

Spectra of cylindrical quantum dots: the effect of electrical and magnetic fields together with AB flux field

Sameer M. Ikhdaire^{1,*} Majid Hamzavi^{2,†} and Ramazan Sever^{3,‡}

¹*Physics Department, Near East University,
922022, Nicosia, North Cyprus, Mersin 10, Turkey*

²*Department of Basic Sciences, Shahrood Branch,
Islamic Azad University, Shahrood, Iran*

³*Physics Department, Middle East Technical University, 06531, Ankara, Turkey*

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Abstract

We study the spectral properties of electron quantum dots (QDs) confined in 2D parabolic harmonic oscillator influenced by external uniform electrical and magnetic fields together with an Aharonov-Bohm (AB) flux field. We use the Nikiforov-Uvarov method in our calculations. Exact solutions for the energy levels and normalized wave functions are obtained for this exactly soluble quantum system. Based on the computed one-particle energetic spectrum and wave functions, the interband optical absorption GaAs spherical shape parabolic QDs is studied theoretically and the total optical absorption coefficient is calculated.

Keywords: Parabolic harmonic oscillator, Quantum dots, Electrical and magnetic fields, AB flux field, Light interband transition, Threshold frequency of absorption

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*E-mail: sikhdaire@neu.edu.tr

†E-mail: majid.hamzavi@gmail.com (Corresponding author) Tel.: +98 273 3395270, fax: +98 273 3395270

‡E-mail: sever@metu.edu.tr

I. INTRODUCTION

In the recent years, the subject of quantum dots (QDs) as low-dimensional quantum systems have been the focus of extensive theoretical investigations. Much efforts have recently been done into understanding their electronic, optical and magnetic properties. The application of magnetic field is equivalent to introducing an additional confining potential which modifies the transport and optical properties of conduction-band electrons in QDs. In addition, introducing electric field gives rise to electron redistribution that makes change to the energy of quantum states which experimentally control and modulate the intensity of optoelectronic devices [1,2]. Indeed, it is worthwhile to investigate the influence of electric and magnetic fields on the electrons in QDs. Experimental research is currently made to investigate the nonlinear optical and quantum properties of low-dimensional semiconducting structures for the fabrication purposes and subsequent working of electronic and optical devices [3-14]. A number of works takes the effects of an electric or a magnetic field into account in studying quantum wells, quantum wires and QDs [9-14]. For practical and theoretical reasons, more works analyzing these structures have been focused on the interband light absorption coefficient and magnetic properties with restricted geometries [15] of spherical [16,17,18], parabolic, cylindrical and rectangular [19] QDs and other nanostructures such as superlattices, quantum wires, wells, antidots, well wires and antiwells [20,21,22] in the presence and absence of magnetic field [1,2]. In recent years, the rapid development in semiconductor physics and in nanostructures technology provides wide techniques for the possibility of fabrication of low-dimensional quantum structures like quantum wells, quantum wires and quantum dots which can be treated with high accuracy as two-,one- and zero-dimensional nanostructures, respectively [23].

Harmonic oscillator belongs to the most important and most commonly used physical models. Due to the formal simplicity, it is considered as one of the exactly solvable quantum mechanical problems. It is used to model a wide variety of phenomena ranging from molecular vibrations to the behaviour of quantized fields. The Schrödinger equation for an electron in a uniform magnetic field confined by a harmonic oscillator type potential was solved in 1928 by Fock [24] and Darwin [25]. There are many recent studies on n -particle systems confined in a nonrelativistic harmonic oscillator potential [26] and rotation-vibration spectra of diatomic molecules [27]. Harmonic oscillator potential may be used to describe

spatial confinement of quantum objects, the effects of embedding particles in nano-cavities, in fullerenes, in liquid helium [28,29].

A relativistic harmonic oscillator is far from being trivial and is not unique. Nikolsky [30] and Postepska [31] studied Dirac equation for an electron in the field of a quadratic potential. The eigenvalue problem reduces to a quartic equation with no bound solutions. Toyama and Nogami [32] discussed the relativistic systems which have infinite number of bound states whose energy states are all equally spaced using the inverse scattering method [33]. An approach leading to the Dirac oscillator based on construction of exactly solvable Dirac equation which in the non-relativistic limit reduces to Schrödinger harmonic oscillator equation [34,35].

Recently, interband optical absorption in GaAs spherical shape parabolic QDs in the presence of electrical and magnetic fields was investigated by Atoyan et al [1,2]. They solved the Schrödinger equation for a spinless particle confined by a two-dimensional (2D) cylindrical harmonic oscillator potential. Su and Ma [36] solved the 3D and 1D Dirac equations with both scalar and vector harmonic oscillator potentials. Qiang [37] obtained the bound state energies and normalized wave functions for the Klein-Gordon (KG) and Dirac equations with equal scalar and vector harmonic oscillator potentials. The 2D quantum systems can probe the connection between classical and quantum chaos and has application in a number of surface systems such as atomic corrals [38]. For this reason, Qiang [39] studied the energy formulas and their corresponding normalized wave functions of a 2D relativistic quantum harmonic oscillator system for the first time. In particular, Qiang solved the spinless (spin-0) KG and spin-1/2 Dirac equations with equal scalar and vector harmonic oscillator potentials in 2D space and obtained the normalized wave functions and formulas for energy.

Very recently, we have studied the exact analytical bound state energy eigenvalues and normalized wave functions of the spinless relativistic equation with equal scalar and vector pseudoharmonic interaction under the effect of external uniform magnetic field and AB flux field [40] in the framework of the Nikiforov-Uvarov (NU) method [41,42,43,44]. The non-relativistic limit of our solution is obtained by making an appropriate mapping of parameters. Further, the KG-pseudoharmonic and KG-harmonic oscillator special cases are also treated. Furthermore, we carried out detailed exact energetic spectrum and wave functions of the Schrödinger equation with a pseudoharmonic potential in the presence of external magnetic

and AB flux fields [45]. The low-lying energy levels serve as a base for calculating the corresponding interband light (optical) absorption coefficient and the threshold frequency value of absorption for the given model. In addition, the effect of the temperature on the effective mass is also calculated for GaAs semiconductor.

An attempt is made in this paper to investigate the Schrödinger equation describing a spinless particle confined by a 2D parabolic harmonic oscillator potential when external uniform electrical and magnetic fields are applied together with Aharonov-Bohm (AB) flux field. We obtain the energy spectra and wave functions in the non-relativistic harmonic oscillators. So, we solve the Schrödinger equation in Refs. [1,2] in the presence of AB flux field too. The NU method [41,42,43,44] is used in the present solution. In addition, the interband light absorption coefficient and the threshold frequency value of absorption are calculated.

The structure of the paper is organized as follows. In Sec. 2, we investigate the Schrödinger particle in QDs confinement 2D parabolic harmonic oscillator potential when electrical and magnetic fields together with Aharonov-Bohm (AB) flux fields are applied in the framework of the NU method. The exact analytical expressions for the energy formulas and normalized wave functions are calculated. We also calculate the direct interband light absorption coefficient and the threshold frequency of absorption. The paper ends with a brief concluding remarks in Sec. 3.

II. THEORY AND CALCULATIONS

In this section, we shall consider the solution of spinless Schrödinger equation for the harmonic oscillator potential influenced by electrical, magnetic and AB flux fields. The NU method [41,42,43,44] which has been proved its success is used in our treatment.

A. Bound-state solutions of the 2D harmonic oscillators

Consider a two-dimensional ($2D$) single charged electron, e , with an electronic effective mass μ (for GaAs, $\mu = 0.067m_0$) in the conduction band, confined to a parabolic potential like quantum dots (QDs). We will study the spectral properties with QDs confinement parabolic harmonic oscillator potential influenced by uniform electrical and magnetic fields

together with an Aharonov-Bohm (AB) flux field, applied simultaneously. In cylindrical coordinates, the Schrödinger equation describing a spinless (spin-0) electron in such a quantum system is usually written in the form [46]

$$\left[\frac{1}{2\mu} \left(\vec{p} + \frac{e}{c} \vec{A} \right)^2 - e \vec{\mathcal{E}} \cdot \vec{z} + V_{\text{conf}}(\vec{r}) \right] \psi(\rho, \varphi, z) = E\psi(\rho, \varphi, z), \quad (1)$$

where \vec{p} is the vector momentum with the magnetic field $\vec{H} = \vec{\nabla} \times \vec{A}$ (in the symmetric gauge vector potential $\vec{A} = (A_\rho = A_z = 0, A_\varphi = H\rho/2)$), $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$ is the applied electrostatic field along the z axis and the QDs confinement parabolic potential is taken as [1,2]

$$V_{\text{conf}}(\rho, z) = \frac{1}{2}\mu\omega^2 r^2 = \frac{1}{2}\mu\omega^2 (\rho^2 + z^2), \quad (2)$$

where ω is the frequency of QDs measuring the strength of confinement potential given by

$$\omega \sim \frac{\hbar}{\mu r_0^2}, \quad (3)$$

and r_0 is the oscillator length. The vector potential \vec{A} of the magnetic field may be represented as a sum of two terms, $\vec{A} = \vec{A}_1 + \vec{A}_2$ such that $\vec{\nabla} \times \vec{A}_1 = \vec{H}$ and $\vec{\nabla} \times \vec{A}_2 = 0$, where $\vec{H} = H\hat{z}$ is the applied magnetic field pointing in the positive z direction and is thus parallel to the two plane-parallel electrodes of infinite extent, and \vec{A}_2 describes the additional magnetic flux Φ_{AB} created by a solenoid inserted inside the QDs. Hence, the vector potentials have azimuthal components, in the cylindrical coordinate system, given by [40,45,47,48]

$$\begin{aligned} \vec{A}_1 &= \left(A_{1\rho} = 0, A_{1\varphi} = \frac{H\rho}{2}, A_{1z} = 0 \right), \quad \vec{A}_2 = \left(A_{2\rho} = 0, A_{2\varphi} = \frac{\Phi_{AB}}{2\pi\rho}, A_{2z} = 0 \right), \\ \vec{A} &= \vec{A}_1 + \vec{A}_2 = \left(A_\rho = 0, A_\varphi = \frac{H\rho}{2} + \frac{\Phi_{AB}}{2\pi\rho}, A_z = 0 \right). \end{aligned} \quad (4)$$

The Schrödinger equation (1) with potential (2) in cylindrical coordinates has a form

$$\begin{aligned} & -\frac{\hbar^2}{2\mu} \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \right) \psi(\rho, \varphi, z) - \left(\frac{i\hbar\omega_c}{2} + \frac{i\hbar e\Phi_{AB}}{2\pi\mu c} \frac{1}{\rho^2} \right) \frac{\partial \psi(\rho, \varphi, z)}{\partial \varphi} \\ & + \left(\frac{\mu(\omega_c^2 + 4\omega^2)}{8} \rho^2 + \frac{\mu\omega^2}{2} z^2 + \frac{e^2\Phi_{AB}^2}{8\pi^2\mu c^2} \frac{1}{\rho^2} + \frac{e^2 H\Phi_{AB}}{4\pi\mu c^2} - e\mathcal{E}z \right) \psi(\rho, \varphi, z) = E\psi(\rho, \varphi, z), \\ & E = E_\rho + E_z, \end{aligned} \quad (5)$$

where $\psi = \psi(\rho, \varphi, z)$ is a wave function and $\omega_c = eH/\mu c$ is the cyclotron frequency. .

Let us take the wave function ansatz for an electron as

$$\psi(\rho, \varphi, z) = R(\rho, \varphi)\chi(z), \quad R(\rho, \varphi) = g(\rho)e^{im\varphi}, \quad m = 0, \pm 1, \pm 2, \dots, \quad (6)$$

where m is the magnetic quantum number. Upon inserting the above wave function into Eq. (5), we shall obtain equations whose solutions are $g(\rho)$ and $\chi(z)$ [1,2,49]:

$$g''(\rho) + \frac{1}{\rho}g'(\rho) + \left(\frac{2\mu E_\rho}{\hbar^2} - \frac{\mu\omega_c}{\hbar}(m + \alpha) - \frac{\mu^2\Omega^2}{4\hbar^2}\rho^2 - \frac{(m + \alpha)^2}{\rho^2} \right) g(\rho) = 0, \quad (7)$$

with

$$\Omega = \sqrt{\omega_c^2 + 4\omega^2}, \quad \alpha = \frac{\Phi_{AB}}{\Phi_0}, \quad \Phi_0 = \frac{hc}{e}, \quad (8)$$

where Φ_0 is flux quantum and

$$\chi''(z) + \frac{\mu}{\hbar^2} (2E_z + 2e\mathcal{E}z - \mu\omega^2 z^2) \chi(z) = 0. \quad (9)$$

Consequently, the wave function $g(\rho)$ is required to satisfy the boundary conditions, *i.e.*, $g(0) = 0$ and $g(\rho \rightarrow \infty) = 0$. [1] In order to solve Eq. (7) by means of the NU method, we introduce the new variable $s = \rho^2$, $\rho \in (0, \infty) \rightarrow s \in (0, \infty)$ which recasts Eq. (7) as in the following hypergeometric type differential equation:

$$g''(s) + \frac{2}{(2s)}g'(s) + \frac{1}{(2s)^2} (-\gamma^2 s^2 + \lambda_1^2 s - \beta^2) g(s) = 0, \quad (10)$$

with

$$\lambda_1^2 = \frac{2\mu}{\hbar^2} E_\rho - \frac{\mu\omega_c}{\hbar} (m + \alpha), \quad (11a)$$

$$\beta^2 = (m + \alpha)^2, \quad (11b)$$

$$\gamma = \frac{\mu\Omega}{2\hbar}, \quad (11c)$$

where we have set $g(\rho) \equiv g(s)$. Now, we apply the basic ideas of NU method [41-45]. Comparing Eq. (10) with the standard form of the hypergeometric differential equation:

$$f''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}f'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}f(s) = 0,$$

gives us the polynomials,

$$\tilde{\tau}(s) = 2, \quad \sigma(s) = 2s, \quad \tilde{\sigma}(s) = -\gamma^2 s^2 + \lambda_1^2 s - \beta^2, \quad (12)$$

[1] The solution of Eq. (7) and Eq. (9) are known in [49].

and further substituting the above polynomials into the expression $\pi(s)$,

$$\pi(s) = \frac{\sigma'(s) - \tilde{\tau}(s)}{2} \pm \sqrt{\left(\frac{\sigma'(s) - \tilde{\tau}(s)}{2}\right)^2 - \tilde{\sigma}(s) + k\sigma(s)},$$

gives

$$\pi(s) = \pm \frac{1}{2} \sqrt{\gamma^2 s^2 + (2k - \lambda_1^2)s + \beta^2}. \quad (13)$$

The expression under the square root of the above equation must be the square of a polynomial of first degree. This is possible only if its discriminant is zero and the constant parameter k can be determined from the condition that the expression under the square root has a double zero. Hence, k is obtained as $k_{+,-} = \lambda^2/2 \pm \beta\gamma$. In that case, it can be written in the four possible forms of $\pi(s)$;

$$\pi(s) = \begin{cases} +(\gamma s \pm \beta), & \text{for } k_+ = \frac{1}{2}\lambda_1^2 + \beta\gamma, \\ -(\gamma s \pm \beta), & \text{for } k_- = \frac{1}{2}\lambda_1^2 - \beta\gamma. \end{cases} \quad (14)$$

One of the four possible forms of $\pi(s)$ must be chosen to obtain an energy spectrum formula. Therefore, the most suitable form can be established by the choice:

$$\pi(s) = \beta - \gamma s,$$

for k_- . The trick in this selection is to find the negative derivative of $\tau(s)$ given in

$$\tau(s) = \tilde{\tau}(s) + 2\pi(s),$$

which yields

$$\tau(s) = 2(1 + \beta) - 2\gamma s, \quad \tau'(s) = -2\gamma < 0. \quad (15)$$

In this case, it is necessary to use the quantity $\lambda_n = -n\tau'(s) - \frac{n(n-1)}{2}\sigma''(s)$ to obtain the eigenvalue equation:

$$\lambda_n = 2\gamma n, \quad n = 0, 1, 2, \dots \quad (16)$$

where $n = 0, 1, 2, \dots$ is the radial quantum number. Another eigenvalue equation is obtained via the equality $\lambda = k_- + \pi'$,

$$\lambda = \frac{1}{2}\lambda_1^2 - \gamma(\beta + 1). \quad (17)$$

Thus to find energy equation, we let $\lambda_n = \lambda$ and the result obtained will depend on E_ρ in the closed form:

$$\lambda_1^2 = 2(2n + 1 + \beta)\gamma, \quad (18)$$

Upon the substitution of the terms on the right-hand sides of Eqs. (11a)-(11c) into Eq. (18), we immediately obtain the non-equidistant magneto-optical energy spectrum for the QDs confinement parabolic potential as

$$E_{nm}(\alpha) = E_\rho = \frac{1}{2}\hbar\omega_c(m + \alpha) + \hbar\omega_c \left(n + \frac{|\beta| + 1}{2} \right) \sqrt{1 + 4 \left(\frac{\omega}{\omega_c} \right)^2}, \quad (19)$$

where $|\beta| = |m| + \alpha > 0$ is an integer. It is apparent from Eq. (19) that the electronic energy levels are nondegenerate for all m . We have one set of quantum numbers (n, m, β) for a spinless electron in QDs. Therefore, the energy formula (19) may be readily used to study the thermodynamic properties of quantum structures with QDs confined by the harmonic oscillator potential in the presence and absence of external magnetic field (H) and AB flux field (Φ_{AB}).

Four special cases are of a particular interest:

- In the presence of a strong magnetic field; say , $\omega_c/\omega = 30$ [48], then $\Omega \rightarrow \omega_c \gg \omega$, then $E_{nm}(\alpha) = \hbar\omega_c [n + \alpha + \frac{1}{2}(m + |m| + 1)]$ which is the formula in the presence of magnetic (H) and AB flux (Φ_{AB}) fields. Meanwhile, in the presence of a weak magnetic field; say , $\omega_c/\omega = 3$ [48], then we can resort to Eq. (19).
- If we set $\alpha = 0$, i.e., in the absence of AB flux field and presence of strong magnetic field, we find $E_{nm} = \hbar\omega_c [n + \frac{1}{2}(|m| + m + 1)]$.
- In the absence of magnetic field ($\omega_c = 0$) and an AB flux field ($\alpha = 0$), we find $E_{nm} = \hbar\omega (2n + |m| + 1)$.
- The case $m = 0$ is simply for harmonic oscillator energy spectrum, i.e., $E_n = \hbar\omega (2n + 1)$.

Next, we need to calculate the corresponding wave function for the confinement potential model. We find the first part of the wave function by

$$\phi_m(s) = \exp \left(\int \frac{\pi(s)}{\sigma(s)} ds \right) = s^{|\beta|/2} e^{-\gamma s/2}. \quad (20)$$

Then, the weight function defined by

$$\rho(s) = \frac{1}{\sigma(s)} \exp \left(\int \frac{\tau(s)}{\sigma(s)} ds \right) = s^{|\beta|} e^{-\gamma s}, \quad (21)$$

which gives the second part of the wave function (Rodrigues formula) given by

$$y_{nm}(s) = y^{(n)}(s) = y(s) = \frac{B_n}{\rho(s)} \frac{d^n}{dr^n} [\sigma^n(s)\rho(s)],$$

or alternatively

$$y_{nm}(s) \sim s^{-|\beta|} e^{\gamma s} \frac{d^{n_r}}{ds^{n_r}} (s^{n+|\beta|} e^{-\gamma s}) \sim L_n^{(|\beta|)}(\gamma s), \quad (22)$$

where $L_a^{(b)}(x) = \frac{(a+b)!}{a!b!} F(a, b+1; x)$ is the associated Laguerre polynomial and $F(a, b+1; x)$ is the confluent hypergeometric function. With the formula $g(s) = \phi_m(s)y_{nm}(s)$, we may write the radial wave function as

$$g(\rho) = C_{n,m} \rho^{|m|+\alpha} e^{-\gamma\rho^2/2} F(-n, |m| + \alpha + 1; \gamma\rho^2), \quad (23)$$

and finally the total wave function (6) reads

$$\begin{aligned} R_{n,m}(\rho, \varphi) &= \frac{1}{a^{(1+|m|+\alpha)}} \left[\frac{n!}{\pi 2^{|m|+\alpha+1} (|m| + \alpha + n)!} \right]^{1/2} \rho^{|m|+\alpha} e^{-\rho^2/4a^2} L_n^{(|m|+\alpha)}(\rho^2/2a^2) e^{im\varphi}, \\ &= \frac{a^{-(1+|m|+\alpha)}}{(|m| + \alpha)!} \left[\frac{(|m| + \alpha + n)!}{\pi 2^{|m|+\alpha+1} n!} \right]^{1/2} \rho^{|m|+\alpha} e^{-\rho^2/4a^2} F(-n, |m| + \alpha + 1; \rho^2/2a^2) e^{im\varphi}, \end{aligned} \quad (24)$$

where $a = \sqrt{\hbar/\mu\Omega}$ is the effective length scale. The energy levels (19) with $\alpha = 0$ (i.e., $\Phi_{AB} = 0$) becomes

$$E_{nm} = \frac{1}{2} \hbar \omega_c m + \hbar \Omega \left(n + \frac{|m| + 1}{2} \right), \quad (25)$$

and the wave function (24) becomes

$$R_{n,m}(\rho, \varphi) = \frac{a^{-(1+|m|)}}{|m|!} \left[\frac{(n_\rho + |m|)!}{\pi 2^{|m|+1} n_\rho!} \right]^{1/2} e^{im\varphi} \rho^{|m|} e^{-\rho^2/4a^2} F(-n, |m| + 1; \rho^2/2a^2). \quad (26)$$

which are identical to Eqs. (10) and (8) in Ref. [1], respectively. On the other hand, Eq. (9) can be recasted in the form:

$$\chi''(z) + (-\delta^2 z^2 + \eta^2 z - \epsilon^2) \chi(z) = 0, \quad (27a)$$

$$\delta = \frac{\mu\omega}{\hbar}, \quad \eta = \sqrt{\frac{2\mu\epsilon\mathcal{E}}{\hbar^2}}, \quad \epsilon = \sqrt{-\frac{2\mu E_z}{\hbar^2}}, \quad E_z < 0. \quad (27b)$$

We follow the same procedures of solution by writting

$$\tilde{\tau}(z) = 0, \quad \sigma(z) = 1, \quad \tilde{\sigma}(z) = -\delta^2 z^2 + \eta^2 z - \epsilon^2. \quad (28)$$

In the present case, the polynomial $\pi(z)$ is obtained as

$$\pi(z) = \pm \frac{1}{2} \sqrt{\delta^2 z^2 - \eta^2 z + \epsilon^2 + k}, \quad (29)$$

and thus the two possible forms of $\pi(z)$ are

$$\pi(z) = \begin{cases} +\delta (z - \eta^2/2\delta^2), & \text{for } k = \eta^4/4\delta^2 - \epsilon^2, \\ -\delta (z - \eta^2/2\delta^2), & \text{for } k = \eta^4/4\delta^2 - \epsilon^2. \end{cases} \quad (30)$$

Therefore, the most suitable form can be established by the choice:

$$\pi(z) = -\delta (z - \eta^2/2\delta^2),$$

and $\tau(z)$ is consequently found as

$$\tau(z) = -2\delta z + \eta^2/\delta. \quad (31)$$

A new eigenvalue equation becomes

$$\lambda_{n_z} = 2\delta n_z, \quad n_z = 0, 1, 2, \dots \quad (32)$$

where n_z is the quantum number and another eigenvalue equation is obtained as

$$\lambda = \frac{\eta^4}{4\delta^2} - \epsilon^2 - \delta. \quad (33)$$

Hence, the energy formula reads as

$$E_z = \hbar\omega \left(n_z + \frac{1}{2} \right) - \frac{e^2 \mathcal{E}^2}{2\mu\omega^2}. \quad (34)$$

Next, we calculate the wave function $\chi(z)$. The first part of the wave function is

$$\phi(z) = e^{-\delta(z-\eta^2/2\delta^2)^2/2+\eta^4/8\delta^3}, \quad (35)$$

and the weight function is

$$\rho(z) = e^{-\delta(z-\eta^2/2\delta^2)^2+\eta^4/4\delta^3}, \quad (36)$$

which gives the second part of the wave function:

$$\begin{aligned} \chi_{n_z}(z) &\sim (-1)^{n_z} e^{-\delta(z-\eta^2/2\delta^2)^2/2} H_{n_z} \left[\sqrt{\delta} (z - \eta^2/2\delta^2) \right] \\ &= \frac{1}{\sqrt{2^{n_z} n_z!}} \left(\frac{\mu\omega}{\hbar\pi} \right)^{1/4} e^{-(\mu\omega/\hbar)(z-e\mathcal{E}/\mu\omega^2)^2/2} H_{n_z} \left[\sqrt{\mu\omega/\hbar} (z - e\mathcal{E}/\mu\omega^2) \right], \end{aligned} \quad (37)$$

where $H_{n_z}(x)$ is the Hermite polynomial. As to electronic energy levels, it is the sum of expressions (19) and (34):

$$E_{n,n_z,m}(\alpha, \omega, \omega_c, \mathcal{E}) = \hbar\omega \left[\left(n + \frac{|m| + \alpha + 1}{2} \right) \sqrt{\left(\frac{\omega_c}{\omega} \right)^2 + 4} + \frac{m + \alpha}{2} \left(\frac{\omega_c}{\omega} \right) + n_z + \frac{1}{2} \right]$$

$$-\frac{e^2 \mathcal{E}^2}{2\mu\omega^2}, \quad (38)$$

where $\hbar\omega_c = 0.11571589 H$ (meV) and H is to be in units of Tesla. Additionally, the term $\alpha = \Phi_{AB}/\Phi_0$ reflects the dependence of the electronic levels on the AB flux Φ_{AB} where we take $\alpha = 6$.

As for the wave functions, it is taken as the product of (24) and (37):

$$\begin{aligned} \psi(\rho, \varphi, z) = & \frac{a^{-(1+|\beta|)}}{|\beta|!} \left[\frac{(|\beta| + n_\rho)!}{\pi 2^{|\beta|+1} n_\rho!} \right]^{1/2} e^{im\varphi} \rho^{|\beta|} e^{-\rho^2/4a^2} F(-n_\rho, |\beta| + 1; \rho^2/2a^2) \\ & \times \frac{(b)^{-1/2}}{(\pi)^{1/4} \sqrt{2^{n_z} n_z!}} e^{-(z - e\mathcal{E}/\mu\omega^2)^2/2b^2} H_{n_z} [(z - e\mathcal{E}/\mu\omega^2)/b], \end{aligned} \quad (39)$$

where $b = \sqrt{\hbar/\mu\omega}$. Expressions (19) and (39) obtained above for a charge carrier energy spectrum and wave functions are identical to Eqs. (10) and (11) in Ref. [1] and Eqs. (4) and (3) in Ref. [50], respectively, in the case when $\alpha = 0$.

To show the behaviour of the energy formula (38), we follow Ref. [51] in plotting it versus ω_c/ω under the influence of magnetic and AB flux fields. In figure 1, we plot the eigenenergies (in units $\hbar\omega$) versus the ratio ω_c/ω (a) for various AB field, α (b) for various magnetic quantum number m and (c) for various quantum number n_z . As shown in figure 1a, the ground state $n = n_z = 0$ (singlet state, $m = 0$) under the influence of the AB field leads to the phase transition to the high-lying states $n > 0$. The family of states for various values of $\alpha = 0, 1, 2, 3, 4$ are non-linear in the presence of weak magnetic field $0 < \omega_c/\omega < 5$. In figure 1b, the energy eigenvalues (in units $\hbar\omega$) is found increasing with the increasing magnetic quantum number m when $\omega_c > \omega$. However, when $\omega_c < \omega$, we notice the crossing between $m = 1$ and $m = -1$ states. As shown in figure 1c, the ground state $n = m = 0$ leads to the phase transition to the high-lying states $n > 0$ when the quantum number n_z is increasing. As n_z is increasing we have a family of states for various $n_z = 0, 1, 2, 3, 4$.

B. Interband light absorption coefficient

Expressions (38) and (39), obtained above for electronic energy spectrum and corresponding wave functions in cylindrical QDs influenced by external uniform electrical and magnetic fields along with an AB flux field, allow us to calculate the direct interband light absorption coefficient $K(\bar{\omega})$ in the present system and the threshold frequency of absorption. In case

of strong size quantization in which it is possible to neglect the electron-hole interactions. According to Ref. [52], the light absorption coefficient is

$$\begin{aligned}
K(\bar{\omega}) &= N \sum_{n,m,n_z} \sum_{n',m',n'_z} \left| \int \psi_{n,m,n_z}^e(\rho, \varphi, z) \psi_{n',m',n'_z}^h(\rho, \varphi, z) \rho d\rho d\varphi dz \right|^2 \delta(\Delta - E_{n,m,n_z}^e - E_{n',m',n'_z}^h) \\
&= N \sum_{n,m,n_z} \sum_{n',m',n'_z} \frac{4 \left(\frac{1}{4a_e^2 a_h^2} \right)^{1+|\beta|}}{(|\beta|!)^4} \frac{(|\beta| + n)! (|\beta| + n')! \left(\frac{\mu_e \omega}{\hbar} \right)^{1/2} \left(\frac{\mu_h \omega}{\hbar} \right)^{1/2}}{2^{n_z+n'_z} n! n'_z!} \frac{1}{\pi n_z! n'_z!} \\
&\times \left| \int_0^\infty \rho d\rho \rho^{2|\beta|} e^{-(1/4a_e^2 + 1/4a_h^2)\rho^2} F(-n, |\beta| + 1; \rho^2/2a_e^2) F(-n', |\beta| + 1; \rho^2/2a_h^2) \right|^2 \\
&\times I_{nn'}^2 \delta(\Delta - E_{n,m,n_z}^e - E_{n',m',n'_z}^h), \tag{40}
\end{aligned}$$

where $\Delta = \hbar\bar{\omega} - \varepsilon_g$, ε_g is the width of forbidden energy gap, $\bar{\omega}$ is the frequency of incident light, N is a quantity proportional to the square of dipole moment matrix element modulus, $\psi^{e(h)}$ is the wave function of the electron (hole) and $E^{e(h)}$ is the corresponding energy of the electron (hole).

Now, we use the integrals [53]

$$\int_0^{2\pi} e^{i(m+m')\phi} d\phi = \begin{cases} 2\pi & \text{if } m = -m', \\ 0 & \text{if } m \neq -m', \end{cases} \tag{41}$$

$$\begin{aligned}
\int_0^\infty e^{-\kappa x} x^{\lambda-1} F(-n, \lambda; qx) F(-n', \lambda; q'x) dx &= \Gamma(\lambda) \kappa^{n+n'-\lambda} (\kappa - q)^{-n} (\kappa - q')^{-n'} \\
&\times {}_2F_1 \left(n, n', \lambda; \frac{qq'}{(\kappa - q)(\kappa - q')} \right), \tag{42}
\end{aligned}$$

and

$$\begin{aligned}
I_{nn'} &= \int_{-\infty}^\infty e^{-\left(\frac{\mu_e \omega_e}{2\hbar}\right)\left(z - \frac{e\mathcal{E}}{\mu_e \omega_e^2}\right)^2} e^{-\left(\frac{\mu_h \omega_h}{2\hbar}\right)\left(z + \frac{e\mathcal{E}}{\mu_h \omega_h^2}\right)^2} \\
&\times H_n \left[\left(\frac{\mu_e \omega_e}{\hbar} \right) \left(z - \frac{e\mathcal{E}}{\mu_e \omega_e^2} \right) \right] H_{n'} \left[\left(\frac{\mu_h \omega_h}{\hbar} \right) \left(z + \frac{e\mathcal{E}}{\mu_h \omega_h^2} \right) \right] dz,
\end{aligned}$$

where $\Gamma(x)$ is the Euler-Gamma function and ${}_2F_1(a, b, c; z)$ is the hypergeometric function, to calculate the light absorption coefficient:

$$K(\bar{\omega}) = N \sum_{n,m,\beta} \sum_{n',m',\beta'} P_{n,n'}^\beta Q_{n',n'}^\beta \delta(\Delta - E_{n,m,\beta}^e - E_{n',m',\beta'}^h), \tag{43}$$

where

$$P_{n,n'}^\beta = \frac{1}{(|\beta|!)^2} \left(\frac{\mu_e \omega}{\hbar}\right)^{1/2} \left(\frac{\mu_h \omega}{\hbar}\right)^{1/2} (\gamma \gamma')^{|\beta|+1} \left(\frac{\gamma + \gamma'}{\gamma - \gamma'}\right)^{2(n+n')} \frac{(n + |\beta|)! (n' + |\beta|)!}{n! n'! \pi n_z! n'_z! 2^{n_z+n'_z}}, \quad (44)$$

and

$$Q_{n,n'}^\beta = \left[\left(\frac{2}{\gamma + \gamma'}\right)^{|\beta|+1} {}_2F_1 \left(n, n', |\beta| + 1; -\frac{4\gamma\gamma'}{(\gamma - \gamma')^2} \right) \right]^2, \quad \gamma = \frac{1}{2a_e^2}, \quad \gamma' = \frac{1}{2a_h^2}. \quad (45)$$

Using Eq. (38), we find the threshold frequency value of absorption as

$$\begin{aligned} \hbar\bar{\omega} = & \varepsilon_g + \hbar \left(n + \frac{|m| + \alpha + 1}{2} \right) \left(\frac{e^2 H^2}{\mu^2 c^2} + 4\omega^2 \right)^{1/2} + \hbar \frac{eH}{\mu c} \left(\frac{m + \alpha}{2} \right) + \hbar\omega \left(n_z + \frac{1}{2} \right) - \frac{e^2 \mathcal{E}^2}{2\mu\omega^2} \\ & + \hbar \left(n' + \frac{|m'| + \alpha + 1}{2} \right) \left(\frac{e^2 H^2}{\mu'^2 c^2} + 4\omega^2 \right)^{1/2} + \hbar \frac{eH}{\mu' c} \left(\frac{m' + \alpha}{2} \right) + \hbar\omega \left(n'_z + \frac{1}{2} \right) - \frac{e^2 \mathcal{E}^2}{2\mu'\omega'^2}. \end{aligned} \quad (46)$$

For ground state, we set $n = m = 0$ in the above expression to obtain the threshold frequency of absorption:

$$\begin{aligned} \bar{\omega}_{00} = & \frac{\varepsilon_g}{\hbar} + \left(\frac{\alpha + 1}{2} \right) \left(\frac{e^2 H^2}{\mu^2 c^2} + 4\omega^2 \right)^{1/2} - \frac{e^2 \mathcal{E}^2}{2\hbar\omega^2} \left(\frac{1}{\mu} + \frac{1}{\mu'} \right) \\ & + \left(\frac{\alpha + 1}{2} \right) \left(\frac{e^2 H^2}{\mu'^2 c^2} + 4\omega^2 \right)^{1/2} + \frac{eH\alpha}{2c} \left(\frac{1}{\mu} + \frac{1}{\mu'} \right) + \omega. \end{aligned} \quad (47)$$

We follow Ref. [54] in plotting the threshold frequency of absorption $\bar{\omega}_{00}$ (in units of ε_g) versus the magnetic field strength H and quantum dot size considering various AB magnetic flux values $\alpha = 0, 1, 2, 3$. In figure 2, we plot the variations of threshold frequency of absorption $\bar{\omega}_{00}$ (in units of ε_g) as a function of applied (a) large magnetic field and (b) small magnetic field in unit of $h = (e\hbar H/\mu c \varepsilon_g)$ with $\rho = 89.53$. It is seen from figure 2a (figure 2b) that the dependence of $\bar{\omega}_{00}$ on H is linear (nonlinear) for large (small) applied magnetic fields. The main feature in the application of the AB flux field leads to a family of the phase transition for the ground state $n = 0$, mainly $\alpha = 0, 1, 2, 3$ leads to a phase transitions for the high-lying states $n > 0$. In figure 3, we plot the threshold frequency of absorption $\bar{\omega}_{00}$ (in units of ε_g) as a function of quantum dot size (in unit of $\rho = \sqrt{\varepsilon_g/\omega\hbar} = \sqrt{\mu\varepsilon_g/\hbar^2 r_0}$) (see Eq. (3)) with $h = 0.062$. It is seen in figure 3 that the threshold frequency of absorption decreases when the quantum dot size increasing. The application of AB flux field Φ_{AB} generates a family of state transitions for $\alpha = \Phi_{AB}/\Phi_0 = 0, 1, 2, 3$.

III. CONCLUDING REMARKS

In this work, we have obtained the bound state solutions of the Schrödinger spinless particle in QDs confined to non-relativistic harmonic oscillator in presence of electrical, magnetic and AB flux fields. The electron (hole) energy spectrum and the corresponding wave functions are used to calculate the the interband light absorption coefficient and the threshold frequency of absorption. Also, the energy spectrum of the electron may be used to study the thermodynamic properties of quantum structures with dot in electrical, magnetic and AB flux fields. The electronic energy levels make a shift under the effect of an external electrical field by an amount $\Delta E = -e^2\mathcal{E}^2 / (2\mu\omega^2)$. It explains the Stark splitting quadratic dependence on $\vec{\mathcal{E}}$. The energy levels in the presence of external electrical field of different strengths are nondegenerate. The threshold frequency of absorption $\bar{\omega}_{00}$ rises on the field $\vec{\mathcal{E}}$ by quadratic law and has also more complicated dependence on the magnetic field \vec{H} . Further, it is noticed that the spinless particle (electron) is localized along the z-axis inside the QDs.

In the quantum mechanics there is a relevant relationship between 2D and 3D harmonic oscillator [35] in Schrödinger theory. We find that there are corresponding relationship between 2D and 3D harmonic oscillators, $\rho \leftrightarrow r$ and $|m| \leftrightarrow l + 1/2$.

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FIG. 1: Eigenenergies (in units $\hbar\omega$) versus the ratio ω_c/ω for a) various AB field α , b) various magnetic quantum number m and c) various quantum number n_z .

FIG. 2: The variations of threshold frequency of absorption $\bar{\omega}_{00}$ (in units of ε_g) as a function of applied (a) large magnetic field and (b) small magnetic field (in unit of h).

FIG. 3: The variations of the threshold frequency of absorption $\bar{\omega}_{00}$ (in units of ε_g) as a function of quantum dot size (in unit of ρ).











