Gravitational Energy-momentum Density in Bianchi Type II Space-times

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Abstract. In this paper, using Einstein and Landau and Lifshitz's energy-momentum complexes in both general relativity and teleparallel gravity, we calculate the total energy distribution (due to matter and fields including gravitation) associated with Locally Rotationally Symmetric(LRS) Bianchi type II cosmological models. We show that energy density in these different gravitation theories is the same, so agree with each other. We obtain that the total energy is zero. This result agrees with previous works of Cooperstock and Israelit, Rosen, Johri *et al.*, Banerjee and Sen, Vargas, Aydogdu and Salti. Moreover, our result supports the viewpoints of Albrow and Tryon.

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1. Introduction

Energy-momentum associated with a symmetry of space-time is regarded as the most fundamental conserved quantity in physics. Also, because of its unusual nature and various points of view, the definition of an energy-momentum density for the gravitational field is one of the oldest and thorny problem of gravitation. Both Einstein's theory of general relativity and teleparallel gravity consider the problem of obtaining energy-momentum definition. To find a generally accepted expression, there are different attempts. However, there is still no generally accepted expression known. The first of such attempts was made by Einstein who suggested a definition for energy-momentum distribution[1]. Following his definition, different people proposed different energy-momentum complexes: e.g. Tolman[2], Papapetrou[3], Landau and Lifshitz[4], Bergmann and Thomson[5], Møller[6], Weinberg[7], Qadir and Sharif[8] and teleparallel gravity analogs of Einstein, Landau and Lifshitz, Bergmann and Thomson[9] and Møller[10] definitions. Except for the Møller definition, others are restricted to calculate the energy-momentum distribution in quasi-cartesian coordinates. Therefore, they can only give reasonable and meaningful result if calculations are carried out in cartesian

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coordinates. For a long time, attempts to deal with this problem were made only by proposers of quasi-local approach[11, 12].

Virbhadra and his collaborators reviewed the problem of the energy-momentum density, using energy-momentum complexes. For a general non-static spherical symmetric metric of the Kerr-Schild class, Virbhadra shown that Einstein, Landau and Lifshitz, Papapetrou, and Weinberg's complexes give the same energy distribution as in the Penrose energy-momentum complex[13, 14]. Rosen and Virbhadra[15] obtained good results for Rosen-Einstein space-time. Using the Einstein's energy-momentum definition, Chamarro and Vibhadra[16] evaluated the energy of a charged dilation black hole. Also, there are many papers about energy-momentum distributions, and in those papers[17], the authors showed that these definitions could give the same results for a given space-time.

Rosen[18], using Einstein energy-momentum complex, shown that the total energy of the closed homogeneous isotropic universe described by Friedmann-Robertson-Walker (FRW) metric is zero. Banerjee and Sen[19] computed the total energy of the Bianchi type I space-times, with Einstein's prescription. They found that the total energy is zero everywhere. The total energy of a FRW spatially closed universe was calculated by Johri *et al.*[20]. They obtained that it is zero at all times irrespective of the equations of state of the cosmic fluid. Using Landau and Lifshitz, Papapetrou and Weinberg prescriptions, Xulu[21] calculated that energy-momentum distribution for the Bianchi type I space-times vanishes everywhere. Vargas[9], using teleparallel gravity analogs of Einstein and Landau and Lifshitz energy-momentum definitions, found that the energy is zero in FRW space-times. This result agree with the previous works of Cooperstock and Israelit[22], Rosen[18], Banerjee and Sen[19], Johri *et al.*[20]. In recent papers, Saltı and Havare[23] considered Bergmann-Thomson's complex in both general relativity and teleparallel gravity for the viscous Kasner-type metric and in another work, Saltı[24], using the Einstein and Landau and Lifshitz complexes in teleparallel gravity for the same metric, found that energy-momentum densities are zero. At the last, Aydogdu and Saltı[25] used the teleparallel gravity analog of Møller's definition for the Bianchi type I metric and found that the total energy is zero.

The basic purpose of this paper is to find the total energy in the LRS Bianci type II universes, with the energy-momentum definitions of Einstein and Landau and Lifshitz in both general relativity and teleparallel gravity. We will proceed according to the following scheme. In the next section, we introduce LRS Bianchi type II cosmological models. In section III, we give review Einstein and Landau Lifshitz energy-momentum complexes in general relativity and than calculate energy density. In section IV, Einstein and Landau Lifshitz energy-momentum complexes in teleparallel gravity are given and than calculate the energy density. The final section is devoted to the discussion and conclusion. Throughout this paper, Latin indices (i,j,k,...) represent the vector number and Greek indices $(\mu, \nu, \lambda, ...)$ represent the vector components. All indices run from 0 to 3 and we use the convention that $G = 1, c = 1$ units.

2. LRS Bianchi type II cosmological models

The FRW models have a significant role in cosmology. Whether these models correctly represent the universe or not isn't known, but it is believed that they are good global approximations of the present universe. Spatial homogeneity and isotropy characterize these models. In last decades, theoretical interest in anisotropic cosmological models has been increased. In current modern cosmology, the spatial homogeneous and anisotropic Bianchi models which present a medium way between FRW models and completely inhomogeneous and anisotropic universes play an important role. Here, we consider the LRS model of Bianchi type II. The metric for Bianchi type II in the LRS case is given by[26]

$$ds^2 = dt^2 - D(t)^2dx^2 - H(t)^2dy^2 - [D(t)^2 + x^2H(t)^2]dz^2 - 2xH(t)^2dydz \quad (1)$$

D(t) and H(t) which are expansion factors could be determined via Einstein's field equations. The non-vanishing components of the Einstein tensor $G_{\mu\nu}(\equiv 8\pi T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor for the matter field described by a perfect fluid with density ρ, pressure p) are

$$G_{11} = D\ddot{D} + D^2\frac{\ddot{H}}{H} + \frac{\dot{D}}{D}\frac{\dot{H}}{H} + \frac{H^2}{4D^2} \quad (2)$$

$$G_{22} = 2H^2\frac{\ddot{D}}{D} + H^2\frac{\dot{D}^2}{D^2} - \frac{3H^4}{4D^4} \quad (3)$$

$$G_{33} = (2x^2H^2 + D^2)\frac{\ddot{D}}{D} + D^2\frac{\ddot{H}}{H} + x^2H^2\frac{\dot{D}^2}{D^2} + \frac{D}{H}\dot{D}\dot{H} + \frac{H^2}{4D^2} - x^2\frac{3H^2}{4D^4} \quad (4)$$

$$G_{00} = \frac{\dot{D}^2}{D^2} - 2\frac{\dot{D}}{D}\frac{\dot{H}}{H} + \frac{H^2}{4D^4} \quad (5)$$

$$G_{23} = H^2(2\frac{\ddot{D}}{D} + \frac{\dot{D}^2}{D^2} - \frac{3H^2}{4D^4}) \quad (6)$$

where dot represents derivation with respect to time.

For the line element (1)

$$g_{\mu\nu} = \delta^0_\mu\delta^0_\nu - D^2\delta^1_\mu\delta^1_\nu - H^2\delta^2_\mu\delta^2_\nu - (D^2 + x^2H^2)\delta^3_\mu\delta^3_\nu - xH^2(\delta^3_\mu\delta^2_\nu + \delta^2_\mu\delta^3_\nu) \quad (7)$$

$$g^{\mu\nu} = \delta^\mu_0\delta^\nu_0 - D^{-2}\delta^\mu_1\delta^\nu_1 - \frac{D^2 + x^2H^2}{H^2D^2}\delta^\mu_2\delta^\nu_2 - D^{-2}\delta^\mu_3\delta^\nu_3 + xD^{-2}(\delta^\mu_3\delta^\nu_2 + \delta^\mu_2\delta^\nu_3) \quad (8)$$

The riemannian metric arises as

$$g_{\mu\nu} = \eta_{ij}h^i_\mu h^j_\nu \quad (9)$$

Using this relation, we obtain the tetrad components

$$h^i_\mu = \delta^i_0\delta^0_\mu + D\delta^i_1\delta^1_\mu + H\delta^i_2\delta^2_\mu + D\delta^i_3\delta^3_\mu + xH\delta^i_2\delta^3_\mu \quad (10)$$

and its inverse is

$$h_i^\mu = \delta_i^0\delta_0^\mu + D^{-1}\delta_i^1\delta_1^\mu + H^{-1}\delta_i^2\delta_2^\mu + D^{-1}\delta_i^3\delta_3^\mu - \frac{x}{D}\delta_i^3\delta_2^\mu \tag{11}$$

From the Christoffel symbols defined by

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\tau}(\partial_\mu g_{\tau\nu} + \partial_\nu g_{\tau\mu} - \partial_\tau g_{\mu\nu}) \tag{12}$$

we obtain non-vanishing components:

$$\begin{aligned}
&\Gamma^0_{11} = D\dot{D}, \qquad \Gamma^0_{22} = H\dot{H}, \qquad \Gamma^0_{33} = D\dot{D} + x^2 H\dot{H}, \qquad \Gamma^1_{01} = \frac{\dot{D}}{D}\\
&\Gamma^2_{02} = \frac{\dot{H}}{H}, \qquad \Gamma^2_{03} = \frac{x(D\dot{H} - H\dot{D})}{HD}, \qquad \Gamma^2_{12} = -\frac{x^2H^2}{2D^2}\\
&\Gamma^2_{13} = \frac{D^2 - x^2H^2}{2D^2}, \qquad \Gamma^3_{03} = \frac{\dot{D}}{D}, \qquad \Gamma^3_{12} = \frac{x^2H^2}{2D^2}, \qquad \Gamma^3_{13} = \frac{x^2H^2}{2D^2}
\end{aligned} \tag{13}$$

3. Energy in general relativity

3.1. Energy in the Einstein prescription

Einstein's[1] energy-momentum complex is given by

$$\Theta^\nu_\mu = \frac{1}{16\pi}\Sigma^{\nu\beta}_{\mu,\beta} \tag{14}$$

where

$$\Sigma^{\nu\beta}_\mu = \frac{g_{\mu\alpha}}{\sqrt{-g}}\left[-g(g^{\nu\alpha}g^{\beta\sigma} - g^{\beta\alpha}g^{\nu\sigma})\right]_{,\sigma} \tag{15}$$

Θ^0_0 is the energy density, Θ^0_α are the momentum density components, and Θ^α_0 are the components of energy current density. Einstein's energy-momentum density satisfies the local conservation laws

$$\partial_\nu\Theta^\nu_\mu = 0. \tag{16}$$

Energy and momentum components are given by

$$P_\nu = \int\int\int \Theta^0_\nu dxdydz. \tag{17}$$

Further Gauss's theorem furnishes

$$P_\nu = \frac{1}{16\pi}\int\int \Sigma^{0\alpha}_\nu \mu_\alpha dS. \tag{18}$$

μ_α stands for the 3-components of unit vector over an infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy.

From eq. (15) with eqs. (7) and (8), we obtain that the required non-vanishing $\Sigma^{\nu\alpha}_\mu$ component is

$$\Sigma^{01}_1 = 2D(H\dot{D} + D\dot{H}) \tag{19}$$

Using above result, we find

$$\Theta^0_0 = \Theta^0_i = 0 \tag{20}$$

From this result, we easily see that Einstein's energy in the LRS Bianchi type II space-time is

$$E^{E}_{GR} = 0 \tag{21}$$

3.2. Energy in the Landau and Lifshitz prescription

Landau and Lifshitz[4] energy-momentum complex is given by

$$L^{\mu\nu} = \frac{1}{16\pi} N^{\mu\nu\alpha\beta}{}_{,\alpha\beta} \tag{22}$$

where

$$N^{\mu\nu\alpha\beta} = -g(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}) \tag{23}$$

The Landau and Lifshitz energy-momentum complex satisfies the local conservation laws

$$\partial_\nu L^{\mu\nu} = 0 \tag{24}$$

in any coordinate system. The energy and momentum components are given by

$$P^{\mu} = \int\int\int L^{\mu 0} dxdydz \tag{25}$$

Further Gauss's theorem furnishes

$$P^{\mu} = \frac{1}{16\pi}\int\int N^{\mu\alpha 0\nu}{}_{,\nu}\eta_\alpha dS. \tag{26}$$

η_α stands for the 3-components of unit vector over an infinitesimal surface element dS. P^i give momentum components P^1, P^2, P^3 and P^0 gives the energy.

From eq. (23) with eqs. (7) and (8), we obtain that the required non-vanishing $N^{\mu\nu\alpha\beta}$ component are

$$N^{1001} = -N^{1010} = D^2H^2 \tag{27}$$

Using above result, we find

$$L^{00} = L^{i0} = 0 \tag{28}$$

As a result of this, we easily see that Landau and Lifshitz's energy in the LRS Bianchi type II space-time is

$$E^{LL}_{GR} = 0 \tag{29}$$

4. Energy in teleparallel gravity

Teleparallel gravity which corresponds to a gauge theory for the translation group based on the Weitzenböck geometry[27] is an alternative approach to gravitation[28]. In this theory gravitation is attributed to torsion[29], which past the role of a force[30], whereas the curvature tensor vanishes identically. The fundamental field is represented by a non-trivial tetrad field, which gives rise to the metric as a by-product. The last translational gauge potentials appear as the nontrivial part of the tetrad field, thus induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting point of teleparallel gravity is that it can reveal a more appropriate approach to consider same specific problem due to gauge structure. This is the case, for example, of the energy-momentum problem, which becomes more transparent when considered from the teleparallel point of view.

The energy-momentum complexes of Einstein and Landau and Lifshitz in teleparallel gravity[9] are given by the following definitions, respectively:

$$hE^{\mu}_{\nu} = \frac{1}{4\pi}\partial_{\lambda}(U_{\nu}{}^{\mu\lambda}) \tag{30}$$

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_{\xi}(hg^{\mu\kappa}U_{\kappa}{}^{\nu\xi}) \tag{31}$$

where $h = \mathtt{det}(h^{i}{}_{\mu})$ and $U_{\kappa}{}^{\nu\xi}$ is the Freud's super-potential, which is given by:

$$U_{\kappa}{}^{\nu\xi} = hS_{\kappa}{}^{\nu\xi}. \tag{32}$$

Here $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = k_1 T^{\mu\nu\lambda} + \frac{k_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{k_3}{2}(g^{\mu\lambda}T^{\beta\nu}{}_{\beta} - g^{\nu\mu}T^{\beta\lambda}{}_{\beta}) \tag{33}$$

with k_1, k_2 and k_3 the three dimensionless coupling constants of teleparallel gravity[29]. For the teleparallel equivalent of general relativity, the specific choice of these three constants are given

$$k_1 = \frac{1}{4}, \qquad k_2 = \frac{1}{2}, \qquad k_3 = -1 \tag{34}$$

To calculate this tensor, firstly, we must compute Weitzenböck connection:

$$\Gamma^{\sigma}{}_{\zeta\beta} = h_{i}{}^{\sigma}\partial_{\beta}h^{i}{}_{\zeta} \tag{35}$$

After this calculation, we get torsion of the Weitzenböck connection:

$$T^{\mu}{}_{\nu\lambda} = \Gamma^{\mu}{}_{\lambda\nu} - \Gamma^{\mu}{}_{\nu\lambda} \tag{36}$$

For the Einstein and Landau and Lifshitz's definitions, energy and momentum components are given as

$$P^{E}_{\mu} = \int_{\Sigma} hE^{0}_{\mu}dxdydz \tag{37}$$

$$P^{\mu}_{L} = \int_{\Sigma} hL^{\mu 0}dxdydz \tag{38}$$

where P_i give momentum components P_1, P_2, P_3 while P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = t =$constant.

From eq. (35) the non-vanishing Weitzenböck connection components are obtained as

$$\begin{aligned} &\Gamma^{1}{}_{10} = \Gamma^{3}{}_{30} = \frac{\dot{D}}{D}, \qquad \Gamma^{2}{}_{20} = \frac{\dot{H}}{H}, \\ &\Gamma^{2}{}_{30} = \frac{x}{DH}(D\dot{H} - H\dot{D}), \qquad \Gamma^{2}{}_{31} = 1 \end{aligned} \tag{39}$$

The corresponding non-vanishing torsion components are found

$$\begin{aligned}
&T^1{}_{01} = -T^1{}_{10} = T^3{}_{03} = -T^3{}_{30} = \frac{\dot{D}}{D}, \qquad T^2{}_{02} = -T^2{}_{20} = \frac{\dot{H}}{H},\\
&T^2{}_{13} = -T^2{}_{31} = 1,\\
&T^2{}_{03} = -T^2{}_{30} = \frac{x}{DH}(D\dot{H} - H\dot{D}) \qquad (40)
\end{aligned}$$

Using these results with eq. (33), the non-vanishing components of the tensor $S^{\mu\nu\lambda}$ are obtained as

$$S^{023} = -\frac{x}{4D^2}(\frac{\dot{H}}{H} - \frac{\dot{D}}{D}) \qquad (41)$$

$$S^{101} = -\frac{x}{2D^2}(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}), \qquad S^{123} = \frac{1}{4D^4} \qquad (42)$$

$$S^{202} = \frac{1}{H^2 D} + \frac{x^2\dot{R}}{D^3}, \qquad S^{203} = \frac{3x}{4D^2}(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}), \qquad S^{213} = \frac{1}{4D^4} \qquad (43)$$

$$S^{303} = \frac{1}{2D^2}(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}), \qquad S^{302} = -\frac{-x}{2D^2}(\frac{\dot{H}}{H} + \frac{\dot{D}}{D}), \qquad S^{312} = \frac{1}{4D^4} \qquad (44)$$

From eq. (32) the required non-vanishing components of Freud's super-potential are calculated as

$$U_0{}^{01} = 0, \qquad U_1{}^{01} = H^2 D\dot{D} + D^2 H\dot{H} \qquad (45)$$

Using above results with eqs. (30) and (31), we find the total energy in teleparallel gravity:

$$E^{E}_{TP} = L^{LL}_{TP} = 0 \qquad (46)$$

5. Discussions

The subject of energy-momentum localization in both general relativity and teleparallel gravity has been very exciting and interesting although it has been associated with some debates. Recently, a large number researches have interested in studying the energy content of the universe in various models. Using Einstein's energy-momentum definitions, Rosen calculated the total energy of a FRW metric and obtained that to be zero. The total energy of the same universe is found by Johri *et al.* with Landau and Lifshitz's energy-momentum complex. They obtained that it is zero at all times. Moreover, they shown that the total energy enclosed within any finite volume of spatially flat FRW universe is vanishing. Banerjee and Sen who considered Bianchi type I space-times obtained that energy-momentum density is zero everywhere, with the energy-momentum definition of Einstein.

In present paper, we used LRS Bianchi type II metric and calculated energy-momentum density for this universe model with Einstein and Landau and Lifshitz's energy-momentum definitions in both general relativity and teleparallel gravity. We found that the total energy in these different gravitation theories give same result:

$$E^{E}_{GR} = L^{LL}_{GR} = E^{E}_{TP} = L^{LL}_{TP} = 0 \tag{47}$$

which agree with the result that obtained by Cooperstock and Israelit, Rosen, Johri *et al.*, Banerjee and Sen, Vargas and Aydogdu and Salti. Furthermore, our result supports the view points of Albrow[31] and Tryon[32]. Although Einstein's energy-momentum tensor has non-vanishing components, the total energy for the LRS Bianchi type II space-time is zero; because the energy-momentum contributions from the matter and field inside arbitrary two surfaces, in the case of the anisotropic model based on the LRS Bianchi type II metric, cancel each other. Finally, our result that total energy density is vanishing everywhere maintains the importance of the energy-momentum complexes.

Furthermore, the result obtained is also independent of the three teleparallel dimensionless coupling constants, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model.

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