

ON THE ENERGY-MOMENTUM IN CLOSED UNIVERSES*

Mustafa Saltı†

*Department of Physics, Graduate School of Natural and Applied Science,
Middle East Technical University, 06531, Ankara-Turkey*

Using the Møller, Einstein, Bergmann-Thomson and Landau-Lifshitz energy-momentum definitions both in general relativity and teleparallel gravity, we find the energy-momentum of the closed universe based on the generalized Bianchi-type I metric.

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I. INTRODUCTION

A large number of formulations of the gravitational energy, momentum and angular momentum have been given since. Some of them are coordinate independent and other are coordinate-dependent. It is possible to evaluate the energy distribution and momentum by using various energy-momentum pseudotensors. There lies a dispute on the importance of non-tensorial energy-momentum complexes whose physical interpretations have been questioned by a number of physicists, including Weyl, Pauli and Eddington. Also, there exists an opinion that the energy-momentum pseudo-tensors are not useful to find meaningful results in a given geometry. Ever since the Einstein's energy-momentum complex[1], used for calculating energy and momentum in a general relativistic system, many attempts have been made to evaluate the energy distribution for a given space-time[2]. Except the one which was defined by Møller, these definitions only give meaningful results if the calculations are performed in "Cartesian" coordinates. Møller constructed an expression which enables one to evaluate energy and momentum in any coordinate system.

Several examples of particular space-times have been investigated and different energy-momentum pseudo-tensor are known to give the same energy distribution for a given space-time[3]. In Phys. Rev. D60-104041 (1999), Virbhadra, using the energy and momentum complexes of Einstein, Landau-Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr-Schild class, showed that all of these energy-momentum formulations give the same energy distribution as in the Penrose energy-momentum formulation.

Albrow[4] and Tryon[5] assumed that the net energy of the universe may be equal to zero. The subject of the energy-momentum distributions of closed and open universes was initiated by an interesting work of Cooperstock and Israelit[6]. They found the zero value of energy for any homogenous isotropic universe described by a Friedmann-Robertson-Walker metric in the context of

general relativity. This interesting result influenced some general relativists[7].

Recently, the problem of energy-momentum localization was also considered in teleparallel gravity. There are a few papers on this topic[8]. The authors found that energy-momentum also localize in this alternative theory, and their results agree with the some previous papers which were studied in the general theory of relativity. In Gen. Relat. Gravit. 36, 1255(2004); Vargas, using the definitions of Einstein and Landau-Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann-Robertson-Walker space-times. Later, with my collaborators I considered a few space-times in both general relativity and teleparallel gravity and obtained the results which agree with each other and the previous ones[9].

The paper is organized as follows: In the next section, we introduce the generalized Bianchi type I metric. In Sec. 3, we give the energy-momentum definitions of Møller, Einstein, Bergmann-Thomson and Landau-Lifshitz in both general relativity and teleparallel gravity. Sec. 4 gives us the energy-momentum in the Heckmann-Schucking metric. Section 5 is devoted to summary and conclusions.

Notations and conventions: $c = h = 1$, metric signature $(-, +, +, +)$, Greek indices run from 0 to 3 and Latin ones from 1 to 3. Throughout this paper, Latin indices (i, j, \dots) number the vectors, and Greek indices (μ, ν, \dots) represent the vector components.

II. THE GENERALIZED BIANCHI-TYPE I METRIC

The Friedmann-Robertson-Walker cosmological model has attracted considerable attention in the relativistic cosmology literature. Maybe one of the most important properties of this model is, as predicted by inflation[10], the flatness, which agrees with the observed cosmic microwave background radiation.

In the early universe the sorts of the matter fields are uncertain. The existence of anisotropy at early times is a very natural phenomenon to investigate, as an attempt to clarify among other things, the local anisotropies that we observe today in galaxies, clusters and superclusters.

*Czech. J. Phys. 56 (2006) 177-190.

†Electronic address: musts6@yahoo.com

So, at the early time, it appears appropriate to suppose a geometry that is more general than just the isotropic and homogenous Friedmann-Robertson-Walker geometry. Even though the universe, on a large scale, appears homogenous and isotropic at the present time, there are no observational data that guarantee this in an epoch prior to the recombination. The anisotropies defined above have many possible sources; they could be associated with cosmological magnetic or electric fields, long-wave length gravitational waves, Yang-Mills fields[11].

The Bianchi-I type universe which is filled with the mixture of three perfect fluids: dust, stiff matter and a cosmological constant. The presence of the cosmological constant in combination with other types of matter is of special interest, because the recent discovery of the phenomenon of the cosmic acceleration[12] makes the presence of the cosmological constant or some other exotic type of matter which mimics some basic features of the cosmological constant unavoidable for any realistic cosmological model. The Bianchi type-I model has the following form:

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad (1)$$

filled with dust whose equation of state is $p = 0$. The functions $A(t)$, $B(t)$ and $C(t)$ are given by the formulae

$$A(t) = a_0 t^{p_1} (t + t_0)^{\frac{2}{3} - p_1}, \quad (2)$$

$$B(t) = a_0 t^{p_2} (t + t_0)^{\frac{2}{3} - p_2}, \quad (3)$$

$$C(t) = a_0 t^{p_3} (t + t_0)^{\frac{2}{3} - p_3} \quad (4)$$

where the exponents p_1 , p_2 and p_3 are well known Kasner exponents[2, 9] satisfying the relations

$$p_1 + p_2 + p_3 = 1, \quad (5)$$

$$p_1^2 + p_2^2 + p_3^2 = 1. \quad (6)$$

When $t \ll t_0$, the solution is close to Kasner solution. In the limit $t \gg t_0$ all the functions $A(t)$, $B(t)$ and $C(t)$ are proportional to $t^{\frac{2}{3}}$, i.e., their behavior coincides with that of the flat Friedmann universe filled with dust.

The metric given by (1) reduces to some well-known space-times in special cases.

1. Defining $A = B = C = R(t)$ and transforming the line element (1) to t, x, y, z coordinates according to

$$\begin{aligned} x &= r \sin \theta \cos \phi, \\ y &= r \sin \theta \sin \phi, \\ z &= r \cos \theta \end{aligned} \quad (7)$$

gives

$$ds^2 = -dt^2 + R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)] \quad (8)$$

which describes the well-known spatially flat Friedmann-Robertson-Walker space-time.

2. Defining $A = e^{l(t)}$, $B = e^{m(t)}$ and $C = e^{n(t)}$, then the line element describes the well-known Bianchi-I type metric.
3. Writing $A = t^{p_1}$, $B = t^{p_2}$ and $C = t^{p_3}$ (where a, b, c are constants), then we obtain the well-known viscous Kasner-type model.
4. When the functions $A(t)$, $B(t)$ and $C(t)$ are given by the following equations[13]

$$\begin{aligned} A(t) &= \left(\frac{M}{H_0^2}\right)^{\frac{1}{3}} \left(1 + \frac{3H_0 t_0}{2}\right)^{k_1 - \frac{1}{3}} \left(\sinh \frac{3H_0 t}{2}\right)^{k_1} \\ &\quad \times \left(\sinh \frac{3H_0 t}{2} + \frac{3H_0 t_0}{2} \cosh \frac{3H_0 t}{2}\right)^{\frac{2}{3} - k_1} \end{aligned} \quad (9)$$

$$\begin{aligned} B(t) &= \left(\frac{M}{H_0^2}\right)^{\frac{1}{3}} \left(1 + \frac{3H_0 t_0}{2}\right)^{k_2 - \frac{1}{3}} \left(\sinh \frac{3H_0 t}{2}\right)^{k_2} \\ &\quad \times \left(\sinh \frac{3H_0 t}{2} + \frac{3H_0 t_0}{2} \cosh \frac{3H_0 t}{2}\right)^{\frac{2}{3} - k_2} \end{aligned} \quad (10)$$

$$\begin{aligned} C(t) &= \left(\frac{M}{H_0^2}\right)^{\frac{1}{3}} \left(1 + \frac{3H_0 t_0}{2}\right)^{k_3 - \frac{1}{3}} \left(\sinh \frac{3H_0 t}{2}\right)^{k_3} \\ &\quad \times \left(\sinh \frac{3H_0 t}{2} + \frac{3H_0 t_0}{2} \cosh \frac{3H_0 t}{2}\right)^{\frac{2}{3} - k_3} \end{aligned} \quad (11)$$

then the line element describes the Heckmann-Schucking solution of the Bianchi-I type space-time. Here $H_0 = \sqrt{\Lambda}$ is nothing but a Hubble parameter for the de Sitter universe with a cosmological constant Λ , while the constant M characterizes the quantity of the dust in the universe.

III. GRAVITATIONAL ENERGY-MOMENTUM DEFINITIONS

A. Energy-Momentum in General Relativity

1. Møller's Definition

In general theory of relativity the energy-momentum complex of Møller[2] is given by

$$M_\mu^\nu = \frac{1}{8\pi} \chi_{\mu,\alpha}^{\nu\alpha} \quad (12)$$

satisfying the local conservation laws:

$$\frac{\partial M_\mu^\nu}{\partial x^\nu} = 0 \quad (13)$$

where the antisymmetric super-potential $\chi_\mu^{\nu\alpha}$ is

$$\chi_\mu^{\nu\alpha} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta}. \quad (14)$$

The locally conserved energy-momentum complex M_μ^ν contains contributions from the matter, non-gravitational and gravitational fields. M_0^0 is the energy density and M_a^0 are the momentum density components. The energy-momentum components are given by

$$P_\mu = \int \int \int M_\mu^0 dx dy dz. \quad (15)$$

Using Gauss's theorem, the energy and momentum are

$$P_\mu = \frac{1}{8\pi} \int \int \chi_\mu^{\nu\alpha} \mu_\alpha dS. \quad (\alpha = 1, 2, 3) \quad (16)$$

where μ_α is the outward unit normal vector over the infinitesimal surface element dS . P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy.

2. Einstein's Definition

The energy and momentum prescription of Einstein[1] in general theory of relativity is given by

$$\Theta_\mu^\nu = \frac{1}{16\pi} H_{\mu,\alpha}^{\nu\alpha} \quad (17)$$

where

$$H_\mu^{\nu\alpha} = \frac{g_{\mu\beta}}{\sqrt{-g}} [-g(g^{\nu\beta}g^{\alpha\xi} - g^{\alpha\beta}g^{\nu\xi})]_{,\xi} \quad (18)$$

Θ_0^0 is the energy density, Θ_a^0 are the momentum density components, and Θ_0^a are the components of energy current density. The Einstein energy and momentum density satisfies the local conservation laws

$$\frac{\partial \Theta_\mu^\nu}{\partial x^\nu} = 0. \quad (19)$$

and energy-momentum components is given by

$$P_\mu = \int \int \int \Theta_\mu^0 dx dy dz. \quad (20)$$

P_μ is called the momentum four-vector, P_a give momentum components P_1, P_2, P_3 and P_0 gives the energy.

3. Bergmann-Thomson's Definition

The energy and momentum complex of Bergmann-Thomson[2] is given by:

$$B^{\mu\nu} = \frac{1}{16\pi} \Pi_{,\alpha}^{\mu\nu\alpha} \quad (21)$$

where

$$\Pi^{\mu\nu\alpha} = g^{\mu\beta} V_\beta^{\nu\alpha} \quad (22)$$

with

$$V_\beta^{\nu\alpha} = \frac{g_{\beta\xi}}{\sqrt{-g}} [-g(g^{\nu\xi}g^{\alpha\rho} - g^{\alpha\xi}g^{\nu\rho})]_{,\rho}. \quad (23)$$

It is clear that

$$V_\beta^{\nu\alpha} = -V_\beta^{\alpha\nu}. \quad (24)$$

The energy and momentum(energy current) density components are respectively represented by B^{00} and B^{a0} . The Bergmann-Thomson energy-momentum prescription satisfies the following local conservation laws:

$$\frac{\partial B^{\mu\nu}}{\partial x^\nu} = 0 \quad (25)$$

in any coordinate system. The energy and momentum components are given by

$$P^\mu = \int \int \int B^{\mu 0} dx dy dz \quad (26)$$

For the time-independent metric one has

$$P^\mu = \frac{1}{16\pi} \int \int \Pi^{\mu 0 a} \kappa_a dS. \quad (a = 1, 2, 3) \quad (27)$$

here κ_a is the outward unit normal vector over the infinitesimal surface element dS . P^i give momentum components P^1, P^2, P^3 and P^0 gives the energy.

4. Landau-Lifshitz's Definition

The energy and momentum complex of Landau-Lifshitz[2] is

$$L^{\mu\nu} = \frac{1}{16\pi} S^{\mu\nu\alpha\beta}_{,\alpha\beta} \quad (28)$$

where

$$S^{\mu\nu\alpha\beta} = -g(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}). \quad (29)$$

$L^{\mu\nu}$ is symmetric in its indices. L^{00} is the energy density and L^{a0} are the momentum(energy current) density components. $S^{\mu\nu\alpha\beta}$ has symmetries of Riemann curvature tensor. The energy-momentum of Landau and Lifshitz satisfies the local conservation laws:

$$\frac{\partial L^{\mu\nu}}{\partial x^\nu} = 0 \quad (30)$$

with

$$L^{\mu\nu} = -g(T^{\mu\nu} + t^{\mu\nu}). \quad (31)$$

Here $T^{\mu\nu}$ is the energy-momentum tensor of the matter and all non-gravitational fields, and $t^{\mu\nu}$ is known as Landau-Lifshitz energy-momentum pseudotensor. Thus the locally conserved quantity $L^{\mu\nu}$ contains contributions from the matter, non-gravitational fields and gravitational fields. The energy-momentum components are given by

$$P^\mu = \int \int \int L^{\mu 0} dx dy dz. \quad (32)$$

B. Energy-Momentum in Tele-parallel Gravity

The teleparallel gravity is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry[14]. In the theory of the teleparallel gravity, gravitation is attributed to torsion[15], which plays the role of a force[16], and the curvature tensor vanishes identically. The role of the essential field in played by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting place of teleparallel gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent.

1. Møller's Definition

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian space[17]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez[18] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer[19] showed that Møller theory is a special case of Poincaré gauge theory[20, 21].

Møller constructed a gravitational theory based on this space-time. In this gravitation theory the field variables are the 16 tetrad components $h_i{}^\mu$, from which the metric tensor is defined by

$$g^{\alpha\beta} = h_i{}^\alpha h_j{}^\beta \eta^{ij} \quad (33)$$

The superpotential of Møller in teleparallel gravity is given by Mikhail *et al.*[8] as

$$U_\mu^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}^{\tau\nu\beta} [\Phi^\rho g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} \gamma^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu} \gamma^{\sigma\rho\chi}] \quad (34)$$

where $P_{\chi\rho\sigma}^{\tau\nu\beta}$ is

$$P_{\chi\rho\sigma}^{\tau\nu\beta} = \delta_\chi^\nu g_{\rho\sigma}^{\nu\beta} + \delta_\rho^\tau g_{\sigma\chi}^{\nu\beta} - \delta_\sigma^\tau g_{\chi\sigma}^{\nu\beta} \quad (35)$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_\rho^\nu \delta_\sigma^\beta - \delta_\sigma^\nu \delta_\rho^\beta \quad (36)$$

and $\gamma_{\mu\nu\beta}$ is the con-torsion tensor given by

$$\gamma_{\mu\nu\beta} = h_{i\mu} h^i{}_{\nu;\rho} \quad (37)$$

where the semicolon denotes covariant differentiation with respect to Christoffel symbols. g is the determinant of the $g_{\mu\nu}$, Φ_μ is the basic vector field defined by

$$\Phi_\mu = \gamma^\rho{}_{\mu\rho} \quad (38)$$

κ is the Einstein constant and λ is the free dimensionless parameter. The energy-momentum density is defined by[17]

$$\Xi_\alpha^\beta = U_{\alpha,\zeta}^{\beta\zeta} \quad (39)$$

where comma denotes ordinary differentiation. The energy E and momentum components P_i are expressed by the volume integral[17];

$$E = \lim_{r \rightarrow \infty} \int_{r=\text{constant}} \Xi_0^0 dx dy dz \quad (40)$$

$$P_i = \lim_{r \rightarrow \infty} \int_{r=\text{constant}} \Xi_i^0 dx dy dz \quad (41)$$

here the index of i is takes the value from 1 to 3.

2. Einstein, Bergmann-Thomson and Landau-Lifshitz's Definitions

The energy-momentum complexes of Einstein, Bergmann-Thomson and Landau-Lifshitz in teleparallel gravity[8] are given by the following expressions, respectively:

$$hE^\mu{}_\nu = \frac{1}{4\pi} \partial_\lambda (U_\nu{}^{\mu\lambda}) \quad (42)$$

$$hB^{\mu\nu} = \frac{1}{4\pi} \partial_\lambda (g^{\mu\beta} U_\beta{}^{\nu\lambda}) \quad (43)$$

$$hL^{\mu\nu} = \frac{1}{4\pi} \partial_\lambda (h g^{\mu\beta} U_\beta{}^{\nu\lambda}) \quad (44)$$

where $h = \det(h^a{}_\mu)$ and $U_\beta{}^{\nu\lambda}$ is the Freud's superpotential, which is given by:

$$U_\beta{}^{\nu\lambda} = h S_\beta{}^{\nu\lambda}. \quad (45)$$

Here $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = m_1 T^{\mu\nu\lambda} + \frac{m_2}{2} (T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2} (g^{\mu\lambda} T^{\beta\nu}{}_\beta - g^{\nu\mu} T^{\beta\lambda}{}_\beta) \quad (46)$$

with m_1 , m_2 and m_3 the three dimensionless coupling constants of teleparallel gravity[15]. For the teleparallel equivalent of general relativity the specific choice of these three constants are:

$$m_1 = \frac{1}{4}, \quad m_2 = \frac{1}{2}, \quad m_3 = -1 \quad (47)$$

To calculate this tensor firstly we must calculate Weitzenböck connection:

$$\Gamma^{\alpha}_{\mu\nu} = h_a^{\alpha} \partial_{\nu} h^a_{\mu} \quad (48)$$

and after this calculation we get torsion of the Weitzenböck connection:

$$T^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\lambda\nu} - \Gamma^{\mu}_{\nu\lambda} \quad (49)$$

For the Einstein, Bergmann-Thomson and Landau-Lifshitz complexes, we have the following relations for the momentum four-vector,

$$P_{\mu}^E = \int_{\Sigma} h E^0_{\mu} dx dy dz \quad (50)$$

$$P_{\mu}^B = \int_{\Sigma} h B^0_{\mu} dx dy dz \quad (51)$$

$$P_{\mu}^L = \int_{\Sigma} h L^0_{\mu} dx dy dz \quad (52)$$

where P_i give momentum components P_1, P_2, P_3 while P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = t = \text{constant}$.

IV. CALCULATIONS

For the line element (1), $g_{\mu\nu}$ and $g^{\mu\nu}$ are defined by

$$g_{\mu\nu} = -\delta_{\mu}^0 \delta_{\nu}^0 + A^2 \delta_{\mu}^1 \delta_{\nu}^1 + B^2 \delta_{\mu}^2 \delta_{\nu}^2 + C^2 \delta_{\mu}^3 \delta_{\nu}^3 \quad (53)$$

$$g^{\mu\nu} = -\delta_0^{\mu} \delta_0^{\nu} + A^{-2} \delta_1^{\mu} \delta_1^{\nu} + B^{-2} \delta_2^{\mu} \delta_2^{\nu} + C^{-2} \delta_3^{\mu} \delta_3^{\nu} \quad (54)$$

The non-trivial tetrad field induces a teleparallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as given by (33). Using this relation, we obtain the tetrad components:

$$h^a_{\mu} = \delta_0^a \delta_{\mu}^0 + A \delta_1^a \delta_{\mu}^1 + B \delta_2^a \delta_{\mu}^2 + C \delta_3^a \delta_{\mu}^3 \quad (55)$$

and its inverse is

$$h_a^{\mu} = \delta_a^0 \delta_0^{\mu} + A^{-1} \delta_a^1 \delta_1^{\mu} + B^{-1} \delta_a^2 \delta_2^{\mu} + C^{-1} \delta_a^3 \delta_3^{\mu} \quad (56)$$

From the Christoffel symbols definition which are given by equation

$$\{\alpha_{\mu\nu}\} = \frac{1}{2} g^{\alpha\beta} (\partial_{\mu} g_{\beta\nu} + \partial_{\nu} g_{\beta\mu} - \partial_{\beta} g_{\mu\nu}). \quad (57)$$

we get following non-vanishing components:

$$\{\alpha_{11}\} = A\dot{A}, \quad \{\alpha_{22}\} = B\dot{B}, \quad \{\alpha_{33}\} = C\dot{C} \quad (58)$$

$$\{\alpha_{10}\} = \{\alpha_{01}\} = \frac{\dot{A}}{A}, \quad \{\alpha_{22}\} = -\frac{B^2}{A^2} \quad (59)$$

$$\{\alpha_{33}\} = -\frac{C^2}{A^2}, \quad \{\alpha_{20}\} = \{\alpha_{02}\} = \frac{\dot{B}}{B} \quad (60)$$

$$\{\alpha_{12}\} = \{\alpha_{21}\} = \{\alpha_{13}\} = \{\alpha_{31}\} = 1 \quad (61)$$

$$\{\alpha_{30}\} = \{\alpha_{03}\} = \frac{\dot{C}}{C}. \quad (62)$$

A. Solution in General Relativity

The energy-momentum in all formulations of Møller, Einstein, Bergmann-Thomson, Landau-Lifshitz is found easily

$$P_{\mu} = 0 \quad (\mu = 0, 1, 2, 3) \quad (63)$$

which means, energy $E = 0$ and momentum components $P_1 = P_2 = P_3 = 0$.

B. Solution in Teleparallel Gravity

1. Møller's Definition

Considering the results given above, we find following non-vanishing components of basic vector field:

$$\Phi^0 = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (64)$$

From equation (34) with the results which are given by the equations (58), (59), (60), (61), (62) and (64) we find the required components of superpotential of Møller's are zero. Substituting these values into the equations (39), (40) and (41), then we find

$$E = P_i = 0 \quad (65)$$

2. Einstein, Bergmann-Thomson and Landau-Lifshitz's Definitions

Using equations (55) and (56), we find the following non-vanishing components of the Weitzenböck connection:

$$\Gamma^1_{10} = \frac{\dot{A}}{A}, \quad \Gamma^2_{20} = \frac{\dot{B}}{B}, \quad \Gamma^3_{30} = \frac{\dot{C}}{C} \quad (66)$$

where a dot indicates derivative with respect to t . The corresponding non-vanishing torsion components are found

$$T^1_{01} = -T^1_{10} = \frac{\dot{A}}{A} \quad (67)$$

$$T^2_{02} = -T^2_{20} = \frac{\dot{B}}{B} \quad (68)$$

$$T^3_{03} = -T^3_{30} = \frac{\dot{C}}{C} \quad (69)$$

Taking these results into equation (46), the non-zero components of the tensor $S_\mu^{\nu\lambda}$ are found as:

$$S_1^{10} = -S_1^{01} = \left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) \frac{\dot{A}}{A} + \frac{m_3 A^2}{2} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (70)$$

$$S_2^{20} = -S_2^{02} = \left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) \frac{\dot{B}}{B} + \frac{m_3 B^2}{2} \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) \quad (71)$$

$$S_3^{30} = -S_3^{03} = \left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) \frac{\dot{C}}{C} + \frac{m_3 C^2}{2} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \quad (72)$$

Now, using equation (45) the non-vanishing components of Freud's superpotential are:

$$U_1^{10} = -U_1^{01} = \dot{A}BC \left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) + \frac{m_3 A^3}{2} (C\dot{B} + B\dot{C}) \quad (73)$$

$$U_2^{20} = -U_2^{02} = \dot{B}AC \left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) + \frac{m_3 B^3}{2} (C\dot{A} + A\dot{C}) \quad (74)$$

$$U_3^{30} = -U_3^{03} = \dot{C}AB \left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) + \frac{m_3 C^3}{2} (B\dot{A} + A\dot{B}) \quad (75)$$

By using the Einstein, Bergmann-Thomson and Landau-Lifshitz energy-momentum definitions, we find

$$P_0 = P_i = 0 \quad (76)$$

V. SUMMARY AND DISCUSSIONS

The problem of energy and momentum localization in the general theory of relativity and teleparallel gravity has been very exciting and interesting; however, it has been associated with some debate. Recently, some researchers interested in studying the energy content of the universe in various models. It is usually suspected that different energy-momentum prescriptions could give different result for a given geometry. Using in the context

of the teleparallel theory of gravity, we have calculated energy and momentum of the Heckmann-Schucking metric. In order to compute the gravitational part of energy and momentum, we have considered the teleparallel analogs of Møller, Einstein, Bergmann-Thomson and Landau-Lifshitz energy-momentum definitions.

Our first result is that energy-momentum vanishes whatever be the pseudotensor used to describe the gravitational energy-momentum in the Heckmann-Schucking metric. It is also independent of the teleparallel dimensionless coupling constants, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model. The second result is that our study supports the viewpoints of Albrow and Tryon and this work agree with the previous works of Cooperstock-Israelit, Rosen, Johri *et al.*, Banerjee-Sen, Xulu, Vargas and Salti *et al.*

APPENDIX: The energy-momentum tensors of the gravitational fields and the material field

Varying with respect to metric tensor $g_{\mu\nu}$ one finds the gravitational field equations which have the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \kappa T_1^1 \quad (77)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = \kappa T_2^2 \quad (78)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \kappa T_3^3 \quad (79)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \kappa T_0^0 \quad (80)$$

Here κ is the Einstein gravitational constant. The energy-momentum tensor of the material field is given by

$$T_\mu^\rho = \frac{i}{4} g^{\rho\nu} (\bar{\psi}\gamma_\mu \nabla_\nu \psi + \bar{\psi}\gamma_\nu \nabla_\mu \psi - \nabla_\mu \bar{\psi}\gamma_\nu \psi - \nabla_\nu \bar{\psi}\gamma_\mu \psi) + (1 - \lambda F)\varphi_{,\mu}\varphi^{,\rho} - \delta_\mu^\rho L + T_{\mu(m)}^\nu \quad (81)$$

where L and F are Lagrangian density and some arbitrary functions, respectively and $T_{\mu(m)}^\nu = (\epsilon, -p, -p, -p)$ is the energy-momentum tensor of a perfect fluid. Energy ϵ is related to the pressure p by the equation of state $p = \varrho\epsilon$. Here ϱ varies between the interval $0 \leq \varrho \leq 1$, whereas $\varrho = 0$ describes the dust universe, $\varrho = \frac{1}{3}$ presents radiation universe, $\frac{1}{3} < \varrho < 1$ ascribes hard universe and $\varrho = 1$ corresponds to the stiff matter.

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