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# The effects of outside options on optimal auction outcomes[1]

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**Abstract**

In this paper, we study the seller-optimal auctions for auction design environments with pure informational externalities, and investigate the effects of outside options on auction outcomes. If the seller's outside option is endogenous and depends on the information structure of the market, we show that the seller sells the good more often as the auction design environment merges into the standard private-value auction setting. If the seller's outside option does not depend on the information structure of the market, we show that the relationship between the degree of informational externalities and sale decisions in seller-optimal auctions is ambiguous.

*Keywords*: Auction design, informational externalities, outside options.

*JEL Classification*: D44, D62.

## 1. Introduction

This study first presents the robust methodology to construct seller-optimal auctions for the auction design problems with pure informational externalities, and then analyzes the effects of outside options' structure on the seller-optimal auction outcomes. In these environments, each buyer's valuation for an auctioned good has two components: a private value component, and a common value component. The private value component depends only on each agent's own private information, and the common value component depends on the other agents' private information as well. The seller's valuation for the auctioned good also depends on buyers' private information for our first case. That is, all agents have private information, which is not completely informative, about the real value of the auctioned good. This is to say that preferences are informationally interdependent. One example of such environments is FCC spectrum auctions where bidders formed their valuations for a given spectrum based on the beliefs of the

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other bidders, see Cramton (1997) for more on spectrum auctions conducted by Federal Communications Commission, an independent U.S. government agency. Other examples are take-overs, oil-drilling rights, timber auctions, art auctions, and treasury bill auctions, see for example Klemperer (1999) for more on these different auction environments. In this framework, we describe the set of seller-optimal auctions, and examine the relationship among auction outcomes, outside options, and the degree of informational externalities. Our contribution is to show that the correlation between the degree of informational externalities and sale decisions depends on different specifications of the seller's outside option.

In the mechanism design literature, the optimal auction design problem with or without externalities has been studied in many papers for different environments under different assumptions. Myerson (1981) provides a characterization of seller-optimal or revenue maximizing auctions in private-value settings. See also Bulow and Roberts (1989) for a simple way to characterize the optimal auctions. Levin (1997) extends this characterization to multi-unit auction environments with complementaries among the units. We also use similar tools of mechanism design to define the set of seller-optimal auctions with informational externalities. Our paper is also closely related to the literature on auction design with externalities. See, e.g., Jehiel, Moldovanu, and Stacchetti (1996), Jehiel, Moldovanu, and Stacchetti (1999), Das Varma (2002), Goeree (2003), Maasland and Onderstal (2007), Aseff and Chade (2008), and Lu (2012). In most of these papers, the basic idea is that all market participants or bidders other than the seller are influenced by the identity of the auction winner and/or the sale price. There are mainly two different types of externalities: (i) allocative externalities, and (ii) informational externalities. In our paper, we do not assume allocative externalities among buyers, but we have a simple structure of informational externality in which the real value of the auctioned good depends on the private information of all participants in a given market. Unlike the above mentioned papers, we assume that the seller also has value for the auctioned item (her outside option is endogenous and hence not normalized to a constant), and her valuation also depends on the information structure of the economy for our first case. For our second case, we assume that the seller's outside option is exogenous and normalized to a constant. We try to observe the relationship among these two different specifications of the seller's outside option, optimal auction outcomes, and the degree of informational externalities. Among others, Brocas (2007), Figueroa and Skreta (2009), Chen and Potipiti (2010), Figueroa and Skreta (2011), and Brocas (2013) also analyze revenue maximizing auctions with externalities, and examine the effects of identity dependent outside options on revenue maximizing auctions for different auction design environments. The main focus of these papers is on the buyers' outside options rather than the sellers' outside options. Moreover, identity dependent externalities or allocative externalities are not present in our current paper.

The remainder of the paper is organized as follows: We present the auction design environment and introduce the notation in the next section. Section 3 provides the results for the derivation of seller-optimal auctions. Section 4 provides numerical examples of our main characterization to provide an intuition for our main result and to highlight the effects of outside options. Finally, we summarize the findings of the paper in Section 5.

## 2. The model

We follow the standard notation and presentation to describe the auction design environment. We refer the reader to Krishna (2009) for more details about the environment and notation. There is a risk-neutral seller, s, who owns an indivisible item, and there are $n \geq 2$ risk-neutral potential buyers. The set of all buyers or bidders is denoted by the set $B = \{1,...,n\}$. Each agent receives a private signal $\theta^i \in \Theta^i = [\underline{\theta}^i, \overline{\theta}^i]$, where each signal is a random variable with a cumulative distribution function $F^i(\theta^i)$, and a density function $f^i(\theta^i)$ which is positive and bounded for all $\theta^i$. A nonmonetary auction outcome is a vector $x = (x^s, x^1, ..., x^n)$ such that $x^i$ is a binary variable for all $i \in B \cup \{s\}$, and $\sum_{i \in B \cup \{s\}} x^i = 1$. We denote the set of all possible outcomes by $X$.

Each buyer $i \in B$ has a quasilinear utility function such that

$$u^i(x, t^i, \theta) = x^i v^i(\theta) - t^i = x^i \left(\rho\theta^i + (1-\rho)\frac{\sum_{j \in B \cup \{s\}} \theta^j}{n+1}\right) - t^i. \quad (1)$$

For our first case, the seller also has a quasilinear utility function such that,

$$u^s(x, t^s, \theta) = x^s v^s(\theta) + t^s = x^s \left(\rho\theta^s + (1-\rho)\frac{\sum_{j \in B \cup \{s\}} \theta^j}{n+1}\right) + t^s. \quad (2)$$

The constant $\rho \in (0,1)$ is a weight assigned to each agents' private information; $\theta$ is an private signal profile or an information structure of the market; $t^i$ is a payment made by a buyer $i \in B$, and $t^s$ is the payment received by the seller. More general preferences that are non-linear in possible auction outcomes, and these types of more structured preferences are also used in Küçükşenel (2012a) to provide comparative statics results about the whole set of interim efficient auction mechanisms. Even though this type of utility functions can be restrictive for some settings, it is one of the most intuitive way to represent the main idea of informational externalities. For these preferences, an agent's real value for the auctioned good, $v^i(\theta)$, depends on a private value component, her private signal, $\theta^i$, and a common value component, average information of all agents in the market, $\frac{\sum_{j\in B\cup\{s\}}\theta^j}{n+1}$. Two extreme cases, $\rho = 0$ (pure common-value auction setting) and $\rho = 1$ (private-value auction setting), are well studied in the auction design literature. We name $\rho$ as the degree of informational externalities in the rest of the paper. For the second case, we assume that the seller has no valuation for the item; that is, $v^s(\theta)$=0. For these two different cases, we analyze the relationship among the seller-optimal auction outcomes, outside options, and the degree of informational externalities.

The auction design problem is to choose an auction mechanism $(y,t)$ such that an auction outcome rule, $y:\Theta\rightarrow X$, specifies who gets the item, and a payment rule, $t:\Theta\rightarrow\mathbb{R}_+^{B\cup\{s\}}$, defines a payment for each buyer, and the total amount that the seller receives. As it is standard in the mechanism design literature, we are concentrating on direct mechanisms in which the set of announced types is equivalent to the set of possible types in the rest of the paper.[2] By the revelation principle, we can study these simple incentive compatible, direct auction mechanisms without loss of generality (Myerson, 1981). Note that for other environments where the seller has private information or signal and also chooses an auction mechanism to implement, this result, revelation principle, may not be true. The seller may want to signal her private information by offering different auction mechanisms.[3] It is beyond the scope of our current paper to analyze this type of more demanding problems. In our setting, the only difference between the seller and a buyer is their outside options. In a sense, an outside agent, mechanism designer or planner, offers a trade mechanism for these agents to use for exchanging the item.

We now introduce three standard axioms (incentive compatibility, individual rationality, and budget balance) that auction mechanisms may satisfy in our auction design environment. See surveys on the mechanism design literature by Jackson (2003) and by Myerson (2008) for more details about these axioms.

An auction mechanism, $(y,t)$, is interim *incentive compatible* if for all $i\in B\cup\{s\}$, and all $\sigma^i,\theta^i\in\Theta^i$

$$U^i(\theta^i)\geq U^i(\theta^i,\sigma^i)=\int_{\Theta^{-i}}u^i(y(\sigma^i,\theta^{-i}),t^i(\sigma^i,\theta^{-i}),\theta)dF^{-i}(\theta^{-i}), \quad (3)$$

where $U^i(\theta^i,\sigma^i)$ is agent i's expected utility when he reports that his type is $\sigma^i\neq\theta^i$ given that his private information or actual type is $\theta^i$. An auction mechanism, $(y,t)$, is interim *individually rational* if all players on the market want to participate in the auction mechanism. That is to say that each

[2] We refer the reader to Krishna (2009) for more details on this.
[3] We thank an anonymous referee for pointing this out.

agent's interim expected utility from participating in the auction mechanism is higher than his or her expected utility of not participating in the auction (or outside option) at the interim stage. For the first case in which the seller also has a private information, the seller's outside option is $U^{0s} = \int_{\Theta^{-s}} (\rho\theta^s + (1-\rho)\frac{\sum_{j\in B\cup\{s\}}\theta^j}{n+1}) dF^{-s}(\theta^{-s})$. This implies that the seller has a valuation for the item, and her outside option at the interim stage is equal to interim expected utility of not entering into the auction, which depends on her information (or signal) and the priors of the buyers. Then, for our first case this axiom requires that for all $\theta^s \in \Theta^s$,

$$U^s(\theta^s) \geq U^{0s} > 0, \tag{4}$$

and for all $i \in B$, and all $\theta^i \in \Theta^i$,

$$U^i(\theta^i) \geq U^{0i} = 0. \tag{5}$$

To understand the effects of outside options, we assume that the seller has no valuation for the item in the second case. Then, for the second case this axiom requires, for all auction participants $j \in B \cup \{s\}$, and all $\theta^j \in \Theta^j$,

$$U^j(\theta^j) \geq U^{0j} = 0. \tag{6}$$

This implies that the only difference between these two cases is about the seller's outside option. In the first case, the seller also has a valuation for the item and receives a partial information about the true value of the item. Note that true value of the item is estimated just by using the information structure of the market and the degree of informational externalities for all agents. In the second case, the seller does not receive any signal about the real value of item, and hence her valuation for the item is normalized to zero.

An auction mechanism, $(y,t)$, is ex-ante *budget balanced* if

$$\int_{\Theta} \sum_{i\in B\cup\{s\}} t^i(\theta) dF(\theta) \geq 0. \tag{7}$$

We call an auction mechanism *desirable* if it satisfies the three axioms (incentive compatibility, individual rationality, and budget balance) stated above.

## 3. Seller-optimal auctions

An auction designer chooses a desirable auction mechanism which maximizes the seller's net expected utility. Therefore, the auction designer maximizes

$$\int_{\underline{\theta}^s}^{\overline{\theta}^s} U^s(\theta^s) dF^s(\theta^s) \tag{8}$$

subject to desirability conditions that we defined at the end of the previous section. We follow the familiar mathematical machinery from the mechanism design literature to solve this constrainted optimization problem, and to find a direct auction mechanism that maximizes the objective function in this section. Note that seller-optimal auction mechanisms are belong to a larger set of auction mechanisms which is called the set of ex-ante efficient auction mechanisms. We also know that the set of interim efficient auction rules is larger than the set of ex-ante efficient auction rules. See Ledyard and Palfrey (1999), Perez-Nievas (2000), Ledyard and Palfrey (2007) and Küçükşenel (2012b) for more on a mathematical description of the set of interim efficient mechanisms for different mechanism design environments. See also Kajii and Ui (2009) for extension of interim efficient allocations to environments under uncertainty, and Hahn and Yannelis (1997) for more on different efficiency concepts for Bayesian environments. The current paper complements these papers, related to a description of the set of interim efficient or optimal auctions, by explicitly studying the effects of outside options on seller-optimal auctions. The steps for the complete description of the seller optimal auctions are very similar to the steps in the description of interim efficient auctions and to the steps in the description of optimal auctions for the private-value auction settings. Therefore, we follow the usual steps in the literature to find a solution to the stated constrainted optimization problem. Thus, the general mathematical structure of seller-optimal auction mechanisms in this class of Bayesian environments is not very surprising. However, we provide the characterization in this section for the completeness of the paper. Our main contribution is to highlight and prove the effects of different specifications of outside options on seller-optimal auction outcomes, and hence sale decisions.

Interim incentive compatibility requires that none of the agents can obtain a strictly higher payoff by individually lying about his or her type given that the other agents report their types truthfully. Let for all $i \in B \cup \{s\}$, $Q^i(\theta^i) = \int_{\Theta^{-i}} \frac{\partial v^i(\theta)}{\partial \theta^i} dF^{-i}(\theta^{-i})$. For all incentive compatible mechanisms, we need to have $Q^i(\theta^i)$ is monotone increasing, and for all $\theta^i \in \Theta^i$

$$U^i(\theta^i) = U^i(\underline{\theta}^i) + \int_{\underline{\theta}^i}^{\theta^i} Q^i(a)da. \tag{9}$$

This characterization of the set of incentive compatible mechanisms and the proof for this result are well-known in the mechanism design literature, and similar to the environments without informational externalities. Rochet (1987) provides a complete proof for independent quasi-linear environments.

Interim individual rationality for both cases requires that for all $i \in B$, and all $\theta^i \in \Theta^i$

$$U^i(\theta^i) - U^{0i}(\theta^i) = U^i(\underline{\theta}^i) + \int_{\underline{\theta}^i}^{\theta^i} Q^i(a)da - U^{0i}(\theta^i) \geq 0\,, \tag{10}$$

and for the seller, and all $\theta^s \in \Theta^s$

$$\begin{aligned} U^s(\theta^s) - U^{0s}(\theta^s) &= U^s(\overline{\theta}^s) - \int_{\underline{\theta}^s}^{\overline{\theta}^s} Q^s(a)da + \int_{\underline{\theta}^s}^{\theta^s} Q^s(a)da - U^{0s}(\theta^s) \\ &= U^s(\overline{\theta}^s) + \int_{\theta^s}^{\overline{\theta}^s} Q^s(a)da - U^{0s}(\theta^s) \geq 0. \end{aligned} \tag{11}$$

The worst type of a buyer is her lowest possible type, $\underline{\theta}^i$, and hence individual rationality condition reduces to $U^i(\underline{\theta}^i) \geq 0$ for a buyer in both cases since $Q^i(\theta^i)$ is non-decreasing. For the seller, we need to consider two different cases. In both cases the worst type of the seller is her highest possible type, $\overline{\theta}^s$. This is because individual rationality condition for both cases requires that $U^s(\theta^s) - U^{0s}(\theta^s) \geq 0$. For the first case in which the outside option of the seller is $U^{0s}(\theta) = \int_{\Theta^{-s}} (\rho\theta^s + (1-\rho)\frac{\sum_{j\in B\cup\{s\}}\theta^j}{n+1}) dF^{-s}(\theta^{-s}) \neq 0$, the requirement of the individual rationality condition for the seller is $U^s(\overline{\theta}^s) + \int_{\theta^s}^{\overline{\theta}^s} Q^s(a)da - U^{0s}(\theta^s) \geq 0$ for all $\theta^s \in \Theta^s$ (Equation 11). This inequality holds for all possible types of the seller only if $\min\{U^s(\overline{\theta}^s) + \int_{\theta^s}^{\overline{\theta}^s} Q^s(a)da - U^{0s}(\overline{\theta}^s)\} = U^s(\overline{\theta}^s) - U^{0s}(\overline{\theta}^s) \geq 0$. For the second case, the outside option of the seller is $U^{0s}(\theta) = 0$, it is easy to see that individual rationality condition for all possible types of the seller is satisfied if $U^s(\overline{\theta}^s) \geq 0$ since the worst type of the seller is her highest possible type in this environment. Therefore, the individual rationality constraint can be binding for the lowest possible type of a buyer and/or for the highest possible type of the seller. These necessary and sufficient conditions for our initial constraints can now be used in the original optimization problem.

As stated at the beginning of this section, the auction designer's aim is to find the seller-optimal auction subject to the desirability constraints. The Lagrangian function for this optimization problem using the above results can be written as

$$\int_{\underline{\theta}^s}^{\overline{\theta}^s} (\frac{1-F^s(\theta^s)}{f^s(\theta^s)})Q^s(\theta^s)dF^s(\theta^s) + \delta B(y,t) + \sum_{i\in B}\mu^i U^i(\underline{\theta}^i) + (1+\mu^s)\ U^s(\overline{\theta}^s) - U^{0s}(\overline{\theta}^s) \tag{12}$$

In the 12th equation, $B(y,t)$ represents the expected budget surplus; the constants $\delta$, $\mu^i$ and $\mu^s$ represent the Lagrangian multipliers respectively for the ex-ante budget balance condition, a buyer i's individual rationality constraint, and the seller's individual rationality constraint. The first order conditions for this maximization problem imply that the budget balance constraint is always binding, and hence $B(y,t)=0$. This result is due to the fact that $\delta \geq 1$ since the multipliers for individual rationality constraints are nonnegative, and $1+\mu^s=\delta$. Up to now, we have ignored one of the requirements for incentive compatibility condition, $Q^i(\theta^i)$ is monotone increasing. Using the same idea in Myerson (1981), we call our main optimization program as a *regular problem* if this monotonicity condition is not binding. That is, the solution without considering the monotonicity requirement is also a solution to our initial optimization problem. Given the above results, we can convert our maximization problem to a more familiar one that works with virtual valuations by substituting the modified constraints to our initial objective function. That is, an auction mechanism $(y,t)$ is a seller-optimal auction if and only if the auction outcome rule, $y$, solves the following maximization problem,

$$\max_{y\in Y}\sum_{i\in B}\int_{\Theta} y^i W^i(\rho,\theta)dF(\theta)+\int_{\Theta} y^s W^s(\rho,\theta)dF(\theta), \tag{13}$$

$$0\leq \int_{\Theta}[\sum_{i\in B} y^i(v^i(\theta)-\frac{\partial v^i(\theta)}{\partial\theta^i}(\frac{1-F^i(\theta^i)}{f^i(\theta^i)}))+y^s(v^s(\theta)+\frac{\partial v^s(\theta)}{\partial\theta^s}\frac{F^s(\theta^s)}{f^s(\theta^s)})]dF(\theta), \tag{14}$$

$$0=(\delta-1)(\int_{\Theta}[\sum_{i\in B} y^i(v^i(\theta)-\frac{\partial v^i(\theta)}{\partial\theta^i}(\frac{1-F^i(\theta^i)}{f^i(\theta^i)}))+y^s(v^s(\theta)+\frac{\partial v^s(\theta)}{\partial\theta^s}\frac{F^s(\theta^s)}{f^s(\theta^s)})]dF(\theta)), \tag{15}$$

where we denote $W^i(\rho,\theta)$ as the *modified* virtual valuation of a buyer $i\in B$;

$$W^i(\rho,\theta)=\rho\theta^i+(1-\rho)\frac{\sum_{j\in B\cup\{s\}}\theta^j}{n+1}+\frac{\rho n+1}{n+1}\left(\frac{F^i(\theta^i)-1}{f^i(\theta^i)}\right), \tag{16}$$

and $W^s(\rho,\theta)$ as the *modified* virtual valuation of the seller for the first case;

$$W^s(\rho,\theta)=\rho\theta^s+(1-\rho)\frac{\sum_{j\in B\cup\{s\}}\theta^j}{n+1}+\frac{\rho n+1}{n+1}\left(1-\frac{1}{\delta}\right)\frac{F^s(\theta^s)}{f^s(\theta^s)}. \tag{17}$$

Notice that $W^s(\rho,\theta)=0$ for the second case. The solution to this problem can be stated in a more familiar way. In the seller-optimal auction, the seller keeps the good if there is no buyer with positive modified virtual valuation which is higher than the modified virtual valuation of the seller. Otherwise, a buyer gets the good. Thus, any agent $i\in B\cup\{s\}$ such that $i\in argmax\{W^i(\rho,\theta)-\max\{0,W^s(\rho,\theta),\max_{k\neq i,k\neq s}W^k(\rho,\theta)\}\}$ gets the auctioned item. The structure of the solution is similar to the structure of optimal auctions in private value environments. As in Myerson (1981), we need to work with virtual valuations. However, these virtual valuations are modified due to informational externalities and desirability conditions. We find this solution for the regular problems. A standard simple sufficient condition, which also guarantees that the monotonicity condition is not binding in our environment, is to assume that modified virtual valuations are monotone increasing in type.

Given this general mathematical structure of the seller-optimal auctions, we next show that comparative statics results that examine the impact of a change in the degree of informational externalities in preferences, $\rho$, on seller-optimal auction outcomes depend on different specifications of the seller's outside option for our two different cases. We can actually show for the first case that as the auction setting merges to the standard private-value auction setting, the seller sells the private good more often in the seller-optimal auction. However, the relationship between the degree of informational externalities and sale decisions in the seller-optimal auctions is ambiguous for the second case. The next result summarizes these findings.

*Proposition 1:* The relationship between the degree of informational externalities and sale decisions in the seller-optimal auctions depends on the way of specification of outside options in two different cases.

*Proof:* For the first case, suppose the seller sells the good in $(\rho,\theta)$ to the buyer $i$ in the seller-optimal auction. Then,

$$\theta^i - \theta^s \geq \frac{\rho n+1}{\rho(n+1)}\left(\frac{1-F^i(\theta^i)}{f^i(\theta^i)} + \left(\frac{1}{\delta}-1\right)\frac{F^s(\theta^s)}{f^s(\theta^s)}\right) = \Pi(\rho,\theta) \geq 0.$$

Let without loss of generality $\rho' > \rho$. Then, the seller also sells the good in $(\rho',\theta)$, since $\theta^i - \theta^s \geq \Pi(\rho,\theta) \geq \Pi(\rho',\theta)$. This is true independent of the seller's individual rationality constraints. Now, suppose the seller does not sell the good in $(\rho,\theta)$ and let $i \in B$ a buyer with the highest private signal, $\theta^i \geq \theta^j$ for all $i,j \in B$. Then $\theta^i - \theta^s < \Pi(\rho,\theta)$. However, it is easy to see that there exist $\rho' \in (0,1)$ and $\theta \in \Theta$ such that $\Pi(\rho',\theta) \leq \theta^i - \theta^s$ and $\theta^i - \theta^s < \Pi(\rho,\theta)$, which implies that the seller sells the good in $(\rho',\theta)$. Therefore, the seller sells the good more often in the seller-optimal auction when $\rho$ increases. Note that if the individual rationality constraint is binding for the seller, and she does not sell the good in $(\rho,(\bar{\theta}^s,\theta^{-s}))$, she will also not sell it in $(\rho',(\bar{\theta}^s,\theta^{-s}))$. This implies that $\rho$ and sale decisions are positively correlated for the first case. We provide examples in the next section to prove that the relationship is ambiguous for the second case. This will complete the proof by showing that the relationship depends on outside options. □

The idea behind the result for the first case is that the seller's valuation becomes more interdependent with the buyers' valuations, and the ratio of information profiles where sale is profitable decreases as $\rho$ decreases. Küçükşenel (2012a) has established the same result for all interim efficient mechanisms for the first case. Therefore, an identical result and intuition holds for all interim efficient auction mechanisms for the first case if valuation functions are linear in auction outcomes, which is the case in this environment. See, Küçükşenel (2012a) for more details on this and on the set of all interim efficient auction mechanisms for more general environments with allocative and informational externalities.

## 4. Numerical examples

In this section, we complete the proof of Proposition 1 for the second case by providing an example for our auction design environment. This result shows that an auction designer should be careful when modelling the outside options which is the main topic of this paper. The following examples are also used to explicitly show the difference between two cases.

Suppose that there are four agents (3 buyers and a seller), and we are in the first case. Then, the valuation functions have the following form:

$$v^i(\theta) = \left( \rho\theta^i + (1-\rho)\frac{\sum_{j\in B\cup\{s\}} \theta^j}{4} \right). \tag{18}$$

We also assume that $\Theta^j \sim \text{Uniform}[0,1]$ for each agent $j \in B \cup \{s\}$. Therefore, modified virtual valuations are equivalent to

$$W^i(\rho,\theta) = \left( \rho\theta^i + (1-\rho)\frac{\theta^1+\theta^2+\theta^3+\theta^s}{4} + (\rho + \frac{1-\rho}{4})(\theta^i - 1) \right), \tag{19}$$

$$W^s(\rho,\theta) = \left( \rho\theta^s + (1-\rho)\frac{\theta^1+\theta^2+\theta^3+\theta^s}{4} + (\rho + \frac{1-\rho}{4})(\theta^s - \frac{\theta^s}{\delta}) \right). \tag{20}$$

Now, fix a type profile $\theta^* = (\theta^s, \theta^1, \theta^2, \theta^3) = (0.4, 0.8, 0.7, 0.5)$. Note that the seller's individual rationality constraint is not binding for this information structure. The following table shows the modified virtual valuations of all agents for different degrees of informational externalities:

**Table 1**

Optimal Auction, Modified Virtual Valuations and Trade

| $\rho$ | $W^s(\rho,\theta^*)$ | $W^1(\rho,\theta^*)$ | $W^2(\rho,\theta^*)$ | $W^3(\rho,\theta^*)$ | Trade |
|---|---|---|---|---|---|
| 0 | 0.6 | 0.55 | 0.53 | 0.48 | No, s keeps the item |
| 0.1 | 0.58 | 0.56 | 0.51 | 0.43 | No, s keeps the item |
| 0.5 | 0.5 | 0.58 | 0.46 | 0.24 | Yes, 1 wins |
| 0.75 | 0.45 | 0.59 | 0.43 | 0.12 | Yes, 1 wins |
| 1 | 0.4 | 0.6 | 0.4 | 0 | Yes, 1 wins |

As the above table suggests, for a given profile there is a certain cut off value for $\rho$ after which the trade starts to occur. If we repeat the same process for all possible profiles, we can conclude that the seller sells the good more often in the seller optimal auction as the auction setting merges to the standard private-value auction setting for our first case. The above example explicitly shows the main idea for the first case.

Now, suppose that the seller has no valuation for the item (the valuation functions and priors are identical to the example above), and hence we are in the second case. In this case, the above observation may no longer be true since the seller's valuation is no longer interdependent with the buyers' valuations (i.e., $v^s(\theta)=0$). If the seller has zero value for the item, the analysis requires only slight changes to modified virtual valuations. The modified virtual valuations for this case can be written as follows:

$$W^i(\rho,\theta)=\left(\rho\theta^i+(1-\rho)\frac{\theta^1+\theta^2+\theta^3}{3}+(\rho+\frac{1-\rho}{3})(\theta^i-1)\right), \qquad (21)$$

$$W^s(\rho,\theta)=0. \qquad (22)$$

Now, fix a type profile $\theta^{\Diamond}=(\theta^1,\theta^2,\theta^3)=(0.4,0.3,0.2)$. The following table shows the modified virtual valuations of all agents for different degrees of informational externalities:

**Table 2**

Optimal Auction, Modified Virtual Valuations and Trade When The Seller Has No Valuation for the Auctioned İtem

| $\rho$ | $W^s(\rho,\theta^*)$ | $W^1(\rho,\theta^*)$ | $W^2(\rho,\theta^*)$ | $W^3(\rho,\theta^*)$ | Trade |
|---|---|---|---|---|---|
| 0 | 0 | 0.1 | 0.07 | 0.03 | Yes, 1 wins |
| 0.1 | 0 | 0.03 | 0.02 | -0.03 | Yes, 1 wins |
| 0.5 | 0 | -0.05 | -0.17 | -0.28 | No, s keeps the item |
| 0.75 | 0 | -0.13 | -0.28 | -0.44 | No, s keeps the item |
| 1 | 0 | -0.2 | -0.4 | -0.6 | No, s keeps the item |

The seller sells the good only if $\max\{W^i(\rho,\theta)\,|\,i\in B\}\geq 0$. The relationship between the degree of informational externalities (or different environments) and the probability of trade depends on whether $\max\{W^i(\rho,\theta)\,|\,i\in B\}$ is increasing or decreasing in $\rho$. It is easy to see that the sign of $\frac{\partial\max\{W^i(\rho,\theta)\,|\,i\in B\}}{\partial\rho}$ depends on priors. This implies that it is impossible to get a general comparative statics result for the second case. In this example, the seller sells the good less often in the optimal auction as $\rho$ increases given $\theta^{\Diamond}$. We cannot get a general result for the second environment because the ratio of type profiles in which the highest virtual valuation is decreasing in $\rho$ depends on priors. Note that in the above example the modified virtual valuation of buyer one is decreasing in $\rho$ given $\theta^{\Diamond}$. This implies that it is possible to construct an example in which the highest virtual valuation changes sign from negative to positive or positive to negative. Therefore, the correlation between auction outcomes and the degree of informational externalities, $\rho$, is ambiguous (or depends on the set of possible priors) for the second case.

## 5. Conclusion

In this paper, we review the mathematical description of the set of seller-optimal auctions for auction design environments with pure informational externalities using familiar tools of the conventional mechanism design literature. We show that seller-optimal auction rules choose an auction outcome that maximizes the total of modified virtual valuations in this auction design environment with pure informational externalities. Our main finding is that the correlation between the degree of informational externalities and sale decisions depends on different specifications of the seller's outside option. We also provide numerical examples to explicitly show the intuition behind this result. Extension of this result to more general environments without the specific assumptions about agents' utility functions will be a part of future research.

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# Özet

## Dış seçeneklerin optimal ihale sonuçlarına etkisi

Bu makalede, enformasyonel dışsallıklar içeren ihale tasarımı problemleri için satıcı-optimal ihale kuralları tanımlanmaktadır ve dış seçeneklerin bu ihale kuralları sonuçları üzerine olan etkisi incelenmektedir. Eğer satıcının dış seçeneği içsel ve piyasanın enformasyon durumuna bağlı ise, ihale standart özel-değer ihale ortamına yaklaştıkça satıcının ihale edilen malı daha sık sattığı gösterilmektedir. Eğer satıcının dış seçeneği piyasanın enformasyon durumuna bağlı değil ise, enformasyonel dışsallık derecesi ile satış kararları arasındaki bağlantı belirsizdir.

*Anahtar Kelimeler*: İhale tasarımı, enformasyonel dışsallık, dış seçenekler.
*JEL Sınıflaması*: D44, D62.