B_c and heavy meson spectroscopy in the local approximation of the Schrödinger equation with relativistic kinematics

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Abstract

We present bound state masses of the self-conjugate and non-self-conjugate mesons in the context of the Schrödinger equation taking into account the relativistic kinematics and the quark spins. We apply the usual interaction by adding the spin dependent correction. The hyperfine splittings for the 2S charmonium and 1S bottomonium are calculated. The pseudoscalar and vector decay constants of the B_c meson and the unperturbed radial wave function at the origin are also calculated. We have obtained a local equation with a complete relativistic corrections to a class of three attractive static interaction potentials of the general form $V(r) = -Ar^{-\beta} + \kappa r^{\beta} + V_0$, with $\beta = 1, 1/2,\ 3/4$ which can also be decomposed into scalar and vector parts in the form $V_V(r) = -Ar^{-\beta} + (1-\epsilon)\kappa r^{\beta}$ and $V_S(r) = \epsilon\kappa r^{\beta} + V_0$; where $0 \leq \epsilon \leq 1$. The energy eigenvalues are carried out up to the third order approximation using the shifted large-N-expansion technique.

I. INTRODUCTION

Theoretical interest has risen in the study of the spectroscopy of B$_c$ meson in the framework of heavy quarkonium theory [1]. Moreover, the discovery of the B_c (the lowest pseudoscalar $^1S_0$ state) was reported in 1998 by the Collider Detector at Fermilab (CDF) collaboration in 1.8 TeV p-$\overline{p}$ collisions at the Fermilab [2] with an observed mass $M_{B_c} = 6.40 \pm 0.39 \pm 0.13\ GeV$ has inspired new theoretical interest in the subject [3-6]. Further, Kwong and Rosner [7] predicted the masses of the lowest vector (triplet) and pseudoscalar (singlet) states of the B_c systems using an empirical mass formula and a logarithmic potential. Eichten and Quigg [1] calculated the energies and decays of the B_c system that was based on the QCD-motivated potential of Buchmüller and Tye [8]. Gershtein *et al.* [9] also presented a detailed account of the energies and decays of the B_c system and used a QCD sum-rule calculations. Baldicchi and Prosperi [6] have computed the $c\overline{b}$ and entire light-heavy quarkonium spectrum based on an effective mass operator with full relativistic kinematics. Fulcher [4] extended the treatment of the spin-dependent potentials to the full radiative one-loop level and thus included effects of the running coupling constant in these potentials. He also used the renormalization scheme developed by Gupta and Radford [10]. On the other hand, Motyka and Zalewiski [11] proposed a nonrelativistic potential model to reproduce the masses of the known $b\overline{b}$ spectrum within the experimental errors using a new proposed potential form for quarkonia. They also extended their work [11] by suplementing the Hamiltonian with the standard spin-dependent terms and produced the $c\overline{c}$ and $c\overline{b}$ quarkonium mass spectra, leptonic decay constant and also decay widths. The shifted large-N expansion technique (SLNET) [12] was applied to get the spin-averaged data (SAD) of both $Q\overline{Q}$ and $q\overline{Q}$ mesons using a group of quarkonium potentials [13] and very recently was utilized to study the $c\bar{b}$ system in the context of Schrödinger equation and also semi-relativistic quark model [14].

Recently, in 2002, the ALEPH collaboration has searched for the pseudoscalar bottomonium meson, the η_b in two-photon interactions at LEP2 with an integrated luminosity of 699 pb^{-1} collected at e^+e^- centre-of mass energies from 181 GeV to 209 GeV. One candidate event is found in the six-charged-particle final state and none in the four-charged-particle final state. The candidate η_b ($\eta_b \rightarrow K_S K^-\pi^+\pi^-\pi^+$) has reconstructed invariant mass of $9.30 \pm 0.02 \pm 0.02$ GeV [15]. Theoretical estimates (from perturbative QCD and lattice nonrelativistic QCD of the mass splitting between $\eta_b(1\mathrm{S})$ and $\Upsilon(1\mathrm{S})$, $M(\Upsilon(1^3\mathrm{S}_1)) = 9.460$ GeV, are reported (cf. [15] and references therein).

Further, in 2002, the Belle Collaboration [16] has observed a new pseudoscalar charmonium state, the $\eta_c(2\mathrm{S})$, in exclusive $B \longrightarrow KK_SK^-\pi^+$ decays. The measured mass of the $\eta_c(2\mathrm{S})$, $M(\eta_c(2\mathrm{S})) = 3654 \pm 14$ MeV. It is close to the $\eta_c(2\mathrm{S})$ mass observed by the same group in the experiment $e^+e^- \longrightarrow J/\psi\eta_c$ where $M(\eta_c(2\mathrm{S})) = 3622 \pm 12$ MeV was found [17]. It is giving rise to a small hyperfine splitting for the 2S state, $\Delta_{\mathrm{hfs}}(2\mathrm{S},\mathrm{exp}) = M(2^3\mathrm{S}_1) - M(2^1\mathrm{S}_0) = 32 \pm 14$ MeV [18]. Badalian and Bakker [19] calculated the hyperfine splitting for the 2S charmonium state, $\Delta_{\mathrm{hfs}}(2\mathrm{S}) = 57 \pm 8$ MeV, in a recent work. Recksiegel and Sumino developed a new formalism [20] based on perturbative QCD to compute the hyperfine splittings of the bottomonium spectrum as well as the fine and hyperfine splittings of the charmonium spectrum [21].

The motivation of the present calculations is to extend the SLNET [12-14] to the treatment of the Schrödinger equation [13,14] by considering the spin dependent term $V_{SD}(r)$ that gives the splitting of the singlet and triplet states and of each $L \geq 1$ level into the four states ${}^1L_1$, ${}^3L_{L-1}$, ${}^3L_L$ and ${}^3L_{L+1}$. We also present solution for the Schrödinger equation to determine the bound state masses of the $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ mesons taking into account the spin-spin, spin-orbit and tensor interactions [22-29]. The spin effects are treated as perturbation to the static potential. We also calculate the masses of the recently found new charmonium $\eta_c(2\mathrm{S})$ and the searched bottomonium $\eta_b(1\mathrm{S})$ mesons together with the hyperfine splittings of their states.

The outline of this paper is as following: In Section II, we first review briefly the analytic solution of the Schrödinger equation for unequal mass case ($m_q \neq m_Q$) [14]. Section III is devoted for the class of three static potentials, which are decomposed into scalar and vector parts and also for their spin corrections. The cases of pure vector, pure scalar and equal mixture of vector-scalar coupling interactions are investigated. The pseudoscalar and vector decay constants of the B_c meson are briefly presented in Section IV. Finally, Section V contains our conclusions. Appendix A, and B contain some definitions as well as the formulas necessary to carry out the above mentioned computations.

II. WAVE EQUATION

We shall consider bound states consisting of fermions with masses m_q and m_Q and their spins $\mathbf{S}_1,\ \mathbf{S}_2$, interacting via a spherically symmetric central potential $V(r)$. Radial part of the Schrödinger equation in the N-dimensional space (in units $\hbar = 1$) [12-14] is:

$$\left\{-\frac{1}{4\mu}\frac{d^2}{dr^2} + \frac{[\bar{k}-(1-a)][\bar{k}-(3-a)]}{16\mu r^2} + V_{eff}(r)\right\} u(r) = E_{n,\ell} u(r), \tag{1}$$

where $\mu = (m_q m_Q)/(m_q + m_Q)$ denotes the reduced mass for the two bound interacting particles. Here $E_{n,\ell}$ denotes the Schrödinger binding energy, and $\bar{k} = N + 2l - a$, with a representing a proper shift to be calculated later on and l is the angular quantum number. We follow the shifted $1/\bar{k}$ expansion method [13,14] by defining

$$V(r(x)) = \frac{\bar{k}^2}{Q}\left[V(r_0) + \frac{V'(r_0) r_0 x}{\bar{k}^{1/2}} + \frac{V''(r_0) r_0^2 x^2}{2\bar{k}} + \cdots\right], \tag{2}$$

and also the energy eigenvalue expansion [13]

$$E_{n,\ell} = \frac{\bar{k}^2}{Q}\left[E_0 + E_1/\bar{k} + E_2/\bar{k}^2 + E_3/\bar{k}^3 + O\left(1/\bar{k}^4\right)\right], \tag{3}$$

where $x = \bar{k}^{1/2}(r/r_0 - 1)$ with r_0 is an arbitrary point where the Taylor expansions is being performed about and Q is a scale to be set equal to $\bar{k}^2$ at the end of our calculations. Following our previous works [13,14], we rewrite down the results as

$$E_0 = V(r_0) + \frac{Q}{16\mu r_0^2}, \tag{4}$$

$$E_1 = \frac{Q}{r_0^2}\left[\left(n_r + \frac{1}{2}\right)\omega - \frac{(2-a)}{8\mu}\right], \tag{5}$$

$$E_2 = \frac{Q}{r_0^2}\left[\frac{(1-a)(3-a)}{16\mu} + \alpha^{(1)}\right], \tag{6}$$

and

$$E_3 = \frac{Q}{r_0^2}\alpha^{(2)}, \tag{7}$$

where $\alpha^{(1)}$ and $\alpha^{(2)}$ are listed in Appendix A. Here the quantity r_0 is chosen to minimize the leading term, E_0 [13,14]

$$\frac{dE_0}{dr_0} = 0 \quad \text{and} \quad \frac{d^2E_0}{dr_0^2} > 0. \tag{8}$$

Therefore, r_0 satisfies the relation

$$Q = 8\mu r_0^3 V'(r_0), \tag{9}$$

and to solve for the shifting parameter a, the next contribution to the energy eigenvalue E_1 is chosen to vanish [12].

$$a = 2 - 4(2n_r + 1)\mu\omega, \tag{10}$$

with

$$\omega = \frac{1}{4\mu}\left[3 + \frac{r_0 V''(r_0)}{V'(r_0)}\right]^{1/2}. \tag{11}$$

Once r_0 is being determined, with the choice $\overline{k} = \sqrt{Q}$ which rescales the potential, we get an analytic expression the energy eigenvalues (3). The Coulomb potential is considered as a testing case, the results are found to be strongly convergent and highly accurate. The calculations of the energy eigenvalues were carried out up to the second order correction. Therefore, the bound state energy to the third order becomes

$$E_{n,l} = E_0 + \frac{1}{r_0^2}\left[\frac{(1-a)(3-a)}{16\mu} + \alpha^{(1)} + \frac{\alpha^{(2)}}{\overline{k}} + O\left(\frac{1}{\overline{k}^2}\right)\right]. \tag{12}$$

Once the problem is collapsed to its actual dimension $N = 3$, it simply rests the task of relating the coefficients of our equation to the one-dimensional anharmonic oscillator in order to read the energy spectrum. One obtains

$$1 + 2l + (2n_r + 1)\left[3 + \frac{r_0 V''(r_0)}{V'(r_0)}\right]^{1/2} = \left[8\mu r_0^3 V'(r_0)\right]^{1/2}. \tag{13}$$

We finally write the bound state mass for spinless particles as

$$M(q\overline{Q}) = m_q + m_Q + 2E_{n,l}. \tag{14}$$

where m_q and m_Q are the constituent meson masses whereas $n = n_r + 1$ is the principal quantum number. As stated before [13,14], for a fixed n the computed energies become more accurate as l increases. This is expected since the expansion parameter $1/\overline{k}$ becomes smaller as l becomes larger since the parameter $\overline{k}$ is proportional to n and appears in the denominator in higher-order correction.

III. HEAVY QUARKONIUM AND B_C MESON MASS SPECTRA

The spin-independent potential (which may be velocity dependent) essentially yields SAD. Furthermore, the spin-dependent term $V_{SD}(r)$ gives the splitting both of the ${}^3S_1$ and ${}^1S_0$, with $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$ is 1 and 0 for triplet and singlet states, respectively, and of each level into the four states ${}^3L_{L-1}$, ${}^3L_L$, ${}^3L_{L+1}$ and ${}^1L_1$. Thus the potential takes [23,26,28-30]

$$V_{eff}(r) = V_{static}(r) + V_{SD}(r) + V_{SI}(r), \tag{15}$$

with spin-dependent and spin-independent perturbation terms are given in Refs. [26,28,30]. Further, the static potential [14,31] takes the general form

$$V_{static}(r) = -Ar^{-\beta} + \kappa r^{\beta} + V_0;\ \beta = 1, 1/2,\ 3/4,\ A, \kappa \geq 0 \tag{16}$$

which has a limited character of Ref. [11,32], (i.e., same β), where V_0 may be of either sign. The form (16) includes three types of static potentials. The first static potential we consider is the Cornell [33] potential ($\beta = 1$) which is one of the earliest QCD-motivated potentials in the literature

$$V_C(r) = -\frac{A}{r} + \kappa r + V_0, \tag{17}$$

where $A = 4\alpha_s/3$, is a short range gluon exchange, and κ is a confinement constant. The second potential is that of Song and Lin [34] ($\beta = 1/2$) which is given by

$$V_{S-L}(r) = -\frac{A}{r^{1/2}} + \kappa r^{1/2} + V_0. \tag{18}$$

The third potential is an intermediate case between the last mentioned potentials and is called Turin potential [31] ($\beta = 3/4$) which has the form

$$V_T(r) = -\frac{A}{r^{3/4}} + \kappa r^{3/4} + V_0. \tag{19}$$

The class of static potentials in Eq. (16) must satisfy the following conditions [31]

$$\frac{dV}{dr} > 0, \ \frac{d^2V}{dr^2} \leq 0. \tag{20}$$

On the other hand, the expression (16) can be rewritten in a more general form with two different power parameters α and β as [32]:

$$V(r) = -Ar^{-\alpha} + \kappa r^{\beta} + V_0, \tag{21}$$

where $\alpha \neq \beta$. Motyka and Zalewiski [11] utilized the form (21) by setting $\alpha = 1$ and $\beta = 1/2$; that is,

$$V(r) = -\frac{A}{r} + \kappa\sqrt{r} + V_0, \tag{22}$$

The potential form (22) belongs to the class of generality (21) and was successfuly used by Motyka *et al.* in fitting the $c\bar{c}$ spectrum and later on extended to the $b\bar{b}$ and B_c spectroscopy [11]. In this work we devote our study to the first class of generality (16) leaving the second class of generality (21) for further study. We will use a fairly flexible parameterization of the potentials of (16) in fitting the data and take the nonrelativistic interaction as a sum of scalar and vector terms as it follows from the Lorentz invariance theory [26,29]

$$V_V(r) = -Ar^{-\beta} + (1-\epsilon)\kappa r^{\beta}, \tag{23}$$

and

$$V_S(r) = \epsilon\kappa r^{\beta} + V_0, \tag{24}$$

where ϵ is the mixing coefficient. The vector term incorporates the expected short-distance behavior from single-gluon exchange. We have also included a multiple of the long-range interaction in $V_V(r)$ to see the nature of the confining interaction. Here, we investigate the cases of pure scalar confinement ($\epsilon = 1$), equal mixture of scalar-vector couplings ($\epsilon = 1/2$) and a pure vector case ($\epsilon = 0$).

The total spin-dependent potential $V_{SD}(r)$ given by [23-25,28-30]

$$V_{SD}(r) = V_A + V_S = \frac{1}{4}\left[\frac{1}{m_q^2} - \frac{1}{m_Q^2}\right]\left[\frac{V_V'(r) - V_S'(r)}{r}\right]\mathbf{L}\cdot\mathbf{S}_-$$

$$+\frac{\mathbf{L}\cdot\mathbf{S}}{m_q m_Q}\frac{V_V'(r)}{r} + \frac{1}{2}\left[\frac{\mathbf{L}\cdot\mathbf{S}_1}{m_q^2} + \frac{\mathbf{L}\cdot\mathbf{S}_2}{m_Q^2}\right]\left[\frac{V_V'(r) - V_S'(r)}{r}\right]$$

$$+\frac{2}{3}\frac{\mathbf{S}_1\cdot\mathbf{S}_2}{m_q m_Q}\left[\nabla^2 V_V(r)\right] + \frac{S_{12}}{m_q m_Q}\left[-V_V''(r) + \frac{V_V'(r)}{r}\right], \tag{25}$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ are the quark spins, $\mathbf{S}_- = \mathbf{S}_1 - \mathbf{S}_2$, $\mathbf{L} = \mathbf{x}\times\mathbf{p}$ is the relative orbital angular momentum, and $S_{12} = T - (\mathbf{S}_1\cdot\mathbf{S}_2)/3$ where $T = (\mathbf{S}_1\cdot\widehat{\mathbf{r}})(\mathbf{S}_2\cdot\widehat{\mathbf{r}})$ is the tensor operator with the versor $\widehat{\mathbf{r}} = \mathbf{r}/r$. The spin dependent correction (25), which is responsible for the hyperfine splitting of the mass levels, in the short-range is generally used in the form for S-wave ($L = 0$) (cf. e.g., [23,30]):

$$V_{hfs}(r) = \frac{2}{3}(\mathbf{S}_1\cdot\mathbf{S}_2)\nabla^2\left[-\frac{4\alpha_s}{3r^\beta}\right], \tag{26}$$

and the one responsible for the fine splittings used for P- and D-waves ($L \neq 0$) is:

$$V_{fs}(r) = \frac{1}{m_q m_Q}\left\{\frac{\mathbf{L}\cdot\mathbf{S}}{r}\left[\left(1 + \frac{1}{4}\frac{m_q^2 + m_Q^2}{m_q m_Q}\right)V_V'(r) - \frac{1}{4}\frac{m_q^2 + m_Q^2}{m_q m_Q}V_S'(r)\right]\right.$$

$$\left.+\frac{2}{3}(\mathbf{S}_1\cdot\mathbf{S}_2)\nabla^2\left[\kappa(1-\epsilon)r^\beta\right] + \left[T - \frac{1}{3}(\mathbf{S}_1\cdot\mathbf{S}_2)\right]\left[V_V''(r) + \frac{V_V'(r)}{r}\right]\right\}, \tag{27}$$

where the matrix element can be evaluated in terms of the expectation values $\langle\mathbf{L}\cdot\mathbf{S}_1\rangle = \langle\mathbf{L}\cdot\mathbf{S}_2\rangle = \frac{1}{2}\langle\mathbf{L}\cdot\mathbf{S}\rangle$. Hence, Eq. (27) is the complete spin-dependent potential in QCD through order m^2. For bound state constituents of spin $S_1 = S_2 = 1/2$, the scalar product of their spins $\mathbf{S}_1\cdot\mathbf{S}_2$ and $\mathbf{L}\cdot\mathbf{S}$ are to be found in the Appendix B. The appearance of a Coulomb-like contribution $\sim 1/r$ in the vector part of the potential causes some problems due to the relation $\nabla^2(1/r) = -4\pi\delta^{(3)}(x)$, in the spin-spin interaction (26) involves a delta function of the S-wave ($L = 0$). Thus, for Cornell potential, the hyperfine splitting potential (26) gives

$$V_{eff}(r) = -\frac{A}{r} + \kappa r + \frac{32\pi\alpha_s}{9m_q m_Q}\delta^{(3)}(\mathbf{r})(\mathbf{S}_1 \cdot \mathbf{S}_2) + V_0; \text{ where } \beta = 1. \tag{28}$$

Therefore for the energy of spin-spin interaction we have approximately:

$$E_{ss} = \frac{1}{2M_{n,0}}\Delta M_{ss}^2 \langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle, \tag{29}$$

where $M_{n,0}$ is given in Eq. (14) and the singlet-triplet mass squared difference

$$\Delta M_{ss}^2 = M_{S=1}^2 - M_{S=0}^2 \simeq \frac{32}{9}\alpha_s\kappa, \tag{30}$$

for light $q\overline{q}$ systems (in the instantaneous-limit approximation) [23], and

$$\Delta M_{ss}^2 = M_{S=1}^2 - M_{S=0}^2 \simeq \frac{256}{3\pi^2}\alpha_s\kappa, \tag{31}$$

for heavy quarkonia (hydrogen-like trial functions) [23]. All these predictions for the mass-squared difference are independent of the mass of the particles which constitute the bound state. Further, for the Song-Lin and Turin potentials, it also give

$$V_{eff}(r) = -\frac{A}{r^\beta} + \kappa r^\beta + \frac{8\beta(1-\beta)\pi\alpha_s}{9m_q m_Q r^2}r^{-\beta}\mathbf{S}_1 \cdot \mathbf{S}_2 + V_0; \text{ where } \beta = 1/2, 3/4. \tag{32}$$

Like most authors (cf. [1]), we determine the coupling constant $\alpha_s(m_c^2)$ from the well measured hyperfine splitting for the 1S($\overline{c}c$) state [18]

$$\Delta E_{\mathrm{HF}}(1\mathrm{S}, \exp) = M_{J/\psi} - M_{\eta_c} = 117.2 \pm 1.5 \text{ MeV}, \tag{33}$$

and also for the 2S($\overline{c}c$) state [16-19]

$$\Delta E_{\mathrm{HF}}(2\mathrm{S}, \exp) = M_{\psi'} - M_{\eta_{c'}} = 32 \pm 14 \text{ MeV}, \tag{34}$$

for each desired potential. On the other hand, the Eq. (27), for P, $D, \cdots$ waves ($L \neq 0$) case, gives

$$V_{eff}(r) = V_{static}(r) + g(r)\left[F_{LS_-}\left(\mathbf{L}\cdot\mathbf{S}_-\right) + F_{LS}\left(\mathbf{L}\cdot\mathbf{S}\right) + F_{SS}\left(\mathbf{S}_1\cdot\mathbf{S}_2\right) + F_T T\right], \qquad (35)$$

with a given set of spin-dependent quantities

$$F_{LS_-} = \left[\frac{1}{4}\frac{m_Q^2 - m_q^2}{m_q m_Q}\left[Ar^{-\beta} + (1-\epsilon)\,\kappa r^{\beta}\right]\right], \qquad (36)$$

$$F_{LS} = \left[\left(1 + \frac{1}{4}\frac{m_q^2 + m_Q^2}{m_q m_Q}\right)\left[Ar^{-\beta} + (1-2\epsilon)\,\kappa r^{\beta}\right] + \epsilon\kappa r^{\beta}\right], \qquad (37)$$

$$F_{SS} = \left[-\frac{(2+\beta)}{3}Ar^{-\beta} + \beta\,(1-\epsilon)\,\kappa r^{\beta}\right], \qquad (38)$$

and

$$F_T = \left[(2+\beta)Ar^{-\beta} + (2-\beta)\,(1-\epsilon)\,\kappa r^{\beta}\right], \qquad (39)$$

where $g(r) = \frac{\beta}{m_q m_Q r^2}$ is a necessary coupling function. The spin-independent corrections in Eq. (15) are explicitly given in Refs. [28,30] which are not treated in our present work.

A. Singlet states

For parastates ($L = J$) or ($S = 0$) case, we have parity $P = (-1)^{J+1}$ and charge conjugation $C = (-1)^L$. Thus, the potential (35) can be rewritten as

$$V_{eff}(r) = V_{static}(r) - \frac{1}{4}\left(3F_{SS} + F_T\right) + \sqrt{\frac{1}{10}(2L+3)(2L-1)}F_{LS_-}, \qquad (40)$$

which can be substituted in Eq. (1) and also by setting $\overline{k} = N + 2J - a$ therein. Further, Eqs. (28) and (32) give

$$V_{eff}(r) = -\frac{A}{r} + \kappa r - \frac{8\pi\alpha_s}{3m_q m_Q}\delta^{(3)}(\mathbf{r}) + V_0, \qquad (41)$$

and

$$V_{eff}(r) = -\frac{A}{r^{\beta}} + \kappa r^{\beta} - \frac{2\beta(1-\beta)\pi\alpha_s}{3m_q m_Q r^2}r^{-\beta} + V_0;\ \text{where } \beta = 1/2, 3/4, \qquad (42)$$

respectively, which generate singlet states with opposite quark and antiquark spins of the signature n^1S_0. Furthermore, Eq. (40) can be rewritten simply as

$$V_{J=L}(r) = g(r)\left\{\frac{1}{4}\frac{m_Q^2 - m_q^2}{m_q m_Q}\sqrt{\frac{1}{10}(2L+3)(2L-1)}\left[Ar^{-\beta} + (1-\epsilon)\kappa r^{\beta}\right]\right.$$

$$\left. - \frac{1}{2}(1+\beta)(1-\epsilon)\kappa r^{\beta}\right\} + V_{static}(r) \tag{43}$$

which generates states of the signatures n^1P_1, n^1D_2, n^1F_3, $n^1G_4, \cdots$.

B. Triplet states

For triplet ($S = 1$) case, we have the known inequality $|L - S| \leq J \leq L + S$ that gives $J = L$ and $L \pm 1$:

1. States $J = L$

Here, the parity $P = (-1)^{J+1}$and the charge conjugation $C = (-1)^{L+1}$. The potential in Eq. (35) takes the following simple form

$$V_{eff}(r) = V_{static}(r) + \frac{1}{4}\left(F_{SS} + F_T - 4F_{LS}\right) + \sqrt{\frac{1}{10}(2L+3)(2L-1)}F_{LS_-}, \tag{44}$$

which can be substituted in (1) together with $\overline{k} = N + 2J - a$ therein. Further, the potential (44) reads

$$V_{J=L}(r) = -\frac{g(r)}{2}\left\{\left[\left(\frac{4-\beta}{3} + \frac{1}{2}\frac{m_q^2 + m_Q^2}{m_q m_Q}\right)Ar^{-\beta} + \left(1 + \frac{1}{2}\frac{m_q^2 + m_Q^2}{m_q m_Q}\right)(1-2\epsilon)\kappa r^{\beta} + \epsilon\kappa r^{\beta}\right]\right.$$

$$\left. - \frac{1}{2}\frac{m_Q^2 - m_q^2}{m_q m_Q}\sqrt{\frac{1}{10}(2L+3)(2L-1)}\left[Ar^{-\beta} + (1-\epsilon)\kappa r^{\beta}\right]\right\} + V_{static}, \tag{45}$$

which generates states like n^3P_1, n^3D_2, n^3F_3, $n^3G_4, \cdots$.

2. States $J = L \pm 1$

We have the parity $P = (-1)^J$ and the charge conjugation $C = (-1)^{L+1}$. The eigenfunction is a superposition of two components with orbital momentum $L = J + 1$ and $L = J - 1$ which have equal space parity

$$\psi_{S,J}(r) = u_{J-1}(r) Y^m_{J-1,1,J}(\theta, \varphi) + u_{J+1}(r) Y^m_{J+1,1,J}(\theta, \varphi). \tag{46}$$

The action of the tensor operator, T, on the two components of the wavefunction in Eq. (46) is

$$T u_{J\pm1} Y^m_{J\pm1,1,J}(\widehat{\mathbf{r}}) = \mp \frac{1}{4(2J+1)} u_{J\pm1} Y^m_{J\pm1,1,J}(\widehat{\mathbf{r}}) + \frac{1}{2} \frac{\sqrt{J(J+1)}}{2J+1} u_{J\mp1} Y^m_{J\mp1,1,J}(\widehat{\mathbf{r}}). \tag{47}$$

Therefore, a set of equations are obtained

$$\left\{ -\frac{1}{4\mu} \frac{d^2}{dr^2} + \frac{\left[\overline{k} - (1-a)\right]\left[\overline{k} - (3-a)\right]}{16\mu r^2} + V_{static}(r) - E_{n,J+1} - (J+2) F_{LS} \right.$$

$$\left. + \frac{1}{4} \left(F_{SS} - \frac{F_T}{(2J+1)} \right) \right\} u_{n,J+1}(r) = \frac{\sqrt{J(J+1)}}{2(2J+1)} F_T \; u_{n,J-1}(r), \tag{48}$$

and

$$\left\{ -\frac{1}{4\mu} \frac{d^2}{dr^2} + \frac{\left[\overline{k} - (1-a)\right]\left[\overline{k} - (3-a)\right]}{16\mu r^2} + V_{static}(r) - E_{n,J-1} + (J-1) F_{LS} \right.$$

$$\left. + \frac{1}{4} \left(F_{SS} + \frac{F_T}{(2J+1)} \right) \right\} u_{n,J-1}(r) = \frac{\sqrt{J(J+1)}}{2(2J+1)} F_T \; u_{n,J+1}(r), \tag{49}$$

where $\overline{k} = N + 2J + 2 - a$. Therefore, Eqs. (48) and (49) describe states such as $n^3P_2,\ n^3D_3,\ n^3F_2,\ n^3H_4, n^3P_0,\ n^3D_1, \cdots$. Here we may consider numerically the system obtained and separate equations by dropping out the mixed terms to see their effect on the spectrum of the masses. Consequently one can rewrite (48) and (49) in the following simplest forms

$$V_{J=L-1}(r) = -g(r) \left\{ (L+1) \left[\left(1 + \frac{1}{4} \frac{m_q^2 + m_Q^2}{m_q m_Q} \right) \left[A r^{-\beta} + (1 - 2\epsilon) \kappa r^{\beta} \right] + \epsilon \kappa r^{\beta} \right] \right.$$

$$+\frac{1}{4}\frac{1}{(2L-1)}\left[(2+\beta)Ar^{-\beta}+(2-\beta)\left(1-\epsilon\right)\kappa r^{\beta}\right]$$

$$+\frac{1}{4}\left[\frac{(2+\beta)}{3}Ar^{-\beta}-\beta\left(1-\epsilon\right)\kappa r^{\beta}\right]\Bigg\}+V_{static}(r), \tag{50}$$

for states $n^3P_0,\ n^3D_1,\ n^3F_2,\ n^3H_4,\cdots$ and

$$V_{J=L+1}(r)=g(r)\left\{\left[\left(1+\frac{1}{4}\frac{m_q^2+m_Q^2}{m_qm_Q}\right)\left[Ar^{-\beta}+(1-2\epsilon)\,\kappa r^{\beta}\right]+\epsilon\kappa r^{\beta}\right]L\right.$$

$$+\frac{1}{4}\frac{1}{(2L+3)}\left[(2+\beta)Ar^{-\beta}+(2-\beta)\left(1-\epsilon\right)\kappa r^{\beta}\right]$$

$$-\frac{1}{4}\left[\frac{(2+\beta)}{3}Ar^{-\beta}-\beta\left(1-\epsilon\right)\kappa r^{\beta}\right]\Bigg\}+V_{static}(r), \tag{51}$$

for states $n^3P_2,\ n^3D_3,\cdots$. Further, for triplet S-wave, we have

$$V_{eff}(r)=-\frac{A}{r}+\kappa r+\frac{8\pi\alpha_s}{9m_qm_Q}\delta^{(3)}(\mathbf{r})+V_0, \tag{52}$$

and

$$V_{eff}(r)=-\frac{A}{r^{\beta}}+\kappa r^{\beta}+\frac{2\beta(1-\beta)\pi\alpha_s}{9m_qm_Qr^2}r^{-\beta}+V_0;\ \text{where } \beta=1/2,3/4, \tag{53}$$

which describe states such as $n^3S_{1.}$

3. State $J=0$

Equations (50) and (51) degenerate into a single equation with an effective potential

$$V_{eff}(r)=V_{static}(r)+\frac{1}{4}(F_{SS}-F_T-8F_{LS}), \tag{54}$$

and also by setting $\overline{k}=N+2-a$ therein. Further, Eq. (54) becomes

$$V_{J=0}(r)=-\left\{1+g(r)\left[2+\frac{1}{2}\frac{m_q^2+m_Q^2}{m_qm_Q}+\frac{2+\beta}{3}\right]\right\}Ar^{-\beta}$$

$$+\left\{1-\frac{g(r)}{2}\left[(5-\beta)\left(1-\epsilon\right)+\left(\frac{m_q^2+m_Q^2}{m_qm_Q}\right)(1-2\epsilon)\right]\right\}\kappa r^{\beta}+V_0. \tag{55}$$

which only describes states such as n^3P_0.

IV. PSEUDOSCALAR AND VECTOR DECAY CONSTANTS OF THE B_C MESON

The B_c can decay via electromagnetic and pionic transitions into the lightest pseudoscalar ground state B_c. The significant contribution to the B_c total decay rate comes from the annihilation of the c quark and $\bar{b}$ antiquark into the vector boson W^+ which decays into a lepton and a neutrino or a quark-antiquark pair. The weak annihilation decay rate is determined by the pseudoscalar constant of the B_c meson.

The nonrelativistic expression for the decay constants is given by [35-37]

$$f_P^{NR} = f_V^{NR} = \sqrt{\frac{12}{M_{P,V}(q\overline{Q})}}\, |\Psi_{P,V}(0)|\,, \tag{56}$$

where $\Psi_{P,V}(0)$ is the meson wave function at the origin ($r = 0$), f_P and f_V, P corresponds to the pseudoscalar B_c and V to to the vector B_c^* mesons and $M_{P,V}(q\overline{Q})$ are the masses of the B_c and B_c^* mesons.

V. RESULTS AND CONCLUSIONS

We have given further tests for the potential model in the context of Schrödinger equation using SLNET and also extended our earlier formalism for the SAD spectra [14] to the calculation of all the states by introducing the spin corrections. The obtained mass formula (14) include fine and hyperfine splitting of the energy levels. This mass formula is able to describe with some accuracy the spectra of all quark-antiquark bound states. We have obtained the self-conjugate meson spectroscopy using a group of three static potential model. This model has also been extended to comprise various cases of pure scalar confinement ($\epsilon = 1$), scalar-vector couplings ($\epsilon = 1/2$) and the vector confinement ($\epsilon = 0$) interactions. The parameters used are shown in Table I. Our results for mass spectrum of $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ systems with the static potentials, in the flavour-independent case are presented in Tables II-IV. Different sets of parameters for the Cornell potential are used to produce the binding masses of heavy quarkonium states as shown in Tables V. Further, the $c\bar{b}$ mass spectrum is given in Table VI. In the equal scalar and vector couplings, we have found that our fits are very good with level values and accurate to a few MeV. For convenience we compare explicitly the predicted and measured spin splitting energy for different L states. We find that the apparent success is achieved for the predicted $\chi_{b2} - \chi_{b1} = 24$ MeV and $\chi_{b1} - \chi_{b0} = 33$ MeV in the average for the three potentials and are very close to the experimental values 21 MeV and 32 MeV respectively. Furthermore, the predicted $\chi'_{b2} - \chi'_{b1} = 13$ MeV and $\chi'_{b1} - \chi'_{b0} = 16$ MeV in the average for the three potentials which are exactly same as the experimental value 13 MeV and close to 23 MeV, respectively. The predicted hyperfine splitting $\Delta_{\rm hfs}(1S) = M(\Upsilon(1S)) - M(\eta_b(1S)) = 80^{+6}_{-8}$ MeV, (cf. [15]), $\Delta_{\rm hfs}(2S) = M(\Upsilon'(2S)) - M(\eta'_b(2S)) = 22^{+3}_{-2}$ MeV, and $\Delta_{\rm hfs}(3S) = M(\Upsilon''(3S)) - M(\eta''_b(3S)) = 14^{+2}_{-1}$ MeV are nearly close to the theoretically calculated values 62 MeV, 40 MeV, and 15 MeV, respectively, (cf. [15,21]). Further, The hyperfine splitting for the 2S charmonium state is calculated and the predicted number is $\Delta_{\rm hfs}(2S) = M(\psi(2S)) - M(\eta_c(2S)) = 56^{+4}_{-8}$ MeV for flavour dependent case and 56^{+18}_{-8} MeV for flavour independent case, (cf. [16-19]). Badalian and Bakker in their recent work [19] calculated and predicted the number as $\Delta_{\rm hfs}$(2S,theory) $=57 \pm 8$ MeV giving $M(\eta_c(2S)$ =3630±8 MeV. Clearly, the precision of the experiments [2] requires a very substantial improvement to be sensitive to the bound-state mass differences between the various calculations. In this regard, from the global chi-square fitting values, there is a clear preference for the Song-Lin ($\epsilon = 1$), Turin ($\epsilon = 1/2$) and Cornell ($\epsilon = 1/2$) potentials, respectively. Therefore, we have found that the Song-Lin ($\epsilon = 1$) potential is the best one fitting the $c\bar{c}$ and $b\bar{b}$ quarkonia.whereas the Cornell ($\epsilon = 1/2$) potential is the worst one. Further, the Cornell ($\epsilon = 1/2$) potential seems to be the best fitting one for the $c\bar{c}$ quarkonium. In the pure scalar confinement ($\epsilon = 1$) couplings, we have found that our fits are fairly good with level values and accurate to several MeV. Further, the case of the vector confinement ($\epsilon = 0$) interaction is being ruled out in our study since it gives the worst fit to the spectra. We make the general remark as once β value increases, the ϵ value decreases. We have compared explicitly the predicted and measured spin splitting energy for different L states and found that splitting approximation can be improved significantly by increasing the quantum number L.

The deviations from experiment are more considerable. The calculation and parameters are also model dependent [14]. Moreover, we tried another set of parameters for the Cornell potential without permitting any additive constant, that is, $V_0 = 0$ (cf. last column in Table I). We have also found that the $\epsilon = 1/2$ case is the best fitting one, in this work, for the $c\bar{c}$ quarkonium and the worst one for the $b\bar{b}$ quarkonium (cf. Table V). It is clear that the coulomb-like parameter A is in accordance with the ideas of asymptotic freedom is expected for the strong gauge-coupling constant of QCD [14]. For better fit to the quarkonium spectra, the QCD coupling constant $\alpha_s(\mu^2)$ should be dependent on the quark-flavour. The consideration of the variation of the effective Coulomb interaction constant becomes especially essential for the Υ particle, for which $\alpha_s(\Upsilon) \neq \alpha_s(\psi)$[1].

The calculated values of the pseudoscalar and vector decay constants of the B_c meson using the nonrelativistic expression (56) are displayed in Table VII. They are compared with the ones calculated using the relativistic, nonrelativistic [1,35,38,39]. The radial wave function at the origin has also been calculated in Table VIII and compared to the other works available in literature [1,4]. These approximations have been calculated without permitting any additive constant, that is, $V_0 = 0$ and they also appear to be fairly good.

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[1]For the best fit to the quarkonium spectra, the QCD coupling constant $\alpha_s(\mu^2)$ must be dependent on the quark flavour [11,19,35]. Motyka and Zalewiski [11] found $\frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)} \simeq 11/18$ whereas Kiselev *et al.* [38] have found $\Delta M_\Upsilon(1S) = \frac{\alpha_s(\Upsilon)}{\alpha_s(\psi)} \Delta M_\psi(1S)$ with $\alpha_s(\Upsilon)/\alpha_s(\psi) \simeq 3/4$.

APPENDIX A: SLNET PARAMETERS FOR THE SCHRÖDINGER EQUATION:

Here, we list the analytic expressions of $\alpha^{(1)}$, $\alpha^{(2)}$, ε_i and δ_j for the Schrödinger equation:

$$\alpha^{(1)} = \frac{(1-a)(3-a)}{16\mu} + \Big[(1+2n_r)\ \bar{\varepsilon}_2\ + 3\ (1+2n_r+2n_r^2)\ \bar{\varepsilon}_4\Big] - \omega^{-1}\Big[\ \bar{\varepsilon}_1^2 + 6\ (1+2n_r)\ \bar{\varepsilon}_1\ \bar{\varepsilon}_3 +\ (11+30n_r+30n_r^2)\ \bar{\varepsilon}_3^2\Big], \tag{A1}$$

$$\begin{aligned}\alpha^{(2)} = &\Big[(1+2n_r)\ \bar{\delta}_2 + 3\ (1+2n_r+2n_r^2)\ \bar{\delta}_4\ + 5\ (3+8n_r+6n_r^2+4n_r^3)\ \bar{\delta}_6 \\ &- \omega^{-1}\ \ (1+2n_r)\ \bar{\varepsilon}_2^2\ + 12\ (1+2n_r+2n_r^2)\ \bar{\varepsilon}_2\ \bar{\varepsilon}_4\ +\ 2\ \bar{\varepsilon}_1\ \bar{\delta}_1 \\ &+ 2\ (21+59n_r+51n_r^2+34n_r^3)\ \bar{\varepsilon}_4^2\ + 6\ (1+2n_r)\ \bar{\varepsilon}_1\ \bar{\delta}_3 \\ &+ 30\ (1+2n_r+2n_r^2)\ \bar{\varepsilon}_1\ \bar{\delta}_5\ + 2\ (11+30n_r+30n_r^2)\ \bar{\varepsilon}_3\ \bar{\delta}_3 \\ &\quad +\ 10\ (13\ +\ 40n_r+42n_r^2\ + 28n_r^3)\ \bar{\varepsilon}_3\ \bar{\delta}_5 + 6\ (1+2n_r)\ \bar{\varepsilon}_3\ \bar{\delta}_1\Big] \\ &+ \omega^{-2}\Big[\ 4\ \bar{\varepsilon}_1^2\ \bar{\varepsilon}_2\ + 36\ (1+2n_r)\ \bar{\varepsilon}_1\ \bar{\varepsilon}_2\ \bar{\varepsilon}_3 + 8\ (11+30n_r+30n_r^2)\ \bar{\varepsilon}_2\ \bar{\varepsilon}_3^2 \\ &+ 24\ (1+2n_r)\ \bar{\varepsilon}_1^2\ \bar{\varepsilon}_4 + 8\ (31+78n_r+78n_r^2)\ \bar{\varepsilon}_1\ \bar{\varepsilon}_3\ \bar{\varepsilon}_4 \\ &\quad + 12\ (57+189n_r+225n_r^2+150n_r^3)\ \bar{\varepsilon}_3^2\ \bar{\varepsilon}_4\Big] \\ &- \omega^{-3}\Big[\ 8\ \bar{\varepsilon}_1^3\ \bar{\varepsilon}_3 + 108\ (1+2n_r)\ \bar{\varepsilon}_1^2\ \bar{\varepsilon}_3^2\ + 48\ (11+30n_r+30n_r^2)\ \bar{\varepsilon}_1\ \bar{\varepsilon}_3^3 \\ &+\ 30\ (31+109n_r+141n_r^2+94n_r^3)\ \bar{\varepsilon}_3^4\Big],\end{aligned} \tag{A2}$$

where

$$\bar{\varepsilon}_i\ =\ \frac{\varepsilon_i}{(4\mu\omega)^{i/2}}, \qquad i = 1,2,3,4. \tag{A3}$$

and

$$\bar{\delta}_j = \frac{\delta_j}{(4\mu\omega)^{j/2}}, \qquad j = 1,2,3,4,5,6. \tag{A4}$$

$$\varepsilon_1 = \frac{(2-a)}{4\mu}\ , \quad \varepsilon_2 = -\frac{3}{8\mu}\ (2-a), \tag{A5}$$

$$\varepsilon_3 = -\frac{1}{4\mu}\ + \frac{r_0^5 V'''(r_0)}{6Q}; \quad \varepsilon_4 = \frac{5}{16\mu} + \frac{r_0^6\ V''''(r_0)}{24Q} \tag{A6}$$

$$\delta_1 = -\frac{(1-a)(3-a)}{8\mu}; \quad \delta_2 = \frac{3\ (1-a)(3-a)}{16\mu}, \tag{A7}$$

$$\delta_3 = \frac{(2-a)}{2\mu}\ ; \quad \delta_4 = -\frac{5\ (2-a)}{8\mu}, \tag{A8}$$

$$\delta_5 = -\frac{3}{8\mu} + \frac{r_0^7\ V'''''(r_0)}{120Q}; \quad \delta_6 = \frac{7}{16\mu} + \frac{r_0^8\ V''''''(r_0)}{720Q}. \tag{A9}$$

APPENDIX B: THE SPIN-CORRECTION TERMS:

For parastates ($S = 0$) case we have:

$$J = L \tag{B1}$$

For triplet ($S = 1$) case we have the following:

$$J = \begin{cases} L-1, \ \mathbf{S}\cdot\mathbf{L} = -(L+1) \\ L, \ \mathbf{S}\cdot\mathbf{L} = -1 \\ L+1, \ \mathbf{S}\cdot\mathbf{L} = L \end{cases} \tag{B2}$$

The independent operators $\mathbf{S}_1 \cdot \mathbf{S}_2$,:$(\mathbf{S}_1 \pm \mathbf{S}_2) \cdot \mathbf{L}$ and T :

$$\langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle = \begin{cases} -3/4, \ \text{for spin singlets } S = 0, \\ +1/4, \ \text{for spin triplets } S = 1. \end{cases} \tag{B3}$$

$$\langle \mathbf{S} \cdot \mathbf{L} \rangle = \begin{cases} 0, \ \text{for spin singlets } S = 0, \\ \frac{1}{2}\left[J(J+1) - L(L+1) - 2\right], \ \text{for spin triplets } S = 1. \end{cases} \tag{B4}$$

$$(\mathbf{S}_1 \cdot \widehat{\mathbf{r}} \mathbf{S}_2 \cdot \widehat{\mathbf{r}}) u_J(r) Y_{J,0,J}(\widehat{\mathbf{r}}) = -\frac{1}{4} u_J(r) Y_{J,0,J}(\widehat{\mathbf{r}}), \tag{B5}$$

$$(\mathbf{S}_1 \cdot \mathbf{S}_2) Y_{J,S,L}^m(\widehat{\mathbf{r}}) = \frac{1}{2}\left[S(S+1) - S_1(S_1+1) - S_2(S_2+1)\right] Y_{J,S,L}^m(\widehat{\mathbf{r}}), \tag{B6}$$

$$(\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L} Y_{J,S,L}^m(\widehat{\mathbf{r}}) = \frac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right] Y_{J,S,L}^m(\widehat{\mathbf{r}}), \tag{B7}$$

$$(\mathbf{S}_1-\mathbf{S}_2)\cdot\mathbf{L}Y^m_{J,S,L}(\widehat{\mathbf{r}})=\sqrt{\frac{1}{10}\left[2L+3)(2L-1)\right]}\delta_{J,L}\left(\delta_{S,0}Y^m_{J,1,L}(\widehat{\mathbf{r}})+\delta_{S,1}Y^m_{J,0,L}(\widehat{\mathbf{r}})\right), \tag{B8}$$

$$TY^m_{J,1,L}(\widehat{\mathbf{r}})=\frac{1}{4}\delta_{J,L}Y^m_{J,1,L}(\widehat{\mathbf{r}})-\frac{1}{4(2L-1)}\delta_{J,L-1}Y^m_{J,1,L}(\widehat{\mathbf{r}})+\frac{1}{4(2L+3)}\delta_{J,L+1}Y^m_{J,1,L}(\widehat{\mathbf{r}})$$

$$-\frac{\sqrt{(L+1)(L+2)}}{2(2L+3)}\delta_{J,L+1}Y^m_{J,1,L+2}(\widehat{\mathbf{r}})-\frac{\sqrt{L(L-1)}}{2(2L-1)}\delta_{J,L-1}Y^m_{J,1,L-2}(\widehat{\mathbf{r}}). \tag{B9}$$

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TABLES

TABLE I. Fitted parameters of the class of static central potentials.

Parameters	Cornell[a]	Song-Lin[a]	Turin[a]	Cornell[b]
m_c	1.840 GeV	1.820 GeV	1.790 GeV	1.3205 GeV
m_b	5.232 GeV	5.199 GeV	5.171 GeV	4.7485 GeV
A	0.520	0.923 $GeV^{1/2}$	0.620 $GeV^{1/4}$	0.472
κ	0.1756 GeV^2	0.511 $GeV^{3/2}$	0.304 $GeV^{7/4}$	0.191 GeV^2
V_0	-0.8578 GeV	-0.798 GeV	-0.823 GeV	0 GeV

[a]These parameter fits [14] are used to produce masses in Tables II–IV.

[b]These parameter fits [4] are used to produce masses in Tables V-VI.

TABLE II. Heavy−meson mass spectra (in MeV) for the Cornell potential.

State	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	[1]	[4]
	$c\bar{c}$			$b\bar{b}$			$c\bar{b}$				
1^1S_0	3068.0	3047.0	3025.6	9424.5	9419.4	9414.2	6314.6	6305.3	6295.9	6264	6286
2^1S_0	3658.5	3647.3	3635.9	10023.7	10021.4	10019.1	6888.0	6883.3	6878.5	6856	6882
3^1S_0	4075.7	4067.6	4059.5	10370.2	10368.6	10367.0	7271.2	7267.8	7264.5	7244	
4^1S_0	4426.2	4419.6	4413.1	10642.4	10641.2	10640.0	7587.2	7584.6	7581.9		
1^1P_1	3487.8	3477.4	3467.0	9918.1	9916.0	9913.9	6742.9	6738.5	6734.2	6730	6737
2^1P_1	3921.8	3914.0	3906.2	10266.9	10265.4	10263.9	7137.8	7134.7	7131.5	7135	
1^1D_2	3766.0	3758.6	3751.1	10162.7	10161.3	10159.9	7003.1	7000.1	6997.0	7009	7028
2^1D_2	4142.4	4136.2	4130.0	10448.0	10446.9	10445.7	7340.2	7337.7	7335.2		
1^3P_1	3480.5	3464.4	3448.1	9910.1	9906.9	9903.7	6736.1	6726.8	6717.8	6736	6760
2^3P_1	3920.6	3908.8	3896.9	10264.1	10261.9	10259.6	7136.4	7129.6	7122.8	7142	
1^3D_2	3768.3	3757.1	3745.9	10161.8	10159.7	10157.5	7004.2	6997.8	6991.4	7012	7028
2^3D_2	4144.7	4135.4	4126.1	10447.5	10445.8	10444.1	7341.4	7336.2	7330.9		
1^3S_1	3068.0	3074.9	3081.8	9424.5	9426.3	9428.0	6314.6	6317.7	6320.7	6337	6341
2^3S_1	3658.5	3662.2	3665.9	10023.7	10024.5	10025.3	6888.0	6889.5	6891.1	6899	6914
3^3S_1	4075.7	4078.4	4081.1	10370.2	10370.7	10371.2	7271.2	7272.3	7273.4	7280	
4^3S_1	4426.2	4428.3	4430.8	10642.4	10642.9	10643.3	7587.2	7588.1	7589.0		
1^3P_2	3496.0	3522.7	3544.7	9928.7	9933.3	9937.9	6757.4	6765.8	6777.4	6747	6772
2^3P_2	3926.3	3943.7	3962.8	10271.2	10274.6	10277.9	7141.2	7150.3	7159.3	7153	
1^3D_3	3765.4	3796.4	3827.0	10166.3	10172.2	10178.1	7003.1	7019.4	7035.7	7005	7032
2^3D_3	4141.7	4168.8	4192.3	10450.3	10455.2	10460.0	7339.4	7353.0	7366.5		
1^3P_0	3419.1	3365.0	3307.1	9874.7	9864.9	9855.0	6700.1	6673.6	6646.4	6700	6701
2^3P_0	3899.8	3866.3	3832.1	10252.2	10246.0	10239.7	7124.0	7106.5	7088.8	7108	
1^3D_1	3761.7	3716.4	3669.9	10154.9	10146.5	10138.1	7000.3	6976.4	6952.1	7012	7019
2^3D_1	4141.3	4104.3	4066.7	10443.1	10436.3	10429.6	7339.6	7320.1	7300.4		

TABLE III. Heavy−meson mass spectra (in MeV) for the Song-Lin potential.

State	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	[35]	[38]
	$c\bar{c}$			$b\bar{b}$			$c\bar{b}$				
1^1S_0	3020.8	2991.4	2960.1	9417.7	9410.3	9402.7	6279.1	6266.4	6253.3	6270	6253
2^1S_0	3634.3	3624.8	3615.0	10010.5	10008.0	10005.5	6870.4	6866.1	6861.6	6835	6867
3^1S_0	3983.6	3978.2	3972.7	10334.1	10332.7	10331.3	7206.7	7204.2	7201.7	7193	
4^1S_0	4242.6	4238.9	4235.3	10567.8	10566.8	10565.9	7454.3	7452.6	7451.0		
1^1P_1	3488.5	3480.6	3472.5	9879.3	9877.2	9875.1	6730.2	6726.6	6722.9	6734	6717
2^1P_1	3873.9	3869.1	3864.2	10239.2	10237.9	10236.7	7103.0	7100.7	7098.5	7126	7113
1^1D_2	3761.6	3757.2	3752.9	10141.7	10140.6	10139.5	6996.6	6994.6	6992.6	7077	7001
2^1D_2	4061.1	4058.0	4054.8	10413.1	10412.3	10411.5	7283.7	7282.3	7280.8		
1^3P_1	3483.3	3466.7	3449.6	9875.1	9870.8	9866.5	6725.7	6715.1	6704.2	6749	6729
2^3P_1	3872.4	3862.5	3852.5	10237.4	10234.9	10232.4	7101.6	7095.2	7088.8	7145	7124
1^3D_2	3762.2	3753.4	3744.6	10141.0	10138.8	10136.5	6996.8	6991.2	6985.6	7079	7016
2^3D_2	4061.8	4055.5	4049.1	10412.7	10411.1	10409.5	7284.0	7280.0	7276.0		
1^3S_1	3081.8	3089.3	3096.7	9447.8	9450.0	9452.2	6313.8	6317.5	6321.1	6332	6317
2^3S_1	3646.0	3649.0	3652.0	10015.8	10016.7	10017.5	6877.0	6878.5	6879.9	6881	6902
3^3S_1	3988.1	3989.9	3991.6	10336.1	10336.6	10337.0	7209.2	7210.0	7210.8	7235	
4^3S_1	4245.0	4246.2	4247.4	10568.8	10569.1	10569.4	7455.6	7456.2	7456.7		
1^3P_2	3509.1	3533.0	3553.3	9892.1	9898.0	9898.5	6746.0	6758.0	6767.7	6762	6743
2^3P_2	3885.5	3900.1	3912.1	10245.3	10248.9	10252.4	7110.0	7118.0	7125.9	7156	7134
1^3D_3	3771.0	3794.3	3816.3	10147.9	10153.9	10159.9	7002.3	7016.1	7027.0	7081	7007
2^3D_3	4067.6	4084.8	4100.6	10417.2	10421.4	10425.7	7287.3	7297.3	7308.4		
1^3P_0	3422.7	3360.6	3288.7	9849.6	9836.2	9822.4	6693.1	6661.1	6627.0	6699	6683
2^3P_0	3847.6	3816.4	3783.1	10226.1	10218.6	10211.1	7087.6	7070.1	7052.0	7091	7088
1^3D_1	3746.1	3708.2	3667.9	10132.5	10123.2	10113.8	6988.0	6966.2	6943.5	7072	7008
2^3D_1	4051.6	4025.0	3997.2	10407.3	10400.8	10394.2	7278.6	7263.2	7247.3		

TABLE IV. Heavy−meson mass spectra (in MeV) for the Turin potential.

State	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	$\epsilon=1$	$\epsilon=1/2$	$\epsilon=0$	[1]	[4]
	$c\bar{c}$			$b\bar{b}$			$c\bar{b}$				
1^1S_0	3041.9	3014.6	2986.1	9418.0	9411.3	9404.6	6290.5	6278.7	6266.7	6264	6286
2^1S_0	3653.5	3642.0	3630.3	10005.1	10002.5	10000.0	6877.3	6872.3	6867.3	6856	6882
3^1S_0	4046.8	4039.3	4031.7	10343.3	10341.7	10340.1	7244.9	7241.8	7238.6	7244	
4^1S_0	4360.3	4354.7	4349.1	10601.6	10600.4	10599.3	7534.5	7532.2	7529.8		
1^1P_1	3489.4	3479.2	3468.8	9881.0	9878.7	9876.4	6729.2	6724.8	6720.4	6730	6737
2^1P_1	3910.7	3903.7	3896.6	10241.2	10239.7	10238.2	7122.6	7119.6	7116.7	7135	
1^1D_2	3771.9	3765.4	3758.9	10136.7	10135.3	10134.0	6997.7	6995.0	6992.2	7009	7028
2^1D_2	4120.7	4115.6	4110.5	10421.9	10420.8	10419.8	7319.2	7317.1	7315.0		
1^3P_1	3484.5	3466.2	3447.6	9875.5	9871.5	9867.5	6724.6	6713.6	6702.4	6736	6760
2^3P_1	3910.1	3898.0	3885.6	10239.1	10236.6	10234.0	7121.7	7114.4	7107.1	7142	
1^3D_2	3774.1	3762.9	3751.6	10136.1	10133.7	10131.3	6999.0	6992.3	6985.5	7012	7028
2^3D_2	4122.7	4114.0	4105.2	10421.5	10419.7	10417.9	7320.4	7315.3	7310.0		
1^3S_1	3075.4	3083.3	3091.1	9441.3	9443.3	9445.4	6311.8	6315.4	6319.0	6337	6341
2^3S_1	3659.0	3662.8	3666.5	10008.0	10008.8	10009.7	6880.5	6882.1	6883.8	6899	6914
3^3S_1	4048.9	4051.3	4053.8	10344.3	10344.8	10345.3	7246.1	7247.2	7248.2	7280	
4^3S_1	4361.4	4363.3	4365.1	10602.0	10602.4	10602.8	7535.1	7535.9	7536.7		
1^3P_2	3502.7	3529.7	3553.5	9891.9	9897.5	9903.1	6740.0	6753.8	6767.3	6747	6772
2^3P_2	3917.7	3936.8	3952.7	10246.2	10249.9	10253.7	7127.1	7136.6	7146.0	7153	
1^3D_3	3773.9	3804.8	3834.2	10141.1	10147.6	10154.1	6999.2	7016.0	7032.0	7005	7032
2^3D_3	4122.8	4145.7	4168.4	10424.7	10429.7	10434.7	7319.7	7332.8	7345.8		
1^3P_0	3426.8	3362.9	3291.7	9847.9	9835.7	9823.3	6692.8	6661.0	6627.6	6700	6701
2^3P_0	3888.3	3852.1	3814.3	10227.9	10220.5	10213.1	7109.3	7089.9	7070.1	7108	
1^3D_1	3764.8	3718.1	3669.3	10129.0	10119.4	10109.6	6994.1	6968.6	6942.4	7012	7019
2^3D_1	4117.3	4081.6	4044.9	10416.9	10409.7	10402.4	7317.7	7298.3	7278.4		

TABLE V. $c\bar{c}$ and $b\bar{b}$ mass spectra (in MeV) using the Cornell potential.

State	Meson[a]	[35]	$\epsilon = 1$	$\epsilon = 1/2$	$\epsilon = 0$	Meson	[35]	$\epsilon = 1$	$\epsilon = 1/2$	$\epsilon = 0$
1^1S_0	η_c	2979	3068.5	3031.4	2993.3	η_b	9400	9447.4	9441.3	9435.3
2^1S_0	η_c'	3588	3704.7	3683.8	3662.6	η_b'	9993	10021.5	10018.6	10015.7
3^1S_0	η_c''	3991	4177.2	4161.8	4146.3	η_b''	10328	10378.4	10376.4	10374.4
4^1S_0			4580.6	4568.1	4555.6			10665.8	10664.2	10662.6
1^1P_1	h_c	3526	3497.6	3478.1	3458.4	h_b	9901	9899.7	9897.1	9894.5
2^1P_1	h_c'	3945	3993.2	3978.4	3963.5	h_b'	10261	10263.3	10261.4	10259.5
1^1D_2		3811	3806.7	3792.6	3778.4		10158	10147.2	10145.4	10143.6
2^1D_2			4242.9	4231.0	4219.2			10451.0	10449.5	10448.1
1^3P_1	χ_{c1}	3510	3496.0	3465.7	3434.9	χ_{b1}	9892	9893.2	9889.2	9885.2
2^3P_1	χ_{c1}'	3929	3996.7	3974.2	3951.4	χ_{b1}'	10255	10261.0	10258.2	10255.3
1^3D_2		3813	3814.3	3793.0	3771.5		10158	10146.7	10144.0	10141.2
2^3D_2			4249.6	4231.9	4214.0			10450.8	10448.6	10446.4
1^3S_1	J/ψ	3096	3068.5	3080.6	3092.7	Υ	9460	9447.4	9449.4	9451.4
2^3S_1	ψ'	3686	3704.7	3711.6	3718.5	Υ'	10023	10021.5	10022.4	10023.4
3^3S_1	ψ''	4088	4177.2	4182.3	4187.4	Υ''	10355	10378.4	10379.0	10379.7
4^3S_1	ψ'''		4580.6	4584.8	4588.9	Υ'''		10665.8	10666.3	10666.8
1^3P_2	χ_{c2}	3556	3505.2	3547.6	3589.1	χ_{b2}	9913	9908.9	9914.7	9920.5
2^3P_2	χ_{c2}'	3972	3994.0	4027.1	4059.8	χ_{b2}'	10268	10267.1	10271.4	10275.7
1^3D_3		3815	3796.9	3855.8	3913.4		10162	10150.0	10157.6	10165.2
2^3D_3			4233.3	4283.1	4331.9			10452.8	10459.0	10465.2
1^3P_0	χ_{c0}	3424	3430.9	3327.5	3209.5	χ_{b0}	9863	9862.9	9851.0	9839
2^3P_0	χ_{c0}'	3854	3975.7	3911.7	3845.3	χ_{b0}'	10234	10249.9	10242.0	10234.1
1^3D_1		3798	3815.4	3728.7	3638.2		10153	10140.7	10130.0	10119.2
2^3D_1			4252.9	4181.8	4108.7			10447.0	10438.3	10429.5

[a]Same parameter fits of Ref. [4] in Table I with $V_0 = 0$.

TABLE VI. B_c meson mass spectrum (in MeV) for the Cornell potential.

State[a]	$\epsilon = 1$	$\epsilon = 1/2$	$\epsilon = 0$	[35]	[38]	[1]	[4]	[40]
1^1S_0	6338.7	6325.7	6312.6	6270	6253	6264	6286	≥ 6219.6
2^1S_0	6930.5	6923.5	6916.5	6835	6867	6856	6882	
3^1S_0	7352.2	7347.1	7342.0	7193		7244		
4^1S_0	7707.2	7703.2	7699.1					
1^1P_1	6756.0	6749.5	6743.0	6734	6717	6730	6737	≥ 6701.2
2^1P_1	7195.2	7190.3	7185.5	7126	7113	7135		
1^1D_2	7036.3	7031.7	7027.1	7077	7001	7009	7028	
2^1D_2	7418.1	7414.3	7410.4					
1^3P_1	6753.6	6737.4	6720.9	6749	6729	6736	6760	≥ 6701.2
2^3P_1	7196.9	7184.9	7172.8	7145	7124	7142		
1^3D_2	7040.9	7029.5	7018.1	7079	7016	7012	7028	
2^3D_2	7422.2	7412.8	7403.4					
1^3S_1	6338.7	6342.9	6347.2	6332	6317	6337	6341	≥ 6278.6
2^3S_1	6930.5	6932.8	6935.1	6881	6902	6899	6914	
3^3S_1	7352.2	7353.8	7355.5	7235		7280		
4^3S_1	7707.2	7708.6	7710.0					
1^3P_2	6761.2	6780.6	6799.8	6762	6743	6747	6772	≥ 6734.7
2^3P_2	7195.5	7211.0	7226.3	7156	7134	7153		
1^3D_3	7029.7	7057.3	7085.5	7081	7007	7005	7032	
2^3D_3	7411.8	7435.6	7458.6					
1^3P_0	6724.1	6680.2	6634.5	6699	6683	6700	6701	≥ 6638.6
2^3P_0	7187.5	7157.5	7127.0	7091	7088	7108		
1^3D_1	7043.6	7002.3	6960.1	7072	7008	7012	7019	
2^3D_1	7425.9	7391.9	7357.4					

[a]Same parameter fits of Ref. [4] in Table I with $V_0 = 0$.

TABLE VII. Pseudoscalar and vector decay constants ($f_P = f_{B_c},\ f_V = f_{B_c^*}$) of the B_c meson (in MeV) using the Cornell potential.

Constants[a]	SLNET[b]	SLNET[c]	SLNET[d]	Rel[35]	[35]	[1]	[38]	[4]	[39]
f_{B_c}	511.4	503.2	495.2	433	562	479-687	460±60	517	420±13
$f_{B_c^*}$	490.0	492.6	495.2	503	562	479-687	460±60	517	-

[a]For parameter fits we cite Ref. [4].

[b]Here ($\epsilon = 0$).

[c]Here ($\epsilon = 1/2$).

[d]Here ($\epsilon = 1$).

TABLE VIII. The radial wave function at the origin (in GeV^3) calculated in our model and by the other authors using the Cornell potential.

Level[a]	SLNET[b]	SLNET[c]	SLNET[d]	Martin	[1]	[4]	[1][e]
$\lvert R_{B_c}(0)\rvert^2$	1.729	1.677	1.628	1.716	1.638	1.81	1.508-3.102
$\lvert R_{B_c^*}(0)\rvert^2$	1.596	1.612	1.628	-	-	-	-

[a]For parameter fits we cite Ref. [4].

[b]Here ($\epsilon = 0$).

[c]Here ($\epsilon = 1/2$).

[d]Here ($\epsilon = 1$).

[e]For the 1S level.