

# The coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$ as derived from QCD sum rules

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## Abstract

We employ QCD sum rules to calculate the coupling constants  $g_{\rho\pi\gamma}$  and  $g_{\omega\pi\gamma}$  by studying the three point  $\rho\pi\gamma$ - and  $\omega\pi\gamma$ -correlation functions. Our results for the decay widths  $\Gamma(\rho^0 \rightarrow \pi^0\gamma)$  and  $\Gamma(\omega \rightarrow \pi^0\gamma)$  calculated using the obtained coupling constants are in good agreement with the experimental values of these decay widths.

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Radiative transitions of the type  $V \rightarrow P\gamma$  where V and P belong the lowest multiplets of vector (V) and pseudoscalar (P) mesons have been a subject of continuous interest both theoretically and experimentally [1]. These transitions have been considered from the point of view of a large variety of theoretical models, such as phenomenological quark models [2], potential models [3], bag models [4], and effective Lagrangian approaches [5]. All these approaches provide effective methods of investigation of these hadronic phenomena for which a formulation for the application of QCD from the first principles has not been possible so far. The effective Lagrangian approach provide a framework to study in general the physics of light neutral vector mesons,  $\rho^0$ ,  $\omega$  and  $\phi$ , by combining the Vector Meson Dominance and Chiral Dynamics which are the two principles governing low energy QCD in a suitably constructed effective Lagrangian [6].

On the other hand, vector meson-pseudoscalar meson-photon  $VP\gamma$ -vertex also plays a role in photoproduction reactions of vector mesons on nucleons. Although, at sufficiently high energies and low momentum transfers electromagnetic production of vector mesons on nucleon targets has been explained by Pomeron exchange models, at low energies near threshold scalar and pseudoscalar meson exchange mechanisms become important [7]. For the photoproduction reactions involving  $\rho^0$  and  $\omega$  mesons, the effective coupling constants  $g_{\rho\pi\gamma}$  and  $g_{\omega\pi\gamma}$  are among the physical inputs that are used in the analyzes of these reactions. In these studies, an effective Lagrangian describing the  $VP\gamma$ -vertex is assumed, which also defines the coupling constant  $g_{VP\gamma}$ , and these coupling constants are then determined utilizing the experimental decay widths  $\Gamma(V \rightarrow P\gamma)$  of the vector mesons. However, it should be noted that in these decays the four-momentum of the pseudoscalar meson P is time-like,  $p'^2 > 0$ , whereas in the pseudoscalar exchange amplitude contributing to the photoproduction of vector mesons it is space-like  $p'^2 < 0$ . Therefore, it is of interest to study the effective coupling constants  $g_{VP\gamma}$  from another point of view as well.

In this work, we estimate the coupling constants  $g_{\rho\pi\gamma}$  and  $g_{\omega\pi\gamma}$  by employing QCD sum rules which provide an efficient and model-independent method to study many hadronic observables, such as decay constants and form factors, in terms of nonperturbative con-

tributions proportional to the quark and gluon condensates [8,9]. Using the techniques of QCD sum rules, the nonperturbative QCD physics is incorporated systematically as power corrections in the short-distance operator product expansion.

In order to derive the QCD sum rule for the coupling constant  $g_{V\pi\gamma}$  where V denotes  $\rho^0$  or  $\omega$  meson, we begin by considering the three point correlation function

$$T_{\mu\nu}(p, p') = \int d^4x d^4y e^{ip' \cdot y} e^{-ip \cdot x} \langle o | T \{ j_\mu^\gamma(0) j_\nu^V(x) j_5(y) \} | 0 \rangle \quad (1)$$

where the interpolating currents  $j_\nu^V$  for  $\rho^0$  and  $\omega$  meson are  $j_\nu^\rho = \frac{1}{2}(\bar{u}\gamma_\nu u - \bar{d}\gamma_\nu d)$ ,  $j_\nu^\omega = \frac{1}{6}(\bar{u}\gamma_\nu u + \bar{d}\gamma_\nu d)$ , respectively,  $j_5 = \frac{1}{2}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)$  is the interpolating current for  $\pi^0$  [8], and  $j_\mu^\gamma = e_u \bar{u}\gamma_\mu u + e_d \bar{d}\gamma_\mu d$ , where  $e_u$  and  $e_d$  denote the quark charges, is the quark electromagnetic current. In accordance with QCD sum rules techniques, we consider the three point correlation function  $T_{\mu\nu}(p, p')$  in the Euclidean region defined by  $p^2 = -Q^2 \sim -1 \text{ GeV}^2$ ,  $p'^2 = -Q'^2 \sim -1 \text{ GeV}^2$ .

The theoretical part of the sum rule for the coupling constant  $g_{V\pi\gamma}$  is obtained in terms of QCD degrees of freedom by calculating the perturbative contribution and the power corrections from operators of different dimensions to the three point correlation function. In the region  $Q^2$ ,  $Q'^2 \sim 1 \text{ GeV}^2$  the perturbative contribution can be approximated by the lowest order free-quark loop diagram shown in Fig. 1-a. Furthermore, we consider the power corrections from operators of different dimensions, resulting in contributions to the three point correlation function that are proportional to the terms  $\langle \bar{q}q \rangle$ ,  $\langle \sigma \cdot G \rangle$  and  $\langle (\bar{q}q)^2 \rangle$ . We do not consider the gluon condensate contribution proportional to  $\langle G^2 \rangle$  since it is estimated to be negligible for light quark systems. The calculations of the power corrections are performed in the fixed point gauge [10]. We work in the SU(2) flavour context with  $m_u=m_d=m_q$ , moreover we perform our calculations of the perturbative and power correction contributions in the limit  $m_q = 0$ . In this limit, the perturbative bare-loop diagram does not make any contribution, and only operators of dimensions  $d=3$  and  $d=5$  make contributions that are proportional to  $\langle \bar{q}q \rangle$  and  $\langle \sigma \cdot G \rangle$ , respectively. The relevant Feynman diagrams for the calculation of these power corrections are shown in Fig. 1-b and

Fig. 1-c.

We then turn to the calculation of the three point correlation function through phenomenological considerations. The vertex function  $T_{\mu\nu}(p, p')$  satisfies a double dispersion relation. In general such a dispersion relation can be written in three ways by choosing two of the three channels. For our purpose, we choose the vector and pseudoscalar channels and by saturating this dispersion relation by the lowest lying meson states in these channels we obtain the physical part of the sum rule as

$$T_{\mu\nu}(p, p') = \frac{\langle 0|j_\nu^V|V \rangle \langle V(p)|j_\mu^\gamma|\pi(p') \rangle \langle \pi|j_5|0 \rangle}{(p^2 - m_V^2)(p'^2 - m_\pi^2)} + \dots \quad (2)$$

where the contributions from the higher states and the continuum is denoted by dots. In this expression, the overlap amplitudes for vector and pseudoscalar mesons are  $\langle 0|j_\nu^V|V \rangle = \lambda_V u_V$  where  $u_V$  is the polarization vector of the vector meson and  $\langle \pi|j_5|0 \rangle = \lambda_\pi$ . The matrix element of the electromagnetic current is given as

$$\langle V(p)|j_\mu^\gamma|\pi(p') \rangle = -i \frac{e}{m_V} g_{V\pi\gamma} K(q^2) \varepsilon^{\mu\alpha\beta\delta} p_\alpha u_\beta q_\delta \quad (3)$$

where  $q = p - p'$  and  $K(0)=1$ . This expression defines the coupling constant  $g_{V\pi\gamma}$  through the effective Lagrangian

$$\mathcal{L}_{V\pi\gamma}^{eff.} = \frac{e}{m_V} g_{V\pi\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu V_\nu \partial_\alpha A_\beta \pi^0 \quad (4)$$

describing the  $V\pi\gamma$ -vertex [7].

After performing the double Borel transform with respect to the variables  $Q^2$  and  $Q'^2$ , we obtain the sum rule for the coupling constant  $g_{V\pi\gamma}$

$$g_{V\pi\gamma} = \frac{3m_V}{\lambda_V \lambda_\pi} e^{\frac{m_V^2}{M^2}} e^{\frac{m_\pi^2}{M'^2}} C_V \langle \bar{q}q \rangle \left( -\frac{3}{4} + \frac{5}{32} m_0^2 \frac{1}{M^2} - \frac{3}{32} m_0^2 \frac{1}{M'^2} \right) \quad (5)$$

where we use the relation  $\langle \sigma \cdot G \rangle = m_0^2 \langle \bar{q}q \rangle$ . The constant  $C_V$  that results in our calculation is  $C_V = 1$  for  $\rho^0$  meson and  $C_V = 3$  for  $\omega$  meson. For the numerical evaluation of the sum rule we use the values  $m_0^2 = 0.8 \text{ GeV}^2$ ,  $\langle \bar{u}u \rangle = -0.014 \text{ GeV}^3$  [11], and  $m_\rho = 0.770 \text{ GeV}$ ,  $m_\omega = 0.782 \text{ GeV}$ ,  $m_\pi = 0.138 \text{ GeV}$  [12]. For the overlap amplitude for

the vector meson states, we use the values that are obtained from the experimental leptonic decay widths [12] by noting that neglecting the electron mass the  $e^+e^-$  decay width of vector meson is given as  $\Gamma(V \rightarrow e^+e^-) = \frac{4\pi\alpha^2}{3} \frac{\lambda_V^2}{m_V^3}$ , this way we obtain the values  $\lambda_\rho = 0.118 \text{ GeV}^2$  and  $\lambda_\omega = 0.036 \text{ GeV}^2$ . We note that these values do obey the SU(3) relation  $\lambda_\rho = 3\lambda_\omega$  within 10% accuracy. The overlap amplitude  $\lambda_\pi$  for the  $\pi$  meson state is given by the relation  $\lambda_\pi = f_\pi \frac{m_\pi^2}{m_u+m_d}$  [14]. We use the experimental value  $f_\pi=0.132 \text{ GeV}$  and the physical mass  $m_{\pi^0}=0.138 \text{ GeV}$  along with  $m_u + m_d=0.014 \text{ GeV}$ , and obtain this amplitude as  $\lambda_\pi=0.18 \text{ GeV}^2$ . In order to analyze the dependence of the coupling constant  $g_{V\pi\gamma}$  on the Borel parameters  $M^2$  and  $M'^2$ , we study independent variations of  $M^2$  and  $M'^2$  in the interval  $0.6 \text{ GeV}^2 \leq M^2, M'^2 \leq 1.4 \text{ GeV}^2$  as these limits determine the allowed interval for the vector channel [13]. The variation of the coupling constant  $g_{\rho\pi\gamma}$  and  $g_{\omega\pi\gamma}$  as a function of Borel parameters  $M^2$  for different values of  $M'^2$  is shown in Fig. 2 and in Fig. 3, respectively. The examination of these figures indicate that the sum rule is quite stable with these reasonable variations of  $M^2$  and  $M'^2$ . Besides those due to variations of  $M^2$  and  $M'^2$ , the other sources contributing to the uncertainty in the coupling constants are the uncertainties in the estimated values of the vacuum condensates. If we take these uncertainties into account by a conservative estimate, we obtain the coupling constants  $g_{\rho\pi\gamma} = 0.63 \pm 0.07$  and  $g_{\omega\pi\gamma} = 1.85 \pm 0.15$ . These values of the coupling constant are consistent with their values used in the analysis of  $\rho^0$  and  $\omega$  photoproduction reactions through pseudoscalar exchange amplitudes which are  $g_{\rho\pi\gamma} = 0.54$  and  $g_{\omega\pi\gamma} = 1.82$ , respectively [15]. If we use the effective Lagrangian given in Eq. 4, then the decay width for  $V \rightarrow \pi^0\gamma$  is obtained as

$$\Gamma(V \rightarrow \pi^0\gamma) = \frac{\alpha}{24} \frac{(m_V^2 - m_{\pi^0}^2)^3}{m_V^5} g_{V\pi\gamma}^2 \quad . \quad (6)$$

Therefore, from our analysis we determine  $\Gamma(V \rightarrow \pi^0\gamma)$  decay widths for  $\rho^0$  and  $\omega$  mesons as  $\Gamma(\rho^0 \rightarrow \pi^0\gamma) = (84 \pm 15) \text{ KeV}$  and  $\Gamma(\omega \rightarrow \pi^0\gamma) = (740 \pm 80) \text{ KeV}$ . The measured decay widths [12] are  $\Gamma(\rho^0 \rightarrow \pi^0\gamma) = (102 \pm 26) \text{ KeV}$  and  $\Gamma(\omega \rightarrow \pi^0\gamma) = (717 \pm 43) \text{ KeV}$  which roughly follow the SU(3) prediction for their ratio. Our results are in good agreement with the experimental values of these decay widths. We also note that the electromagnetic

decays  $V \rightarrow P\gamma$  of vector mesons in the flavour  $SU(3)$  sector was studied previously [14] by employing the method of QCD sum rules in the presence of the external electromagnetic field. Our results which are obtained by QCD sum rules utilizing three point correlation functions are consistent with the values obtained in that analysis and therefore supplements the study of these decays using QCD sum rules method.

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**Figure Captions:**

**Figure 1:** Feynman Diagrams for the  $V\pi\gamma$ -vertex: a- bare loop diagram, b- d=3 operator corrections, c- d=5 operator corrections. The dotted lines denote gluons.

**Figure 2:** The coupling constant  $g_{\rho\pi\gamma}$  as a function of the Borel parameter  $M^2$  for different values of  $M'^2$ .

**Figure 3:** The coupling constant  $g_{\omega\pi\gamma}$  as a function of the Borel parameter  $M^2$  for different values of  $M'^2$ .



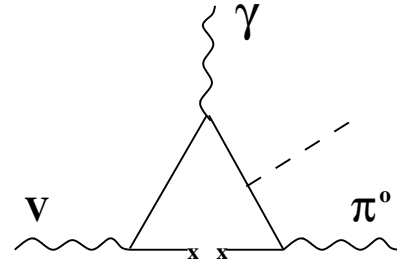
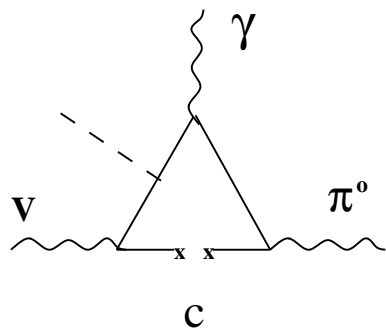
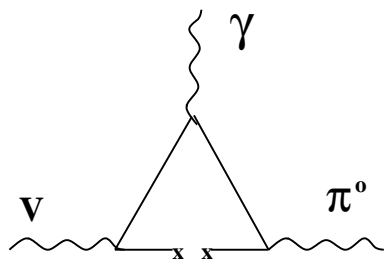
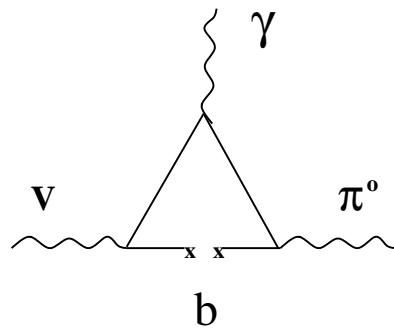
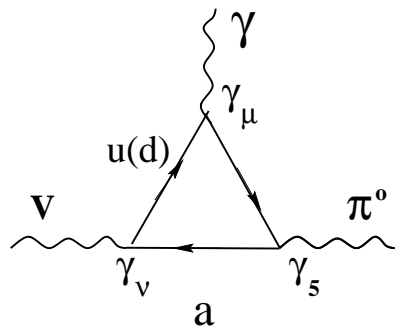


Figure 1

Figure 2

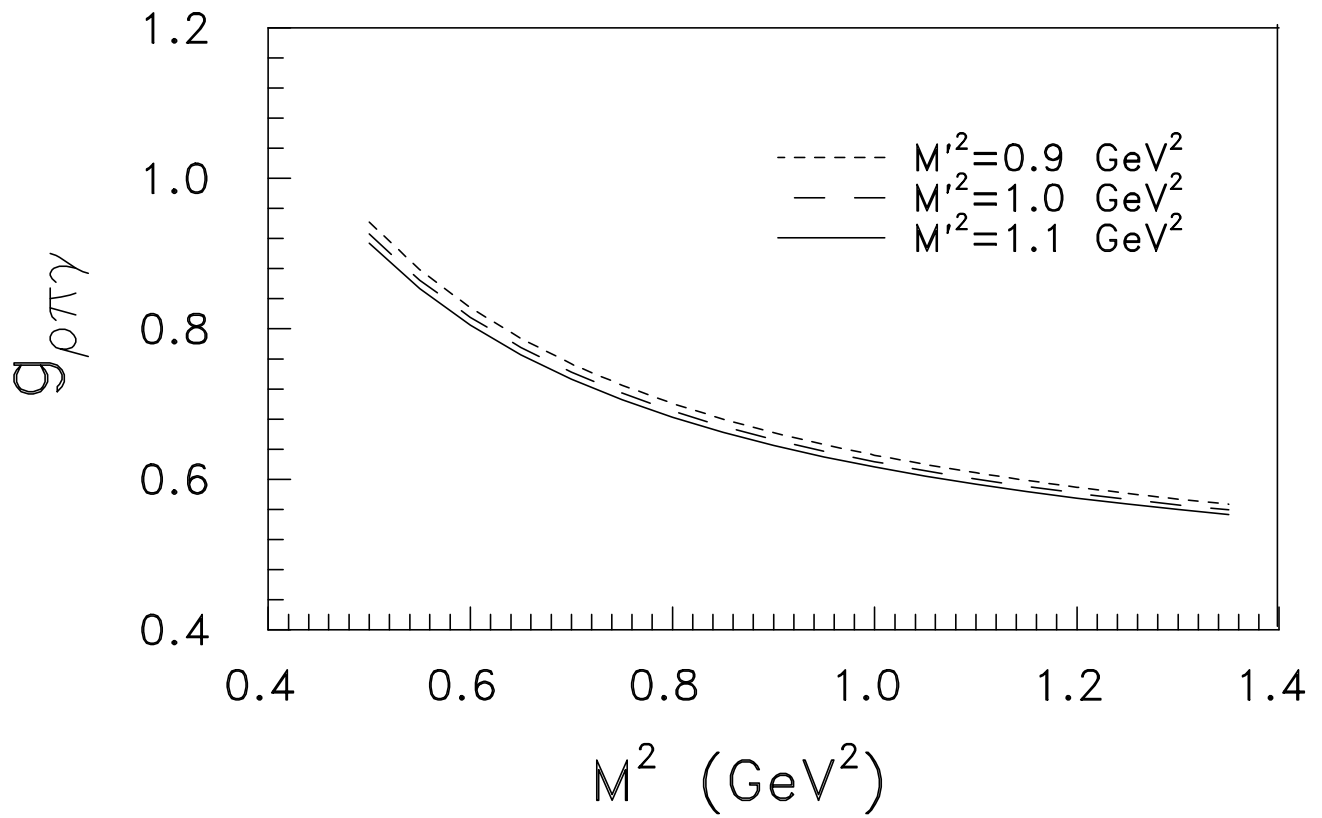


Figure 3

