# The $\phi a_0 \gamma$- and $\phi \sigma \gamma$-vertices in light cone QCD

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## Abstract

We study $\phi a_0 \gamma$- and $\phi \sigma \gamma$-vertices in the framework of the light cone QCD sum rules and we estimate the coupling constants $g_{\phi a_0 \gamma}$ and $g_{\phi \sigma \gamma}$ utilizing $\omega\phi$-mixing. We compare our results with the previous estimations of these coupling constants in the literature obtained from phenomenological considerations.

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The studies of $\phi(1020)$ meson and in particular its radiative decays have been important sources of information in hadron physics. The Novosibirsk CMD [1] and SND [2] collaborations have reported measurements of radiative $\phi \to \pi^0\pi^0(\eta)\gamma$ and $\phi \to \pi^+\pi^-\gamma$ decays, and the high luminosity Frascati $\phi$-factory $DA\Phi NE$ will soon be performing precise measurements of radiative $\phi$ decays. These information will help us to increase our understansing of the complicated dynamics of meson physics in the 1 GeV energy region. In this energy region, low-mass scalar mesons may also play an important role. The incorporation of the role of the scalar resonances in processes involving vector mesons provides an opportunity to increase our insight of the dynamics of the meson physics. Among the processes involving the vector and scalar mesons, the $\phi a_0\gamma$- and $\phi\sigma\gamma$-vertices are interesting and important for several reasons. The $\phi a_0\gamma$-vertex plays a role in the study of the radiative $\phi \to \pi^0\eta\gamma$ decay [3], and the knowledge of the $\phi\sigma\gamma$-vertex is needed in the analysis of the decay mechanism of the $\phi \to \pi^0\pi^0\gamma$ decay [4]. Furthermore, the coupling constant $g_{\phi\sigma\gamma}$ is needed in the study of the structure of the $\phi$ meson photoproduction amplitude on nucleons in the near threshold region based on the one-meson exchange and Pomeron-exchange mechanisms [5]. In the present work, we employ the light cone QCD sum rules method to study the $\phi a_0\gamma$- and $\phi\sigma\gamma$-vertices, and utilizing $\omega\phi$-mixing we estimate the $g_{\phi a_0\gamma}$ and $g_{\phi\sigma\gamma}$ coupling constants.

In order to analyze the $\phi a_0\gamma$- and $\phi\sigma\gamma$-vertices using light cone QCD sum rules, we begin with the observation that the $\phi \to \pi^0\gamma$ decay width vanishes if the $\phi$ meson is a pure $s\bar{s}$ state. The measured width $\Gamma(\phi \to \pi^0\gamma) = (5.6 \pm 0.5)$ KeV [6] is significantly different from zero which is explained as primarily being due to $\omega - \phi$ mixing. Bramon et al. [7] have recently studied radiative $VP\gamma$ transitions between vector (V) mesons and pseudoscalar (P) mesons, and using the available experimental information they have determined the mixing angle for $\omega - \phi$ as well as other relevant parameters of $\omega - \phi$ and $\eta - \eta'$ systems. In this work, we follow their treatment and we write the physical $\omega$ and $\phi$ meson states as

$$
\begin{aligned}
\mid \omega > \; &= \cos\theta_v \mid \omega_0 > - \sin\theta_v \mid \phi_0 > \\
\mid \phi > \; &= \sin\theta_v \mid \omega_0 > + \cos\theta_v \mid \phi_0 > \quad ,
\end{aligned} \tag{1}
$$

where $|\ \omega_0>=\frac{1}{\sqrt{2}}\ |\ u\overline{u}+d\overline{d}>$ and $|\ \phi_0>=|\ s\overline{s}>$ are the non-strange and the strange basis states. The mixing angle has been determined from the available experimental data by Bramon et al. as $\theta=(3.4\pm 0.2)^o$ [7]. We, therefore, choose the interpolating currents for $\omega$ and $\phi$ mesons defined in the quark flavour basis as

$$j^{\omega}_{\mu}=\cos\theta_v j^{\omega_0}_{\mu}-\sin\theta_v j^{\phi_0}_{\mu}$$
$$j^{\phi}_{\mu}=\sin\theta_v j^{\omega_0}_{\mu}+\cos\theta_v j^{\phi_0}_{\mu}\quad , \tag{2}$$

where $j^{\omega_0}_{\mu}=\frac{1}{6}(\overline{u}\gamma_\mu u+\overline{d}\gamma_\mu d)$ and $j^{\phi_0}_{\mu}=-\frac{1}{3}\overline{s}\gamma_\mu s$ [8].

In order to study the $\phi s\gamma$-vertex and to estimate the coupling constant $g_{\phi s\gamma}$ with s denoting $a_0$ or $\sigma$ meson, we consider the two point correlation function with photon

$$T_\mu(p,q)=i\int d^4x e^{ip\cdot x}<\gamma(q)|T\{j^{\phi}_{\mu}(0)j_s(x)\}|0> \tag{3}$$

where $j^{\phi}_{\mu}=\sin\theta_v j^{\omega_0}_{\mu}+\cos\theta_v j^{\phi_0}_{\mu}$ and $j_s=\frac{1}{2}(\overline{u}u+(-1)^I\overline{d}d)$ are the interpolating currents for $\phi$ and for isoscalar I=0 $\sigma$ meson, and for isovector I=1 $a_0$ meson. The overlap amplitudes of these interpolating currents with the meson states are defined as

$$<0|j^{\phi}_{\mu}|\phi> \ =\lambda_\phi u_\mu$$
$$<0|j_s|s> \ =\lambda_s \tag{4}$$

where $u_\mu$ is the polarization vector of $\phi$ meson and s denotes $\sigma$ or $a_0$ meson. The $e^+e^-$ leptonic decay width of $\phi$ meson neglecting electron mass can be written as $\Gamma(\phi\rightarrow e^+e^-)=\frac{4\pi\alpha^2}{3m^3_\phi}\lambda^2_\phi$. We use the experimental value for the branching ratio $B(\phi\rightarrow e^+e^-)=(2.91\pm 0.07)\times 10^{-4}$ of $\phi$ meson [6], and this way we determine the overlap amplitude $\lambda_\phi$ of $\phi$ meson as $\lambda_\phi=(0.079\pm 0.016)$ GeV$^2$. We have employed the QCD sum rules method in previous works and we determined the overlap amplitudes $\lambda_\sigma$ and $\lambda_{a_0}$ as $\lambda_\sigma=(0.12\pm 0.03)\ \ GeV^2$ [9] and $\lambda_{a_0}=(0.21\pm 0.005)\ \ GeV^2$ [10], since they are not available experimentally.

The theoretical part of the sum rule for the coupling constants $g_{\phi s\gamma}$ is obtained in terms of QCD degrees of freedom by calculating the two point correlator in the deep Euclidean region where $p^2$ and $(p+q)^2$ are large and negative. In this calculation we use the full light quark propagator with both perturbative and nonperturbative contributions, and it is given as [11]

$$\begin{aligned} iS(x,0) &= <0|T\{\overline{q}(x)q(0)\}|0> \\ &= i\frac{\not x}{2\pi^2 x^4} - \frac{<\overline{q}q>}{12} - \frac{x^2}{192} m_0^2 <\overline{q}q> \\ &\quad - ig_s \frac{1}{16\pi^2}\int_0^1 du \left\{ \frac{\not x}{x^2}\sigma_{\mu\nu}G^{\mu\nu}(ux) - 4iu\frac{x_\mu}{x^2}G^{\mu\nu}(ux)\gamma_\nu \right\} + ... \end{aligned} \tag{5}$$

where terms proportional to light quark mass $m_u$ or $m_d$ are neglected. We note that it is the $j_\mu^{\omega_0}$ part of the $\phi$ meson interpolating current, that is $j_\mu^\phi = \sin\theta_v j_\mu^{\omega_0} = (1/6)\sin\theta_v(\overline{u}\gamma_\mu u + \overline{d}\gamma_\mu d)$, that makes a contribution in the calculation of the theoretical part of the sum rule. After a straightforward computation we obtain

$$T_\mu(p,q) = 4i\int d^4x e^{ipx} A(x_\sigma g_{\mu\tau} - x_\tau g_{\mu\sigma}) < \gamma(q) \mid \overline{q}(x)\sigma_{\tau\sigma}q(0) \mid 0 > \tag{6}$$

where $A = \frac{i}{2\pi^2 x^4}$. In order to evaluate the two point correlation function further, we need the matrix elements $<\gamma(q)|\overline{q}\sigma_{\alpha\beta}q|0>$. These matrix elements are defined in terms of the photon wave functions [12–14]

$$\begin{aligned} <\gamma(q)|\overline{q}\sigma_{\alpha\beta}q|0> &= ie_q <\overline{q}q> \int_0^1 du e^{iuqx}\{(\epsilon_\alpha q_\beta - \epsilon_\beta q_\alpha)\left[\chi\phi(u) + x^2[g_1(u) - g_2(u)]\right] \\ &\quad + \left[q\cdot x(\epsilon_\alpha x_\beta - \epsilon_\beta x_\alpha) + \epsilon\cdot x(x_\alpha q_\beta - x_\beta q_\alpha)\right] g_2(u)\} \quad , \end{aligned} \tag{7}$$

where the parameter $\chi$ is the magnetic susceptibility of the quark condensate and $e_q$ is the quark charge, $\phi(u)$ stands for the leading twist-2 photon wave function, while $g_1(u)$ and $g_2(u)$ are the two-particle photon wave functions of twist-4. In the further analysis the path ordered gauge factor is omitted since we work in the fixed point gauge [15].

In order to construct the phenomenological part of the two point function in Eq. 3, we note that the two point function $T_\mu(p,q)$ satisfies a dispersion relation, and we saturate this dispersion relation by inserting a complete set of one hadron states into the correlation function. This way we construct the phenomenological part of the two point correlation function as

$$T_\mu(p,q)=\frac{< s\gamma|\phi >< \phi|j_\mu^\phi|0 >< 0|j_s|s >}{(p^2-m_\phi^2)(p'^2-m_s^2)}+... \tag{8}$$

where we denote the contributions from the higher states and the continuum starting from some threshold $s_0$ by dots. The coupling constant $g_{\phi s\gamma}$ is defined through the effective Lagrangian

$$\mathcal{L}=\frac{e}{m_\phi}g_{\phi s\gamma}\partial^\alpha\phi^\beta(\partial_\alpha A_\beta-\partial_\beta A_\alpha)s \tag{9}$$

describing the $\phi s\gamma$-vertex [16]. The $< s\gamma|\phi >$ matrix can therefore be written as

$$< s(p')\gamma(q)\mid\phi(p) >= i\frac{e}{m_\phi}g_{\phi s\gamma}K(q^2)(p\cdot q\ u_\mu-u\cdot q\ p_\mu) \tag{10}$$

where $q=p-p'$ and $K(q^2)$ is a form factor with K(0)=1. In order to take the contributions coming from the higher states and the continuum into account, we invoke the quark-hadron duality prescription and replace the hadron spectral density with the spectral density calculated in QCD. In accordance with the QCD sum rules method strategy, we equate the two representations of the two point correlation function, theoretical and phenomenological, and we construct the sum rule for the coupling constant $g_\phi s\gamma$. After evaluating the Fourier transform and then performing the double Borel transformation with respect to the variables $Q_1^2=-p'^2$ and $Q_2^2=-(p'+q)^2$, we finally obtain the following sum rule for the coupling constant $g_{\phi s\gamma}$

$$g_{\phi s\gamma}=\frac{1}{6}\frac{m_\phi(e_u+(-1)^I e_d)<\overline{u}u>}{\lambda_\phi\lambda_s}e^{m_s^2/M_1^2}e^{m_\phi^2/M_2^2}\left\{-M^2\chi\phi(u_0)f_0(s_0/M^2)+4g_1(u_0)\right\}\sin\theta \tag{11}$$

where the function $f_0(s_0/M^2)=1-e^{-s_0/M^2}$ is the factor used to subtract the continuum, $s_0$ being the continuum threshold, and

$$u_0=\frac{M_1^2}{M_1^2+M_2^2}\qquad ,M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2} \tag{12}$$

with $M_1^2$ and $M_2^2$ are the Borel parameters.

For the numerical evaluation of the sum rules, we use the value $<\overline{u}u>=(-0.014\pm 0.002)\ \ GeV^3$ [8] for the vacuum condensate, and $\chi=-4.4\ \ GeV^{-2}$ [13,17] for the magnetic susceptibility of the quark condensate. The leading twist-2 photon wave function is given as $\phi(u) = 6u(1-u)$ and the two-particle photon wave function of twist-4 is given by the expression $g_1(u) = -(1/8)(1-u)(3-u)$ [13]. We use for the overlap amplitudes the values $\lambda_\sigma = (0.12 \pm 0.03) \;\; GeV^2$, $\lambda_{a_0} = (0.21 \pm 0.05) \;\; GeV^2$, and $\lambda_\phi = (0.079 \pm 0.016) \;\; GeV^2$ as we have discussed previously, and we use $m_\phi = 1.02 \;\; GeV$, $m_\sigma = 0.5 \;\; GeV$, and $m_{a_0} = 0.98 \;\; GeV$. In order to analyze the dependence of the coupling constants $\mathrm{g}_{\phi\sigma\gamma}$ and $\mathrm{g}_{\phi a_0\gamma}$ on the Borel parameters $M_1^2$ and $M_2^2$, we study independent variations of $M_1^2$ and $M_2^2$. We find that the sum rule for the coupling constant $\mathrm{g}_{\phi\sigma\gamma}$ is quite stable for $M_1^2 = 1.2 \;\; GeV^2$ and for $1.0 \;\; GeV^2 < M_2^2 < 1.4 \;\; GeV^2$, and that for the coupling constant $\mathrm{g}_{\phi a_0\gamma}$ for the values $M_2^2 = 2.0 \;\; GeV^2$ and for $1.0 \;\; GeV^2 < M_2^2 < 1.4 \;\; GeV^2$. We note that these limits on $M_2^2$ are within the allowed interval for the vector channel [18]. Moreover, we study the dependence of the sum rules on the threshold parameter $s_0$. The variation of the coupling constants $\mathrm{g}_{\phi a_0\gamma}$ and $\mathrm{g}_{\phi\sigma\gamma}$ as a function of the Borel parameter $M_2^2$ for the values of $s_0 = 1.5, 1.6, 1.7 \;\; GeV^2$ with $M_1^2 = 2.0 \;\; GeV^2$ for $\mathrm{g}_{\phi a_0\gamma}$, and for the values of $s_0 = 1.1, 1.2, 1.3 \;\; GeV^2$ with $M_1^2 = 1.2 \;\; GeV^2$ in the case of $\mathrm{g}_{\phi\sigma\gamma}$ are shown in Fig. 1 and in Fig. 2, respectively, from which we conclude that the variations are quite stable. The sources contributing to the uncertainties are those due to variations in the Borel parameters $M_1^2$, $M_2^2$, in the threshold parameter $s_0$, and in the estimated values of the vacuum condensate and the magnetic susceptibility. If we take these uncertainties into account by a conservative estimate, we obtain the coupling constants $\mathrm{g}_{\phi a_0\gamma}$ and $\mathrm{g}_{\phi\sigma\gamma}$ as $\mathrm{g}_{\phi a_0\gamma} = (0.11 \pm 0.03)$ and $\mathrm{g}_{\phi\sigma\gamma} = (0.036 \pm 0.008)$.

In a previous work [4], we studied the radiative $\phi \to \pi^0\pi^0\gamma$ decay. We considered $\rho$-pole vector meson dominance amplitude as well as scalar $\sigma$-pole and $f_0$-pole amplitudes, and by employing the experimental value for this decay rate and by analyzing the interference effects between different contributions in the experimental $\pi^0\pi^0$ invariant mass spectrum for the decay $\phi \to \pi^0\pi^0\gamma$, we estimated the coupling constant $\mathrm{g}_{\phi\sigma\gamma}$ as $\mathrm{g}_{\phi\sigma\gamma} = (0.025 \pm 0.009)$ which is in reasonable agreement with our present calculation utilizing light cone QCD sum rules method. On the other hand, Friman and Soyeur [16] in their study of the photoproduction of $\rho^0$ mesons on proton targets near threshold showed that photoproduction cross section is given mainly by $\sigma$-exchange. They calculated the $\rho\sigma\gamma$-vertex assuming vector meson dominance of the electromagnetic current and then they performed a fit to the experimental $\rho^0$ photoproduction data. Their result when described using an effective Lagrangian for the $\rho\sigma\gamma$-vertex gave the value $\mathrm{g}_{\rho\sigma\gamma} = 2.71$ for this coupling constant. In their study of the structure of the $\phi$ meson photoproduction amplitude on nucleons near threshold based on the one-meson exchange and Pomeron-exchange mechanisms, Titov et al. [5] used this value of the coupling constant $\mathrm{g}_{\rho\sigma\gamma}$ to calculate the coupling constants $\mathrm{g}_{\phi\sigma\gamma}$ and $\mathrm{g}_{\phi a_0\gamma}$ by invoking unitary symmetry arguments. They assumed that $\sigma$, $f_0$, and $a_0$ are members of a unitary nonet, and they obtained the results $\mathrm{g}_{\phi\sigma\gamma} = 0.047$ and $\mid g_{\phi a_0\gamma} \mid = 0.16$ for these coupling constants. Our results $\mathrm{g}_{\phi\sigma\gamma} = (0.036 \pm 0.008)$ and $\mathrm{g}_{\phi a_0\gamma} = (0.11 \pm 0.03)$ are in agreement with the values of these coupling constants calculated by Titov et al. and used in their analysis. However, it should be noted that in our work we do not make any assumption about the assignment of scalar states into various unitary nonets, which is not without problems.

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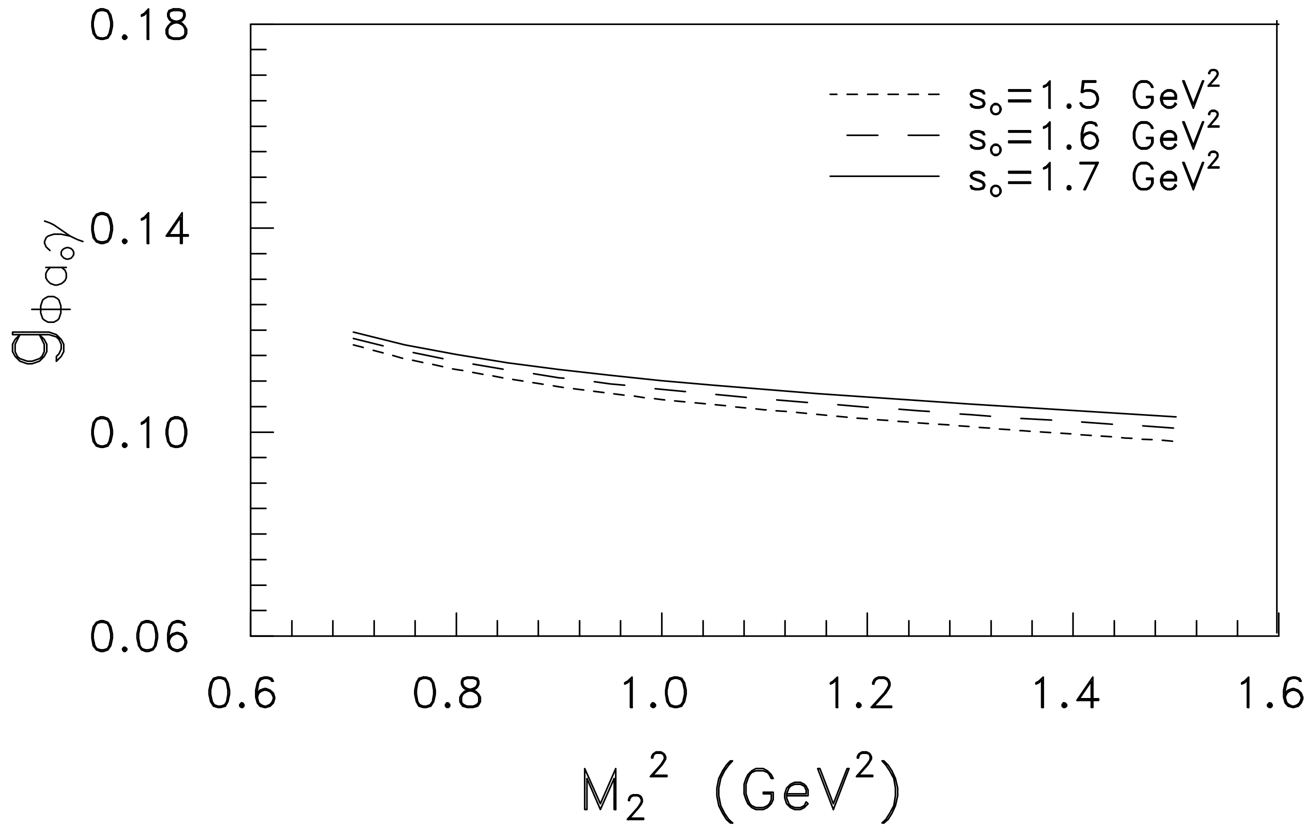
**Figure Caption:**

**Figure 1:** The coupling constant $g_{\phi a_0 \gamma}$ as a function of the Borel parameter $M_2^2$ for different values of the threshold parameters $s_0$ with $M_1^2$=2.0 GeV$^2$.

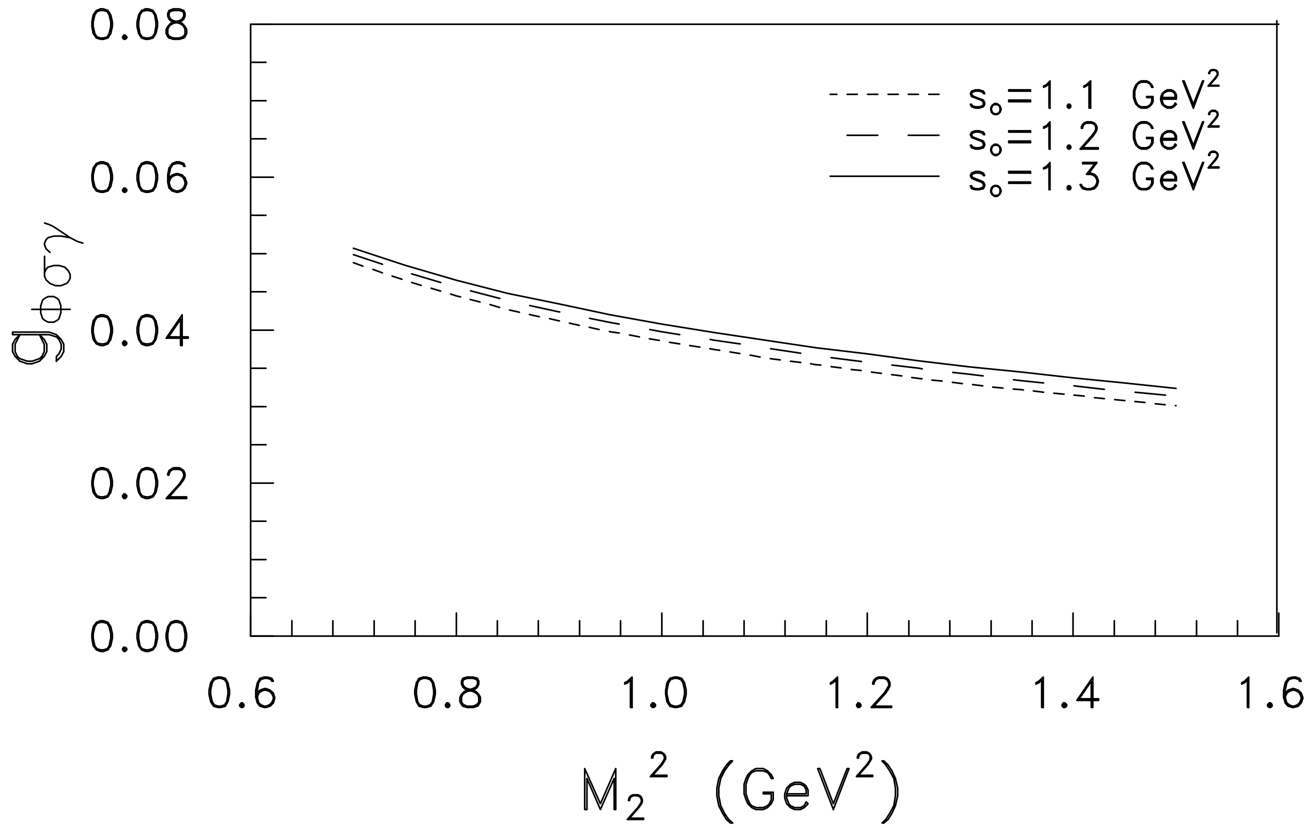
**Figure 2:** The coupling constant $g_{\phi \sigma \gamma}$ as a function of the Borel parameter $M_2^2$ for different values of the threshold parameters $s_0$ with $M_1^2$=1.2 GeV$^2$.

Figure 1



s_o=1.5 GeV^2
s_o=1.6 GeV^2
s_o=1.7 GeV^2
0.18
0.14
0.10
0.06
0.6
0.8
1.0
1.2
1.4
1.6
M_2^2 (GeV^2)

Figure 2



$s_o$=1.1 GeV$^2$
$s_o$=1.2 GeV$^2$
$s_o$=1.3 GeV$^2$
$g_{\phi\sigma\gamma}$
0.08
0.06
0.04
0.02
0.00
0.6
0.8
1.0
1.2
1.4
1.6
$M_2^2$ (GeV$^2$)