

The role of σ -meson in $\omega \rightarrow \pi\pi\gamma$ decays and the coupling constant $g_{\omega\sigma\gamma}$

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Abstract

We study the $\omega \rightarrow \pi\pi\gamma$ decays by adding to the amplitude calculated within the framework of chiral perturbation theory and vector meson dominance the amplitude of σ -meson intermediate state. We estimate the coupling constant $g_{\omega\sigma\gamma}$ utilizing the experimental value of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay rate.

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The radiative decay processes of the type $V \rightarrow PP\gamma$ where P and V belong to the lowest multiplets of vector (V) and pseudoscalar (P) mesons have been a subject of continuous interest both theoretically and experimentally. Although the decay rates of those of these decays that do not involve bremsstrahlung radiation are small and consequently such decays are difficult to detect, their study nevertheless offers new physics features about the interesting mechanisms involved in these decays.

Among such radiative $V \rightarrow PP\gamma$ decays, the branching ratio for the decay $\omega \rightarrow \pi^0\pi^0\gamma$ has been measured to be $\text{Br}(\omega \rightarrow \pi^0\pi^0\gamma) = (7.2 \pm 2.5) \times 10^{-5}$ [1], whereas for the charged mode only an upper limit exists as $\text{Br}(\omega \rightarrow \pi^+\pi^-\gamma) < 3.6 \times 10^{-3}$ [2]. Thus by using the well determined ω full width of (8.41 ± 0.09) MeV [1], we obtain for the decay rates of the radiative decays $\omega \rightarrow \pi^0\pi^0\gamma$ and $\omega \rightarrow \pi^+\pi^-\gamma$ the values $\Gamma^{(exp)}(\omega \rightarrow \pi^0\pi^0\gamma) = (0.61 \pm 0.22)$ KeV and $\Gamma^{(exp)}(\omega \rightarrow \pi^+\pi^-\gamma) < 31$ KeV.

The decay $\omega \rightarrow \pi\pi\gamma$ as well as other radiative vector meson decays was first studied by Singer [3], who postulated a mechanism involving the dominance of the intermediate vector meson contribution (VDM), thus assumed that this decay proceeds through an intermediate $(\rho\pi)$ state as $\omega \rightarrow (\rho)\pi \rightarrow \pi\pi\gamma$. Singer also noticed the relation $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = \frac{1}{2}\Gamma(\omega \rightarrow \pi^+\pi^-\gamma)$ for the decay rates of these reactions calculated using Born-term amplitudes of VDM contributions where the factor 1/2 is a result of charge conjugation invariance to order α which imposes pion pairs of even angular momentum. Renard [4] later studied radiative decays $V \rightarrow PP\gamma$ in a gauge invariant way with current algebra, hard-pion and Ward-identities techniques. He, moreover, established the correspondence between these current algebra results and the structure of the amplitude calculated in the single particle approximation for the intermediate states. He obtained the value $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = 350$ eV for the contribution of the VDM amplitude to the decay $\omega \rightarrow \pi^0\pi^0\gamma$ which when corrected by the present day data for the relevant masses and coupling constants becomes 227 eV [5]. The contribution of intermediate vector mesons to the decay $V \rightarrow PP\gamma$ was also considered by Bramon et al. [5] using standard Lagrangians obeying the SU(3)-symmetry. Their result for the decay rate of the $\omega \rightarrow \pi^0\pi^0\gamma$ was $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = 235$ eV. Fajfer and Oakes [6] using

a low energy effective Lagrangian approach with gauged Wess-Zumino terms calculated the rate for the decay $\omega \rightarrow \pi^0\pi^0\gamma$ as $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = 690$ eV. However, it should be noted that Bramon et al. [5] disagree with the numerical predictions of Fajfer and Oakes [6] for the decays $V \rightarrow PP\gamma$ in general, even if the initial expressions for the Lagrangians are the same. Guetta and Singer [7] recently calculated the $\omega \rightarrow \pi^0\pi^0\gamma$ decay rate using the Born amplitude for VDM mechanism as $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = (344 \pm 85)$ eV. In their calculation they noted that the decay rate of $\omega \rightarrow \pi^0\pi^0\gamma$ is proportional to the coupling constant $g_{\omega\rho\pi}^2$ and $g_{\rho\pi\gamma}^2$, and they assumed that the decay $\omega \rightarrow 3\pi$ proceeds with the same mechanism as $\omega \rightarrow \pi^0\pi^0\gamma$, that is as $\omega \rightarrow (\rho)\pi \rightarrow \pi\pi\pi$ [8]. They use the experimental input for $\Gamma(\omega \rightarrow 3\pi)$, $\Gamma(\rho^0 \rightarrow \pi^0\gamma)$, and $\Gamma(\rho \rightarrow \pi\pi)$ [2], and furthermore they employ a momentum dependent width for ρ -meson. If a constant width is used for ρ -meson, then the decay rate is obtained as $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = 306$ eV using the Born term for the VDM mechanism.

Recently, on the other hand, a new theoretical approach has been developed for the calculation of $V \rightarrow PP\gamma$ decays within the framework of chiral effective Lagrangians using chiral perturbation theory [9]. Bramon et al. [9] calculated the decay rates for various decays of the type $V \rightarrow PP\gamma$ using this approach. They noted that using chiral perturbation theory Lagrangians there is no tree-level contribution to the amplitudes for the decay processes $V \rightarrow PP\gamma$ and that the one-loop contributions are finite and to this order no counterterms are required. They considered both $\pi\pi$ and $K\bar{K}$ intermediate loops. In the good isospin limit the π -loop contributions to the $\omega \rightarrow \pi^0\pi^0\gamma$ amplitude vanish and the contribution of the K -loops is very small, $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma)_K = 1.8$ eV.

Since, in addition to the chiral loop contribution, there is always the term in the decay amplitude given by the intermediate vector meson dominance (VDM) mechanism [5,6], we obtain for the amplitude of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ the theoretical result

$$A(\omega \rightarrow \pi^0\pi^0\gamma) = A_\chi(\omega \rightarrow \pi^0\pi^0\gamma) + A_{VDM}(\omega \rightarrow \pi^0\pi^0\gamma) \simeq A_{VDM}(\omega \rightarrow \pi^0\pi^0\gamma) \quad (1)$$

where A_χ and A_{VDM} are the chiral and VDM amplitudes, respectively. However, the central value of the experimental result $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma) = 0.61$ KeV for the decay rate of the decay

$\omega \rightarrow \pi^0\pi^0\gamma$ is nearly twice the calculated value employing VDM amplitude, hence twice the theoretical result.

Therefore, the mechanism of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ should be reexamined. To this end, within the theoretical framework of chiral perturbation theory and vector meson dominance, Guetta and Singer [7] considered a neglected feature so far, that is the possibility of $\omega - \rho$ mixing. However, their calculation showed that the $\omega - \rho$ mixing increases the $\omega \rightarrow \pi^0\pi^0\gamma$ decay rate by 5% only which is even less than 12% increase provided by using a momentum dependent width for ρ -meson in the calculation using the VDM amplitude as also noted by these authors.

In this work, we extend our previous studies of the role of σ -meson in $\rho^0 \rightarrow \pi^+\pi^-\gamma$ [10] and $\rho^0 \rightarrow \pi^0\pi^0\gamma$ [11] decays to $\omega \rightarrow \pi^0\pi^0\gamma$ and $\omega \rightarrow \pi^+\pi^-\gamma$ decays. We follow a phenomenological approach and attempt to calculate the decay rate for the decay $\omega \rightarrow \pi^0\pi^0\gamma$ by considering ρ -pole vector meson dominance amplitude as well as the σ -pole amplitude. By employing the experimental value for this decay rate we calculate the coupling constant $g_{\omega\sigma\gamma}$ as a function of the experimental σ -meson parameters M_σ and Γ_σ of the $f_0(400 - 1200)$ meson [2] which is the candidate for σ -meson. We also consider for the σ meson parameters the values suggested by two recent experiments, that is $M_\sigma = 555$ MeV $\Gamma_\sigma = 540$ MeV from CLEO [12], and $M_\sigma = 478$ MeV $\Gamma_\sigma = 324$ MeV from Fermilab E791 [13]. We then use the coupling constant $g_{\omega\sigma\gamma}$ we calculate this way to predict the decay rate for the $\omega \rightarrow \pi^+\pi^-\gamma$ decay assuming that this decay also proceeds through the same mechanism as well, that is its amplitude is provided by ρ -meson and σ -meson intermediate state amplitudes. Therefore, the measurement of the $\omega \rightarrow \pi^+\pi^-\gamma$ decay rate may provide us with insight about the mechanism of $V \rightarrow PP\gamma$ decays.

Our calculation is based on the Feynman diagrams shown in Fig. 1 for $\omega \rightarrow \pi^0\pi^0\gamma$ decay and on those shown in Fig. 2 for $\omega \rightarrow \pi^+\pi^-\gamma$ decay. We describe the $\omega\rho\pi$ -vertex by the effective Lagrangian [14]

$$\mathcal{L}_{\omega\rho\pi}^{int.} = g_{\omega\rho\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\vec{\rho}_\beta \cdot \vec{\pi} \quad (2)$$

which also conventionally defines the coupling constant $g_{\omega\rho\pi}$. Since there is no phase space to measure an $\omega \rightarrow \rho\pi$ transition, this vertex should be extracted from theoretical models. Vector Meson Dominance and current-field identities [15] gives $g_{\omega\rho\pi} \simeq 12 \text{ GeV}^{-1}$ while approximate SU(3) symmetry suggests $g_{\omega\rho\pi} \simeq 16 \text{ GeV}^{-1}$ [16]. On the other hand, QCD sum rule calculations obtain the value $g_{\omega\rho\pi} \simeq (15 - 17) \text{ GeV}^{-1}$ [14], and the light cone QCD sum rules method extracts the value $g_{\omega\rho\pi} = 15 \text{ GeV}^{-1}$ [17]. Recently QCD sum rules method for the polarization operator in an external field yields the value $g_{\omega\rho\pi} \simeq 16 \text{ GeV}^{-1}$ [18]. In this work we use this coupling constant as $g_{\omega\rho\pi} = 15 \text{ GeV}^{-1}$. For the $\sigma\pi\pi$ -vertex we use the effective Lagrangian [19]

$$\mathcal{L}_{\sigma\pi\pi}^{int.} = \frac{1}{2}g_{\sigma\pi\pi}M_{\sigma}\vec{\pi} \cdot \vec{\pi}\sigma \quad . \quad (3)$$

The decay width of the σ -meson that follows from this effective Lagrangian is given as

$$\Gamma_{\sigma} \equiv \Gamma(\sigma \rightarrow \pi\pi) = \frac{g_{\sigma\pi\pi}^2}{4\pi} \frac{3M_{\sigma}}{8} \left[1 - \left(\frac{2M_{\pi}}{M_{\sigma}} \right)^2 \right]^{1/2} \quad . \quad (4)$$

The $\rho\pi\gamma$ -vertex is described by the effective Lagrangian [15]

$$\mathcal{L}_{\rho\pi\gamma}^{int.} = \frac{e}{M_{\rho}}g_{\rho\pi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_{\mu}\vec{\rho}_{\nu} \cdot \vec{\pi}\partial_{\alpha}A_{\beta} \quad . \quad (5)$$

The coupling constant $g_{\rho\pi\gamma}$ can then be obtained from the experimental partial width of the radiative decay $\rho \rightarrow \pi\gamma$ [2]. However, at present there appears to be a discrepancy between the experimental widths of the $\rho^0 \rightarrow \pi^0\gamma$ and $\rho^+ \rightarrow \pi^+\gamma$ decays [2]. We use the experimental rate for the decay $\rho^0 \rightarrow \pi^0\gamma$ to extract the coupling constant $g_{\rho\pi\gamma}$ as $g_{\rho\pi\gamma}^2 = 0.485$ since in our calculation we use the experimental value for the decay rate of $\omega \rightarrow \pi^0\pi^0\gamma$ to estimate the coupling constant $g_{\omega\sigma\gamma}$. Finally, we describe the $\omega\sigma\gamma$ -vertex by the effective Lagrangian [20]

$$\mathcal{L}_{\omega\sigma\gamma}^{int.} = \frac{e}{M_{\omega}}g_{\omega\sigma\gamma}[\partial^{\alpha}\omega^{\beta}\partial_{\alpha}A_{\beta} - \partial^{\alpha}\omega^{\beta}\partial_{\beta}A_{\alpha}]\sigma \quad , \quad (6)$$

which also defines the coupling constant $g_{\omega\sigma\gamma}$.

In our calculation of the invariant amplitudes for the decays $\omega \rightarrow \pi^0\pi^0\gamma$ and $\omega \rightarrow \pi^+\pi^-\gamma$, in the σ -meson propagator we make the replacement $M_{\sigma} \rightarrow M_{\sigma} - \frac{1}{2}i\Gamma_{\sigma}$, where Γ_{σ} is given

by Eq. 4. Since the experimental candidate for σ -meson $f_0(400 - 1200)$ has a width of (600-1000) MeV [2], we estimate the coupling constant $g_{\omega\sigma\gamma}$ from the experimental decay rate of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay for a set of values of σ -meson parameters M_σ and Γ_σ . We furthermore consider the results of two recent experiments for σ meson parameters [12]- [13]. On the other hand, for ρ -meson propagator we also make the replacement $M_\rho \rightarrow M_\rho - \frac{1}{2}i\Gamma_\rho$ but use the constant experimental width of ρ -meson.

In terms of the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$, the differential decay probability of $\omega \rightarrow \pi\pi\gamma$ decay for an unpolarized ω -meson at rest is then given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3} \frac{1}{8M_\omega} |\mathcal{M}|^2, \quad (7)$$

where E_γ and E_1 are the photon and pion energies respectively. We perform an average over the spin states of ω -meson and a sum over the polarization states of the photon. The decay width $\Gamma(\omega \rightarrow \pi\pi\gamma)$ is then obtained by integration

$$\Gamma = \left(\frac{1}{2}\right) \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad (8)$$

where now the factor $(\frac{1}{2})$ is included in the calculation of the width of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay because of the $\pi^0\pi^0$ pair in the final state. The minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M_\omega^2 - 4M_\pi^2)/2M_\omega = 341$ MeV. The maximum and minimum values for pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_\omega - M_\omega^2)} [-2E_\gamma^2 M_\omega + 3E_\gamma M_\omega^2 - M_\omega^3 \pm E_\gamma \sqrt{(-2E_\gamma M_\omega + M_\omega^2)(-2E_\gamma M_\omega + M_\omega^2 - 4M_\pi^2)}].$$

We first consider the $\omega \rightarrow \pi^0\pi^0\gamma$ decay and using the experimental value for its decay rate we estimate the coupling constant $g_{\omega\sigma\gamma}$. Since the theoretical decay rate we calculate using Feynman diagrams in Fig. 1 results in a quadric equation for the coupling constant $g_{\omega\sigma\gamma}$, for a given set of σ -meson parameters M_σ and Γ_σ we obtain two values for this coupling constant, one being positive and one being negative. We present the results of our calculation in the first two columns of Table 1. We then consider the $\omega \rightarrow \pi^+\pi^-\gamma$ decay and using the

experimental upper limit for its decay rate and the theoretical value we calculate from Feynman diagrams in Fig. 2 this time we obtain upper and lower limits, in other words an interval, for the coupling constant $g_{\omega\sigma\gamma}$. The results are presented in the last two columns of Table 1 again for different sets of parameters M_σ and Γ_σ . These results determine an interval for $g_{\omega\sigma\gamma}$, for example for $M_\sigma = 500$ MeV and $\Gamma_\sigma = 600$ MeV the interval is $-1.73 < g_{\omega\sigma\gamma} < 1.58$. Examination of the results in the first two and the last two columns of Table 1 show that these results are consistent with each other. We then, by using the values of the coupling constant $g_{\omega\sigma\gamma}$ we estimate from the $\omega \rightarrow \pi^0\pi^0\gamma$ decay, calculate the decay rate of $\omega \rightarrow \pi^+\pi^+\gamma$ decay using a given set of σ -meson parameters M_σ and Γ_σ . We like to note, however, that our results for the $\omega \rightarrow \pi^+\pi^-\gamma$ decay rate does not change for different set of M_σ and Γ_σ as expected, so that we only give the result for the set $M_\sigma = 478$ MeV and $\Gamma_\sigma = 374$ MeV obtained in the recent Fermilab experiment E791 [13]. If we choose the positive value for $g_{\omega\sigma\gamma} = 0.13$ the resulting branching ratio for the decay $\omega \rightarrow \pi^+\pi^-\gamma$ is $Br(\omega \rightarrow \pi^+\pi^-\gamma) = 14.6 \times 10^{-5}$ and for $g_{\omega\sigma\gamma} = -0.27$ it is also $Br(\omega \rightarrow \pi^+\pi^-\gamma) = 14.6 \times 10^{-5}$ in accordance with Singer's theorem [3].

The photon spectra for the decay rate of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ are plotted in Fig. 3 and in Fig. 4. In these figures we use the σ -meson parameters $M_\sigma = 478$ MeV, $\Gamma_\sigma = 324$ MeV resulting in the coupling constant $g_{\sigma\pi\pi} = 5.29$. In Fig. 3 we use the positive value of the coupling constant $g_{\omega\sigma\gamma} = 0.13$ and in Fig. 4 the negative value $g_{\omega\sigma\gamma} = -0.27$. The general shape of the spectrum as well as the relative contributions of different terms for positive and negative values of $g_{\omega\sigma\gamma}$ are quite different. These figures clearly show the importance of the σ -amplitude term and of the interference term between σ -amplitude and VDM-amplitude. As a matter of fact for these values of M_σ and Γ_σ , and for $g_{\omega\sigma\gamma} = 0.13$ we obtain for the decay rate of $\omega \rightarrow \pi^0\pi^0\gamma$ calculated using σ - and VDM-amplitudes the results $\Gamma_{VDM}(\omega \rightarrow \pi^0\pi^0\gamma) = 291$ eV and $\Gamma_\sigma(\omega \rightarrow \pi^0\pi^0\gamma) = 156$ eV, and the interference term between σ - and VDM-amplitudes contributes $\Gamma_{inter}(\omega \rightarrow \pi^0\pi^0\gamma) = 167$ eV to the decay rate. On the other hand for the negative value of the coupling constant $g_{\omega\sigma\gamma} = -0.27$ we obtain for the decay rate of $\omega \rightarrow \pi^0\pi^0\gamma$ calculated using σ - and VDM-amplitudes the results

$\Gamma_{VDM}(\omega \rightarrow \pi^0\pi^0\gamma) = 291$ eV and $\Gamma_{\sigma}(\omega \rightarrow \pi^0\pi^0\gamma) = 674$ eV, and the interference term between σ - and VDM-amplitudes now makes a negative contribution $\Gamma_{inter}(\omega \rightarrow \pi^0\pi^0\gamma) = -347$ eV to the decay rate.

We like to stress that by considering the contribution of the σ -meson intermediate state to the amplitudes for the $\omega \rightarrow \pi\pi\gamma$ decays we may resolve the discrepancy between the experimental value and the theoretical result calculated within the framework of chiral perturbation theory and Vector Meson Dominance for the $\omega \rightarrow \pi^0\pi^0\gamma$ decay rate. Moreover, since we also make prediction for the photon spectra of $\omega \rightarrow \pi^+\pi^-\gamma$ decay its measurement can provide a test for the mechanism of $\omega \rightarrow \pi\pi\gamma$ decays, and the sign of the coupling constant $g_{\omega\sigma\gamma}$.

Furthermore, the coupling constant $g_{\omega\sigma\gamma}$ is an important physical input for studies of ω -meson photoproduction on nucleons [21]. Although at sufficiently high energies and low momentum transfers electromagnetic production of vector mesons on nucleon targets has been explained by Pomeron exchange models, at low energies near threshold scalar and pseudoscalar meson exchange mechanisms becomes important [20], so that the coupling constant $g_{\omega\sigma\gamma}$ is required for the analysis of photoproduction reactions of ω -meson on nucleons near threshold within the framework of meson-exchange mechanism.

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TABLES

TABLE I. The coupling constant $g_{\omega\sigma\gamma}$ for different set of σ -meson parameters. First two columns show $g_{\omega\sigma\gamma}$ estimated from the experimental rate of the $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma)$ decay, and the last two columns show the upper and lower limits for $g_{\omega\sigma\gamma}$ estimated from the experimental upper limit of the $\Gamma(\omega \rightarrow \pi^+\pi^-\gamma)$ decay rate.

$g_{\omega\sigma\gamma}(\omega \rightarrow \pi^0\pi^0\gamma)$		M_σ (MeV)	Γ_σ (MeV)	$g_{\sigma\pi\pi}$	$g_{\omega\sigma\gamma}(\omega \rightarrow \pi^+\pi^-\gamma)$	
0.18	-0.33	500	600	6.97	1.58	-1.73
0.21	-0.36	500	800	8.04	1.80	-1.95
0.24	-0.43	600	800	7.11	2.10	-2.30
0.27	-0.51	700	800	6.46	2.45	-2.69
0.30	-0.59	800	600	5.18	2.73	-3.02
0.32	-0.61	800	900	6.34	2.91	-3.19
0.37	-0.71	900	900	5.94	3.31	-3.65
0.18	-0.36	555	540	6.15	1.68	-1.85
0.13	-0.27	478	324	5.29	1.20	-1.34

Figure Captions:

Figure 1: Feynman Diagrams for the decay $\omega \rightarrow \pi^0\pi^0\gamma$

Figure 2: Feynman Diagrams for the decay $\omega \rightarrow \pi^+\pi^-\gamma$

Figure 3: The photon spectra for the decay width of $\omega \rightarrow \pi^0\pi^0\gamma$ for $g_{\omega\sigma\gamma} > 0$. The contributions of different terms are indicated.

Figure 4: The photon spectra for the decay width of $\omega \rightarrow \pi^0\pi^0\gamma$ for $g_{\omega\sigma\gamma} < 0$. The contributions of different terms are indicated.

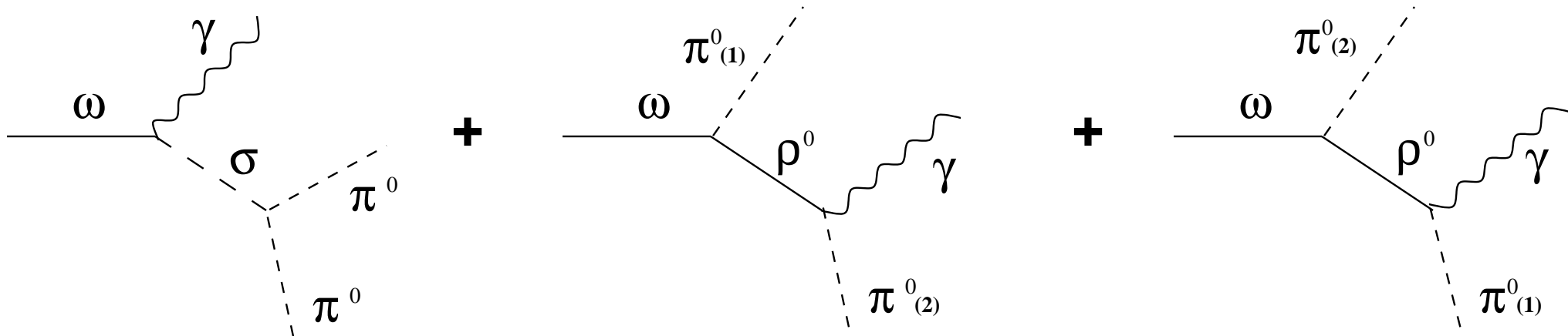


Figure 1

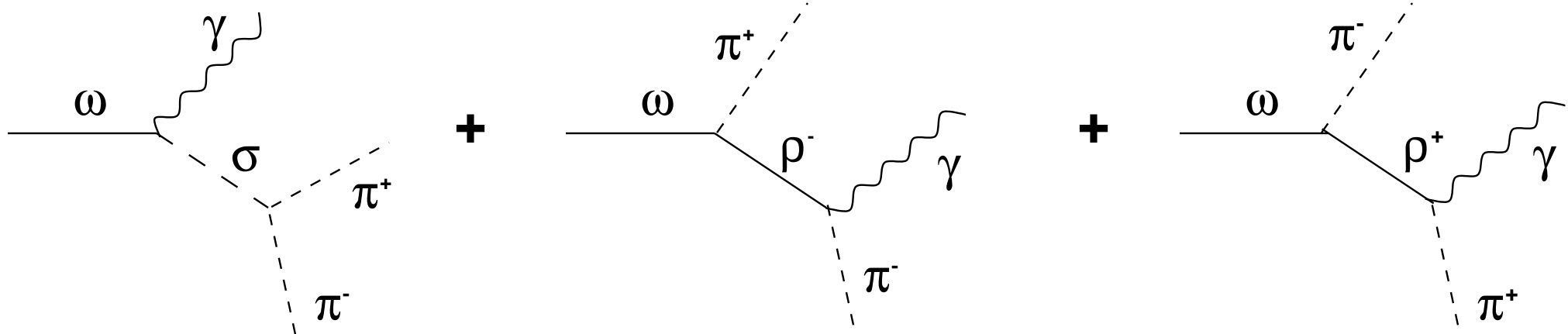


Figure 2

Fig. 3

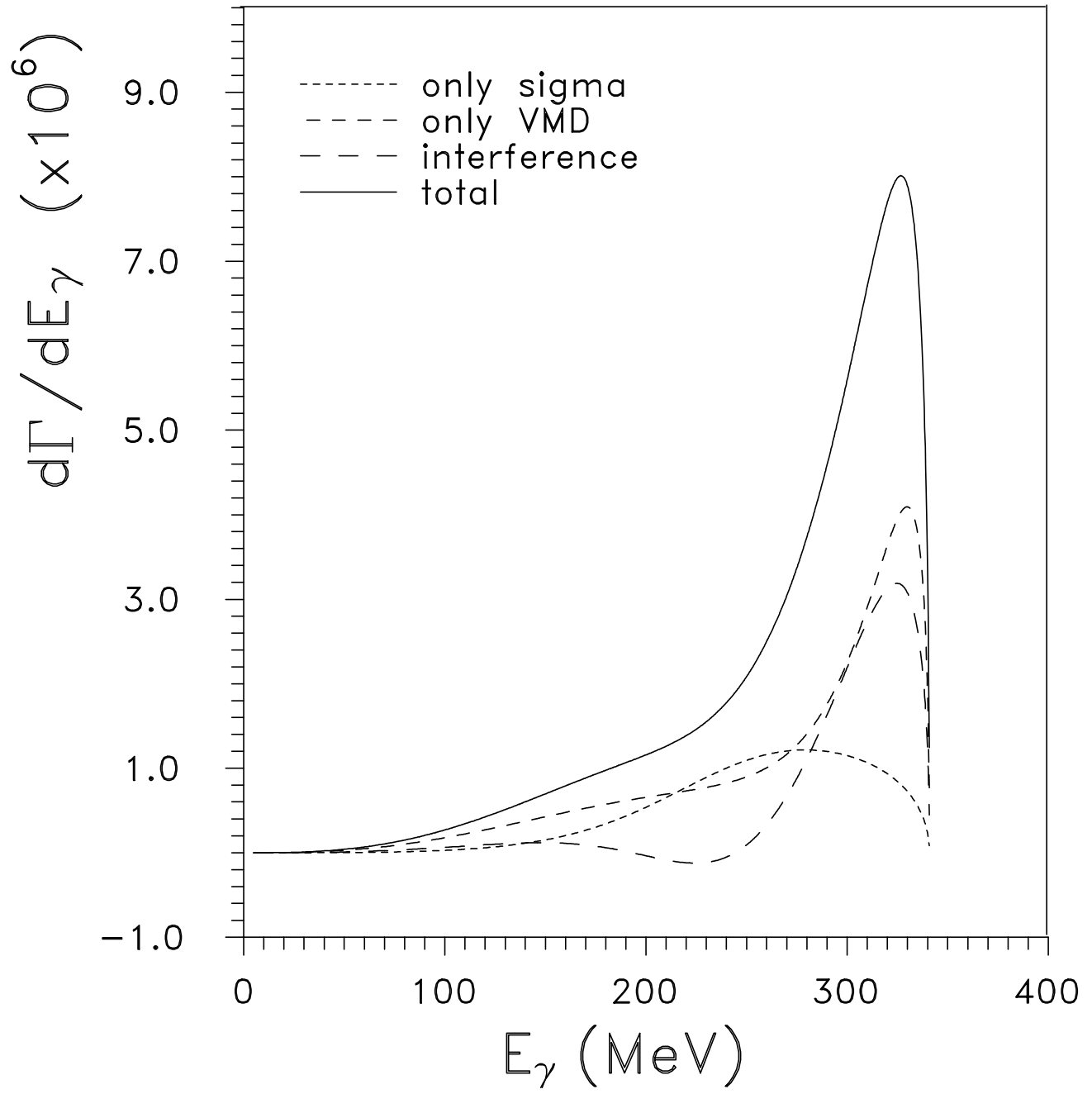


Fig. 4

