An analysis of $f_0 - \sigma$ mixing in light cone QCD sum rules

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Abstract

We investigate $f_0 - \sigma$ mixing in the framework of light cone QCD sum rules and by employing the experimental results about the decay widths and the masses of these scalar mesons we estimate the scalar mixing angle by using σ meson data and f_0 meson data. The two values we thus obtain of the scalar mixing angle are not entirely consistent with each other, possibly indicating that the structure of these mesons cannot be simple quark-antiquark states.

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The light scalar mesons have been the subject of continuous interest in hadron spectroscopy. In addition to well established isoscalar $f_0(980)$ and isovector $a_0(980)$ scalar mesons, the recent experiments and theoretical analysis find evidence for the existence of an isoscalar $\sigma(600)$ [1], and an isodoublet $\kappa(900)$ scalar mesons [2]. Although the structure of these light scalar mesons have not been unambiguously determined yet [3], the possibility may be suggested that these nine scalar mesons below 1 GeV may form a scalar SU(3) flavor nonet [4]. In this case, one would expect a mixing in the scalar sector similar to that occurs in the pseudoscalar sector. One thus expresses the physical $\sigma(600)$ and $f_0(980)$ states as linear combinations of isoscalar $s\overline{s}$ and $(u\overline{u}+d\overline{d})/\sqrt{2}$ states as

$$\sigma = \cos\theta_s \; \frac{u\overline{u}+d\overline{d}}{\sqrt{2}} - \sin\theta_s \; s\overline{s}$$
$$f_0 = \sin\theta_s \; \frac{u\overline{u}+d\overline{d}}{\sqrt{2}} + \cos\theta_s \; s\overline{s} \quad , \tag{1}$$

where θ_s is the scalar mixing angle.

In this work, we estimate the scalar mixing angle θ_s by employing light cone QCD sum rules. The method has been used for the calculation of hadronic coupling constants and for the studies of hadronic properties [5]. We study the hadronic decays $f_0 \to \pi\pi$ and $\sigma \to \pi\pi$ within the framework of this method and by utilizing the experimental data about these decays and about the masses and the widths of f_0 and σ mesons we deduce the scalar mixing angle θ_s.

In order to study $S\pi\pi$-vertex, where S denotes the scalar meson f_0 or σ, we consider the two point correlation function

$$T_\mu(p,q) = i\int d^4x \; e^{ip\cdot x} < \pi^+(q)|T\{j_\mu^{\pi^-}(x) j^S(0)\}|0> \quad , \tag{2}$$

where p and $j_\mu^{\pi^-}$ are the four-momentum and the interpolating current for π^- meson, j^S is the interpolating current for scalar f_0 or σ meson, and q is the four-momentum of π^+ state. The interpolating currents in terms of quark fields are the axial vector $j_\mu^{\pi^-}(x) = \overline{u}(x)\gamma_\mu\gamma_5 d(x)$ for pion, and the scalar currents $j^{f_0}(x) = (\sin\theta_s/\sqrt{2})[\overline{u}(x)u(x) + \overline{d}(x)d(x)] + \cos\theta_s \; \overline{s}(x)s(x)$ for f_0 meson and $j^\sigma(x) = (\cos\theta_s/\sqrt{2})[\overline{u}(x)u(x) + \overline{d}(x)d(x)] - \sin\theta_s \; \overline{s}(x)s(x)$ for σ meson. The scalar current j^S for f_0 and σ meson is assumed to have a non-vanishing matrix element between the vacuum and a scalar meson state, $< S \mid j^S \mid 0 >= \lambda_S$, where λ_S is called the overlap amplitude. The overlap amplitude λ_σ for σ meson [6] and the overlap amplitude λ_{f_0} for f_0 meson [7] were previously determined by using two-point QCD sum rules method.

The correlation function $T_\mu(p,q)$ can be written in terms of two independent invariant functions T_1 and T_2 by using Lorentz invariance as

$$T_\mu(p,q) = iT_1\left((p+q)^2,\; p^2\right)p_\mu + T_2\left((p+q)^2,\; p^2\right)q_\mu \quad . \tag{3}$$

We consider the invariant function $T_1((p+q)^2,\; p^2)$. In order to construct the theoretical part of the sum rule in terms of QCD degrees of freedom we calculate the function T_1 by evaluating the correlation function T_μ in the deep Euclidean region, where $(p+q)^2$ and p^2 are large and negative, as an expansion near the light cone $x^2 = 0$. This expansion involves matrix elements of non-local quark-gluon operators between pion and vacuum states which defines pion distribution amplitudes of increasing twist. We retain terms up to twist four accuracy since higher twist amplitudes are suppressed by powers of $1/[-(p+q)^2]$ or $1/[-(p)^2]$ and thus are known to make a small contribution [8]. In our calculation we use the full light quark propagator with both perturbative and nonperturbative contributions which is given as [9]

$$\begin{aligned} iS(x,0) &= <0|T\{\overline{q}(x)q(0)\}|0> \\ &= i\frac{\not{x}}{2\pi^2 x^4} - \frac{<\overline{q}q>}{12} - \frac{x^2}{192}m_0^2 <\overline{q}q> \\ &-ig_s\frac{1}{16\pi^2}\int_0^1 du\left\{\frac{\not{x}}{x^2}\sigma_{\mu\nu}G^{\mu\nu}(ux) - 4iu\frac{x_\mu}{x^2}G^{\mu\nu}(ux)\gamma_\nu\right\} + \ldots \ . \end{aligned} \tag{4}$$

We note that the two-point correlation function T_μ, and therefore the invariant function T_1, have contributions coming only from the first terms of the interpolating currents for f_0 or σ mesons. In other words, since the pion does not have any strangeness content only the $(\sin\theta_s/\sqrt{2})(\overline{u}u+\overline{d}d)$ part of the interpolating current for f_0 meson and the $(\cos\theta_s/\sqrt{2})(\overline{u}u+\overline{d}d)$ part of the interpolating current for σ meson make any contribution in the evaluation of time-ordered products of pion and scalar meson currents in Eq. 2. We, therefore, in our calculation of the theoretical part of the sum rule consider the scalar current as $j^S(x) = [\overline{u}(x)u(x)+\overline{d}(x)d(x)]/\sqrt{2}$, and denote the resulting expression for the invariant function T_1 as T_1'. After a straightforward computation and after performing the Fourier transforms the invariant function T_1' is obtained to twist four accuracy as

$$\begin{aligned} T_1'\left((p+q)^2,\ p^2\right) &= \frac{f_\pi M_\pi^2}{2m_q}\int_0^1 du\left\{\left[-\frac{1}{(p+q-uq)^2}\ \varphi_{P\pi}(u) - \frac{1}{3}\ \frac{(p+q-uq)\cdot q}{(p+q-uq)^4}\ \varphi_{\sigma\pi}(u)\right]\right. \\ &\left. + \left[-\frac{1}{(p+uq)^2}\ \varphi_{P\pi}(u) - \frac{1}{3}\ \frac{(p+uq)\cdot q}{(p+uq)^4}\ \varphi_{\sigma\pi}(u)\right]\right\} \\ &+f_{3\pi}\int_0^1 dv\left\{\int D\alpha_i\left[\frac{M_\pi^2}{[p+q(1-\alpha_1-v\alpha_3)]^4}\ \varphi_{3\pi}(\alpha_i) - \frac{M_\pi^2}{[p+q(\alpha_1+v\alpha_3)]^4}\ \varphi_{3\pi}(\alpha_i)\right]\right\} \\ &+f_{3\pi}\int_0^1 2v\ dv\left\{\int D\alpha_i\left[-\frac{M_\pi^2}{[p+q(1-\alpha_1-v\alpha_3)]^4}\ \varphi_{3\pi}(\alpha_i) + \frac{M_\pi^2}{[p+q(\alpha_1+v\alpha_3)]^4}\ \varphi_{3\pi}(\alpha_i)\right]\right\}\ . \end{aligned} \tag{5}$$

In the above expression the functions $\varphi_{\sigma\pi}(u)$ and $\varphi_{p\pi}(u)$ are the twist 3 quark-antiquark pion distribution amplitudes which are defined by the matrix elements [10]

$$<\pi^+(q)|\overline{u}(x)i\gamma_5 d(0)|0> = f_\pi\mu_\pi\int_0^1 du e^{iuq\cdot x}\varphi_{p\pi}(u)\ \ , \tag{6}$$

$$<\pi^+(q)|\overline{u}(x)\sigma_{\mu\nu}\gamma_5 d(0)|0> = i\frac{f_\pi\mu_\pi}{6}\left(1-\frac{M_\pi^2}{\mu_\pi^2}\right)(q_\mu x_\nu - x_\mu q_\nu)\int_0^1 du e^{iuq\cdot x}\varphi_{\sigma\pi}(u)\ \ , \tag{7}$$

where $\mu_\pi = M_\pi^2/2m_q$ with $m_q = (m_u+m_d)/2$ is the twist 3 distribution amplitude normalization factor. Although in the evaluation of the light quark propagator given in Eq. 4 we put $m_u = m_d = 0$, we like to note that m_q in the normalization factor μ_π is obtained by making use of the Gell-Mann-Oakes-Renner relation [11] as $m_q = -f_\pi^2 M_\pi^2/4\langle\overline{q}q\rangle$.

We work in the fixed-point gauge $x^\mu A_\mu = 0$, consequently the path-ordered gauge factors $P\ exp\left\{ig_s \int_0^1 d\alpha\ x_\mu A^\mu(\alpha x)\right\}$ which appear in between the quark fields and which assure gauge invariance are not included in the matrix elements [9]. The twist 3 quark-antiquark-gluon pion distribution amplitude $\varphi_{3\pi}(\alpha_i)$ is defined as [10]

$$
\begin{aligned}
< \pi^+(q)|\overline{u}(x) g_s \gamma_5 \sigma_{\alpha\beta} G\mu\nu(vx) d(0)|0 > &= i f_{3\pi}\left[(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})\right] \\
&\times \int D\alpha_i \varphi_{3\pi}(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)} \qquad (8)
\end{aligned}
$$

where $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$. After performing the Borel transformation with respect to the variables $Q_1^2 = -(p+q)^2$ and $Q_2^2 = -p^2$, we obtain the theoretical expression for the invariant function T_1' in the form

$$
\begin{aligned}
T_1'(M_1^2,\ M_2^2) = & \frac{f_\pi M_\pi^2 M^2}{2m_q}\left[\varphi_{p\pi}(u_{01}) + \frac{1}{6}\frac{d\varphi_{\sigma\pi}}{du}\ |_{u=u_{01}}\right] \\
& + f_{3\pi} M_\pi^2 \int_0^{u_{01}} d\alpha_1 \int_{u_{01}-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3}\ \varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\left(2\frac{u_{01}-\alpha_1}{\alpha_3} - 1\right) \\
& + \frac{f_\pi M_\pi^2 M^2}{2m_q}\left[\varphi_{p\pi}(u_{02}) - \frac{1}{6}\frac{d\varphi_{\sigma\pi}}{du}\ |_{u=u_{02}}\right] \\
& - f_{3\pi} M_\pi^2 \int_0^{u_{02}} d\alpha_1 \int_{u_{02}-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3}\ \varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\left(2\frac{u_{02}-\alpha_1}{\alpha_3} - 1\right) \qquad (9)
\end{aligned}
$$

where M_1^2 and M_2^2 are the Borel parameters and

$$
u_{01} = \frac{M_2^2}{M_1^2 + M_2^2} \quad , \quad u_{02} = \frac{M_1^2}{M_1^2 + M_2^2} \quad , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \quad .
$$

The correlation function $T_\mu(p,q)$ satisfies a dispersion relation, therefore we can represent the invariant function T_1 as

$$
T_1\left((p+q)^2,\ p^2\right) = \int\int ds ds'\ \frac{\rho^{had}(s,s')}{[s-(p+q)^2)](s'-p^2)} \quad , \qquad (10)
$$

where the hadronic spectral density $\rho^{had}(s,s')$ receives contributions from the single-particle states and also from higher resonances and continuum states. We therefore saturate this dispersion relation by inserting a complete set of one hadron-states into the correlation function and we consider the single-particle π and scalar meson S, where S denotes f_0 or σ, states. This way we obtain the expression for the invariant function T_1 in the form

$$
\begin{aligned}
T_1\left((p+q)^2,\ p^2\right) = & \frac{< 0 \mid j_\mu^{\pi^-} \mid \pi^+(p) >< \pi^+\pi^- \mid S >< S(p+q)|j^S \mid 0 >}{[(p+q)^2 - M_S^2]\,(p^2 - M_\pi^2)} \\
& + \int_{s_0} ds \int_{s_0'} ds'\ \frac{\rho^{cont}(s,s')}{[s-(p+q)^2](s'-p^2)} \quad , \qquad (11)
\end{aligned}
$$

where now the hadronic spectral density $\rho^{had}(s,s')$ includes the contributions of higher resonances and the hadronic continuum which contribute in a domain $\mathcal{D}$ of the (s,s') plane starting from two thresholds s_0 and s_0'. The matrix element $< \pi^+\pi^- \mid S >$ defines the coupling constant $g_{S\pi\pi}$

$$< \pi^+(q)\pi^-(p) \mid S(p+q) >= g_{S\pi\pi} \tag{12}$$

and the current-particle matrix elements are given as

$$< S(p+q) \mid j^S \mid 0 >= \lambda_S \quad , \tag{13}$$

$$< 0 \mid j_\mu^{\pi^-} \mid \pi^+(p) >= i f_\pi p_\mu \quad . \tag{14}$$

The contribution coming from the continuum can be identified by using global quark-hadron duality [12] with the QCD contribution above the thresholds s_0 and s_0'. This way the pole contribution in which the coupling constant $g_{S\pi\pi}$ appears is isolated. After performing the same Borel transformation that was applied to the theoretical expression for the invariant function T_1' we obtain for the hadronic representation of the same invariant function the result

$$\begin{aligned} T_1(M_1^2, M_2^2) = {} & \lambda_S \; f_\pi \; g_{S\pi\pi} \; e^{-M_S^2/M_1^2} \; e^{-M_\pi^2/M_2^2} \\ & + \int_{s_0} ds \int_{s_0'} ds' \; \rho^{cont}(s,s') \; e^{-s/M_1^2} \; e^{-s'/M_2^2} \quad . \end{aligned} \tag{15}$$

The sum rule for the coupling constant $g_{S\pi\pi}$ then follows in accordance with the QCD sum rules method strategy by equating the expressions $T_1(M_1^2, M_2^2)$ obtained for the invariant function $T_1((p+q)^2, p^2)$ by theoretical calculation given in Eq. 9 and by physical considerations which is given in Eq. 15. For this purpose we note that the expression for T_1' given in Eq. 9 should be multiplied by $\cos\theta_s$ to obtain the sum rule for $g_{\sigma\pi\pi}$ and by $\sin\theta_s$ for the corresponding sum rule for $g_{f_0\pi\pi}$. In order to do this we have to identify the second term in Eq. 15 representing the contribution of the continuum with a part of the term calculated theoretically in QCD. We follow the prescription, for this so called the subtraction of the continuum, for the cases where the Borel parameters corresponding to channels with different mass scales cannot be constrained to be equal [13,14]. This prescription is based on the observation that the distribution amplitudes $\varphi_{p\pi}(u)$ and $\varphi_{\sigma\pi}(u)$ are polynomials in $(1-u)$, therefore in order to compute their contribution in the duality region $\mathcal{D}$ we can write

$$\varphi_{p\pi}(u) + \frac{1}{6}\,\frac{d\varphi_{\sigma\pi}}{du} = \sum_{k=0}^{N} b_k (1-u)^k \quad , \qquad \varphi_{p\pi}(u) - \frac{1}{6}\,\frac{d\varphi_{\sigma\pi}}{du} = \sum_{k=0}^{N} b_k' (1-u)^k \quad . \tag{16}$$

Since the contribution of the twist 3 quark-antiquark-gluon term in Eq. 9 is small, we thus affect the continuum subtraction in the leading twist 3 quark-antiquark term. Therefore, we finally obtain the sum rules for the coupling constants $g_{\sigma\pi\pi}$ and $g_{f_0\pi\pi}$ in the form

$$g_{\sigma\pi\pi} = \cos\theta_s \; g'_{\sigma\pi\pi} \quad , \qquad g_{f_0\pi\pi} = \sin\theta_s \; g'_{f_0\pi\pi} \tag{17}$$

where

$$g'_{S\pi\pi} = \frac{1}{\lambda_S} e^{M_S^2/M_1^2} e^{M_\pi^2/M_2^2} \left\{ \frac{M^2 M_\pi^2}{2m_q} \sum_{k=0}^{N} b_k \left(\frac{M^2}{M_2^2}\right)^k \left[1 - e^{-A} \sum_{i=0}^{k} \frac{A^i}{i!} + e^{-A} \frac{M^2 M_\pi^2}{M_1^2 M_2^2} \frac{A^{(k+1)}}{(k+1)!}\right] \right.$$
$$+ \frac{f_{3\pi} M_\pi^2}{f_\pi} \int_0^{u_{01}} d\alpha_1 \int_{u_{01}-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3} \varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) \left(2\frac{u_{01}-\alpha_1}{\alpha_3} - 1\right)$$
$$+ \frac{M^2 M_\pi^2}{2m_q} \sum_{k=0}^{N} b'_k \left(\frac{M^2}{M_1^2}\right)^k \left[1 - e^{-A} \sum_{i=0}^{k} \frac{A^i}{i!} + e^{-A} \frac{M^2 M_\pi^2}{M_1^2 M_2^2} \frac{A^{(k+1)}}{(k+1)!}\right]$$
$$\left. - \frac{f_{3\pi} M_\pi^2}{f_\pi} \int_0^{u_{02}} d\alpha_1 \int_{u_{02}-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3} \varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) \left(2\frac{u_{02}-\alpha_1}{\alpha_3} - 1\right) \right\} \quad , \quad (18)$$

where $A = s_0/M^2$ with s_0 the smallest continuum threshold.

In the numerical evaluation of the sum rule we use the twist 3 pion distribution amplitudes given by [10]

$$\varphi_{p\pi}(u) = 1 + \left(30\frac{f_{3\pi}}{\mu_\pi f_\pi} - \frac{5}{2}\frac{M_\pi^2}{\mu_\pi^2}\right) C_2^{1/2}(2u-1)$$
$$+ \left[-3\frac{f_{3\pi}\omega_{3\pi}}{\mu_\pi f_\pi} - \frac{27}{20}\frac{M_\pi^2}{\mu_\pi^2}(1+6a_2^\pi)\right] C_4^{1/2}(2u-1) \quad (19)$$

$$\varphi_{\sigma\pi}(u) = 6u\overline{u}\left\{1 + \left[5\frac{f_{3\pi}}{\mu_\pi f_\pi}(1 - \frac{1}{10}\omega_{3\pi}) - \frac{7}{20}\frac{M_\pi^2}{\mu_\pi^2}(1 + \frac{12}{7}a_2^\pi)\right] C_2^{3/2}(2u-1)\right\} \quad (20)$$

$$\varphi_{3\pi}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3^2\left[1 + \frac{\omega_{3\pi}}{2}(7\alpha_3 - 3)\right] \quad (21)$$

where $C_m^k(2u-1)$ are the Gegenbauer polynomials. The overlap amplitudes λ_S were determined previously by employing two-point QCD sum rules method as $\lambda_\sigma = (0.17\pm0.02)\ GeV^2$ [6] and $\lambda_{f_0} = (0.18 \pm 0.02)\ GeV^2$ [7]. We also adopt the values of the parameters in the distribution amplitudes at the renormalization scale 1 GeV as $f_{3\pi}(1GeV) = 0.0035\ GeV^2$, $\omega_{3\pi}(1GeV) = -2.88$, and $a_2^\pi = 0$ [10]. Moreover, we also use the values $\langle\overline{q}q\rangle(1GeV) = -(0.240\ GeV)^3$ and $f_\pi = 0.132\ GeV$ [13]. The mass of the σ meson is taken as $\mathrm{M}_\sigma = (483\pm31)\ MeV$ [1] in which statistical and systematic errors are added in quadrature.

We then study the dependencies of the sum rules for the coupling constants $\mathrm{g}'_{\sigma\pi\pi}$ and $\mathrm{g}'_{f_0\pi\pi}$ on the Borel parameters M_1^2 and M_2^2 by considering independent variations of these parameters. As for the continuum threshold parameter s_0 we use the value $s_0 = 1.2\ GeV^2$ for σ meson [6] and $s_0 = 1.1\ GeV^2$ for f_0 meson [7] that were obtained in the calculation of the overlap amplitudes λ_σ and λ_{f_0} using two-point QCD sum rules method. We find that the sum rules are quite stable for the variation of the Borel parameters in the ranges $2 \le M_1^2 \le 6\ GeV^2$ and $0.4 \le M_2^2 \le 0.8\ GeV^2$. The variation of the coupling constant $\mathrm{g}'_{\sigma\pi\pi}$ as a function of Borel parameters M_1^2 and M_2^2 is shown in Fig. 1 and that of $\mathrm{g}'_{f_0\pi\pi}$ is shown in Fig. 2. The values of the coupling constants thus can be obtained by choosing the value of M_1^2 about at the middle of the stability region as $3.2 \le g'_{\sigma\pi\pi} \le 3.9\ GeV$ and $3.4 \le g'_{f_0\pi\pi} \le 3.8\ GeV$.

On the other hand, the decay widths of σ and f_0 mesons can be calculated in terms of the coupling constants $\mathrm{g}_{\sigma\pi\pi}$ and $\mathrm{g}_{f_0\pi\pi}$ which are defined through the matrix elements given in Eq. 12. The decay width $S \to \pi^+\pi^-$ can thus be obtained in terms of the coupling constant $g_{S\pi\pi}$ as

$$\Gamma(S \to \pi^+\pi^-) = \frac{1}{8\pi M_S^2} g_{S\pi\pi}^2 \; p^* \quad , \tag{22}$$

where $p^* = \sqrt{M_S^2 - (M_{\pi^+} + M_{\pi^-})^2}/2$ is the magnitude of the three-momentum of the either of the pions in the C.M. frame. The experimental width

$$\Gamma^{tot}(S \to \pi\pi) = \frac{3}{2}\; \Gamma(S \to \pi^+\pi^-) = 3\; \Gamma(S \to \pi^0\pi^0) \tag{23}$$

is given as $\Gamma_{f_0} = 40 - 100$ MeV with $M_{f_0} = (980 \pm 10)$ MeV [15] for f_0 meson, and a best fit to the Dalitz plot of $D^+ \to \pi^+\sigma \to \pi^+\pi^+\pi^-$ decay in E791 Collaboration experiment [1] results in $M_\sigma = (483 \pm 31)$ MeV and $\Gamma_\sigma = (338 \pm 48)$ MeV for σ meson. If we use these experimental values we then obtain the coupling constants $g_{S\pi\pi}$ as $g_{\sigma\pi\pi} = (2.6 \pm 0.2)\;\; GeV$ and $g_{f_0\pi\pi} = (1.6 \pm 0.8)\;\; GeV$. The scalar mixing angle θ_s can then be obtained by using Eq. 17 and utilizing on the one hand the values of the coupling constants $g_{S\pi\pi}$ determined from experimental results and on the other hand the values of $g'_{S\pi\pi}$ estimated by light cone QCD sum rules method. This way we determine the scalar mixing angle θ_s from σ meson results as $\theta_s = (41 \pm 11)^o$ and from f_0 meson results as $\theta_s = (27 \pm 13)^o$. These two values do not exclude each other, since they have a small overlap region, but given the large difference in the average values of the scalar mixing angle θ_s determined using σ meson and f_0 meson properties we may assert that our results indicate that the structure of σ and f_0 mesons cannot be simple quark-antiquark states.

It should be mentioned that the previous determinations of the scalar mixing angle θ_s through experimental data coming from charmed meson decays also lead to a somewhat inconsistent picture. The $J/\psi \to f_0(980)\phi$ and $J/\psi \to f_0(980)\omega$ decays were used to estimate the mixing angle [16] and the result $\theta_s = (34 \pm 6)^o$ was obtained, which is consistent with our results. The analysis of the experimental results $D_S^+ \to f_0(980)\pi^+$ and $D_S^+ \to \phi\pi^+$ [17] on the other hand yields the mixing angle in the range $35^o \leq \theta_s \leq 55^0$. In these analysis, it is the mixing angle in f_0 meson state that is determined. However, such a mixing angle implies that σ meson also has a strange component, therefore the decay $D_S^+ \to \sigma\pi^+$ should also occur, but this decay is not observed to make any contribution to the $D_S^+ \to \pi^+\pi^+\pi^-$ decay by the E791 Collaboration [18]. Therefore, this apparent inconsistency about the scalar mixing angle may be taken to indicate that the structure of scalar mesons σ and f_0 are more complicated than simple quark-antiquark states, a conclusion that is also supported by our calculation.

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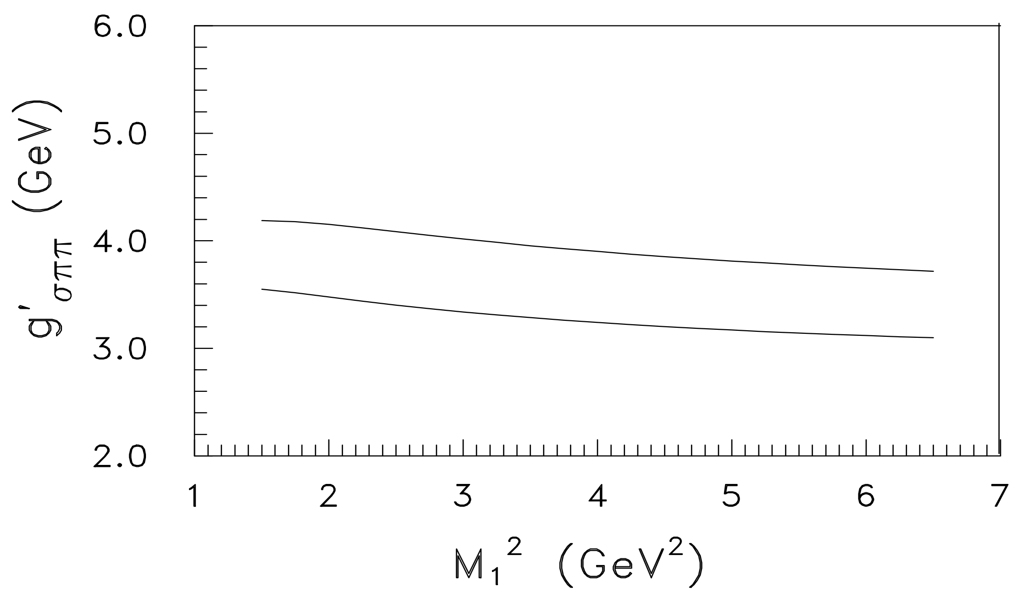

FIG. 1. The coupling constant $g'_{\sigma\pi\pi}$ as a function of the Borel parameter M_1^2 for different values of the Borel parameter M_2^2. The curves denote the limits of the stability region.

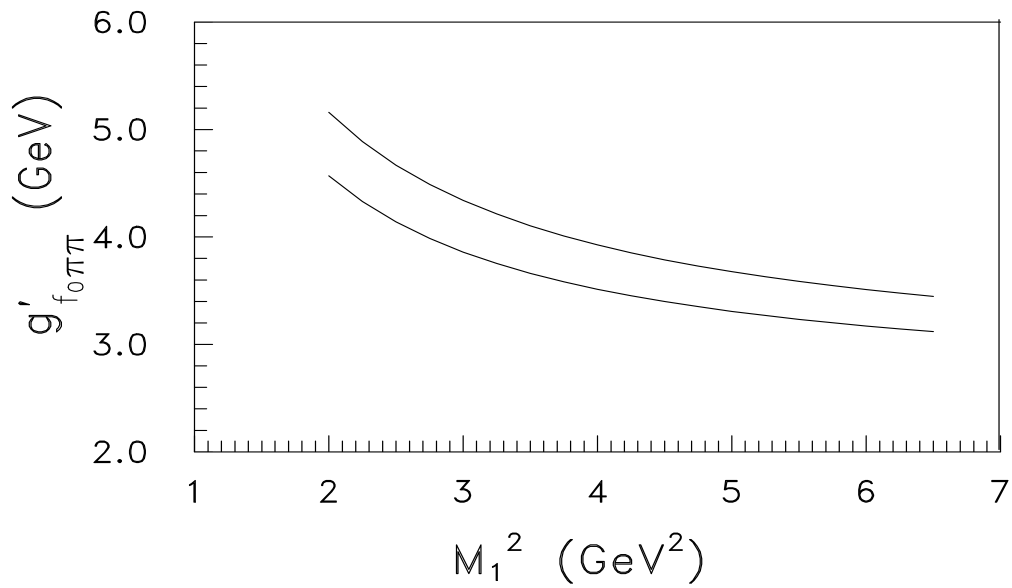

FIG. 2. The coupling constant $g'_{f_0\pi\pi}$ as a function of the Borel parameter M_1^2 for different values of the Borel parameter M_2^2. The curves denote the limits of the stability region.