The effect of the location of the new Higgs doublet on the radiative lepton flavor violating decays in the split fermion scenario.

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Abstract

We study the branching ratios of the lepton flavor violating processes $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ in the split fermion scenario, with the assumption that the new Higgs doublet is restricted to the 4D brane (thin bulk) in one and two extra dimensions, in the framework of the two Higgs doublet model. We observe that the branching ratios are sensitive to the location of the 4D brane and, in the second case, the width of the thin bulk, especially for the $\mu \to e\gamma$ decay.

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1 Introduction

Since the existence of the radiative lepton flavor violating (LFV) decays depends on the flavor changing currents, they appear in the loop level and, therefore, they are rich from the theoretical point of view. In the standard model (SM), even with the neutrino mixing with non zero neutrino masses, their calculated branching ratios (BRs) are too small to reach the experimental limits. This motivates one to search new models beyond the SM and to extend the particle spectrum in order to enhance the BRs of these decays.

The discoveries of heavy leptons stimulated the experimental work for the LFV decays. The current limits for the (BRs) of $\mu \to e\gamma$ and $\tau \to e\gamma$ decays are 1.2×10^{-11} [1] and 3.9×10^{-7} [2], respectively. To search for the LFV decay $\mu \to e\gamma$ [3] a new experiment at PSI has been described and aimed to reach to a sensitivity of $BR \sim 10^{-14}$, improved by three order of magnitudes with respect to previous searches. At present the experiment (PSI-R-99-05 Experiment) is still running in the MEG [4]. On the other hand, the BR of $\tau \to \mu\gamma$ decay has been measured as 1.1×10^{-6} [5], and recently, an upper limit of $BR = 9.0\,(6.8)\,10^{-8}$ at 90% CL has been obtained [6] ([7]), which is an improvement almost by one order of magnitude with respect to previous one.

Besides the experimental work, there is an extensive theoretical analysis done on the radiative LFV decays in the literature [8]-[14]. These decays are studied in the supersymmetric models [8], in the framework of the two Higgs doublet model (2HDM) [9, 10, 11, 12] and in a model independent way [13]. Recently, they are analyzed in the framework of 2HDM and the supersymmetric model in [14].

In the present work, we study the LFV processes $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ in the 2HDM and we respect the idea that the hierarchy of fermion masses is coming from the overlap of the fermion Gaussian profiles in the extra dimensions, so called the split fermion scenario [15]. Here, the extension of the Higgs sector and the permission of the flavor changing neutral currents (FCNC's) at tree level, ensure the BRs of the radiative decays under consideration to be enhanced, theoretically. In addition to this, the extension of the space-time, the inclusion of one (two) extra spatial dimension, causes certain modifications in the BRs.

In the split fermion scenario, the fermions are assumed to locate at different points in the extra dimensions with the exponentially small overlaps of their wavefunctions and there are various studies in the literature [15]-[25]. One of the phenomenologically reliable set of explicit positions of left and right handed components of fermions in a single extra dimension have been predicted in [16] and this is the one we use in our numerical calculations. In [17], the restrictions on the split fermions in the extra dimensions have been obtained by using the leptonic W decays and the lepton violating processes, and the CP violation in the quark sector has been studied in [18]. [19] is devoted to find stringent bounds on the size of the compactification scale 1/R, the physics of kaon, neutron and B/D mesons and, in [20], the rare processes in the split fermion scenario have been studied. The shapes and overlaps of the fermion wave functions in the split fermion model has been estimated in [21] and the work in [12] ([22],[23]) is related to the the radiative LFV decays (electric dipole moments of charged leptons, the LFV $Z \to l_i\, l_j$ decays) in the split fermion scenario. Recently, the Higgs localization in the split fermion models has been studied in [24].

In our calculations, we consider that the leptons have Gaussian profiles in the extra dimension(s). Furthermore, we first assume that the new Higgs doublet lies in 4D brane, whose coordinate(s) in one (two) extra dimension(s) is arbitrary, $y_p\, \sigma$ $(y_p \sigma, z_p \sigma)$ where σ is the width of the Gaussian lepton profile in the extra dimension(s). Second, we take that the new Higgs doublet lies in one (two) extra dimension(s) but restricted into the thin bulk which has width $w\, R\, (w_y\, R, w_z\, R)$, $w \leq 2\,\pi$ $(w_y \leq 2\,\pi,\, w_z \leq 2\,\pi)$. We observe that the BRs are sensitive to the location of the 4D brane and, in the second case, the width of the thin bulk, especially for the $\mu \to e\gamma$ decay.

The paper is organized as follows: In Section 2, we present the lepton-lepton-new Higgs scalar vertices and the BRs of the radiative LFV decays in the split fermion scenario with the assumption that the new Higgs doublet is restricted to the 4D brane (thin bulk) in the one and two extra dimensions, in the framework of the 2HDM. Section 3 is devoted to discussion and our conclusions.

2 The possible effects on the radiative LFV decays in the split fermion scenario, due to the different locations of the new Higgs doublet in the extra dimensions, in the 2HDM

The radiative LFV decays $l_i \to l_j\, \gamma$ exist at least at one loop level in the SM and the numerical values of the BRs of these decays are extremely small. To enhance them, one goes beyond the SM and the version of the 2HDM, permitting the existence of the FCNCs at tree level, is one of the candidate to obtain relatively large BRs, since the extended Higgs sector brings additional contributions. These contributions are controlled by the new Yukawa couplings, which are complex in general. Besides, the inclusion of the spatial extra dimensions causes to enhance the BRs, since the particle spectrum is further extended after the compactification. In our analysis, we consider effects of the extension of the Higgs sector and the extra dimensions. Here we respect the split fermion scenario which is based on the idea that the hierarchy of lepton masses are coming from the lepton Gaussian profiles in the extra dimensions.

Now, we present the Yukawa Lagrangian responsible for these interactions in a single (two) extra dimension, respecting the split fermion scenario,

$$\mathcal{L}_Y = \xi^{E}_{5\,(6)\,ij} \bar{\hat{l}}_{iL} \phi_2 \hat{E}_{jR} + h.c. \;, \tag{1}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet and $\xi^{E}_{5\,(6)\,ij}$ are the flavor violating complex Yukawa couplings in five (six) dimensions. Here, $\hat{l}_{iL}$ ($\hat{E}_{jR}$), with family indices i, j, are the zero mode[1] lepton doublets (singlets) with Gaussian profiles in the extra dimension(s) y (y, z) and, in a single extra dimensions, they read

$$\begin{aligned} \hat{l}_{iL} &= N\, e^{-(y-y_{iL})^2/2\sigma^2}\, l_{iL} , \\ \hat{E}_{jR} &= N\, e^{-(y-y_{jR})^2/2\sigma^2}\, E_{jR} , \end{aligned} \tag{2}$$

with the normalization factor $N = \frac{1}{\pi^{1/4}\,\sigma^{1/2}}$. In the case that the leptons are accessible two both dimensions with Gaussian profiles, we get

$$\begin{aligned} \hat{l}_{iL} &= N\, e^{-\left((y-y_{iL})^2+(z-z_{iL})^2\right)/2\sigma^2}\, l_{iL} , \\ \hat{E}_{jR} &= N\, e^{-\left((y-y_{jR})^2+(z-z_{jR})^2\right)/2\sigma^2}\, E_{jR} , \end{aligned} \tag{3}$$

with the normalization factor $N = \frac{1}{\pi^{1/2}\,\sigma}$. Here, l_{iL} (E_{jR}) are the lepton doublets (singlets) in four dimensions and the parameter σ represents the Gaussian width of the leptons, satisfying the property $\sigma << R$, where R is the compactification radius. In eq. (2) (eq. (3)), the parameters y_{iL}- y_{iR} (y_{iL} and z_{iL} - y_{iR} and z_{iR}) are the fixed positions of the peaks of left-right handed parts of i^{th} lepton in the fifth (fifth and sixth) dimension and they are obtained by taking the observed lepton masses into account (see [16] for a single extra dimension case). The underlying idea is that the mass hierarchy of leptons are coming from the relative positions of the Gaussian peaks of the wave functions located in the extra dimension [15, 16]. One possible set of locations for the lepton fields in the fifth dimension read (see [16] for details)

$$P_{l_i} = \sqrt{2}\,\sigma \begin{pmatrix} 11.075 \\ 1.0 \\ 0.0 \end{pmatrix} \;, \quad P_{e_i} = \sqrt{2}\,\sigma \begin{pmatrix} 5.9475 \\ 4.9475 \\ -3.1498 \end{pmatrix} \;. \tag{4}$$

[1]Notice that we take only the zero mode lepton fields in our calculations.

For two extra dimensions, a possible positions of left handed and right handed leptons can be obtained by using the observed masses [2]. With the assumption that the lepton mass matrix is diagonal, one of the possible set of locations for the Gaussian peaks of the lepton fields in the two extra dimensions are [22]

$$P_{l_i} = \sqrt{2}\,\sigma \left(\begin{array}{c} (8.417, 8.417) \\ (1.0, 1.0) \\ (0.0, 0.0) \end{array} \right) , \quad P_{e_i} = \sqrt{2}\,\sigma \left(\begin{array}{c} (4.7913, 4.7913) \\ (3.7913, 3.7913) \\ (-2.2272, -2.2272) \end{array} \right) , \tag{5}$$

where the numbers in the parenthesis denote the y and z coordinates of the location of the Gaussian peaks of lepton flavors in the extra dimensions. Notice that we choose the same numbers for the y and z locations of the Gaussian peaks.

Here, we take that the new Higgs sector does not mix with the old one and collect SM (new) particles in the first (second) doublet. We choose the Higgs doublets ϕ_1 and ϕ_2 as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array} \right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array} \right) \right] ; \phi_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2}H^+ \\ H_1 + iH_2 \end{array} \right) , \tag{6}$$

with the vacuum expectation values,

$$< \phi_1 > = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v \end{array} \right) ; < \phi_2 > = 0 \,, \tag{7}$$

and H_1 and H_2 are the mass eigenstates h^0 and A^0 respectively since no mixing occurs between two CP-even neutral bosons H^0 and h^0 at tree level. In this case, the first Higgs doublet is responsible for the hierarchy of lepton masses and the LFV interaction at tree level is carried by the new Higgs field ϕ_2. Now, we follow two possibilities:

- The new Higgs doublet lies in 4D brane, whose coordinate(s) in one (two) extra dimension(s) is arbitrary, $y_p\,\sigma$ $(y_p\,\sigma, z_p\,\sigma)$ and leptons are living in one (two) extra dimension(s);

- The new Higgs doublet lies in one (two) extra dimension(s) but restricted into the thin bulk which has width $w\,R\,(w_y\,R, w_z\,R)$, $w \leq 2\,\pi$ $(w_y \leq 2\,\pi,\ w_z \leq 2\,\pi)$ and leptons are living in one (two) extra dimension;

and present the lepton-lepton-S, $S = h^0, A^0$ vertex factors after the integration over the extra dimension(s).

[2]The calculation is similar to the one presented in [16] which is done for a single extra dimension.

2.1 The vertex factors for the case that new Higgs doublet lives in the 4D brane

The LFV processes $l_i \to l_j \gamma$ are carried by the lepton-lepton-S ($S = h^0, A^0$) vertices. The vertex factors $V_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\, S(y_p)\, \hat{f}_{jR\,(L)}$ with the right (left) handed i^{th} flavor lepton fields $\hat{f}_{jR\,(L)}$ (see eq. (2)) and the new Higgs bosons S on the 4D brane with the location $y = y_p\,\sigma$ in five dimensions , are obtained by the integration over the fifth dimension and they read

$$V^{br}_{LR\,(RL)\,ij} = V^{br,0}_{LR\,(RL)\,ij}\, V^{br,extr}_{LR\,(RL)\,ij} \; , \tag{8}$$

where

$$V^{br,extr}_{LR\,(RL)\,ij} = e^{-y_p\,(-y_p\,\sigma + y_{iL\,(R)} + y_{jR\,(L)})/\sigma} \; . \tag{9}$$

Here the factor $V^{br,0}_{LR\,(RL)\,ij}$ is included in the definition of the coupling in four dimensions

$$\xi^E_{ij}\left((\xi^{E\dagger}_{ij})^\dagger\right) = V^{br,0}_{LR\,(RL)\,ij}\, \xi^E_{5\,ij}\left((\xi^E_{5\,ij})^\dagger\right)/2\pi R \; , \tag{10}$$

and it is obtained as

$$V^{br,0}_{LR\,(RL)\,ij} = \frac{2\,\sqrt{\pi}\,R}{\sigma}\, e^{-(y^2_{iL\,(R)} + y^2_{jR\,(L)})/2\,\sigma^2} \; , \tag{11}$$

by choosing that the new Higgs bosons place on the 4D brane at $y = 0$. In the case of two extra dimensions where the leptons feel, the lepton-lepton-S vertex factors $V^{br,(2)}_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\, S(y_p, z_p)\, \hat{f}_{jR\,(L)}$, with the right (left) handed i^{th} flavor lepton fields $\hat{f}_{jR\,(L)}$ (see eq. (3)) and the Higgs bosons S restricted on the brane at $y = y_p\,\sigma, z = z_p\,\sigma$ in six dimensions, are obtained by the integration over the fifth and sixth dimensions as

$$V^{br,(2)}_{LR\,(RL)\,ij} = V^{br,0,\,(2)}_{LR\,(RL)\,ij}\, V^{br,\,extr2}_{LR\,(RL)\,ij} \; , \tag{12}$$

where

$$V^{br,\,extr2}_{LR\,(RL)\,ij} = \mathbf{e}^{\left(y_p\,(-y_p\,\sigma + y_{iL\,(R)} + y_{jR\,(L)}) + z_p\,(-z_p\,\sigma + z_{iL\,(R)} + z_{jR\,(L)})\right)/\sigma} \; . \tag{13}$$

Here the factor $V^{br,0\,(2)}_{LR\,(RL)\,ij}$ is included in the definition of the coupling in four dimensions

$$\xi^E_{ij}\left((\xi^E_{ij})^\dagger\right) = V^{br,0\,(2)}_{LR\,(RL)\,ij}\, \xi^E_{6\,ij}\left((\xi^E_{6\,ij})^\dagger\right)/(2\pi R)^2 \, , \tag{14}$$

and it is obtained by choosing that the new Higgs bosons place in the 4D brane at $y, z = 0$:

$$V^{br,0,\,(2)}_{LR\,(RL)\,ij} = \frac{4\,\pi\,R^2}{\sigma^2}\, e^{-(y^2_{iL\,(R)} + y^2_{jR\,(L)} + z^2_{iL\,(R)} + z^2_{jR\,(L)})/2\,\sigma^2} \; . \tag{15}$$

2.2 The vertex factors in the case that the new Higgs doublet lies in the thin bulk placed around the origin, in the bulk

If we consider that the new Higgs doublet lies in a single extra dimension and is restricted into the thin bulk, having a width $w\,R$, $w \leq 2\,\pi$, which is placed symmetrically around the origin, after the compactification on the orbifold S^1/Z_2, it reads

$$\phi_2(x,y) \;=\; \frac{1}{\sqrt{w\,R}}\left\{\phi_2^{(0)}(x)+\sqrt{2}\sum_{n=1}^{\infty}\phi_2^{(n)}(x)\cos\big(\frac{2\,\pi\,n\,y}{w\,R}\big)\right\}\,, \tag{16}$$

where $\phi_2^{(0)}(x)$ ($\phi_2^{(n)}(x)$) is the Higgs doublet in the four dimensions (the KK modes) including the charged Higgs boson H^+ ($H^{(n)+}$), the neutral CP even-odd Higgs bosons h^0- A^0 ($h^{0(n)}$-$A^{0(n)}$). The non-zero n^{th} KK mode of the charged Higgs mass is $\sqrt{m_{H^\pm}^2+m_n^2}$, and the neutral CP even (odd) Higgs mass is $\sqrt{m_{h^0}^2+m_n^2}$, ($\sqrt{m_{A^0}^2+m_n^2}$), with the n'th level KK particle mass $m_n=\frac{2\,\pi\,n}{w\,R}$. In the two extra dimensions, after the compactification on the orbifold $(S^1\times S^1)/Z_2$, the new Higgs field ϕ_2 is expanded as

$$\phi_2(x,y,z) \;=\; \frac{1}{\sqrt{w_y\,w_z}\,R}\left\{\phi_2^{(0,0)}(x)+2\sum_{n,s}^{\infty}\phi_2^{(n,s)}(x)\cos\Big(\frac{2\,\pi}{R}\big(\frac{ny}{w_y}+\frac{sz}{w_z}\big)\Big)\right\}\,. \tag{17}$$

where $w_y\,R$ and $w_z\,R$ are widths of thin rectangular bulk volume, having the center at the origin, with, $w_y \leq 2\,\pi$ and $w_z \leq 2\,\pi$. The KK modes of charged (neutral CP even, neutral CP odd) Higgs fields existing in the new Higgs doublet have the masses $\sqrt{m_{H^\pm}^2+m_n^2+m_s^2}$, ($\sqrt{m_{h^0}^2+m_n^2+m_s^2}$, $\sqrt{m_{A^0}^2+m_n^2+m_s^2}$) where $m_{n\,(s)}=\frac{2\,\pi\,n\,(s)}{w_{y\,(z)}\,R}$ are the masses of $n\,(s)$'th level KK modes. Now, we present the lepton-lepton-S vertex factors $V^{bulk}_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\,S^{(0)}(x,y)\,\hat{f}_{jR\,(L)}$, with $S^{(0)}=h^0,A^0$ zero modes and the right (left) handed i^{th} flavor lepton fields $\hat{f}_{jR\,(L)}$ in five dimensions (see eq. (2)). After the integration over the fifth dimension we get

$$V^{bulk}_{LR\,(RL)\,ij}=V^{bulk,0}_{LR\,(RL)\,ij}\,V^{bulk,extr}_{LR\,(RL)\,ij}\,, \tag{18}$$

where

$$V^{bulk,extr}_{LR\,(RL)\,ij}=\sqrt{\frac{\pi}{2\,w}}\,\Big(Erf[f^{(0)}_{LR\,(RL)\,ij}(y_p)]-Erf[f^{(w)}_{LR\,(RL)\,ij}(y_p)]\Big)\,, \tag{19}$$

with

$$\begin{aligned} f^{(w)}_{LR\,(RL)\,ij}(y_p) &= \frac{-2\,(y_p+w)\,R+y_{iL\,(R)}+y_{jR\,(L)}}{2\,\sigma}\,,\\ f^{(0)}_{LR\,(RL)\,ij}(y_p) &= f^{(w)}_{LR\,(RL)\,ij}(y_p)|_{w\to 0}\,, \end{aligned} \tag{20}$$

and $y_p = -w/2$. Here the function $Erf[z]$ is the so called error function and it is defined as

$$Erf[z] = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt \; . \tag{21}$$

Notice that the factor $V^{bulk,0}_{LR\,(RL)\,ij}$ is included in the definition of the coupling in four dimensions as

$$\xi^E_{ij}\left((\xi^E_{ij})^\dagger\right) = V^{bulk,0}_{LR\,(RL)\,ij}\, \xi^E_{5\,ij}\left((\xi^E_{5\,ij})^\dagger\right)/\sqrt{2\pi R}\;, \tag{22}$$

where

$$V^{bulk,0}_{LR\,(RL)\,ij} = e^{-(y_{iL\,(R)}-y_{jR\,(L)})^2/4\sigma^2}\;, \tag{23}$$

which is responsible for the hierarchy of lepton masses and it is the only factor appearing in the case that the size of the thin bulk, restricting the new Higgs bosons, is $2\pi R$ (This is the case that the factor $V^{bulk,extr}_{LR\,(RL)\,ij}$ is unity.). In the case of the interaction of the KK modes for the thin bulk of S boson with the leptons, the lepton-lepton-$S^{(n)}$ vertex factor $V^{bulk,n}_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\, S^{(n)}(x,y)\, \hat{f}_{jR\,(L)}$ is obtained by the integration over the fifth dimension and it reads

$$V^{bulk,n}_{LR\,(RL)\,ij} = V^{bulk,0}_{LR\,(RL)\,ij}\, V^{bulk,(n),\,extr}_{LR\,(RL)\,ij}\;, \tag{24}$$

where

$$\begin{aligned} V^{bulk,(n),extr}_{LR\,(RL)\,ij} \;&=\; \frac{1}{2}\sqrt{\frac{\pi}{2\,w}}\Bigg\{ \\ & e^{+(n)}_{LR\,(RL)\,ij}\Big(Erf[f^{(0)}_{LR\,(RL)\,ij}(y_p)+\frac{i\,n\,\pi\,\rho}{w}] - Erf[f^{(w)}_{LR\,(RL)\,ij}(y_p)+\frac{i\,n\,\pi\,\rho}{w}]\Big) \\ & + e^{-(n)}_{LR\,(RL)\,ij}\Big(Erf[f^{(0)}_{LR\,(RL)\,ij}(y_p)-\frac{i\,n\,\pi\,\rho}{w}] - Erf[f^{(w)}_{LR\,(RL)\,ij}(y_p)-\frac{i\,n\,\pi\,\rho}{w}]\Big)\Bigg\}, \end{aligned} \tag{25}$$

and

$$e^{\pm(n)}_{LR\,(RL)\,ij} = Exp\Big[-\frac{n^2\pi^2\,\rho^2}{w^2} \pm \frac{i\,n\,\pi\,((y_{iL\,(R)}+y_{iR\,(L)})}{R\,w}\Big]\;. \tag{26}$$

Here the function $f^{(w)}_{LR\,(RL)\,ij}(y_p)$ is defined in eq. (20) and the parameter ρ is defined as $\rho = \sigma/R$.

If the new Higgs bosons and leptons are accessible to two extra dimensions, the lepton-lepton-S vertex factors $V^{bulk,(2)}_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\, S^{(0,0)}(x,y,z)\, \hat{f}_{jR\,(L)}$, with $S^{(0,0)} = h^0, A^0$ zero modes and the right (left) handed i^{th} flavor lepton fields $\hat{f}_{jR\,(L)}$ in six dimensions (see eq. (3)), are obtained by the integration over the fifth and sixth dimensions and as

$$V^{bulk,(2)}_{LR\,(RL)\,ij} = V^{0,0}_{LR\,(RL)\,ij}\, V^{bulk,\,extr2}_{LR\,(RL)\,ij}\;, \tag{27}$$

where

$$V^{bulk,\,extr2}_{LR\,(RL)\,ij} = \frac{\pi}{2}\sqrt{\frac{1}{w_y\,w_z}}\prod_{r=y,z}\Big(Erf[f^{(0)}_{LR\,(RL)\,ij}(r_p)] - Erf[f^{(w_r)}_{LR\,(RL)\,ij}(r_p)]\Big)\,. \tag{28}$$

Notice that the factor $V^{bulk,\,0,0}_{LR\,(RL)\,ij}$ is included in the definition of the coupling in four dimensions:

$$\xi^E_{ij}\left((\xi^E_{ij})^\dagger\right) = V^{bulk,\,0,0}_{LR\,(RL)\,ij}\,\xi^E_{6\,ij}\left((\xi^E_{6\,ij})^\dagger\right)/(2\pi R)\,, \tag{29}$$

where

$$V^{bulk,\,0,0}_{LR\,(RL)\,ij} = e^{-\Big((y_{iL\,(R)}-y_{iR\,(L)})^2+(z_{iL\,(R)}-z_{iR\,(L)})^2\Big)/4\sigma^2}\,, \tag{30}$$

which is responsible for the hierarchy of lepton masses in the case of two extra dimensions and it is the only factor appearing when the size of the thin bulk, restricting the new Higgs bosons, is $2\pi R$ in both directions (This is the case that the factor $V^{bulk,\,extr2}_{LR\,(RL)\,ij}$ is unity.). On the other hand, the lepton-lepton-$S^{(n,s)}$ vertex factor $V^{bulk,n,s}_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\,S^{(n,s)}(x,y)\,\hat{f}_{jR\,(L)}$ is obtained by the integration over the fifth and six dimensions and it reads

$$V^{bulk,n,s}_{LR\,(RL)\,ij} = V^{bulk,\,0,0}_{LR\,(RL)\,ij}\,V^{bulk,(n,s),\,extr2}_{LR\,(RL)\,ij}\,, \tag{31}$$

with

$$\begin{aligned} V^{bulk,(n,s),\,extr2}_{LR\,(RL)\,ij} &= \frac{\pi}{4}\sqrt{\frac{1}{w_y\,w_z}}\Big\{ \\ & e^{+(n,s)}_{LR\,(RL)\,ij}\prod_{(r;t)=\binom{(y;n)}{(z;s)}}\Big(Erf[f^{(0)}_{LR\,(RL)\,ij}(r_p)+\frac{i\,t\,\pi\,\rho}{w_r}] \\ &- Erf[f^{(w_r)}_{LR\,(RL)\,ij}(r_p)+\frac{i\,t\,\pi\,\rho}{w_r}]\Big) \\ &+ e^{-(n,m)}_{LR\,(RL)\,ij}\prod_{(r;t)=\binom{(y;n)}{(z;s)}}\Big(Erf[f^{(0)}_{LR\,(RL)\,ij}(r_p)-\frac{i\,t\,\pi\,\rho}{w_r}] \\ &- Erf[f^{(w_r)}_{LR\,(RL)\,ij}(r_p)-\frac{i\,t\,\pi\,\rho}{w_r}]\Big)\Big\}\,, \end{aligned} \tag{32}$$

where

$$e^{\pm(n,s)}_{LR\,(RL)\,ij} = Exp\Big[-\pi^2\,\rho^2\,(\frac{n^2}{w_y^2}+\frac{s^2}{w_z^2}) \pm \frac{i\,\pi}{R}\Big(\frac{n\,((y_{iL\,(R)}+y_{iR\;(L)})}{w_y}+\frac{s\,((z_{iL\,(R)}+z_{iR\;(L)})}{w_z}\Big)\Big]\,. \tag{33}$$

2.3 The decay widths of LFV decays $l_i \to l_j \gamma$

Now, we will present the decay widths of the LFV processes $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ for both cases. Since they exist at loop level, the logarithmic divergences appear in the calculations and we eliminate them by using the on-shell renormalization scheme [3]. Taking only τ lepton for the internal line, [4] the decay width Γ for the $l_i \to l_j \gamma$ decay reads

$$\Gamma(l_i \to l_j \gamma) = c_1(|A_1|^2 + |A_2|^2) \, , \tag{34}$$

for $l_i\,(l_j) = \tau; \mu\,(\mu \text{ or } e; e)$. Here $c_1 = \frac{G_F^2 \alpha_{em} m_{l_i}^3}{32\pi^4}$, A_1 (A_2) is the left (right) chiral amplitude.

If the new Higgs doublet lies in 4D brane, with the coordinate $y_p\,\sigma$,in a single extra dimension, the amplitudes read,

$$\begin{aligned}
A_1 &= Q_\tau \frac{1}{48\, m_\tau^2} \Bigg(6\, m_\tau\, \bar{\xi}^{E*}_{N,\tau f_2}\, \bar{\xi}^{E*}_{N,f_1\tau}\, V^{br,extr}_{LR\, f_1\, \tau}\, V^{br,extr}_{RL\, f_2\, \tau} \Big(F(v_{h^0}) - F(v_{A^0}) \Big) \\
&+ m_{f_1}\, \bar{\xi}^{E*}_{N,\tau f_2}\, \bar{\xi}^{E}_{N,\tau f_1}\, V^{br,extr}_{RL\, f_1\, \tau}\, V^{br,extr}_{RL\, f_2\, \tau} \Big(G(v_{h^0}) + G(v_{A^0}) \Big) \Bigg) \, , \\
A_2 &= Q_\tau \frac{1}{48\, m_\tau^2} \Bigg(6\, m_\tau\, \bar{\xi}^{E}_{N,f_2\tau}\, \bar{\xi}^{E}_{N,\tau f_1}\, V^{br,extr}_{LR\, f_2\, \tau}\, V^{br,extr}_{RL\, f_1\, \tau} \Big(F(v_{h^0}) - F(v_{A^0}) \Big) \\
&+ m_{f_1}\, \bar{\xi}^{E}_{N,f_2\tau}\, \bar{\xi}^{E*}_{N,f_1\tau}\, V^{br,extr}_{LR\, f_2\, \tau}\, V^{br,extr}_{LR\, f_1\, \tau} \Big(G(v_{h^0}) + G(v_{A^0}) \Big) \Bigg) \, ,
\end{aligned} \tag{35}$$

where $v_S = \frac{m_\tau^2}{m_S^2}$, and $V^{br,extr}_{LR\,(RL)\, f_1\, \tau}$ and $V^{br,extr}_{RL\,(LR)\, f_2\, \tau}$ are defined in eq. (9). Here the functions $F(w)$ and $G(w)$ are given by

$$\begin{aligned}
F(w) &= \frac{w\,(3 - 4\,w + w^2 + 2\,ln\,w)}{(-1+w)^3} \, , \\
G(w) &= \frac{w\,(2 + 3\,w - 6\,w^2 + w^3 + 6\,w\,ln\,w)}{(-1+w)^4} \, .
\end{aligned} \tag{36}$$

In the case that new Higgs doublet lies in 4D brane, with the location $y_p\,\sigma, z_p\,\sigma$ and the leptons have gaussian profiles in the two extra dimensions, the amplitudes would be the same except the vertex factors $V^{br,extr}_{LR\, f_1\, \tau}$ and $V^{br,extr}_{RL\, f_2\, \tau}$ are replaced by $V^{br,\,extr2}_{LR\, f_1\, \tau}$ and $V^{br,\,extr2}_{RL\, f_2\, \tau}$ (see eq. (13)).

[3] In this scheme, the self energy diagrams for on-shell leptons vanish since they can be written as $\sum(p) = (\hat{p} - m_{l_1})\bar{\sum}(p)(\hat{p} - m_{l_2})$, however, the vertex diagrams (see Fig.1) give non-zero contribution. In this case, the divergences can be eliminated by introducing a counter term V^C_μ with the relation $V^{Ren}_\mu = V^0_\mu + V^C_\mu$, where V^{Ren}_μ (V^0_μ) is the renormalized (bare) vertex and by using the gauge invariance: $k^\mu V^{Ren}_\mu = 0$. Here, k^μ is the four momentum vector of the outgoing photon.

[4] We take into account only the internal τ-lepton contribution since, we respect the Sher scenerio [26], results in the couplings $\bar{\xi}^E_{N,ij}$ $(i,j = e,\mu)$, are small compared to $\bar{\xi}^E_{N,\tau\, i}$ $(i = e,\mu,\tau)$, due to the possible proportionality of them to the masses of leptons under consideration in the vertices. Here, we use the dimensionful coupling $\bar{\xi}^E_{N,ij}$ with the definition $\xi^E_{N,ij} = \sqrt{\frac{4\,G_F}{\sqrt{2}}}\,\bar{\xi}^E_{N,ij}$ where N denotes the word "neutral".

As another possibility, we consider that the new Higgs doublet lies in the one extra dimension but restricted into the thin bulk which has width $w\,R$, $w \leq 2\,\pi$ and leptons have gaussian profiles in one extra dimension. Respecting this scenario, the amplitudes read,

$$\begin{aligned}
A_1 &= Q_\tau \frac{1}{48\,m_\tau^2}\Bigg\{6\,m_\tau\,\bar{\xi}^{E*}_{N,\tau f_2}\,\bar{\xi}^{E*}_{N,f_1\tau}\Bigg(V^{bulk,extr}_{LR\,f_1\,\tau}\,V^{bulk,extr}_{RL\,f_2\,\tau}\Big(F(v_{h^0})-F(v_{A^0})\Big)\\
&+ 2\sum_{n=1}^{\infty} V^{bulk,(n),extr}_{LR\,f_1\,\tau}\,V^{bulk,n,extr}_{RL\,f_2\,\tau}\Big(F(v_{n,h^0})-F(v_{n,A^0})\Big)\Bigg)\\
&+ m_{f_1}\,\bar{\xi}^{E*}_{N,\tau f_2}\,\bar{\xi}^{E}_{N,\tau f_1}\Bigg(V^{bulk,extr}_{RL\,f_1\,\tau}\,V^{bulk,extr}_{RL\,f_2\,\tau}\Big(G(v_{h^0})+G(v_{A^0})\Big)\\
&+ 2\sum_{n=1}^{\infty} V^{bulk,(n),extr}_{RL\,f_1\,\tau}\,V^{bulk,n,extr}_{RL\,f_2\,\tau}\Big(G(v_{n,h^0})+G(v_{n,A^0})\Big)\Bigg)\Bigg\}\,,\\
A_2 &= Q_\tau \frac{1}{48\,m_\tau^2}\Bigg\{6\,m_\tau\,\bar{\xi}^{E}_{N,f_2\tau}\,\bar{\xi}^{E}_{N,\tau f_1}\Bigg(V^{bulk,extr}_{LR\,f_2\,\tau}\,V^{bulk,extr}_{RL\,f_1\,\tau}\Big(F(v_{h^0})-F(v_{A^0})\Big)\\
&+ 2\sum_{n=1}^{\infty} V^{bulk,(n),extr}_{LR\,f_2\,\tau}\,V^{bulk,(n),extr}_{RL\,f_1\,\tau}\Big(F(v_{n,h^0})-F(v_{n,A^0})\Big)\Bigg)\\
&+ m_{f_1}\,\bar{\xi}^{E}_{N,f_2\tau}\,\bar{\xi}^{E*}_{N,f_1\tau}\Bigg(V^{bulk,extr}_{LR\,f_2\,\tau}\,V^{bulk,extr}_{LR\,f_1\,\tau}\Big(G(v_{h^0})+G(v_{A^0})\Big)\\
&+ 2\sum_{n=1}^{\infty} V^{bulk,(n),extr}_{LR\,f_2\,\tau}\,V^{bulk,(n),extr}_{LR\,f_1\,\tau}\Big(G(v_{n,h^0})+G(v_{n,A^0})\Big)\Bigg)\Bigg\}\,, \qquad (37)
\end{aligned}$$

where $v_{n,S} = \frac{m_\tau^2}{m_S^2+m_n^2}$, $m_n = \frac{2\,\pi\,n}{w\,R}$ and Q_τ is the charge of τ lepton. Here the vertex factors $V^{bulk,extr}_{LR\,(RL)\,f_1\,\tau}$ and $V^{bulk,extr}_{RL\,(LR)\,f_2\,\tau}$ ($V^{bulk,(n),extr}_{LR\,(RL)\,f_1\,\tau}$ and $V^{bulk,(n),extr}_{RL\,(LR)\,f_2\,\tau}$) are defined in eq. (19) (eq. (25)).

If the new Higgs doublet lies in the two extra dimensions but restricted into the thin bulk which has widths $w_y\,R, w_z\,R$, $w_y \leq 2\,\pi, w_z \leq 2\,\pi$ and leptons have gaussian profiles in two extra dimensions, the amplitudes become,

$$\begin{aligned}
A_1 &= Q_\tau \frac{1}{48\,m_\tau^2}\Bigg\{6\,m_\tau\,\bar{\xi}^{E*}_{N,\tau f_2}\,\bar{\xi}^{E*}_{N,f_1\tau}\Bigg(V^{bulk,extr2}_{LR\,f_1\,\tau}\,V^{bulk,extr2}_{RL\,f_2\,\tau}\Big(F(v_{h^0})-F(v_{A^0})\Big)\\
&+ 4\sum_{(n,s)}^{\infty} V^{bulk,(n,s),\,extr2}_{LR\,f_1\,\tau}\,V^{bulk,(n,s),\,extr2}_{RL\,f_2\,\tau}\Big(F(v_{(n,m),h^0})-F(v_{(n,m),A^0})\Big)\Bigg)\\
&+ m_{f_1}\,\bar{\xi}^{E*}_{N,\tau f_2}\,\bar{\xi}^{E}_{N,\tau f_1}\Bigg(V^{bulk,extr2}_{RL\,f_1\,\tau}\,V^{bulk,extr2}_{RL\,f_2\,\tau}\Big(G(v_{h^0})+G(v_{A^0})\Big)\\
&+ 4\sum_{(n,s)}^{\infty} V^{bulk,(n,s),\,extr2}_{RL\,f_1\,\tau}\,V^{bulk,(n,s),\,extr2}_{RL\,f_2\,\tau}\Big(G(v_{(n,s),h^0})+G(v_{(n,s),A^0})\Big)\Bigg)\Bigg\}\,,\\
A_2 &= Q_\tau \frac{1}{48\,m_\tau^2}\Bigg\{6\,m_\tau\,\bar{\xi}^{E}_{N,f_2\tau}\,\bar{\xi}^{E}_{N,\tau f_1}\Bigg(V^{bulk,extr2}_{LR\,f_2\,\tau}\,V^{bulk,extr2}_{RL\,f_1\,\tau}\Big(F(v_{h^0})-F(v_{A^0})\Big)\\
&+ 4\sum_{(n,s)}^{\infty} V^{bulk,(n,s),extr2}_{LR\,f_2\,\tau}\,V^{bulk,(n,s),extr2}_{RL\,f_1\,\tau}\Big(F(v_{(n,s),h^0})-F(v_{(n,s),A^0})\Big)\Bigg)
\end{aligned}$$

$$
\begin{aligned}
&+ \quad m_{f_1}\, \bar{\xi}^{E}_{N,f_2\tau}\, \bar{\xi}^{E*}_{N,f_1\tau} \Bigg(V^{bulk,extr2}_{LR\, f_2\, \tau}\, V^{bulk,extr2}_{LR\, f_1\, \tau} \Big(G(v_{h^0}) + G(v_{A^0}\Big) \\
&+ \quad 4 \sum_{(n,s)}^{\infty} V^{bulk,(n,s),extr2}_{LR\, f_2\, \tau}\, V^{bulk,(n,s),extr2}_{LR\, f_1\, \tau} \Big(G(v_{(n,s),h^0}) + G(v_{(n,s),A^0})\Big)\Bigg)\Bigg\}\;, \qquad (38)
\end{aligned}
$$

where $v_{(n,s),S} = \frac{m_\tau^2}{m_S^2+m_n^2+m_s^2}$, $m_{n\,(s)} = \frac{2\,\pi\, n\,(s)}{w_{y\,(z)}\, R}$ and the vertex factors $V^{bulk,extr2}_{LR\,(RL)\, f_1\, \tau}$ and $V^{bulk,extr2}_{RL\,(LR)\, f_2\, \tau}$ ($V^{bulk,(n,s),extr2}_{LR\,(RL)\, f_1\, \tau}$ and $V^{bulk,(n,s),extr2}_{RL\,(LR)\, f_2\, \tau}$) are defined in eq. (28) (eq. (32)). In eq. (38) the summation would be done over $n, s = 0, 1, 2...$, except $n = s = 0$.

3 Discussion

The radiative LFV decays $l_i \to l_j \gamma$ are based on the FCNC currents and such currents are permitted at tree level in the framework of the general 2HDM. Since these decays exist at least in the one loop of level, the theoretical expressions of the physical parameters like the BR contains number of free parameters belonging to the model used and the Yukawa couplings, $\xi^{E}_{N,ij}$, $i, j = e, \mu, \tau$, which are strengths of lepton-lepton-S ($S = h^0, A^0$) vertices, playing essential role on the physical parameters, are among them. In the present work, we follow the split fermion scenario, which is among the possible solutions of the hierarchy of lepton masses, with the assumption that the lepton Gaussian profiles have different locations in the extra dimension. Here, we consider two possibilities: The first one is that the new Higgs doublet lies in 4D brane, whose coordinate(s) in one (two) extra dimension(s) is arbitrary, $y_p\, \sigma$ ($y_p\, \sigma$, $z_p\, \sigma$) and the split leptons are living in one (two) extra dimension(s). As a second case, we take that the new Higgs doublet lies in one (two) extra dimension(s) but restricted into the thin bulk, which has width, $w\, R\, (w_y\, R\, , w_z\, R\,)$, $w \leq 2\,\pi$ ($w_y \leq 2\,\pi$, $w_z \leq 2\,\pi$) and leptons are living in one (two) extra dimension. To get the interaction vertices lepton-lepton-S ($S = h^0, A^0$) in four dimensions, we make the integration over extra dimensions(s) and we calculate the additional factors to the Yukawa couplings. In the case that the new Higgs doublet lies in 4D brane, we take the additional effects $V^{br,extr}_{LR\,(RL)\, ij}$ (see eq. (9)) $\Big(V^{br,\,extr2}_{LR\,(RL)\, ij}$ (see eq. (13))$\Big)$, as the ones coming from the calculations for the positions $y_p \neq 0$ ($y_p \neq 0$, $z_p \neq 0$) of the 4D brane in one (two) extra dimenion(s), over the ones, which are obtained for the positions $y_p = 0$ ($y_p = 0$, $z_p = 0$) of the 4D brane, namely $V^{br,0}_{LR\,(RL)\, ij}$ (see eq. (11)) $\Big(V^{br,0,(2)}_{LR\,(RL)\, ij}$ (see eq. (15))$\Big)$, included in the original Yukawa coupling ξ^{E}_{ij} (see eq. (10) $\Big($eq. (14)$\Big)$. On the other hand, if the new Higgs doublet lies in one (two) extra dimension(s) but it is restricted into the thin bulk, the additional effects, $V^{bulk,extr}_{LR\,(RL)\, ij}$- $V^{bulk,(n),extr}_{LR\,(RL)\, ij}$ (see eq. (19-25)) $\Big(V^{bulk,extr2}_{LR\,(RL)\, ij}$-$V^{bulk,(n,s),extr2}_{LR\,(RL)\, ij}$ (see eq. (28-32))$\Big)$, are the ones coming from the calculations for $w \leq 2\,\pi$ ($w_y \leq 2\,\pi$, $w_z \leq 2\,\pi$) in the bulk, over the ones, which are obtained for the $w = 2\,\pi$ ($w_y = 2\,\pi$, $w_z = 2\,\pi$), in zero-KK mode level. We include the zero level results, $V^{bulk,0}_{LR\,(RL)\,ij}$ (see eq. (23)) $\Big(V^{bulk,0,0}_{LR\,(RL)\,ij}$ (see eq. (30))$\Big)$ in the original Yukawa coupling, $\xi^E_{N,ij}$ (see eq. (22) $\Big($eq. (29)$\Big)$.

In our calculations, we take split leptons in a single and two extra dimensions and use a possible set of locations to calculate the contributions under consideration. In the case of a single extra dimension (two extra dimensions) we use the estimated locations of the leptons given in eq. (4) (eq. (5)) to calculate the lepton-lepton-Higgs scalar vertices. For the parameter $\rho = \sigma/R$, where σ is the Gaussian width of the fermions (see [16] for details), we use the numerical value $\rho = 0.001$. Furthermore, we choose the appropriate numerical values for the Yukawa couplings, by respecting the current experimental measurements of these decays (see Introduction section) and the muon anomalous magnetic moment (see [27] and references therein). Notice that, for the Yukawa coupling $\bar{\xi}^E_{N,\tau\tau}$, we use the numerical value which is greater than the upper limit of $\bar{\xi}^E_{N,\tau\mu}$. The compactification scale $1/R$ is another free parameter of the model and there are numerous constraints for a single extra dimension in the split fermion scenario. The direct limits from searching for KK gauge bosons imply $1/R > 800\ GeV$, the precision electro weak bounds on higher dimensional operators generated by KK exchange place a far more stringent limit $1/R > 3.0\ TeV$ [28] and from $B \to \phi\, K_S$ the lower bounds for the scale $1/R$ have been obtained as $1/R > 1.0\ TeV$, from $B \to \psi\, K_S$ one got $1/R > 500\ GeV$, and from the upper limit of the BR, $BR\,(B_s \to \mu^+\mu^-) < 2.6\,\times 10^{-6}$, the estimated limit was $1/R > 800\ GeV$ [20]. We make our analysis by choosing two different values of the compactification scale $1/R$, by respecting these limits in the case of a single extra dimension. For two extra dimensions, we used the same values of the scale $1/R$.

In Fig. 2, we plot the ratio $V^{br,extr}_{LR\,(RL)\,ij} = \frac{V^{br}_{LR\,(RL)\,ij}}{V^{br,0}_{LR\,(RL)\,ij}}$ with respect to $y_p = y/\sigma$, which is the location of the 4D brane, including the new Higgs bosons, in the single extra dimension. Here the solid (dashed, small dashed, dotted, dot-dashed) line represents the ratio for $V^{br,extr}_{LR\,(RL)\,\tau\tau}$ ($V^{br,extr}_{LR\,\tau\mu}$, $V^{br,extr}_{RL\,\tau\mu}$, $V^{br,extr}_{LR\,\tau e}$, $V^{br,extr}_{RL\,\tau e}$). This figure shows that the ratios $V^{br,extr}_{LR\,(RL)\,ij}$ (therefore, the effective Yukawa couplings $V^{br,extr}_{LR\,(RL)\,ij}\,\bar{\xi}^E_{N,ij}$) are sensitive the location y_p of the 4D brane. The ratios for $\tau_{L(R)}\tau_{R(L)}S$ and $\tau_R\mu_L S$ interactions decrease and the others increase with the increasing values of the location y_p. The enhancement in the values of the ratios for the light flavors increases more compared to the heavy ones. Respecting that the couplings $\bar{\xi}^E_{N,ij}$ with heavier flavors are larger compared to lighter ones and with the similar assumption for the effective Yukawa couplings, the location y_p of the 4D brane, containing the new Higgs doublet should be in the possible range, $0 \leq y_p \leq\sim 0.3$. This is an interesting result since the considered brane should be at the origin or near to that point for the chosen free parameters and the location of the lepton Gaussian profiles in the extra dimension, if the above scenario is possible.

Fig. 3 represents the ratio $V^{br,extr2}_{LR\,(RL)\,\tau\tau} = \frac{V^{br,(2)}_{LR\,(RL)\,\tau\tau}}{V^{br,0,(2)}_{LR\,(RL)\,\tau\tau}}$ with respect to $y_p = y/\sigma$, for four different values of the $z_p = z/\sigma$, which is the location of the 4D brane in the second extra dimension. Here solid (dashed, small dashed, dotted) line represents the ratios for $z_p = 0\,(0.1, 0.3, 0.5)$. It is shown that the ratio decreases with the increasing values of the locations y_p and also z_p. In Fig. 4 (5) we present the ratio $V^{br,extr2}_{LR\,(RL)\,\tau\mu}$ ($V^{br,extr2}_{LR\,(RL)\,\tau e}$) with respect to $y_p = y/\sigma$, for four different values of the z_p. Here the upper-lower solid (dashed, small dashed, dotted) lines represent the ratios $V^{br,extr2}_{LR\,\tau\mu}$-$V^{br,extr2}_{RL\,\tau\mu}$ and $V^{br,extr2}_{LR\,\tau e}$-$V^{br,extr2}_{RL\,\tau e}$ for $z_p = 0\,(0.1, 0.3, 0.5)$, in Fig. 4 and 5, respectively. Fig. 4 shows that the ratios $V^{br,extr2}_{LR\,\tau\mu}$ increase, while $V^{br,extr2}_{RL\,\tau\mu}$ decrease with the increasing values of the location y_p and also z_p. On the other hand, both the ratios $V^{br,extr2}_{LR\,\tau e}$ and $V^{br,extr2}_{RL\,\tau e}$ are extremely sensitive to the location of the 4D brane in two extra dimensions and increase with the increasing values of the locations y_p and z_p, especially for $V^{br,extr2}_{LR\,\tau e}$. These enhancements switch on the increase in the values of the effective couplings for the light flavors and, again, respecting that the couplings $\bar{\xi}^E_{N,ij}$ and the effective Yukawa couplings with heavier flavors are larger compared to lighter ones, the location y_p, z_p of the 4D brane, containing the new Higgs doublet, should be in the possible range, $0 \leq y_p,\, z_p \leq\sim 0.2$. Similar to one extra dimension case, the brane, including the new Higgs doublet, should be at the origin or near to that point for the chosen free parameters and lepton profiles.

Now, we would like to study the effects of the brane location on the BRs of the LFV $l_i \to l_j \gamma$ decays. In Fig. 6, we plot the BR of the decay $\mu \to e\gamma$ with respect to the $y_p = y/\sigma$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^E_{N,\tau\mu} = 10\,GeV$, $\bar{\xi}^E_{N,\tau e} = 0.001\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR for a single extra dimension (for two extra dimensions and $z_p = 0, 0.1, 0.3$). It is observed that the BR is strongly sensitive to the location of the 4D brane and increases with the increasing values of y_p (y_p, z_p). Not to exceed the experimental value of the BR($\mu \to e\gamma$), namely 1.2×10^{-11} [1], one expects that the 4D brane, including the new Higgs doublet, should be in the location $0 \leq y_p \leq\sim 0.05$ ($0 \leq y_p, z_p \leq\sim 0.05$ in two dimensions) [5]

[5]It is obvious that these numerical values change with the different values of the couplings $\bar{\xi}^E_{N,\tau\mu}$ and $\bar{\xi}^E_{N,\tau e}$. However, the location of the 4D brane under consideration should not be so far than the origin, if such scenario exists.

Fig. 7 is devoted to the BR of the decay $\tau \to e\gamma$ with respect to the $y_p = y/\sigma$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau e} = 1\,GeV$ [6]. Here the solid (dashed, small dashed, dotted) line represents the BR for a single extra dimension (for two extra dimensions and $z_p = 0, 0.1, 0.3$). This figure shows that the BR is sensitive to the location of the 4D brane and increases with the increasing values of y_p (y_p, z_p). In this case, the BR ($\tau \to e\gamma$) does not exceed the experimental value 3.9×10^{-7} [2] in the given region of the 4D brane, for the free parameters used.

Fig. 8 represents the BR of the decay $\tau \to \mu\gamma$ with respect to the $y_p = y/\sigma$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau\mu} = 10\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR for a single extra dimension (for two extra dimensions and $z_p = 0, 0.1, 0.3$). We see that the BR is more sensitive to the location of the 4D brane compared to the $\tau \to e\gamma$ decay and decreases with the increasing values of y_p (y_p, z_p). Not to be so much below the experimental value of the BR, 1.1×10^{-6} [5] (also the recent upper limit of $9.0\,(6.8)\,10^{-8}$ at $90\%CL$ [6] ([7])), the 4D brane including the new Higgs doublet should be near to the origin. [7]

As another possibility, we assume that the new Higgs doublet in the extra dimension(s) is localized into the thin bulk and we try to examine the the effects of the width of the thin bulk to the BRs of the LFV $l_i \to l_j\gamma$ decays.

Fig. 9 is devoted to the BR of the decay $\mu \to e\gamma$ with respect to the the parameter $w = width/R$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\mu} = 10\,GeV$, $\bar{\xi}^{E}_{N,\tau e} = 0.001\,GeV$. Here the solid (dashed) line represents the BR for one-two extra dimensions and $1/R = 5000\,GeV$ (for two extra dimensions and $1/R = 500\,GeV$). It is observed that the BR is strongly sensitive to the width of the thin bulk and it should be $w > 3$, not to exceed the experimental value of the BR.

In Fig. 10 we present the BR of the decay $\tau \to e\gamma$ with respect to the the parameter $w = width/R$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau e} = 1\,GeV$. Here the solid (dashed) line represents the BR for one-two extra dimensions and $1/R = 5000\,GeV$ (for two extra dimensions and $1/R = 500\,GeV$). We see that the BR is sensitive to the width of the thin bulk and the width of the thin bulk should be $w > 2$, not to exceed the experimental value of the BR of the decay considered.

[6] For $\tau \to e\gamma$ we take the numerical value of the coupling $\bar{\xi}^{E}_{N,\tau e}$, $\bar{\xi}^{E}_{N,\tau e} = 1\,GeV$. Here we try to reach the new experimental result of the BR of this decay (see [2]). With the more sensitive future measurements of the BRs of these decays these couplings would be fixed more accurately.

[7] With the more accurate future experimental measurements, the upper limit can be pulled to the smaller values and one could obtain more stringent restriction for the location of the 4D brane.

Fig. 11 represents the BR of the decay $\tau \to \mu\gamma$ with respect to the the parameter $w = width/R$, for $m_{h^0} = 100\, GeV$, $m_{A^0} = 200\, GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\, GeV$, $\bar{\xi}^{E}_{N,\tau\mu} = 10\, GeV$. Here the solid (dashed) line represents the BR for one-two extra dimensions and $1/R = 5000\, GeV$ (for two extra dimensions and $1/R = 500\, GeV$). We see that the BR is strongly sensitive to the width of the thin bulk, similar to the $\mu \to e\gamma$ decay and the width of the thin bulk should be $w > 3$.

At this stage we would like to summarize our results:

- If the new Higgs doublet is located in the 4D brane, in the extra dimension(s), the BRs of LFV $l_i \to l_j\gamma$ decays are strongly sensitive to the location of the 4D brane. For $\mu \to e\gamma$ and $\tau \to e\gamma$ ($\tau \to \mu\gamma$) decays the BRs increase (decrease) with the increasing values of y_p and, in two extra dimensions, y_p, z_p. The considered 4D brane should be located near to the origin in the extra dimension(s) not to have a conflict with the current experimental results of the BRs.

- If we assume that the new Higgs doublet in the extra dimension(s) is localized into the thin bulk, it is estimated that the BR is strongly sensitive to the width of the thin bulk and its width should be approximately $w > 3$, not to exceed the current experimental results of the BRs of LFV $l_i \to l_j\gamma$ decays.

With the help of the forthcoming most accurate experimental measurements of the radiative LFV decays, a considerable information can be obtained to restrict the free parameters and to check the split fermion scenarios with the new Higgs sector.

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References

[1] M. L. Brooks et. al., MEGA Collaboration, *Phys. Rev. Lett.* **83**, 1521 (1999).

[2] K. Hayasaka et al.., *Phys. Lett.* **B613** (2005) 20.

[3] Donato Nicolo, MUEGAMMA Collaboration, *Nucl. Instrum. Meth* **A503** (2003) 287.

[4] S. Yamada, *Nucl. Phys. Proc. Suppl.* **144** (2005) 185.

[5] S. Ahmed et.al., CLEO Collaboration, *Phys. Rev.* **D61**, 071101 (2000).

[6] J.M. Roney and the BABAR Collaboration, *Nucl. Phys. Proc. Suppl.* **144** (2005) 155.

[7] B. Aubert et. al., BABAR Collaboration, SLAC-PUB-11028, BABAR-PUB-04-049, Feb. 2005, 7. pp, *Phys. Rev. Lett.* **95** (2005) 041802.

[8] R. Barbieri and L. J. Hall, *Phys. Lett.* **B338**, 212 (1994); R. Barbieri, L. J. Hall and A. Strumia, *Nucl. Phys.* **B445**, 219 (1995); R. Barbieri, L. J. Hall and A. Strumia, *Nucl. Phys.* **B449**, 437 (1995); P. Ciafaloni, A. Romanino and A. Strumia, IFUP-YH-42-95; T. V. Duong, B. Dutta and E. Keith, *Phys. Lett.* **B378**, 128 (1996); G. Couture, et. al., *Eur. Phys. J.* **C7**, 135 (1999); Y. Okada, K. Okumara and Y. Shimizu, *Phys. Rev.* **D61**, 094001 (2000).

[9] E. O. Iltan, *Phys. Rev.* **D64**, 115005, (2001); *Phys. Rev.* **D64** 013013, (2001).

[10] R. Diaz, R. Martinez and J-Alexis Rodriguez, Phys.Rev. D63 (2001) 095007.

[11] E. O. Iltan, *JHEP* **0408**, 20, (2004).

[12] E. O. Iltan, hep-ph/0504013, (2005).

[13] D. Chang, W. S. Hou and W. Y. Keung, *Phys. Rev.* **D48**, 217 (1993).

[14] P. Paradisi, hep-ph/0508054, (2005).

[15] N. Arkani-Hamed and M. Schmaltz, *Phys. Rev.* **D61**, 033005 (2000); N. Arkani-Hamed, Y. Grossman and M. Schmaltz, *Phys. Rev.* **D61** (2000) 115004 .

[16] E. A. Mirabelli, Schmaltz, *Phys. Rev.* **D61** (2000) 113011.

[17] W. F. Chang, I. L. Ho and J. N. Ng, *Phys. Rev.* **D66** 076004 (2002).

[18] G. C. Branco,A. Gouvea, M. N. Rebelo, *Phys. Lett.* **B506** (2001) 115.

[19] W. F. Cang, J. N. Ng, *JHEP* **0212** (2002) 077.

[20] B. Lillie, J. L. Hewett, *Phys. Rev.* **68** (2003) 116002.

[21] Y. Grossman and G. Perez, *Pramana* **62** (2004) 733.

[22] E. O. Iltan, hep-ph/0503001.

[23] E. O. Iltan, hep-ph/0507213.

[24] Z. Surujon, hep-ph//0507036

[25] A. Delgado, A. Pomarol, M. Quiros, *JHEP* **0001** (2000) 030; Y. Grossman, *Int. J. Mod. Phys.* **A15** (2000) 2419; D. E. Kaplan and T. M. Tait, *JHEP* **0111** (2001) 051; G. Barenboim, *et. al.*, *Phys. Rev.* **D64** (2001) 073005; W. F. Chang, I. L. Ho and J. N. Ng, *Phys. Rev.* **D66** (2002) 076004; W. Skiba and D. Smith, *Phys. Rev.* **D65** (2002) 095002; Y. Grossman, R. Harnik, G. Perez, M. D. Schwartz and Z. Surujon, *Phys. Rev.* **D71** (2005) 056007; P. Dey, G. Bhattacharya, *Phys. Rev.* **D70** (2004) 116012; Y. Nagatani, G. Perez, *JHEP* **0502** (2005) 068; R. Harnik, G. Perez, M. D. Schwartz, Y. Shirman, *JHEP* **0503** (2005) 068.

[26] T. P. Cheng and M. Sher, *Phy. Rev.* **D35** (1987) 3383.

[27] E. O. Iltan, H. Sundu, *Acta Phys.Slov.* **53** (2003) 17.

[28] T. G. Rizzo, J. D. Wells, *Phy. Rev.* **D61** (2000) 016007. .

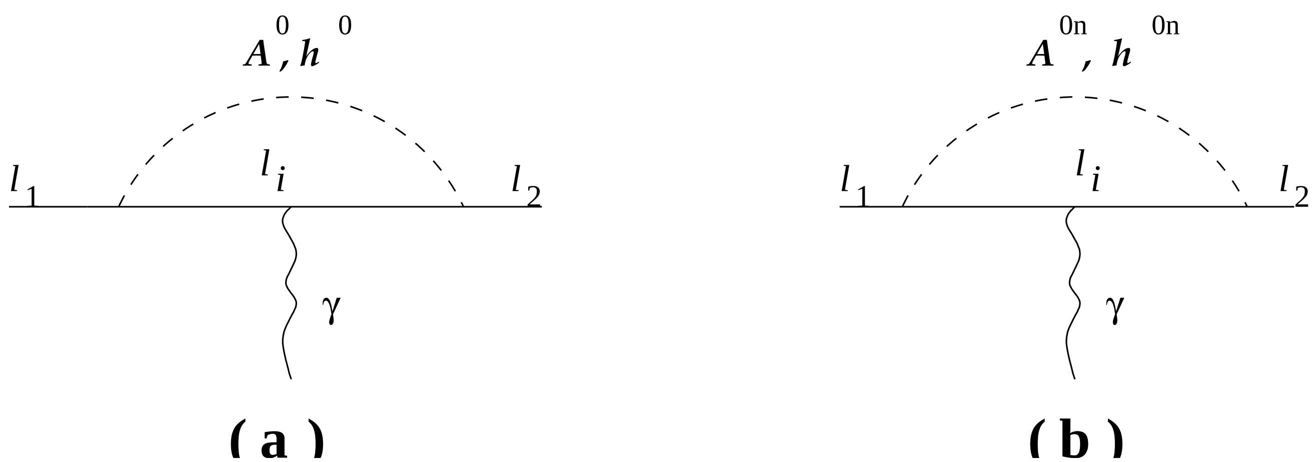

Figure 1: One loop diagrams contribute to $l_1 \rightarrow l_2\gamma$ decay due to the zero mode (KK mode) neutral Higgs bosons h^0 and A^0 (h^{0n} and A^{0n}) in the 2HDM, for a single extra dimension. Here l_i represents the internal charged lepton.

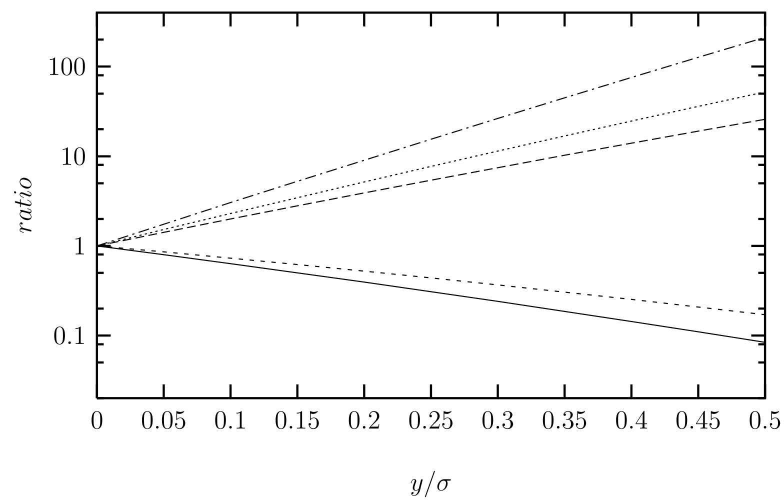

Figure 2: $V^{br,extr}_{LR\,(RL)\,ij}$ with respect to $y_p = y/\sigma$. Here the solid (dashed, small dashed, dotted, dot-dashed) line represents the ratio for $V^{br,extr}_{LR\,(RL)\,\tau\tau}$ ($V^{br,extr}_{LR\,\tau\mu}$, $V^{br,extr}_{RL\,\tau\mu}$, $V^{br,extr}_{LR\,\tau e}$, $V^{br,extr}_{RL\,\tau e}$).

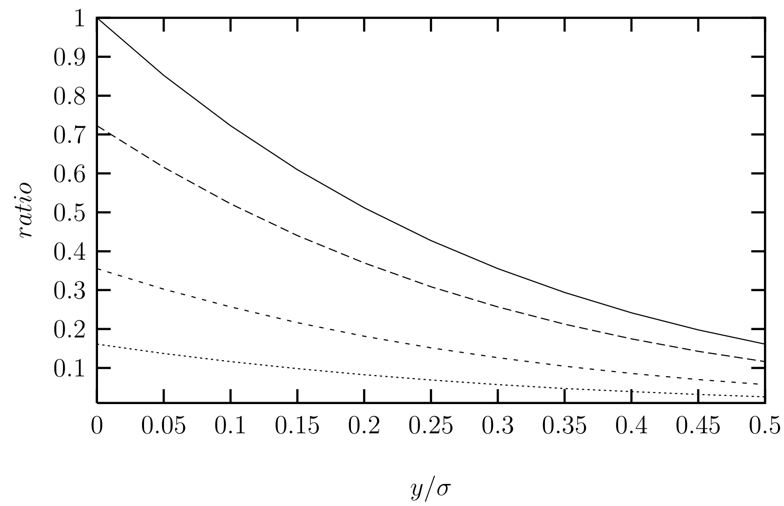

Figure 3: $V^{br,extr2}_{LR\,(RL)\,\tau\tau}$ with respect to $y_p = y/\sigma$, for four different values of the z_p. Here solid (dashed, small dashed, dotted) line represents the ratios for $z_p = 0(0.1, 0.3, 0.5)$.

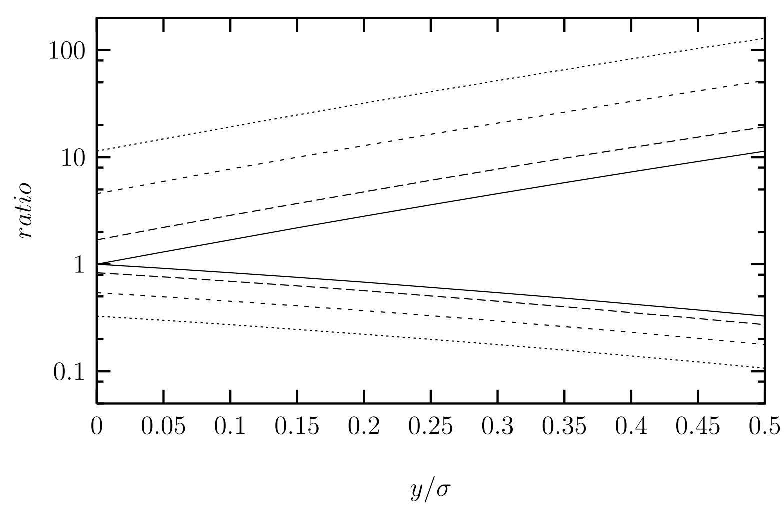

Figure 4: The same as Fig. 3 but for $V^{br,extr2}_{LR\,(RL)\,\tau\mu}$.

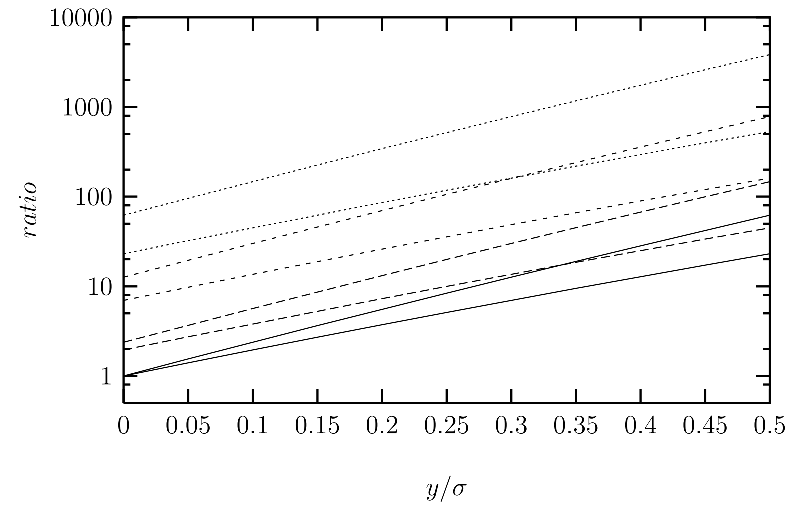

Figure 5: The same as Fig. 3 but for $V^{br,extr2}_{LR\,(RL)\,\tau e}$.

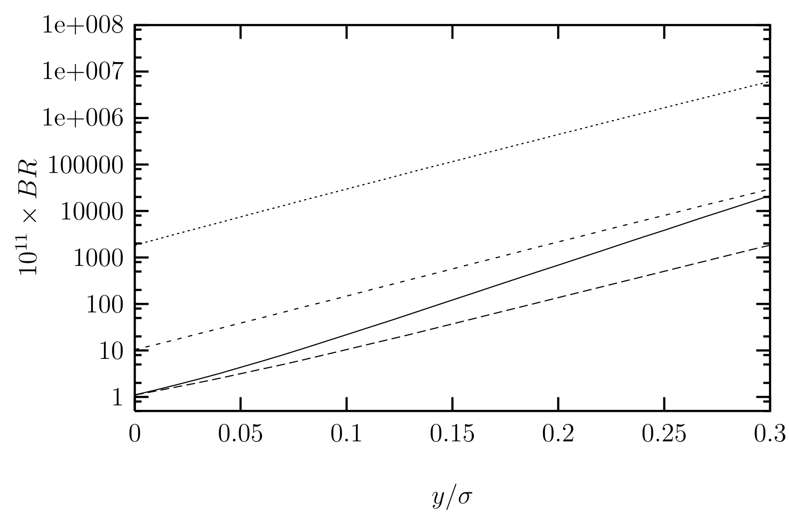

Figure 6: BR ($\mu \to e\gamma$) with respect to the $y_p = y/\sigma$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^E_{N,\tau\mu} = 10\,GeV$, $\bar{\xi}^E_{N,\tau e} = 0.001\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR for a single extra dimension (for two extra dimensions and $z_p = 0, 0.1, 0.3$).

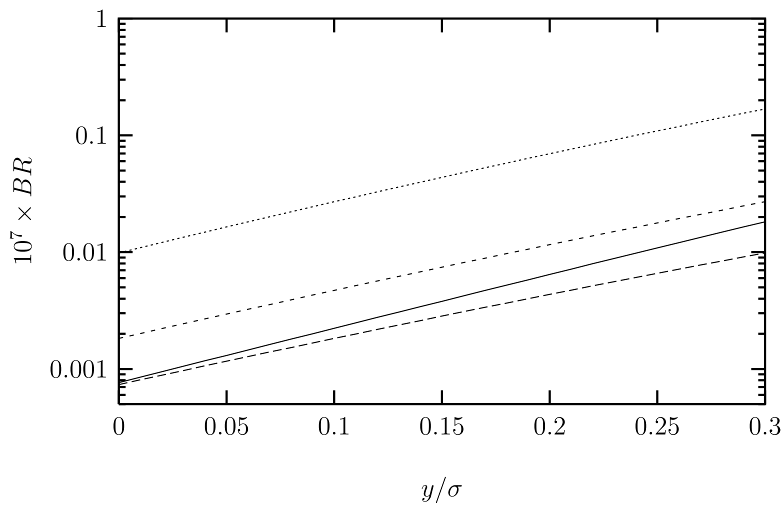

Figure 7: BR ($\tau \to e\gamma$) with respect to the $y_p = y/\sigma$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau e} = 1\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR for a single extra dimension (for two extra dimensions and $z_p = 0, 0.1, 0.3$).

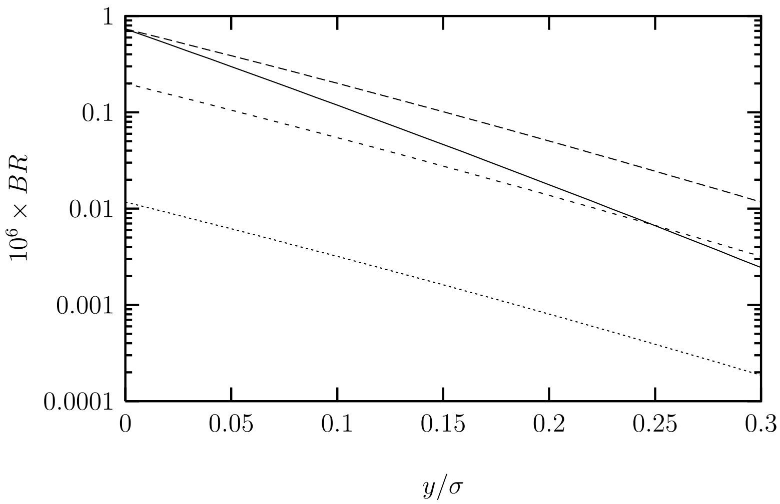

Figure 8: BR $(\tau \to \mu\gamma)$ with respect to the $y_p = y/\sigma$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau\mu} = 10\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR for a single extra dimension (for two extra dimensions and $z_p = 0, 0.1, 0.3$).

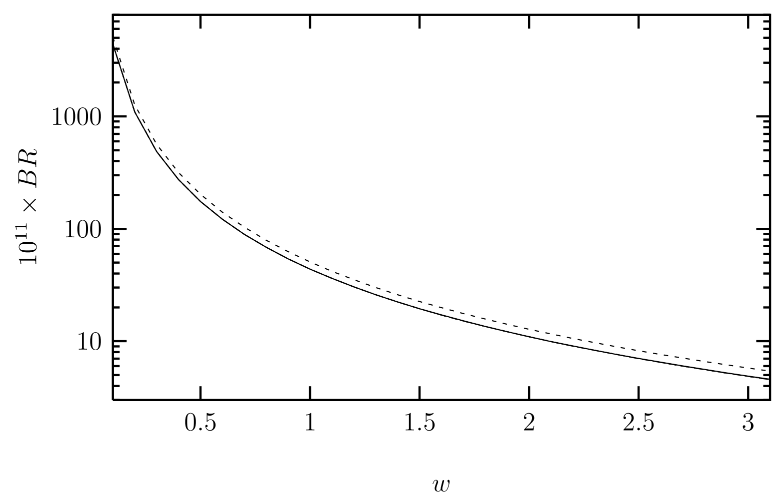

Figure 9: BR($\mu \to e\gamma$) with respect to the parameter $w = width/R$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\mu} = 10\,GeV$, $\bar{\xi}^{E}_{N,\tau e} = 0.001\,GeV$. Here the solid (dashed) line represents the BR for one-two extra dimensions and $1/R = 5000\,GeV$ (for two extra dimensions and $1/R = 500\,GeV$).

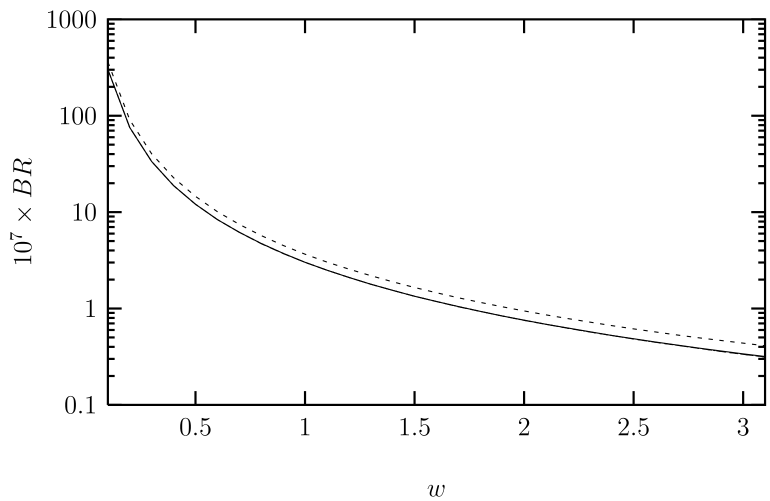

Figure 10: BR($\tau \to e\gamma$) with respect to the parameter $w = width/R$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau e} = 1\,GeV$. Here the solid (dashed) line represents the BR for one-two extra dimensions and $1/R = 5000\,GeV$ (for two extra dimensions and $1/R = 500\,GeV$).

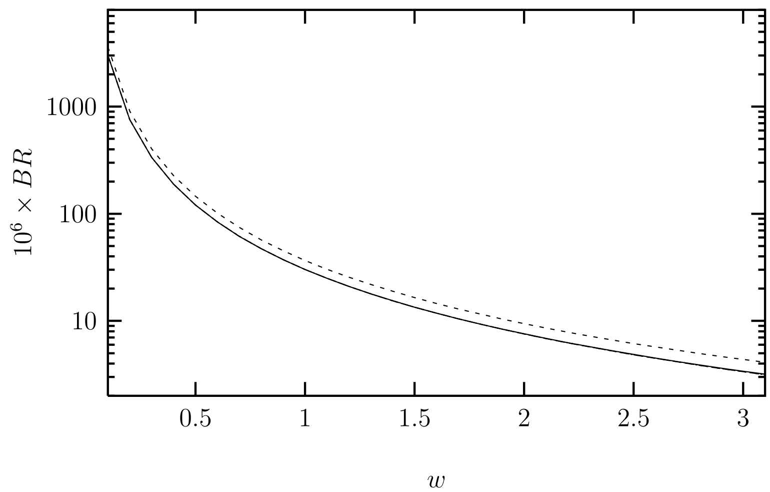

Figure 11: BR($\tau \rightarrow \mu\gamma$) with respect to the parameter $w = width/R$, for $m_{h^0} = 100\,GeV$, $m_{A^0} = 200\,GeV$ and the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$, $\bar{\xi}^{E}_{N,\tau\mu} = 10\,GeV$. Here the solid (dashed) line represents the BR for one-two extra dimensions and $1/R = 5000\,GeV$ (for two extra dimensions and $1/R = 500\,GeV$).