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Q–ball formation in the MSSM with explicit CP violation

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Abstract

Q–balls generically exist in the supersymmetric extensions of the standard model. Taking into account the additional sources of CP violation, which are naturally accomodated by the supersymmetric models, it is shown that the Q–ball matter depends additively on individual CP phases, whereas mass per unit charge in the Q–ball depends only on the relative phases. There are regions of the parameter space where there is no stable Q–ball solution in the CP–conserving limit whereas finite CP phases induce a stable Q ball.

1. Introduction

Non-topological solitons, in particular Q–balls, are extended objects with finite mass and spatial extension, and arise in scalar field theories when there is an exact continious symmetry and some kind of attractive interaction, as already classified by Coleman [1].

As first pointed out by Kusenko [2], Q-balls naturally exist in supersymmetric theories thanks to global baryon (B) and lepton (L) number symmetries. Besides, theories with an extended scalar sector can support non–baryonic Q–balls [3]. Q–balls have found applications in modelling several physical processes ranging from leptoquarks to dark matter [4,5].

It is well known that the MSSM has unremovable physical phases which can be identified with the phases of μ parameter and A terms [6]. According to the vacuum stability arguments these phases relax to CP conserving points [7]. However, the same arguments are not sufficient to relax the phases in the minimal extensions [8]. Hence, it is plausible to take these phases finite and look for their effects in low energy processes.

As summarized above, the MSSM predicts the existence of both non–topological solitons and finite CP violation. However, so far the investigations on the Q–ball formation in the MSSM have not dealt with the effects of the CP violation. In this short note we investigate the effects of explicit CP violation in the MSSM on the Q–ball formation. In the next section we discuss this issue in detail using the MSSM scalar potential. We particularly analyze the effects of the phases in the trilinear couplings and the μ parameter. In the last section we summarize the main findings.

2. MSSM with explicit CP violation and Q-balls

The MSSM scalar sector contains two Higgs doublets (with opposite hypercharge) and scalar partners of quarks and leptons. The supersymmetry and gauge symmetry are broken by the soft supersymmetry breaking terms which introduce a number of mass parameters to the potential. Both the Higgsino Dirac mass term μ and the trilinear couplings in soft terms are complex, and they lead to CP violation beyond the CKM matrix already present in the SM. Denoting the neutral components of the Higgs doublets as ϕ_1 and ϕ_2, the MSSM scalar potential for one generation of sfermions reads as:

$$
\begin{aligned}
V_{MSSM} &= \left[(h_u A_u \phi_2 - h_u \mu^* \phi_1^*) \tilde{u}_L \tilde{u}_R^* - (h_d A_d \phi_1 - h_d \mu^* \phi_2^*) \tilde{d}_L \tilde{d}_R^* \right. \\
&\quad \left. - (h_e A_e \phi_1 - h_e \mu^* \phi_2^*) \tilde{e}_L \tilde{e}_R^* + h.c. \right] \\
&\quad + m_{\tilde{Q}}^2 |\tilde{Q}|^2 + m_{\tilde{u}}^2 |\tilde{u}_R|^2 + m_{\tilde{d}_R}^2 |\tilde{d}_R|^2 + m_{\tilde{L}}^2 |\tilde{L}|^2 + m_{\tilde{e}_R}^2 |\tilde{e}_R|^2 \\
&\quad + m_1^2 |\phi_1|^2 + m_2^2 |\phi_2|^2 + |\mu|^2 |\phi_2|^2 + |\mu|^2 |\phi_1|^2 + h_e^2 |\tilde{L}^2| |\tilde{e}_R|^2 \\
&\quad + h_d^2 |\tilde{Q}^2| |\tilde{d}_R|^2 + h_e^2 |\tilde{L}^2| |\tilde{e}_R|^2 + h_u^2 |\tilde{Q}^2| |\tilde{u}_R|^2 + h_e^2 |\phi_1|^2 |\tilde{e}_R|^2 \\
&\quad + h_d^2 |\phi_1|^2 |\tilde{d}_R|^2 + h_e^2 |\phi_1 \tilde{e}_L|^2 + h_u^2 |\phi_2 \tilde{u}_L|^2 + h_d^2 |\phi_1 \tilde{d}_L|^2 \quad (1)
\end{aligned}
$$

plus the D term contributions which will not be shown explicitely. This scalar potential has two global symmetries: $U(1)_B$ and $U(1)_L$ corresponding to baryon number and lepton number symmetries, respectively. These are the exact symmetries of the theory and their breaking (spontaenous and otherwise) lead to B– and L– violating processes. As was emphasized in Refs. 1 and 2 it is mainly the cubic couplings that generate B–ball or L–ball type solitonic solutions. It is appearent that slepton doublet $\tilde{L}$ and right–handed slepton $\tilde{e}_R$ contribute to L–balls whereas one needs the squark doublet $\tilde{Q}$ and right–handed squarks $\tilde{u}_R$ and $\tilde{d}_R$ to form B–balls. In both cases Higgs fields are necessary. In this form the scalar potential involves several scalar fields, and a true analysis of the Q–ball formation requires a minimization of the multi–field quantity [1]

$$
m_{eff}^2(\phi_1, \cdots, \tilde{e}_R) \equiv 2 V_{MSSM} / \sum_{Ball} charge \times |field|^2 \quad (2)
$$

that guarantees the stability of the Q–ball against decaying into its constitutents. In this formula *charge* and *field* denote the baryon number (lepton number) and squark fields (slepton fields). Instead of dealing with coupled equations of motion for fields contributing to a particular Q–ball, practically one can describe the nature of the Q–ball by using a single scalar degree of freedom [2]. This approximation is quite accurate especially for D– and F–flat potentials as the degrees of freedom orthogonal to flat directions will be much more massive [4,5]. For this purpose it is convenient to introduce a scalar field φ representing the Q-ball matter, and decompose the component fields in terms of φ using the dimensionless parameters ξ_i, with $\xi < 1$ and $\sum_i \xi_i^2 = 1$, as follows:

$$
\tilde{e}_{L,R} = \xi_{e_{L,R}} \varphi, \quad \tilde{u}_{L,R} = \xi_{u_{L,R}} \varphi, \quad \tilde{d}_{L,R} = \xi_{d_{L,R}} \varphi, \quad \phi_{1,2} = \xi_{1,2} \varphi . \quad (3)
$$

Using this decomposition, the scalar potential V_{MSSM} takes the form

$$V_\varphi = M_\varphi^2 |\varphi|^2 + M_c Re\varphi |\varphi|^2 - M_s Im\varphi |\varphi|^2 + \lambda |\varphi|^4 \qquad (4)$$

for both L– and B–balls. Here M_φ^2 is the linear combination of the scalar quadratic mass parameters in V_{MSSM}. The quartic coupling λ is a linear combination of Yukawa couplings $h_{u,d,e}$ and gauge couplings $g_{3,2,1}$ following, respectively, from the F–term and D–term contributions.

As the general analyses of Q–ball formation Refs. 1, 2, 3 show explicitly, the crucial parameters in Eq. (4) are the trilinear mass parameters M_c and M_s. For L–balls one has

$$M_c = 2 \left[-\cos(\phi_{A_e}) \tilde{A}_e + \cos(\phi_\mu) \tilde{\mu} \right] \qquad (5)$$

$$M_s = 2 \left[-\sin(\phi_{A_e}) \tilde{A}_e + \sin(\phi_\mu) \tilde{\mu} \right] \qquad (6)$$

where

$$\begin{aligned} \tilde{A}_e &= h_e |A_e| \xi_1 \xi_{e_L} \xi_{e_R} \\ \tilde{\mu} &= h_e |\mu| \xi_2 \xi_{e_L} \xi_{e_R} \end{aligned} \qquad (7)$$

where ϕ_{A_e} and ϕ_μ are the phases of A_e and μ parameter, respectively.

For B–balls, however, M_c and M_s have the following expressions

$$M_c = 2 \left[\cos(\phi_{A_u}) \tilde{A}_u - \cos(\phi_{A_d}) \tilde{A}_d + \cos(\phi_\mu) \tilde{\mu} \right] \qquad (8)$$

$$M_s = 2 \left[\sin(\phi_{A_u}) \tilde{A}_u - \sin(\phi_{A_d}) \tilde{A}_d + \sin(\phi_\mu) \tilde{\mu} \right] \qquad (9)$$

where

$$\begin{aligned} \tilde{A}_u &= h_u |A_u| \xi_2 \xi_{u_L} \xi_{u_R} \\ \tilde{A}_d &= h_d |A_d| \xi_1 \xi_{d_L} \xi_{d_R} \\ \tilde{\mu} &= |\mu| (h_d \xi_2 \xi_{d_L} \xi_{d_R} - h_u \xi_1 \xi_{u_L} \xi_{u_R}) \ . \end{aligned} \qquad (10)$$

The form of the scalar potential Eq. (4) is such that $U(1)_B$ or $U(1)_L$ symmetries are not manifest at all. In particular, $Im\varphi$ and $Re\varphi$ refer to Higgs fields $\phi_{1,2}$ which do not contribute to the charge of the Q–ball. For instance, in thin–wall approximation and in the notation of Ref. 2, the total charge and the effective mass of the B–ball are respectively given by

$$\begin{aligned} B &= 2a\omega |\varphi|^2 V \\ m_{eff}^2 &= \frac{1}{a} \left[M_\varphi^2 + M_c Re\varphi - M_s Im\varphi + \lambda\{(Re\varphi)^2 + (Im\varphi)^2\} \right] \\ \text{where} \ \ a &= \frac{1}{3} \left[\xi_{u_L}^2 + \xi_{u_R}^2 + \xi_{d_L}^2 + \xi_{d_R}^2 \right] \end{aligned} \qquad (11)$$

Thus, the total charge vanishes if squarks are absent; that is, the Higgs fields do not play a role in charge accumulation. However, there is no stable Q–ball if the Higgs fields are absent, as can be seen from vanishing of the trilinear couplings M_c and M_s. In this sense, Re and Im parts of φ in Eq. (4) refer to the time–independent phase of φ generated by the non–trivial phases in μ and $A_{u,d,e}$ parameters.

As is seen from Eq. (4), the main effect of complex μ and $A_{u,d,e}$ is to introduce $M_s \neq 0$ which is proportional to $Im\varphi$. There are three distinct limits in which the Q–ball matter gains different CP characteristics depending on the values of M_c and M_s.

So far discussions of Q–balls have been based on purely real μ and $A_{u,d,e}$ in which case $M_s \equiv 0$. Then only $Re\varphi$ has a trilinear coupling and stable Q–matter is thus composed of $Re\varphi$ which is purely CP even.

In the opposite limit, that is, for purely imaginary μ and $A_{u,d,e}$ one has $M_c \equiv 0$. Then only $Im\varphi$ has a trilinear coupling, and thus, the resulting Q–ball is composed of $Im\varphi$, and Q–matter is purely CP odd.

In the general case, where μ and $A_{u,d,e}$ are complex parameters with nonvanishing real and imaginary parts, both $Im\varphi$ and $Re\varphi$ contribute to the Q–matter. Namely, m_{eff}^2 is minimized for $Re\varphi = -M_c/2\lambda$ and $Im\varphi = M_s/2\lambda$, so that mass per unit charge for B–ball reads as

$$m_{eff}^2(B-ball) = \frac{1}{a}\left[M_\varphi^2 - \frac{1}{4\lambda}(M_c^2 + M_s^2)\right] \ . \tag{12}$$

According to Coleman's theorem [1], if $0 < m_{eff}^2 < M_\varphi^2/a$ then the resulting B-ball is stable. One also notices that m_{eff}^2 depends explicitly on the relative phase between any pair of $\tilde{A}_d$, $\tilde{A}_u$ and $\tilde{\mu}$, using Eq.(8):

$$m_{eff}^2(B-ball) = \frac{1}{a}\left[M_\varphi^2 - \frac{1}{\lambda}\left(\tilde{A}_u^{\ 2} + \tilde{A}_d^{\ 2} + \tilde{\mu}^2 - 2\tilde{A}_u\tilde{A}_d\cos(\phi_{A_u} - \phi_{A_d})\right.\right.$$

$$\left.\left. +2\tilde{A}_u\tilde{\mu}\cos(\phi_{A_u} - \phi_\mu) - 2\tilde{A}_d\tilde{\mu}\cos(\phi_{A_d} - \phi_\mu)\right)\right] \tag{13}$$

Depending on the values of these CP phases m_{eff}^2 takes on a range of values. Therefore, mass per unit charge in the B–ball varies with the CP violating phases. To illustrate this, one can consider the simple case of $\tilde{u}_L\tilde{u}_R$ B–ball, that is, $A_d = 0$. In this case m_{eff}^2 varies from its minimum value ($\phi_{A_u} - \phi_\mu = 0$)

$$\left[m_{eff}^2(B-ball)\right]_{min} = \frac{1}{a}\left[M_\varphi^2 - \frac{1}{\lambda}\left(\tilde{A}_u + \tilde{\mu}\right)^2\right] \tag{14}$$

to the maximal value ($\phi_{A_u} - \phi_\mu = \pi$)

$$\left[m_{eff}^2(B-ball)\right]_{max} = \frac{1}{a}\left[M_\varphi^2 - \frac{1}{\lambda}\left(\tilde{A}_u - \tilde{\mu}\right)^2\right] \tag{15}$$

with the mean ($\phi_{A_u} - \phi_\mu = \pi/2$)

$$\left[m_{eff}^2(B-ball)\right]_{mean} = \frac{1}{a}\left[M_\varphi^2 - \frac{1}{\lambda}\left(\tilde{A}_u^{\ 2} + \tilde{\mu}^2\right)\right] \tag{16}$$

taking $\tilde{\mu}$ positive. One notices that none of the conditions above implies a specific value for the phases ϕ_{A_u} and ϕ_μ. Indeed the mass parameters M_c and M_s take on the following values

$$M_c = 2\left(\tilde{A}_u + \tilde{\mu}\right)\cos\phi_{A_u} \ , \quad M_s = 2\left(\tilde{A}_u + \tilde{\mu}\right)\sin\phi_{A_u}$$

$$M_c = 2\left(\tilde{A}_u - \tilde{\mu}\right)\cos\phi_{A_u} \ , \quad M_s = 2\left(\tilde{A}_u - \tilde{\mu}\right)\sin\phi_{A_u} \tag{17}$$

$$M_c = 2\left(\tilde{A}_u\cos\phi_{A_u} + \tilde{\mu}\sin\phi_{A_u}\right) \ , \quad M_s = 2\left(\tilde{A}_u\sin\phi_{A_u} - \tilde{\mu}\cos\phi_{A_u}\right)$$

4

for $\phi_{A_u} - \phi_\mu = 0$, π and $\pi/2$, respectively. Hence, despite the phase independence of mass per unit charge (14)–(16), the value of the condensate, determined by M_c and M_s, is an explicit function of the CP phases. This follows from the fact that the Q–matter φ depends on the individual phases additively whereas m^2_{eff} depends only on the relative phases. This particular pattern of phase structure can be important for Q–ball formation. In the purely CP–conserving limit, one would have $m^2_{eff} = \left[m^2_{eff}(B - ball) \right]_{min}$ together with $M_c = 2 \left(\tilde{A}_u + \tilde{\mu} \right)$ and $M_s = 0$, correspondig to the first line of (17) with $\phi_{A_u} = 0$. However, as the Eqs. (14)–(16) and (17) suggest clearly the nonvanishing CP phases offer more alternatives.

To see the implications of such a phase dependence, one can consider the special case of $\tilde{A}_u = \tilde{\mu}$ and $\tilde{\mu}^2/\lambda = M^2_\varphi/4$. Then, for $\phi_{A_u} - \phi_\mu = 0$, one has $m^2_{eff} = 0$ with nonvanishing M_c and M_s. Therefore, for this parameter set one has a massless Q–ball, or equivalently, a Q–matter distribution over entire space. In this case, at least in the thin–wall approximation, there is no macroscopic structure with finite size. On the other hand, for $\phi_{A_u} - \phi_\mu = \pi$, $m^2_{eff} = M^2_\varphi/a$, and this corresponds to the critical value of m^2_{eff} below which there would be a stable Q–ball solution. A careful look at M_c and M_s shows that they vanish identically and Q–ball formation without the trilinear couplings is already out of question. Therefore this possibility leaves no room for Q–ball formation. Finally, for $\phi_{A_u} - \phi_\mu = \pi/2$, however, one obtaines $m^2_{eff} = M^2_\varphi/2a$ leaving both M_c and M_s nonvanishing. This is a regular Q–ball solution, and it corresponds to maximal CP–violation, for instance, in the Higgs sector [6]. Indeed, M_c and M_s can vary with ϕ_{A_u} further and this does not affect the Q–ball structure obtained for $\left[m^2_{eff}(B - ball) \right]_{mean}$. Therefore, depending on the amount of CP violation, there may be regions of the parameter space that support a stable Q–ball solution though the strictly CP–conserving MSSM does not.

Another implication of these phases would be on the scattering of fermions from the Q–ball. Indeed, a typical cross section has the form $\sigma \sim 4\pi R^2$ where $R^2 \sim 1/m^2_{eff}$. Therefore, the supersymmetric CP phases influence the formation as well as interactions of the Q–balls with surrounding plasma. In fact, this expectation is confirmed by the recent analysis of the scattering of the dark matter particles from the nucleons [10].

So far we have discussed Q–ball formation without imposing any D– and/or F–flatness. However, the scalar potential of the low–energy supersymmetric theories has many flat directions. Such flat potentials are phenomenologically relevant as it is possible to produce large enough Q–balls that can resist the evaporation on time scales of the order of the age of the universe [4,5]. As has been listed in Ref. [9], there are slepton as well as squark flat directions with corresponding Q–balls. In fact, radiatively corrected flat directions in the MSSM induce a potential of the form

$$V_\varphi = M^2_\varphi |\varphi|^2 + \lambda^2 \frac{|\varphi|^{2(d-1)}}{M^{2(d-3)}_{Pl}} + \left(\lambda A \frac{\varphi^d}{M^{(d-3)}_{Pl}} + h.c. \right) \tag{18}$$

where d is the mass dimension of the nonrenormalizable operator in the potential, and A is a typical trilinear coupling in the soft supersymmetry breaking part. For the purpose of this work the essential piece in this formula is A dependent part. As discussed above, if A–terms are complex parameters, in general, the $Re\varphi$ as well as $Im\varphi$ can develop nonvanishing vacuum expectation values leading to CP violating Q–matter.

3. Discussions

5

In this work we have discussed Q-ball formation in the MSSM with explicit CP violation. It is seen that the complex μ parameter and A–terms can affect the Q-ball formation process. In particular, it is shown that the scalar vacuum expectation value in the Q–ball depends additively on the soft CP phases whereas the mass per unit charge of the Q–ball depends only on the relative phases. There are regions of the parameter space where finite CP phases induce a stable Q–ball where the strictly CP–conserving does not.

Once the Q–ball is formed, as long as the Q–matter is CP violating one, one expects that the scatterings of the light fermions from the Q–ball as well as its decay to fermions (neutralino LSP, for example) can affect their phenomenology. Particularly interesting is the Q–matter contributing to dark matter where the detection rates will change with the CP violating phases considerably [10].

It is worthy of reemhasizing that:

1. The vacuum expectation values of the fields, that is, the Q–ball matter φ depends on the individual relative CP phases additively; however,

2. The mass per unit charge in the Q-ball depends only on the relative phases.

Hence, there are regions of the parameter space where the Q–matter is nonvanishing; however, the Q–ball solutions are not stable. Moreover, there are regions of the parameter space where finite CP phases suport a stable Q–ball whereas the CP conserving MSSM does not. These discussions equally apply to L–balls as well, and one may obtain new kinds of Q–ball solutions by including appropriate soft terms such as R–parity violating ones.

Acknowledgments

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