The possible effects of non-universal extra dimensions on $t \to c \, l_i^- l_j^+$ and $H^0 \to h^0(A^0) l_i^- l_j^+$ decays in the general two Higgs Doublet model

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Abstract

We study the effects of non-universal extra dimensions on the decay widths of the lepton flavor violating processes, $t \to c l_i^- l_j^+$ and $H^0 \to h^0(A^0) l_i^- l_j^+$ in the general two Higgs doublet model. We consider that the extra dimensions are accessible to the standard model gauge fields and the new Higgs doublet. We observe that the lepton flavor violating $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ and $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ Higgs decays are sensitive to the extra dimensions, especially, in the case of two spatial ones. This result may ensure a test to determine the compactification scale and the possible number of extra dimensions, with the accurate experimental measurements.

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1 Introduction

Lepton flavor violating (LFV) interactions are worthwhile to study since they ensure comprehensive information about the possible new physics effects beyond the standard model (SM) and the free parameters existing in these models. Such processes occur with the help of the tree level flavor changing neutral currents (FCNC), which appear in the models beyond the SM, like the two Higgs doublet model (2HDM). It is well known that the model III version of the 2HDM possesses the FCNC at tree level and the strengths of the flavor changing (FC) interactions are regulated by the Yukawa couplings, appearing as free parameters which should be restricted by the experimental data. The addition of extra dimensions into the theory brings new extension and the search of the effects of the possible new dimensions would be illustrative to test their existence.

Extra dimensions are introduced for solving the gauge hierarchy problem of the SM and there are various studies on this subject in the literature [1]-[11]. The idea is that there is a fundamental theory lying in higher dimensions and the ordinary four dimensional SM is its low energy effective theory. This is achieved by considering that the extra dimension (two extra dimensions) over four dimensions is compactified on orbifold S^1/Z_2 ($(S^1 \times S^1)/Z_2$) with small radius R, which is a typical size of the extra dimension(s). This compactification results in the production of Kaluza-Klein (KK) states of the fields with masses regulated by the parameter R. If the extra dimensions are accessible to all fields in the model, they are called as universal extra dimensions (UED) in the literature [1]-[5]. In this case, the extra dimensional momentum is conserved at each vertex and the interactions with only one KK state are forbidden, i.e., the KK number is conserved. The conservation of the KK number leads to the appearance of heavy stable particles. Furthermore, this conservation causes that the KK modes enter into the calculations as loop corrections and, therefore, the constraints on the size of the extra dimensions which are obtained from SM precision measurements are less stringent. The size of compactification scale has been studied by taking into account the loop effects induced by the internal top quark and it has been estimated in the range of $200 - 500 \, GeV$, using the electroweak precision measurements [1], the $B - \overline{B}$ -mixing [2],[3] and the flavor changing process $b \to s\gamma$ [4]. In several works [6, 7, 8, 9, 10], this size has been estimated as large as few hundreds of GeV. In the case of non-universal extra dimensions, where some of the particles are confined on 4D brane and do not feel the new dimensions, the coupling of two zero modes with the KK mode is permitted and this ensures to predict the effects of extra dimensions even at tree level.

Our work is devoted to the analysis of the LFV t-quark $t \to c l_i^- l_j^+$ and Higgs boson $H^0 \to h^0 l_i^- l_j^+$ and $H^0 \to A^0 l_i^- l_j^+$ decays in the framework of the model III, with the addition of a single (two) extra dimension(s). We consider that the new Higgs doublet and the gauge sector of the SM feel the extra dimensions, however, the other SM fields are confined on 4D brane. Since these decays can exist at tree level in the model III, the higher dimensional effects for non-universal case under consideration appear with the intermediate (virtual) neutral Higgs fields H, namely h^0 and A^0 , which can create "two zero modes-KK mode" vertices, in contrast to the case of UED. In the present analysis, we try to predict the additional effects due to a single and two spatial extra dimensions. In the case of a single extra dimension, the KK modes of the neutral Higgs fields H, with masses $\sqrt{m_H^2 + n^2/R^2}$, appear after the compactification on orbifold S^1/Z_2 . Here $m_n = n/R$ is the mass of n'th level KK particle where R is the compactification radius. If there exist two spatial extra dimensions which are accessible to the new Higgs doublet, the non-zero KK modes of the neutral Higgs fields H have the masses $\sqrt{m_H^2 + m_n^2 + m_k^2}$, where the mass terms $m_n = n/R$ and $m_k = k/R$ are due to the compactification of the extra dimensions on orbifold $(S^1 \times S^1)/Z_2$ [1].

In the numerical calculations we see that the extra dimension contribution to the FV $t \rightarrow c (\tau^+ \mu^- + \tau^- \mu^+)$ decay is negligible, at least, up to two extra dimensions. However, the LFV $H^0 \rightarrow h^0 (\tau^+ \mu^- + \tau^- \mu^+)$ and $H^0 \rightarrow A^0 (\tau^+ \mu^- + \tau^- \mu^+)$ Higgs decays are sensitive to the extra dimensions and the predictions of additional effects to their decay widths are almost comparable with the decay widths obtained without extra dimensions, in the case of two extra dimensions. This result may ensure a test to determine the compactification scale and the possible number of extra dimensions.

The paper is organized as follows: In Section 2, we present the decay widths of LFV interactions $t \to c (l_i^- l_j^+ + l_j^- l_i^+)$ and $H^0 \to h^0(A^0)(l_i^- l_j^+ + l_j^- l_i^+)$ in the model III version of the 2HDM, with the inclusion of non-universal extra dimensions. Section 3 is devoted to discussion and our conclusions.

2 The LFV interactions $t \rightarrow c (l_i^- l_j^+ + l_j^- l_i^+)$ and $H^0 \rightarrow h^0(A^0)(l_i^- l_j^+ + l_j^- l_i^+)$ in the general two Higgs Doublet model with the inclusion of non-universal extra dimensions

The FV interactions are worthwhile to investigate and, among them, the LFV interactions receive great interest since the theoretical predictions of their branching ratios (BR's) in the

framework of the SM are small and this forces one to go the new models beyond. The model III version of the 2HDM, permitting tree level neutral currents, is one of the candidate that can ensure additional contributions to the physical quantities with the appropriate choice of free parameters, such as Yukawa couplings, masses of new particles. The inclusion of spatial extra dimensions causes to enhance the BR's of these decays and these enhancements depend on the compactification scale 1/R, where R is the radius of the compactification.

Now, we assume that the second Higgs doublet feels the extra dimensions. We start with the part of the Lagrangian which is responsible for the FV vertex, the so called Yukawa Lagrangian, in a single extra dimension:

$$\mathcal{L}_{Y} = \eta_{ij}^{U} \bar{Q}_{iL} \tilde{\phi}_{1} U_{jR} + \eta_{ij}^{D} \bar{Q}_{iL} \phi_{1} D_{jR} + \xi_{5\,ij}^{U\dagger} \bar{Q}_{iL} (\tilde{\phi}_{2}|_{y=0}) U_{jR} + \xi_{5\,ij}^{D} \bar{Q}_{iL} (\phi_{2}|_{y=0}) D_{jR} + \eta_{ij}^{E} \bar{l}_{iL} \phi_{1} E_{jR} + \xi_{5\,ij}^{E} \bar{l}_{iL} (\phi_{2}|_{y=0}) E_{jR} + h.c. , \qquad (1)$$

where y represents the extra dimension, L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for i = 1, 2, are two scalar doublets, \bar{Q}_{iL} are left handed quark doublets, $U_{jR}(D_{jR})$ are right handed up (down) quark singlets, l_{iL} (E_{jR}) are lepton doublets (singlets), with family indices i, j. The Yukawa couplings ξ_{5ij}^E are dimensionful and rescaled to the ones, $\xi_{ij}^{U,D,E}$, in four dimensions as $\xi_{5ij}^{U,D,E} = \sqrt{2\pi R} \xi_{ij}^{U,D,E}$.

In the present work, we assume that the Higgs doublet lying in the four dimensional brane has non-zero vacuum expectation value which ensures the ordinary masses of the gauge fields and the fermions. On the other hand, the second doublet, which is accessible to the extra dimension, does not receive the vacuum expectation value. Namely, we choose the doublets ϕ_1 and ϕ_2 and the their vacuum expectation values as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right]; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} , \qquad (2)$$

and

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0\\v \end{pmatrix} ; \langle \phi_2 \rangle = 0 .$$
 (3)

With the choice under consideration the mixing between neutral scalar Higgs bosons is switched off and it would be possible to separate the particle spectrum so that the SM particles are collected in the first doublet and the new particles in the second one. Here H_1 (H_2) is the well known mass eigenstate h^0 (A^0). Notice that both Higgs doublets can have non-zero vacuum expectation values in general and this leads to the mixing between the neutral Higgs bosons H^0 and h^0 in the CP even sector. In the CP odd one, the mixing appears between χ^0 and H_2 . There exist new parameters which include the mixing angle of CP even neutral Higgs bosons and the ratio of the vacuum expectation values of each Higgs doublet, in the vertices (for example $H^0 - h^0 - h^0$ and $H^0 - A^0 - A^0$ vertices, lepton-lepton Higgs boson vertices (see [12] and [13] for review). Therefore, in general, the mixing angle and the ratio of the vacuum expectation values appear in the physical quantities.

The part which produce FCNC at tree level

$$\mathcal{L}_{Y,FC} = \xi_{5ij}^{U\dagger} \bar{Q}_{iL} (\tilde{\phi}_2|_{y=0}) U_{jR} + \xi_{5ij}^D \bar{Q}_{iL} (\phi_2|_{y=0}) D_{jR} + \xi_{5ij}^E \bar{l}_{iL} (\phi_2|_{y=0}) E_{jR} + h.c. , \qquad (4)$$

carries the information about the extra dimension over the second Higgs doublet ϕ_2 and it can be expanded into its KK modes after the compactification of the extra dimension on orbifold S^1/Z_2 as

$$\phi_2(x,y) = \frac{1}{\sqrt{2\pi R}} \left\{ \phi_2^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \phi_2^{(n)}(x) \cos(ny/R) \right\},$$
(5)

where $\phi_2^{(0)}(x)$ is the four dimensional Higgs doublet which contains the charged Higgs boson H^+ , the neutral CP even (odd) h^0 (A^0) Higgs bosons and R is the compactification radius. Furthermore, each non-zero KK mode of Higgs doublet ϕ_2 includes a charged Higgs of mass $\sqrt{m_{H^\pm}^2 + m_n^2}$, a neutral CP even Higgs of mass $\sqrt{m_{h^0}^2 + m_n^2}$, a neutral CP odd Higgs of mass $\sqrt{m_{A^0}^2 + m_n^2}$ where $m_n = n/R$ is the mass of *n*'th level KK particle, emerging from compactification.

Now, we start to investigate the LFV inclusive $t \to c l_i^- l_j^+$ decay where l_i , l_j are different lepton flavors (see Fig. 1) in the model III, where only the new Higgs doublet feels a single extra dimension. This process can exist at tree level, by taking non-zero $t - c (l_i^- l_j^+)$ transition driven by the neutral bosons h^0 and A^0 . There are FV vertex in the quark sector, $t - c h^{0*}(A^{0*})$ and it is connected to the $l_i^- l_j^+$ outgoing leptons. Since only the new Higgs doublet, and therefore, the h^0 and A^0 bosons, feels extra dimension, the KK modes of them contribute to the process in addition to their zero modes (see Fig. 1). Notice that in the case of UED there would be no contribution coming from the extra dimension at tree level due to the KK number conservation.

Here we present the matrix element square of the process $t \to c \left(l_i^- l_j^+ + l_j^+ l_i^- \right)$ (see [14])

$$\begin{split} |M|^2 &= 8 m_t^2 (1-s) \sum_{H=h^0, A^0} |p_H|^2 \left(|a_H^{(q)}|^2 + |a_H'^{(q)}|^2 \right) \left((s m_t^2 - (m_{l_i^-} - m_{l_j^+})^2) |a_H'^{(l)}|^2 \right) \\ &+ (s m_t^2 - (m_{l_i^-} + m_{l_j^+})^2) |a_H'^{(l)}|^2 \right) \\ &+ 16 m_t^2 (1-s) \left((s m_t^2 - (m_{l_i^-} - m_{l_j^+})^2) Re[p_{h^0} p_{A^0}^* a_{h^0}^{(l)} a_{A^0}^{*(l)} (a_{h^0}^{(q)} a_{A^0}^{*(q)} + a_{h^0}' a_{A^0}^{*(q)})] \right] \end{split}$$

$$+ (s m_t^2 - (m_{l_i^-} + m_{l_j^+})^2) Re[p_{h^0} p_{A^0}^* a_{h^0}^{\prime(l)} a_{A^0}^{\prime*(l)} (a_{h^0}^{(q)} a_{A^0}^{*(q)} + a_{h^0}^{\prime(q)} a_{A^0}^{\prime*(q)})] \bigg),$$
(6)

where

$$p_H = \frac{i}{k^2 - m_H^2 + im_H \Gamma_{tot}^H} + 2 \sum_{n=1}^{\infty} \frac{i}{k^2 - m_{H^n}^2}, \qquad (7)$$

and Γ_{tot}^{H} is the total decay width of H boson, for $H = h^{0}$, A^{0} . In eq. (7), the parameter s is $s = \frac{k^{2}}{m_{t}^{2}}$, with the intermediate H boson momentum square k^{2} and $m_{H^{n}}$ is the mass of n^{th} KK mode of H boson, $m_{H^{n}} = \sqrt{m_{H}^{2} + \frac{n^{2}}{R^{2}}}$. Here the functions $a_{h^{0},A^{0}}^{(l)}$, $a_{h^{0},A^{0}}^{\prime(l)}$ read,

$$a_{h^{0}}^{(l)} = -\frac{i}{2\sqrt{2}} \left(\xi_{N,l_{i}l_{2}}^{D} + \xi_{N,l_{2}l_{i}}^{*D} \right),$$

$$a_{A^{0}}^{(l)} = \frac{1}{2\sqrt{2}} \left(\xi_{N,l_{i}l_{j}}^{D} - \xi_{N,l_{j}l_{i}}^{*D} \right),$$

$$a_{h^{0}}^{\prime(l)} = -\frac{i}{2\sqrt{2}} \left(\xi_{N,l_{i}l_{j}}^{D} - \xi_{N,l_{j}l_{i}}^{*D} \right),$$

$$a_{A^{0}}^{\prime(l)} = \frac{1}{2\sqrt{2}} \left(\xi_{N,l_{i}l_{j}}^{D} + \xi_{N,l_{j}l_{i}}^{*D} \right),$$

$$a_{h^{0}}^{(q)} = \frac{i}{2\sqrt{2}} \left(\xi_{N,tc}^{U} + \xi_{N,ct}^{*U} \right),$$

$$a_{h^{0}}^{\prime(q)} = -\frac{1}{2\sqrt{2}} \left(\xi_{N,tc}^{U} - \xi_{N,ct}^{*U} \right),$$

$$a_{A^{0}}^{\prime(q)} = -\frac{i}{2\sqrt{2}} \left(\xi_{N,tc}^{U} - \xi_{N,ct}^{*U} \right).$$
(8)

Notice that we replace $\xi^{U,D,E}$ with $\xi_N^{U,D,E}$ where "N" denotes the word "neutral". Using the eq. (6), the differential decay width (dDW) $\frac{d\Gamma}{ds}(t \to c (l_1^- l_2^+ + l_1^+ l_2^-))$ is obtained as

$$\frac{d\Gamma}{ds} = \frac{1}{256 N_c \pi^3} \lambda |M|^2, \qquad (9)$$

where $\underline{\lambda}$ is:

$$\lambda = \frac{\sqrt{\left(m_t^2 \, (s-1)^2 - 4 \, m_c^2\right) \left(m_c^4 + m_{l_i}^4 + (m_{l_j}^2 - m_t^2 \, s)^2 - 2 \, m_c^2 \, (m_{l_i}^2 + m_{l_j}^2 - m_t^2 \, s) - 2 \, m_{l_i}^2 \, (m_{l_j}^2 + m_t^2 \, s)\right)}}{2 \, m_t^2 \, s}.$$
 Here the parameter s is restricted into the region $\frac{(m_{l_i} + m_{l_j})^2}{m_t^2} \le s \le \frac{(m_t - m_c)^2}{m_t^2}.$

s is restricted into the region $\underline{m_i^2} \geq s \geq \underline{m_i^2}$. At this stage we study the processes $H^0 \to h^0 l_i^- l_j^+$ and $H^0 \to A^0 l_i^- l_j^+$ (see [15]) where l_i, l_j are different lepton flavors (see Fig. 2) and we consider the model III version of the 2HDM with the addition of extra dimension that is felt by the new Higgs doublet, similar to the previous calculation. These processes exist at tree level and the extra dimension effects appear in the case of virtual $h^0(A^0)$ transitions (see Fig. 2-c and 2-d). The KK modes of these neutral Higgs bosons contribute to the processes contrary to the UED case where there would be no contribution coming from the extra dimension at tree level.

Using the diagrams Fig. 2-a and Fig. 2-c the matrix element square of the process $H^0 \to h^0 l_i^- l_j^+$ reads

$$|M|^2 = A_1 + A_2 + A_3 \,, \tag{10}$$

where

$$A_{1} = \frac{1}{2(m_{H^{0}}^{2} + 2p.k_{l_{i}})^{2}} \left\{ m_{l_{i}}^{2} |\xi_{N,ji}^{E}|^{2} \left(2(p.k_{l_{i}})^{2} + (m_{H^{0}}^{2} - 4m_{l_{i}}^{2}) q.k_{l_{i}} + p.k_{l_{i}} (m_{H^{0}}^{2} + 4m_{l_{i}}(m_{l_{j}} + 2m_{l_{i}} - 2m_{l_{j}} \sin^{2}\theta_{ij}) - 2p.q) + m_{l_{i}}(4m_{l_{i}}^{2}(m_{l_{i}} + m_{l_{j}} - 2m_{l_{j}} \sin^{2}\theta_{ij}) + m_{H^{0}}^{2}(3m_{l_{i}} + m_{l_{j}} - 2m_{l_{j}} \sin^{2}\theta_{ij}) - 4m_{l_{j}} p.q) \right\},$$

$$A_{2} = \frac{1}{\sqrt{2}(m_{H^{0}}^{2} + 2p.k_{l_{i}})} \left\{ 4m_{l_{i}} m_{h^{0}}^{2} |\xi_{N,ji}^{E}|^{2} Im[p_{h^{0}}] \left((3m_{l_{i}} + m_{l_{j}} - 2m_{l_{j}} \sin^{2}\theta_{ij}) p.k_{l_{i}} + m_{l_{i}} (m_{H^{0}}^{2} + 2m_{l_{i}}^{2} + 2m_{l_{i}}m_{l_{j}} - 4m_{l_{i}}m_{l_{j}} \sin^{2}\theta_{ij} - 2q.(k_{l_{i}} - p)) \right) \right\},$$

$$A_{3} = 4m_{h^{0}}^{4} |\xi_{N,ji}^{E}|^{2} Abs[p_{h^{0}}]^{2} \left(m_{l_{i}}(m_{l_{i}} + m_{l_{j}} - 2m_{l_{j}} \sin^{2}\theta_{ij}) + (p - q).k_{l_{i}} \right),$$
(11)

and

$$p_{h^0} = \frac{i}{k^2 - m_{h^0}^2 + im_{h^0} \Gamma_{tot}^{h^0}} + 2 \sum_{n=1}^{\infty} \frac{i}{k^2 - m_{h^{0^n}}^2},$$
(12)

with the transfer momentum square k^2 , four momentum of incoming H^0 , outgoing h^0 , outgoing l_i^- lepton, p, q, k_{l_i} , respectively. In eq. (11), the parameter θ_{ij} carries the information about the complexity of the Yukawa coupling $\xi_{N,ij}^E$ with the parametrization

$$\xi_{N,ij}^{E} = |\xi_{N,ij}^{E}| \, e^{i\theta_{ij}} \,. \tag{13}$$

Similarly, using the diagrams Fig. 2-b and Fig. 2-d, the matrix element square of the process $H^0 \to A^0 l_i^- l_j^+$ is obtained as

$$|M|^2 = A_1' + A_2' + A_3', (14)$$

where

$$A_1' = \frac{1}{2(m_{H^0}^2 + 2p.k_{l_i})^2} \left\{ m_{l_i}^2 |\xi_{N,ji}^E|^2 \left(2(p.k_{l_i})^2 + (m_{H^0}^2 - 4m_{l_i}^2) q.k_{l_i} + p.k_{l_i} (m_{H^0}^2 - 4m_{H^0}^2) q.k$$

$$+ 4 m_{l_{i}}(-m_{l_{j}} + 2 m_{l_{i}} + 2m_{l_{j}} \sin^{2}\theta_{l_{i}l_{j}}) - 2p.q) + m_{l_{i}}(4 m_{l_{i}}^{2}(m_{l_{i}} - m_{l_{j}} + 2 m_{l_{j}} \sin^{2}\theta_{ij}) + m_{H^{0}}^{2}(3 m_{l_{i}} - m_{l_{j}} + 2 m_{l_{j}} \sin^{2}\theta_{l_{i}l_{j}}) - 4 m_{l_{j}} p.q) \bigg\},$$

$$A'_{2} = \frac{1}{\sqrt{2}(m_{H^{0}}^{2} + 2p.k_{l_{i}})} \bigg\{ 4 m_{l_{i}} m_{A^{0}}^{2} |\xi_{N,ji}^{E}|^{2} Im[p_{A^{0}}] \bigg((3m_{l_{i}} - m_{l_{j}} + 2 m_{l_{j}} \sin^{2}\theta_{ij}) p.k_{l_{i}} + m_{l_{i}} (m_{H^{0}}^{2} + 2 m_{l_{i}}^{2} - 2 m_{l_{i}} m_{l_{j}} + 4 m_{l_{i}} m_{l_{j}} \sin^{2}\theta_{ij} - 2 q.(k_{l_{i}} - p)) \bigg) \bigg\},$$

$$A_{3} = 4 m_{A^{0}}^{4} |\xi_{N,ji}^{E}|^{2} Abs[p_{A^{0}}]^{2} \bigg(m_{l_{i}} (m_{l_{i}} - m_{l_{j}} + 2 m_{l_{j}} \sin^{2}\theta_{ij}) + (p - q).k_{l_{i}} \bigg),$$

$$(15)$$

and q is four momentum of outgoing A^0 . The decay width Γ is obtained in the H^0 boson rest frame by using the well known expression

$$d\Gamma = \frac{(2\pi)^4}{m_{H^0}} |M|^2 \,\delta^4(p - \sum_{i=1}^3 p_i) \,\prod_{i=1}^3 \frac{d^3 p_i}{(2\pi)^3 2E_i} \,, \tag{16}$$

where p (p_i , i=1,2,3) is four momentum vector of H^0 boson, (h^0 (A^0) boson, outgoing l_i^- and l_i^+ leptons).

Finally, we would like to analyze these decays in the two extra spatial dimensions. With the assumption that the second Higgs doublet ϕ_2 feels the extra dimensions, it can be expanded into its KK modes after the compactification of the extra dimensions on orbifold $(S^1 \times S^1)/Z_2$ as

$$\phi_2(x, y, z) = \frac{1}{2\pi R} \left\{ \phi_2^{(0,0)}(x) + 2\sum_{n,k} \phi_2^{(n,k)}(x) \cos(ny/R + kz/R) \right\},$$
(17)

where each circle is considered having the same radius R. In the summation, the indices n and k are positive integers including zero but both are not zero at the same time. Here $\phi_2^{(0,0)}(x)$ is the four dimensional Higgs doublet which contains the charged Higgs boson H^+ , the neutral CP even (odd) h^0 (A^0) Higgs bosons. Each non-zero KK mode of Higgs doublet ϕ_2 includes a charged Higgs of mass $\sqrt{m_{H^{\pm}}^2 + m_n^2 + m_k^2}$, a neutral CP even Higgs of mass $\sqrt{m_{h^0}^2 + m_n^2 + m_k^2}$, a neutral CP odd Higgs of mass $\sqrt{m_{A^0}^2 + m_n^2 + m_k^2}$ where the mass terms $m_n = n/R$ and $m_k = k/R$ exist due to the compactification.

In the decays we consider that there appear KK modes $h^{0n,k}$ and $A^{0n,k}$ on the virtual h^0 and A^0 lines and the parameter p_H in eq. (6) (eqs. (11) and (15)) is redefined as

$$p_H = \frac{i}{s \, m_t^2 - m_H^2 + i m_S \, \Gamma_{tot}^H} + 4 \, \sum_{n,k} \frac{i}{s \, m_t^2 - m_{H^{n,k}}^2}, \qquad (18)$$

where $m_{H^{n,k}} = \sqrt{m_H^2 + \frac{n^2 + k^2}{R^2}}$.

3 Discussion

The LFV $t \to c \, l_i^- l_j^+$ and $H^0 \to h^0(A^0) l_i^- l_j^+$ decays exist at tree level in the model III and the Yukawa couplings $\bar{\xi}_{N,ij}^{U,D,E~1}$, with different quark and lepton flavors i, j, play the main role in the interactions. Since these couplings are free parameters of the theory, they need to be restricted by using the experimental results. Now we will present the assumptions and the numerical values we use for the free parameters under consideration:

- The Yukawa couplings $\bar{\xi}_{N,ij}^{U,D,E}$ are symmetric with respect to the indices *i* and *j*.
- The couplings $\bar{\xi}_{N,ij}^E$, $i, j = e, \mu, \tau$ respect the Cheng-Sher scenerio [16] and, therefore, the couplings with the indices $i, j = e, \mu$ are small compared to the ones with the indices $i = \tau, j = e, \mu, \tau$, since the strength of these couplings are related to the masses of leptons denoted by the indices of them. This forces us to study the $\tau\mu$ output in the above processes.
- For the coupling $\bar{\xi}^E_{N,\tau\mu}$ the numerical values $((1-10) \, GeV)$ are taken by respecting the predicted upper limit $30 \, GeV$ (see [17]) which is obtained by using the experimental uncertainty, 10^{-9} , in the measurement of the muon anomalous magnetic moment [18].
- For the coupling $\bar{\xi}_{N,tc}^U$ we use the constraint region obtained by restricting the Wilson coefficient C_7^{eff} , which is the effective coefficient of the operator $O_7 = \frac{e}{16\pi^2} \bar{s}_{\alpha} \sigma_{\mu\nu} (m_b R + m_s L) b_{\alpha} \mathcal{F}^{\mu\nu}$ (see [19] and references therein), in the range $0.257 \leq |C_7^{eff}| \leq 0.439$. Here upper and lower limits were calculated using the CLEO measurement [20]

$$BR(B \to X_s \gamma) = (3.15 \pm 0.35 \pm 0.32) \, 10^{-4} \,, \tag{19}$$

and all possible uncertainties in the calculation of C_7^{eff} [19]. The above restriction ensures to get upper and lower limits for $\bar{\xi}_{N,tt}^U$ and also for $\bar{\xi}_{N,tc}^U$ (see [19] for details). In our numerical calculations, we choose the upper limit for $C_7^{eff} > 0$, fix $\bar{\xi}_{N,bb}^D = 30 m_b$ and take $\bar{\xi}_{N,tc}^U \sim 0.01 \bar{\xi}_{N,tt}^U \sim 0.45 \, GeV$, respecting the constraints mentioned.

For the Higgs masses m_{h^0} and m_{A^0} , we used the numerical values $m_{h^0} = 85 \, GeV$ and $m_{A^0} = 90 \, GeV$. We respect the appropriate region obtained by using the direct Higgs boson searches and indirect limits coming from the SM measurements, namely, $m_{h^0} > 55 \, GeV$ and $m_{A^0} > 63 \, GeV$ where production of $h^0 A^0$ is kinematically allowed at LEP2 which has center of mass energy 200 GeV (see [21]).

¹We use the parametrization $\xi_{N,ij}^{U,D,E} = \sqrt{\frac{4G_F}{\sqrt{2}}} \bar{\xi}_{N,ij}^{U,D,E}$ for the Yukawa couplings.

The addition of extra dimensions that are felt by the new Higgs doublet results in new contributions to the decay widths of the processes. In the case of the UEDs where all fields experience the extra dimensions, the tree level particle-particle-KK mode interactions are forbidden since the KK number at each vertex should be conserved. This leads to the non-zero contributions due to the extra dimensions at least at one loop level and they are suppressed. However, in the case of NUEDs, there is no need for the conservation of KK modes at each vertex and the tree level fermion-fermion-scalar field KK mode interaction is permitted. In our case, the fields $h^0(A^0)$ feel the extra dimensions and their KK modes are responsible for the additional contributions after the compactification. Our calculations are based on such vertices and the assumption that the new Yukawa couplings existing for the KK modes of $h^0(A^0)$ are the same as the ones existing in the zero-mode case. There is one more parameter R, which is the size of the extra dimension, emerging after the compactification and its restriction has been studied in various works (see [8] for example). Notice that we use a broad range for the compactification scale 1/R, $100 \ GeV \le 1/R \le 5000 \ GeV$ and present the Γ of Higgs decays for $1/R \le 1000 \ GeV$ since they are weakly sensitive the scale 1/R for $1/R > 1000 \ GeV$.

In our work, we investigate the LFV $t \to c l_i^- l_j^+$ and $H^0 \to h^0(A^0) l_i^- l_j^+$ decays in the model III, where the new Higgs doublet and the SM gauge fields feel extra dimension and we take τ , μ for the lepton flavors l_i , l_j since the Yukawa couplings, and, therefore, the decay widths, for other pairs are highly suppressed. Here we choose a single spatial extra dimension and, then, two spatial extra dimensions. In the case of two spatial extra dimensions the compactification is done on orbifold $(S^1 \times S^1)/Z_2$ and we assume that each circle has the same radius R. In contrast to a single extra dimension, the convergence of the KK sum should be examined for two extra dimensions. In our numerical calculations, we get convergent series for the considered range of the compactification scale and make a rough estimate for this sum. The numerical calculations show that the quark decay $t \to c l_i^- l_j^+$ is not sensitive to the extra dimensions, however, the Higgs decays $H^0 \to h^0(A^0) l_i^- l_j^+$ are sensitive, especially, to two extra dimensions.

In Fig. 3, we present the compactification scale 1/R dependence of the ratio $r = \frac{\Gamma_1}{\Gamma_0}(\frac{\Gamma_2}{\Gamma_0})$ for the $t \to c(\tau^+\mu^- + \tau^-\mu^+)$ decay, for $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$. Here $\Gamma_0(\Gamma_1, \Gamma_2)$ is the decay width of the process under consideration without extra dimension (a single extra dimension contribution to the decay width, two extra dimensions contribution to the decay width). The solid (dashed) line represents the ratio $r = \frac{\Gamma_1}{\Gamma_0}(\frac{\Gamma_2}{\Gamma_0})$. This figure shows that the contribution of the extra dimensions is suppressed for the large values of the scale $1/R \ge 200 \, GeV$, especially for a single extra dimension case. For two extra dimensions, there is almost two order enhancement in the ratio compared to the one obtained including only one extra dimension. However, this effect is 0.1 % of the one which is obtained without inclusion of extra dimension and, therefore, the extra dimension contribution in this FV decay is negligible, at least, up to two extra dimensions.

Fig. 4 is devoted to the compactification scale 1/R dependence of the ratio $r = \frac{\Gamma_1}{\Gamma_0}(\frac{\Gamma_2}{\Gamma_0})$ for the LFV Higgs $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ and $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ decays, for $m_{h^0} = 85 \ GeV$, $m_{A^0} = 90 \ GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \ GeV$. Here $\Gamma_0(\Gamma_1, \Gamma_2)$ is the decay width of these processes without extra dimension (a single extra dimension contribution to the decay width, two extra dimensions contribution to the decay width). The solid (dashed) line represents the ratio $r = \frac{\Gamma_1}{\Gamma_0}$ for $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+) \ (H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay and the small dashed (dotted) line represents the ratio $r = \frac{\Gamma_2}{\Gamma_0}$ for $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+) \ (H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay. This figure shows that the contribution of the extra dimension is at the order of 1% of the one without extra dimension for the large values of the scale $1/R \ge 200 \ GeV$ and slightly larger for the A^0 output. In the case of two extra dimensions, the ratio is almost one and the contribution due to the two extra dimensions is comparable with the one without extra dimension. This is an interesting result since these Higgs decays are sensitive to higher dimensions and, with the more accurate measurements, it would be possible to check the effects of extra dimensions and to get a valuable information about the compactification scale.

Now, we would like to examine the effects of extra dimensions on these Higgs decays in detail.

Fig. 5 (6) represents the compactification scale 1/R dependence of the decay width Γ of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ $(H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay, for $m_{H^0} = 150 \, GeV$, $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, and three different values of the coupling $\bar{\xi}^D_{N,\tau\mu}$. The solid (dashed, small dashed) line:curve represents the Γ for $\bar{\xi}^D_{N,\tau\mu} = 1 \, GeV$ (5 GeV, 10 GeV) without:with the inclusion of a single extra dimension. It is shown that the Γ of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ $(H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay is of the order of the magnitude of $10^{-5} \, GeV$ ($10^{-5} \, GeV$) for the coupling $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$ and it enhances almost 20 % ($30 \,$ %) with the inclusion of a single extra dimension, in the range of the compactification scale $200 \, GeV \ge 1/R \ge 300 \, GeV$.

In Fig. 7 (8) we present the compactification scale 1/R dependence of the decay width Γ of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ $(H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay, for $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$ and three different values of the mass m_{H^0} . The solid (dashed, small dashed) line:curve represents the Γ for $m_{H^0} = 100 \, GeV$ (150 GeV, 170 GeV) without:with the inclusion of a single extra dimension. It is shown that for the large values of the Higgs mass m_{H^0} the Γ

of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ $(H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay is of the order of the magnitude of $10^{-2} GeV (10^{-4} GeV)$ and the sensitivity of the extra dimension becomes smaller with the increasing values of the Higgs masses.

Fig. 9 (10) is devoted to the compactification scale 1/R dependence of the decay width Γ of $H^0 \rightarrow h^0(\tau^+\mu^- + \tau^-\mu^+)$ $(H^0 \rightarrow A^0(\tau^+\mu^- + \tau^-\mu^+))$ decay, for $m_{H^0} = 150 \, GeV$, $m_{A^0} = 90 \, GeV$ $(m_{h^0} = 80 \, GeV) \, \bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$ and three different values of the mass $m_{h^0} \, (m_{A^0})$. The solid (dashed, small dashed) line:curve represents the Γ for $m_{h^0} = 75 \, GeV$ ($80 \, GeV$, $85 \, GeV$) $(m_{A^0} = 90 \, GeV \, (100 \, GeV, 120 \, GeV))$ without:with the inclusion of a single extra dimension. Here we see that the increase in the mass values $m_{h^0} \, (m_{A^0})$ causes the decay width to decrease and the sensitivity to the single extra dimension to increase .

Now we would like to present the results briefly:

- For the t → c(τ⁺μ⁻ + τ⁻μ⁺) decay, the contribution of the extra dimensions is small for the large values of the scale 1/R ≥ 200 GeV, especially for a single extra dimension case. In the case of two extra dimensions the additional contribution is almost two order larger compared to the one obtained for a single extra dimension. In any case, this effect is 0.1% of the contribution which is obtained without inclusion of extra dimension and, therefore, the extra dimension contribution is negligible in this FV decay, at least, up to two extra dimensions.
- The decay widths of LFV H⁰ → h⁰(τ⁺μ⁻+τ⁻μ⁺) and H⁰ → A⁰(τ⁺μ⁻+τ⁻μ⁺) decays are sensitive to the extra dimensions. The new effect coming from a single extra dimension is of the order of 1% of the contribution obtained without extra dimension for the large values of the scale 1/R ≥ 200 GeV. In the case of two extra dimensions the new effects are almost comparable with the one obtained without extra dimension.

As a final comment, the Higgs decays $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ and $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ are sensitive to the extra dimensions and with the more accurate future measurements it would be possible to check effects of extra dimensions and predict valuable information about the compactification scale.

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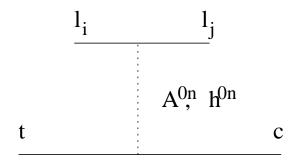


Figure 1: Tree level diagrams contributing to the decay $t \to c l_i^- l_j^+$. Dotted lines represent the h^{0n}, A^{0n} fields where n = 0, 1, 2, ...

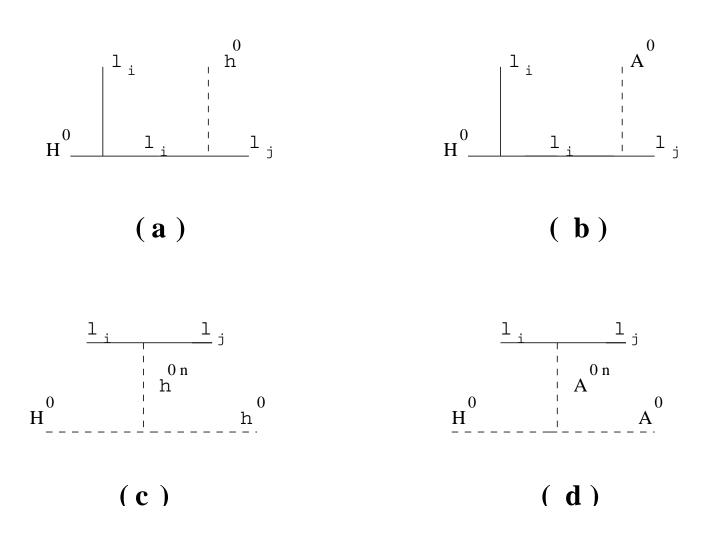


Figure 2: Tree level diagrams contributing to $\Gamma(H^0 \to h^0(A^0) l_i^- l_j^+)$, $i = e, \mu, \tau$ decay in the model III version of 2HDM. Solid lines represent leptons, dashed lines represent the H^0 , h^0 and A^0 fields, where n = 0, 1, 2, ...

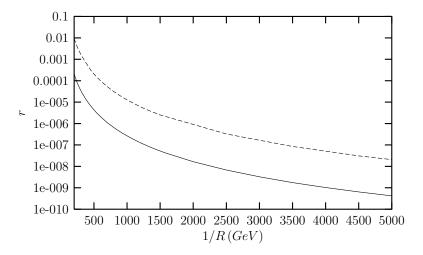


Figure 3: The compactification scale 1/R dependence of the ratio $r = \frac{\Gamma_1}{\Gamma_0} (\frac{\Gamma_2}{\Gamma_0})$ for the decay $t \to c(\tau^+\mu^- + \tau^-\mu^+)$ for $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$. The solid (dashed) line represents the ratio $r = \frac{\Gamma_1}{\Gamma_0} (\frac{\Gamma_2}{\Gamma_0})$.

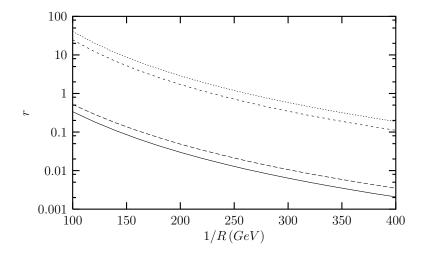


Figure 4: The compactification scale 1/R dependence of the ratio $r = \frac{\Gamma_1}{\Gamma_0} \left(\frac{\Gamma_2}{\Gamma_0}\right)$ for the LFV Higgs $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ and $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ decays for $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$. The solid (dashed) line represents the ratio $r = \frac{\Gamma_1}{\Gamma_0}$ for $H^0 \to h^0(A^0)(\tau^+\mu^- + \tau^-\mu^+)$ decay and the small dashed (dotted) line represents the ratio $r = \frac{\Gamma_2}{\Gamma_0}$ for $H^0 \to h^0(A^0)(\tau^+\mu^- + \tau^-\mu^+)$ decay.

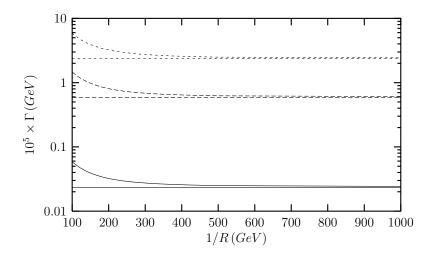


Figure 5: The compactification scale 1/R dependence of the decay width Γ of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ decay, for $m_{H^0} = 150 \, GeV$, $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, and three different values of the coupling $\bar{\xi}^D_{N,\tau\mu}$. The solid (dashed, small dashed) line:curve represents the Γ for $\bar{\xi}^D_{N,\tau\mu} = 1 \, GeV \, (5 \, GeV, 10 \, GeV)$ without:with the inclusion of a single extra dimension.

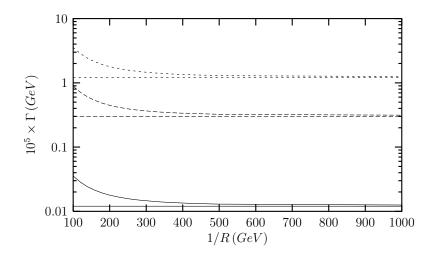


Figure 6: The same as Fig. 5 but for $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ decay.

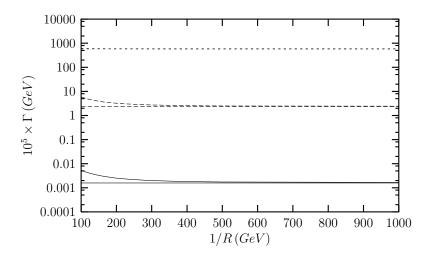


Figure 7: The compactification scale 1/R dependence of the decay width Γ of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ decay, for $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$ and three different values of the mass m_{H^0} . The solid (dashed, small dashed) line:curve represents the Γ for $m_{H^0} = 100 \, GeV$ (150 GeV, 170 GeV) without:with the inclusion of a single extra dimension.

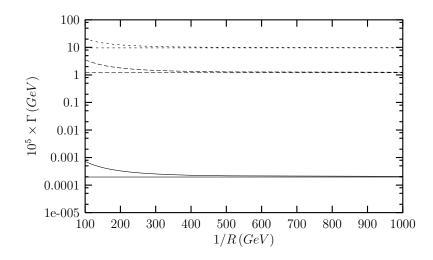


Figure 8: The same as Fig. 7 but for $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ decay.

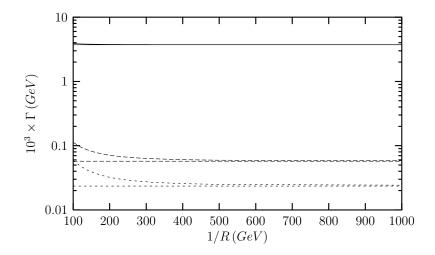


Figure 9: The compactification scale 1/R dependence of the decay width Γ of $H^0 \to h^0(\tau^+\mu^- + \tau^-\mu^+)$ decay, for $m_{H^0} = 150 \, GeV$, $m_{A^0} = 90 \, GeV \, \bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$ and three different values of the mass m_{h^0} . The solid (dashed, small dashed) line:curve represents the Γ for $m_{h^0} = 75 \, GeV$ (80 GeV, 85 GeV) without:with the inclusion of a single extra dimension.

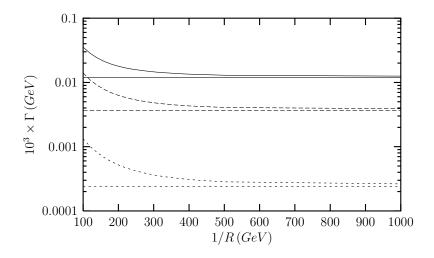


Figure 10: The compactification scale 1/R dependence of the decay width $H^0 \to A^0(\tau^+\mu^- + \tau^-\mu^+)$ decay, for $m_{H^0} = 150 \, GeV$, $m_{h^0} = 80 \, GeV$, $\bar{\xi}^D_{N,\tau\mu} = 10 \, GeV$ and three different values of the mass m_{A^0} . The solid (dashed, small dashed) line:curve represents the Γ for $m_{A^0} = 90 \, GeV$ (100 GeV, 120 GeV) without:with the inclusion of a single extra dimension.