

# The coupling constant $g_{\rho\sigma\gamma}$ as derived from QCD sum rules

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(October 25, 2018)

## Abstract

We employ QCD sum rules to calculate the coupling constant  $g_{\rho\sigma\gamma}$  by studying the three point  $\rho\sigma\gamma$ -correlation function. Our results is consistent with the value of this coupling constant obtained using vector meson dominance of the electromagnetic current and the experimental  $\rho^0$ -photoproduction data.

PACS numbers: 12.38.Lg;13.40.Hq;14.40.Cs

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## I. INTRODUCTION

The existence of the scalar-isoscalar  $\sigma$  meson as a broad  $\pi\pi$  resonance has long been controversial. Recently, an increasing number of theoretical and experimental analyzes point toward the existence of this important meson [1]. Most of these analyzes find a  $\sigma$ -pole position near 500-i250 MeV [2]. A direct experimental evidence seems to emerge from the  $D^+ \rightarrow \sigma\pi^+ \rightarrow 3\pi$  decay channel observed by the Fermilab E791 collaboration, where  $\sigma$  meson is seen as a clear dominant peak with  $M_\sigma=478$  MeV and  $\Gamma_\sigma=324$  MeV [3]. Since  $\sigma$  meson is a relevant hadronic degree of freedom, it must be incorporated into the analysis of hadronic processes.

Although at sufficiently high energies and low momentum transfers electromagnetic production of vector mesons on nucleon targets has been explained by Pomeron exchange models, at low energies near threshold scalar and pseudoscalar meson exchange mechanisms become important [4]. In particular, the presently available data on the photoproduction of  $\rho^0$  meson on proton targets near threshold can be described by a simple one-meson exchange model [5]. In this picture,  $\rho^0$  meson photoproduction cross section on protons is given mainly by  $\sigma$  meson exchange. An important physical input for these studies is the coupling constant  $g_{\rho\sigma\gamma}$ . This coupling constant was estimated by calculating the  $\rho\sigma\gamma$ -vertex assuming vector meson dominance of the electromagnetic current and then performing a fit to the experimental  $\rho^0$ -photoproduction data. When this result was derived using an effective Lagrangian for the  $\rho\sigma\gamma$ -vertex the value  $g_{\rho\sigma\gamma}=2.71$  is obtained for this coupling constant [5]. Since it plays an important role in hadronic processes involving  $\sigma$  meson, it is of interest to study this coupling constant from a different perspective other than vector meson dominance.

In this work, we estimate the coupling constant  $g_{\rho\sigma\gamma}$  by employing QCD sum rules which provide an efficient method to study many hadronic observables, such as decay constants and form factors, in terms of nonperturbative contributions proportional to the quark and gluon condensates [6–8]. In section 2, we give a QCD sum rules analysis of scalar current,

derive a sum rule for the overlap amplitude  $\lambda_\sigma$  of the scalar current to the  $\sigma$  meson state and estimate this amplitude since it is not available experimentally. In section 3, we derive a sum rule for the coupling constant  $g_{\rho\sigma\gamma}$  and by utilizing the experimental value of  $\lambda_\rho$ , the calculated value of  $\lambda_\sigma$  and the known values of condensates we estimate this coupling constant. Let us here note that the nature of  $\sigma$  meson, whether it is a  $\bar{q}q$  state or it is of another quark, gluon structure is still a matter of debate [1,2,9].

## II. QCD SUM RULES ANALYSIS OF SCALAR CURRENT

The QCD sum rules approach [6–8] is a model independent method to study the properties of hadrons through correlation functions of appropriately chosen currents. In the SU(2) flavour limit  $m_u = m_d = m_q$ , we study the scalar-isoscalar mesons by considering the scalar current correlation function

$$\Pi(p^2) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ j_s(x) j_s(0) \} | 0 \rangle \quad (1)$$

where  $j_s = \frac{1}{2}(\bar{u}u + \bar{d}d)$  is the interpolating current for  $\sigma$  meson. The two-loop expression for the scalar current correlation function  $\Pi(p^2)$  in perturbative QCD was calculated [10], and it is given by the expression

$$\Pi_{pert}(p^2) = \frac{3}{16\pi^2} (-p^2) \ln\left(\frac{-p^2}{\mu^2}\right) \left\{ 1 + \frac{\alpha_s}{\pi} \left[ \frac{17}{3} - \ln\left(\frac{-p^2}{\mu^2}\right) \right] \right\} . \quad (2)$$

QCD-vacuum condensate contributions to the scalar current correlation function  $\Pi(p^2)$  were obtained by the operator product method [11], and they were found in the  $m_q=0$  limit as

$$\Pi(p^2 = -Q^2)_{cond} = \frac{3}{2Q^2} \langle m_q \bar{q}q \rangle + \frac{1}{16\pi Q^2} \langle \alpha_s G^2 \rangle - \frac{88\pi}{27Q^4} \langle \alpha_s (\bar{q}q)^2 \rangle . \quad (3)$$

We note that the term  $\langle m_q \bar{q}q \rangle$  is not considered in the order  $m_q$  terms, as this condensate's magnitude, which is obtained by making use of Gell-Mann-Oakes-Renner relation as  $-f_\pi^2 m_\pi^2/4$ , is independent of quark mass [6].

On the other hand, the correlation function  $\Pi(p^2)$  satisfies the standard subtracted dispersion relation [6]

$$\Pi_{pert}(p^2) = p^2 \int_0^\infty \frac{ds}{s(s-p^2)} \rho(s) + \Pi(0) \quad (4)$$

where the spectral density function is given as  $\rho(s) = (1/\pi)Im\Pi(s)$ . We parameterize the spectral density as a single sharp pole  $\pi\lambda_\sigma\delta(s-m_\sigma^2)$ , where the overlap amplitude is defined as  $\lambda_\sigma = \langle 0|j_s|\sigma \rangle$ , plus a smooth continuum representing the higher states. The continuum contribution to the spectral density is estimated in the form  $\rho = \rho_h(s)\theta(s-s_0)$  where  $s_0$  denote the continuum threshold and  $\rho_h$  is given by the expression  $\rho_h(s) = (1/\pi)Im\Pi_{OPE}(s)$  with  $\Pi_{OPE}(s)$  obtained from Eq. (2) and Eq. (3) as  $\Pi_{OPE}(s) = \Pi_{pert}(s) + \Pi_{cond}(s)$ . After performing the Borel transformation we obtain the following QCD sum rule for the overlap amplitude  $\lambda_\sigma$  of the scalar current

$$\lambda_\sigma e^{-\frac{m_\sigma^2}{M^2}} = \frac{3}{16\pi^2} M^2 \left\{ \left[ 1 - \left( 1 + \frac{s_0}{M^2} \right) e^{-\frac{s_0}{M^2}} \right] \left( 1 + \frac{\alpha_s(M)}{\pi} \frac{17}{3} - \frac{\alpha_s(M)}{\pi} \int_0^{s_0/M^2} w \ln w e^{-w} dw \right) \right. \\ \left. + \frac{3}{2M^2} \langle m_q \bar{q}q \rangle + \frac{1}{16\pi M^2} \langle \alpha_s G^2 \rangle - \frac{88\pi}{27M^4} \langle \alpha_s (\bar{q}q)^2 \rangle \right. \quad (5)$$

For the numerical evaluation of the above sum rule, we use the values  $\langle m_q \bar{q}q \rangle = (-0.82 \pm 0.10) \times 10^{-4} \text{ GeV}^4$ ,  $\langle \alpha_s G^2 \rangle = (0.038 \pm 0.011) \text{ GeV}^4$ ,  $\langle \alpha_s (\bar{q}q)^2 \rangle = -0.18 \times 10^{-3} \text{ GeV}^6$  [8,12], and  $m_\sigma = 0.5 \text{ GeV}$ . For the continuum threshold we choose  $s_0 = 1.1, 1.2, 1.3 \text{ GeV}^2$  and we study the  $M^2$  dependence between  $0.5 \text{ GeV}^2$  and  $1.4 \text{ GeV}^2$ . The overlap amplitude as a function of  $M^2$  for different values of  $s_0$  is shown in Fig. 1 from which by choosing the middle value  $M^2 = 0.9 \text{ GeV}^2$  in its interval of variation we obtain the overlap amplitude as  $\lambda_\sigma = (0.12 \pm 0.01) \text{ GeV}^2$  where we include the uncertainty due to the variation of the continuum threshold and the Borel parameter  $M^2$ . Other sources of uncertainty are the errors attached to the estimated values of condensates as quoted above. If we take these errors into account in our analysis as well, we then obtain the value  $\lambda_\sigma = (0.12 \pm 0.03) \text{ GeV}^2$  for the overlap amplitude.

### III. QCD SUM RULES FOR THE $\rho\sigma\gamma$ -VERTEX FUNCTION

In order to derive the QCD sum rule for the coupling constant  $g_{\rho\sigma\gamma}$ , we consider the three point correlation function

$$T_{\mu\nu}(p, p') = \int d^4x d^4y e^{ip'.y} e^{-ip.x} \langle 0 | T \{ j_\mu^\gamma(0) j_\nu^\rho(x) j_s(y) \} | 0 \rangle . \quad (6)$$

The interpolating current for  $\rho$  meson [6] is  $j_\nu^\rho = \frac{1}{2}(\bar{u}\gamma_\nu u - \bar{d}\gamma_\nu d)$ , and  $j_\mu^\gamma = \bar{u}\gamma_\nu u + \bar{d}\gamma_\nu d$ ,  $j_s = \frac{1}{2}(\bar{u}u - \bar{d}d)$  are the electromagnetic and scalar currents, respectively.

The theoretical part of the sum rule is obtained by calculating the perturbative contribution and the power corrections from operators of different dimensions to the three point correlation function  $T_{\mu\nu}$ . For the perturbative contribution we consider the lowest order bare-loop diagram. Furthermore, we consider the power corrections from the operators of different dimensions,  $\langle \bar{q}q \rangle$ ,  $\langle \sigma \cdot G \rangle$  and  $\langle (\bar{q}q)^2 \rangle$ . We do not consider the gluon condensate contribution proportional to  $\langle G^2 \rangle$  since it is estimated to be negligible for light quark systems. We perform the calculations of the power corrections in the fixed point gauge [13]. We work in the limit  $m_q=0$ , and in this limit perturbative bare-loop diagram does not make any contribution. Moreover, in this limit only operators of dimensions  $d=3$  and  $d=5$  make contributions that are proportional to  $\langle \bar{q}q \rangle$  and  $\langle \sigma \cdot G \rangle$ , respectively. The relevant Feynman diagrams for the power corrections are shown in Fig. 2.

In order to obtain the phenomenological part, we consider a double dispersion relation for the vertex function  $T_{\mu\nu}$ , and we saturate this dispersion relation by the lowest lying meson states in the vector and scalar channels. We obtain this way for the physical part

$$T_{\mu\nu}(p, p') = \frac{\langle 0 | j_\nu^\rho | \rho \rangle \langle \rho(p) | j_\mu^\gamma | \sigma(p') \rangle \langle \sigma | j_s | 0 \rangle}{(p^2 - m_\rho^2)(p'^2 - m_\sigma^2)} + \dots \quad (7)$$

where the contributions from the higher states and the continuum is denoted by dots. In this expression,  $\lambda_\sigma = \langle \sigma | j_s | 0 \rangle$  has been determined in Section 2. The overlap amplitude  $\lambda_\rho$  of rho meson is defined as  $\langle 0 | j_\nu^\rho | \rho \rangle = \lambda_\rho u_\nu$  where  $u_\nu$  is the polarization vector of the  $\rho$  meson. The matrix element of the electromagnetic current is given as

$$\langle \rho(p) | j_\mu^\gamma | \sigma(p') \rangle = -i \frac{e}{m_\rho} g_{\rho\sigma\gamma} K(q^2) (p \cdot k u_\mu - u \cdot k p_\mu) \quad (8)$$

where  $q = p - p'$  and  $K(q^2)$  is a form factor with  $K(0)=1$ . This expression defines the coupling constant  $g_{\rho\sigma\gamma}$  through the effective Lagrangian

$$\mathcal{L} = \frac{e}{m_\rho} g_{\rho\sigma\gamma} \partial^\alpha \rho^\beta (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \sigma \quad (9)$$

describing the  $\rho\sigma\gamma$ -vertex [5].

After performing double Borel transform with respect to the variables  $Q^2 = -p^2$  and  $Q'^2 = -p'^2$ , and by considering the gauge-invariant structure  $(p_\mu q_\nu - p \cdot q g_{\mu\nu})$  we obtain the sum rule for the coupling constant  $g_{\rho\sigma\gamma}$

$$g_{\rho\sigma\gamma} = \frac{3m_\rho}{\lambda_\rho \lambda_\sigma} e^{\frac{m_\rho^2}{M^2}} e^{\frac{m_\sigma^2}{M'^2}} \langle \bar{u}u \rangle \left( -\frac{3}{4} + \frac{5}{32} m_0^2 \frac{1}{M^2} - \frac{3}{32} m_0^2 \frac{1}{M'^2} \right) \quad (10)$$

where we use the relation  $\langle \sigma \cdot G \rangle = m_0^2 \langle \bar{q}q \rangle$ . For the numerical evaluation of the sum rule we use the values  $m_0^2 = (0.8 \pm 0.02) \text{ GeV}^2$ ,  $\langle \bar{u}u \rangle = (-0.014 \pm 0.002) \text{ GeV}^3$  [8,14], and  $m_\rho = 0.77 \text{ GeV}$ ,  $m_\sigma = 0.5 \text{ GeV}$ . For the overlap amplitude  $\lambda_\sigma$  we use the value  $\lambda_\sigma = (0.12 \pm 0.03) \text{ GeV}^2$  that we have estimated in Section 2 and for  $\lambda_\rho$  we use its experimental value  $\lambda_\rho = (0.107 \pm 0.003) \text{ GeV}^2$  as obtained from the measured leptonic width  $\Gamma(\rho^0 \rightarrow e^+e^-)$  of  $\rho$  meson [15]. In order to analyze the dependence of  $g_{\rho\sigma\gamma}$  on Borel parameters  $M^2$  and  $M'^2$ , we study the independent variations of  $M^2$  and  $M'^2$  in the interval  $0.6 \text{ GeV}^2 \leq M^2, M'^2 \leq 1.4 \text{ GeV}^2$  as these limits determine the allowed interval for the vector channel [16]. The variation of the coupling constant  $g_{\rho\sigma\gamma}$  as a function of Borel parameters  $M^2$  for different values of  $M'^2$  is shown in Fig. 3, examination of which indicates that it is quite stable with these reasonable variations of  $M^2$  and  $M'^2$ . We choose the middle value  $M^2 = 1 \text{ GeV}^2$  for the Borel parameter in its interval of variation and obtain the coupling constant  $g_{\rho\sigma\gamma}$  as  $g_{\rho\sigma\gamma} = 3.2 \pm 0.2$  where only the error arising from the numerical analysis of the sum rule is considered. The other sources contributing to the uncertainty in the coupling constant besides those due to variations of  $M^2$  and  $M'^2$  are the uncertainties in the estimated values of the vacuum condensates. If we take these uncertainties into account, we then obtain the coupling constant  $g_{\rho\sigma\gamma}$  as  $g_{\rho\sigma\gamma} = 3.2 \pm 0.6$ .

Our estimate of the coupling constant  $g_{\rho\sigma\gamma}$  is in agreement with its value deduced from the analysis using the vector meson dominance of the electromagnetic current for the  $\rho\sigma\gamma$ -vertex. On the other hand, an independent analysis [17] utilizing the experimental value of

the decay rate  $\Gamma(\rho^0 \rightarrow \pi^+\pi^-\gamma)$  and using the values for the  $\sigma$  meson parameters  $m_\sigma=478$  MeV and  $\Gamma_\sigma=324$  MeV that are obtained from the Fermilab E791 experiment gives the value  $g_{\rho\sigma\gamma} = 5.92 \pm 1.34$  for this coupling constant. However, we like to note that in the present work we consider  $\sigma$  meson in the narrow resonance limit and do not take the finite width of  $\sigma$  meson into account.

## ACKNOWLEDGMENTS

We thank T. M. Aliev for helpful discussions during the course of our work.

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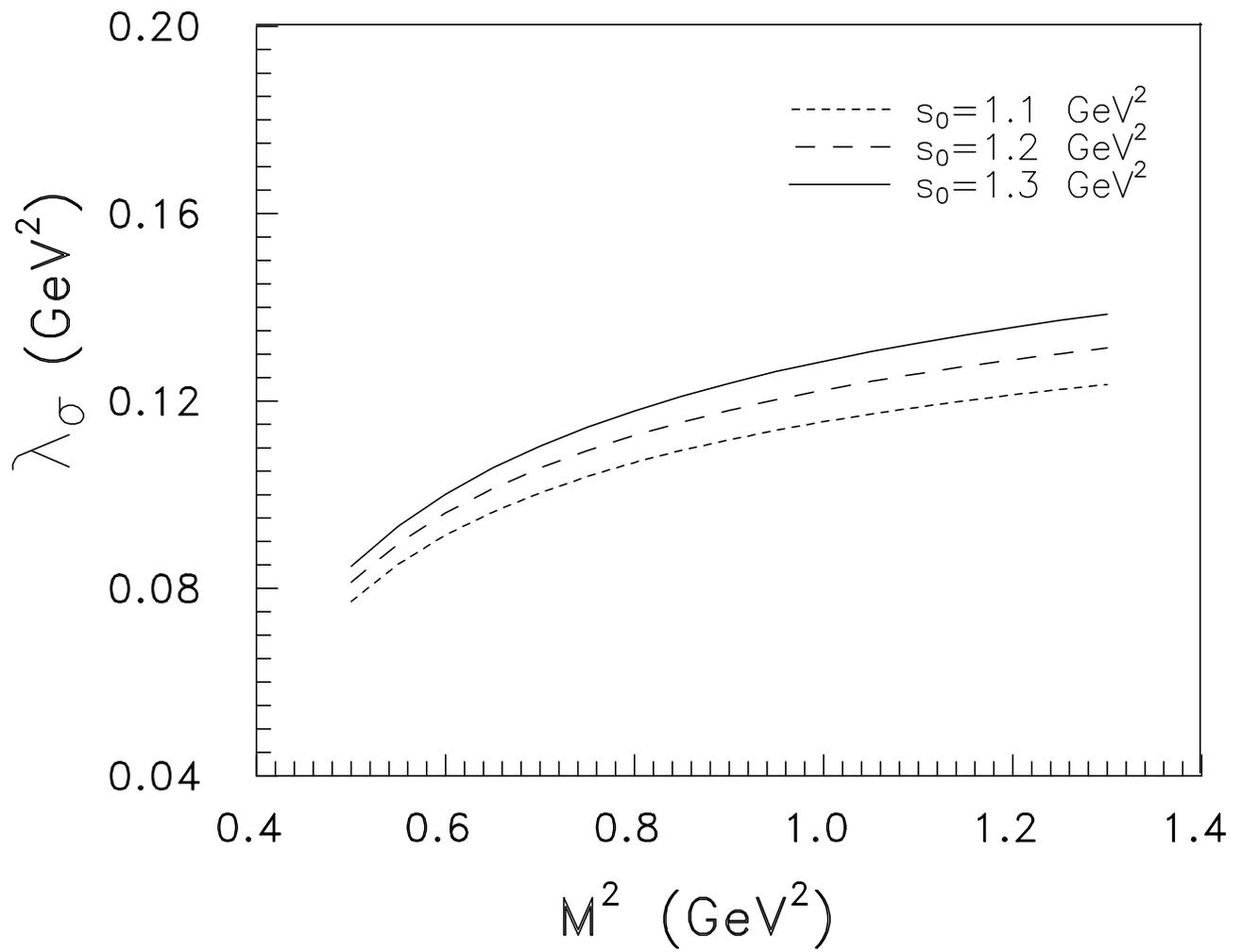
**Figure Captions:**

**Figure 1:** The overlap amplitude  $\lambda_\sigma$  as a function of Borel parameter  $M^2$ .

**Figure 2:** Feynman Diagrams for the  $\rho\sigma\gamma$ -vertex: a- bare loop diagram, b- d=3 operator corrections, c- d=5 operator corrections. The dotted lines denote gluons.

**Figure 3:** the coupling constant  $g_{\rho\sigma\gamma}$  as a function of the Borel parameter  $M^2$  for different values of  $M'^2$ .

Figure 1



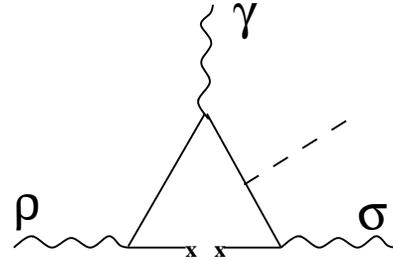
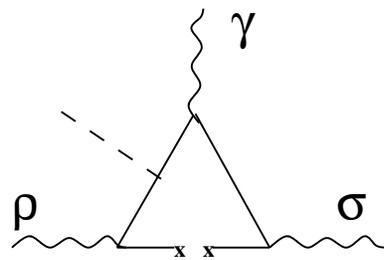
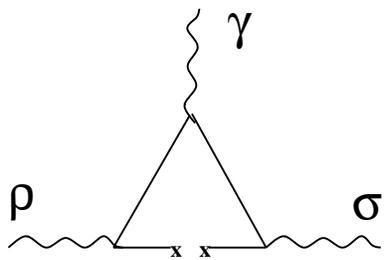
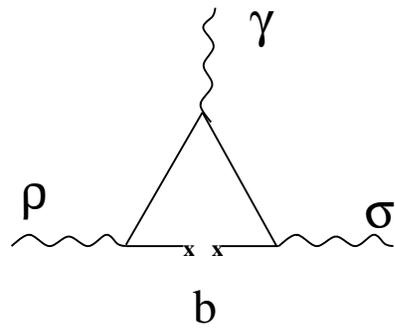
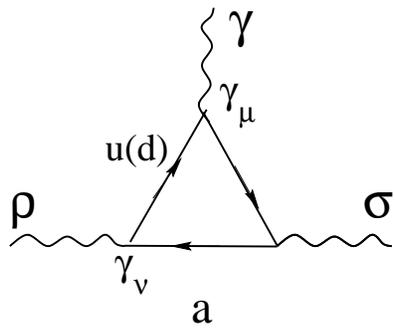


Figure 2

Figure 3

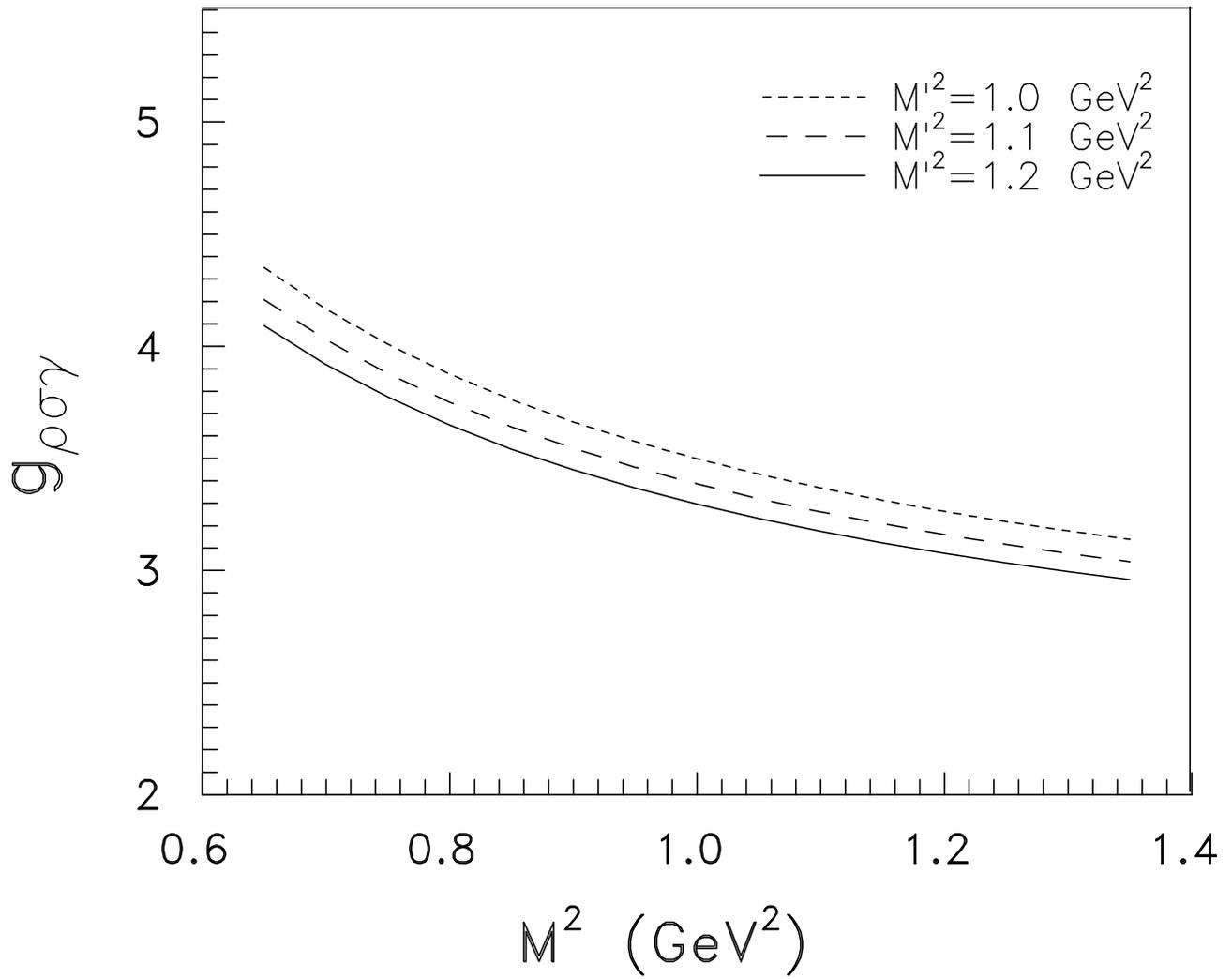


Figure 3

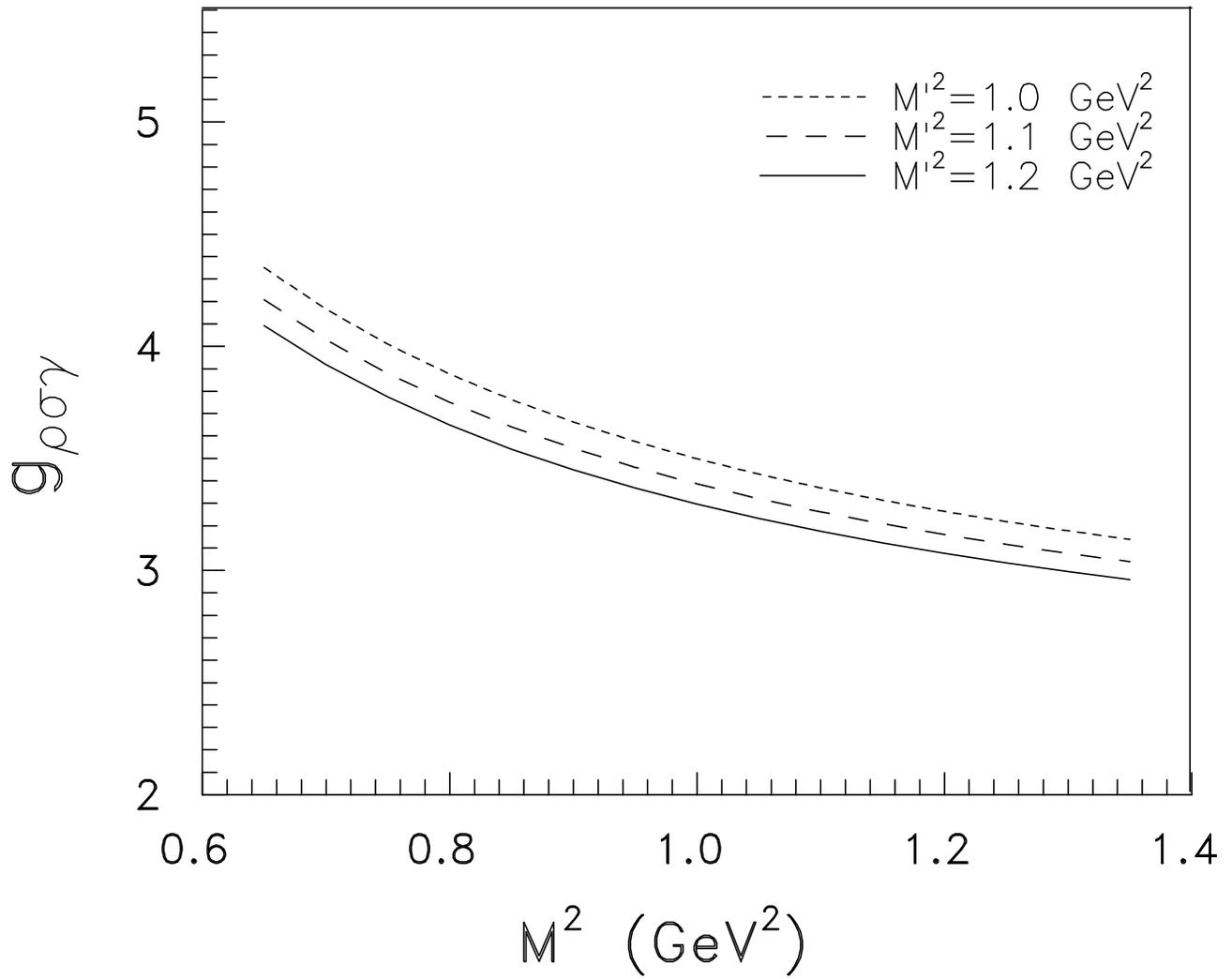


Figure 3

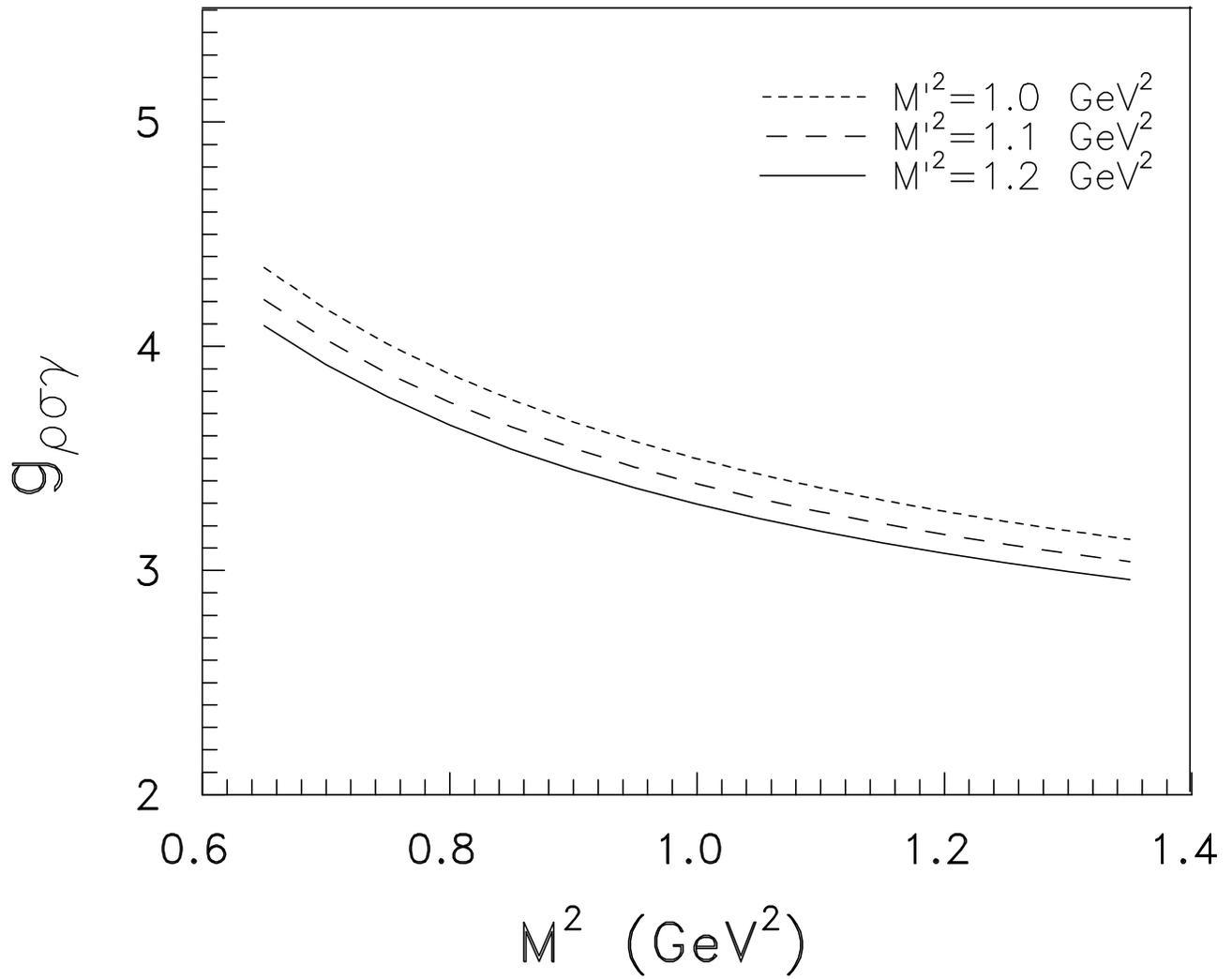


Figure 3

