgage components for a 95% LTV mortgage with 2% early prepayment penalty is illustrated in Table 6. Interest rate volatility directly affects the value of the prepayment option. As the volatility increases, borrowers tend to exercise their prepayment option, consequently the value of the mortgage for the lender declines significantly.

<sup>15</sup> We thank the anonymous referee who suggested us to discuss the Turkish bank's higher yield (or lower mortgage interest rate) may be attributable to life insurance product.

**Table 5**  
House Price Volatility and the Value of Mortgage Components

95% LTV Fixed-Rate Mortgage With Early Termination Penalty					
House Price Volatility	Future Payments	Mortgage	Default	Prepayment	Insurance
( $\sigma_H$ )	(A)	(V)	(D)	(C)	(I)
3.0%	90557	86804	2	3750	4
6.0%	90557	87343	35	3179	64
9.0%	90557	88186	170	2201	322
12.0%	90557	89225	471	861	920
15.0%	90557	90321	941	0	1871

The following parameters were used in the construction of this table: the contract rate ( $c$ ) is 18%; the early termination penalty ( $\pi$ ) is 2%; the spot interest rate ( $r(0)$ ) is 15%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the interest rate volatility ( $\sigma$ ) is 12%; the house service flow ( $\delta$ ) is 4%; and the correlation coefficient ( $\rho$ ) is 0.

Source: Calculated by the authors.

**Table 6**  
Interest Rate Volatility and the Value of Mortgage Components

95% LTV Fixed-Rate Mortgage With Early Termination Penalty					
Interest Rate Volatility	Future Payments	Mortgage	Default	Prepayment	Insurance
( $\sigma_r$ )	(A)	(V)	(D)	(C)	(I)
6.0%	90147	90504	143	0	436
9.0%	90318	89517	160	641	385
12.0%	90557	88186	170	2201	322
15.0%	90870	86522	167	4180	254
18.0%	91232	84591	153	6488	192

The following parameters were used in the construction of this table: the contract rate ( $c$ ) is 18%; the early termination penalty ( $\pi$ ) is 2%; the spot interest rate ( $r(0)$ ) is 15%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the house price volatility ( $v$ ) is 9%; the house service flow ( $\delta$ ) is 4%; and the correlation coefficient ( $\rho$ ) is 0.

Source: Calculated by the authors.

Table 7A and Table 7B show the effects induced by different types of yield curves in terms of the value of the mortgage-related assets for 95% LTV and 75% LTV fixed-rate mortgages, respectively. By changing the initial level assumed by the spot interest rate,  $r(0)$ , while holding constant the steady state spot interest rates,  $\theta$ , we capture different yield curve shapes. According to our numerical results, there is a direct relationship between the evolution of the level of the initial spot rate and the coupon rate for a fixed  $\theta$ . In fact, higher levels of spot interest rates lead to higher equilibrium coupon rates.

A further important highlight is the effect of increases in interest rate volatility,  $\sigma_r$ , for different slopes of the yield curve. The increase in the interest rate volatility directly affects the evolution of default and, especially, of prepayment. Table 7A shows that as the interest rate volatility

**Table 7-A**  
**Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities**

95% LTV Fixed-Rate Mortgage With Early Termination Penalty																		
Spot rate ( $r_0$ )	Eq. Contract Rate			Future Payments			Mortgage			Default			Prepayment			Insurance		
	(c)			(A)			(V)			(D)			(C)			(I)		
	$\sigma = 12\%$	$\sigma = 15\%$	$\sigma = 18\%$	$v = 9\%$	$v = 12\%$	$v = 15\%$	$v = 9\%$	$v = 12\%$	$v = 15\%$	$v = 9\%$	$v = 12\%$	$v = 15\%$	$v = 9\%$	$v = 12\%$	$v = 15\%$	$v = 9\%$	$v = 12\%$	$v = 15\%$
12%	20.06%	19.56%	19.56%	95275	93675	92500	92500	91980	617	862	2157	834	594	1119				
15%	20.68%	20.19%	20.19%	97651	96063	92500	92500	91992	916	1265	4235	2806	593	1112				
18%	21.28%	20.79%	20.79%	95456	93912	92517	92053	92053	700	955	2239	905	582	1047				
21%	21.97%	21.49%	21.49%	98016	96473	92524	92059	92059	1023	1415	4469	2999	575	1038				
	22.39%	21.92%	21.92%	95779	94316	92522	92111	92111	779	1058	2478	1147	573	986				
	23.15%	22.69%	22.69%	98529	97075	92529	92113	92113	1141	1561	4859	3401	573	982				
	23.92%	23.50%	23.50%	96488	95199	92522	92181	92181	916	1239	3050	1779	572	919				
	24.80%	24.39%	24.39%	99525	98253	92528	92181	92181	1295	1783	5702	4288	563	918				

The following parameters were used in the construction of this table: the arrangement fee ( $\xi$ ) is 2%; the early termination penalty ( $\pi$ ) is 2%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the house service flow ( $\delta$ ) is 4%; and the correlation coefficient ( $\rho$ ) is 0.

Source: Calculated by the authors.

**Table 7-B**  
**Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities**

Spot Rate	75% LTV Fixed-Rate Mortgage With Early Termination Penalty												
	Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance		
	(c)	(c)	(A)	(A)	(V)	(V)	(D)	(D)	(C)	(C)	(I)	(I)	
	v = 9%	v = 12%	v = 9%	v = 12%	v = 9%	v = 12%	v = 9%	v = 12%	v = 9%	v = 12%	v = 9%	v = 12%	
12%	$\sigma = 12\%$	20.13%	19.69%	75408	74282	73490	73479	2	15	1917	788	2	21
	$\sigma = 15\%$	20.75%	20.31%	77278	76149	73494	73479	3	21	3780	2648	2	20
15%	$\sigma = 12\%$	21.34%	20.90%	75526	74401	73490	73477	3	16	2033	909	2	17
	$\sigma = 15\%$	22.03%	21.59%	77548	76432	73498	73484	4	22	4047	2927	2	16
18%	$\sigma = 12\%$	22.46%	22.02%	75791	74689	73498	73485	3	16	2290	1188	2	15
	$\sigma = 15\%$	23.22%	22.79%	77951	76868	73495	73484	4	23	4452	3361	2	14
21%	$\sigma = 12\%$	24.00%	23.57%	76353	75324	73495	73485	3	19	2855	1820	2	12
	$\sigma = 15\%$	24.87%	24.47%	78755	77763	73491	73489	4	23	5261	4251	1	11

The following parameters were used in the construction of this table: the arrangement fee ( $\xi$ ) is 2%; the early termination penalty ( $\pi$ ) is 2%; the long term average of the interest rate ( $\theta$ ) is 24%; the speed of reversion ( $\kappa$ ) is 56%; the house service flow).

Source: Calculated by the authors.



increases from 12% to 15%, the value of default and, especially value of prepayment option increases significantly for 95% LTV mortgages. On the other hand, as seen from Table 7B, interest rate volatility is only effective on prepayment but not the default option for 75% LTV mortgages. For the 95% LTV mortgages, increases both in default and prepayment option values contribute to the reduction in the value of the mortgage to the borrower,  $V_B$ . However, value of insurance (I) tends to move in a direction opposite to the movement in  $\sigma_r$ . Consequently, the overall result in terms of the evolution of mortgage value to the borrower,  $V_B$  (the relationship between increases in A, D, and C) and the evolution in I. Both tend to decrease with increases in  $\sigma_r$ . Under these circumstances in order to reach an equilibrium it is necessary to increase the coupon rate and consequently the value of A to compensate for those declines in I and increases in C+D.

Lastly, we examine the joint effect generated by increases in house price volatility and changes in the slope of the yield curve. Given that the effects induced by increases in house price volatility on the evolution of default and prepayment are of opposite nature and tend partially compensate each other, the influence of house price volatility in terms of the equilibrium coupon rates is moderate. The overall effect of house price volatility seems to be translated into a slight reduction of the equilibrium coupon rate, for all the yield curves studied in the present work (see Table 7A and 7B).

### 5.3. *Equilibrium mortgage coupon rates and the prevailing coupon rates in Turkey: September 2007*

This section mainly examines if Turkish economy ready for originating long-term FRMs? More specifically, we answer the question of whether or not the currently observed FRM coupon rates represent a fair transaction between the banks and the borrowers in Turkey?

In section 5.2, we have calculated the equilibrium coupon rates for a typical 10-year FRM contract with 75%, 95%, and 100% LTV ratios separately. As almost every bank in Turkey charges 2% prepayment penalty for the FRM contracts, we evaluate the variation of equilibrium coupon rates by taking into account the arrangement fee and the mortgage default insurance policy. As a next step, we compare the monthly equilibrium coupon rates with the prevailing mortgage coupon rates in September 2007.

For a representative 10-year FRM with 2% arrangement fee but without mortgage default insurance, we find that the equilibrium coupon rates range between 1.497% and 1.531% for 75% LTV mortgages, and between 1.531% and 1.601% for 100% LTV mortgages. However, the prevailing mortgage coupon rates in September 2007 range between 1.29% for Finansbank and 1.53% for Is Bank. This result shows that, except for Is Bank, all other banks offer mortgage interest rates that are significantly below the equilibrium coupon rates, involving arbitrage profit for the borrowers. In other words, we claim that these contracts do not represent a

fair deal between the lenders and borrowers in Turkey. In fact, due to significantly low mortgage coupon rates, large numbers of borrowers took out mortgages throughout this period. The value of mortgage loans was approximately 20.67 billion TRY as of September 2007, which was a remarkably high figure since 2000.

Although the prevailing mortgage coupon rates are below the equilibrium rates, 20% to 25% mortgage interest rates per annum are extremely high for establishing a well-functioning primary mortgage market in any economy. In comparison with other developing countries, such as India (9.25%-13.25% per annum), Mexico (12% per annum), Korea (6.29% per annum), Bulgaria (6% per annum), etc, Turkish banks offer significantly high mortgage interest rates, which tremendously increase the cost of mortgage financing for the borrowers.

**Table 8**  
Equilibrium Mortgage Coupon Rates and the Prevailing Mortgage Coupon Rates of the Turkish Banks: September, 2007

<b>PANEL-A: September, 2007</b>			
<b>Spot interest rate 15%, Long-term interest rate 24%, Interest rate volatility =12%</b>			
	No fee & No Mortgage Insurance [ $V_B-L$ ]	2% Arrangement Fee & No Insurance [ $V_B-(1-\xi)L$ ]	2% Arrangement Fee & Mortgage Insurance [ $V_B-(1-\xi)L+I$ ]
Equilibrium Coupon Rates: <b>LTV = 75%</b>	1.531% – 1.601%	1.497% – 1.531%	1.497% – 1.531%
Equilibrium Coupon Rates: <b>LTV = 95%</b>	1.531% – 1.601%	1.490% – 1.531%	1.460% – 1.490%
Equilibrium Coupon Rates: <b>LTV = 100%</b>	No equilibrium coupon	1.531% – 1.601%	1.480 %
Prevailing Coupon Rates		Oyak Bank: 1.35% HSBC: 1.44% Akbank: 1.31% Garanti: 1.34% Vakifbank: 1.30% Is Bank: 1.53% Finansbank: 1.29% Yapi Kredi: 1.33%+	

*Source:* Calculated by the authors.

Furthermore, we examine the FRM mortgage coupon rates and contract maturities of our sample banks over the last one-year time period, from September 2007 to November 2008. We observe that every bank in our sample significantly increased their coupon rates and most of the banks drastically reduced their contract maturities. In particular, observed monthly coupon rates in November 2008 range between 1.79% (Oyak Bank) and 2.40% (HSBC Bank). İş Bank reduced its contract maturity from 120 months to 60 months. Akbank, Yapı Kredi Bank and Vakıf Bank, with 240-

month contract maturities, currently originate FRM mortgages with a maturity of up to 60-120 months. Similarly, HSBC and Finansbank, with 360-month contract maturities, currently originate FRM mortgages with a maturity of up to 120-180 months.

Since the mortgage coupon rates and contract maturities have changed drastically over a very short time period, from September 2007 to November 2008, we argue that the effects of the global financial crisis are started to be felt in Turkish mortgage market. The primary mortgage market in Turkey is still in its infancy stage. Turkish banks, including the private, public and foreign deposit banks, have constructed their own mortgage portfolios, without the sub-prime loans.<sup>16</sup> Borrowers, who take out mortgages, are not low income citizens. They are not likely to default on their mortgages. However, the global financial crisis is likely to exert indirect and powerful negative effects on Turkey as it has been doing indiscriminately so far on several developing and emerging economies.

## 6. Concluding remarks

In this paper we use traditional option-pricing model to evaluate the current 10-year fixed rate mortgage (FRM) contracts with embedded default and prepayment options in Turkey. In fact, theoretical and empirical research on mortgage design and pricing has been widely conducted on the fixed and adjustable rate mortgages used in the United States and United Kingdom. One of the key questions for any aspect of research on emerging markets is whether the models and theories put forth and tested several times in developed financial markets also describe the realities observed in emerging markets. To our knowledge, this paper is the very first attempt to use a structural approach (option pricing model) to price the FRM contracts in an emerging economy with its different and unique dynamics.

Due to the recent improvements in Turkish economy, the current government in collaboration with the Capital Market Board initiated a legal framework to contract, for the first time ever, long-term fixed rate mortgages. As of December 2006, the value of mortgage debt in Turkey was approximately 12.24 € billion, which made it the second largest mortgage market among the new members and candidate members of the EU. Currently, the Turkish banks offer a variety of mortgage products, including Turkish Lira (TRY) - denominated fixed-rate, adjustable rate, and graduated payment mortgages and US Dollar-, and, Euro-denominated mortgages.

We employ explicit finite difference methodology as the appropriate procedure to valuing mortgages with embedded default and prepayment options. For each component of a typical mortgage contract, including the promised future monthly payments, default option, prepayment option, and mortgage default insurance, we present numerical results in 3-dimensional

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<sup>16</sup> Recently, the chairman of the Capital Markets Board commented that there was not any problem in Turkey's mortgage markets since there were no sub-prime mortgages in Turkey.

figures. We demonstrate that the numerical solution is smooth over the grid and there is economic consistency of the results.

In order to understand if option pricing models and theories put forth and tested several times in developed financial markets are also successful in describing the realities observed in Turkish mortgage market, we attempt to answer the following questions. 1. Do the currently observed FRM coupon rates represent a fair transaction between the banks and the borrowers in Turkey? 2. What will be the value of bank's (lender's) position if they have mortgage default insurance policy, especially for the high loan-to-value (LTV) housing loans?

Our results show that, except for İş Bank, all other banks offer mortgage interest rates that are significantly below the equilibrium coupon rates, involving arbitrage profit for the borrowers. In other words, we claim that these contracts do not represent a fair deal between the lenders and borrowers in Turkey. We also conclude that even if the prevailing mortgage coupon rates are below the equilibrium rates, 20%-25% mortgage interest rates per annum are extremely high for establishing a well-functioning primary mortgage market in any economy. In comparison with other developing countries, such as India (9.25%-13.25% per annum), Mexico (12% per annum), Korea (6.29% per annum), Bulgaria (6% per annum), etc, Turkish banks offer significantly high mortgage interest rates, which tremendously increase the cost of mortgage financing for the borrowers.

Most of the banks in Turkey originate maximum 75% LTV mortgages. We find that 75% LTV mortgages do not have high default risk and, correspondingly, adding a mortgage insurance policy to the contract does not add any value to the lender's position. Thus, it is not beneficial for the lenders to have mortgage default insurance for the 75% LTV loans. On the other hand, our findings show that it is beneficial for the lenders to have mortgage default insurance, especially for the high LTV ratio mortgages.

Finally, we argue that the effects of the global financial crisis are started to be felt in Turkish mortgage market as the banks have increased their mortgage coupon rates and shortened the contract maturities drastically over a very short time period, from September 2007 to November 2008. The primary mortgage market in Turkey is still in its infancy stage. Turkish banks, including the private, public and foreign deposit banks, have constructed their own mortgage portfolios, without the sub-prime loans. Borrowers, who take out mortgages, are not low income citizens. They are not likely to default on their mortgages. However, the global financial crisis is likely to exert indirect and powerful negative effects on Turkey as it has been doing indiscriminately so far on several developing and emerging economies.

### Appendix 1

#### Estimation of the CIR Process Parameters

CIR model (1985) assumes that the time evolution of the short term interest rate under real probabilities  $P$  has the following representation

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r r_t^{1/2} dW_t^P \tag{1}$$

Under risk neutral measure  $Q$  it follows

$$dr_t = \kappa(\theta - r_t - \lambda\sigma_r)dt + \sigma_r r_t^{1/2} dW_t^Q \tag{2}$$

where  $\lambda$  is the market price of risk. The unique positive solution to the short rate stochastic differential equation (2) is

$$r(t) = \theta^* + (r(s) - \theta^*)e^{-\kappa(t-s)} + \sigma_r e^{-\kappa(t-s)} \int_s^t e^{\kappa(u-s)} \sqrt{r(u)} dW(u) \tag{3}$$

for any  $t \geq s$  and  $\theta^* = \theta - \lambda\sigma_r$ . The probability density of interest rate at time  $t$ , conditional on its value at current time  $s$ , is given by

$$f(r(t), t; r(s), s) = ce^{-u-v} \left(\frac{v}{u}\right)^{q/2} I_q(2(uv)^{1/2}), \tag{4}$$

where

$$c = \frac{2\kappa}{\sigma_r^2(1 - e^{-\kappa(t-s)})}, \quad u = cr(s)e^{-\kappa(t-s)}, \quad v = cr(t), \quad q = \frac{2\kappa\theta^*}{\sigma_r^2} - 1$$

and  $I_q(\cdot)$  is the modified Bessel function of the first kind of order  $q$ .

Notice that short term interest rate is distributed as chi-squared conditioned on the filtration at time  $s$ ,

$$r(t) | F_s \sim \chi(2cr(t), 2q + 2, 2u) \tag{5}$$

with  $2q + 2$  degrees of freedom and parameter of non-centrality  $2u$  proportional to the current short rate.

Assuming independency, one can write the likelihood function for the short rate series with  $T$  observations

$$L(\Theta | r(t)) = \prod_{t=1}^T f(r(t), t; r(s), s) \tag{6}$$

Then log-likelihood function is

$$\text{Log}L(\Theta | r(t)) = (T-1)\text{ln}c + \sum_{t=1}^T \left\{ -u_t - v_{t+1} + 0.5\text{ln}\left(\frac{v_{t+1}}{u_t}\right) + \text{ln}\left(I_q\left(2\sqrt{u_t v_{t+1}}\right)\right) \right\} \tag{7}$$

Then the estimation of the parameter is done by maximizing equation (7) over the control variables  $\Theta = \{\kappa, \sigma_r, \theta, \lambda\}$ .

$$\hat{\Theta} = \arg \max_{\Theta} \text{Log}L(\Theta | r(t))$$

We choose initials implementing the Direct optimization algorithm (see Dan finkel) and then we make use of Nelder-Mead algorithm to solve the nonlinear optimization problem.

We use daily 3-Month treasury yields obtained from the secondary market to estimate the parameters of the CIR process. Data covers the period between 28 Feb 2002 and 21 Nov 2007. Data set has 1518 observations in total.

## Appendix 2

### Descriptions of the Model Variables

- $H$  = the value of the mortgaged housing property,  
 $r$  = the instantaneous risk-free interest rate,  
 $\mu$  = average rate of house price appreciation,  
 $\sigma_H$  = the volatility of disturbances in actual house price appreciation,  
 $z_H$  = the standardized Wiener process that drives the uncertainty in house prices,  
 $\theta$  = the long-term mean value for the interest rate,  
 $\kappa$  = the speed of adjustment in the mean reverting process,  
 $\sigma_r$  = the standard deviation of the interest rate disturbance,  
 $z_r$  = the standardized Wiener process,  
 $\rho$  = the instantaneous correlation coefficient,  
 $V$  = the mortgage value,  
 $n$  = the life of the mortgage in months,  
 $L$  = amount of the loan,  
 $c$  = the fixed coupon rate,  
 $\pi$  = early termination penalty,  
 $f$  = fraction for insurance coverage,  
 $\xi$  = arrangement fee,  
 $\eta(i)$  =  $i$  th payment date,  
 $OB(i)$  = outstanding balance after  $i$  th payment date,  
 $MP$  = monthly payment for a fixed-rate mortgage,  
 $TD(t)$  = borrower's total dept at time  $t$ .  
 $V_B(r; H; t)$  = value of the mortgage to the borrower at time  $t$  for given  $r$  and  $H$ ,  
 $A(r; t)$  = value of the remaining mortgage payments at time  $t$  for given  $r$ ,  
 $D(r; H; t)$  = value of the default option at time  $t$  for given  $r$  and  $H$ ,  
 $C(r; H; t)$  = value of the prepayment option at time  $t$  for given  $r$  and  $H$ ,  
 $J(r; H; t)$  = value of the joint option at time  $t$  for given  $r$  and  $H$ .

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## Özet

### Gelişmekte olan ekonomilerde uzun vadeli, sabit faiz oranlı Mortgage Kredilerinin piyasaya sunumu: Türkiye’de son gelişmeler

Bu makale Türkiye mortgage piyasasındaki son gelişmeleri ve mortgage piyasası ile sermaye piyasalarının entegrasyonunu sayısal olarak incelemektedir. Sermaye Piyasası Kurulu birincil mortgage piyasası ve mortgage teminatlının menkul kıymetlere dayanan ikincil mortgage piyasası gelişimine ilişkin kanun tasarısını hazırlamış ve bu tasarı Mart 2007 tarihinde yürürlüğe girmiştir. Konut Finansmanı Kanunu ile birlikte Türkiye’deki bankalar ilk defa oldukça uzun vadeli konut kredilerini piyasaya sunmaya başlamışlardır. Bu makale Opsiyon Fiyatlama Modeli’ni (OFM) kullanarak, Türkiye’deki 10-yıl vadeli, sabit faiz oranlı mortgage kredilerini borçlanan tarafın temerrüt ve erken ödeme opsiyonları ile birlikte fiyatlamaktadır. Bu çalışma, Türkiye gibi gelişmekte olan ekonomilerdeki sabit faiz oranlı mortgage kredilerini klasik OFM ile fiyatlandırılan ilk çalışmadır. Sonuçlara göre, çalışmamızın başlangıcı olan 2007 yılında İş Bankası dışındaki tüm bankaların uyguladığı aylık mortgage faiz oranları, fiyatlama modelindeki denge faiz oranlarının oldukça altındadır. Bu durumda, 2007 yılında sabit faizli mortgage kredisi alan kişilerin oldukça büyük bir arbitraj karı elde ettiklerini söylemek mümkündür. 2007 yılındaki aylık mortgage faiz oranları, OFM denge faiz oranlarından düşük olsa da, bu oranlar güvenli ve verimli işleyen bir mortgage piyasası için oldukça yüksektir. Makalenin diğer önemli sonucu ise, küresel finansal kriz etkilerinin ülkemiz birincil mortgage piyasasında net bir şekilde gözlenmesidir. Eylül 2007 ile Kasım 2008 arasındaki yaklaşık bir yıllık dönemde, mortgage faiz oranlarında belirgin bir artış görülürken, kredi vadelerinde hızlı bir düşüş yaşanmıştır.

*Anahtar kelimeler:* Sabit faiz oranlı krediler, gelişmekte olan ekonomiler, opsiyon fiyatlama modeli (OFM), Türkiye mortgage piyasası, açık sonlu farklar yöntemi.

*JEL kodları:* G01 ; G13 ; G21 ; C63.