

Non-relativistic quark-antiquark potential: spectroscopy of heavy-quarkonia and exotic SUSY quarkonia

Sameer M. Ikhdair^{1,*} and Ramazan Sever^{2,†}

¹*Department of Physics, Near East University, Nicosia, Cyprus, Turkey*

²*Department of Physics, Middle East Technical University, 06531, Ankara, Turkey*

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Abstract

The experiments at LHC have shown that the SUSY (exotic) bound states are likely to form bound states in an entirely similar fashion as ordinary quarks form bound states, i.e., quarkonium. Also, the interaction between two squarks is due to gluon exchange which is found to be very similar to that interaction between two ordinary quarks. This motivates us to solve the Schrödinger equation with a strictly phenomenological static quark-antiquark potential: $V(r) = -Ar^{-1} + \kappa\sqrt{r} + V_0$ using the shifted large N -expansion method to calculate the low-lying spectrum of a heavy quark with anti-sbottom ($c\bar{b}, b\bar{b}$) and sbottom with anti-sbottom ($\tilde{b}\bar{b}$) bound states with $m_{\tilde{b}}$ is set free. To have a full knowledge on spectrum, we also give the result for a heavier as well as for lighter sbottom masses. As a test for the reliability of these calculations, we fix the parameters of this potential by fitting the spin-triplet (n^3S_1) and center-of-gravity $l \neq 0$ experimental spectrum of the ordinary heavy quarkonia $c\bar{c}, c\bar{b}$ and $b\bar{b}$ to few MeV. Our results are compared with other models to gauge the reliability of these predictions and point out differences.

Keywords: Bound state energy, exotic quarkonia, conventional heavy quarkonia, shifted large N -expansion technique.

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*E-mail: sameer@neu.edu.tr; sikhhdair@neu.edu.tr

†E-mail: sever@metu.edu.tr

I. INTRODUCTION

Weak-scale Supersymmetric Standard Model (SSM) is the leading candidate for physics beyond the Standard Model (SM) [1,2]. Supersymmetry (SUSY) is built on a solid theoretical and mathematical foundation. It is also well-motivated as an elegant solution to the gauge hierarchy problem and has merits of gauge coupling unification, dynamical electroweak symmetry and providing a legitimate candidate for dark matter. SUSY predicts the existence of a super partner called SUSY particles (sparticles) corresponding to each ordinary particle of SM. These sparticles should be accessible at the exist and constructing colliders such as Tevatron and LHC. Over the past years, great effort has been made to search for such sparticles. So far, no direct signal for SUSY has been observed and some lower mass bounds have been established for sparticles. The experimental results at LEP [3-8,9,10] and Tevatron [11-14], squarks must be heavier than about 100 GeV. However, most experimental searches for sparticles are performed with model-dependent assumptions and rely on a large missing energy cut. A long-lived light SUSY bottom quark (sbottom), \tilde{b} , and its anti-sbottom, $\tilde{\bar{b}}$, with a mass $m_{\tilde{b}}$ close to m_b (~ 4.9 GeV), roughly half the $\Upsilon(1S)$ mass, has not been excluded by experiments [15,16]. Hence, a light sbottom and its anti-sbottom, are not excluded so far partly because of the ALEPH collaboration indication [3-8] and partly because of interesting scenario to explain the excess of $b\bar{b}$ pair production in hadron collisions than theoretical prediction by a factor two. Some analyses [15] showed that if the light \tilde{b} is an appropriate admixture of left-handed and right-handed sbottom quark, its coupling to Z boson can be small enough to avoid LEP-I Z decay bounds. In addition, a scenario with light gluino and long-lived light sbottom with mass close to the bottom quark was proposed in [16] with which the excess of measured $b\bar{b}$ pair production in hadron collision over QCD theoretical prediction by a factor two is explained successfully (cf. [17-19]). The data about $b\bar{b}$ pair production in hadron collision given by CDF and D0 can be explained well by QCD theoretical production: e.g., we can learn the details from the web address in [17-19]. The CLEO exclusion of a \tilde{b} with mass 3.5 to 4.5 GeV [20] can also be loosed even avoided, since their analysis depends on the assumption for semi-leptonic decays of the light sbottom. Moreover, since sbottom is a scalar, based on the spin freedom counting only, its pair production rate at collision will be smaller than the bottom quark by a factor four, so the sbottom samples must be rarer than those of bottom quark in experiments.

In contrary, it is interesting to point out that some experiments seemingly favor such a light sbottom. The ALEPH collaboration has reported experimental hints for a light sbottom with a mass around 4 GeV and lifetime of 1 ps [21]. A recent analysis of old anomaly in the MARK-I data for cross section of $e^+e^- \rightarrow \text{hadrons}$ shows that the existence of such a light sbottom can bring the measured cross section into agreement with the theoretical prediction [22]. As mentioned by Berger *et al.* [16], a light-gluino analysis was done by Baer *et al.* [23] in which the gluino is assumed LSP. Cheung and Keung [24] modified the analysis [23] by letting the light gluino decay into b and \tilde{b} and study the possible constraint and implication at LSP. Therefore, the light gluino and light sbottom scenario will certainly give rise to other interesting signals, e.g., decay of χ_b into the light sbottom [25], enhancement of $t\bar{t}b\bar{b}$ production at hadron colliders [26], decay of Υ into a pair of light sbottoms [27] and flavor-changing effects in radiative decays of B mesons [28].

The phenomenology of a very light sbottom has been studied by many authors recently [24-26,28-32]. If such a light sbottom indeed exist, new meson-like bound states is formed by a pair of the sbottom and anti-sbottom ($\tilde{b}\bar{b}$) and fermion-like ones by an ordinary quark with anti-sbottom ($q\bar{b}$) (*e.g.*, heavy quark $q = c, b$) may be also formed.

Some quarkonium binding systems like $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ have been studied with encouraging success [33-35], in the framework of the potential model using a strictly phenomenological static heavy quark-antiquark potential belonging to the generality $V(r) = -Ar^{-\alpha} + \kappa r^\beta + V_0$, ($\alpha = 1, \beta = 1/2$). Hence, the parameters of this potential are fixed by fitting the experimentally measured triplet S-states and the center-of-gravity (c.o.g.) non S-states of $c\bar{c}$ and $b\bar{b}$ spectra to their calculated levels and taking into consideration their hyperfine splittings in the framework of non-relativistic quarkonium model. It has been found that the potential description is flavor-independent, i.e., the same potential describes equally well the $c\bar{c}$ and the $b\bar{b}$ systems. Therefore, if we take the potential parameters obtained from the fitting of the mass spectra $c\bar{c}$ and the $b\bar{b}$ systems, we may predict the Schrödinger bound-state masses of the exotic states. We take the very same values of potential parameters used for the observed bound states of ordinary quarks to predict the unknown exotic squark bound states. It is well-known that the interaction between two squarks is due to the gluon exchange [36] which is found to be very similar to that interaction between two ordinary quarks, and an interaction due to Higgs particle exchange [37]. Thus, the gluon exchange interaction in squarkonium motivates us to use very similar parameter set of present potential

model as in quarkonium [33-35].

Over the past years, the experiments at LHC have shown that the exotic bound states are likely to form bound states in an entirely similar fashion as ordinary quarks form bound state, i.e., quarkonium. The long-lived sbottom is not excluded by conventional searches and an analysis should be done to verify that there are no additional constraints on the allowed range of sbottom masses and lifetimes. In addition, it is well-known that the interaction between two squarks is due to gluon exchange which is found to be very similar to that interaction between two ordinary quarks, and an interaction due to Higgs particle exchange. Thus, the gluon exchange interaction in squarkonium motivates us to use very similar parameter set of present potential model as in quarkonium. The purpose is to calculate the spectroscopy of $(Q\bar{b})$ $Q = c, b$ and $(\tilde{b}\bar{b})$ in terms of potential model with Coulomb plus square-root potential [33-35] in which the parameters are fixed by heavy quarkonia $(c\bar{c})$ and $(b\bar{b})$ with sbottom mass, $m_{\tilde{b}}$ is set as a free parameter. Furthermore, in order to have a full knowledge on such a spectrum, we also give the result for a heavier sbottom masses from 3.0 GeV to 60.0 GeV.

The outline of this paper is as following: In section 2, we first review the analytic solution of the Schrödinger equation for non-equal mass case for the spin-triplet S -states and c.o.g (center of gravity) P - and D -states. In section 3, we briefly present the squarkonium production through the leptonic decay width and through the Z^0 decay. We present the observed states and reproduce the calculated spectrum of the $c\bar{c}$, $c\bar{b}$ and $b\bar{b}$ spectra in section 4. Also, the corresponding triplet S -states and the c.o.g. $l = 1, 2$ for an ordinary SM heavy quark with anti-sbottom $(c\bar{b}, b\bar{b})$ and sbottom with anti-sbottom $\tilde{b}\bar{b}$ meson-like binding system with sbottom mass, $m_{\tilde{b}}$ is set as a free parameter. The conclusions are also given in section 5.

II. SPIN-AVERAGED BINDING MASS SPECTRUM

We limit our discussion to the following generality of potentials [33,38-49]:

$$V(r) = -Ar^{-\alpha} + \kappa r^{\beta} + V_0, \quad \alpha, \beta > 0 \quad (1)$$

where A and κ are positive constants whereas V_0 is taking any sign. These static quarkonium potentials are monotone nondecreasing, and concave functions satisfying the condition [41-49]

$$V'(r) > 0 \quad \text{and} \quad V''(r) \leq 0. \quad (2)$$

At least ten potentials of this generality, but with various values of the parameters, have been proposed in the literature (see, for example, Ref. [33] and references therein). Motyka and Zalewski [34,35] have also explored the quality of fit in the region $0 \leq \alpha \leq 1.2$, $0 \leq \beta \leq 1.1$ of the α, β plane reasonably well with coordinates $\alpha = 1, \beta = 0.5$. Therefore, the five parameters $(A, \kappa, V_0, m_c, m_b)$ are fixed, in fitting the $c\bar{c}$ and the $b\bar{b}$ experimental triplet states in flavor-independent model, as the below values:

$$V(r) = -\frac{0.325250}{r} + 0.70638\sqrt{r} - 0.78891, \quad (3)$$

with fitted quark masses:

$$m_c = 1.3959 \text{ GeV}, \quad m_b = 4.8030 \text{ GeV}, \quad (4)$$

where $V(r)$, \sqrt{r} and r^{-1} are all in units of GeV. Notice that for the c quark, m_c is roughly half the J/ψ mass and for b quark it is roughly half of the $\Upsilon(1S)$ mass. Thus, the potential model (3) is convincing as it approaches to the perturbative QCD formula in the short-distance region and approached to the confining potential in the long-distance region. Consequently, in short-distance region, this potential involves the r^{-1} (Coulombic part) corresponding to one gluon exchange which is approaching to the perturbative QCD formula. The linear confinement part of the potential is $\sim r$, as in Cornell potential [50,51], is not seen. Such a linearly rising potential [50,51] is capable of confining quarks permanently and it can give rise to spectrum of particles containing light quarks in rough accord with experiment [52,53]. Probably the heavy quarkonia like $b\bar{b}$, $c\bar{c}$ (B_c meson) and $\tilde{b}\tilde{b}$ are too small to reach sufficiently far into the asymptotic region of linear confinement. On the other hand, the charmonium $c\bar{c}$ is too large to reach sufficiently far into the confining potential in the large distance particularly for excited states near and above the open flavor threshold. Perhaps a more flexible potential would exhibit the linear part, but one may be observing an effect of the expected screening of the interaction between the heavy quarks by the light sea quarks [54].

We choose the corresponding spin triplet c.o.g. states for the practical reasons that the masses of the spin singlet pseudoscalars for the bottomonium $b\bar{b}$ are currently unknown or very poorly measured [55] and the two unknown charmonium $c\bar{c}$ states in the 3S and 4S multiplets are the 3^1S_0 and 4^1S_0 pseudoscalars.

For a system of two composite particles, we shall consider the D-dimensional space Schrödinger equation for any spherically symmetric central potential in (3). Using $\psi(\mathbf{r}) =$

$Y_{l,m}(\theta, \phi)u(r)/r^{(D-1)/2}$, it is straightforward to find the $l \neq 0$ radial wave equation (in the usual units $\hbar = c = 1$) :

$$\left\{ -\frac{1}{4\mu} \frac{d^2}{dr^2} + \frac{[\bar{k} - (1-a)][\bar{k} - (3-a)]}{16\mu r^2} + V(r) \right\} u(r) = E_{n,l} u(r), \quad \bar{k} = D + 2l - a, \quad (5)$$

where $u(r)$ is the radial wave function, $E_{n,l}$ is the Schrödinger binding energy of meson and a is a proper shift. We follow the shifted $1/N$ or $1/\bar{k}$ expansion method by defining

$$V(x(r_0)) = \sum_{m=0}^{\infty} \left(\frac{d^m V(r_0)}{dr_0^m} \right) \frac{(r_0 x)^m}{m! Q} \bar{k}^{(4-m)/2}, \quad Q = \bar{k}^2, \quad (6)$$

and the binding energy expansion

$$E_{n,l} = \sum_{m=0}^{\infty} \frac{\bar{k}^{(2-m)}}{Q} E_m, \quad (7)$$

where $x = \bar{k}^{1/2}(r/r_0 - 1)$, r_0 is an arbitrary point where the Taylor's expansions is being performed around. Following Refs. [33,38-49], we give the necessary expressions for calculating the binding energies to the third order:

$$E_0 = V(r_0) + \frac{\beta}{16\mu}, \quad (8)$$

$$E_1 = \beta \left[\left(n_r + \frac{1}{2} \right) \omega - \frac{(2-a)}{8\mu} \right], \quad (9)$$

$$E_2 = \beta \left[\frac{(1-a)(3-a)}{16\mu} + \alpha^{(1)} \right], \quad (10)$$

$$E_3 = \beta \alpha^{(2)}, \quad \beta = \left(\frac{\bar{k}}{r_0} \right)^2, \quad (11)$$

where $\alpha^{(1)}$ and $\alpha^{(2)}$ are two useful expressions given by Imbo *et al.* [56-58] and also the parameter \bar{k} is

$$\bar{k} = \sqrt{8\mu r_0^3 V'(r_0)}. \quad (12)$$

Hence, the total binding energy of the three-dimensional ($D = 3$) Schrödinger equation to the third order is

$$E_{n,l} = V(r_0) + \frac{1}{2} r_0 V'(r_0) + \frac{1}{r_0^2} \left[\frac{(1-a)(3-a)}{16\mu} + \alpha^{(1)} + \frac{\alpha^{(2)}}{\bar{k}} + O\left(\frac{1}{\bar{k}^2}\right) \right], \quad (13)$$

and the shift parameter is

$$a = 2 - (2n_r + 1) \left[3 + \frac{r_0 V''(r_0)}{V'(r_0)} \right]^{1/2}. \quad (14)$$

The root, r_0 , in Eqs. (13)-(14) can be found through the relation:

$$1 + 2l + (2n_r + 1) \left[3 + \frac{r_0 V''(r_0)}{V'(r_0)} \right]^{1/2} = [8\mu r_0^3 V'(r_0)]^{1/2}, \quad n_r, l = 0, 1, 2, 3, \dots, \quad (15)$$

where the radial number $n_r = n - 1$ with $n = 1, 2, 3, \dots$ is the principal quantum number. Once r_0 is determined through Eq. (15), hence finding the Schrödinger binding energy of any quarkonium system from Eq. (13) becomes relatively simple and straightforward. Finally, the corresponding ordinary or exotic bound states become

$$\begin{aligned} M(q_i \bar{q}_j)_{nl} &= m_{q_i} + m_{q_j} + 2E_{n,l}, \\ M(q_i \bar{\bar{q}}_j)_{nl} &= m_{q_i} + m_{\bar{\bar{q}}_j} + 2E_{n,l}, \end{aligned} \quad (16)$$

where m_{q_i} and m_{q_j} are the composite masses of the quark with antiquark and $m_{\bar{q}_i}$ and $m_{\bar{\bar{q}}_j}$ squark with anti-squark. Details of the model and the method of solution may be found in Refs. [33,38-48].

Now let us turn to the investigation of the spin-spin interaction. It is well-known that the system under study is a nonrelativistic, the treatment is based on the Schrödinger equation with a Hamiltonian [59,60]

$$H_o = -\frac{\nabla^2}{2\mu} + V(r) + V_{SS}, \quad (17)$$

where V_{SS} is the spin-spin contact hyperfine interaction which is one of the spin-dependent terms predicted by one-gluon exchange (OGE) forces [59,60]. Recently, the spin-spin part, in momentum space ($q = \mu$), was found to be [61-63]

$$V_{SS}(m_1, m_2, q) = \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{3m_1 m_2} g_s^2(q) \left[\frac{N_c^2 - 1}{N_c} c_3(q, m_1) c_3(q, m_2) - 6N_c d(q) \right], \quad (18)$$

with Wilson coefficient

$$c_3(q, m) = \left(\frac{\alpha_s(q)}{\alpha_s(m)} \right)^{-9/25} \quad \text{and} \quad d(q) = \frac{N_c^2 - 1}{8N_c^2} \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{-9/25} \left[1 - \left(\frac{\alpha_s(q)}{\alpha_s(m_2)} \right)^{-18/25} \right], \quad (19)$$

where N_c is the number of colors, $g_s(q)$ is the running coupling constant [61-65]. The formula (19) improves upon the one-loop perturbative calculation in two important respects: (i) it is independent of μ and (ii) also includes the higher order logarithmic terms.

If the coefficients are calculated at tree level; i.e., $c_3(\mu, m) = 1$, $d(\mu) = 0$, the potential reduces to the Eichten-Feinberg result [66,67]. And if these coefficients are expanded to order $\alpha_s(\mu)$ then reduced to a one-loop quarkonium spin-spin interaction in the nonrelativistic case [68-70] which is responsible for the hyperfine splitting of the mass levels [71-84]

$$V_{SS} \longrightarrow V_{\text{HF}} = \frac{32\pi\alpha_s}{9m_{q_i}m_{\bar{q}_j}}\delta^3(\mathbf{r})\left(\mathbf{s}_1\cdot\mathbf{s}_2 - \frac{1}{4}\right), \quad (20)$$

adapted from the Breit-Fermi Hamiltonian. The number $\frac{1}{4}$ substituted from the product of the spins corresponds to the recent assumption that the unperturbed nonrelativistic Hamiltonian gives the energy of the triplet states. Since for the states with orbital angular momentum $L > 0$ the wave function vanishes at the origin, the shift affects only the S states. Thus, the only first order effect of this perturbation is to shift to the pseudoscalar 1S_0 states down in energy:

$$\Delta E_{\text{HF}} = \frac{32\pi\alpha_s}{9m_{q_i}m_{\bar{q}_j}}|\psi(0)|^2, \quad (21)$$

with the wave function at the origin is calculated by using the expectation value of the potential derivative via [33,41-46,49,82-84]

$$|\psi(0)|^2 = \frac{\mu}{2\pi}\left\langle\frac{dV(r)}{dr}\right\rangle. \quad (22)$$

An application of the last formula needs the value of the wavefunction at the origin. This can be achieved by solving the Schrödinger equation with the nonrelativistic Hamiltonian and the coupling constant. In such an approach, the QCD strong coupling constant $\alpha_s(4\mu^2)$, on the renormalization point μ^2 is not an independent parameter. It can be connected (in the \overline{MS} renormalization scheme) through the two-loop relation [34,35,85-87]

$$\alpha_s(\mu^2) = \frac{2\pi\eta^2 - 1}{\beta_0 \ln(\eta)}, \quad \eta = \frac{2\mu}{\Lambda_{\overline{MS}}^{(n_f)}}, \quad (23)$$

where $\beta_0 = 11 - \frac{2}{3}n_f$. Like most other authors (see, for example, Refs. [33-35,72-75]), the strong coupling constant $\alpha_s(m_c^2)$, is fitted to the experimental charmonium hyperfine splitting numbers $\Delta_{\text{HF}}(1\text{S},\text{exp}) \approx 116.5 \pm 1.2$ MeV and $\Delta_{\text{HF}}(2\text{S},\text{exp}) \approx 48.1 \pm 4$ MeV [41-46,55], which yields

$$\alpha_s(m_c^2) = 0.254. \quad (24)$$

Knowing the coupling at the scale m_c^2 , we obtain the couplings at other scales as follows. The number of flavours (n_f) is put equal to three for $4\mu^2 \leq m_c^2$ (we are not interested in the

region $4\mu^2 \leq m_s^2$), equal to four for $m_b^2 \geq 4\mu^2 \geq m_c^2$ and equal to five for $4\mu^2 \geq m_b^2$ (we are not interested in the region $4\mu^2 \geq m_t^2$). Then the value of $\alpha_s(4\mu^2 = m_c^2)$ from (23) is used to calculate $\Lambda_{\overline{MS}}^{(n_f=3)}$ and $\Lambda_{\overline{MS}}^{(n_f=4)}$. Using the known value of $\Lambda_{\overline{MS}}^{(n_f=4)}$ and Eq. (23) we find the value

$$\alpha_s(m_b^2) = 0.200 \text{ and } \alpha_s(4\mu_{cb}^2) = 0.224, \quad (25)$$

and

$$\alpha_s(m_{\tilde{b}}^2) = \alpha_s(m_b^2) \text{ and } \alpha_s(4\mu_{\tilde{c}\tilde{b}}^2) = \alpha_s(4\mu_{cb}^2). \quad (26)$$

We follow these analysis in order to calculate the experimental binding masses of the heavy quarkonia $c\bar{c}$, $c\bar{b}$ and $b\bar{b}$. At the end, it is worth to note that the sbottom \tilde{b} is a scalar, there is no spin-spin interaction (hyperfine splitting) for $(Q\tilde{b})$ and $(\tilde{b}\tilde{b})$, where $Q = c, b$.

III. SQUARKONIUM PRODUCTION

A. Production Through the Leptonic Decay

The squarks might be discovered by detecting ordinary quark-antiquark bound states as resonances at LHC. (This is of course one of the main ways of studying charm and bottom quarks). The bound states of squarks are narrow resonances depend primarily on their leptonic widths (unless the squark itself has a very large width). Therefore, our aim will be to compute Γ_e . The leptonic decay widths of the heavy quarkonia and squarkonium are proportional to the squares of the wave functions at the origin. Therefore, they are significant only for the S states. To compute the decay rate for this process, we use non-relativistic bound state techniques since the squarks are expected to be very heavy objects (massive particles) and therefore their bound states are not relativistic systems. For the $c\bar{c}$ and $b\bar{b}$ quarkonium and $\tilde{b}\tilde{b}$ sbottomonium systems, we shall consider the decays of the n^3S_1 (vector, $J^{PC} = 1^{--}$) states decay into charged lepton pairs, e.g. e^+e^- pairs are usually calculated from the QCD corrected Van Royen-Weiskopf formula

$$\Gamma_e(n^3S_1 \rightarrow \bar{l}l) = 16\pi\alpha^2 e_q^2 \frac{|\psi(0)|^2}{M_V^2} \left(1 - \frac{16\alpha_s(m_q^2)}{3\pi} \right), \quad (27)$$

where $|\psi(0)|$ is the bound state radial wavefunction at the origin, M_V the mass of the bound triplet (vector) state, α the fine-structure constant and e_q the charge of the quark in units of the electron charge. In the computation we have taken for ordinary quarks

$e_c = \frac{2}{3}$ and $e_b = -\frac{1}{3}$, however, for squarks $e_{\tilde{c}} = \frac{1}{3}$. For the $c\bar{b}$ quarkonium and $\tilde{c}\tilde{b}, \tilde{b}\tilde{b}$ squarkonium, we consider the decays of the n^1S_0 (pseudoscalar) states into $\tau\nu_\tau$ pairs. Since the probability of such decays contains as a factor the square of the lepton mass, the decays into lighter leptons are much less probable [33-35]. For vector mesons containing light quarks (squarks) this formula leads to paradoxes (cf., e.g., Ref. [33] and references therein). For quarkonia, however, the main problem seems to be the QCD correction. Thus, in order to get quantitative predictions it is necessary to include higher order corrections which are not known. In order to estimate the missing terms we tried two simple forms. Exponentialization of the first correction

$$C_1(\alpha_s(m_q^2)) = \exp\left(-\frac{16\alpha_s(m_q^2)}{3\pi}\right), \quad (28)$$

and Padeization

$$C_2(\alpha_s(m_q^2)) = \frac{1}{1 + \frac{16\alpha_s(m_q^2)}{3\pi}}. \quad (29)$$

We use the average of these two estimates as our estimate of the QCD correction factor extended to higher orders. The difference between C_1 and C_2 is our crude evaluation of the uncertainty of this estimate. Further, we have the relation

$$\Gamma_e(n^3S_1 \rightarrow \bar{l}l) = \frac{9}{8} \frac{4m_q^2}{M_V^2} \frac{\alpha^2 e_q^2}{\alpha_s(m_q^2)} C_{av} \Delta E_{\text{HF}}, \quad (30)$$

where C_{av} is the averaged QCD correction factor. With our choice of parameters this formula reduces to

$$\Gamma_e(n^3S_1 \rightarrow \bar{l}l) = F(c) \frac{4m_q^2}{M_V^2} \Delta E_{\text{HF}}, \quad (31)$$

where m_q ($m_{\tilde{q}}$) is the quark (squark) mass and $F(c) = 7.07 \times 10^{-5}$ and $F(b) = F(\tilde{b}) = 2.43 \times 10^{-5}$, see Eq. (34). The leptonic width is a small fraction of the total width. Experimentally, for narrow resonances (whose width is much smaller than the beam-energy spread) one measures the integrated area of the resonance cross-section. For a Breit-Wigner-type resonance this is connected to the leptonic width by the formula [88]

$$\int \sigma_{res} dE = \frac{2\pi^2(2J+1)}{M_V^2} \frac{\Gamma_e \Gamma_h}{\Gamma}, \quad (32)$$

and $\Gamma_h/\Gamma \simeq 1$ if the hadronic width predominates. The formula for the leptonic widths of the pseudoscalar $c\bar{b}$ quarkonium reads

$$\Gamma_{\tau\nu_\tau} = \frac{G^2}{8\pi} f_{B_c}^2 |V_{cb}|^2 M_{B_c} m_\tau^2 \left(1 - \frac{m_\tau^2}{M_{B_c}^2}\right)^2, \quad (33)$$

where G is the Fermi constant, $V_{cb} \approx 0.04$ is the element of the Cabibbo-Kobayashi-Masakawa matrix and the decay constant f_{B_c} is given by the formula (cf., e.g., Ref. [33] and references therein)

$$f_{B_c}^2 = \frac{12 |\psi(0)|^2}{M_{B_c}^2} \overline{C}^2(\alpha_s), \quad (34)$$

where $\overline{C}(\alpha_s)$ is QCD correction factor. Formally this decay constant is defined in terms of the element of the axial weak current

$$\langle 0 | A_\mu(0) | B_c(q) \rangle = i f_{B_c} V_{cb} q_\mu. \quad (35)$$

The QCD correction factor is

$$\overline{C}(\alpha_s) = 1 - \frac{\alpha_s(\mu_{cb}^2)}{\pi} \left[2 - \frac{m_b - m_c}{m_b + m_c} \ln \frac{m_b}{m_c} \right]. \quad (36)$$

With our parameters $\overline{C}(\alpha_s) \approx 0.905$ and since this is rather close to unity, we use it without trying to estimate the higher order terms.

Let us note the convenient relation

$$f_{B_c}^2 = \frac{27 \mu_{cb}}{8 \pi \alpha_s (4 \mu_{cb}^2)} \frac{m_b + m_c}{M_{B_c}} \overline{C}^2(\alpha_s) \Delta E_{\text{HF}}, \quad (37)$$

which for our values of the parameters yields

$$f_{B_c} = 65.2 \sqrt{\frac{6199}{M_{B_c}}} \sqrt{\Delta E_{\text{HF}}}, \quad (38)$$

where all the parameters are in suitable powers of MeV.

B. Production Through Z^0 Decay

We compute first the width of squarkonium $J^{PC} = 0^{++}$. The 0^{++} state decays almost entirely into two gluons and $q\bar{q}$ pairs. Explicit calculations give [89]

$$\Gamma(0^{++} \rightarrow gg) = \left(\frac{16 \pi \alpha_s(m^2)}{3m} \right)^2 |\psi(0)|^2, \quad (39)$$

with $m = m_q$ ($m_{\bar{q}}$) is the constituent mass of quark (squark) of the bound state system and

$$\Gamma(0^{++} \rightarrow q\bar{q}) = \left(\frac{512 \pi \alpha_s(m^2)}{9m^2} \right) \frac{4R^2}{(1 + 4R^2)^2} |\psi(0)|^2, \quad (40)$$

with $R = \tilde{m}/m$, $\tilde{m} = m_g$ ($m_{\tilde{g}}$) is the gluon (gluino) mass. In conventional quarkonium, one can extract the value of $|\psi(0)|^2$ from experimental data on the leptonic width of the quarkonium state (except, of course, for toponium). We choose to use a tentative input for $|\psi(0)|^2$ obtained by a Coulomb like potential ($1/r$ gluonic behaviour), see Eq. (3). This is justified, since as the mass of the constituents of the bound state system goes higher (high mass regime), the short-range forces should approach the Coulomb like interaction whereas the confining linear potential should be negligible in the short distance limit. We shall stick to a coulombic wavefunction, namely, [82-84]

$$|\psi(0)|^2 = \frac{1}{\pi} \left(\frac{m\alpha_s(m^2)}{3} \right)^3. \quad (41)$$

The relative branching ratio of Eqs. (39) and (40) is

$$B = \frac{\Gamma(0^{++} \rightarrow gg)}{\Gamma(0^{++} \rightarrow q\bar{q})}. \quad (42)$$

Grifols and Méndez [89] computed $\Gamma(0^{++} \rightarrow \tilde{g}\tilde{g}) \approx 0.6$ MeV and $B > 3/8$ for $\alpha_s(m_{\tilde{q}}^2) \approx 0.1609$, $m_{\tilde{q}} \approx 65$ GeV and $\tilde{m} = m_{\tilde{g}} \geq 87$ GeV or $\tilde{m} \leq 12$ GeV.

IV. OBSERVED ORDINARY QUARK AND PREDICTED EXOTIC SQUARK SPECTROSCOPY

The levels are labeled by S, P, D , corresponding to relative orbital angular momentum $L = 0, 1, 2$ between quark and antiquark. (No candidates for $L \geq 3$ states have been seen yet.) The spin of the quark and antiquark can couple to either $S = 0$ (spin singlet) or $S = 1$ (spin triplet) states. The parity of quark-antiquark or squark-anti-squark states with orbital angular momentum L is $P = (-1)^{L+1}$, the charge-conjugation is $C = (-1)^{L+S}$. Thus, $L = 0$ states can be 1S_0 or 3S_1 ; $L = 1$ states can be 1P_1 or $^3P_{0,1,2}$; $L = 2$ states can be 1D_2 or $^3D_{1,2,3}$. The radial quantum number is denoted by n [55].

The experimentally clear spectrum of relatively narrow states below the open-charm DD threshold of 3730 MeV can be identified with the 1S, 1P and 2S $c\bar{c}$ levels predicted by potential models, which incorporates a color Coulomb term at short distances and a linear confining term at large distances [50,51].

A recent interest in charmonium spectroscopy [90] is revived because of the recent discovery of the long missing $\eta'_c(2^1S_0)$ state of binding mass 3638 ± 4 MeV by the Belle Collaboration [91,92], which has since then been confirmed by BABAR [93] and has also been

observed by CLEO in $\gamma\gamma$ collisions [94]. The observation of the 2^3P_2 state with binding mass 3929 ± 5 MeV. The reported 1^1P_1 h_c signal is in the decay chain $\psi' \rightarrow \pi^0 h_c$, $h_c \rightarrow \gamma \eta_c$. The masses found in two different inclusive analysis were 3524.8 ± 0.7 MeV and 3524.4 ± 0.9 MeV (with an estimated systematic error of ~ 1 MeV) in exclusive decay (with six different identified η_c final states.) Additional interest in $c\bar{c}$ spectroscopy has followed the discovery of the remarkable $X(3872)$ by Belle [95] and CDF [96] in B decays to $J/\psi\pi^+\pi^-$; assuming that this is a real resonance rather than a threshold effect, the $X(3872)$ is presumably either a DD^* charmed meson molecules [97-99] or a narrow $J = 2$ D -wave $c\bar{c}$ state [100,101]. Very recent observations of the $X(3872)$ in $\gamma J/\psi$ and $\omega J/\psi$ by Belle support 1^{++} DD^* molecule assignment [102,103]. Experimental activity in the spin-singlet P -wave, with recent reports of the observation of exclusive 1^1P_J h_c state by CLEO measurement of mass 3524.65 ± 0.55 MeV [104,105]. The $\psi(3770)$ is generally assumed to be 3D_1 $c\bar{c}$ state, perhaps with a significant 2^3S_1 component [106]. The four known $c\bar{c}$ states above the DD threshold, $\psi(3770)$, $\psi(4040)$, $\psi(4159)$ and $\psi(4415)$ are of special interest because they are easily produced at e^+e^- machines. The mass $\psi(4040)$ which is a very interesting case for the study of strong decays. The 1^{--} $\psi(4159)$ is 2^3D_1 $c\bar{c}$ assignment. The final known above DD threshold is the 1^{--} $\psi(4415)$ has the assignment 4^3S_1 .

On the other hand, the searched pseudoscalar bottomonium $\eta_b(1^1S_0)$ meson, the new observed meson $\Upsilon(4^3S_1) = 10579.4 \pm 1.2$ MeV and the 1^3D_J (probably all or mostly $J = 2$) state with bound mass 10161.1 ± 1.7 MeV [55] and the pseudoscalar charmed bottom meson with an averaged experimental mass $B_c(0^{-+}) = 6286 \pm 5$ MeV by Yao *et al.* have also revived this interest [55].

We first apply this model described above to the ordinary quarkonia known through the SM. Consequently, we calculate the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}(b\bar{c})$ quarkonium binding mass spectra in close agreement with above up-to-date experimental findings. The theoretically calculated quarkonium binding masses together with their S-states hyperfine splittings are listed in Tables 1-2. The hyperfine mass splittings of the $1S$ $c\bar{b}$ state is predicted by the present potential model and other models are listed in Table 3. The calculated binding masses are found to be in close agreement to the recently observed ones. This allows us to extend this study to the unknown spectra of the squarks to predict their binding masses in a unified way. Further, in Tables 1-3, all hyperfine splitting calculations of the potential model try to reproduce the old experimental values, while the lattice calculations and perturbative

QCD favor the new values. No confirmed experimental data to check these predictions are available yet. The experimental finding $\Delta_{\text{HF}}(2\text{S}, c\bar{c}) = \psi'(2^3\text{S}_1) - \eta'_c(2^1\text{S}_0) \approx 48.093 \pm 3.97$ MeV is found to be lower than the calculated values from the potential models [19,26]. In all cases, where comparison with the other models are significantly smaller than the splittings found by Eichten and Quigg [71] and Gupta and Johnson [107]. The QCD sum rules [108,109] finds the hyperfine splitting of the bottomonium $\Delta_{\text{HF}}(1\text{S}, b\bar{b}, \text{theory}) = 63_{-51}^{+29}$ MeV with the central value agrees well to several MeV with expectation. However, the uncertainty is too large to distinguish between the potential models. A lattice calculation [110] gives the hyperfine splitting $\Delta_{\text{HF}}(1\text{S}, b\bar{b}, \text{theory}) = 60$ MeV with a large uncertainty. The central value seems to be close to our model, but the uncertainty is big enough to be consistent with all the potential models quoted here. Furthermore, our model predicts nearly an approximate hyperfine splitting for the 1S bottomonium and 2S charmonium as in the other potential models [34,35,49,71], lattice [111,112] and perturbation QCD [113,114].

The level fine and hyperfine splittings in charmonium and bottomonium together with the experimental and other models findings are listed in Table 4. The fine splitting in charmonium is found to be $M_{\psi'(2\text{S})} - M_{J/\psi(1\text{S})} = 597$ MeV. It is within 7.8 MeV from the experimental value. However, the splitting for bottomonium is found to be $M_{\Upsilon'(2\text{S})} - M_{\Upsilon(1\text{S})} = 571$ MeV. It is within 8 MeV from the experimental value.

Motivated by the great success of our earlier applications [33,38-49], we extend this study to produce the binding masses of the sbottom with anti-sbottom and heavy quark with anti-sbottom. In Table 5, we show the results about $\widetilde{b\bar{b}}$ states. The numerical results about $(q\bar{q})$, $(q = b, c)$ states are shown in Table 6 and Table 7, respectively. Because sbottom is a spin zero particle, the spectrum of the corresponding bound states is simpler than that of $(q\bar{q})$ states. We can see from these tables that when $m_{\widetilde{b}} \approx m_b$, the binding masses of the lowest states $(b\bar{b})$, $(\widetilde{b\bar{b}})$ and $\widetilde{b\bar{b}}$ are very close to each other. This is reasonable because the strong interaction for these states is similar. Also, the leptonic decay widths of the sbottomonium system resonances for different values of the resonance sbottom mass and $e_q = \frac{1}{3}$ are shown in Table 8.

In this analysis, taking the unknown mass of the sbottom quark close to the ordinary bottom quark in the $\widetilde{b\bar{b}}$ pair with $m_{\widetilde{b}} = 4.5$ MeV, we find out the fine splittings for the S-states as follows: 569, 335, 252 MeV and for $m_{\widetilde{b}} = 5.0$ MeV as: 571, 332, 249 MeV. Thus, we remark that the fine splittings of the ordinary $c\bar{c}$, $b\bar{b}$ and $c\bar{b}(b\bar{c})$ quarkonium systems and

the $(q\bar{q})$, $(q = b, c)$ and $\tilde{b}\tilde{b}$ exotic quarkonium states are nearly same [130]. Finally, in general, the potential models seem to reproduce the experimental values much better. This feature would be understandable, since the potential models contain much more input parameters than the lattice or perturbative QCD models.

We point out that Chang *et al.* [78] used a relativistic model to calculate part of these bound states for some $J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}$ corresponding to n^3P_0, n^3D_1, n^3P_2 and n^3D_3 , $n = 0, 1, 2, 3$, respectively, which is entirely different than our c.o.g. calculations. Calculating states like n^3P_1, n^3P_2 and n^3D_2 could help us to compare with other models [130].

V. CONCLUSIONS

We have obtained the bound state masses and hyperfine energy splittings of a flavor-independent static quarkonium potential model for few ordinary quarkonium and scalar squarkonium mesons within the framework of the shifted N -expansion technique for $L = 0, 1, 2$ states (see Tables 1-7). In Tables 1-4, we have shown the effectiveness of the employed static quarkonium potential model in producing the quarkonium bound-state masses to several MeV. Encouraged by this success of a flavor-independent potential model, we have also predicted the bound state masses of few unknown squarkonium systems for low- to- high squark masses (3.0 GeV-150.0 GeV) as shown in Tables 5-7. In finding the unknown squarkonium energy splittings, we have used the quarkonium strong coupling constant $\alpha_s(m^2)$ values to predict the squarkonium energy splittings. Essentially, this is because the type of interaction between two squarks is very similar to the interaction between two quarks (for reviews see, for example, [36] and references therein). Such an interaction is part of the one gluon exchange interaction and is responsible for the mass differences. Apparently, the squarkonium fine and hyperfine splittings are found to be nearly same as their quarkonium counterparts if the squark mass is chosen near the quark mass (i.e., $m_{\tilde{q}} \simeq m_q$). Our conclusions are also consistent with the conclusions made by Ref. [130]. This is reasonable because the strong interaction for these states is similar (see, for example, [130]). The exotic bound states are likely to form bound states in an entirely similar fashion as ordinary quarks form bound state, i.e., quarkonium [89]. Since the same potential model is used for conventional QCD bound states and sbottomonium, so it is obvious, from the present work and Ref. [130],

that the mechanism responsible for the binding of a $b\bar{b}$ couple is the same one responsible for the binding a scalar $\widetilde{b\bar{b}}$. In general, the type of interaction in squarkonium is very similar to that in quarkonium [36].

Furthermore, the calculation of the leptonic decay constant is important in predicting the cross-section. Therefore, we have calculated the leptonic decay constants in Table 8. We conclude that sbottomonium bound states can be detected as resonances at LHC. Above quark masses of about 2 GeV, Γ_e is much higher than the experimental upper limits even for squarks of charge $\frac{2}{3}$. Moreover, Γ_e increases rapidly with the squark mass. In addition, in the supersymmetric front, squarks, sleptons and gauginos do also have a probability to be pairwise produced at LHC [89]. Squarks are likely to form bound states in an entirely similar fashion as conventional quarks from bound state, i.e., quarkonium [89]. The model-dependent squark leptonic decay widths are bigger than the ordinary quarks for $m_{\tilde{q}} < m_b$ and $m_{\tilde{q}} > 10$ GeV and give decay smaller than the energy splitting between bound states which is assumed to be a narrow resonance. As an illustrative example, with given squark mass $m_{\tilde{q}} = 65$ GeV and strong coupling constant $\alpha_s(m_{\tilde{q}}^2) = 0.1609$ in Ref. [89], we solve the Schrödinger equation for the strictly phenomenological scalar potential (3) to obtain the squarkonium binding masses $M(1^3S_1) = 127.745$ GeV and $M(1^1S_0) = 127.577$ GeV for the singlet and triplet ground states, respectively. We, further, calculate the hyperfine splitting energy $\Delta E = 167.72$ MeV and decay constant $\Gamma_e = 0.556$ KeV which is smaller than the energy-beam width. Hence, we conclude that this decay width is a wide resonance. On the other hand, the hidden supersymmetry showing up the squarkonium production through the Z^0 decay into a photon and the lowest lying state of the squarkonium system, the $J^{PC} = 0^{++}$ state (1^1S_0 in spectroscopic notation). The integrated area of the resonance cross-section, Eq. (32), has a large (small) value for light (heavy) squark mass, respectively. For example, from Table 8, we find $0.008 \times 10^{-6} GeV^{-1} \leq \int \sigma_{res} dE \leq 2.56 \times 10^{-6} GeV^{-1}$ for the range 3.0 GeV $\leq m_{\tilde{q}} \leq 150.0$ GeV sbottom quark mass resonance. Hence, the detection of a low-mass squarks at LHC are much favored.

Meanwhile, from the width of squarkonium 0^{++} , Eqs. (39)-(42), we see that the relative branching ratio, $B = 0.662$ for $m_{\tilde{q}} = 60$ GeV and $m_{\tilde{g}} = 87$ GeV by using the Coulomb potential result for $|\psi(0)|^2$ we get $\Gamma(0^{++} \rightarrow \widetilde{g\bar{g}}) = 2.54$ MeV with the strong coupling constant is taken relatively larger than Ref. [89]. For low-mass case $m_{\tilde{q}} = 10$ GeV and $m_{\tilde{g}} = 12$ GeV, we obtain $B = 0.498 \approx 4/8$ which is above $3/8$ and $\Gamma(0^{++} \rightarrow \widetilde{g\bar{g}}) = 0.424$

MeV which is consistent with [89]. This result also favors low bottom squark mass [16].

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TABLE I: Observed and calculated binding mass spectrum of $c\bar{c}$ states (in MeV). ΔX denotes the mass shift of the spin-singlet state (n^1S_0) from the spin-triplet state (n^3S_1).

State	$n(J^{PC})$	Our work	NR [59]	GI [60]	EQ [71]	GJ [107]	MZ [34,35]	PDG [55]
$J/\psi(1^3S_1)$	$1(1^{--})$	3097	3090	3098	3097	3097	3097	3096.916 ± 0.011
$\Delta 1^1S_0$	$1(0^{-+})$	-117	-108	-123	-117	-117	-117	-116.516 ± 1.189
$1P$ (c.o.g)	$1(0, 1, 2)^{++}$	3521	3524.3	3525	3492	3526	3521	3525.3 ± 0.11
$\psi'(2^3S_1)$	$2(1^{--})$	3694	3672	3676	3686	3685	3690	3686.093 ± 0.034
$\Delta 2^1S_0$	$2(0^{-+})$	-65.9	-42	-53	-78	-68	-72	-48.093 ± 3.966
$1D$ (c.o.g) ^a	$1(1, 2, 3)^{--}$	3806	3801	3842	-	-	-	3771.1 ± 2.4
$2P$ (c.o.g) ^b	$2(0, 1, 2)^{++}$	3944	3943	3963.3	-	-	-	3929 ± 5
$\psi(3^3S_1)$	$3(1^{--})$	4078	4072	4100	-	-	-	4040 ± 10
$2D$ (c.o.g)	$2(1, 2, 3)^{--}$	4150	4161.2	4211.4	-	-	-	4159 ± 20
$3P$ (c.o.g)	$3(0, 1, 2)^{++}$	4265	4288.9	4325.3	-	-	-	-
$\psi(4^3S_1)$	$4(1^{--})$	4377	4406	4450	-	-	-	4415 ± 6
$3D$ (c.o.g)	$3(1, 2, 3)^{--}$	4430	-	-	-	-	-	-
$\psi(5^3S_1)$	$5(1^{--})$	4628	-	-	-	-	-	-

^a 1^3D_1 state.

^b 2^3P_2 state.

TABLE II: Observed and calculated binding mass spectrum of $b\bar{b}$ states (in MeV). ΔX denotes the mass shift of the spin-singlet state (n^1S_0) from the spin-triplet state (n^3S_1).

State	$n(J^{PC})$	Our work	EQ [71]	KR [115]	MZ [34,35]	PDG [55]
$\Upsilon(1^3S_1)$	$1(1^{--})$	9460	9464	—	9460	9460.30 ± 0.26
$\Delta 1^1S_0$	$1(0^{-+})$	-57.9	-87	—	-56.7	(160)?
$1P(\text{c.o.g.})$	$1(0, 1, 2)^{++}$	9900	9873	9903	9900	9899.87 ± 0.41
$\Upsilon(2^3S_1)$	$2(1^{--})$	10031	10007	—	10023	10023.26 ± 0.31
$\Delta 2^1S_0$	$2(0^{-+})$	-23.2	-44	—	-28	—
$1D(\text{c.o.g.})$ (1^3D_J state) ^a	$1(1, 2, 3)^{--}$	10155	10127	10156	10155	10161.1 ± 1.7
$2P(\text{c.o.g.})$	$2(0, 1, 2)^{++}$	10261	10231	10259	10260	10260.237 ± 0.56
$\Upsilon(3^3S_1)$	$3(1^{--})$	10364	10339	—	10355	10355.2 ± 0.5
$2D(\text{c.o.g.})$	$2(1, 2, 3)^{--}$	10438	—	10441	10438	—
$3P(\text{c.o.g.})$	$3(0, 1, 2)^{++}$	10527	—	10520	10525	—
$\Upsilon(4^3S_1)$	$4(1^{--})$	10614	—	—	—	10579.4 ± 1.2
$3D(\text{c.o.g.})$	$3(1, 2, 3)^{--}$	10666	—	—	—	—
$\Upsilon(5^3S_1)$	$5(1^{--})$	10820	—	—	—	—

^aProbably all or mostly $J = 2$.

TABLE III: The calculated $\bar{b}c$ binding masses of the lowest S-states and its splitting compared with the other authors (in MeV).

Work	$M_{B_c}(1^1S_0)^a$	$M_{B_c^*}(1^3S_1)$	Δ_{1S}
PDG (Yao et al.) [55]	6286 ± 5	—	—
Our work	6290.8	6349	58.2
Motyka and Zalewski [34,35]	6291	6349	58
Eichten and Quigg [71]	6264	6337	73
Colangelo and Fazio [117]	6280	6350	70
Chabab [109]	6250 ± 200	—	—
Baker et al. [118]	6287	6372	85
Roncaglia et al. [119]	6255	6320	65
Godfrey et al. [120]	6270	6340	70
Bagan et al. [121]	6255 ± 20	6330 ± 20	75
Brambilla et al. [122]	—	6326_{-9}^{+29}	60
Baldicchi and Prospero [85-87]	$6194 \sim 6292$	$6284 \sim 6357$	$65 \leq \Delta_{1S} \leq 90$
SLET [49] ^b	6253_{-6}^{+13}	6328_{-9}^{+7}	$68 \leq \Delta_{1S} \leq 83$
SLET [49] ^c	6258_{-11}^{+8}	6333_{-14}^{+2}	—
Chen and Kuang [61,62]	6310	6355	45
Gershtein et al. [123]	6253	6317	64
Gupta and Johnson [107]	6267	6308	41

^aThe averaged observed mass.

^bAveraging over the five values in Table 1 of [41].

^cWe treat results of [71] in the same manner.

TABLE IV: Level hyperfine and fine splittings in charmonium and bottomonium (in MeV).

Level splitting	Our work	[71] ^a	[34,35] ^a	[124-127] ^a	[128] ^a	[111] ^b	[129] ^b	[113,114] ^c	PDG [55]
$\Delta_{HF}^{(c\bar{c})}(2S) = M_{\psi'(2S)} - M_{\eta'_c(2S)}$	66	78	72	98	92	43	—	38	48.093 ± 3.966
$\Delta_{HF}^{(b\bar{b})}(1S) = M_{\Upsilon(1S)} - M_{\eta_b(1S)}$	58	87	57	60	45	—	51	44	(160)?
$\Delta_{HF}^{(b\bar{b})}(2S) = M_{\Upsilon'(2S)} - M_{\eta'_b(2S)}$	23	44	28	30	28	—	—	21	—
$\Delta_F^{(c\bar{c})} = M_{\psi'(2S)} - M_{J/\psi(1S)}$	597								589.177 ± 0.023
$\Delta_F^{(c\bar{c})} = M_{\psi'(3S)} - M_{\psi'(2S)}$	384								353.907 ± 9.966
$\Delta_F^{(b\bar{b})} = M_{\Upsilon'(2S)} - M_{\Upsilon(1S)}$	571								562.96 ± 0.05
$\Delta_F^{(b\bar{b})} = M_{\Upsilon'(3S)} - M_{\Upsilon'(2S)}$	333								331.94 ± 0.19
$\Delta_F^{(b\bar{b})} = M_{\Upsilon'(4S)} - M_{\Upsilon'(3S)}$	250								224.2 ± 0.7

^aPotential model.

^bLattice.

^cPerturbative QCD.

TABLE V: The calculated binding energy masses of $\bar{b}\bar{b}$ pair (in MeV).

State	$m_{\bar{b}}^a$	nS	nP^b	nD^c	$m_{\bar{b}}^a$	nS	nP	nD
$n = 1$	3.0	6033	6455	6718	3.5	6976	7402	7661
$n = 2$		6601	6835	7021		7543	7775	7958
$n = 3$		6950	7119	7267		7886	8053	8198
$n = 4$		7216	7352	7477		8146	8280	8402
$n = 1$	4.0	7925	8356	8613	4.5	8880	9316	9572
$n = 2$		8493	8724	8904		9449	9679	9858
$n = 3$		8831	8997	9139		9784	9948	10088
$n = 4$		9087	9219	9339		10036	10167	10284
$n = 1$	5.0	9839	10280	10535	5.5	10800	11248	11501
$n = 2$		10410	10640	10817		11375	11604	11780
$n = 3$		10742	10905	11043		11705	11866	12003
$n = 4$		10991	11120	11236		11951	12079	12193
$n = 1$	6.0	11764	12219	12471	8.0	15639	16121	16373
$n = 2$		12343	12572	12747		16236	16468	16639
$n = 3$		12671	12831	12967		16560	16719	16851
$n = 4$		12915	13041	13155		16797	16921	17031
$n = 1$	10.0	19533	20044	20298	20.0	39120	39793	40066
$n = 2$		20152	20388	20558		39879	40142	40314
$n = 3$		20476	20634	20764		40215	40379	40506
$n = 4$		20709	20831	20938		40442	40563	40666
$n = 1$	40.0	78472	79496	79821	60.0	117887	119278	119663
$n = 2$		79559	79884	80072		119329	119717	119925
$n = 3$		79942	80126	80259		119766	119973	120114
$n = 4$		80179	80307	80410		120019	120157	120264

^aThe mass is in GeV .

^bThe calculated c.o.g. binding energy mass for states $n(0)^{++}, n(1)^{++}$ and $n(2)^{++}$.

^cThe calculated c.o.g. binding energy mass for states $n(1)^{--}, n(2)^{--}$ and $n(3)^{--}$.

TABLE VI: The calculated binding energy masses of $c\bar{b}$ pair (in MeV).

State	$m_{\bar{b}}^a$	nS	nP	nD	$m_{\bar{b}}^a$	nS	nP	nD
$n = 1$	4.0	5562	5982	6254	4.5	6052	6471	6743
$n = 2$		6140	6381	6576		6629	6869	7063
$n = 3$		6504	6682	6837		6992	7168	7323
$n = 4$		6785	6929	7061		7271	7415	7546
$n = 1$	4.8	6346	6766	7037	4.9	6445	6864	7135
$n = 2$		6923	7162	7356		7021	7261	7454
$n = 3$		7285	7461	7616		7383	7559	7714
$n = 4$		7564	7707	7838		7662	7805	7935
$n = 1$	5.0	6543	6963	7233	5.5	7036	7456	7726
$n = 2$		7119	7359	7552		7611	7851	8044
$n = 3$		7481	7657	7812		7972	8148	8302
$n = 4$		7760	7903	8033		8250	8393	8523
$n = 1$	6.0	7530	7950	8219	8.0	9513	9933	10201
$n = 2$		8105	8344	8536		10086	10324	10515
$n = 3$		8465	8640	8794		10444	10618	10771
$n = 4$		8742	8884	9014		10719	10860	10989
$n = 1$	10.0	11502	11922	12189	20.0	21479	21900	22166
$n = 2$		12074	12312	12503		22050	22286	22476
$n = 3$		12431	12605	12757		22404	22577	22727
$n = 4$		12705	12846	12974		22676	22815	22942
$n = 1$	40.0	41467	41888	42153	60.0	61463	61884	62149
$n = 2$		42037	42273	42462		62033	62269	62457
$n = 3$		42390	42562	42712		62385	62557	62706
$n = 4$		42660	42799	42925		62655	62794	62920

^aThe mass is in GeV .

TABLE VII: The calculated binding energy masses of $b\bar{b}$ pair (in MeV).

State	m_b^a	nS	nP	nD	$m_{\bar{b}}^a$	nS	nP	nD
$n = 1$	4.0	8695	9130	9385	4.5	9170	9608	9863
$n = 2$		9263	9494	9673		9740	9970	10148
$n = 3$		9599	9763	9904		10074	10238	10377
$n = 4$		9852	9983	10101		10325	10455	10572
$n = 1$	4.8	9458	9897	10152	4.9	9554	9994	10248
$n = 2$		10028	10258	10435		10124	10354	10531
$n = 3$		10361	10524	10663		10457	10620	10759
$n = 4$		10611	10741	10857		10707	10836	10953
$n = 1$	5.0	9650	10090	10345	5.5	10131	10575	10829
$n = 2$		10221	10450	10627		10704	10933	11110
$n = 3$		10553	10716	10855		11035	11197	11336
$n = 4$		10803	10932	11048		11283	11412	11528
$n = 1$	6.0	10616	11061	11315	8.0	12567	13022	13274
$n = 2$		11189	11419	11594		13145	13375	13550
$n = 3$		11520	11681	11819		13474	13634	13770
$n = 4$		11767	11895	12010		13717	13844	13957
$n = 1$	10.0	14534	14995	15247	20.0	24457	24935	25187
$n = 2$		15117	15347	15520		25051	25283	25454
$n = 3$		15444	15603	15738		25376	25534	25667
$n = 4$		15685	15811	15923		25613	25737	25847
$n = 1$	40.0	44410	44900	45153	60.0	64393	64888	65140
$n = 2$		45013	45246	45417		64999	65233	65404
$n = 3$		45337	45495	45627		65323	65481	65613
$n = 4$		45572	45696	45805		65558	65681	65789

^aThe mass is in GeV .

TABLE VIII: Leptonic widths of the sbottomonium system resonances for different values of the resonance sbottom mass and $e_q = \frac{1}{3}$.

State	$m(\text{GeV})$	$\Gamma_e(\text{KeV})$	$m(\text{GeV})$	$\Gamma_e(\text{KeV})$	$m(\text{GeV})$	$\Gamma_e(\text{KeV})$	$m(\text{GeV})$	$\Gamma_e(\text{KeV})$
$n = 1$	2.0	1.715	2.5	1.635	3.0	1.571	3.5	1.523
$n = 2$		0.694		0.657		0.620		0.587
$n = 3$		0.464		0.445		0.423		0.401
$n = 1$	4.0	1.487	4.5	1.461	4.8	1.450	4.9	1.447
$n = 2$		0.557		0.530		0.516		0.512
$n = 3$		0.381		0.362		0.352		0.349
$n = 1$	5.0	1.444	5.5	1.433	10.0	1.504	20.0	2.032
$n = 2$		0.507		0.487		0.381		0.315
$n = 3$		0.345		0.330		0.244		0.174
$n = 1$	40.0	3.405	60.0	4.869	100.0	7.863	150.0	11.641
$n = 2$		0.324		0.380		0.533		0.749
$n = 3$		0.138		0.131		0.142		0.172